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Report on

COMPARATIVE STUDY OF FLEXURAL CAPACITY OF PIPES

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by

Donald R. Sherman and Anand S. Tanavde
Department of Civil Engineering
University of Wisconsin--Milwaukee

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EXECUTIVE SUMMARY

This report compares the data on the flexural capacity of tubes and pipes that have been obtained in six programs conducted since 1964 in North America. In all but three of the fifty-two tests, the diameter to thickness ratio was less than one hundred. The data has been restricted to unstiffened specimens loaded in a manner to produce a region of constant moment.

Three basic types of comparisons are included. The first separates the data according to yield strength, test program, type of pipe and size. The conclusions of this analysis are that lower strength pipe have higher normalized capacities than high strength pipe and that hot formed pipe have slightly higher capacities than either Electric Resistance Welded (ERW) or fabricated pipe. Size does appear to be a major factor.

In the second comparison the correlation between the flexural data and six different predictor equations was studied. The forms of the equations and the associated slenderness parameters (e.g. D/t, $F_y D/t$, etc.) were adopted from equations used for critical stresses under axial loads. Correlation coefficients did not vary substantially among the equations. Although a quadratic equation with a complex slenderness parameter gave the best correlation, the most satisfactory equation of a simpler form is a linear expression with $(E/F_y)/(D/t)$ as the slenderness parameter.

The final comparison is with seven current design standards. All of these standards yield conservative predictions of strength. Allowable stress standards are very conservative in predicting the strength of pipes with low D/t but are quite reasonable for larger values. Newer Load and Resistance Factor Design (LRFD) specifications are not as conservative for low D/t values.

The report concludes by recommending the following expression for predicting the flexural capacity of pipe with a 95% confidence level that the real strength will exceed the predicted strength

$$M_R = 0.90 M_u$$

where $$M_u/M_p = (0.775 + 0.016\alpha)$$

and $$\alpha = (E/F_y)/(D/t)$$

This expression implies that for E = 29000 ksi, the full plastic moment can be achieved when

$$D/t \leq 2060/F_y$$

and that the buckling will be elastic when

$$D/t \geq 8970/F_y$$

INTRODUCTION

Since the 1960's, a number of experimental projects have been undertaken to investigate the bending capacity of circular tubes with diameter to thickness ratios of 100 and less. Tubes with these proportions are typical of those that would be used as structural members in offshore construction. In order to develop design standards that reflect the true behavior of tubes rather than wide flange shapes, it was necessary to determine the moment capacity as a function of wall slenderness and to establish the slenderness limits at which yield or fully plastic moments could be achieved. The wall-slenderness parameters that have historically been used are the diameter to thickness ratio (D/t) or a modification that includes the yield strength, $F_y D/t$.

The most recent project (23) involved the testing of pipe fabricated in a series of short cans formed from structural plates as is typical in offshore construction. This project was undertaken because all of the previous tests involved hot formed seamless or ERW manufactured pipes usually in diameters of 12 inches or less. Not only were there significant inconsistencies in the previous data, but there was also an uncertainty as to whether the results could be applied to fabricated pipe with its weld seams and different imperfection and material characteristics.

The primary purposes of this report are to compare and analyze the data obtained in the various experimental programs. The analysis consists of identifying the parameters that significantly affect the moment capacity and to determine the appropriate wall-slenderness parameter that best expresses the strength. The test results are also compared with existing North American design specifications (e.g. API, AISI, AISC, etc.) in order to determine the degree of conservatism in current practice.

In the United States, present provisions for bending capacities of circular tubes are based primarily on the data obtained in two studies. In 1965 [20] Schilling published the results of tests on manufactured (hot-formed seamless) tubes, establishing the limit below which local buckling would not occur for tubes in bending as $(F_y/E)(R/t)$ of 0.06. Based on his study the AISI code [3] recommended a D/t limit of $3300/F_y$ as a compact section criterion. This limit subsequently carried over into other AISI [21] and AISC [1] codes and was in the early versions of API RP2A [5]. In 1974, Sherman [22] tested hot formed seamless (HFS) and electric resistance welded tubes. Based on these tests, the AISI Criteria [21] recommended a much more restrictive limit of $1300/F_y$ for moment redistribution. Since Schilling's report, two other investigations in the U.S. [7,8] and one Japanese project [13] were oriented toward buckling of pipelines during laying. Some of the data in these studies can be compared to tests of structural members. CIDECT-sponsored research in Canada [14-16], oriented toward plastic design, has also provided data on the bending of pipe. Another recent Canadian project [24] included the bending of two thin pipes with D/t ratios well beyond the range that historically has been used for structural members but still not as great as that encountered in storage tank or aircraft construction. As mentioned previously, the most recent project studying bending capacity of fabricated tubes was jointly sponsored at the University of Wisconsin--Milwaukee by fourteen organizations [23].

The test results included in this report have been restricted to those which can be directly compared to the fabricated pipe bending project [22]. Therefore, the results are limited to the following:

1. Recent experimental investigations dealing only with flexural-buckling behavior of unstiffened tubes subjected to a constant moment region.
2. D/t ratios less than 400. However, most of the tests concern D/t less than 100.
3. Test results in which the strength and material yield data is clearly reported or can be easily deduced from the information in the report.

DESCRIPTION OF TEST PROGRAMS

The tables in Appendix A list the results of the six test programs included in this report. The following is a brief description of each program, identifying various important features such as the type of pipe and the method of testing. Important conclusions contained in the reports are also summarized.

Tests by Schilling (1964)

Schilling's landmark paper [20], published in 1965, summarized the available information on the buckling of circular steel tubes in axial compression, flexure and torsion. Sizes and loading conditions were limited to tubes that could be used as structural members and various design methods and formulas were compared. A brief review of the shape of the stress-strain curves and magnitudes of imperfections that can be expected in tubes produced by different methods was also presented.

The paper graphically presented the results of ten flexural tests that had been conducted by Schilling and his co-workers the previous year. These apparently were the only flexural tests available at the time. Little description of the tests was provided in the paper. However, an in-house report [12] provided the following details.

The test specimens were 4-1/2 inch diameter hot formed seamless pipe. The specimens were cut from two pieces of longer pipe with yield strengths equal to 39 and 52 psi. Both had sharp yielding stress-strain curves. A series of diameter/thickness ratios were obtained by machining a six inch length of the pipe to a reduced thickness. A two point loading system on a simple span was used so that the constant moment length was one foot in length. Consequently, the test data was based on a relatively short unstiffened length which was part of the constant moment length. The report indicates that variation in the wall thicknesses were as much as 25% in some specimens.

The published paper contains a plot of (M_{ult}/M_p) vs. $(F_y/E)(R/t)$ and recommends that for a value of 0.06 for the slenderness parameter, it is reasonable to assume that cylindrical tubes of yield point steels will reach the full plastic moment without premature local buckling. It was also observed that the test points indicated a tendency to group according to yield strength. Schilling therefore concludes that there may be a more appropriate slenderness parameter using F_y to other than the first power.

Tests by Jirsa et al. (1972)

In this investigation [7] the influence of ovaling on the flexural behavior of pipelines stressed beyond the elastic limit was considered. The testing program consisted of four tests on bare pipes and on two coated pipes. The report does not state how the pipe was produced and the investigators can no longer recall. Therefore, for this report it has been that the pipe was hot formed seamless.

Each pipe was tested as a simple beam and loads were applied to produce a region of pure flexure. Both the loading frame and the supports were assembled to enable the frame to rotate freely as deflections and rotations of the pipe increased. Since both the loading and supporting rods were pinned at each end, there was no restraint to lateral movement at the supports or the load points and the introduction of axial forces in the pipe was avoided. In order to hold the pipe in place and to distribute load over the surface of the pipe, a high strength gypsum mortar was cast between the pipes and the support and loading frames.

Each specimen was instrumented with strain gages at a number of locations (over the constant moment span) at both the top and the bottom surfaces. These were used to determine curvatures. Deflections of the pipe in the pure flexure span were also measured to calculate curvatures. To determine the ovalization of the pipe, changes in horizontal and vertical diameters were measured at a number of locations with large micrometers. Load was applied in small increments and after each load increment, diameter change, strain-gage, and deflection readings were taken.

It was concluded that ovaling did not significantly reduce the moment capacity of the pipes until strains well into the plastic region were reached. The data given in Table A2 of the Appendix was determined from stress strain and moment-curvature plots.

Tests by Korol (1974)

Korol's report [17] deals with the experimental investigation of the strength of circular tubular beams in bending. Tubes having D/t ratios from

28 to 80 were used with the higher D/t having larger diameters. The specimens were cold formed ERW pipes without the benefit of stress relieving.

A symmetrical two point loading was applied on a simple span with the help of a centrally located hydraulic jack and spreader beam. Inverted V-shaped blocks were used to distribute the load to the tube at the support and load points. In some cases a circular curved plate, 6" in length and conforming to the outer profile of the section was placed between the block and the specimen (ref. 16, Table 3-1), in order to further cushion the effects of concentrated loads and insure that the local buckle occurred near the center of the constant moment length. There was, however, one test in which the local buckle did develop at the load point. Although the resulting low capacity is reported in the Appendix, it has not been included in the comparisons in this report, because, as Korol states, it is a premature failure condition resulting from the nature of loading.

Electric resistance strain gages were placed at midspan, mounted on the top and bottom fibers of the test specimen. At load points strain gages were located only on the underside of the section.

It was observed that the load distribution at the support plate had an effect on the post buckling behavior of the tube and that geometrical imperfections reduced the moment capacity of the section as the slenderness ratio increased. Reasonable agreement was found between the strength results of tests and the theoretical prediction [16,17].

Tests by Sherman (1976)

Sherman's paper [22] deals with the flexural behavior up to and beyond the ultimate moment of tubes with proportions often encountered in bracing members. It presents the moment redistribution capabilities of round tubes and applicability of plastic design principles to tubes subjected to flexure. Six specimens with a 10.75 inch O.D. and D/t ranging from 18 to 102 were tested as simple beams subjected to constant bending moment. Hot formed seamless pipes were used for the two heaviest specimens and ERW for the remainder.

Loads were applied through stiffening plates, which spread the concentrated loads and stiffened the section sufficiently to prevent premature local failure even in the thinnest tube. The applied loads were measured with standard strain gage load cells. Curvatures were deduced from the measurements of top and bottom surface strains with electric resistance strain gages. To determine the degree of ovalization, changes in horizontal and vertical diameters were measured with a micrometer at the same locations as the curvature measurements. Maximum deflections were measured with a 0.001 inch dial indicator. Data readings were taken at load intervals of about 0.1 times the expected ultimate load. Tests were terminated when large deflections of the beam mechanism caused significant side forces on loading jacks.

It was observed that the plastic moment capacity was not quite attained at D/t of $3300/F_y$ as would be implied from Schilling's conclusions. Restrained beams with this D/t exceeded the full plastic capacity (including effects of moment redistribution) but this may have been due to the development of axial tensions at the large deflections corresponding to the ultimate load. From the simple span tests, it was concluded that a D/t limit of $1540/F_y$ would be required to develop a full plastic moment.

Tests by Stephens et al. (1982)

This report [24] deals in part with the local elastic buckling strength of thin-walled unstiffened cylinders subjected to flexural loads. The type of members tested are used extensively in the materials handling operations of industrial plants. The two fabricated circular steel cylinders used in the flexural tests had 60 inch diameters and D/t ratios of 300 and 450.

The specimens were fabricated with two relatively thick end sections that acted as stiffeners to the central test section. All welds, both circumferential and longitudinal, were full penetration groove welded butt joints. The cylinder was attached to end frames which provided an additional length for the simply supported beam. One end of the system was free to translate horizontally so as not to induce axial restraint. The two symmetric loads were applied to the thicker end sections of the cylinder so that the entire test section was in a region of constant moment.

Longitudinal strains were measured with electric resistance gages mounted on the outer top and bottom surfaces of the central test section. Rotation meters were attached at the neutral axis of the tube to permit indirect measurement of the curvature during the loading. Transducers were placed inside the central test section to measure flattening of the cross-section and dial gages were positioned at the level of the neutral axis to measure the vertical deflection at various locations.

