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ABSTRACT

The provisions regarding analysis and, especially, stability in the 2005 AISC Specification for Structural Steel Buildings represent a significant departure from earlier editions. The changes were intended to reflect the current state of knowledge and also to make the specification more transparent to the user. The new provisions spell out the general safety- and reliability-based requirements that must be satisfied by all structural designs, giving designers the freedom to select or devise their own methods of analysis and design within these constraints, and also provide “prescriptive” methods for those who prefer that approach.

This paper discusses the logical basis of the new Specification requirements for stability, and outlines the three alternative prescriptive methods that are specified. The most versatile and powerful of these methods is the Direct Analysis Method. An Appendix to this paper offers a model specification reformulated around the Direct Analysis Method alone, making it easier to understand and use. This represents the direction in which the AISC Specification appears to be evolving; the stability section of the next edition is likely to resemble this model specification.
INTRODUCTION

In today’s engineering practice, there is no such thing as a “normal” or “standard” structural analysis: Advanced analysis methods that were regarded as research tools a few years ago have entered some design offices while other practices are still using the same (except bigger and faster) analysis tools they had a generation ago. This is especially true in the area of stability, where direct, rigorous second-order analysis is routine in some practices but not in others. This range in analysis options is especially important in the area of stability because of the close interrelationship between stability design and analysis.

The provisions regarding analysis and, especially, stability in the 2005 AISC Specification for Structural Steel Buildings (AISC, 2005) represent a significant departure from earlier editions. The new specification recognizes the wide range of analyses in common use. It spells out the general safety- and reliability-based requirements that must be satisfied by all structural designs, giving designers the freedom to select or devise their own methods of analysis and design within these constraints, and also provides “prescriptive” methods for those (possibly a large majority of designers) who prefer that approach.

This paper discusses the logical basis of the new specification requirements for stability, and outlines the three alternative prescriptive methods that are specified. The most versatile and powerful of these methods is the Direct Analysis Method. An appendix to this paper offers a model specification reformulated around the Direct Analysis Method alone, making it easier to understand and use. This represents the direction in which the AISC Specification appears to be evolving; the stability section of the next edition is likely to resemble this model specification.

GENERAL REQUIREMENTS

The chapter of the Specification on Design Requirements (Chapter B) specifies that the design of structural components must be consistent with the assumptions made in the structural analysis used to determine the required strengths of the components. There are no other constraints on the method of analysis.

The chapter on Stability Analysis and Design (Chapter C) specifies that the design of the structure for stability must consider all of the following:

- Flexural, shear and axial deformations of members.
- All other component and connection deformations that contribute to displacements of the structure.
- P-Δ effects, which are the effects of loads acting on the displaced location of points of intersection of members in the structure. (In typical building structures, this is the effect of loads acting on the laterally displaced location of floors and roofs.)
- P-𝛿 effects, which are the effects of loads acting on the deformed shape of individual members.
- Geometric imperfections, such as initial out-of-plumbness.
- The reduction in member stiffness due to inelasticity (including residual stress effects) and, in particular, the effect of this stiffness reduction on the stability of the structure.
When the required strengths of members have been determined from an analysis that considers all the above effects, the members can be designed using the provisions for design of individual members (provided in Chapters D, E, F, G, H and I).

The Specification states explicitly that any method of analysis and design that considers all the specified effects is permissible, and then presents certain specific approaches that account for the last four of the listed effects (P-Δ effects, P-δ effects, geometric imperfections, inelasticity).

