DISCUSSION

Limit State Response of Composite Columns and Beam-Columns Part II: Application of Design Provisions for the 2005 AISC Specification

Paper by ROBERTO T. LEON and JEROME F. HAJJAR
(First Quarter, 2008)

Discussion by LOUIS F. GESCHWINDNER

The authors discuss the application of a set of equations for analysis and design of composite columns subjected to combined compression and bending. These equations were presented in the CD that accompanied the 13th edition *Steel Construction Manual* (AISC, 2005). The CD presents, in Figures I-1a through I-1d, sets of equations to be used to determine specific points on a simplified interaction diagram for encased W-shapes with bending about either the strong or the weak axes and filled rectangular and round HSS. These figures are used as the basis for Tables 2 through 5 in the paper. However, the authors have altered the figures from the CD for presentation in their paper.

The most significant difference between the authors' tables and the AISC figures occurs for the round HSS. The authors correctly point out a typographical error in Figure I-1d in the equation for θ where the terms $f'_c A_c$ should be removed. Clearly, if these variables were included in a calculation, the units, as well as the value, would be incorrect. The authors also point to "a discrepancy in the computation of Z_{sB}." However, the two equations that the authors provided for the plastic section modulus of the steel, Z_{sB}, appear to contain approximations that can be replaced with simple derivations that provide better accuracy. The paper does not include derivations for these equations.

In this discussion, three equations for use in determining Z_{sB} are developed and compared to those of the authors. The first equation is developed using the segment of a circle; the second, considered as a usable lower bound representation, is developed using the sector of a circle; and the third solution is developed as an exact solution.

Figure 1 shows the geometry of a concrete-filled round HSS. The plastic neutral axis is shown in the location that would result if the member were to undergo pure bending. This is point B in Table 5 of the paper and this figure is similar to that shown for point B in Table 5. The development of the flexural strength of the composite member requires the determination of several different properties of portions of the steel and concrete. One is the plastic section modulus, Z_{sB}, of that portion of the steel beyond the plastic neutral axis on the compression side and the symmetrically placed steel section on the tension side. These areas are shown shaded in Figure 1. The different solutions for Z_{sB} result from different approaches to modeling these two areas.

CIRCULAR SEGMENT

Figure 2(a) shows the geometric properties of a circular segment. Using these properties, the moment of the area of this circular segment taken about the circle center is

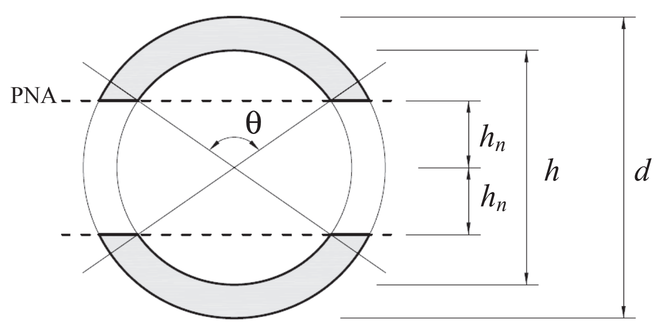

Fig. 1. Plastic neutral axis of concrete-filled round HSS in pure bending.

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$$\left(\text{area}\right)\left(\text{arm}\right) = \frac{r^2}{2}\left(\theta - \sin\theta\right)\left(\frac{4r}{3}\right)\left(\frac{\sin^3\left(\theta/2\right)}{\left(\theta - \sin\theta\right)}\right) = \left(\frac{2r^3}{3}\right)\sin^3\left(\theta/2\right) \tag{1}$$

Using $R = d/2$, the plastic section modulus for the pair of circular segments in tension and compression is twice the moment of the area of one circular segment. Thus,

$$Z_{seg} = 2\left(\frac{2\left(d/2\right)^3}{3}\right)\sin^3\left(\theta/2\right) = \frac{d^3}{6}\sin^3\left(\theta/2\right) \tag{2}$$

Similarly, the plastic section modulus for the matching segments of concrete with diameter, h, is

$$Z_{cB} = \frac{h^3}{6}\sin^3\left(\theta/2\right) \tag{3}$$

The plastic section modulus of the steel areas shown shaded in Figure 1, Z_{sB}, can then be determined as the plastic section modulus of the segment minus the plastic section modulus of the concrete. Thus,

$$Z_{sB} = Z_{seg} - Z_{cB} = \frac{\left(d^3 - h^3\right)}{6}\sin^3\left(\theta/2\right) \tag{4}$$

Equation 4 is the equation given in AISC Figure I-1d. This is not an exact solution since the two circle segments are not properly aligned. Figure 3 shows the areas that are used to determine Z_{seg} and Z_{cB} and where they are located with respect to each other. It also shows the area of steel that should have been included but is not, $A_{s,\text{missing}}$, and the area of concrete that was subtracted that should not have been, $A_{c,\text{extra}}$. As the thickness of the steel section gets smaller or the angle, θ, approaches π, Equation 4 approaches the correct value.