The flexural test results along with the results of axial tests are reported as critical elastic stresses due to the large D/t ratio. Failures in the flexural specimens were in the form of a diamond shaped buckle pattern in the extreme compression region of the central test sections adjacent to the circumferential groove welds joining them to the end sections. Pretest imperfection measurements indicated that the presence of the circumferential weld had introduced a greater degree of imperfection in the cylinder.

Comparisons of critical stresses were made with theoretical predictions and with previously obtained test results for axially loaded cylinders. Alternate ultimate strength equations suitable for design purposes were also developed.

Tests on Fabricated Pipes by Sherman (1982)

This extensive program [23] consisted of sixteen specimens with D/t ratios ranging from 18 to 96 and was subsequently supplemented with five additional tests. The specimens were fabricated in accordance with offshore design standards and practice. The objectives of this investigation were to study the strength and behavior of members that reach the full plastic moment or fail by inelastic local buckling prior to achieving the plastic moment. In addition to D/t, the other major variables were two nominal yield strengths and two diameters.

The test specimens consisted of two cans welded together with full penetration girth welds so that the longitudinal welds were staggered by at least 90 degrees. These cans were welded to two heavier end pieces to obtain the full length of the simply supported beam. Two symmetric loads were applied on the center cans at a spacing of four pipe diameters. Therefore, the entire constant moment region was in the test cans and contained a girth weld. The load system was carefully designed not to develop secondary axial forces even at large deflections. Loads were applied through rotationally free plates that acted as stiffeners and prevented local buckling from occurring at the point of application.

Pretest measurements included out-of-roundness measurements by Kendrick's method [11] and straightness profiles. Longitudinal and circumferential strain gages were on the top and bottom surfaces of the two test cans. Five deflection measurements were made in the constant moment region to permit the calculations of curvatures and total rotations even after a local buckle formed. Changes in vertical diameters were also measured.

Size and out-of-roundness did not appear to be significant factors affecting the strength of the members. It was noted that the presence of the girth weld tended to influence the moment capacity and that the absence of good strain hardening characteristics in the pipe material could inhibit the ability of pipe with small D/t ratios to achieve the plastic moment.

BASIS FOR COMPARISONS

In evaluating and comparing the strength results from the various test programs, it is necessary to deal with them in nondimensional form. Therefore, the ultimate experimental moments are normalized by the theoretical plastic moment calculated from the true diameter, thickness and yield strength of the test piece. Ideally, the yield strength should be obtained from a coupon removed from the specimen and tested at a strain rate that corresponds to that used to obtain the reported ultimate moment. The surest way to do this is to use static values [19], which are those observed when the coupon or specimen has been held at a constant deformation for several minutes. Dynamic yield stresses (those obtained at a constant strain rate typically used in ASTM A307 tests) are about 4 to 10 percent higher. Therefore, strain rate can affect both the ultimate test moment and the yield value used to calculate the plastic moment. In the reports presented in this study, it is not always clear whether static or dynamic values have been reported. It is assumed, however, that the strain rates for the reported ultimate moment, M_{ult} and the computed plastic moment, M_p, correspond.

The relatively large number of variables and few test results precludes the use of sophisticated statistical theory to determine whether a particular parameter has a significant effect. Therefore, a simpler basis for comparison has been used in this report. A regression curve (second degree polynomial) is fitted to the data in the plot of (M_{ult}/M_p) vs. (D/t). This curve is shown in Fig. 1 by a dotted line. The equation for the curve is as shown below:

$$(M_{ult}/M_p) = 1.074 - 9.84 \times 10^{-4}(D/t) - 9.15 \times 10^{-6}(D/t)^2 \quad (1)$$

A similar curve is fitted through the data points in the plot of (M_{ult}/M_p) vs. (F_yD/t) as shown by a dotted line in Fig. 2. The equation of this curve is as follows:

$$(M_{ult}/M_p) = 1.089 - 2.14 \times 10^{-5}(\lambda) - 4.42 \times 10^{-9}(\lambda^2) \quad (2)$$

$$\lambda = (F_yD/t)$$

The percent deviation of each test point from one of the regression curves is calculated. The data is then grouped according to various parameters so that the maximum, minimum and mean deviations can be obtained to provide a quantitative indication of where a particular group of data lies.

The regression curves have been developed solely as a basis for comparison and their form is somewhat arbitrary. The slenderness parameters of D/t and F_yD/t were chosen because they are the ones typically used in the reports and specifications dealing with circular tubes. Second degree polynomials are used because curves with that shape generally fit the trend of the data and third order polynomials only increased the correlation coefficient from 0.636 to 0.641. A further explanation of the statistical terminology is given in Appendix B.

A second part of the data analysis concerns an investigation of various slenderness parameters and equations used to predict the moment capacity. A number of different suggestions appear in the literature for the related topic of critical stress for local buckling under axial load. The numerical constants in these equations have been adjusted to provide a reasonable fit to the bending data and comparisons have been made on the basis of correlation coefficients and deviations.

The final comparison is with existing North American specifications. In this case deviations provide a better basis for comparison than correlation coefficients because these specifications tend to be based on lower bounds rather than best fits. Direct comparisons can be made with ultimate strength specifications but allowable stress provisions must be increased by the inherent safety factor. Since this value is not specifically stated in the specifications, a value of 1.67, as is used for members in the AISC and AISI specifications, has been assummed.

COMPARISONS OF DATA

Identification of Key Parameters

The parameters that were investigated to determine if they produced significant differences in deviations from the regression curve are:

1. Minimum Specified Yield Strength. Two groups are considered; one has specified yield strengths less than 50 ksi and the other has specified yield strength equal to 50

ksi. No tests involved material with a specified yield strength greater than 50 ksi, although actual yield strengths did exceed this value.

2. Test Program. The data is further grouped according to the five test programs to determine if there is any indication that the method of testing used by the various investigators biased the results. The two tests on pipe with D/t of 350 and 400 are not included in this part of the study since they represent elastic buckling and are well outside the range of the slenderness parameters of the other tests.
3. Type of Pipe. The results are divided according to whether the specimens were hot-formed seamless, electric resistance welded or fabricated pipes. This division reflects differences in residual stresses and the amount of cold working.
4. Size. There has been a question as to whether there is a scale effect that prevents the direct application of tests of small diameter tubes to the larger sizes used in offshore construction. Therefore, the results have been grouped by diameters less than 10 inches, between 10 and 13 inches and greater than 13 inches.

The results of the comparisons are presented in a series of bar graphs with the mean and range of the percent deviation from one of the regression curves plotted as ordinates.

Figures 3 and 4 are the bar graphs for separating specified yield strengths. The former graph is based on regression equation 1 that uses the slenderness parameter D/t while the latter is based on equation 2 with $F_y D/t$

as the parameter. The bars separate the two yield strength groups for the total data base and for the five individual studies. In Figure 3, it is evident that the high strength grouping always has a lower mean deviation than the low strength group. From this it can be concluded that the yield strength is an important parameter affecting the normalized moment capacity that should be included in the slenderness parameter.

In Figure 4 where the regression curve is based on $F_y D/t$, the overall range in the deviation is not significantly reduced, but the differences between the means of the high strength and lower strength materials is reduced in the total and in the individual studies. Since this parameter is better than D/t in reducing the influence of the yield stress parameter, the remaining bar graphs are based on deviations from equation 2 only.

The separation on the basis of the type of pipe is shown in Figure 5. It would appear from the total data base that hot-formed pipe tends to have higher normalized capacities than ERW or fabricated pipe which have essentially the same mean deviation.

The final grouping shown in Figure 6 is on the basis of size. There is a tendency for the small diameter specimens to yield higher test data but size does not appear to be a factor in pipe larger than 10 inches in diameter.

It should be reemphasized that due to interrelations of parameters and lack of a large data base, these conclusions cannot be based on a more firm statistical evaluation. Although there is considerable scatter in the data, with deviations from -14% to +21%, mean deviations in the various programs range from about -4% to +6%. Probably the most significant conclusions pertinent to offshore structures are:

1. Yield strength is a parameter that influences the normalized bending capacity of pipes.

2. The test data on hot formed pipe produces higher normalized capacities than fabricated pipe.

3. The mean strength levels of fabricated pipes do not vary significantly from tests of smaller ERW pipes. However, there is a tendency for greater scatter in the fabricated pipe data.

Ultimate Moment Predictions

The topics of local buckling due to flexure and due to axial loads are closely related. Many allowable stress specifications use the same critical stress in thin cylinders for both situations. While this may be true for elastic buckling, the situation is different for inelastic buckling. Under axial load, the maximum capacity of a section occurs when it is fully yielded. Therefore, inelastic local buckling will occur at strains near the yield strain. In flexure, on the other hand, the maximum capacity is the plastic moment which requires that strains several times the yield strain must develop. Therefore, the wall slenderness limits for achieving the full capacity in flexure are more severe than that for axial loads and as a consequence, the relation for predicting the capacity near the ultimate will also differ.

Historically, tubes loaded in axial compression have been under investigation for a longer period than tubes in flexure. All of the variables that affect flexure also influence axial capacity and the axial test data also show considerable scatter. As a result, a number of expressions have been proposed for predicting the axial capacity. In some cases, the wall slenderness parameter takes different forms. In this section of the report, several different expressions for predicting flexural capacity will be examined. Some of them will have the general form of equations for axial capacity but with modification to account for the different limits for achieving the full flexural capacity. A comparison is based on mean deviations of the data and on statistical correlation coefficients. However, since it was shown earlier that the data for hot formed pipe is higher than fabricated or ERW pipe date, it has been excluded in the comparisons of Figures 7-11.

In addition to the slenderness parameters of D/t and $F_y D/t$ which are used in the plots of Figures 1 and 2, one parameter frequently encountered for axial loads is α equal to $(E/F_y)/(D/t)$. This is essentially the inverse of $F_y D/t$ but as can be seen in Figure 7, the elastic buckling region is included. In Figure 8, D/t is the slenderness parameter (Figure 8 is similar to Figure 1). Another slenderness parameter is similar to the one used for sections made with plate elements, $\beta = (D/t)\sqrt{F_y/E}$, and the data is plotted in Figure 9 on this basis. A more complex parameter $\alpha(F_y/E)^{2/3}$ provided the best correlation for recent tests of axially loaded fabricated cylinders (19), and the bending data is plotted against this parameter in Figure 10. Finally, a good fit can be obtained if the moment capacities are normalized on the yield moment and D/t is used as the slenderness parameter. This plot is shown in Figure 11.

The objective of the following analysis is to determine the best inelastic buckling expression for the flexural data. In order to establish limits for the inelastic equations, it is assumed that the predicted capacity cannot exceed the plastic moment nor the elastic buckling moment given by $.33SE/(D/t)$ where S is the elastic section modulus. The critical elastic stress of $.33E/(D/t)$, suggested by Plantema for axial loads on manufactured tubes, is only slightly conservative when compared to the two flexural tests on fabricated tubes with high D/t. Its limit of applicability has been extended from Plantema's suggestion to the points where it intersects the various inelastic equations. The inelastic expressions that have been considered are the following:

A. Plantema's linear expression in terms of α

$$M_u/M_p = A + B\alpha \tag{3}$$

B. The American Water Works Association's quadratic equation in terms of α that includes both the inelastic and elastic regions

$$M_u/M_p = A\alpha + B\alpha^2 \tag{4}$$

C. The API equation that is a function of D/t

$$M_u/M_p = A - B(D/t)^N \tag{5a}$$

which can be converted to

$$M_u/M_p = A - C(F_y\alpha)^N \tag{5b}$$

D. Ostapenko's [19] more complex equation in terms of α and F_y

$$M_u/M_p = A(F_y/E)^{2/3}\alpha - B(F_y/E)^{4/3}\alpha^2 + C(F_y/E)^2\alpha^3 \tag{6}$$

E. A linear expression for the parameter involving F_y

$$M_u/M_p = A + B \quad F_y/E \quad D/t \tag{7}$$

F. For the data expressed in terms of yield moment and D/t, a form similar to API,

$$M_u/M_y = A - B(D/t)^N \tag{8}$$

With E taken equal to 29,000 ksi, the coefficients for each equation were determined by a regression analysis considering the data base excluding the hot formed seamless pipe. The resulting constants, correlation coefficients and deviations are given in Table 1.

Current Design Specifications

The specific inclusion of pipes as structural members in design standards is a relatively recent occurrence. The flexural requirements are based on a mix of allowable stresses established for wide flange shapes and critical buckling stresses for cylinders loaded in axial compression. Some modifications have been made as flexural test data became available. It is now possible to compare existing standards with a substantial body of test data in order to determine the state of current practice.

