Building Floor Vibrations

THOMAS M. MURRAY, 1991 T. R. Higgins Award

INTRODUCTION

Annoying floor motion induced by building occupants is probably the most persistent floor serviceability problem encountered by designers. According to Allen and Rainer (1975), Tredgold in 1828 wrote that girders over long spans should be "made deep to avoid the inconvenience of not being able to move on the floor without shaking everything in the room." If the response of a floor system from normal activities is such that occupants are uneasy or annoyed, the intended use of the building can be radically affected. Correcting such situations is usually very difficult and expensive, and success has been limited.

A number of procedures have been developed by researchers which allow a structural designer to analytically determine occupant acceptability of a proposed floor system. Generally, the analytical procedures require the calculation of the first natural frequency of the floor system and either maximum amplitude, velocity, or acceleration for a reference excitation. An estimate of the damping in the floor system is also required in some instances. A human perceptibility scale is then used to determine if the floor system meets serviceability requirements.

The purpose of this paper is to present an overview of analytical tools and concepts for controlling annoying floor movement in residential, office, commercial, and gymnasium type environments.

OVERVIEW OF NORTH AMERICAN DESIGN PROCEDURES

Murray (1975, 1981, 1985) has developed an analytical procedure to determine the acceptability of proposed floor systems supporting residential or office-type environments. The procedure utilizes a human response scale which was developed using field measurements taken on approximately 100 floor systems. The scale relates occupant acceptability of floor motion to three parameters: first natural frequency, amplitude, and damping. The amplitude is the maximum displacement of the floor system due to a reference heel-drop excitation. Guidelines for estimating damping in the system are provided as part of the procedure. The procedure is widely used and no instances of unacceptable performance of floor systems which satisfy the criterion have been reported.

The Canadian Standards Association provides a design pro-

Thomas M. Murray is Montague-Betts professor of structural steel design at Virginia Polytechnic Institute and State University in Blacksburg, VA.

cedure to ensure satisfactory performance of floor systems in Appendix G, Canadian Standards Association (1984). This procedure includes a human response scale based on the work of Allen and Rainer (1976). The scale was developed using test data from 42 long-span floor systems. The data for each test floor includes initial amplitude from a heel-drop impact, frequency, damping ratio, and subjective evaluation by occupants or researchers. The procedure requires the calculation of peak acceleration, first natural frequency, an estimate of system damping, and evaluation using the human response scale. Apparently, as part of a Canadian Standards Association Specification, this procedure must be followed in all Canadian building designs.

To provide sufficient static stiffness against floor motions during walking, Ellingwood and Tallin (1984) have suggested a stiffness criterion of 1 mm due to a concentrated load of 1 kN be used. The criterion is recommended by them for floors used for normal human occupancy (e.g., residential, office, school), particularly for light residential floors. This criterion does not include damping, which many researchers believe to be the most important parameter in controlling transient vibrations. In addition, no test data are presented to substantiate the criterion. Since the criterion is relatively new, acceptance by structural designers and performance of floor systems so designed is unknown at this time.

Allen, Rainer and Pernica (1985) and Allen (1990, 1990a) published criteria for the acceptability of vibration of floor systems that are subjected to rhythmic activities such as dancing and jumping exercises. Values for dynamic load parameters and acceleration limits are suggested for various activities. Using the suggested values, a set of minimum natural frequencies for different occupancies and floor constructions are recommended. For dance floors and gymnasia, the recommended minimum frequencies are 7, 9, and 11 hz for solid concrete, steel joist-concrete slab, and wood supported structures, respectively.

In the following section, specific recommendations, based on the writer's experience, are made for floor serviceability design. Three types of occupancy are considered: (1) residential and office environments, (2) commercial environments, and (3) gymnasium environments.

RECOMMENDED DESIGN CRITERIA

Residential and Office Environments

Ellingwood, et al. (1986) is a critical appraisal of structural

serviceability. The criteria developed by Murray (1981) and by Ellingwood and Tallin (1984) with modifications are recommended for controlling objectionable floor vibrations due to walking. Because of this recommendation and the wide use of the writer's criterion, the former procedure is recommended for floor motion control in office and residential environments.