CIRCULAR SECTOR

Figure 2(b) shows the geometric properties of a circular sector. The moment of the area of the circular sector about the circle center is

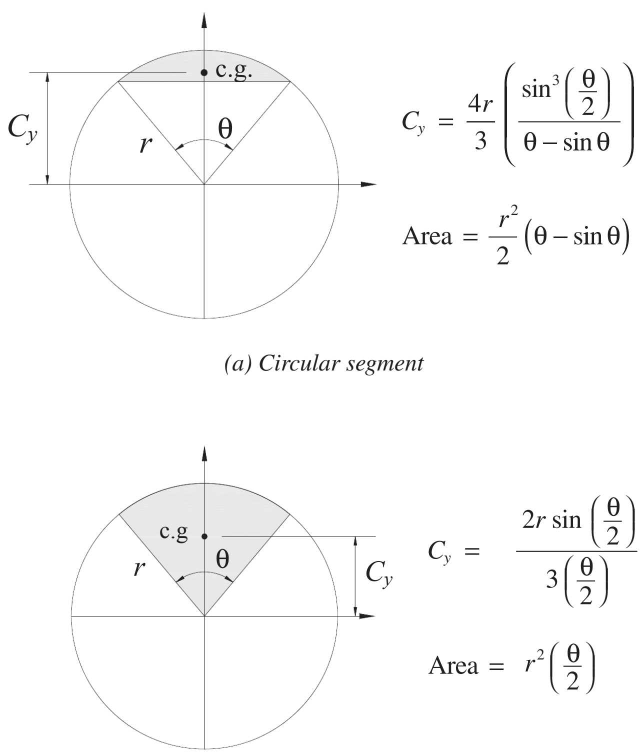
$$\left(\text{area}\right)\left(\text{arm}\right) = r^2\left(\frac{\theta}{2}\right)\left(\frac{2r\sin\left(\theta/2\right)}{3\left(\theta/2\right)}\right) = \frac{2}{3}r^3\sin\left(\theta/2\right) \tag{5}$$

Using $r = d/2$, the plastic section modulus for the pair of circular sectors in tension and compression is twice the moment of the area of one circular sector. Thus,

$$Z_{sec} = \frac{d^3}{6}\sin\left(\theta/2\right) \tag{6}$$

Similarly, the plastic section modulus for the matching sectors of concrete with diameter, h, is

$$Z_{conc} = \frac{h^3}{6}\sin\left(\theta/2\right) \tag{7}$$



(a) Circular segment

(b) Circular sector

Fig. 2. Properties of a circle.

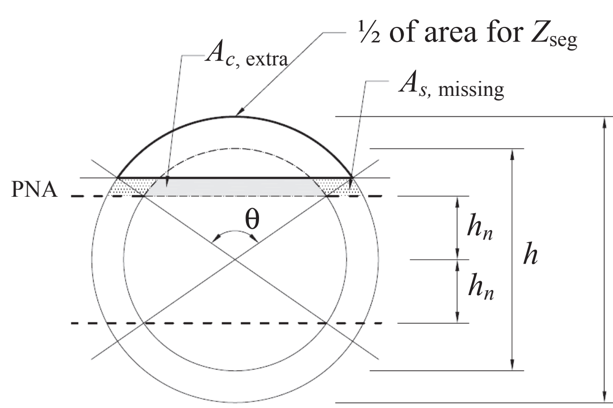


Fig. 3. Geometry for circular segment solution.

Subtracting the Z_{conc} from Z_{sec} will give the plastic section modulus of the steel. Thus,

$$Z_{sB} = \frac{\left(d^3 - h^3\right)}{6}\sin\left(\theta/2\right) \tag{8}$$

As was the case with the derivation of Equation 4, this is not an exact solution. Figure 4 shows the areas that are used to determine Z_{sec} and Z_{conc}. It also shows the area of steel that has not been included in the final calculation for Z_{sB}. Since the only approximation included in this derivation is the steel that has been ignored, this approach can be considered a "lower bound" solution.

EXACT SOLUTION

An exact solution is possible using the geometry of the circular segment and properly accounting for the two angles needed to describe the steel and concrete geometry. Figure 5(a) shows the concrete-filled round HSS with two circular segments defined by the angles, θ and θ_s. The angle, θ, is the same angle as defined for the earlier two derivations. The angle, θ_s, is the angle that defines the location of the plastic neutral axis at the outer face of the steel. Using the plastic section modulus as defined by Equation 2 and θ_s, yields

$$Z_{seg} = \frac{d^3}{6}\sin^3\left(\theta_s/2\right) \tag{9}$$

For the concrete segment, using Equation 2 and θ, yields

$$Z_{cB} = \frac{h^3}{6}\sin^3\left(\theta/2\right) \tag{10}$$

The exact plastic section modulus for the steel is then

$$Z_{sB} = Z_{seg} - Z_{cB} \tag{11}$$

In order to combine Equations 9 and 10, the relationship between θ and θ_s is needed. From Figure 5(b), the following relationship is seen

$$\frac{h}{2}\cos\left(\theta/2\right) = \frac{d}{2}\cos\left(\theta_s/2\right) \tag{12}$$

and combined with the basic trigonometric relationship, $\sin A = \sqrt{1-\cos^2 A}$, yields

$$\sin\left(\theta_s/2\right) = \left(1 - \frac{h^2}{d^2}\cos^2\left(\theta/2\right)\right)^{1/2} \tag{13}$$