The standards considered in this report are summarized in the following. The formulas that are listed for each standard are taken from the specification but have been converted to give moments instead of stresses. For ultimate strength specifications, these moments are predicted capacities of the sections and can be compared directly with the data. The moments in allowable stress standards are design moments and contain an implied safety factor. A safety factor of 1.67 has been assumed and has been included in the equations plotted for comparison with the data. Since the various standards are inconsistent in the wall slenderness parameter, it is difficult to compare them on a single plot. Therefore, they are compared with the data on the most appropriate plot.

1) API RP2A. This is an allowable stress specification for fixed offshore platforms that uses D/t as the slenderness ratio.

$M_D = .66F_{xc}S$

$F_{xc} = F_y$ for $D/t < 60$

$F_{xc} = (1.64 - .23(D/t)^{.25})F_y$ for $60 < D/t < 300$ but not to exceed $.6E/(D/t)$.

See Figure 12 for $1.67M_D$.

2) AISC. This familiar allowable stress specification is intended for building construction. It uses F_yD/t as the slenderness parameter.

$M_D = .66F_yS$ for $F_yD/t < 3300$

$M_D = (\frac{662}{F_yD/t} + .40)F_yS$ for $3300 < F_yD/t < 13000$

See Figure 14 for $1.67M_D$.

3) AISI Cold Formed. This allowable stress specification was written for thin walled sections where plastic conditions are seldom achieved. Its provisions are similar to AISC except that no allowance is made for a shape factor.

$M_D = .60F_yS$ for $F_yD/t < 3300$

$M_D = (\frac{662}{F_yD/t} + .399)F_yS$ for $3300 < F_yD/t < 13000$

See Figure 14 for $1.67M_D$.

4) AISI Tentative Criteria. This criteria was written specifically to take full advantage of the properties of tubes and the research available in the early 1970s. It is a modification of AISC provisions.

$M_D = .72F_yS$ for $F_yD/t < 3300$

$M_D = (\frac{662}{F_yD/t} + .40)F_yS$ for $3300 < F_yD/t < 13000$

See Figure 14 for $1.67M_D$.

5) API LRFD. This is an ultimate strength design specification for fixed offshore platforms. Its flexure criteria is somewhat a mix of RP2A and AISC provisions. Both D/t and F_yD/t slenderness parameters are included.

$M_R = \phi M_u$, $\phi = .92$

$M_u = F_{xc}Z$ for $F_yD/t < 1740$

$M_u = (\frac{848}{F_yD/t} + .51)F_{xc}Z$ for $1740 < F_yD/t < 3300$

$M_u = F_{xc}S$ for $3300 < F_yD/t$

where F_{xc} = smallest of
F_y
$0.6ED/t$
$F_y[1.64 - .23(D/t)^{.25}]$

For the range F_y between 36 and 50 considered in this report and the limit of D/t less than 360 implied in the standard for these provisions, some simplification can be made.

$M_u = F_y Z = M_p$ for F_y D/t < 1740

$M_u = F_{xc} Z$ for 1740 < F_yD/t < 3300

$M_u = F_{xc} S$ for 3300 < F_yD/t

where $F_{xc} = F_y$ for D/t < 60

$F_{xc} = F_y[1.64 - .23(D/t)^{.25}]$ for D/t > 60

The value of M_u is plotted in Figure 13 for the yield strengths of 36 and 50 ksi and M_R is included in Figure 14.

6) AISC LRFD. This ultimate strength standard is applicable to building design and uses F_yD/t as the slenderness parameter.

$M_R = \phi M_u$, $\phi = .90$

$M_u = F_y Z = M_p$ for F_yD/t < 1300

$M_u = M_p - (M_p - M_r)(\frac{F_y D/t - 1300}{2000})$ for 1300 < F_yD/t < 3300

$M_r = F_y S$

$M_u = (\frac{1100}{F_y D/t} + \frac{2}{3}) F_y S$ for 3300 < F_yD/t < 13000

See Figure 14 for M_R.

7) Canadian LSD. This ultimate strength standard has been in use since 1976. It uses F_yD/t as a slenderness parameter

but makes no provision for thin tubes. Limits in the specifications are for SI units but are converted to ksi in this report.

$M_R = \phi M_u$, $\phi = .90$

$M_u = ZF_y = M_p$ for $F_y D/t < 2610$

$M_u = SF_y$ for $2610 < F_y D/t < 3335$

See Figure 14 for M_R.

In general this review of seven specifications indicates that current North American practice in the design of tubular beams is conservative with respect to the test data. The large degree of conservatism found for tubes with low D/t in most allowable stress design specifications is probably due to applying the stress limits developed for wide flange sections without adjusting for the higher shape factors in pipes. For tubes with high D/t, where some local buckling data is available, the provisions are not as conservative when a 1.67 factor of safety is assumed. The ultimate strength criteria inherently includes the shape factor. Therefore they are less conservative for tubes with low D/t when compared to the test data and the allowable stress design specifications.

RESULTS AND CONCLUSIONS

Conclusions related to the design of flexural members in offshore structures are:

1. Since data from tests on hot formed pipe tend to give higher results than tests of ERW or fabricated pipe, it is conservative to exclude the former from the data base used to establish design criteria.

2. Seven North American specifications present flexural criteria in a variety of forms. They are, however, generally conservative with respect to the test data. LRFD specifications are less conservative than allowable stress design specifications (with an assumed safety factor of 1.67) for pipe with small D/t that reach M_p.
3. As indicated in Figure 14, major differences occur in the specifications at D/t ratios less than $3300/F_y$. The early suggestion that this limit was satisfactory for achieving a plastic moment was optimistically based on the results of hot formed pipe tests.
4. Although six different expressions for predicting the ultimate moment were evaluated, correlation coefficients among the six did not vary significantly. However, it would appear that the slenderness parameter should involve the yield strength of the material.

RECOMMENDATIONS

Based on the results and conclusions of this study, it is recommended that a relatively simple equation should be used for predicting the ultimate flexural capacity. For the inelastic range, the recommendation is equation 9 of Table 1

$$\frac{M_u}{M_p} = .775 + .016(E/F_y)/(D/t) \leq 1 \qquad (9)$$

or with E = 29000 ksi

$$M_u = (.775 + \frac{464}{F_y(D/t)})\, M_p \leq M_p$$

with this equation, the full value of M_p is the predicted capacity only for $D/t < 2060/F_y$. The elastic buckling equation

$$M_u = .33ES/(D/t)$$

will control for $D/t > 8970/F_y$ if the shape factor is assumed to be 1.29 for the thin cylinder.

If the ultimate resisting moment is obtained using a resistance factor of 0.90

$$M_R = \phi M_u, \qquad \phi = 0.90$$

capacities close to the 95% confidence limit in Figure 7 will be predicted.

These recommendations are related to strength only. Rotational capacities to determine if a pipe section is suitable for plastic analysis were not considered in the study.

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NOMENCLATURE

A,B,C	coefficients in equations
D	outside tube diameter
E	modulus of elasticity (for steel assumed = 29000 ksi)
F_{xc}	local buckling stress from equations in API Standard
F_{ult}	material ultimate stress
F_y	material yield stress (nominal)
F_{ys}	material static yield strength
M	bending moment
M_D	design moment in allowable stress criteria
M_p	computed plastic moment
M_{ps}	M_p based on static yield, F_{ys}
M_R	resisting moment in LRFD criteria
M_u	ultimate moment predicted by equations
M_{ult}	experimental ultimate moment based on static ultimate load
M_y	computed yield moment
M_{ys}	M_y based on static yield, F_{ys}
N	power coefficient in Equations
R	radius of cylinder
S	elastic section modulus
t	tube thickness
Z	plastic section modulus
ϵ	strain
α, β, λ	slenderness parameters
ϕ	resistance factor in LRFD criteria

TABLE 1 - COMPARISONS WITH EQUATIONS FOR PREDICTING MOMENT CAPACITIES (1)

EQU. #	SLENDERNESS PARAMETER	EQUATION FORM	COEFFICIENTS (2) A	COEFFICIENTS (2) B	COEFFICIENTS (2) C or N	CORRELATION COEFFICIENT (2)	DEVIATIONS (3) MEAN	DEVIATIONS (3) RANGE	SEE FIGURE
9	$\alpha = (E/F_y)/(D/t)$	$M_u/M_p = A + B\alpha$	0.775	0.016		0.779	0.0162	0.1460 -0.0788	7
10	α	$= A + B\alpha + C\alpha^2$	0.669	0.037	-0.001	0.794	0.0171	0.1460 -0.0833	7
11	D/t	$= A + B(D/t)^N$	1.15	-0.0123	0.7	0.737	0.0175	0.1460 -0.0972	8
12	$\lambda = \alpha(F_y/E)^{2/3}$	$= A + B\alpha + C\alpha^2$	0.605	3.40	-7.61	0.796	0.0198	0.1460 -0.0917	10
13	$\beta = (F_y/E)^{1/2}(D/t)$	$= A + B\beta$	1.10	-0.063		0.769	0.0145	0.1460 -0.0946	9
14	D/t	$M_u/M_y = A - B(D/t)^N$	1.65	-0.057	0.50	0.773	0.0282	0.2090 -0.1279	11

NOTES: (1) Comparison is with ERW and fabricated pipe data only

(2) Regression coefficients and correlation coefficients are for data in the slenderness range where the predicted moment capacity is less than M_p

