

## The design of laterally unsupported angles

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### 1. Introduction

The steel angle is a common and almost traditional member in building construction. Its popularity stems from its relative lightness and compactness and the ease with which it can be connected to other members. In view of its long and widespread use it is surprising to find that little is known of many major aspects of its performance as a structural member. In these areas design guidance is only available to a limited extent and consists mainly of empirical extrapolations of solutions for other sections<sup>1</sup> and continued misconceptions about non-principal axis loading and shear centre eccentricities.

The behaviour of angles as compression members has been studied relatively extensively (e.g. <sup>2, 3, 4</sup>) as a result of their widespread use in such structures as transmission towers. These towers are usually precisely analysed<sup>5</sup> for actual failure under well defined load factors and an accurate knowledge of member load capacity has been essential. Even here, however, the underlying research has frequently been highly empirical with strut load capacities given for each member size under practical field conditions<sup>6</sup>.

The case which presents the designer with his current major information gap occurs when the angle is used as a laterally unsupported beam. For example, the S.A.A. Steel Structures Code AS CA17 states in Rule 5.4.3:

'The Standards Association of Australia is not prepared at this stage to make recommendations for angles which are not supported laterally.'

The British Code permits its standard beam rules to be used for angles, but the technique developed can not be rationally defended<sup>8, 9</sup> and does not lead to consistent design solutions. The U.S. steel design specification<sup>10</sup> does not specifically cover the case.

The logical question to ask at this stage is why the problem of the laterally unsupported angle used as a beam has remained without a practical solution for so long. The answer is, basically, that although the angle is a very simple section to the lay-

man and the producer, it is a difficult one for the stress analyst. The principal axes of the cross-section do not coincide with common loading directions and any routine loading will therefore cause biaxial bending deflections which are not in the same plane as the applied loads. To further complicate the problem, the shear centre is not at the centroid and is not on the line of most major applied loads. Thus most loads will cause the cross-section to twist and to deflect out of its loading plane. Finally, common end connections are usually eccentric because of the lack of symmetry of the cross-section.

## 2. Current investigations

The purpose of the current investigation is to develop rational but simple formulas for the design of laterally unsupported angles in bending. This should help fill the present, previously quoted, void in the S.A.A. Steel Structures Code, CA1, and thus permit the more widespread use of angles in building construction.

The loading case to be considered will be a uniform moment along the entire laterally unsupported span. This will produce the most critical lateral buckling situation<sup>11</sup> and will therefore give results which will be safe for any other bending moment distribution. The same uniform moment basis is used for the other lateral buckling rules of CA1<sup>12, 13</sup>. The lengths under consideration are assumed to be completely unsupported and the solutions may therefore be applied to both fully unsupported beams or restrained beams between restraint points.

Later work will include an experimental examination of various aspects of the problem. However, this article will be confined to a theoretical derivation of design rules.

Solutions are only presented for equal angles (leg lengths equal). Similar solutions can be obtained for unequal angles, but the complete asymmetry of these latter sections produces algebraically involved results which tend to obscure the basic underlying principles.

The range of equal angles produced by BHP are given in <sup>14</sup>. The sections are approximated by the dual rectangle idealisation shown in Fig.1. This linearised sec-

tion ignores fillets and toe radii, but can be made to reproduce actual member properties very precisely by adjusting the idealised leg length,  $B$ , to produce an exact similitude for some chosen geometrical property (such as area). The assumption, therefore, is not critical and is necessary in order to obtain a solvable set of equations.

## 3. Notation and sign convention

The notation to be used is:

- $B$  = Width of angle leg.
- $A, C, D$  = Constants of integration.
- $E$  = Young's Modulus.
- $F$  = Design stress.
- $F_0$  = Critical buckling stress.
- $F_b$  = Maximum permissible bending stress.
- $F_y$  = Yield stress.
- $G$  = Modulus of rigidity (shear or torsion modulus)
- $I_U$  = Second moment of areas about UU axis.
- $I_V$  = Second moment of area about VV axis.
- $I_\omega$  = Warping moment of area.
- $K_T$  = St. Venant torsional constant.
- $\bar{K}$  = Torsional component of the normal stress (see eq.5.4).
- $L$  = Length of span.
- $M$  = Component moment of the applied moment.
- $M_{cr}$  = Critical buckling moment.
- $M_a$  = Applied moment about Y axis + moment due to the dead weight of the beam.
- $S$  = Shear centre.
- $U$  Denotes the major principal axis.
- $V$  Denotes the minor principal axis.
- $W$  Denotes the polar axis.
- $Y, X$  Denotes axes through the centroid, parallel to an angle leg.
- $Z$  = Section modulus.
- $Z_a$  = Section modulus about same axis as  $M_a$ .
- $Z_v$  = Section modulus through the V axis.
- $c$  = Centroid location.
- $t$  = Thickness of angle leg.
- $u$  = U — U axis co-ordinate.
- $v$  = V — V axis co-ordinate.
- $u_0$  = Shear centre co-ordinate.
- $v_0$  = Shear centre co-ordinate.