In these environments, the forcing function is intermittent movement of a few occupants. Movement of groups does not generally occur and thus the floor motion is transient (e.g., motion occurs because of a short duration impact and decays with time). As a result, the most important parameter for residential and office environments is damping.

The recommended criterion (Murray 1981) states that if the following inequality is satisfied, motion of the floor system caused by normal human activity in office or residential environments will not be objectionable to the occupants:

$$D > 35 A_o f + 2.5 \qquad (1)$$

where D = damping in percent of critical, A_o = maximum initial amplitude of the floor system due to a heel-drop excitation (in.), and f = first natural frequency of the floor system (hz). The heel-drop excitation used to develop the criterion can be approximated by a linear decreasing ramp function having a magnitude of 600 lbs and a duration of 50 milliseconds. The criterion was developed using field measurements of approximately 100 floor systems mostly in the frequency range of 5–8 hz. Use of the criterion for floor systems with a first natural frequency above about 10 hz is not recommended. Detailed calculation procedures and an example are given in the Appendix.

Use of this criterion requires careful judgment on the part of the designer. A light office building floor system with hung ceiling and minimal mechanical ductwork will exhibit at least 3–3.5% of critical damping. Additional damping may be provided by a thicker slab, office furniture, partitions, equipment, and the occupants themselves. If the required damping (right hand side of Inequality 1) is less than 3.5–4%, the system will be satisfactory even if the supported areas are completely free of fixed partitions. If the required damping is between 4% and about 5%, the designer must carefully consider the final configuration of the environment and the intended use. For instance, if fixed partitions will not be present, the environment is quiet, and the required damping is 4%, complaints may be received once the building is occupied. If the required damping is much greater than 5%, the designer must be able to identify an exact source of damping or artificially provide additional damping to be sure the floor system will be satisfactory. If this cannot be accomplished, redesign is necessary.

Framed in-place partitions (sheetrock on wood or metal studs) are very effective sources of additional damping if (1) each partition is attached to the floor system in at least three locations and (2) they are located within the effective beam spacing or the effective floor width which is used to calculate system amplitude (Murray 1975, Galambos, undated). The direction of the partitions with respect to the supporting member span does not affect the damping provided. Partitions are equally effective if they are attached below the slab as compared to directly on the floor slab.

If partitions are not part of the architectural plan, either above or below the floor area under investigation, the designer may consider methods to artificially increase damping. If sufficient space exists between the ceiling and the underside of the floor slab, "false" sheetrock partitions of maximum possible depth might be installed in this space. This approach is relatively inexpensive and can provide damping equivalent to a similarly constructed handrail for a pedestrian bridge or crossover. From unreported laboratory tests conducted by the writer, an increase in damping of 0.5% to 1% can be achieved if the "partitions" are 2–3 feet deep.

Attempts to artificially increase damping in a floor system have been periodically reported in the literature. The use of dashpot dampers was shown to be successful in laboratory tests (Lenzen 1966), but successful installation in actual buildings has not been reported. Viscoelastic material has been attached to the bottom flanges of beams in an existing department store building where the floor motion was annoying to shoppers. The effort was reported to be only marginally successful (Nelson 1968). Additional experiments with these materials have not been reported. The use of viscoelastic materials to increase damping is very expensive, typically over $5 per square foot of floor area.

Although not strictly a method to increase system damping, the installation of a second mass system below the floor slab, in theory at least, has the same effect. Laboratory experiments have been reported (Allen and Pernica 1984), but the writer is unaware of any successful field installations. Allen (1990) states that "tuned dampers have so far not been very successful."

Damping devices, dashpots, friction dampers, viscoelastic materials, and second mass systems all require relative movement between the floor system and the device support. Because a vertical floor motion amplitude of only 0.040–0.050 in. can be very annoying to humans, the problem of developing a device which can effectively dampen floor motion is difficult. However, work is currently in progress at Virginia Polytechnic Institute and State University on the development of methods to artificially increase floor system damping.