(3) Deviations include the data where the predicted capacity is M_p

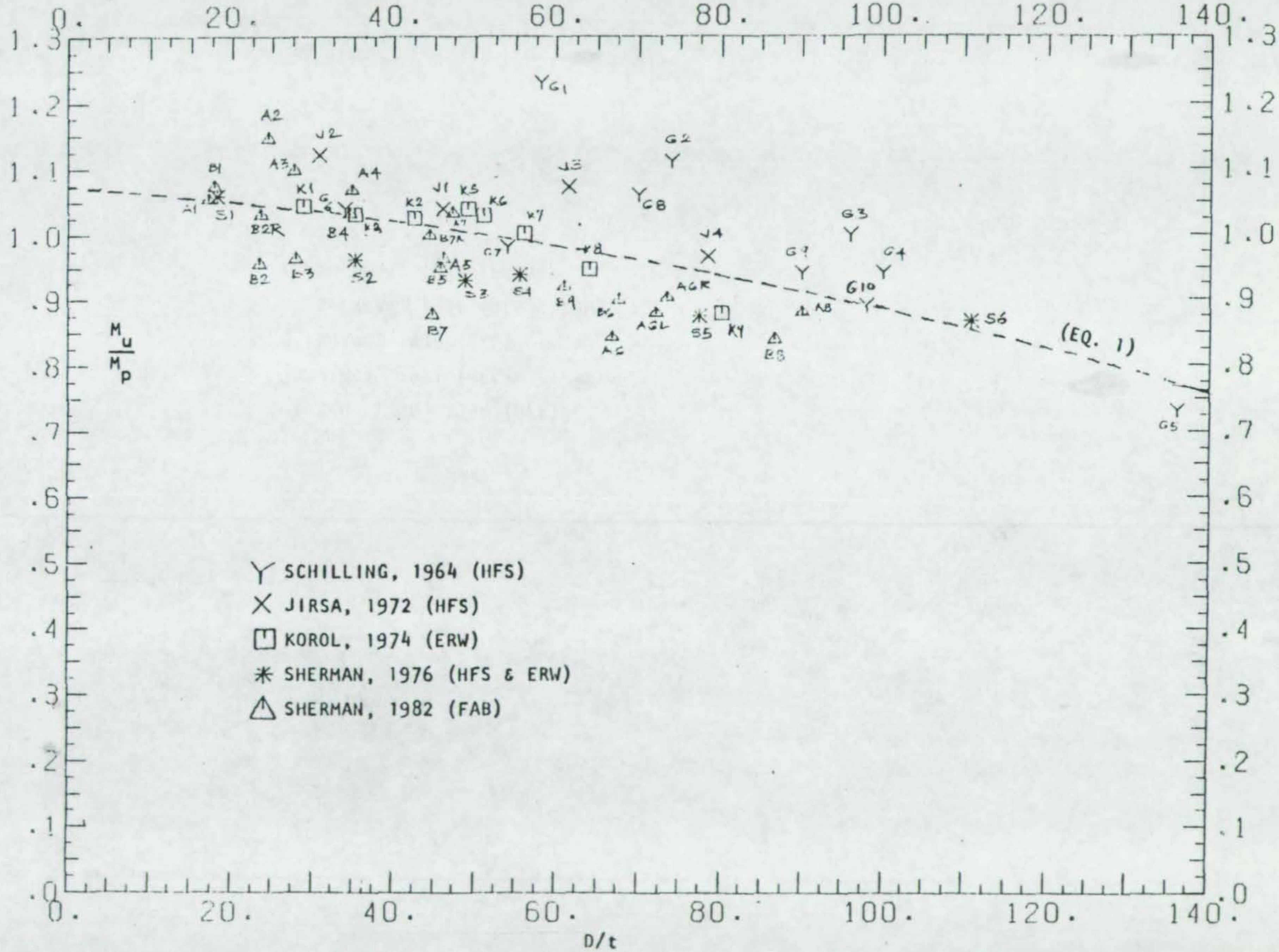

FIGURE 1 - TOTAL DATA BASE AS A FUNCTION OF D/t

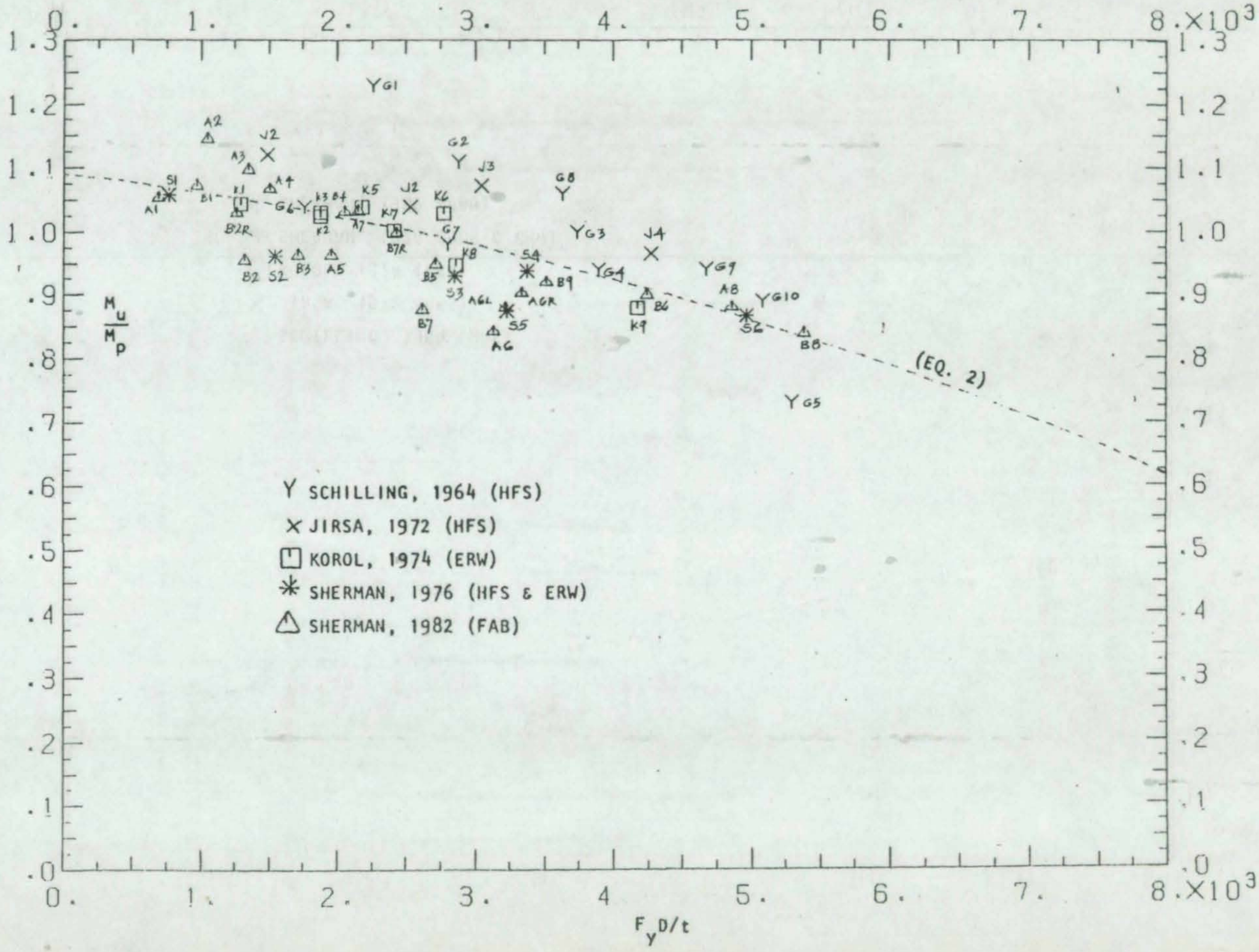

FIGURE 2 - TOTAL DATA BASE AS A FUNCTION OF F_yD/t

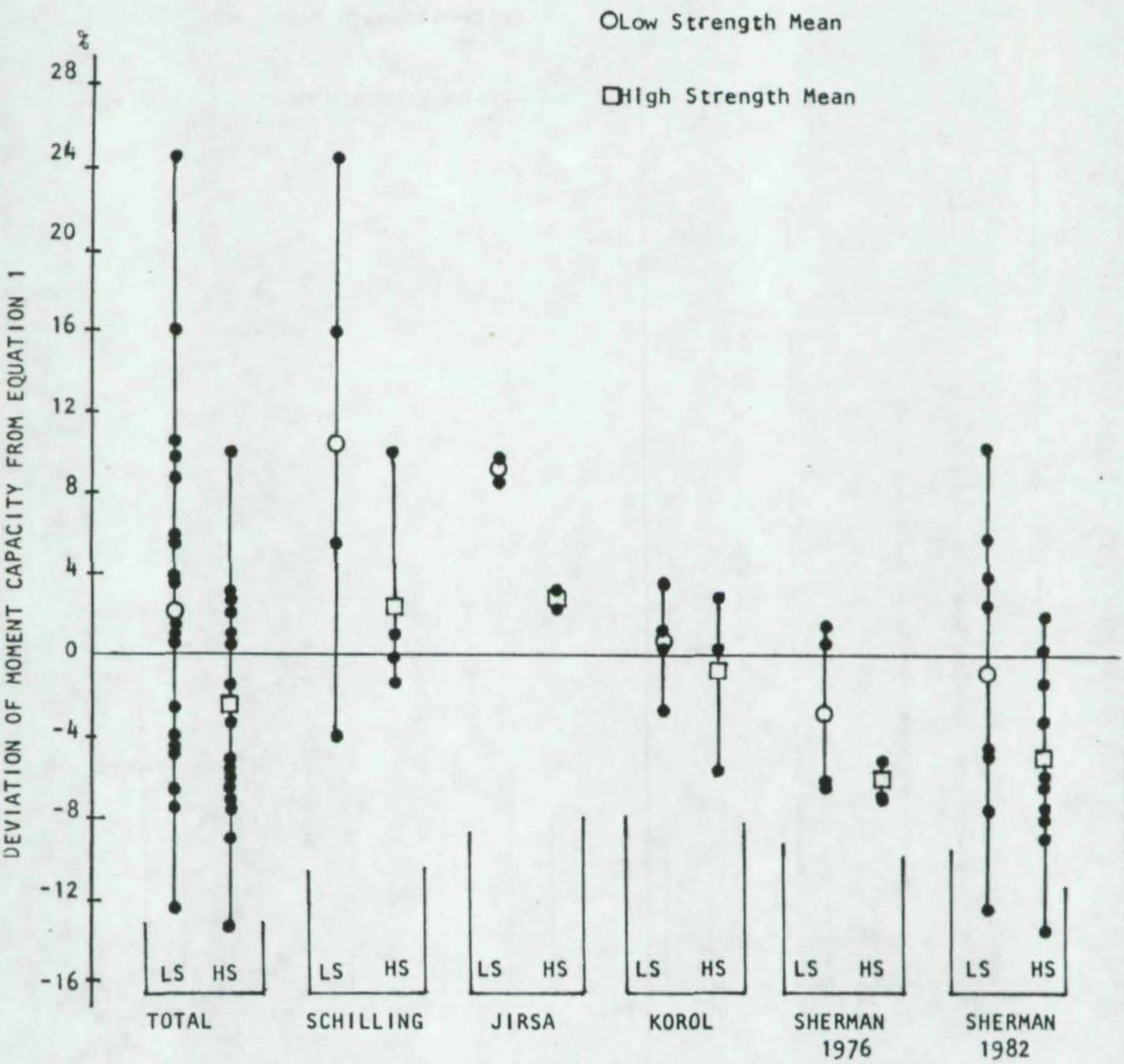

FIGURE 3 - SEPARATION OF DATA ACCORDING TO LOW STRENGTH (LS) AND HIGH STRENGTH (HS) STEEL WITH D/t SLENDERNESS

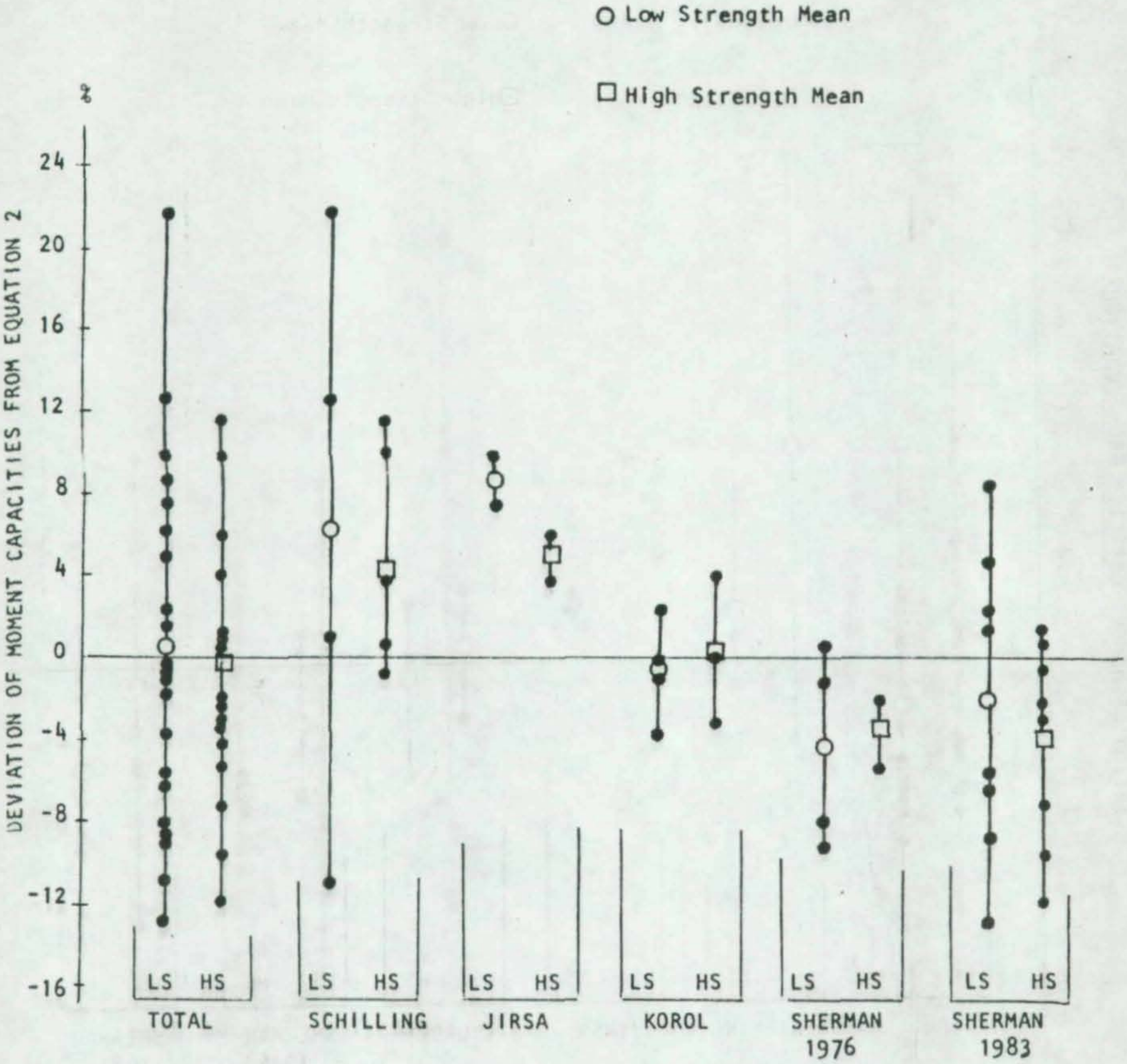

