REASONABLE COLUMN DESIGN EQUATIONS

by

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INTRODUCTION

Over the years numerous empirical column design equations have been proposed. In most cases the equations were an attempt to reasonably represent experimental test data. The historical perspective for many of these equations is available in the literature(1,2,3). The current American Institute of Steel Construction, Inc. (AISC) Specification(4) column equations which have been used since about 1963 reflect this philosophy. They are based on the Column Research Council (CRC),(2) now Structural Stability Research Council (SSRC) equations with a safety factor. The two equations represent different physical phenomena. The first, for slender columns (KL/r), is the Euler buckling equation with a constant safety factor of approximately 1.92. Columns fitting into this group buckle elastically and it is believed that residual stresses will not appreciably influence the results. The second, for less slender columns, is a parabolic equation with a variable safety factor. This equation reflects the effects of residual stresses in the column. Very short columns have a safety factor of about 1.67 since the residual stresses have maximum effect but sudden buckling is less likely to occur. As the slenderness increases to Cc the variable safety factor increases to match the 1.92 value used for the Euler equation. Cc represents the common column length in the two equations when the stress is equal to the yield stress Fy divided by 2. K, L and r are effective length factor, length and radius of gyration as defined in the AISC Specification(4). It is believed that the 1.92 safety factor was chosen to provide added protection against the effect of out-of-straightness in slender columns.

A more direct approach, independent of test results, was developed by Bjorhovde(5) using probabalistic techniques. Available statistical information for column cross-sectional properties, residual stresses and out-of-straightness, etc. was used to develop multiple column curves. It was felt that as more statistical information became available the data base would become more precise and the need for costly and difficult physical testing would be eliminated. Bjorhovde developed two sets of multiple column curves. The first group adopted by SSRC(3) is based on the ASTM A6 maximum out-of-straightness limit of L/1000 and maximum residual stress levels. The second group is based on the then available test data with a mean out-of-straightness of L/1470.

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PREVIOUSLY PROPOSED COLUMN EQUATIONS

During the initial development of AISC's Load and Resistance Factor Design (LRFD) Specification(6) several column equations were considered. Initially, the Specification Committee adopted the Rondal and Maquoi (RM)(7) formulation of what has become known as SSRC Curve 2 with a resistance factor (Ø) of 0.85. The RM equation is a single higher order equation that very closely represents Bjorhovde's (SSRC Curve 2) five segment equation. The elastic buckling (Euler) strength is not readily identified since the equation is one continuous function.

Since the current Allowable Stress Design (ASD) has a different philosophical origin than LRFD, direct comparisons can only be made by careful conditioning. This has been previously pointed out by the author.(8) LRFD employs different load factors for dead and live load. Any comparison depends on the live load to dead load ratio (LL/DL). A realistic way to make this comparison is to establish reasonable upper and lower bounds for the live load to dead load ratios when converting ASD provisions to an equivalent LRFD format. For a LL/DL ratio range of 0.35 to 3.00 the effective load factors are 1.30 and 1.50, respectively. Although, live load and dead load combinations outside this range may occur, these values represent a large percentage of actual design cases.

The SSRC Curve 2 is plotted in Fig. 1 along with two CRC curves, as modified by the AISC variable safety factor, for comparison. Fa represents the AISC allowable compressive stress as given by Equations 1.5-1 and 1.5-2(4). Fy represents the specified minimum yield stress. All three curves use a common resistance factor, Ø, of 0.85. The slenderness ratio λ is obtained from the column properties and is equal to $KL\sqrt{Fy/E}/(\pi r)$. In the intermediate range of slenderness ($0.75 \leq \lambda \leq 1.50$), the SSRC Curve 2 displays a significant drop. This drop is attributed to the combined effect of the maximum residual stresses and the out-of-straightness criteria used. Hall(1,9,10) has shown that the test data does not reflect the drop predicted by the SSRC curves. This is particularly interesting when one considers the data base includes those columns which have not been straightened. The restraint provided by the test end conditions has been recognized as contributing to some increased test column capacity. This effect is maximum in very slender columns, whereas the drop in the SSRC Curve 2 is a maximum in the intermediate range. Recent work in Europe, reported by Bernard(11) and summarized by Hall(1), provides further evidence that combining maximum residual stresses and maximum out-of-straightness is overly conservative.

