Modern Approaches to Solution of the Wind Problems of Long Span Bridges

ROBERT H. SCANLAN

During the process of design, the possible future loads and conditions to which a structure may be subjected must be conceived of and properly accounted for in analysis. It is hoped that, after the structure is erected, the loads and conditions which nature and man eventually impose on it will fall within the “envelope” of those previously conceived. When they do, the design is successful over its life span. Clearly the designer thus assumes responsibility for as wide a conception as possible of future loads. This conception is then translated into the structural form. Meanwhile, from another side, economics dictates that the structure not be overdesigned, which can occur if certain types of loads, though anticipated, are not well identified.

In the case of large bridges, awareness of loading conditions has proceeded both from insight and by accident. Early efforts and failures to make proper assessments of static wind loads, going back a century and more ago, are well known. Even today, new information continues to be sought in this area. The vibratory responses of bridges to wind have also touched off many investigations which have led eventually to greater insights into bridge aerodynamic loading effects. Though history now identifies many related examples which preceded it, the Tacoma Narrows disaster of 1940 signaled the advent of the modern era of investigation into the aeroelastic problems of long span bridges. Until recently, however, this whole area of problems has remained less familiar to the designer than have the traditional static problems.

The decade 1940–1950 witnessed particularly strong activity in identifying the wind-induced flutter problem of long suspended spans. After a lull of about a dozen years, the early 1960’s saw the reopening of vigorous further study centered about the aeroelastic problems of long span bridges. Until recently, however, this whole area of problems has remained less familiar to the designer than have the traditional static problems.

To the problem of bridge flutter have now been added the wind problems known as lateral buckling, torsional divergence, vortex-induced oscillations and buffeting. These, then, are the problems which occupy the center of the stage of modern bridge aerodynamic research to be discussed in the present paper.

Robert H. Scanlan is Professor of Civil Engineering, Department of Civil Engineering, Princeton University, Princeton, N. J., and is the recipient of the 1976 T. R. Higgins Lectureship Award.

WIND ENVIRONMENT

Though not the main preoccupation of the discussions to follow, a basic preliminary in the design studies affecting a major bridge should be an adequate assessment of the wind climatology affecting the site. In brief, this study sets values on the highest winds, from all compass points, expected to occur at the site during the projected lifetime of the span. Thom has published data on the highest winds expected in the United States for various return periods up to 100 years, and has discussed the use of local meteorological studies in assessing the directions of these high winds in a given area.

The wind blowing over the surface of the earth creates a “boundary layer” of flow wherein the air closest to the ground is moving very slowly, whereas the mean wind velocity increases as distance above the ground increases. The horizontal wind is actually turbulent, but its mean or average value $U$ builds up logarithmically, away from the ground, according to the formula:

$$U = 2.5u_\ast \log \frac{z - z_d}{z_0}$$

where $z$ is height above ground, $z_d$ is the “ground plane displacement” based on the average height of surrounding buildings or structures, and $z_0$ is a reference “roughness length” which typifies the kind of surface roughness over which the wind approaches the point of interest. In Eq. (1) $u_\ast$ is a reference “friction velocity,” typically determined by making direct measurements of $U = U(z)$ at some known height $z$, and using Eq. (1) in reverse.

Typical values of $z_d$ are given in cities by the smaller of the two quantities, 20 meters or $0.75H$, where $H$ is the average height of buildings in the surrounding area. Outside of cities $z_d \approx 0$ is a good approximation. Typical values of $z_0$ are given in Table 1. The mean wind speed profile builds up from the ground, with the general appearance of Fig. 1.

In addition to the mean, or average, velocity of the wind, gusts or turbulence are present. These are almost entirely due to the stirring of the wind in its passage over obstacles distributed over the terrain, such as various surface roughnesses, buildings, trees, hills, and the like. One of the standard ways of depicting atmospheric turbulence is through wind spectra.
If one imagines a variable wind velocity "signal" (Fig. 2) as recorded from an anemometer, it appears as a random function of time, varying temporarily about some local mean value. One way of visualizing the gust spectrum inherent in such a random wind signal is to imagine this signal to be rewritten mathematically as a Fourier series, i.e., an infinite sum of sines and cosines of stepwise increasing frequencies. One may then picture the wind velocity spectrum as a plot proportional to the squares of the amplitudes of the successive sine and cosine terms in the Fourier series; this spectral plot will of course be made with frequency as the abscissa. Since wind pressure is proportional to the square of the wind velocity, one may consider a wind spectral plot as representative of the distribution, against frequency, of oscillating wind pressures.