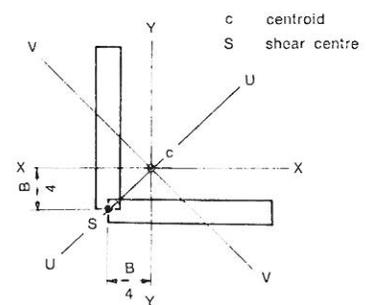


Fig.1 (a). Orientation of axes and locations of centroid and shear centre.

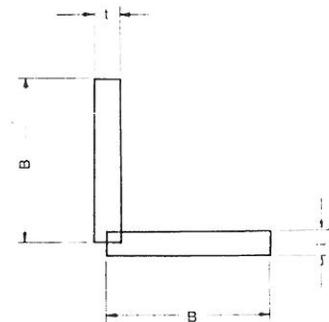


Fig.1 (b). Simplified angle section dimensions

$w$  = Distance measured along the length of the beam.  
 $\sigma$  = Actual section stress.  
 $\sigma_a$  = Stress calculated using conventional beam formula:  

$$\sigma_a = \frac{M_x}{Z_x}$$
 $\sigma_{cr}$  = Critical buckling stress.  
 $\phi$  = Angle of twist.  
 $\phi_0$  = Initial angle of twist due to imperfections.  
 $\alpha$  = Coefficient in solution of differential equations.

' = Differentiation with respect to  $w$ .  
 This notation is coupled with the sign convention shown in Fig.2.

#### 4. Loading cases

The behaviour of the beam is dependent on the axis about which the moment is applied, Fig.2. Four loading conditions are illustrated in Fig.3. These conditions can be used vectorially to represent all possible cross-section loadings. Taken individually they are:

**Case I:** Moment applied about an axis through the shear centre parallel to one leg.

**Case II:** Moment applied about the UU axis (strong axis).

**Case III:** Moment applied about the VV axis (weak axis).

**Case IV:** Moment applied about an axis midway between the UU axis and the YY axis.

Each of these cases will now be individually studied.

#### 5. Case I

The problem to be solved is illustrated in Fig.3(a) and Fig.4. Galambos<sup>15</sup> has shown that for this case the following equations apply\*:

Bending in the V Direction:  
 (3.81)  $EI_v v'' + M\phi = -M$  —(5.1)

Bending in the U Direction:  
 (3.82)  $EI_u u'' + M\phi = M$  —(5.2)

Torsional Equilibrium:  
 (3.83)  $EI_\omega \phi''' - (GK_T + \bar{K})\phi' + Mu' + Mv' = 0$  —(5.3)

where the symbols are as defined in Section 3 and the primes indicate differentiation with respect to  $w$ , the distance along the beam.

The equations are derived from the following set of assumptions:

- The material is elastic.
- The member is straight and prismatic.
- The cross-section is thin walled and open.
- Deflections are small.

All the constants are readily calculable (see Sect.4) with the exception of  $I_\omega$  and  $\bar{K}$ . It has been shown in <sup>18</sup> that warping is insignificant for angle sections, therefore,  $I_\omega = 0$ .  $\bar{K}$  can be determined from the following constitutive equations given by Galambos:

(3.85)  $\bar{K} = M(\beta_u - \beta_v)$  —(5.4)

(3.13)  $\beta_v = \frac{1}{I_v} \int v(v^2 + u^2)tds - 2v_0$  —(5.5)

(3.85)  $\beta_u = \frac{1}{I_u} \int u(u^2 + v^2)tds - 2u_0$  —(5.6)

\*The left hand equation numbers correspond to those in Galambos<sup>15</sup>.