A comparison of the SSRC and ASD curves indicates the former one appears to be calibrated against the lower bound ASD curve. When a survey of market distribution is made, it appears that the SSRC curve, with Ø equal to 0.85, represents roughly 25 percent or less of the building and component categories having this lower LL/DL ratio.

Comparing the ASD equations with SSRC Curve 2 indicates that ASD will be more economical in the higher slenderness ranges, except for those cases with very low LL/DL ratios. Similarly, stocky column designs having low LL/DL ratios will be more economical using the SSRC curve. Figure 1 clearly demonstrates that when practical slenderness and LL/DL ratios are combined, the ASD design procedure is generally found to be more economical over a large range. When nominal market volume and distribution are also included in the comparison, the ASD advantage in column design becomes more pronounced.

Since, to the knowledge of the author, there is no recorded evidence to indicate that there have been any column failures attributed to the proper use of current AISC Specification,(4) it does not seem justifiable to arbitrarily make most column designs, using SSRC Curve 2 as formulated for LRFD, almost 20 percent more conservative. Others shared this same belief. The key objectionable factor is the drop in the middle of the SSRC Curve 2 which is attributed to the use of maximum out-of-straightness combined with maximum levels of residual stresses. It is believed that Bjorhovde's(5) work represented the theoretical state of the art at the time the research was completed. It can be so demonstrated by quoting directly from his work.

> Page 56: "It is believed that further investigations on the influence of cold-straightening will show that many columns that are straightened in this way also may be assigned to Category 1."

> Page 145: "The lack of influence of the residual stresses may be attributed to the fact that the strength and behavior of the column is more influenced by the out-of-straightness than of any other factor, which therefore overrides the influence of the other parameters. It must be stated, however, that the random nature of the overall residual stress distribution in a shape has not been studied, and the effects of differences in the residual stress pattern as influenced by the various manufacturing methods are of profound importance for the column behavior and strength."

Category 1 refers to SSRC Curve 1, which lies above SSRC Curve 2. Many of the columns used in the data base were specifically ordered from the mills in an "as is" condition from the cooling beds bypassing any form of straightening. Presumably, the objective at that time was to determine "as rolled" residual stresses.

Straightening can be accomplished by either the rotary or gag method. Both procedures have the dual advantage of reducing the level of residual stresses and obtaining a straighter column. The nominal L/1000 out-of-straightness actually represents an upper limit allowed by ASTM A6. The steel mills, as a matter of production economics, generally produce columns that are straighter, since any column which does not meet this tolerance can be, and has been, rejected by the purchaser. Occasionally, a fabricator will further straighten a column which does not meet their more restrictive in-house criteria. By converting the relative term L/1000 to actual dimensions, one obtains values of 1/4 in. for 20 ft or 1/2 in. for 40 ft. It is doubtful whether any reasonable fabrication or erection tolerances could be maintained using this out-of-straightness criterion other than as a maximum.

The SSRC Ad hoc committee at the San Francisco meeting in April, 1984, recommended using Bjorhovde's(5) sets of equations 1P, 2P and 3P which have an approximate out-of-straightness of L/1500. It appears that rotary straightened column shapes were assigned to curve 1P; jumbo shapes to 3P and all other hot rolled non-stress relieved W shapes to curve 2P. Rotary straightening is usually limited to W shaped columns weighing 100 pounds per foot or less. Some columns fabricated from high strength steel will be assigned to a design curve later.