Substituting Equation 13 into Equation 9 yields

$$Z_{seg} = \frac{d^3}{6}\left(1 - \frac{h^2}{d^2}\cos^2\left(\theta/2\right)\right)^{3/2} \tag{14}$$

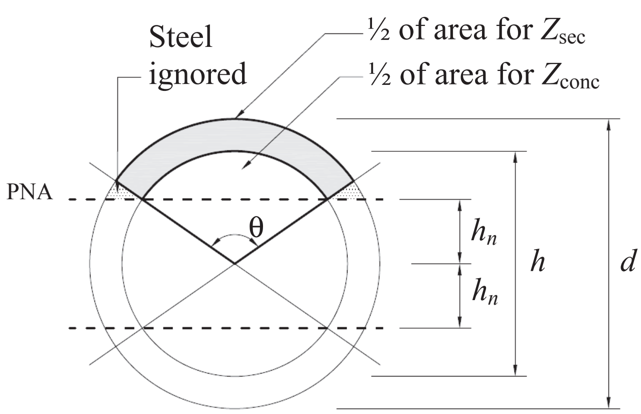
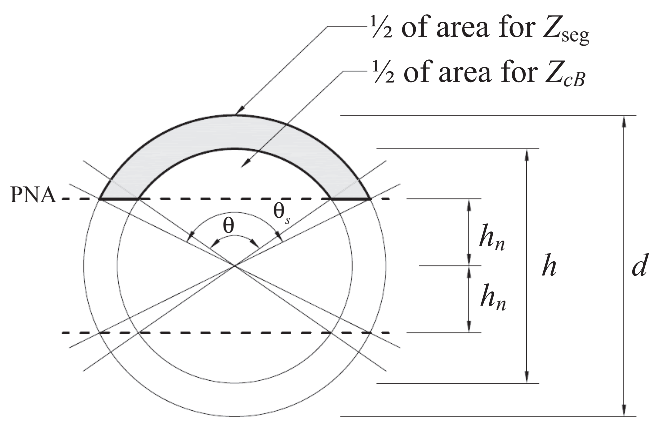
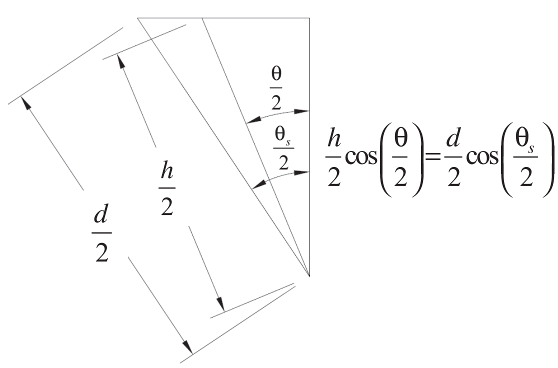


Fig. 4. Geometry for circular sector solution.



(a) Geometry



(b) Relationship between θ and θ_s

Fig. 5. Geometry for exact solution.

and substituting Equations 10 and 14 into Equation 11 yields

$$Z_{sB} = \frac{d^3}{6}\left(1 - \frac{h^2}{d^2}\cos^2\left(\theta/2\right)\right)^{3/2} - \frac{h^3}{6}\sin^3\left(\theta/2\right) \tag{15}$$

Unlike the two previous derivations given for the circular segment and the circular sector, this derivation gives the exact solution for Z_{sB}.

AUTHORS' EQUATIONS

The two equations presented in the paper for Z_{sB} are:

a "correct" formulation

$$Z_{sB} = \left(\frac{d^3 - h^3}{12}\right)\sin^3\left(\theta/2\right) \times \left[\frac{\theta}{\theta - \sin\theta} + \frac{\left(2\pi - \theta\right)}{\left(2\pi - \theta\right) - \sin\left(2\pi - \theta\right)}\right] \tag{16}$$

and a simplified approximation

$$Z_{sB} \approx \frac{\left(d^3 - h^3\right)}{6}\sin^{4/3}\left(\theta/2\right) \tag{17}$$

COMPARISON OF RESULTS

Five equations for the plastic section modulus of the steel for point B, pure bending, of a concrete-filled round HSS have been presented. The results from these five equations are plotted in Figure 6 for an HSS 16.000×0.250 over the full range of angle, θ, from 0 to π.

Equation 4, the original AISC equation, is the least accurate of the equations derived in this discussion. Equation 8, the "lower bound" solution is closer to the exact solution than all of the other equations shown. The two equations presented by the authors, Equations 16 and 17, appear to be unrelated to those derived in this discussion. Although they give values closer to the exact solution than Equation 4, they do not provide a better solution than Equation 8, the "lower bound" solution. The origins of Equations 16 and 17 are not discussed in the paper.