For the idealised section (Fig.1)

$$v = \pm \left( u + \frac{B}{2\sqrt{2}} \right)$$

and  $ds = \pm \sqrt{2}du = \sqrt{2}dv$   
 Integration of equations (5.5) and (5.6) yields  
 $\beta_u = 0$  and  $\beta_v = \sqrt{2}B$

whereupon:

$$\bar{K} = -\sqrt{2}BM \quad \text{---(5.7)}$$

for equal angle sections.

The angle of twist  $\phi$  may now be determined by substituting this solution into eq. (5.3) to give:

$$-(GK_T - \sqrt{2}BM)\phi' + Mu' + Mv' = 0 \quad \text{(5.8)}$$

Differentiating this and substituting values of  $u''$ ,  $v''$  from (5.1), (5.2) gives:

$$\lambda_1 \phi'' + \lambda_2 \phi = -\lambda_3 \quad \text{---(5.9)}$$

$$\lambda_1 = GK_T - \sqrt{2}MB \quad \text{---(5.10)}$$

$$\lambda_2 = \frac{M^2}{E} \left( \frac{1}{I_v} + \frac{1}{I_u} \right) \quad \text{---(5.11)}$$

$$\lambda_3 = -\frac{M^2}{E} \left( \frac{1}{I_v} - \frac{1}{I_u} \right) \quad \text{---(5.12)}$$

The general solution is:—

$$\phi = A \cos \alpha w + D \sin \alpha w - \frac{\lambda_3}{\lambda_2} \quad \text{---(5.13)}$$

where  $\alpha = \left( \frac{\lambda_2}{\lambda_1} \right)^{1/2}$

with boundary conditions:

$$\phi(w=0) = \phi(w=L) = 0$$

one obtains

$$\phi_{max} = \frac{3}{5} \left( 1 - \frac{1}{\cos \alpha L/2} \right) \quad \text{---(5.14)}$$

#### 6. Case I. Critical buckling

For Case I the critical buckling condition occurs when:

$$\alpha L = \pi$$

as at this value

$$\phi = -\infty \quad \text{(see eq. 5.14).}$$

Since

$$\alpha L = \left( \frac{\lambda_2}{\lambda_1} \right)^{1/2} \cdot L$$

the critical moment is given by:

$$\pi = \frac{7.65ML}{B^2 E t^2 \left( 1 - \frac{5.5M}{E t^3} \right)^{1/2}} \quad \text{---(6.1)}$$

or

$$\left[ \frac{M_x}{E t^3} \right]_{cr} = \frac{\pi^2}{15} \cdot \frac{B^2}{L t} \left[ \left( \left( \frac{B^2}{L t} \right)^2 + \frac{10}{1.3\pi^2} \right)^{1/2} - \frac{B^2}{L t} \right] \quad \text{---(6.2)}$$

where

$(M_x)_{cr} = \sqrt{2}M_{cr}$  = the critical applied moment and the dimensionless parameters  $\frac{M_x}{E t^3}$  and  $\frac{L t}{B^2}$  are used to draw the critical buckling curve (Fig.5a). This curve allows an estimation of the critical applied moment for a given length and section. The horizontal lines represent the values of  $\frac{M_x}{E t^3}$  to produce a stress of  $3F_y$  (where  $F_y$  is the material yield stress), for yield stresses of 52 and 36 ksi and  $\frac{B}{L t}$  ratios of 6 and 16.

It has been shown<sup>13, 19</sup> that failure stresses will be unaffected by elastic buckling if the buckling stress is at least three times the material yield stress.

Thus, it can be established that  $F_b$  may be taken as  $0.66 F_y$  for the following cases:

Case	B/t	Range for $F_b = 0.66 F_y$
$F_y = 52$ ksi	6	$0 < L/t < 680$
	11	$0 < L/t < 570$
	16	$0 < L/t < 330$
$F_y = 36$ ksi	6	$0 < L/t < 990$
	11	$0 < L/t < 850$
	16	$0 < L/t < 690$

The critical stress corresponding to the critical moment in eq.6.2 can be obtained by:

$$\sigma_{cr} = \frac{(M_x)_{cr}}{Z} = \frac{9}{\sqrt{2}} \cdot \frac{(M_x)_{cr}}{B^2 t}$$

$$\sigma_{cr} = 0.424\pi^2 \frac{E t}{L}$$

$$\left[ \left( \left( \frac{B^2}{L t} \right)^2 + \frac{10}{1.3\pi^2} \right)^{1/2} - \frac{B^2}{L t} \right] \quad \text{---(6.3)}$$

This stress corresponds to  $F_b$  in Rule 5.4.3 of AS CA1 and the safe bending stress  $F_{bc}$  for the beam can be calculated using eqs.(4) and (5) of those rules as the purpose of these equations is for the purpose of such conversions to be made<sup>(12, 13)</sup>. The result of converting  $\sigma_{cr}$  in eq.(6.3) into  $F_{bc}$  is shown in Fig.5b, which may thus be used directly for design.