FIGURE 4 - SEPARATION OF DATA ACCORDING TO LOW STRENGTH (LS) AND HIGH STRENGTH (HS) STEEL WITH $F_y D/t$ SLENDERNESS

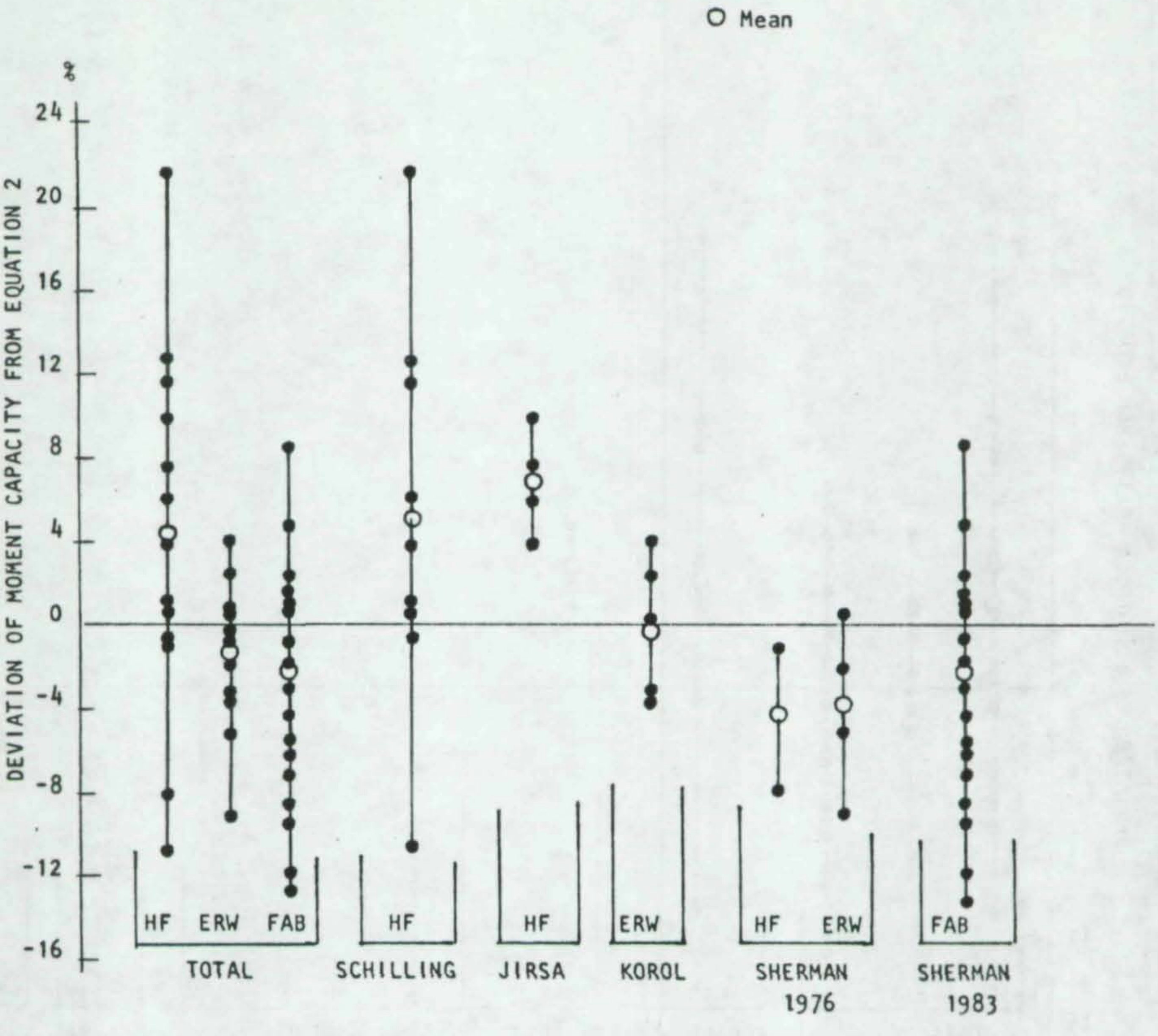

FIGURE 5 - SEPARATION OF DATA ACCORDING TO TYPE OF PIPE WITH $F_y D/t$ SLENDERNESS

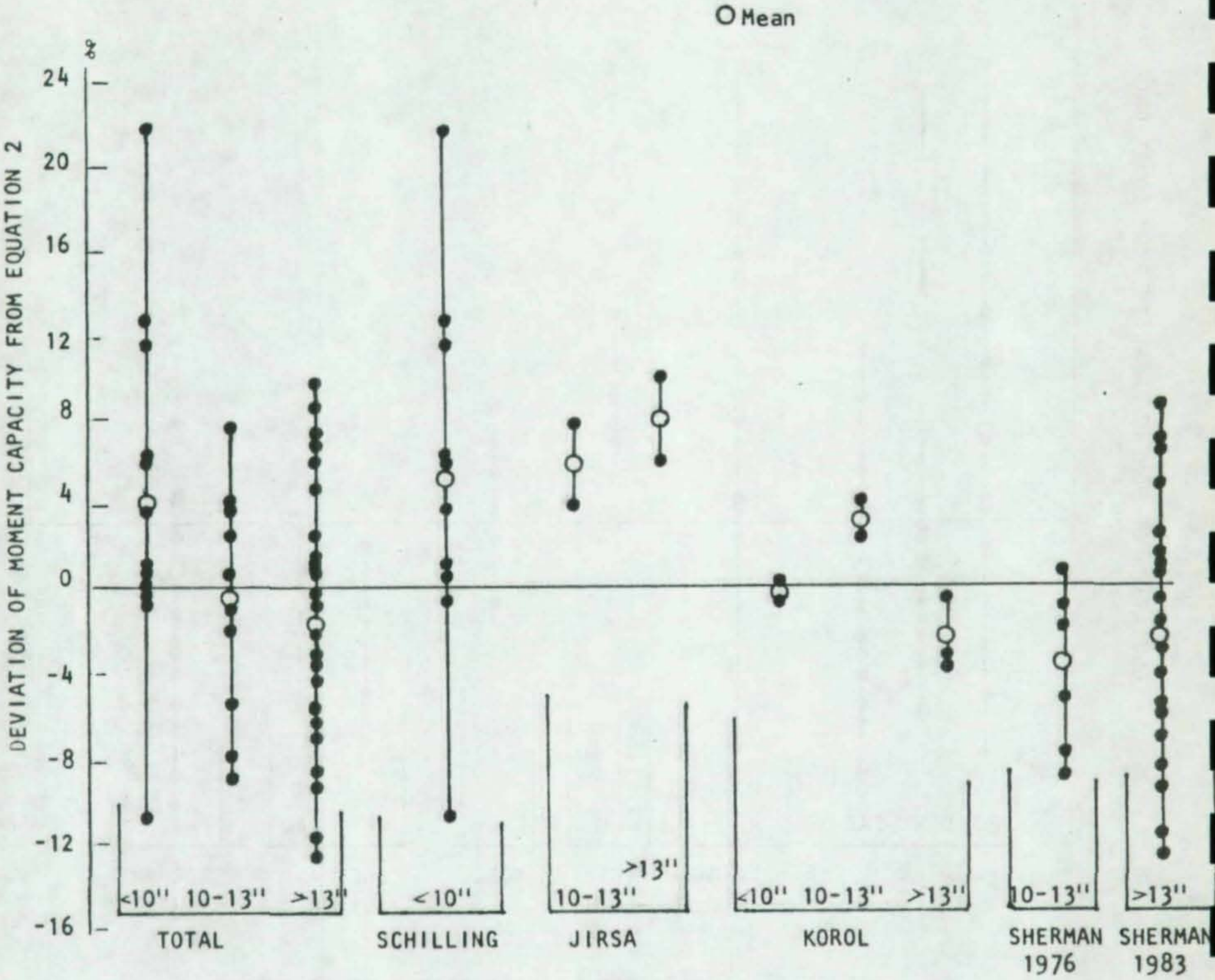

FIGURE 6 - SEPARATION OF DATA ACCORDING TO PIPE SIZE WITH $F_y D/t$ SLENDERNESS

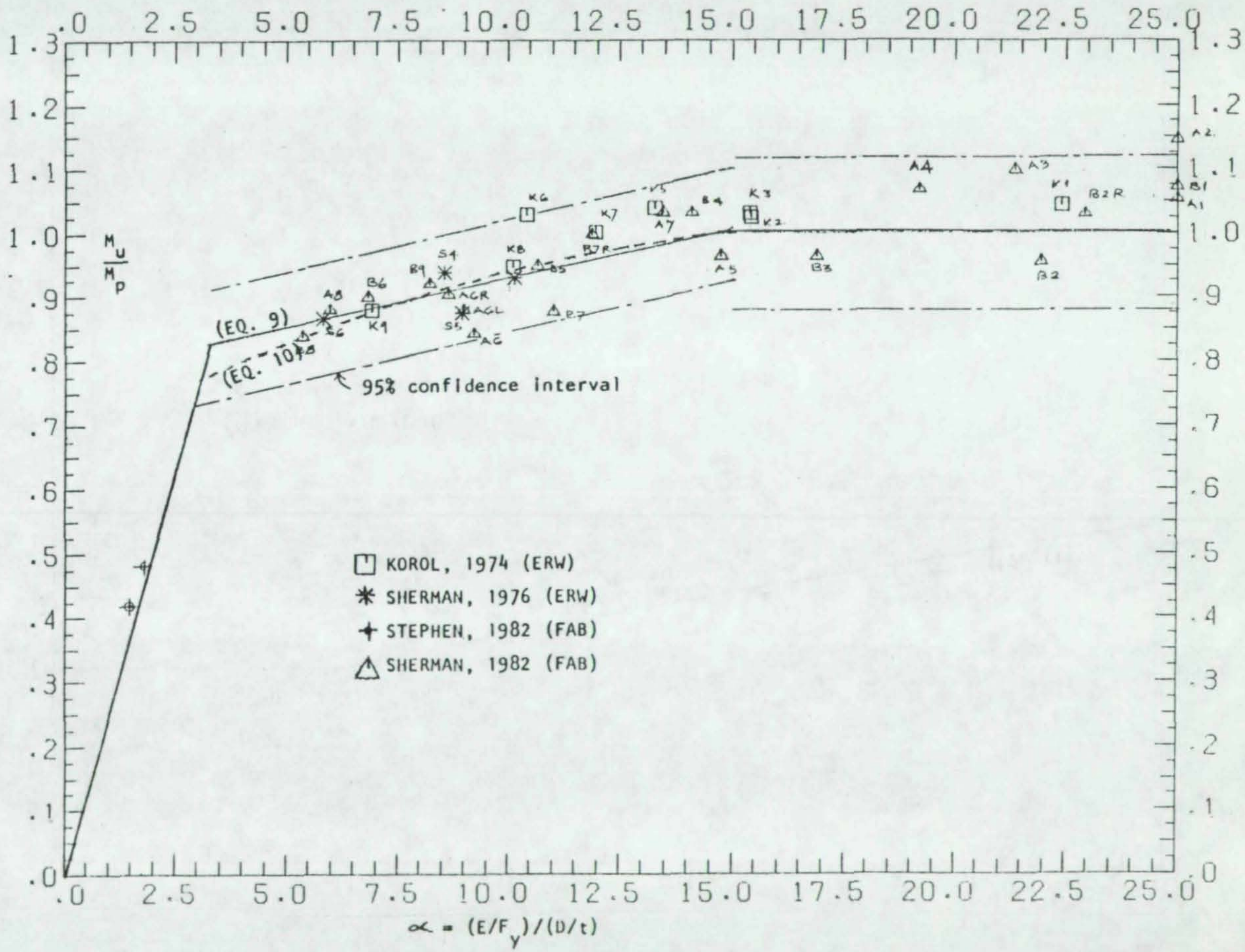