The difference between Equations 8 and 15 is greatest for the lower values of θ. Thus, it would be helpful to know the approximate range of θ for realistic round HSS and acceptable values of concrete strengths. As concrete strength increases, the angle, θ, decreases. Thus, a check was made for all of the concrete filled round HSS listed in the Composite Column Tables of the 13th edition *Steel Construction Manual* (AISC, 2005b) but with a concrete strength, f'_c = 10.0 ksi. For these shapes, with F_y = 42 ksi, the HSS 16.000×0.250 required the smallest angle, θ = 1.77 rad. As seen in Figure 6 for this shape, Equations 4, 8, 15, 16 and 17 give the following values for Z_{sB}:

Eq. No.	Model	Z_{sB} (in.^{3})
4	Circular segment	26.9
8	Circular sector	44.8
15	Exact	45.3
16	Paper "correct"	41.1
17	Paper simplified	41.2

In addition to using the Z_{sB} equations for determining moment strength for the pure bending case, the same basic formulation is used by the authors, with θ_2 to determine Z_{sE}, for moment strength at point E. The realistic range for θ_2 is π to 0 as points between C and somewhere close to A are determined. Thus, the error in not using Equation 15 with

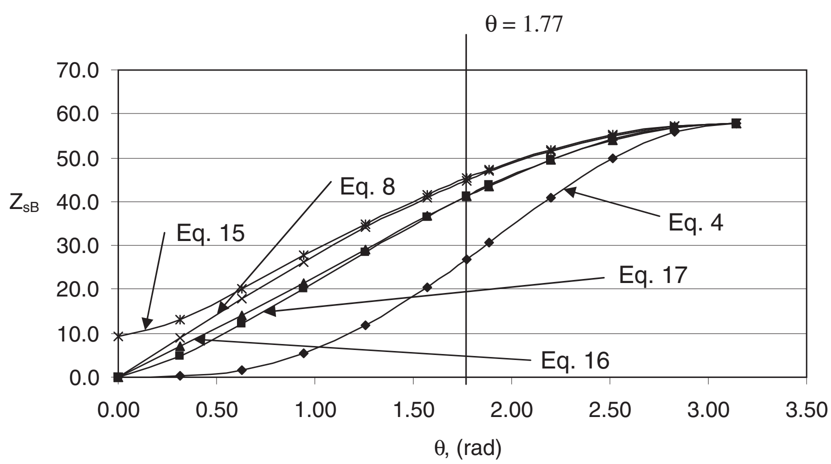


Fig. 6. Comparison of five equations for Z_{sB} for an HSS 16.000×0.250.

θ_2 for these points can be quite substantial. However, the lowest value of θ_2 for point E as defined by the authors for the HSS 16.000×0.250 discussed earlier is 1.23 rad and the error in computing Z_{sE} using the "lower bound" equation is approximately 5%.

RECOMMENDATIONS

Based on the derivations presented in this discussion, it is recommended that either the exact solution, Equation 15, or the circular sector solution, Equation 8, be used in calculations for pure bending, Point B, for a concrete-filled round HSS. Considering the simplicity of the latter and its ability to closely represent the correct value for Z_{sB}, it is further recommended that Equation 8 be adopted for use in place of the currently listed equation in Figure I-1d of the 13th edition companion CD.

In the rare case where point E is to be determined, it is recommended that the lower bound equation, Equation 8 with θ_2, be used. If more points on the interaction curve are to be determined, the exact solution, Equation 15, should be used.

In addition, revised versions of Figures I-1a through I-1d from the CD Companion V.13.0 are presented as Tables A through D of this Discussion. Note that Tables A through D also correspond to Figures 2 through 5 of the Leon and Hajjar paper, but with corrections.

In summary the revisions incorporated are:

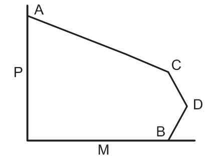
1. No changes to Figure I-1a (Table A).
2. Two editorial changes in Figure I-1b (Table B).
3. Several editorial changes and the inclusion of equations for point E in Figure I-1c (Table C).
4. Several editorial changes, the inclusion of equations for point E, and updated equations for Z_{sB} and Z_{sE} in Figure I-1d (Table D).