Concerning frequency, the designer must be aware of very low first natural frequencies (below about 3 hz) to avoid walking resonance. Further, it is well known from automobile and aircraft comfort studies that humans react adversely to frequencies in the 5–8 hz range (Hanes 1970). The explanation for this phenomenon is that the natural frequencies of internal human organs (heart, kidneys, liver, and bladder)

are in the 5–8 hz range. Consequently, when the human body is subjected to such motion, resonance occurs and annoyance is the result. The writer has investigated over 50 problem floors (none of which satisfied Inequality 1) and, in the vast majority of the cases, the measured first natural frequency of the floor system was between 5 and 8 hz. The writer can state that he has never encountered an annoying residential/office floor where the span was greater than 40 feet, which is contrary to the common belief that long span floors vibrate and should be avoided. Furthermore, an office/residential floor with a natural frequency greater than 10 hz has never been found to be a problem.

In calculating natural frequency, the transformed moment of inertia is to be used, as long as the slab (or deck) rests on the supporting member. This assumption is to be applied even if the slab is not structurally connected to the beam flange or joist chord, since the magnitudes of the impacts are not sufficient to overcome the friction force between the elements. For the case of a girder supporting joists, it has been found that the joist seats are sufficiently stiff to transfer the shear, and the transformed moment of inertia assumption is to be used for the girder. If only the beam moment of inertia is used, a lower frequency results, but the system will actually vibrate at a much higher frequency and, thus, an evaluation using Inequality 1 may be inaccurate.

If the supporting member is separated from the slab (for example, the case of overhanging beams which pass over a supporting girder), the performance of the floor system can be improved if shear connection is made between the slab and supporting girder. Generally, two to four short pieces of the overhanging beam section, placed with their webs in the plane of the web of the girder and attached to both the slab and girder, provide sufficient shear connection.

Annoying vibration of office floors occurs when the floor system is lightly loaded; thus a careful estimate of the supported load must be made. Only the actual dead loads should be included plus 10% to 25% of the design live loads. Annoying vibrations have not been reported when the floor system is supporting the full design live load. One should note that an increase in supported load results in a decrease in frequency, which in turn results in a lower required damping.

In some instances, the beams or joists and the supporting girders will vibrate as a unit. This phenomenon usually occurs when the supporting girders are flexible relative to the beams or joists or when overhanging beams are supported by girders. In these instances the system frequency can be approximated from

$$\frac{1}{f_s^2} = \frac{1}{f_b^2} + \frac{1}{f_g^2} \tag{2}$$

where f_s = system frequency, f_b = beam or joist frequency, and f_g = girder frequency, all in hz.

Commercial Environments

In commercial environments, such as shopping centers, the forcing function can be nearly continuous walking or running movement of the occupants. In this situation, damping is not as critical as for office/residential environments because the floor movement is approximately steady-state. Control of the stiffness of the structural system is the best solution.

The criterion suggested by Ellingwood, et al. (1986) is recommended for commercial floor design. This criterion is based on an acceleration tolerance limit of 0.005 g and walking excitation. The criterion is satisfied if the maximum deflection under a 450 lbs. (2 kN) force applied anywhere on the floor system does not exceed 0.02 in. (0.5 mm).

Because the maximum deflections caused by occupant movements are so small, the floor system will act as if composite construction was used even if structural connection is not provided between the floor slab and the beam. Thus, the transformed moment of inertia should be used when calculating the stiffness of a proposed design.

Although it is doubtful that the floor system which satisfies this criterion will have a very low natural frequency, the possibility of walking resonance must also be checked. First harmonic resonance will occur below 3 hz and second harmonic resonance between 5 and 6 hz. It is recommended that the first natural frequency of the floor system be above 8 hz for commercial environments. The guidelines given in the above subsection and in the Appendix for calculation of frequency and effective floor width of residential/office floors can be used for commercial floors.