**DIRECT ANALYSIS METHOD**

The most generally-applicable method of accounting for P-Δ and P-δ effects, geometric imperfections and inelasticity is the “Direct Analysis Method” (presented in Appendix 7 of the Specification). It is applicable to all types of structural systems; the provisions of the Direct Analysis Method do not distinguish between braced frames, moment-resisting frames, shear wall systems, and combinations of these and other structure types. In the Direct Analysis Method:

- P-Δ and P-δ effects are accounted for through second-order analysis (either explicit second-order analysis or second-order analysis by amplified first-order analysis, for which a procedure is presented in the Specification).
- Geometric imperfections are accounted for either by direct inclusion of imperfections in the analysis model or by the application of “notional loads” (which are a proportion of the gravity load, applied laterally).
- Stiffness reductions due to inelasticity are accounted for by reducing the flexural and axial stiffnesses of members by specified amounts or, at the designer’s option, by a combination of reduced member stiffness and additional notional loads.

When the required strengths of members have been determined from an analysis conforming to the above requirements, individual members can be designed using an effective length factor of unity in calculating the nominal strengths of members subject to compression.

The Specification provides enough direction to allow application of the Direct Analysis Method in “cook book” fashion. But it also lays out the logical basis for the provisions in a way that offers designers the option of tailoring the method to particular situations. For instance, it is spelled out that the specified 0.002 notional load coefficient to account for geometric imperfections is based on a maximum initial story out-of-plumbness ratio of 1/500; a different notional load can be used if the known or anticipated out-of-plumbness is different; the imperfections can even be modeled explicitly instead of applying notional loads.

In time, if not immediately, the Direct Analysis Method will almost certainly become the “standard” method of stability design of steel building structures.

**INDIRECT METHODS**

For structures in which second-order effects are not very large (where the ratio of second-order drift to first-order drift is below a specified threshold), the Specification offers two alternatives to the Direct Analysis Method.
Effective Length Method. In this method, the structure is analyzed using the nominal geometry and nominal elastic stiffness of all members; required member strengths are determined from a second-order analysis (either explicit second-order analysis or second-order analysis by amplified first-order analysis); all gravity-only load combinations include a minimum lateral load at each frame level of 0.002 of the gravity load applied at that level. Effective length factors (K) or buckling stresses for calculating the nominal strengths of compression members must be determined from a sidesway buckling analysis, except that K=1 may be used for braced frames or where the ratio of second-order drift to first-order drift is less than 1.1.

First-Order Analysis Method. This method is applicable only when the required compressive strength is less than half the yield strength in all members whose flexural stiffnesses are considered to contribute to the lateral stability of the structure. In this method, the structure is analyzed using the nominal geometry and nominal elastic stiffness of all members; required member strengths are determined from a first-order analysis; all load combinations include an additional lateral load at each frame level of a magnitude based on the gravity load applied at that level and the lateral stiffness of the structure. The nominal strengths of compression members may be determined assuming K=1; beam-column moments must be adjusted (using a formula that is provided) to account for non-sway amplification.

<table>
<thead>
<tr>
<th>Specification reference</th>
<th>Direct Analysis Method</th>
<th>Effective Length Method</th>
<th>First-Order Analysis Method</th>
</tr>
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<tr>
<td>Limits on applicability?</td>
<td>No</td>
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<td>Yes</td>
</tr>
<tr>
<td>Type of analysis</td>
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<td>First-Order</td>
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<td>Member stiffness</td>
<td>Reduced EI &amp; EA</td>
<td>Nominal EI &amp; EA</td>
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<tr>
<td>Notional lateral load?</td>
<td>Yes</td>
<td>Yes</td>
<td>Additional lateral load</td>
</tr>
<tr>
<td>Column effective length</td>
<td>K=1</td>
<td>Sidesway buckling analysis</td>
<td>K=1</td>
</tr>
</tbody>
</table>

TABLE 1
COMPARISON OF ANALYSIS AND DESIGN OPTIONS

The alternative analysis methods and corresponding stability design requirements in the AISC Specification are summarized in Table 1.

METHODS OF SECOND-ORDER ANALYSIS

As noted in the discussion of alternative analysis-design approaches, the Direct Analysis Method and one of the two indirect methods require a second-order analysis of the structure. The second-order analysis can take the form of an explicit second-order analysis that includes both P-
Δ and P-δ effects. Alternatively, the second-order analysis can consist of amplified first-order analysis, for which a detailed procedure is provided in the *Specification*. (This is the “B1-B2” procedure familiar to designers from previous editions of the *Specification*.)