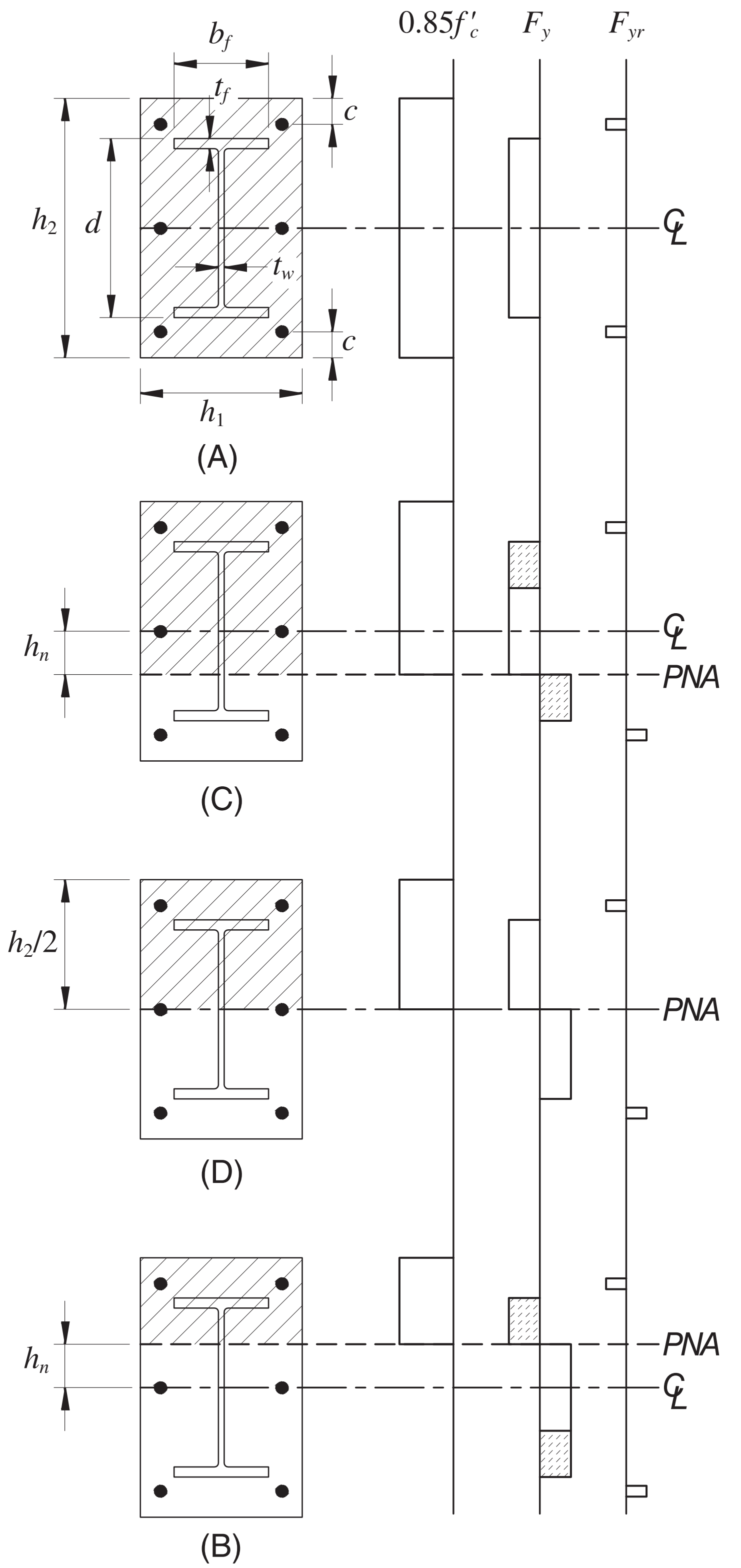
REFERENCE

AISC (2005), *Steel Construction Manual,* 13th Edition, American Institute of Steel Construction, Chicago, IL.

Editor's Note: AISC's Committee on Manuals and Textbooks has decided to incorporate Dr. Geschwindner's recommendations in revisions that will be made with the 14th edition AISC Steel Construction Manual*.*

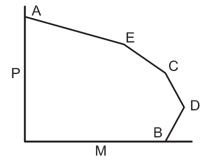
Table A.
Plastic Capacities for Rectangular, Encased W-Shapes Bent About the X-X Axis