Gymnasium Environments

For floor systems supporting dancing or exercise activities, damping is usually not of consequence. The forcing function for these activities generally results in steady-state response and resonance must be avoided. Accompanying music for aerobic exercising usually does not exceed 150 beats per minute. The resulting forcing frequency is then about 2.5 hz. Allen and Rainer (1976) suggest that the first natural frequency of floors supporting such activities be above 7–9 hz to avoid resonance with the first and second harmonics of the forcing function.

More recently, Allen (1990a) has presented specific guidelines for the design of floor systems supporting aerobic activities. He recommends that such floor systems be designed so that

$$f_o \geq f_i\sqrt{1 + \frac{2}{a_o/g}\frac{\alpha_i w_p}{w_t}} \tag{3}$$

with f_o = first natural frequency of the floor structure (hz), f_i = ith multiple of f (hz), i = harmonic of jumping frequency (i = 1,2,3), f = jumping frequency (hz), a_o/g = acceleration limit, α_i = dynamic load factor for the harmonic of the loading function, w_p = equivalent uniformly

distributed load of participants (psf), and w_t = equivalent uniformly distributed floor weight (psf). (The reader is referred to the referenced paper for more details.) Application of Inequality 3 generally results in a required natural frequency greater than 9–10 hz.

To avoid complaints of undesirable motion of floors supporting exercise activities, the following is recommended: (1) provide structural framing so that the first natural frequency satisfies Inequality 3, generally above 9–10 hz; (2) isolate the floor system from the remaining structure using separate columns; (3) separate ceilings and partitions immediately below the exercise floor by supporting the ceiling on its own framing and by not extending partitions to the floor above; and (4) accept the possibility of complaints from non-participating individuals who happen to be on the exercise floor during significant activity by medium-to-large groups (20–60 participants). (It is also recommended that sound insulation be provided between the exercise floor and the ceiling below.) Obviously, only recommendations 3 and 4 are economically feasible once construction is complete.

Structural framing with sufficient stiffness to meet the 9–10 hz criterion can be very expensive, as frequency is proportional to the square root of moment of inertia. The most economical systems result from the use of deep beams or joists and lightweight concrete slabs (a decrease in mass increases frequency). The guidelines given above for calculating frequencies of floor-supporting residential/office activities apply for gymnasium floors.

SPECIAL SITUATIONS

Pedestrian Bridges

Pedestrian footbridges or crossovers require particular attention because damping is usually less than 2.5–3% and resonance with walking impacts can occur. (Recall that the average walking frequency of a human is approximately 2 hz.) If only casual pedestrian traffic is anticipated (for instance, a crossover in a hotel atrium), it is recommended that Inequality 1 be used as the design criterion. For this case, the damping exhibited by the completed structure should be assumed to be less than 3% of critical. If heavy traffic is anticipated (for instance, a footbridge at a sports arena exit), the structure should be designed so that the first natural frequency exceeds 7–9 hz to avoid walking resonance.

The designer of footbridges is cautioned to pay particular attention to the location of the concrete slab. The writer is aware of a situation where the designer apparently "eyeballed" his design based on previous experience with floor systems. Unfortunately, the concrete slab was located between the beams (because of clearance considerations) and the footbridge vibrated at a much lower frequency and at a larger amplitude than anticipated because of the reduced transformed moment of inertia. The result was a very unhappy owner and an expensive retrofit.

Motion Transverse to Supporting Members

Occasionally, a floor system will be judged particularly annoying because of motion transverse to the supporting beams or joists. In these situations, when the floor is impacted at one location, there is the perception that a wave moves from the impact location in a direction transverse to the supporting members. The writer has observed this phenomenon and felt the "wave" 50–70 ft. (15–20 m) from the impact location perhaps up to 1 second after the impact. In at least one instance the "wave" was felt to hit the exterior wall and return almost to the impact location. This phenomenon occurs when the building is rectangular, the floor is free of fixed partitions, and all beams are equally spaced and of the same stiffness, including those at column lines. The resulting motion is very annoying to occupants because the floor moves without apparent reason (the cause is not within sight or hearing). However, a simple remedy is available. The "cure" is to periodically (say every third bay) change one beam spacing or one beam stiffness. The result is that the "wave" simply stops at this location.