Since stability is an inherently nonlinear phenomenon, it is essential that all second-order analyses be carried out at the LRFD load level. To obtain the proper level of reliability when ASD is used, the analysis must be conducted under 1.6 times the ASD load combinations and the results must then be divided by 1.6 to obtain the forces and moments for member design by ASD. (The 1.6 load multiplier must also be used, in ASD, when checking the ratio of second-order drift to first-order drift, as required under certain provisions.)

**SOURCE OF ADDITIONAL INFORMATION**

This outline of the analysis provisions in the 2005 AISC *Specification* is intended primarily as an introduction to these provisions and to show the logical progression of the provisions from general requirements applicable to all structures to specific procedures that designers may choose to use for the design of typical structures. More information on the rational basis of the new *Specification* provisions can be found in the Commentary to the *Specification* and the references listed therein.

**FURTHER DEVELOPMENTS**

The most versatile and powerful of the three alternative methods of stability analysis and design in the 2005 AISC *Specification* is the Direct Analysis Method. An appendix to this paper offers a model specification reformulated around the Direct Analysis Method alone, making it easier to understand and use. This represents the direction in which the AISC Specification appears to be evolving; the stability section of the next edition is likely to resemble this model specification. A second appendix explains the substantive differences between this model specification and the present AISC *Specification*.

**REFERENCE**

APPENDIX

SPECIFICATION FOR STABILITY DESIGN BY DIRECT ANALYSIS

As discussed in the paper to which this is an appendix, the 2005 AISC Specification for Structural Steel Buildings (AISC, 2005) offers three alternatives for the design of structures for stability. The main body of the Specification, in Chapter C, prescribes two methods: the Effective Length Method in Section C2.2a and the First-Order Analysis Method in Section C2.2b. Appendix 7 presents the Direct Analysis Method. The Effective Length and First-Order Analysis Methods are of limited applicability; the Direct Analysis Method is applicable to all structures.

Of the three methods, the Effective Length Method will be most familiar to users of previous editions of the Specification and that is why it was placed in the main body of the current edition. The Direct Analysis Method (now in an Appendix) is, however, the most powerful and versatile of the available methods and, as noted, it is applicable to all structures, unlike the other approaches. There is little doubt that in time the Direct Analysis Method will become the “standard” method of design for stability.

In this appendix, the stability provisions of the 2005 AISC Specification are rewritten around the Direct Analysis Method. The material is presented in the language and format of the AISC Specification, including “User Notes” and the italicizing of terms listed in the glossary. The focus on a single method has offered the opportunity to expand some of the provisions beyond what is in the current Specification, both to improve clarity and to address issues that have arisen from use of the document. Where this involved substantive changes, they are explained in a second appendix.

What follows is not an approved AISC specification. In the author’s judgment, however, a design that conformed to the following “model” specification would also conform to the stability provisions of the 2005 AISC Specification (AISC, 2005). This reformulation represents the direction in which the AISC Specification appears to be evolving. The stability section (currently Chapter C) of the next edition is likely to resemble this model specification, except that it will almost certainly permit variations of today’s Effective Length and First Order Analysis Methods as alternate approaches, specified in appendices.

STABILITY ANALYSIS AND DESIGN

This specification addresses requirements for the analysis and design of structures for stability. It is organized as follows:

1. General Stability Requirements
2. Calculation of Required Strengths
3. Design of Components
1. GENERAL STABILITY REQUIREMENTS

Stability shall be provided for the structure as a whole and for each of its elements. The effects of all of the following on the stability of the structure and its elements shall be considered: (1) flexural, shear and axial member deformations, and all other component and connection deformations that contribute to displacements of the structure; (2) second-order effects (including P-Δ and P-δ effects) calculated at a level of loading corresponding to LRFD load combinations or 1.6 times ASD load combinations; (3) geometric imperfections; and (4) stiffness reductions due to inelasticity.