AISC/LRFD COLUMN EQUATIONS

An agreement was reached whereby one set of LRFD Column equations would approximate the ASD equations with a LL/DL ratio of approximately 1.1 at λ equal to 1.0. The exponential equation in combination with a factored Euler equation were found to satisfy most of the requirements and will be published in AISC's revised LRFD document(6) later this year (1985). The equations with Ø equal to 0.85 and λ as defined earlier are:

$$\lambda \leq 1.5$$

$$F_{cr} = \emptyset \text{ EXP } (-0.419\ \lambda^2)\ F_y \qquad 1)$$

$$\lambda > 1.5$$

$$F_{cr} = \emptyset\ (0.877\ \lambda^{-2})\ F_y \qquad 2)$$

Note: $\text{EXP } (X) = e^x$, and

$\text{EXP } (-0.419\ \lambda^2) = 0.658^{\lambda^2}$

The exponential form of Equation 1 was found to reasonably reflect a lower bound limit for the lower range of λ. The coefficient with Equation 2 (Euler) was obtained by equating the two equations at the common λ of 1.5. Coincidentally, this set of equations yield almost identical results to Bjorhovde's(5) set of equations labeled 2P that was recommended by the SSRC Ad hoc committee.

The equations are shown in Fig. 2 along with the two AISC ASD equations previously discussed and the test data collected by Hall(1) and Lenz(12). With the exception of some of the test data near λ equal to 0.5 all the data points are located above the design equations. The data points that are below the design equation represent five W12 x 161 columns with out-of-straightness exceeding the ASTM A6 limits. A very few data points fall below the upper converted ASD curve. Overall, the design equations appear to be well below the test data. In comparison, the dip in the SSRC Curve 2 with Ø of 0.85 in Fig. 1, where it drops below the lower converted ASD Curve, would appear to be overly conservative.

Obviously, the AISC/LRFD equations result in a slightly more conservative design, than with ASD when LL/DL is greater than about 1.1. It is felt that the overall effect on the industry is acceptable since the difference is within one rolled shape capacity increment for a large percentage of load conditions. When future research increases our knowledge about the effects of out-of-straightness and residual stresses on column design the equations can be adjusted accordingly. Changing the coefficients and common slenderness point permits the two equations to move up or down without changing the basic format.

RELIABILITY

LRFD has introduced new ways of examining individual member or total structure performance. In a sense, reliability, β, is to LRFD what safety factor is to ASD. Reliability is defined as:

$$\beta = \frac{\ln(Rm/Qm)}{\sqrt{Vr^2 + Vq^2}} \qquad 3)$$

Where:
- Rm = mean resistance
- Qm = mean load
- Vq = coefficient of variation, loads
- Vr = coefficient of variation, resistance
- ln = refers to natural logarithm

The consensus of the AISC Specification Committee was that LRFD designs, in general, would not be more conservative than ASD unless there was justification. For columns this would indicate a target reliability of about 3.00.

The projected reliability for the AISC/LRFD design equations is given in Fig. 3. At the high and low λ values, β values exceeding 3 are obtained compared to β of 2.6 at λ of 1.1 This would appear to be a contradiction of the AISC Specification Committee design philosophy. However, since the design equations form a lower bound to the test results the contradiction is mitigated by examining the testing variables and interpretations of the results. Each of the variables that influence β are dependent on the uniformity of sample acquisition, test procedures and identification of raw data.

It has been pointed out indirectly by Galambos(13) that inherent in most, if not all, test data is a contribution of base fixity greater than the theoretical pinned case. AISC(4) in the effective length nomograph given in the commentary recognized this and permitted the use of 10 for the pinned case instead of the theoretical ∞ for joint stiffness (G) when determining effective length factor (K). Obviously, the sophistication of the test end fixtures, if any, varied from one research institution to the next. Similarly, in some cases the test specimen was positioned and adjusted in the test machine until equal strains were initially obtained at low loads. This is often referred to as the "old Lehigh method". In other cases, the specimen was geometrically centered in the test machine. Attempts have been made to predict and correct the effect of the first testing technique on the recorded test loads.

Techniques for determining the yield stress of steel have not been completely documented. The two most important variables are the testing strain rate and cold straightening. Cold straightening techniques employed by the different producers vary and there is variation from one plant to the next within the same organization. As mentioned earlier, some column research had specific interest in determining residual stress levels prior to cold straightening. These conditions would result in a greater spread of data than would normally occur.