EXAMPLES

The following examples illustrate some of the concepts discussed.

EXAMPLE 1

Check the typical interior bay shown for susceptibility to vibration. The floor supports office space. (See Appendix for definition of terms.)

3½-in. lightweight (110 pcf, n = 14) concrete slab
2-in. metal deck (concrete in deck + deck = 9.1 psf)
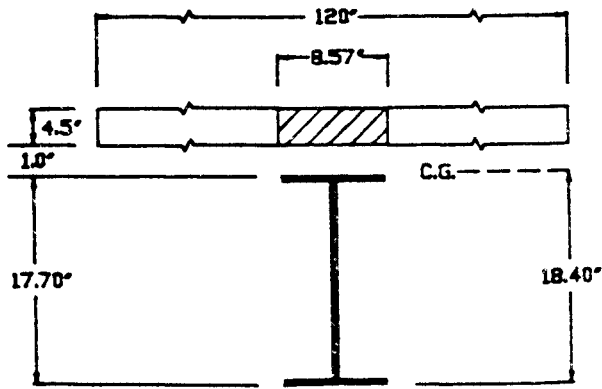Composite construction; hung ceiling; little ductwork
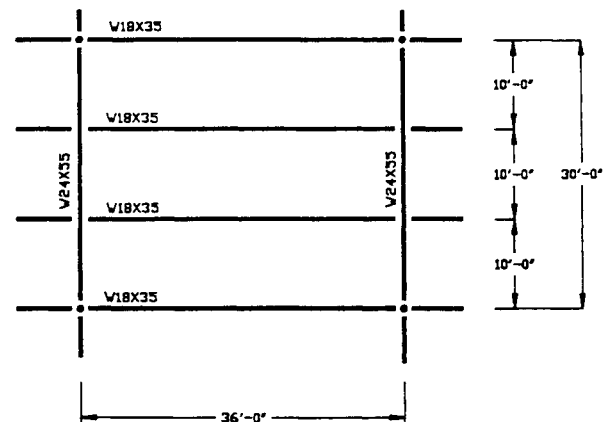
Beam

W18×35 A = 10.3 in.^{2} I_x = 510 in.^{2} d = 17.70 in.

d_e = 3.5 + (9.1 / 110) (12) = 4.5 in.
A_c/n = 120 (4.5) / 14 = 38.57 in.^{2}

$$Y_b = \frac{38.57 \ (20.95) \ + \ 10.3 \ (17.70 \ / \ 2)}{38.57 \ + \ 10.3} = 18.40 \text{ in.}$$

$$I_t = \frac{8.57(4.5)^3}{12} + 38.57(2.55)^2 + 510.0 + 10.30(18.40 - 17.70/2)^2$$

$$= 1,765 \text{ in.}^4$$

Supported weight = slab + deck + beam + actual mechanical (4 psf) + actual ceiling (2 psf) + 20% live load (10 psf)

$W = (4.5/12)(110)(10.0) + 35 + (4 + 2 + 10)(10.0)]36.0 = 21,870$ lbs.

$$f_b = 1.57\left[\frac{gEI_t}{WL^3}\right]^{1/2} = 1.57\left[\frac{(386.0(29 \times 10^6)(1,765)}{(21,870)(36 \times 12)^3}\right]^{1/2} = 5.26 \text{ hz}$$

From Table 1, $DLF = 0.75$

$$A_{ot} = (DLF)_{max}\frac{600L^3}{48EI_t} = \frac{(0.75)(600)(36 \times 12)^3}{(48)(29 \times 10^6)(1,765)} = 0.0148 \text{ in.}$$

$$N_{eff} = 2.97 - \frac{S}{17.3d_e} + \frac{L^4}{1.35EI_t}$$

$$= 2.97 - \frac{120.0}{17.3(4.5)} + \frac{(36 \times 12)^4}{1.35(29 \times 10^6)(1,765)} = 1.93$$

$A_{ob} = A_{ot} \ / \ N_{eff} = 0.0148 \ / \ 1.93 = 0.0077$ in.