#### 7. Case I. Stress solution

The actual maximum section stress is obtained from the stress equation, which gives the stress at any point in the section as:

$$(2.74) \quad \sigma = \frac{M_x v}{I_v} - \frac{M_y u}{I_u} + E\omega_n \phi'' \quad \text{---(7.1)}$$

where  $M_x = M(1 + \phi)$ ,  $M_y = M(1 - \phi)$

It has been shown that the effect of warping is insignificant and since:

$$(2.62) \quad I_\omega = \int \omega_n^2 t ds = 0 \quad \text{---(7.2)}$$

then  $\omega_n = 0$

Equation (7.1) becomes:

$$\sigma = \frac{M(1 + \phi)v}{I_v} - \frac{M(1 - \phi)u}{I_u} \quad \text{---(7.3)}$$

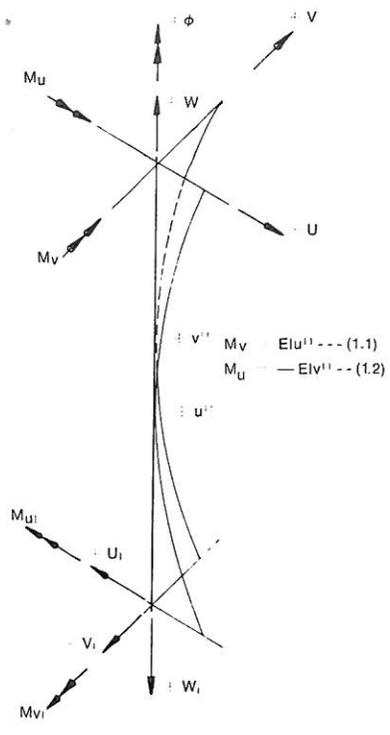
Substituting values for  $I_u$ ,  $I_v$  gives:

$$\sigma = \frac{3M}{B^2 t} (v + 4u)\phi + v - 4u \quad \text{---(7.4)}$$

This equation shows that the stress in the section is a linear function of the amount of twisting to which it has been subjected. The twist resulting from applied loads is given in eq.(5.14). Further twisting will result from initial eccentricities present in the unloaded angle. There are no specification limits for torsional eccentricity; however Massey<sup>17</sup> has measured the torsional eccentricity in steel I beams and suggests an average value of initial twist as:

$$\phi_0 = 0.436 \times 10^{-4} L \text{ radians} \quad \text{---(7.5)}$$

Values measured for two angle lengths are given in Fig.7 together with Massey's general estimate. The method of measurement is shown in Fig.8. The twist due to the weight stress is avoided by measuring the total twist ( $\phi_a$ ,  $\phi_b$ ) of the angle in two positions ninety degrees apart. The measured values are in agreement with Massey's equation.



**Note**  
 Axes are drawn with W or W<sub>1</sub> as the outward drawn normal from the surface under consideration.

Fig. 2. Sign convention

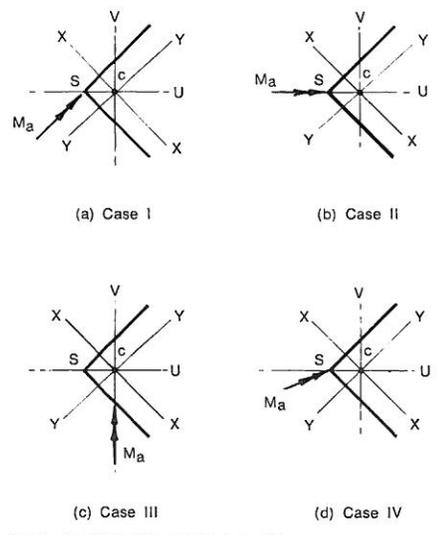


Fig. 3. Loading for cases I to IV  
 (a) Case I  
 (b) Case II  
 (c) Case III  
 (d) Case IV