When wind signals of both horizontal and vertical components of wind velocity are made, they may both be converted into spectra in the manner suggested. Such plots are often constructed in the nondimensional manner sketched in Fig. 3, where typical standard wind spectra $S(n)$ are plotted in the form $nS(n)/u_*^2$ versus the frequency parameter:

$$f = \frac{n z}{U}$$  \hspace{1cm} (2)

where $n$ is frequency in Hz, $S(n)$ is half the square of the Fourier coefficient amplitude at frequency $n$, $u_*$ is the friction velocity, $z$ is the height above ground level, and $D$ is the mean wind horizontal velocity.

In addition to spectra of the wind, the questions of lateral and longitudinal coherence and scale must be considered. These will not be dealt with here in any detail; suffice it to say that coherence reflects the manner in which the wind velocity amplitude associated with any given frequency falls off as distance (across-wind or along-wind) from an observation point increases. Scale concerns the size (physical dimensions) of the conceptual whirls or eddies of the wind which pass the structure under turbulent conditions.

Armed with basic definitions of the local wind climate as briefly alluded to above, study of bridge susceptibility thereto can now begin.
NEED FOR WIND TUNNEL MODELING

A certain body of aerodynamic knowledge has grown up, enabling the theoretical calculation of wind forces on some objects. The class of objects (like airfoils) to which this possibility applies, however, is mainly restricted to the areas of aircraft, spacecraft, and rocketry. For the typically bluff objects which occur in civil engineering (buildings, towers, bridges, etc.), the possibility of theoretical calculation of wind forces is still rather remote, and the engineer must therefore have recourse to experiment.

This situation is not unlike that obtaining in aeronautics for the complex forms of whole aircraft, where obligatory model studies in the wind tunnel are now classical. However, the wind itself, being turbulent instead of smooth, is more difficult to model in the civil engineering context. Thus, considering that buildings, bridges, etc., do in fact all respond dynamically to the wind, the modeling question is at least as complex, and in some respects more complex, than in the corresponding aeronautical case.

We have commented in particular how long suspended-span bridges can be susceptible to wind, and this will be the special focal point of the present discussion. A long bridge is, however, not a convenient object to model in the wind tunnel, and considerable thought must be given to this situation and its alternatives. A few full-length suspension bridge models have indeed been studied at reduced scale in the wind tunnel, notably by Scruton, Davenport, Hirai, and by others as well. Wardlaw, at the National Research Council in Ottawa, Canada, is currently studying such a model.

The point of view espoused in the research to be reported in this paper is, however, to accomplish all that is possible regarding bridge aerodynamic studies with the aid of the very much cheaper and convenient deck section model, which consists of only a short, typical section taken out by cross-roadway cuts. This section model will therefore be the main object of discussion here, as it was in the earlier paper by Scanlan and Tomko. The method of approach to be discussed will be to measure all important aerodynamic facts on such a relatively inexpensive model and later apply these facts, through analysis methods, to the full prototype bridge. Briefly stated, then, this is the rationale behind the studies to be reported.

WIND PROBLEMS OF THE LONG SPAN SUSPENDED BRIDGE

The traditional static (steady) wind problems of bridge structures are normally met by choosing reasonable values of wind pressure to be applied perpendicularly to the structural members, along their lateral surfaces. An example of a relatively new code specifying some values of this type is given by Ref. 10. In a few important cases it may be further desirable to explore wind effects in the wind tunnel through the use of static bridge models placed at various azimuth angles to the wind. These important, basic considerations are primary to any wind loading study program. However, because of the degree to which they are already understood by the profession, the present paper will pass on to a treatment of the dynamic and aeroelastic wind problems of greater immediate interest.

The classical suspension bridge and the newer cable-stayed bridge are susceptible to the wind excitations identified by the names lateral buckling and torsional divergence, flutter, vortex-induced oscillations, galloping, and buffeting. A brief definition of each of these conditions, caused by the distribution of wind pressures over the deck, will be given.