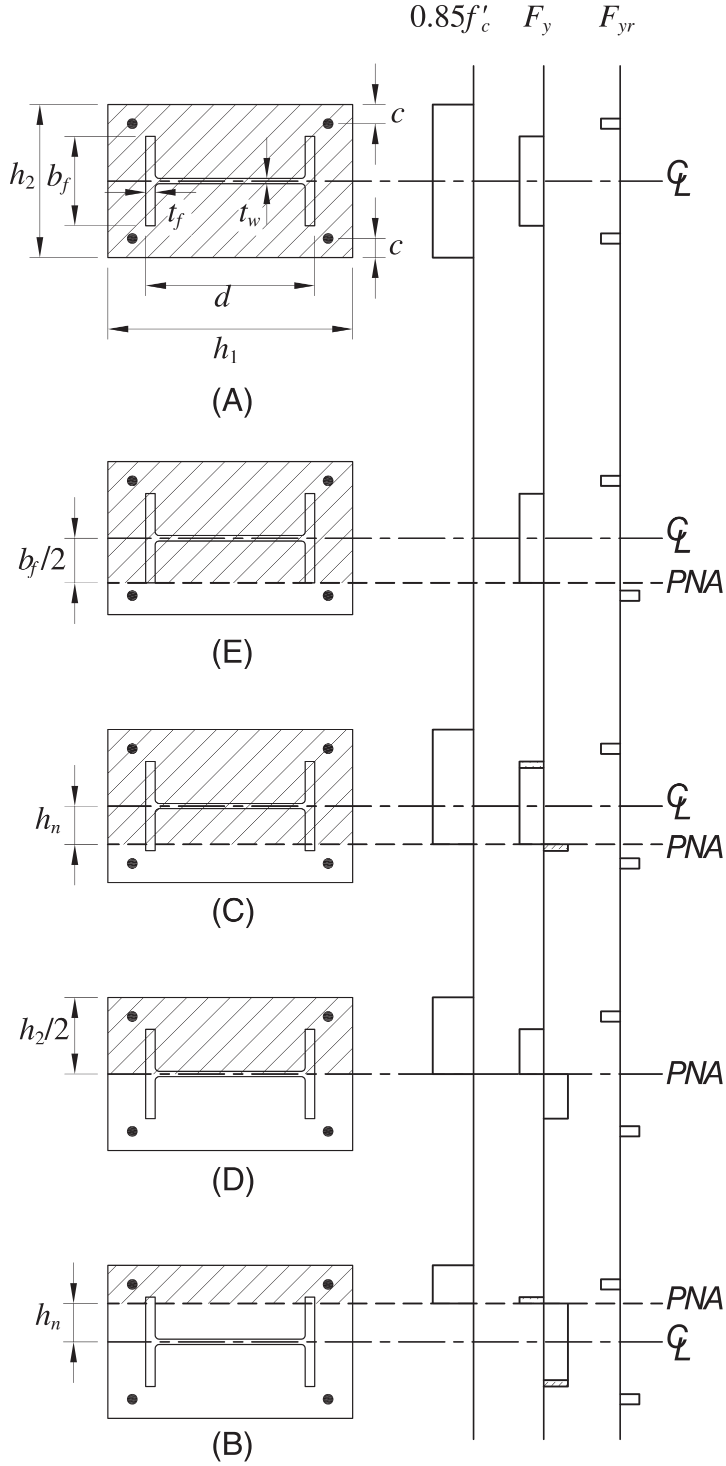




Section	Stress Distribution	Point	Defining Equations
		A	$P_A = A_s F_y + A_{sr} F_{yr} + 0.85 f'_c A_c$ $M_A = 0$ A_s = area of steel shape A_{sr} = area of all continuous reinforcing bars $A_c = h_1 h_2 - A_s - A_{sr}$
		C	$P_C = 0.85 f'_c A_c$ $M_C = M_B$
		D	$P_D = \frac{0.85 f'_c A_c}{2}$ $M_D = Z_s F_y + Z_r F_{yr} + \frac{Z_c}{2}(0.85 f'_c)$ Z_s = full x-axis plastic section modulus of steel shape A_{srs} = area of continuous reinforcing bars at the centerline $Z_r = (A_{sr} - A_{srs})\left(\frac{h_2}{2} - c\right)$ $Z_c = \frac{h_1 h_2^2}{4} - Z_s - Z_r$
		B	$P_B = 0$ $M_B = M_D - Z_{sn} F_y - ½ Z_{cn}(0.85 f'_c)$ $Z_{cn} = h_1 h_n^2 - Z_{sn}$ For h_n below the flange $\left(h_n \le \frac{d}{2} - t_f\right)$ $h_n = \frac{0.85 f'_c (A_c + A_{srs}) - 2F_{yr} A_{srs}}{2\left[0.85 f'_c (h_1 - t_w) + 2F_y t_w\right]}$ $Z_{sn} = t_w h_n^2$ For h_n within the flange $\left(\frac{d}{2} - t_f < h_n \le \frac{d}{2}\right)$ $h_n = \frac{0.85 f'_c \left(A_c + A_s - d b_f + A_{srs}\right) - 2F_y\left(A_s - d b_f\right) - 2F_{yr} A_{srs}}{2\left[0.85 f'_c \left(h_1 - b_f\right) + 2F_y b_f\right]}$ $Z_{sn} = Z_s - b_f\left(\frac{d}{2} - h_n\right)\left(\frac{d}{2} + h_n\right)$ For h_n above the flange $\left(d_n > \frac{d}{2}\right)$ $h_n = \frac{0.85 f'_c (A_c + A_s + A_{srs}) - 2F_y A_s - 2F_{yr} A_{srs}}{2(0.85 f'_c h_1)}$ $Z_{sn} = Z_{sx}$ = full x-axis plastic section modulus of steel shape

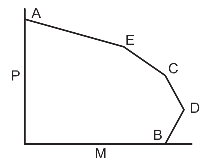
Table B.
Plastic Capacities for Rectangular, Encased W-Shapes Bent About the Y-Y Axis





Point	Defining Equations
A	$P_A = A_sF_y + A_{sr}F_{yr} + 0.85f'_cA_c$ $M_A = 0$ A_s = area of steel shape A_{sr} = area of continuous reinforcing bars $A_c = h_1h_2 - A_s - A_{sr}$
E	$P_E = A_sF_y + (0.85f'_c)\left[A_c - \frac{h_1}{2}(h_2 - b_f) + \frac{A_{sr}}{2}\right]$ $M_E = M_D - Z_{sE}F_y - \frac{1}{2}Z_{cE}(0.85f'_c)$ $Z_{sE} = Z_{sy}$ = full y-axis plastic section modulus of steel shape $Z_{cE} = \frac{h_1b_f^2}{4} - Z_{sE}$
C	$P_C = 0.85f'_cA_c$ $M_C = M_B$
D	$P_D = \frac{0.85f'_cA_c}{2}$ $M_D = Z_sF_y + Z_rF_{sr} + \frac{1}{2}Z_c(0.85f'_c)$ Z_s = full y-axis plastic section modulus of steel shape $Z_r = A_{sr}\left(\frac{h_2}{2} - c\right)$ $Z_c = \frac{h_1h_2^2}{4} - Z_s - Z_r$
B	$P_B = 0$ $M_B = M_D - Z_{sn}F_y - \frac{1}{2}Z_{cn}(0.85f'_c)$ $Z_{cn} = h_1h_n^2 - Z_{sn}$ For h_n below the flange $\left(\frac{t_w}{2} < h_n \le \frac{b_f}{2}\right)$ $h_n = \frac{0.85f'_c(A_c + A_s - 2t_fb_f) - 2F_y(A_s - 2t_fb_f)}{2\left[4t_fF_y + (h_1 - 2t_f)0.85f'_c\right]}$ $Z_{sn} = Z_s - 2t_f\left(\frac{b_f}{2} + h_n\right)\left(\frac{b_f}{2} - h_n\right)$ For h_n above the flange $\left(h_n > \frac{b_f}{2}\right)$ $h_n = \frac{0.85f'_c(A_c + A_s) - 2F_yA_s}{2\left[0.85f'_ch_1\right]}$ $Z_{sn} = Z_{sy}$ = full y-axis plastic section modulus of steel shape