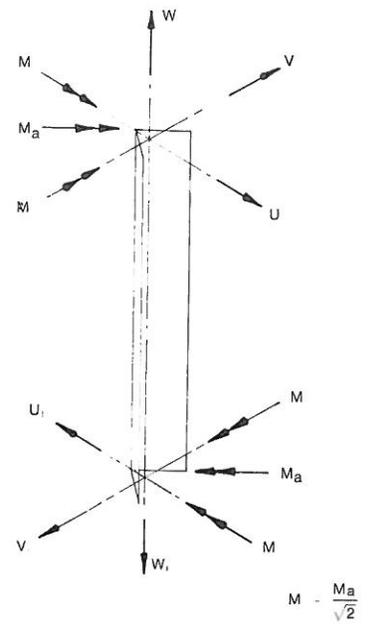


Fig. 4 Loading for case I

$$\left[ \frac{M_a}{E I^3} \right]_{cr} = \frac{\pi^2 B^2}{15 L t} \left[ \sqrt{\left( \frac{B^2}{L t} \right)^2 + \frac{10}{1.3 \pi^2} \frac{B^2}{L t}} \right] \quad \text{--- eq 6.2}$$

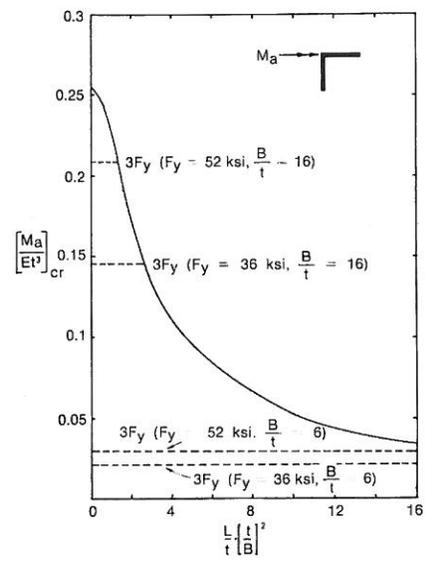


Fig. 5 (a). Critical buckling curve for case I

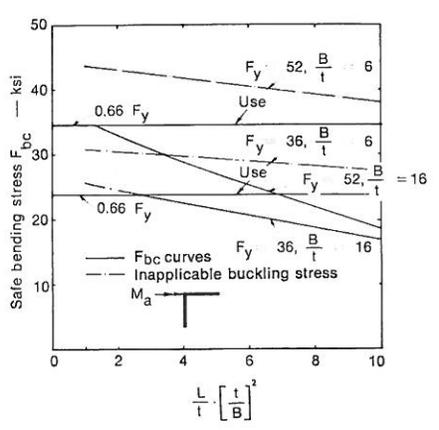


Fig. 5 (b). Graph for determination of F<sub>bc</sub> for case I

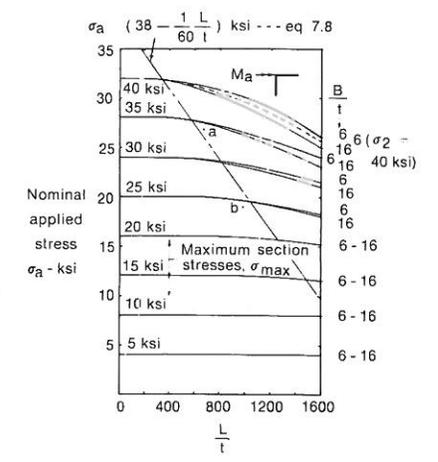


Fig. 6. Graph for determination of actual max. section stress for case I

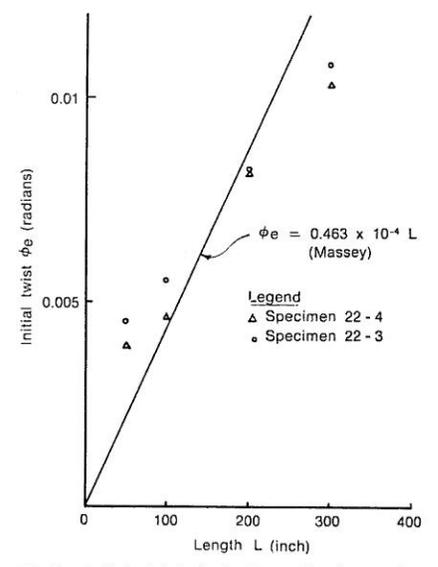
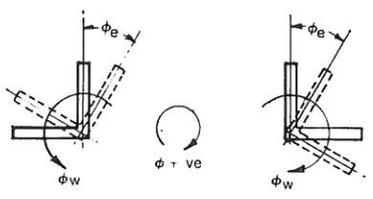
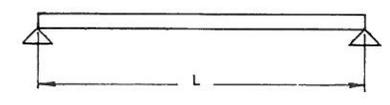