Any rational method of analysis and design that considers all of the listed effects is permitted. Calculation of required strengths in accordance with Section 2 and design of components in accordance with Section 3 is permitted for all structures.

2. CALCULATION OF REQUIRED STRENGTHS

The required strengths of components of the structure shall be determined from an analysis conforming to Section 2.1. The analysis shall include consideration of initial imperfections in accordance with Section 2.2 and adjustments to stiffness in accordance with Section 2.3.

2.1. Method of Analysis

The analysis of the structure shall conform to the following requirements:

(1) The analysis shall be an elastic second-order analysis that considers both P-Δ and P-δ effects, except as provided in (2), below.

**User Note:** The second-order analysis may consist of either a rigorous second-order analysis or a first-order analysis amplified to account for second-order effects.

(2) Methods of analysis that neglect the effects of P-δ on nodal displacements in the structure are permitted where the axial loads satisfy Equation 2-1 in all members whose flexural stiffnesses are considered to contribute to the stability of the structure.

**User Note:** Notwithstanding this exclusion in the analysis, P-δ effects must still be considered in the design of individual members.

\[
\alpha P_r < 0.15P_{el} \quad (2-1)
\]

where

- \( P_r \) = required axial compressive strength under LRFD or ASD load combinations, kips (N)
- \( P_{el} \) = Euler buckling load, \( \pi^2EI/L^2 \), evaluated in the plane of bending, kips (N)

and

\[ \alpha = 1.0 \text{ (LRFD)} \quad \alpha = 1.6 \text{ (ASD)} \]
(3) The analysis shall consider flexural, shear and axial member deformations, and all other component and connection deformations that contribute to displacements of the structure. The analysis shall use reduced stiffnesses for all components whose stiffnesses are considered to contribute to the stability of the structure, as specified in Section 2.3.

(4) For design by LRFD, the second-order analysis shall be carried out under LRFD load combinations. For design by ASD, the second-order analysis shall be carried out under 1.6 times the ASD load combinations, and the results shall be divided by 1.6 to obtain the required strengths of components.

2.2. Consideration of Initial Imperfections

The effect of initial imperfections on the stability of the structure shall be taken into account either by direct modeling of imperfections in the analysis as specified in Section 2.2a or by the application of notional loads as specified in Section 2.2b.

**User Note:** The imperfections considered in this section are imperfections in the locations of points of intersection of members in the unloaded structure. In typical building structures, the important imperfection of this type is the out-of-plumbness of columns. Initial out-ofstraightness of individual members is not addressed in this section; it is accounted for by other aspects of the analysis and design procedure and need not be considered explicitly as long as it is within customary limits.

2.2a. Direct Modeling of Imperfections

In all cases, it is permissible to account for the effect of initial imperfections by including the imperfections in the analysis. The structure shall be analyzed with points of intersection of members displaced from their nominal locations by the maximum amount considered in the design. The pattern of initial displacements shall be such that it provides the greatest destabilizing effect.

**User Note:** Initial displacements similar in configuration to both displacements due to loading and anticipated buckling modes should be considered in the modeling of imperfections.

In the analysis of structures to which Sections 2.2b and 2.2b(4) are applicable, subject to load combinations that would require no application of notional loads under Section 2.2b(4), it is permissible to neglect the effect of initial imperfections.

2.2b. Use of Notional Loads to Represent Imperfections

For building structures that support gravity loads primarily through nominally-vertical columns, walls or frames, it is permissible to use notional loads to represent the effect of initial imperfections in accordance with the requirements of this section.
**User Note:** The notional load concept is applicable to all types of structures, but the specific requirements in 2.2b(1) through 2.2b(4) are applicable only for the particular class of structure identified above.