FIGURE 7 - EQUATIONS USING SLENDERNESS PARAMETER $\alpha = (E/F_y)/(D/t)$

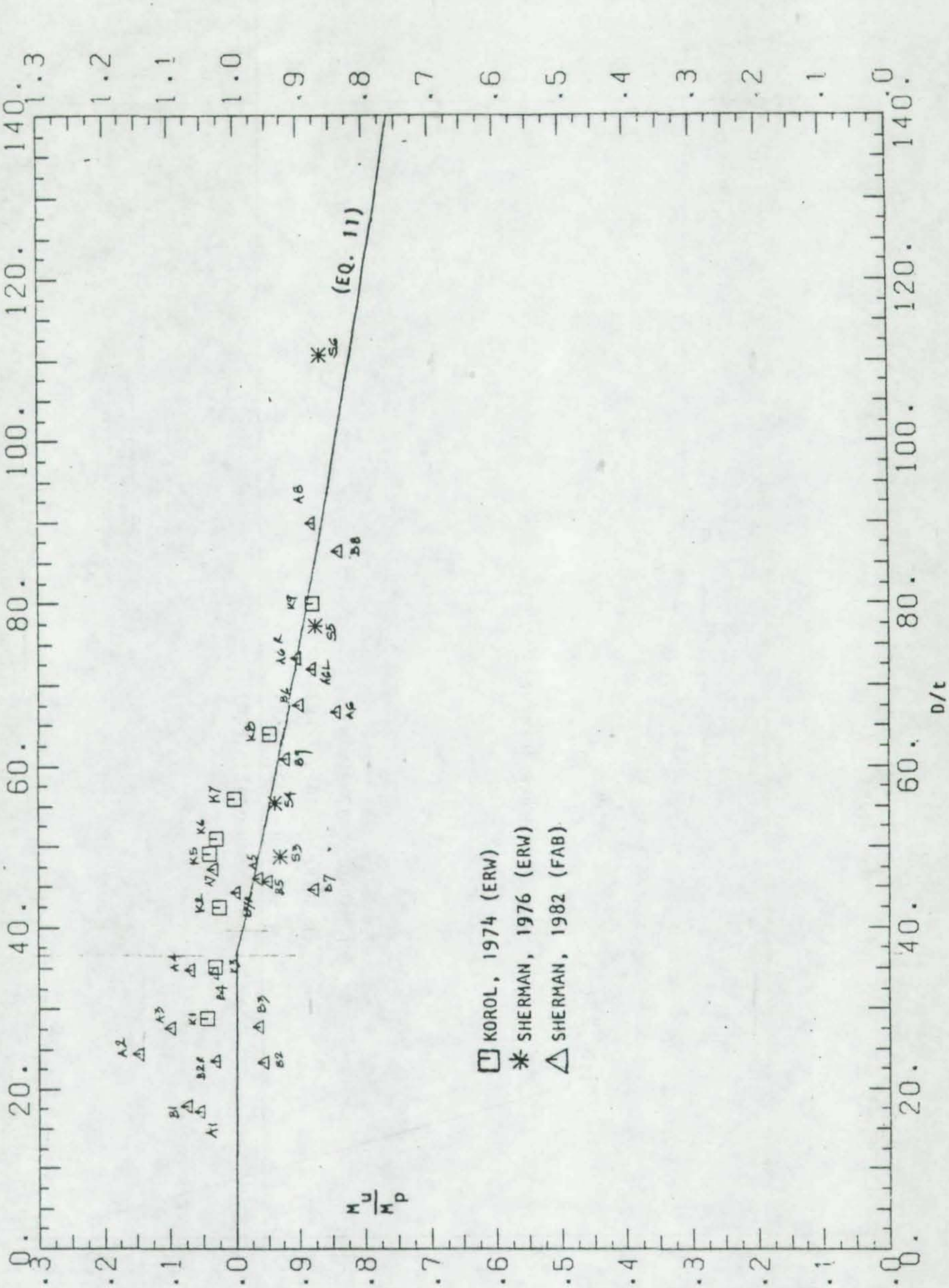

FIGURE 8 - EQUATION 11 WITH D/t SLENDERNESS PARAMETER

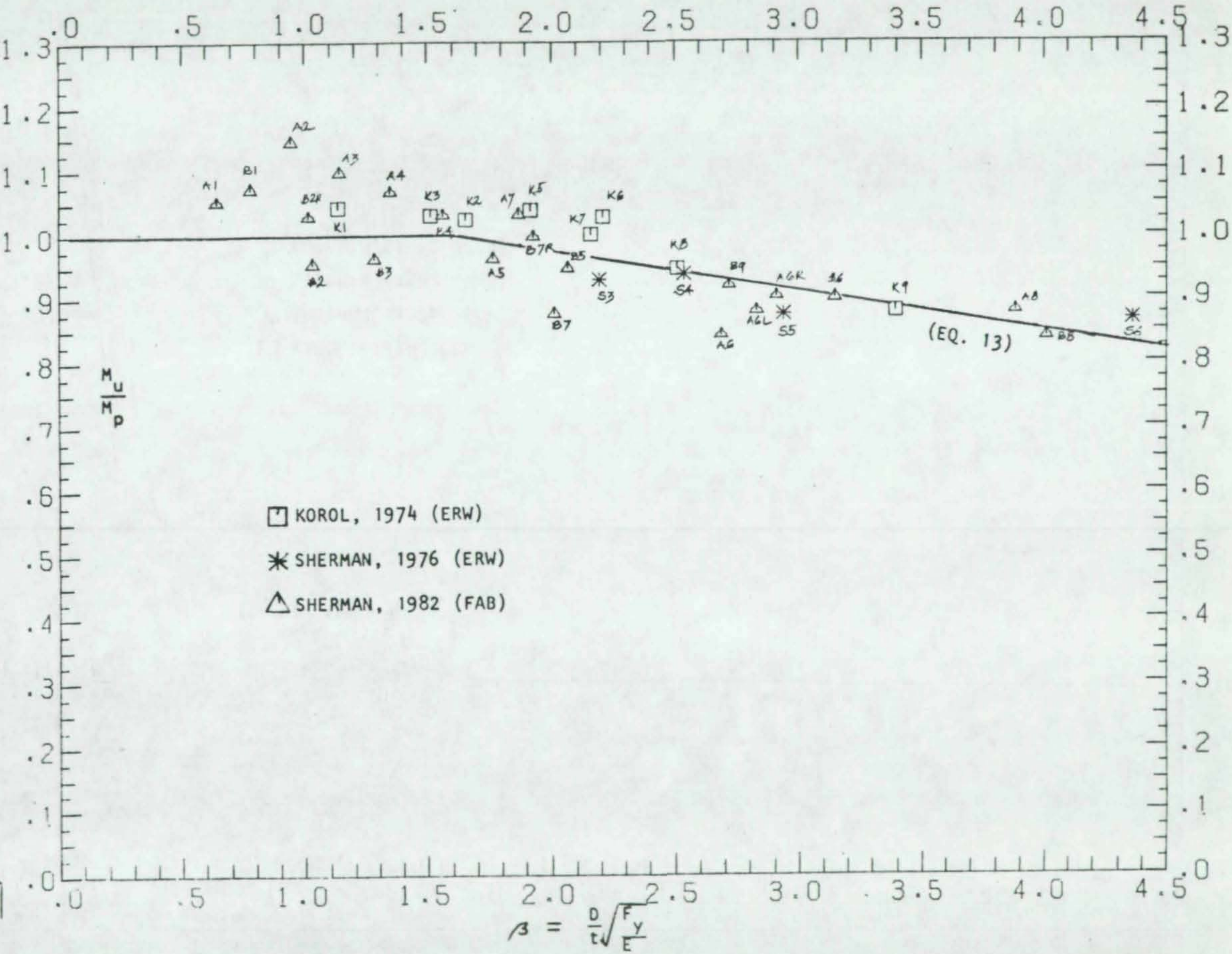

FIGURE 9 - EQUATION USING SLENDERNESS PARAMETER $\beta = \frac{D}{t}\sqrt{\frac{F_y}{E}}$

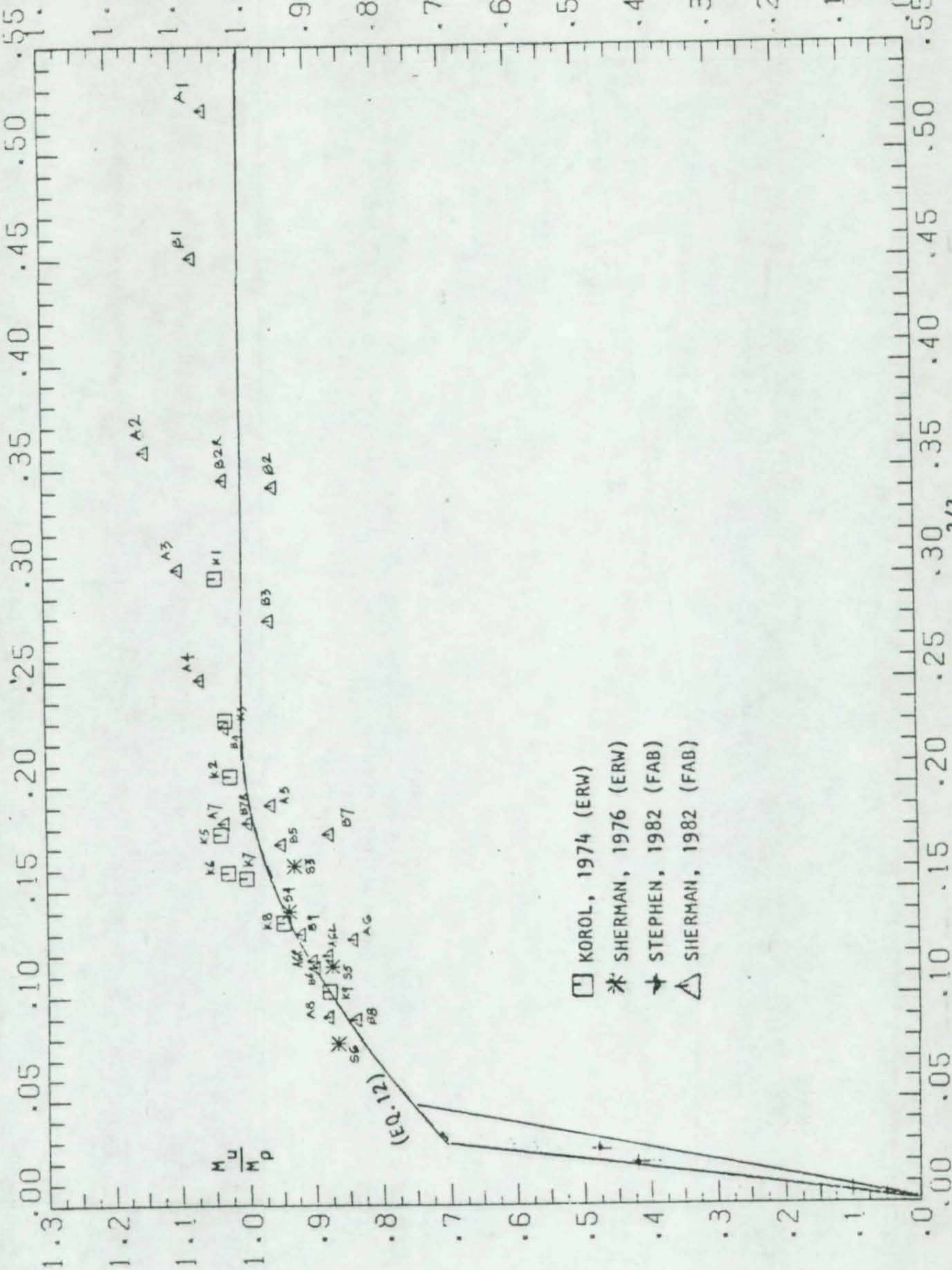

FIGURE 10 - EQUATION USING SLENDERNESS PARAMETER $\lambda = \alpha (E/F_y)^{2/3}$