Required damping $= 35A_0f + 2.5 = 35(0.0077)(5.26) + 2.5 = 3.9\%$

Girder

W24×55 $A = 16.20$ in.^{2} $I_x = 1,350$ in.^{4}

As above, with an assumed effective slab width = 10 ft, $I_t = 4,000$ in.^{4}
Supported weight = $2 \times 21,870 + 30 \times 55 = 45,390$ lbs

$$f_g = 1.57\left[\frac{(386.0)(29 \times 10^6)(4,000)}{(45,390)(30 \times 12)^3}\right]^{1/2} = 7.22 \text{ hz}$$

$(DLF)_{max} = 0.97$

$$A_{ot} = \frac{(0.97)(600)(30 \times 12)^3}{(48)(29 \times 10^6)(4,000)} = 0.0049 \text{ in.}$$

$N_{eff} = 1.0$ $A_{og} = 0.0049$ in.

Required damping $= 35(0.0049)(7.22) + 2.5 = 3.7\%$

System

$$\frac{1}{f_s^2} = \frac{1}{f_b^2} + \frac{1}{f_g^2} = \frac{1}{5.26^2} + \frac{1}{7.22^2}$$

$f_s = 4.25$ hz

$A_{os} = A_{ob} + A_{og} / 2 = 0.0077 + 0.0049 / 2 = 0.0102$ in.

Required damping $= 35 (0.0102) (4.25) + 2.5 = 4.0\%$

Evaluation

	f, hz	A_o, in.	Required Damping, %
Beam	5.26	0.0077	3.9
Girder	7.22	0.0049	3.7
System	4.25	0.0102	4.0

Since the required damping is approximately 4%, the system is judged to be satisfactory unless the office environment is very quiet or sensitive equipment is being operated. Because the girder frequency is greater than the beam frequency, the system will probably vibrate at the beam frequency, 5.26 hz, rather than the system frequency, 4.25 hz.

EXAMPLE 2
Evaluate the framing plan of Example 1 if used in the public areas of a shopping center.

Applying the criterion that the deflection caused by a 450 lbs. force does not exceed 0.02 in. (Ellingwood, et al. 1986):

$$\Delta_b = \frac{450L^3}{48EI_t} \times \frac{1}{N_{eff}} = \frac{450(36 \times 12^3)}{48(29 \times 10^6)(1,765)} \times \frac{1}{1.93} = 0.0077 \text{ in.}$$

$$\Delta_g = \frac{450(30 \times 12)^3}{48(29 \times 10^6)(4,000)} = 0.0038 \text{ in.}$$

$\Delta_s = \Delta_b + \Delta_g/2 = 0.0077 + 0.0038/2 = 0.0096$ in. < 0.02 in. **OK**

However, the natural frequency of the system is estimated to be 5.2 hz, which is considerably less than the recommended minimum value of 8 hz. Redesign is necessary.

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APPENDIX

GUIDELINES FOR ESTIMATION OF PARAMETERS

Damping

Damping in a completed floor system can be estimated from the following ranges:

Bare Floor: 1–3%
Lower limit for thin slab of lightweight concrete; upper limit for thick slab of normal weight concrete.

Ceiling: 1–3%
Lower limit for hung ceiling; upper limit for sheetrock on furring attached to beams or joists.

Ductwork and Mechanical: 1–10%
Depends on amount and attachment.

Partitions: 10–20%
If attached to the floor system and not spaced more than every five floor beams or the effective joist floor width.

Note: The above values are based on observation only.