The variation in dead and live loading also influence the computation of β. It appears that during most of the development of LRFD, load conditions usually associated with human occupancy were considered since live load reductions were incorporated in the computations. These conditions are significantly different than those associated with roof and industrial loading.

Finally, Fig. 2 indicates the distribution of test data accumulated over many years. It is unknown whether this represents the general distribution throughout the total market volume. It is believed that light rotary straightened shapes would numerically dominate the market. Furthermore, there has been no history of unacceptable behavior of columns designed using the higher allowable ASD procedure. This includes cases with LL/DL ratios greater than 1.1. Reliance and confidence in β will require that most of the concerns be discussed and answered forthrightly. As a result, a reliability less than 3.00 is considered satisfactory. The computed β values should form the basis upon which to make further improvements.

SUMMARY AND CONCLUSIONS

A reasonable set of LRFD column design equations has been presented which provide practical and economical results. The design equations form the lower bound to the available test data that satisfied the ASTM A6 out-of-straightness criterion. This appears to be conservative for rotary straightened W shapes. Rotary straightening is common for shapes weighing 100 pounds per foot or less.

In practice, using this lower bound criterion is acceptable since it minimizes the effect of test machine base fixity when effective length factors are computed for other end conditions. Unusual column design geometric configurations and end conditions, which frequently occur, can also be confidently accommodated with these equations.

Reliability computations using available test data imply that the equations do not provide uniform or consistent safety. It is more likely that the data, acquired over many years and from many different research organizations using different procedures and equipment, have statistical flaws. Many years of experience indicate that there have been no problems with current column design practice and therefore reliability computations should only form the foundation for future improved understanding of column behavior.

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REFERENCES

1. Hall, D. H. "Proposed Steel Column Strength Criteria", Journal of the Structural Division, ASCE Vol. 107, No. ST4, April 1981.

2. Column Research Council, "Guide to Design Criteria for Metal Compression Members", Second Edition, Edited by Bruce G. Johnston, John Wiley and Sons. New York, N.Y. 1966.

3. Structural Stability Research Council, "Guide to Design Criteria for Metal Compression Members", Third Edition, Edited by Bruce G. Johnston, John Wiley and Sons New York, N.Y. 1976.

4. American Institute of Steel Construction, Inc. "Specification for the Design, Fabrication and Erection of Structural Steel for Buildings", New York, N.Y., November 1978.

5. Bjorhovde, R., "Deterministic and Probabilistic Approaches to the Strength of Steel Columns", PhD Dissertation in Civil Engineering, Lehigh University, Bethlehem, Pa., 1972.

6. American Institute of Steel Construction, Inc., "Load and Resistance Factor Design Specification for Structural Steel Buildings", 1985 draft release, Chicago, IL 1985.

7. Rondal, J. and Maquoi, R., "Single Equation for SSRC Column - Strength curves," Journal of the Structural Division, ASCE, Vol. 105, No. ST1, Jan. 1979.

8. Lui, E. M., and Chen, W. F., "End Restraint and Column Design Using LRFD". Discussion by R. H. R. Tide, AISC Engineering Journal, Vol. 21, No. 2, 1984.

9. Hall, D. H., "The Column Problem--One Perspective", AISC Proceedings of the National Engineering Conference, Memphis, Tenn., March 4, 1983.

10. Hall, D. H. and H. F. Stup, "A Preliminary Study on the Effect of Residual Stresses on the Strength of H-Columns", Bethlehem Steel Corporation, Report No. PD68-32-1, September 1969.

11. Bernard, A., "Buckling of Section Beams in High-Strength Steel", Draft Final Report, ARBED-Recherches Nov. 24, 1981.

12. Lenz, J. C., "Reliability Based Design Rules for Column Buckling," Master of Science Thesis in Civil Engineering, Washington University, St. Louis, Mo., 1972.