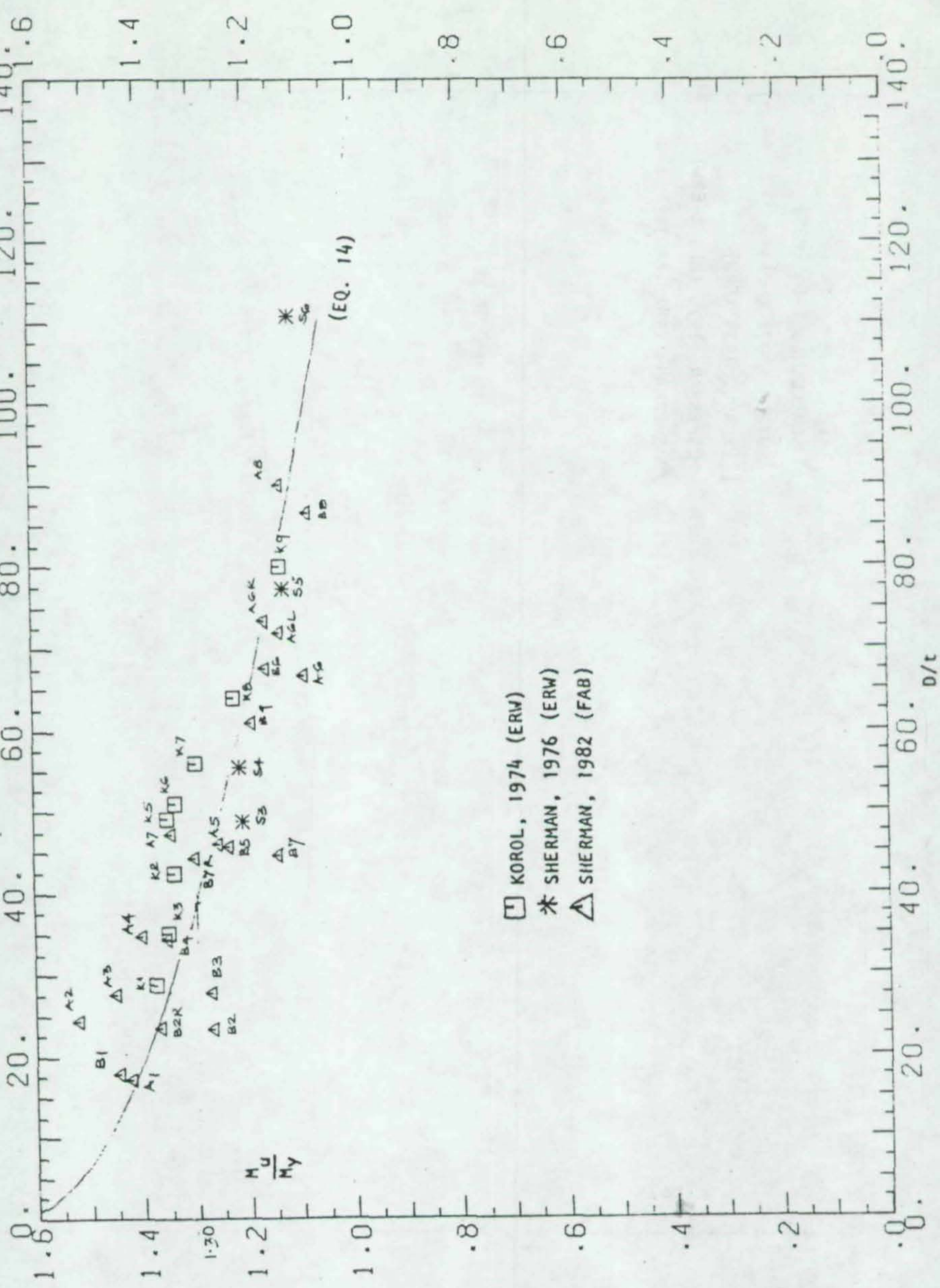

FIGURE 11 - EQUATION USING M_u/M_y AND SLENDERNESS PARAMETER D/t

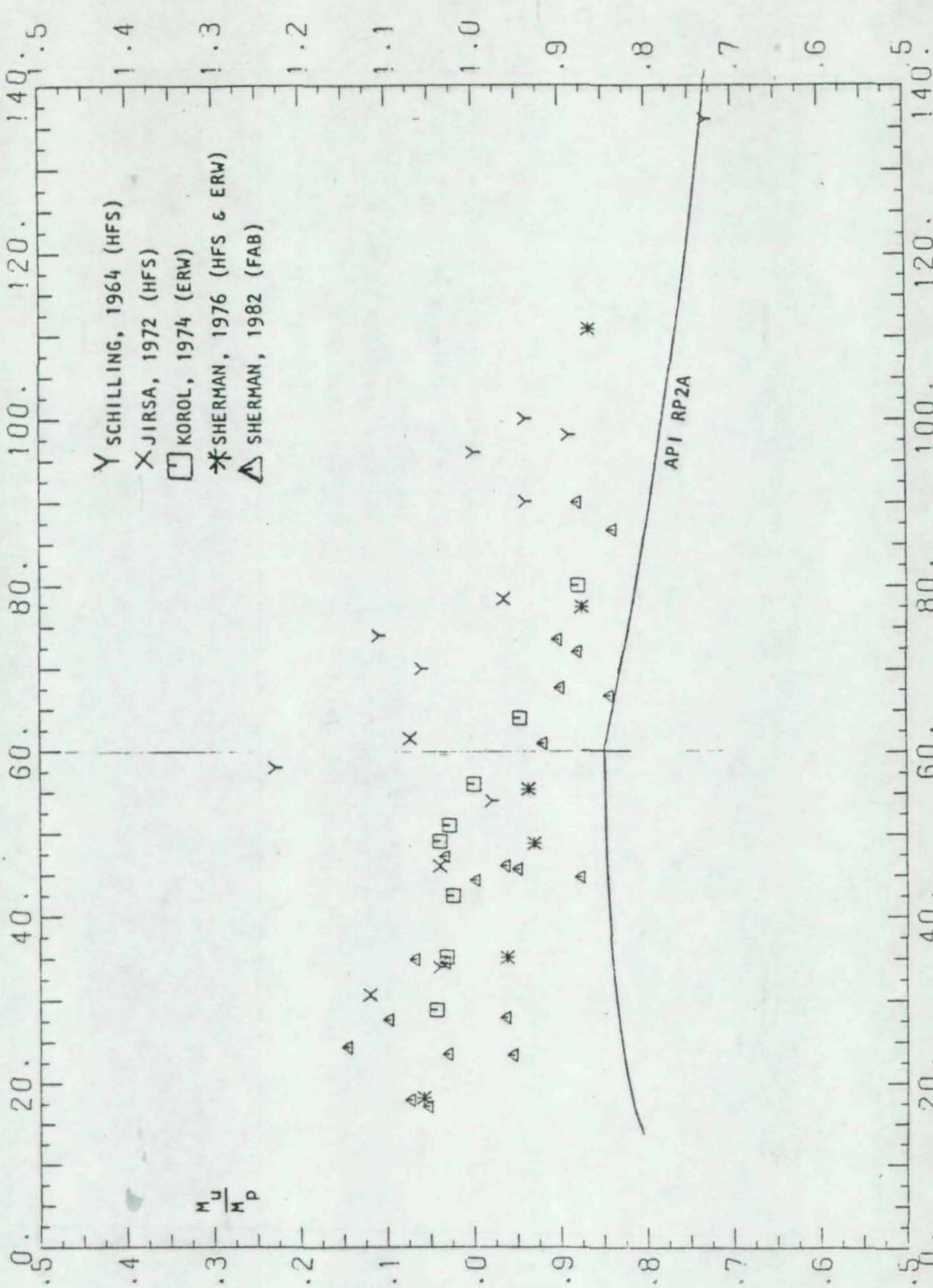

FIGURE 12 - DATA COMPARED TO API RP2A (with 1.67 Safety Factor)