Frequency

Beam or girder frequency can be estimated from

$$f = K\left[\frac{gEI_t}{WL^3}\right]^{1/2} \quad (A.1)$$

where

f = first natural frequency, hz.
K = 1.57 for simply supported beams
= 0.56 for cantilevered beams
= from Fig. 1 for overhanging beams
g = acceleration of gravity, in./sec./sec.
E = modulus of elasticity, psi
I_t = transformed moment of inertia of the tee-beam model, Fig. 2, in.[4]
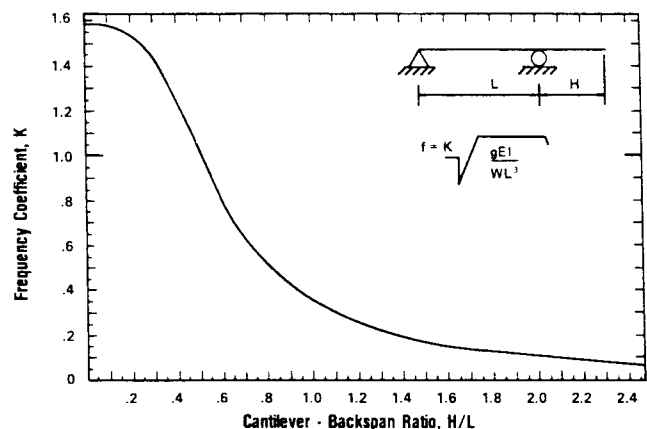

Fig. 1. Frequency coefficients for overhanging beams.
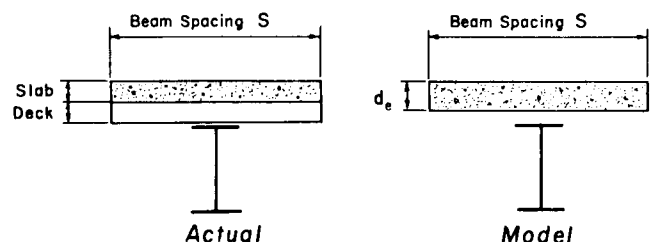

Fig. 2. Tee-beam model for computing transformed moment of inertia.

W = total weight supported by the tee-beam, dead load plus 10–25% of design live load, lbs.

L =
   tee-beam span, in.

System frequency is estimated using

$$\frac{1}{f_s^2} = \frac{1}{f_b^2} + \frac{1}{f_g^2}$$

where

f_s = system frequency, hz
f_b = beam or joist frequency, hz
f_g = girder frequency, hz

Amplitude from a Heel-Drop Impact

$$A_o = \frac{A_{ot}}{N_{eff}} \qquad (A.2)$$

where

A_o = initial amplitude of the floor system due to a heel drop impact, in.
N_{eff} = number of effective tee-beams
A_{ot} = initial amplitude of a single tee-beam due to a heel drop impact, in.
   = $(DLF)_{max} d_s$ \qquad (A.3)

where

$(DLF)_{max}$ = maximum dynamic load factor, Table 1
d_s = static deflection caused by a 600 lbs. force, in.
See (Murray 1975) for equations for $(DLF)_{max}$ and d_s

For girders, N_{eff} = 1.0.

For beams:

1. $S < 2.5$ ft, usual steel joist-concrete slab floor systems.

$$N_{eff} = 1 + 2\Sigma\left(\cos\frac{\pi x}{2x_o}\right) \text{ for } x \le x_o \qquad (A.4)$$

where

x = distance from the center joist to the joist under consideration, in.
x_o = distance from the center joist to the edge of the effective floor, in.
   = 1.06 ϵL
L = joist span, in.
ϵ = $(D_x/D_y)^{0.25}$
D_x = flexural stiffness perpendicular to the joists
   = $E_c t^3 / 12$
D_y = flexural stiffness parallel to the joists
   = EI_t / S
E_c = modulus of elasticity of concrete, psi
E = modulus of elasticity of steel, psi