Table C.
Plastic Capacities for Composite, Filled HSS
Bent About the X-X Axis



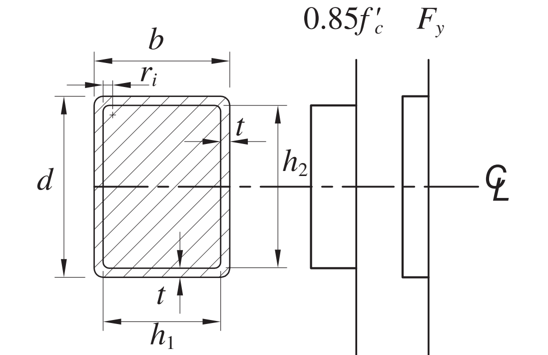
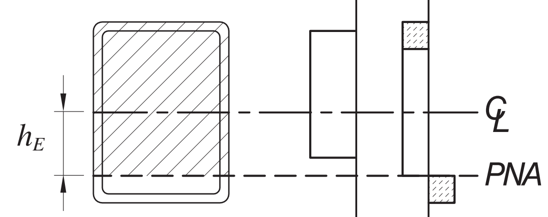
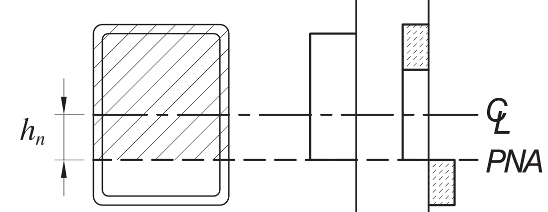
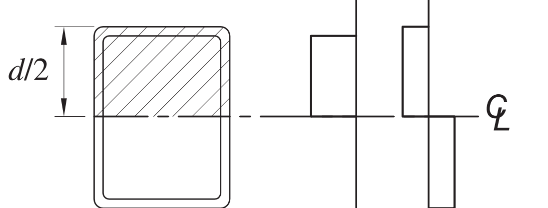
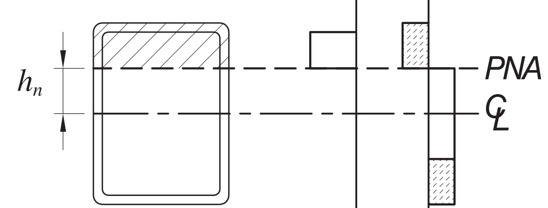
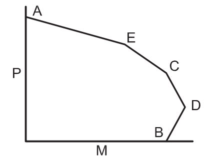
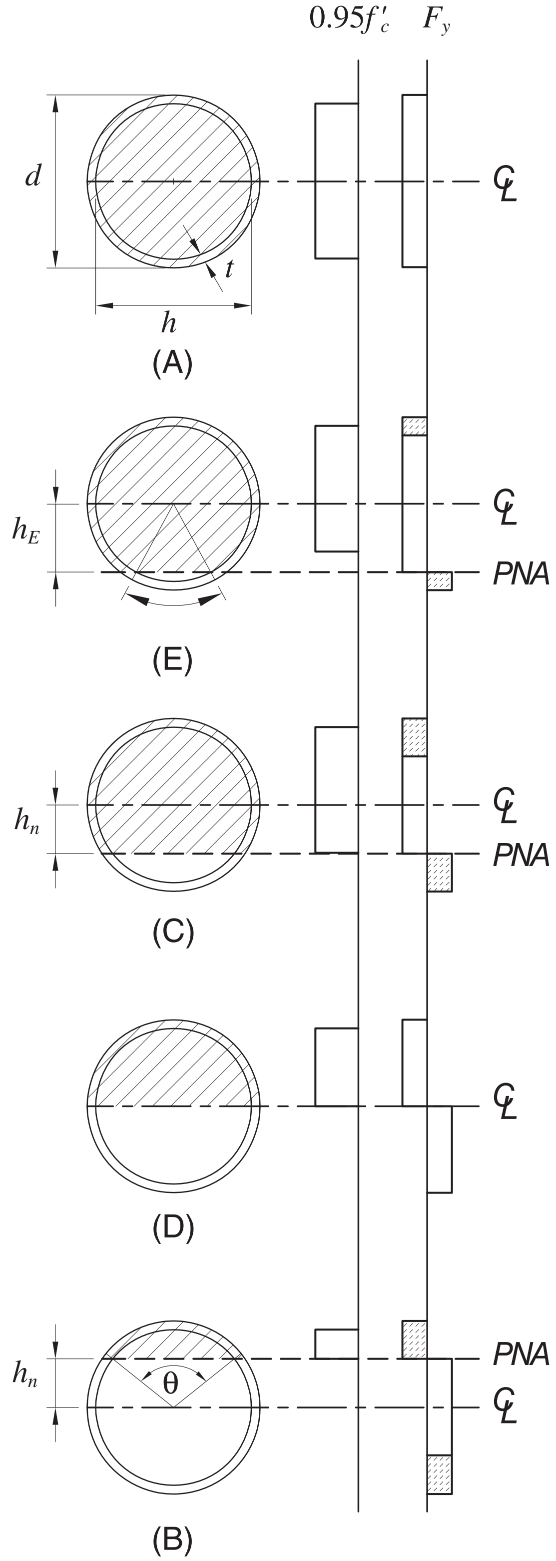
Section	Stress Distribution	Point	Defining Equations
 (A)		A	$P_A = F_y A_s + 0.85 f'_c A_c$ $M_A = 0$ A_s = area of steel shape $A_c = h_1 h_2 - 0.858 r_i^2$ $h_1 = b - 2t$ $h_2 = d - 2t$
 (E)		E	$P_E = \frac{1}{2}\left(0.85 f'_c A_c\right) + 0.85 f'_c h_1 h_E + 4 F_y t h_E$ $M_E = M_D - F_y Z_{sE} - \frac{1}{2}\left(0.85 f'_c Z_{cE}\right)$ $Z_{cE} = h_1 h_E^2$ $Z_{sE} = 2 t h_E^2$ $h_E = \frac{h_n}{2} + \frac{d}{4}$
 (C)		C	$P_C = 0.85 f'_c A_c$ $M_C = M_B$
 (D)		D	$P_D = \frac{0.85 f'_c A_c}{2}$ $M_D = F_y Z_s + \frac{1}{2}\left(0.85 f'_c Z_c\right)$ Z_s = full x-axis plastic section modulus of HSS $Z_c = \frac{h_1 h_2^2}{4} - 0.192 r_i^3$
 (B)		B	$P_B = 0$ $M_B = M_D - F_y Z_{sn} - \frac{1}{2}\left(0.85 f'_c Z_{cn}\right)$ $Z_{sn} = 2 t h_n^2$ $Z_{cn} = h_1 h_n^2$ $h_n = \frac{0.85 f'_c A_c}{2\left[0.85 f'_c h_1 + 4 t F_y\right]} \le \frac{h_2}{2}$