Lateral buckling is the static instability phenomenon wherein the deck, under horizontal drag loads of the wind, acts like a long, thin beam with horizontal web, which buckles out-of-plane when the horizontal load exceeds a certain value.

Torsional divergence is the static instability phenomenon wherein the deck, under aerodynamic twisting moments about its spanwise axis, acts like a long torsion rod, the elastic restoring forces of which, at some wind velocity and corresponding twist angle, can no longer counter the wind-induced moment. The latter grows with twist angle; therefore, at some critical wind velocity the elastic restoring moment is completely overcome, and catastrophic damage is precipitated. This critical wind velocity is called the divergence velocity. It should be remarked here that, in practical cases, the two phenomena of lateral buckling and torsional divergence are not separable and are, in fact, aspects of the same overall wind-structure interaction phenomenon. Lateral buckling-torsional divergence typically does not occur for normal bridges except at unusually high wind speeds.

Flutter is a vibratory instability associated with wind self-excitation forces brought about by the structural motion itself. It typically begins at some critical wind speed, and oscillation intensity increases thereafter with increasing wind speed, until destructive amplitudes are reached. Flutter may occur in a single degree of freedom (typically, torsion) or in two coupled degrees (bending and torsion). In the single degree of freedom, the phenomenon is usually of the stall flutter type; in the binary degree-of-freedom case, the flutter is of the classical or coupled type. In the former, the aerodynamic damping is negative, whereas in the latter, damping in both individual freedoms is positive, but phasing between the motions permits an energy feed-in from the wind stream. Flutter occurs under fairly high-velocity wind unless the form of the bridge deck cross section is particularly vulnerable thereto.

Vortex-induced oscillations are a characteristic bridge response to the alternate shedding of vortices from the aerodynamically bluff cross-section of the deck. As is well known, vortices are rhythmically shed behind circular cylinders placed in a wind stream, and they are likewise shed behind almost all bluff bodies, including bridge decks, most particularly solid-section decks, even those which are partially streamlined.

If the alternating pressures which accompany the vortex shedding happen to excite a natural frequency of the bridge, the structure responds. Such events usually take place in
rather mild, steady winds, and are not normally storm-associated phenomena. Long, slender members, such as the deck hangers of arch bridges, often exhibit vibration due to this phenomenon also, but usually at fairly high wind speeds.

Galloping is a cross-wind, large-amplitude oscillation set up in strong wind by certain combinations of the steady-state lift and drag of a long, bluff body. This is not generally a bridge deck phenomenon, but it can occur to bridge hangers, etc., which have bluff shapes.

Buffeting is a generic name for the “battering” given a bridge by the turbulence in the atmosphere. Bridge response to it is characterized by random oscillations at the natural frequencies of a few fundamental bridge vibration modes. Even stable (non-flutter-prone) bridges may exhibit the unpleasant effects of buffeting by wind gusts.

METHODS OF ATTACK ON THE WIND PROBLEMS OF BRIDGES

The methods open to the designer are both experimental and analytic, with wind tunnel testing playing a major role. Both full-length bridge models and sections thereof are used in testing. Since full bridge models are costly and require faithful scale modeling of mass, elastic, damping, and geometric properties of the prototype, and since models should not be allowed to get too small because of Reynolds number* difficulties, there is a tendency to employ reasonably-scaled section models whenever possible, as these are relatively inexpensive, easy to test and modify, and avoid gross Reynolds number problems. With the testing of section models today, there is a growing trend toward their use more for the abstraction of the indispensable, purely aerodynamic data and the use of such data in subsequent dynamic analyses of prototype bridge performance (see Refs.) rather than as assumed, directly faithful models of that performance. In this role, the bridge section model becomes most valuable for its geometric fidelity rather than any of its other scaling parameters. The paper by Tomko and the writer emphasizes this new trend in section model work.

It is common to employ reasonable geometric scales, from 1/25 to 1/100, for section models. Since most bluff forms used in bridge structures have sharp edges and corners, models at this scale range are generally considered to have only minor Reynolds number problems, since flow separation, the outstanding flow event in their vicinity, occurs at these same sharp edges in either model or prototype. For models at the scale of 1/300 or smaller, however, there may be local viscous effects which influence model results. Drag, for instance, increases radically as Reynolds number drops to low values. Figure 4 emphasizes this fact.