Table 1.
Dynamic Load Factors for Heel-Drop Impact

f, hz	DLF	F, hz	DLF	F, hz	DLF
1.00	0.1541	5.50	0.7819	10.00	1.1770
1.10	0.1695	5.60	0.7937	10.10	1.1831
1.20	0.1847	5.70	0.8053	10.20	1.1891
1.30	0.2000	5.80	0.8168	10.30	1.1949
1.40	0.2152	5.90	0.8282	10.40	1.2007
1.50	0.2304	6.00	0.8394	10.50	1.2065
1.60	0.2456	6.10	0.8505	10.60	1.2121
1.70	0.2607	6.20	0.8615	10.70	1.2177
1.80	0.2758	6.30	0.8723	10.80	1.2231
1.90	0.2908	6.40	0.8830	10.90	1.2285
2.00	0.3058	6.50	0.8936	11.00	1.2339
2.10	0.3207	6.60	0.9040	11.10	1.2391
2.20	0.3356	6.70	0.9143	11.20	1.2443
2.30	0.3504	6.80	0.9244	11.30	1.2494
2.40	0.3651	6.90	0.9344	11.40	1.2545
2.50	0.3798	7.00	0.9443	11.50	1.2594
2.60	0.3945	7.10	0.9540	11.60	1.2643
2.70	0.4091	7.20	0.9635	11.70	1.2692
2.80	0.4236	7.30	0.9729	11.80	1.2740
2.90	0.4380	7.40	0.9821	11.90	1.2787
3.00	0.4524	7.50	0.9912	12.00	1.2834
3.10	0.4667	7.60	1.0002	12.10	1.2879
3.20	0.4809	7.70	1.0090	12.20	1.2925
3.30	0.4950	7.80	1.0176	12.30	1.2970
3.40	0.5091	7.90	1.0261	12.40	1.3014
3.50	0.5231	8.00	1.0345	12.50	1.3058
3.60	0.5369	8.10	1.0428	12.60	1.3101
3.70	0.5507	8.20	1.0509	12.70	1.3143
3.80	0.5645	8.30	1.0588	12.80	1.3185
3.90	0.5781	8.40	1.0667	12.90	1.3227
4.00	0.5916	8.50	1.0744	13.00	1.3268
4.10	0.6050	8.60	1.0820	13.10	1.3308
4.20	0.6184	8.70	1.0895	13.20	1.3348
4.30	0.6316	8.80	1.0969	13.30	1.3388
4.40	0.6448	8.90	1.1041	13.40	1.3427
4.50	0.6578	9.00	1.1113	13.50	1.3466
4.60	0.6707	9.10	1.1183	13.60	1.3504
4.70	0.6835	9.20	1.1252	13.70	1.3541
4.80	0.6962	9.30	1.1321	13.80	1.3579
4.90	0.7088	9.40	1.1388	13.90	1.3615
5.00	0.7213	9.50	1.1434	14.00	1.3652
5.01	0.7337	9.60	1.1519	14.10	1.3688
5.20	0.7459	9.70	1.1583	14.20	1.3723
5.30	0.7580	9.80	1.1647	14.30	1.3758
5.40	0.7700	9.90	1.1709	14.40	1.3793

t = slab thickness, in.
I_t = transformed moment of inertia of the tee-beam, in.[4]
S = joist spacing, in.

2. $S > 2.5$ ft, usual steel beam-concrete slab floor systems.

$$N_{eff} = 2.97 - \frac{S}{17.3 d_e} + \frac{L^4}{135 EI_T} \qquad (A.5)$$

where E is defined above and

S = beam spacing, in.

d_e = effective slab depth, in.

L = beam span, in.

Limitations:

$$15 \leq (S/d_e) < 40; \quad 1 \times 10^6 \leq (L^4/I_T) \leq 50 \times 10^6$$

The amplitude of a two-way system can be estimated from

$$A_{os} = A_{ob} + A_{og} / 2$$

where

A_{os} = system amplitude

A_{ob} = A_{ot} for beam

A_{og} = A_{ot} for girder