Table D.
Plastic Capacities for Composite, Filled Round HSS Bent About Any Axis



Section	Stress Distribution	Point	Defining Equations
		A	$P_A = F_y A_s + 0.95 f_c' A_c$ * $M_A = 0$ $A_s = \pi(dt - t^2)$ $A_c = \dfrac{\pi h^2}{4}$
		E	$P_E = P_A - \tfrac{1}{4}\left[F_y\left(d^2 - h^2\right) + \tfrac{1}{2}\left(0.95 f_c'\right)h^2\right]\left(\theta_2 - \sin\theta_2\right)$ $M_E = F_y Z_{sE} + \tfrac{1}{2}\left(0.95 f_c' Z_{cE}\right)$ $Z_{cE} = \dfrac{h^3}{6}\sin^3\left(\dfrac{\theta_2}{2}\right)$ $Z_{sE} = \dfrac{\left(d^3 - h^3\right)}{6}\sin\left(\dfrac{\theta_2}{2}\right)$ $h_E = \dfrac{h_n}{2} + \dfrac{h}{4}$ $\theta_2 = \pi - 2\arcsin\left(\dfrac{2h_E}{h}\right)$
		C	$P_C = 0.95 f_c' A_c$ $M_C = M_B$
		D	$P_D = \dfrac{0.95 f_c' A_c}{2}$ $M_D = F_y Z_s + \tfrac{1}{2}\left(0.95 f_c' Z_c\right)$ Z_s = plastic section modulus of steel shape $= \dfrac{d^3}{6} - Z_c$ $Z_c = \dfrac{h^3}{6}$
		B	$P_B = 0$ $M_B = F_y Z_{sB} + \tfrac{1}{2}\left(0.95 f_c' Z_{cB}\right)$ $Z_{sB} = \dfrac{\left(d^3 - h^3\right)}{6}\sin\left(\dfrac{\theta}{2}\right)$ $Z_{cB} = \dfrac{h^3 \sin^3\left(\dfrac{\theta}{2}\right)}{6}$ $\theta = \dfrac{0.0260K_c - 2K_s}{0.0848K_c} + \dfrac{\sqrt{\left(0.0260K_c + 2K_s\right)^2 + 0.857K_cK_s}}{0.0848K_c}$ (rad) $K_c = f_c' h^2$ $K_s = F_y\left(\dfrac{d - t}{2}\right)t$ ("thin" HSS wall assumed) $h_n = \dfrac{h}{2}\sin\left(\dfrac{\pi - \theta}{2}\right) \le \dfrac{h}{2}$

*0.95f_c' may be used for concrete filled round HSS.