1. *Notional loads* shall be applied as *lateral loads* at all levels, independently in two orthogonal directions. The *notional loads* shall be additive to other *lateral loads* and shall be applied in all *load combinations*, except as indicated in (4), below. The magnitude of the *notional loads* shall be:

   \[ N_i = 0.002 Y_i \]  

   where
   \( N_i = \) notional load applied at level \( i \), kips (N)
   \( Y_i = \) gravity load from the *LRFD load combination* or 1.6 times the *ASD load combination* applied at level \( i \), kips (N)

   **User Note:** The notional loads do not cause a net horizontal reaction on the foundation but may, in some cases, cause horizontal reactions on individual foundation components. A horizontal force of \( \Sigma N_i \), opposite in direction to the notional loads, may be applied in the analysis at the bases of all columns to yield the correct reactions at the foundation.

2. The *notional load* at any level, \( N_i \), shall be distributed over the level in the same manner as the *gravity load* at that level. The *notional loads* shall be applied independently in opposite directions.

3. The *notional load* coefficient of 0.002 in Equation 2-2 is based on a nominal initial story out-of-plumbness ratio of 1/500. Where the use of a different maximum out-of-plumbness is justified, it is permissible to adjust the notional load coefficient proportionally.

4. For frames in which the ratio of maximum second-order drift to maximum first-order drift (both determined for *LRFD load combinations* or 1.6 times *ASD load combinations*) in all stories is equal to or less than 1.7, it is permissible to apply the notional load, \( N_i \), as a minimum lateral load (such that the total lateral load in any load combination at any level is not less than \( N_i \)) and not in combination with other lateral loads. The specified drift ratio threshold of 1.7 is based on analyses using stiffnesses adjusted as indicated in Section 2.3. If the drift ratio is determined from analyses using nominal, unreduced stiffnesses, the drift ratio threshold for applying the notional loads as minimum lateral loads shall be taken as 1.5.

### 2.3. Adjustments to Stiffness

The analysis of the structure to determine the *required strengths* of components shall use reduced stiffnesses, as follows:
(1) A factor of 0.8 shall be applied to all axial, shear and flexural stiffnesses that are considered to contribute to the stability of the structure. It is permissible to apply this reduction factor to all stiffnesses in the structure.

User Note: Applying the stiffness reduction to some members and not others can, in some cases, result in artificial distortion of the structure under load. This can be avoided by applying the reduction to all members, including those that do not contribute to the stability of the structure.

(2) An additional factor, $\tau_b$, shall be applied to the flexural stiffnesses of all members whose flexural stiffnesses are considered to contribute to the stability of the structure, where:

$$
\tau_b = \begin{cases} 
1.0 & \text{for } \alpha P_r/P_y \leq 0.5 \\
4[\alpha P_r/P_y (1-\alpha P_r/P_y)] & \text{for } \alpha P_r/P_y > 0.5 
\end{cases}
$$

$P_r = \text{required axial compressive strength under LRFD or ASD load combinations}$, kips (N)

$P_y = \text{axial yield strength}$, kips (N)

and

$$
\alpha = 1.0 \text{ (LRFD)} \quad \alpha = 1.6 \text{ (ASD)}
$$

(3) In structures to which Section 2.2b is applicable, in lieu of using $\tau_b < 1.0$ where $\alpha P_r/P_y > 0.5$, it is permissible to use $\tau_b = 1.0$ for all members if a notional load of $0.001Y_i$ (as defined in Section 2.2b(1)) is applied at all levels, independently in two orthogonal directions, in all load combinations. These notional loads shall be added to those, if any, used to account for imperfections and shall not be subject to the limits of Section 2.2b(4).

3. DESIGN OF COMPONENTS

When required strengths have been determined in accordance with Section 2, members and connections shall be designed to satisfy the provisions of Chapters D, E, F, G, H, I and J, as applicable, of the 2005 AISC Specification for Structural Steel Buildings, with no further consideration of overall structure stability. The effective length factor, $K$, of all members shall be taken as unity unless a smaller value can be justified by rational analysis.