* Reynolds number, approximated in air by the formula

\[ R_e = \frac{6400 U B}{\nu} \]

where \( U \) is wind velocity in ft/sec and \( B \) is a typical geometric dimension in feet, is a measure of the relative strengths of the inertial and viscous forces in a given flow.

Lateral buckling and torsional divergence of a deck section can be analyzed if the experimental curves of steady-state lift, drag, and twisting moment (in the nondimensional form of coefficients \( C_L, C_D, \) and \( C_M \), respectively) are available. Figures 5, 6, and 7 exhibit these quantities for the newer bridge at the Tacoma Narrows site. A brief sketch of the problem will be given.
Let \( \alpha \) be a column matrix of all the local angles of attack, section by section, to the wind. Let \( C_T \) be the matrix of structural influence coefficients (torsional flexibility matrix) of the bridge deck. Let the local aerodynamic moment coefficient (due to all aerodynamic pressures) about the section elastic axis be written as:

\[
C_M = \left( \frac{\partial C_M}{\partial \alpha} \right) \alpha + C_{MO}
\]  

(3)

Then the column of aerodynamic moments for all sections is:

\[
\{M_A\} = \frac{1}{2} \rho U^2(2B^2) \left[ \frac{\partial C_M}{\partial \alpha} \right] \Delta L
\]  

(4)

where \( B \) is deck width and \( \Delta L \) is the span length of one section. Equating aerodynamic to structural resistance moments then yields

\[
C_T \{\alpha\} = \frac{1}{2} \rho U^2(2B^2) \Delta L \frac{\partial C_M}{\partial \alpha} \{\alpha\} + \left[ \frac{1}{2} \rho U^2(2B^2) \Delta L C_{MO} \right]
\]  

(5)

This may be condensed to:

\[
[C_T - pI] \{\alpha\} = q
\]  

(6)

where

\[
p = \frac{1}{2} \rho U^2(2B^2) \Delta L \frac{\partial C_M}{\partial \alpha}
\]  

(7)

\[
q = \frac{1}{2} \rho U^2(2B^2) \Delta L C_{MO}
\]  

(8)

This problem may be solved for the case of divergence (\( |\alpha| \to \infty \)) when the determinant below is zero:

\[
|C_T - pI| = 0
\]  

(9)

which yields a value \( p_c \) as solution. This in turn yields the divergence-lateral buckling velocity

\[
U_c = \left[ \frac{p_c}{\rho B^2 \Delta L (\partial C_M/\partial \alpha)} \right]^{1/2}
\]  

(10)

*Flutter* is a problem requiring the full dynamic equations of motion of the system. These full equations appear in Refs. 12, 13, 14, 15. However, a sketch of the problem can be given here. Let \( h, \alpha \) be the respective bending and torsional deflections of the deck section. Full section equations at motion take the form:

\[
M[\dddot{h} + 2 \xi_h \omega_h \dot{h} + \omega_h^2 h] = L_h
\]  

(11)

\[
I[\dddot{\alpha} + 2 \xi_\alpha \omega_\alpha \dot{\alpha} + \omega_\alpha^2 \alpha] = M_\alpha
\]  

(12)

where subscripts \( h, \alpha \) refer to the respective freedoms, \( M \) and \( I \) are the mass and mass moment of inertia per unit span of the deck, \( \omega \) refers to natural frequency, and \( L_h, M_\alpha \) are aerodynamic lift and moment, respectively.

In the self-excited case (flutter), aerodynamic lift and moment are given by:

\[
L_n = \frac{1}{2} \rho U^2(2B) \left[ KH_1 \frac{\dot{h}}{U} + KH_2 \frac{B \ddot{\alpha}}{U} + K^2 H_3 \alpha \right]
\]  

(13)

\[
M_n = \frac{1}{2} \rho U^2(2B^2) \left[ K A_1 \frac{\dot{h}}{U} + K A_2 \frac{B \ddot{\alpha}}{U} + K^2 A_3 \alpha \right]
\]  

(14)

where \( U \) is mean horizontal wind velocity across the bridge deck, \( p \) is air density, \( B \) is deck width, and \( H_i, A_i \) (\( i = 1,2,3 \)) are experimentally obtained aerodynamic coefficients, functions of \( K = B/U \), where \( \omega \) is circular flutter frequency. The main thrust of the paper by Scanlan and Tomko\(^9\) was the measurement and presentation of typical values of such coefficients.