Fig. 7. Initial twist  $\phi_e$  test results for angle sections compared with Massey's relationship



$\phi_a = -\phi_w$      $\phi_b = \phi_w + \phi_e$   
 where  $\phi_e$  = Initial angle of twist  
 $\phi_w$  = Angle of twist due to dead weight  
 hence  $\phi_e = \frac{\phi_a + \phi_b}{2}$

Fig. 8 Method for determination of initial angle of twist

If  $\phi_e$  is considered, the stress equation (7.4) becomes:

$$\phi_2 = \frac{3M}{B^2 t} \left( (V + 4U)\phi_1 + V - 4U \right) \quad (7.6)$$

and if amplification effects near the buckling load are neglected

$$\phi_1 = \phi + \phi_e = \frac{3}{5} \left( 1 - \frac{1}{\cos \alpha L/2} \right) + 0.436 \times 10^{-4} \left( \frac{L}{t} \right) \quad (7.7)$$

where  $t$  in the  $\phi_e$  part of the expression has been put equal to 1 to produce the maximum value of  $\phi_e$  for values of  $\left( \frac{L}{t} \right)$

It can be seen from Section 5 that

$$\phi = f \left( \frac{L}{t}, \frac{\sigma_a}{E}, \frac{B}{t} \right),$$

where  $\sigma_a = \frac{M_a}{Z_a}$  does not include the stress due to twist. Thus, it is possible to produce curves of  $\sigma_a$  against  $L/t$  with contours of  $\sigma_{max}$ , the maximum section stress, as shown in Fig.6. Contours of  $\sigma_2$  (stress including initial twist) are also shown. Although the initial twist does cause a stress increase over  $\sigma_{max}$  for the range examined, the magnitude of this increase is small and only apparent in the graph for large values of  $M$  and  $\left( \frac{L}{t} \right)$ .

If the maximum section stress is calculated for Case I using conventional beam formulas and, if the applied moment is not resolved into components in the U and V axes, the calculated stress may be up to 50% less than the actual stress produced in the member. In terms of the symbols used above,  $\sigma_a$  may be up to 50% less than  $\sigma_{max}$ .

It is clear from the graph that twisting may be ignored if:

$$\sigma_a < \left( 38 - \frac{1}{60} \cdot \frac{L}{t} \right) \text{ ksi} \quad (7.8)$$

The expression is empirically determined from the form of the contours in Fig.6.

The two points 'a' and 'b' on Fig. 6 are obtained from the buckling solution given in Fig.5a, as the points where buckling does not influence the results. It is seen that the two approaches lead to similar results as 'a' and 'b' lie close to eq.(6.11). The buckling approach relies on the  $F_o - F_b$  conversion of eq.(4) and (5) of CA1, whereas the maximum stress approach is based on limiting the true peak stress to permissible values. The two solutions will therefore lead to similar but not identical results and the selection of a method will depend on the formulation of the problem.

### 8. Case II

Galambos<sup>15</sup> has shown that, for singly symmetric sections subject to the loading shown in Fig.3(b), the equations for lateral torsional buckling are:

$$(3.49) \quad EI_v u^{iv} + M\phi'' = 0 \quad (8.1)$$

$$(3.50) \quad EI_w \phi^{iv} - (GK_T + M\beta_T)\phi'' + Mu'' = 0 \quad (8.2)$$

Since  $\beta_u = 0$  (Sect.5) and warping is insignificant then

$$\lambda_a \phi^{iv} + \lambda_b \phi'' = 0 \quad (8.3)$$

where

$$\lambda_a = GK_T \quad (8.4)$$

$$\lambda_b = \frac{M^2}{EI_v} \quad (8.5)$$

The general solution is:

$$\phi'' = A \sin \alpha w + D \cos \alpha w \quad (8.6)$$

where

$$\alpha = \left( \frac{\lambda_b}{\lambda_a} \right)^{1/2} \quad (8.7)$$

Applying the end conditions of zero torsional restraining moment:

$$\phi''(w=0) = \phi''(w=L) = 0$$

gives

$$D = 0$$

and

$$\sin \alpha L = 0$$

The lowest critical moment occurs when:

$$\alpha L = \pi$$

i.e.