Bracing intended to define the unbraced lengths of members shall have sufficient stiffness and strength to control member movement at the braced points. Methods of satisfying this requirement are provided in Appendix 6, Stability Bracing for Columns and Beams, of the 2005 AISC Specification.

User Note: The requirements of Appendix 6 of the 2005 AISC Specification are not applicable to bracing that is included in the analysis of the overall structure as part of the overall load-resisting system.
APPENDIX
EXPLANATION OF CHANGES

The model specification for stability analysis and design presented in the previous appendix is based on Chapter C and Appendix 7 of the 2005 AISC Specification for Structural Steel Buildings (AISC, 2005). Where substantive technical changes have been made in AISC Specification provisions, they are explained in this appendix. The changes are conservative in that a design that conforms to the proposed model specification would also conform to the 2005 AISC Specification.

Type of Structure. Some of the provisions of the 2005 AISC Specification, specifically those related to the use of notional loads, are applicable only to conventional building structures that support gravity loads primarily through nominally-vertical columns, walls or frames. They are not applicable, for instance, to the analysis of laterally-un-supported compression chords of long-span trusses. This limitation is not noted in the Specification, except to the extent that the entire Specification is described in the Scope section of Chapter A as being intended for buildings and building-like structures, which would, typically, fall within the constraint.

The model specification is based on the very versatile Direct Analysis Method and is intended to be applicable to a broader range of structures than just conventional building frames. Therefore, those provisions that can only be used for typical building structures (Sections 2.2b and 2.3(3)) are clearly identified and alternatives usable with all structures are provided. [The notional load concept is broadly applicable, but the specific provisions in Sections 2.2b and 2.3(3) are intended only for the limited class of building structures.]

Load Level for Calculation of Second-Order Effects. The requirement that second-order effects be considered at a level of load corresponding to LRFD Load Combinations or 1.6 times ASD Load Combinations is set forth in the general requirements section of the present work. The 2005 AISC Specification has this requirement only in the sections on specific methods, which could be taken to imply, incorrectly, that it applied only to those methods and was not a general requirement for all designs.

Application of Notional Loads. Requirements regarding the distribution and direction of notional loads are specified in greater detail in the present work (Section 2.2b(2)). These requirements may have been implicit in the 2005 AISC Specification; they are now spelled out.

Drift Ratio Threshold for Applying Notional Loads as Minimum Lateral Loads. In the 2005 AISC Specification, the drift ratio (ratio of second-order drift to first-order drift) of 1.5 below which notional loads are applied as minimum lateral loads (rather than as additive loads) is based on analyses with nominal stiffnesses. In the model specification, the user is offered the option of determining the drift ratio from analyses with either nominal or reduced stiffnesses (reduced per Section 2.3). The threshold value for applying notional loads as minimum lateral loads is 1.5 in the former case, 1.7 in the latter case.
Interpretation of Minimum Lateral Load. Another change involves clarification of the conditions under which the minimum lateral load needs to be applied. The wording of the 2005 AISC Specification could be interpreted as requiring the application of notional loads only in “gravity-only” load combinations and not in combinations that include wind or other applied lateral loads, even if the applied lateral loads are less than the specified minimum. The present work makes it clear that every load combination must include lateral load at least equal to the minimum (see Section 2.2b(4)).

Adjustments to Stiffness. The 2005 AISC Specification requires analysis with reduced axial and flexural stiffnesses of members whose stiffnesses are considered to contribute to the lateral stability of the structure. It offers no explicit guidance, however, about member shear stiffnesses, diaphragm stiffnesses, column base rotational stiffnesses, etc. The present work takes the more conservative approach of applying the basic 0.8 reduction to all stiffnesses that contribute to the stability of the structure (see Section 2.3(1)).