Figure 8 presents results for \( H_i \) and \( A_i \) plotted versus \( U/NB \) for the airfoil and a few representative bridges at zero mean angle of attack under laminar flow conditions. References 9 and 16 contain further data of this type. For complete bridge studies, such data must be gathered at a range of angles of attack. Further, it should be studied also under turbulence (see Ref. 17).

Many approaches exist today for gathering data of the type of Fig. 8. Suffice it to mention that methods used in Japan\(^\text{18}\) and France\(^\text{19}\) have employed mechanical driving of the bridge model through sinusoidal oscillations, while that used in the U.S.\(^9\) uses free oscillations of the model as suspended on four linear springs (see Fig. 9).

The work first carried out in Ref. 9 used simple methods of system identification to establish the coefficients \( H_i, A_i \). Of principal importance among the findings of this paper and subsequent researches are the following points:
1. Bridge decks have aerodynamic stability derivatives $H_a^*$ and $A_t^*$ which differ in a pronounced manner from comparable results for airfoils. Therefore, use of airfoil flutter derivatives for bridge flutter analysis is incorrect.

2. One of the outstanding results for many bridges is the very characteristic change in sign of the $A_t^*$ coefficient (aerodynamic damping in torsion) with increasing values of $1/K$. This characterizes, particularly, the very common single-degree-of-freedom type of flutter, in torsion, of many bridges.

3. The method of exploiting the section model uniquely for its geometric properties frees the testing procedure from all similarity requirements other than geometric. The section model thus becomes an analog computer of dimensionless aerodynamic effects alone; these aerodynamic effects may then be incorporated into any desired subsequent analysis of the prototype bridge.

4. The freely oscillating model technique which is used is very inexpensive, particularly as compared to the driven model technique. Moreover, it permits the intrinsically nonlinear aerodynamics of the situation to be quite reasonably linearized, since the model response remains essentially damped sinusoidal at all times.

5. Bridge structural frequencies are little changed by the aerodynamic forces during flutter.

6. Bridge aerodynamic stability derivatives obtained under laminar flow remain to be verified (through future research) under turbulent flow. However, the lower frequency gusts of horizontal wind appear to affect a bridge in the same manner as quasi-steady laminar flow.

7. The exact values of bridge mechanical damping do not turn out to be of great importance in bridge flutter. It is sufficient to estimate them in some range (say 1 to 5 percent).

8. The geometric forms of bridge decks are of prime importance relative to flutter stability. Figure 10 presents some forms exhibiting good flutter stability. In general, for open-truss forms, the bleeding through of air (as by grids in the roadway lanes) from bottom to top of the deck section serves stability. For streamlined forms, greater streamlining toward a "thin profile" helps stability. In particular, forms enhancing the smooth advance of the flow onto the deck section when it is at an angle of attack to the wind appear to cut down flow separation and instability tendencies. (See Fig. 10.)

9. The increase of the ratio of fundamental torsional to bending frequencies generally enhances stability. In other words, making the bridge as torsionally stiff as possible is stabilizing against flutter.

Vortex-induced oscillation may be set off by wind of even a mild nature blowing across the deck section. Occasionally such oscillation, while not reaching dangerous amplitudes, can cause annoyance to bridge users. It persists with bluff, solid-section deck forms, while open-truss types are affected somewhat less by it.

The vortex-shedding phenomenon from any bluff body takes on a frequency $N$ which is governed, for quite a large range of Reynolds number, by the Strouhal relation:

$$\frac{ND}{U} = \mathcal{S}$$

where $\mathcal{S}$ is the Strouhal number, $D$ is the projected body dimension normal to the flow, and $U$ is the mean flow velocity. For example, for a circular cylinder, when $D$ is the diameter, $\mathcal{S} \approx 0.2$. For other bluff forms $\mathcal{S}$ lies in the range:

$$0.1 \leq \mathcal{S} \leq 0.25$$

When the wind velocity $U$ is such that the frequency $N$ satisfying Eq. (15) happens to fall upon a natural frequency of the bridge deck, the latter responds in its corresponding mode. When the structure itself oscillates in this mode, this changes the local boundary conditions of the flow, and the well-known "lock-in" phenomenon occurs. In this phenomenon the structural natural frequency governs the vortex shedding for a considerable range even when Eq. (15) is not satisfied by $U$. However, wind of a very much lower or higher velocity will not cause such oscillation to take place.