13. Galambos, T. V., "Influence of Partial Base Fixity on Frame Stability", Journal of the Structural Division, ASCE, Vol. 86, No. ST5, May 1960.

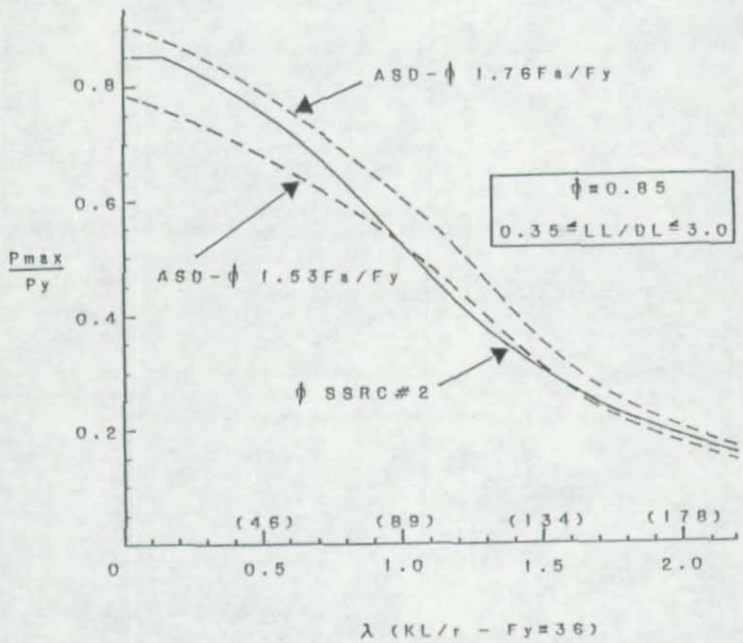

Fig. 1 - Effect of live load to dead load ratio on column capacity when comparing ASD to SSRC Curve 2

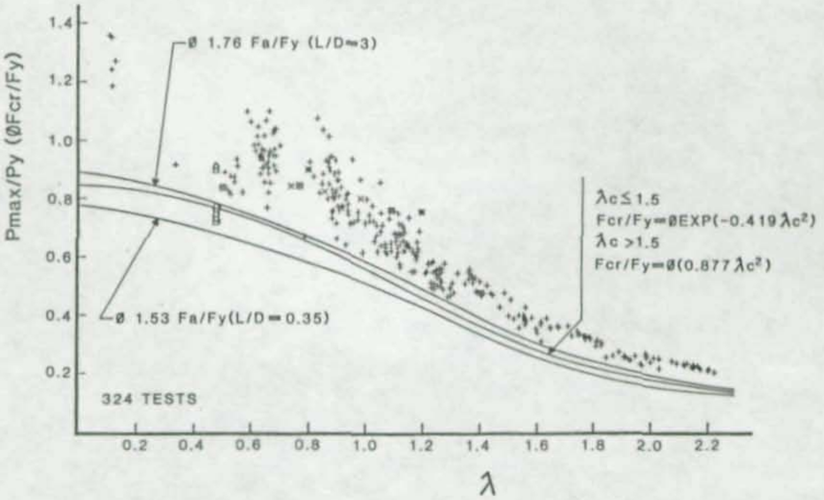

Fig. 2 - Comparing AISC/LRFD column curve to two converted ASD curves.

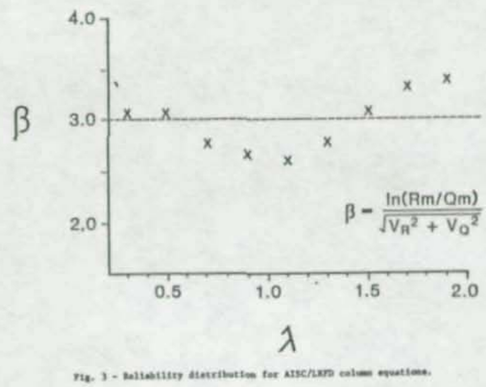

Fig. 3 - Reliability distribution for AISC/LRFD column equations.