$$\left( \frac{\lambda_b}{\lambda_a} \right)^{1/2} \cdot L = \pi$$

or

$$M_{cr} = \frac{\pi E}{6\sqrt{1.3}} \cdot \frac{B^2 t^2}{L} \quad (8.8)$$

This result can also be obtained using the St. Venant buckling solution,

$$M_{cr} > \frac{\pi}{L} \left( EI_y GK_T \right)^{1/2} \quad (8.9)$$

Substituting  $M_{cr} = \sigma_{cr} \cdot Z_v$  in eq.(8.1) gives:

$$\sigma_{cr} = \frac{\pi E t}{2\sqrt{2.6} L} \quad (8.10)$$

which is the critical elastic buckling stress for the member. Using the 'elastic critical stress to design stress' conversion of the SAA Steel Structures Code CA1, Rule 5.4.3, eqs.(4) and (5), together with eq.(8.10), allows Fig.9 to be drawn. This figure shows both the critical buckling stress curve of eq.(8.10) and the curves of the design bending stress for yield stresses of 52 and 36 ksi derived as indicated above.

It is apparent that when  $L/t < 200$  for  $F_y = 52$  and  $L/t < 300$  for  $F_y = 36$  ksi,  $F_b$  may be taken as  $.66 F_y$ . This follows from the  $F_o > 3 F_y$  criterion used earlier.

Fig. 10 has been included to permit rapid estimation of  $F_b$  when the  $\frac{L}{t}$  ratio and the yield stress are known. The maximum stress in a section may be determined directly from the applied moment and the section modulus.

### 9. Case III

The loading for Case III is shown in Fig. 3c. Since the moment is applied about the weak axis there is no possibility of buckling to a more stable configuration and the beam will continue to bend about this axis only. Therefore conventional beam formulas may be used. The maximum stress is given by:

$$\sigma_{max} = \frac{M_v}{Z_v} \quad (9.1)$$

### 10. Loads not through the shear centre

Loads not through the shear centre will cause twisting of the angle section. Such loads will include the weight of the section acting through the centroid. These loads will cause an angle of twist given by:

$$\sigma = \frac{TL^2}{8GK_T} \quad (10.1)$$

For weight twisting, the value of T is:

$$T = \frac{WB}{4} \text{ in lb/in.}$$

where  $w = \text{lb/in. length}$ . The increased stresses due to additional twisting can be calculated from a generalised form of eq. (7.1).

$$\sigma = \frac{(M_u - \phi M_v)v}{I_u} - \frac{(M_v - \phi M_u)u}{I_v} \quad (10.2)$$

A more exact and comprehensive solution to this problem can be found in Ref.20.

### 11. Case IV

The loading for Case IV is shown in Fig. 3d. In this case the moment can be resolved into moments about the U and V axes (principal axes) and the theory of Cases I and II applies.

More generally, if the applied moment acts in any position between the X or Y and U axes, the component moments  $M_N$ ,  $M_M$ , resolved in the U, V directions, will produce stresses  $\sigma_N$  and  $\sigma_M$ . The design is satisfactory if:

$$\frac{\sigma_N}{F_N} + \frac{\sigma_M}{F_M} < 1 \quad (11.1)$$

where  $F_N$  and  $F_M$  are the maximum permissible stresses associated with the axis under consideration.

### 12. Conclusions

It has been shown that for laterally unrestrained angle beams the following relationships apply:

#### Case I:

The stress at any point in the section is:

$$\sigma = \frac{3M}{B^2 t} \left( (V + 4U)\phi_1 + V - 4U \right)$$

The maximum section stress is:

$$\sigma_{max} = \frac{3M(3 - \phi_1)}{\sqrt{2} B^2 t}$$

where the angle of twist  $\phi_1 = \phi + \phi_e$ .