Thus, the phenomenon is not important except in the neighborhood of a natural frequency of the structure. In such a neighborhood, however, the oscillating lift force per unit span is of the form

$$L_{v\ell} = \frac{1}{2} \rho U^2 D C_L \sin 2\pi N t$$

where $N$ satisfies Eq. (15) and $C_L$ is to be determined experimentally. Reference 15 discusses this problem at greater length and offers a conservative calculation of the expected amplitude of vortex-induced oscillation of a full bridge under uniform wind crossing the full span.

---

Fig. 10. Severn, Lillebaelt, Wardlaw designs
Galloping is usually considered to be an oscillation of large amplitude compared to the cross-sectional size of the oscillating body. In such large amplitude excursions it is possible to approximate oscillating lift and drag coefficients by their steady-state values, $C_L$ and $C_D$ respectively. When the so-called “Den Hartog Criterion”

$$
\left( \frac{\partial C_L}{\partial \alpha} + C_D \right) < 0
$$

is satisfied, galloping can occur, the usual cross-wind equation of motion being:

$$
M \ddot{h} + 2 \zeta D \omega_n \dot{h} + \omega_n^2 h = -\frac{1}{2} \rho U^2 D \left( \frac{\partial C_L}{\partial \alpha} + C_D \right) \dot{h}
$$

This results in a complete balancing of the mechanical damping when

$$
2 M \omega_n \dot{h} = -\frac{1}{2} \rho U^2 D \left( \frac{\partial C_L}{\partial \alpha} + C_D \right) \dot{h}
$$

i.e., when

$$
U = U_{crit} = \frac{-4 M \omega_n \dot{h}}{\rho D \left( \frac{\partial C_L}{\partial \alpha} + C_D \right)}
$$

This situation rarely occurs for whole bridge decks, but they may easily be checked out by use of Eq. (19); it does occur more frequently for bluff-section bridge hangers, which can similarly be checked.

Buffeting, the most complicated of the bridge response phenomena, may be considered to be the total response of the full bridge to the natural, turbulent wind. This full response is complicated by the random nature of the wind gusts both in space and in time. One may gain an abridged view of this problem by considering it as similar to the flutter problem, governed for each spanwise section of the bridge by Eqs. (11) through (14), to which must be added the oscillating lift and moment buffeting terms:

$$
L_b(t) = \frac{1}{2} \rho U^2 (2B) C_{Lb}(t)
$$

$$
M_b(t) = \frac{1}{2} \rho U^2 (2B^2) C_{Mb}(t)
$$

where $C_{Lb}(t)$, $C_{Mb}(t)$ are, respectively, time-varying random lift and moment coefficients.

Information on the nature of these coefficients must be gathered from experiments on section models under properly simulated turbulent conditions in the wind tunnel. Theory for the application of the results is given more explicitly in Refs. 14 and 15, including extension to the full span of the bridge. Some calculated results are reported in Ref. 21.

The state of the art of the buffeting problem is not yet fully developed insofar as acquisition of all data from section models is concerned. One of the principal remaining points requiring research is the creation of simulated wind turbulence which has a large enough scale (average eddy size) to match the scales used in typical section models (1/25 to 1/100). Research is going forward on this point, but it appears as if active, rather than passive, turbulence generators may be required for the task.

Alternatively, when passive turbulence initiation occurs, it is difficult to raise the geometric scale above 1/100, the more common results being in the range of 1/600 to 1/300, as presently achieved in boundary layer wind tunnels. Full-bridge models have been tested at these lower scales, however, with some success.

SUMMARY

The present paper briefly reviews the state of the art of the aerodynamic and aeroelastic problems of long suspended-span bridges. In particular, the earlier work of Tomko and the writer is referred to and it is placed in the broader context of the entire field to which it originally contributed. This field is broadening daily with contributions world-wide, and research continues on every aspect of it. This report cannot, therefore, aspire to be more than a summary view of a field to which the research of many is now actively committed.

REFERENCES