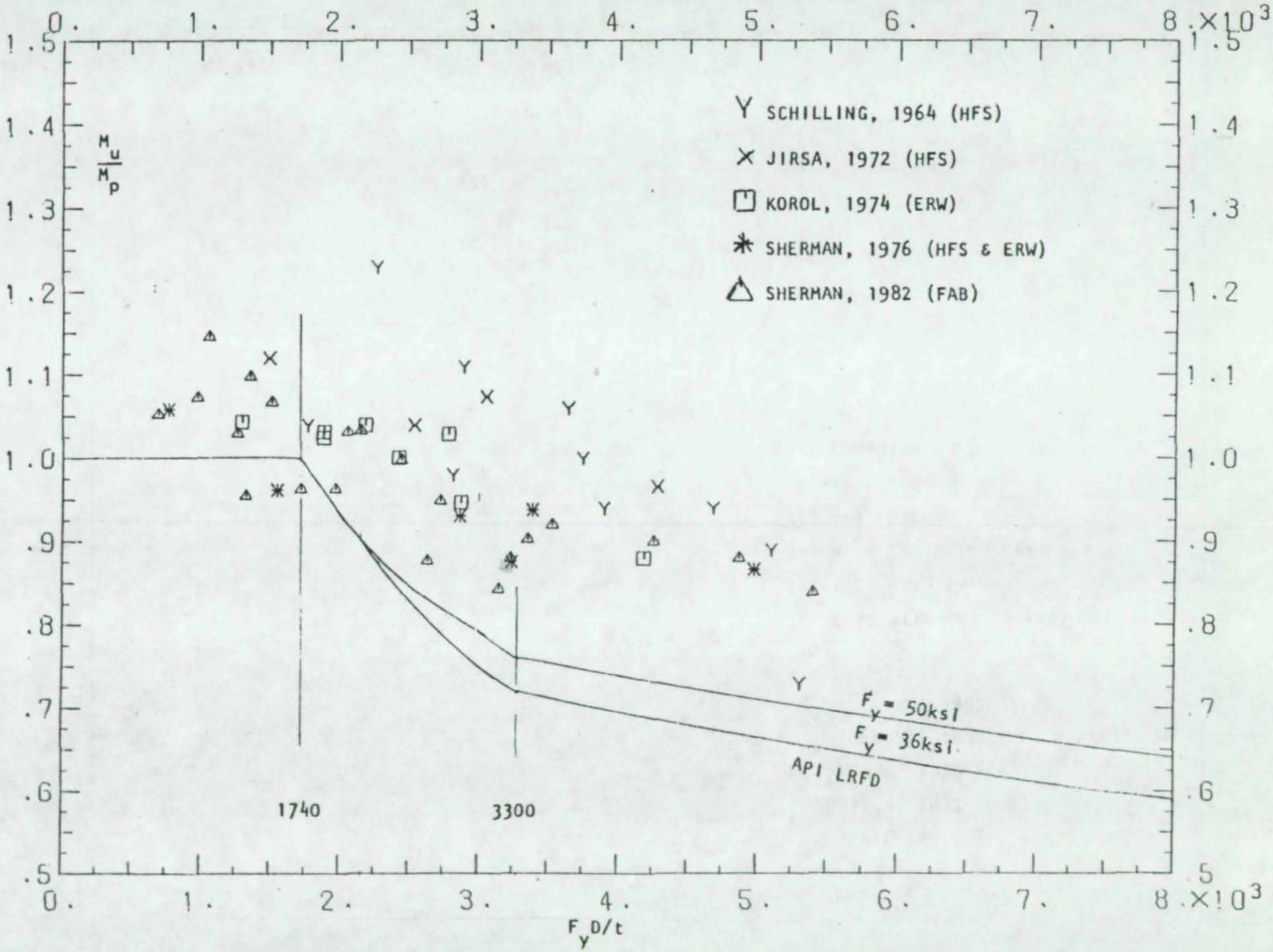

FIGURE 13 - DATA COMPARED TO ULTIMATE MOMENT OF API LRFD

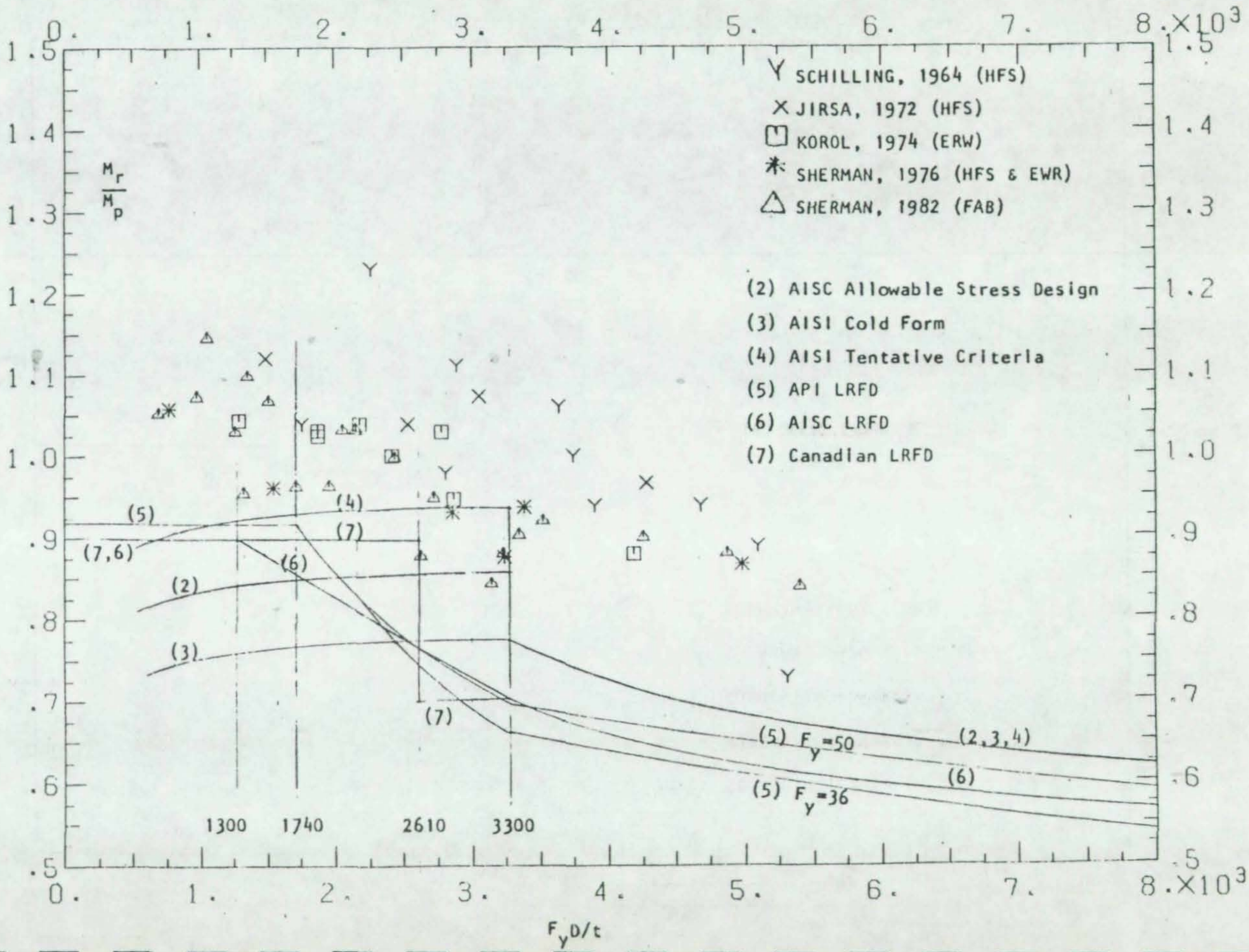

FIGURE 14 - DATA COMPARED TO RESISTING MOMENTS OF SPECIFICATIONS
($M_r = \emptyset M_u$ in LRFD and $1.67 M_p$ in ASD)

APPENDICES

A - TABLES OF EXPERIMENTAL DATA

B - STATISTICAL DEFINITIONS

APPENDIX A. TABLES OF EXPERIMENTAL DATA

Table A1. Tests by Schilling, U. S. Steel Corp., 1965.

Designation	Type	O.D.	t	D/t	F_y	F_yD/t	$\alpha=\frac{Et}{F_yD}$	M_{ult}/M_p
G1	HFS	4.292	0.074	58.0	39.0	2262	12.8	1.23
G2	HFS	4.070	0.055	74.0	39.0	2886	10.0	1.11
G3	HFS	4.128	0.043	96.0	39.0	3744	7.7	1.00
G4	HFS	4.070	0.037	110.0	39.0	3900	7.4	.94
G5	HFS	4.080	0.030	136.0	39.0	5304	5.5	.73
G6	HFS	3.944	0.116	34.0	52.0	1768	16.4	1.04
G7	HFS	4.914	0.091	54.0	52.0	2808	10.3	.97
G8	HFS	4.410	0.063	70.0	52.0	3640	8.0	1.06
G9	HFS	3.510	0.039	90.0	52.0	4860	6.0	.94
G10	HFS	4.606	0.047	98.0	52.0	5096	5.7	.90

Table A2. Tests by Jirsa et al., Rice University, 1972.

Designation	Type	O.D.	t	D/t	F_y	F_yD/t	$\alpha=\frac{Et}{F_yD}$	M_{ult}/M_p
J1	HFS*	10.75	.233	46.1	55.0	2536	11.4	1.04
J2	HFS*	10.75	.350	30.7	48.5	1489	19.5	1.12
J3	HFS*	16.00	.260	61.5	49.6	3050	9.5	1.07
J4	HFS*	20.00	.255	78.4	54.6	4281	6.8	.97

*Type cannot be confirmed. Tension tests show rounded stress-strain curves and yield is by the 0.2% offset method.

Table A3. Tests by Korol, McMaster University, 1974.

Designation	Type	O.D.	t	D/t	F_y	F_yD/t	$\alpha=\frac{Et}{F_yD}$	M_{ult}/M_p
K1	CF	4.5	.156	28.9	44.8	1295	22.4	1.04
K2	CF	6.625	.156	42.5	44.2	1879	15.4	1.02
K3	CF	6.625	.188	35.2	53.5	1883	15.4	1.03
K4	CF	20.0	.250	80.0	54.4	4352	6.7	.60*
K5	CF	10.75	.219	49.1	44.4	2180	13.3	1.04
K6	CF	12.75	.250	51.0	54.5	2780	10.4	1.03
K7	CF	14.00	.250	56.0	43.2	2419	12.0	1.00
K8	CF	16.00	.250	64.0	44.8	2867	10.1	.95
K9	CF	20.00	.250	80.0	52.3	4184	6.9	.88

*Premature failure at loading point. Data point is not included in the figures or analysis of this report.

Table A4. Tests by Sherman, UWM, 1976.

Designation	Type	O.D.	t	D/t	F_y	F_yD/t	$\alpha=\frac{Et}{F_yD}$	M_{ult}/M_p
S1	HFS	10.75	.585	18.3	42.1	770	37.6	1.06
S2	HFS	10.75	.307	35.1	44.1	1548	18.7	.96
S3	ERW	10.75	.221	48.8	58.6	2860	10.1	.93
S4	ERW	10.75	.194	55.4	61.1	3385	8.6	.94
S5*	ERW	10.75	.139	77.4	41.8	3235	9.0	.88
S6*	ERW	10.75	.097	110.8	44.9	4975	5.8	.87

*Yield point stress-strain characteristics. Other specimens have rounded curves.

Table A5. Tests by Stephens et al., University of Alberta, 1982.

Designation	Type	O.D.	t	D/t	F_y	F_yD/t	$\alpha=\frac{Et}{F_yD}$	M_{ult}/M_p
C1	Fab.	60.2	.202	298	54.5	16240	1.79	.48
C2	Fab.	60.2	.135	444	44.4	19710	1.49	.42

Sharp yielding stress-strain curves.

Table A6. Tests by Sherman, UWM, 1983.

Designation	Type	O.D.	t	D/t	F_y	F_yD/t	$\alpha=\frac{Et}{F_yD}$	M_{ult}/M_p
A1	Fab.	18	1.049	17.2	40.5	697	41.6	1.05
A2	Fab.	18	.737	24.4	43.3	1057	27.4	1.15
A3	Fab.	18	.651	27.7	49.0	1357	21.4	1.10
A4	Fab.	18	.516	34.9	43.3	1511	19.2	1.07
A5	Fab.	18	.391	46.1	42.6	1964	14.8	.96
A6	Fab.	18	.270	66.7	47.1	3142	9.2	.84
A6R	Fab.	18	.242	74.4	45.6	3392	8.6	.89
A6L	Fab.	18	.249	72.3	44.8	3239	9.0	.91
A7	Fab.	24	.509	47.2	45.6	2152	13.5	1.03
A8	Fab.	24	.267	89.9	54.1	4864	6.0	.88
B1	Fab.	18	1.000	18.0	54.3	977	29.7	1.07
B2	Fab.	18	.772	23.4	56.5	1322	21.9	.96
B2R	Fab.	18	.745	24.2	53.7	1297	22.4	1.06
B3	Fab.	18	.645	27.9	61.5	1716	16.9	.96
B4	Fab.	18	.522	34.5	59.6	2056	14.1	1.03
B5	Fab.	18	.395	45.7	59.5	2719	10.7	.95
B6	Fab.	18	.267	67.6	62.9	4252	6.8	.90
B7	Fab.	18	.537	44.7	58.8	2628	11.0	.88
B7R	Fab.	24	.541	44.4	55.0	2442	11.9	1.00
B8	Fab.	24	.277	86.6	62.3	5395	5.4	.84
B9	Fab.	24	.394	60.9	57.9	3526	8.2	.92

APPENDIX B

STATISTICAL DEFINITIONS

A regression analysis was used as a statistical tool for comparison of various data. The statistical package 'MINITAB' (B1) was used for the calculation of various statistical parameters, for example correlation and regression coefficients, confidence intervals, standard error (B2) etc.

Correlation Coefficient:

The ratio of the explained variation to the total variation is called the coefficient of determination. If the total variation is completely unexplained, this ratio is zero. If it is totally explained, the ratio is one. In other cases the ratio lies between zero and one. Since the ratio is always positive, we denote it by r^2. The quantity r, called the "coefficient of correlation, is given by

$$r = \pm \frac{\text{explained variation}}{\text{total variation}} = \pm \frac{\Sigma\,(y \text{ predicted} - \bar{y})^2}{\Sigma\,(y - \bar{y})^2} \qquad (1)$$

and varies between -1 and +1. The signs ± are used for positive linear correlation and negative correlation, respectively. Correlation coefficient 'r' is a dimensionless quantity, i.e. it does not depend on the units employed. For the case of linear correlation the quantity 'r' is the same regardless of whether x or y is considered the independent variable.

In case of multiple independent variables, the correlation coefficient between y-actual and y-predicted is calculated.

Regression Analysis:

Often, on the basis of sample data, we wish to estimate the value of a dependent variable 'y' corresponding to a given value of an independent variable 'x'. This can be accomplished by estimating the value of 'y' from a least square curve which fits the sample data. The resulting curve is called a regression curve of 'y' on 'x' since 'y' is estimated or predicted from 'x'.

A regression equation is an equation for estimating a dependent variable, say y from the independent variables x_1, x_2, x_3, . . . and is called a regression equation of y on x_1, x_2, x_3, . . . In functional notation this can be written briefly as $y = F(x_1, x_2, x_3, \ldots)$.

For the case of two variables, the simple regression equation of y on x has the form:

$$y = a + bx$$

where a and b are regression coefficients. The equation represents the equation of a straight line; therefore, it is called a linear regression equation of y on x.

If we have dependent variables as x, x^2, . . ., x^n etc. then it is called a nonlinear multiple regression equation and has the following form:

$$y = a + bx + cx^2 + \ldots + dx^n$$

If we have a dependent variable, x^n, then it has the form:

$$y = a + b(x^n)$$

the equation is called a nonlinear regression equation.

<u>Confidence Interval</u>:

The confidence interval for any parameter can be computed by a formula that has the following form:

$$\text{parameter} = \text{observed or predicted statistic} \pm (A)(B)$$

The A in the first parentheses relates to the specific confidence level chosen and the B in the second parenthesis relates to the precision of estimates or prediction resulting from the sampling procedure. In calculating the confidence interval the normal distribution of the sample is assumed; therefore, for a 95% confidence interval the above formula has the following form:

$$y = \text{y-predicted} \pm t_{0.025} \text{ (Standard Error)}$$

Standard Error $= \dfrac{S}{\Sigma x^2}$

Residual Variance $= S^2 \equiv \frac{1}{n} \Sigma(y - \bar{y})^2$

n = Degrees of freedom to estimate the variance

$\bar{y}$ = Mean value

$t_{0.025}$ = Critical value or confidence coefficient corresponding to 95% confidence limit

REFERENCES: FOR APPENDIX B

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