If the maximum stress is calculated without resolving the applied load into U and V components, the result may be up to 50% less than the actual maximum stress. Twisting may be ignored if:

$$\sigma_a < \left( 38 - \frac{1}{60} \cdot \frac{L}{t} \right) \text{ ksi.}$$

An alternative method of beam design for Case I is to consider the critical buckling moment given by:

$$\left[ \frac{M_a}{Et^3} \right]_{cr} = \frac{\pi^2}{15} \cdot \frac{B^2}{Lt} \left[ \left( \frac{B^2}{Lt} \right)^2 + 0.785 \right]^{1/2} - \frac{B^2}{Lt}$$

and then use Ref.7, Rule 5.4.3, to convert this into a design stress.

The values of  $L/t$  for which the safe bending stress,  $F_b$ , may be taken as  $0.66 F_y$ , are shown in Section 6.

#### Case II:

The angle of twist  $\phi$  has no direct effect in this case and the safe bending stress can be calculated using <sup>7</sup>, Rule 5.4.3, where the critical buckling stress  $F_o$ , is obtained from:

$$F_o = \frac{\pi E}{2\sqrt{2.6}} \cdot \frac{t}{L}$$

This may be ignored if  $\frac{L}{t} < 200$  for  $F_y = 52$  and  $\frac{L}{t} < 300$  for  $F_y = 36$  ksi, and a design stress of  $0.66 F_y$  may be used.

**Case III:**

No secondary effects will occur and conventional beam formulas may be used.

**Case IV:**

The design is satisfactory if:

$$\frac{\sigma_N}{F_N} + \frac{\sigma_M}{F_M} < 1$$

where  $F_N$  and  $F_M$  are the appropriate maximum permissible stresses.

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**13. Summary**

The design criteria for angle beams can be summarised as follows:

Case	Use Simple Principal Axis Loading if:	Additional Effects if Column 2 Not Satisfied
I	(i) Stress Solution: $\sigma_a < 38 - \frac{1}{60} \cdot \frac{L}{t}$ (ii) Critical Buckling Solution: See Table below.	$\sigma_{max} = \frac{2 \cdot 12M}{B \cdot t} (3 - \phi_1)$ (Fig.6) Use $F_o \rightarrow F_{bc}$ conversion of Ref.7.
II	$\frac{L}{t} < 200$ ( $F_y = 52$ ksi) $\frac{L}{t} < 300$ ( $F_y = 36$ ksi)	Use Fig.10.
III	All Sections	—
Inter-mediate Loadings		$\frac{\sigma_N}{F_N} + \frac{\sigma_M}{F_M} < 1$

**Critical Buckling Solution Case I:**

Yield Stress $F_y$	B/t	Range for $F_{bc} = 0.66 F_y$
52	6	$0 < L/t < 680$
	11	$0 < L/t < 570$
	16	$0 < L/t < 330$
36	6	$0 < L/t < 990$
	11	$0 < L/t < 850$
	16	$0 < L/t < 690$

For other B/t values, interpolate.

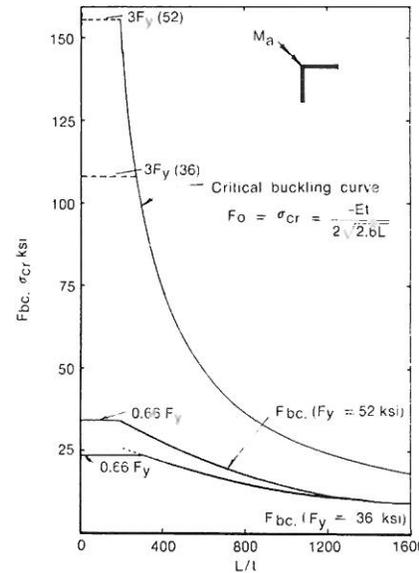


Fig.9. Critical buckling curve and curves for safe bending stress case II

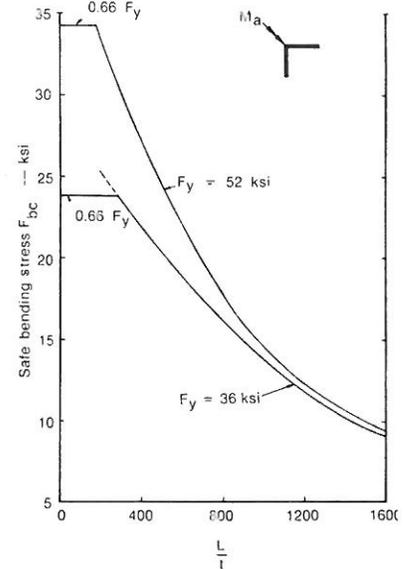


Fig.10 Graph for determination of  $F_{bc}$  for case II