BASIC DESIGN FOR STABILITY

Part I - Columns and Frames

Lecture 3 – Frame Stability - Alignment Charts and Modifications

AMERICAN INSTITUTE OF STEEL CONSTRUCTION INC.
AISC
FOUNDED 1921

STRUCTURAL STABILITY RESEARCH COUNCIL
SSRC
FOUNDED 1944

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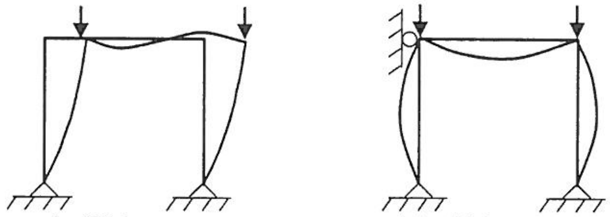
BASIC DESIGN FOR STABILITY
Part I - Columns and Frames

- **Introduction to Stability**
- **Column Stability**
- **Frame Stability – Alignment Chart and Modifications**
- **Frame Stability - P-Δ Method and Frequently Asked Questions**

ALIGNMENT CHART

- Since the 1960's, most column design has been performed using K-factors, which have traditionally been found using the alignment chart.
- To determine the K-factor for a column, engineers must first address the bracing condition of the frame. Two different nomographs are available to evaluate the K-factors:



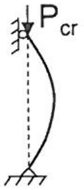
a) Sidesway Permitted

b) Sidesway Prevented

EFFECTIVE LENGTH FACTORS

- A column with pinned ends is often referred to as an Euler column for which the buckling capacity is:

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$



- The use of K-factors permits us to calculate an artificial length that allows us to use the Euler equation to evaluate the buckling capacity of a column with relatively general support conditions.

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$ K ≡ Effective Length Factor

IDEALIZED K-FACTORS

The AISC Commentary provides a number of K-factors for idealized support conditions.

Although these values are useful for gaining a “feel” for the range of K-factors for specific problems, they have very little practical value.

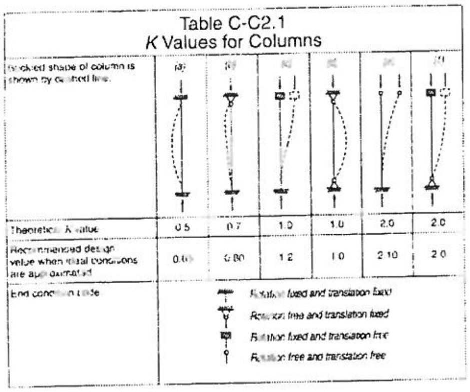
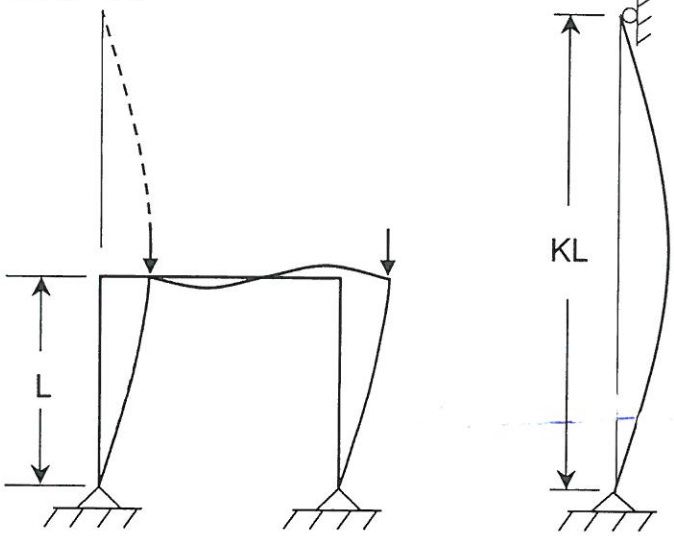


Table C-C2.1
K Values for Columns

	(a)	(b)	(c)	(d)	(e)	(f)
Buckled shape of column is shown by dashed line.						
Theoretical *K* value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.10	2.0
End condition code	Rotation fixed and translation fixed Rotation free and translation fixed Rotation fixed and translation free Rotation free and translation free					

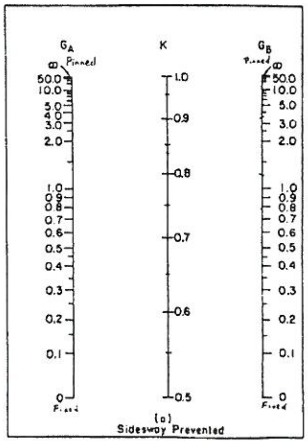
EFFECTIVE LENGTH

Most practical problems don't have ideal support conditions.

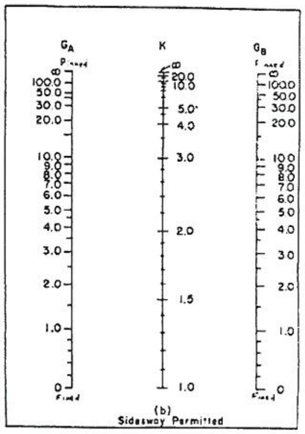


ALIGNMENT CHARTS

- The K-factors can be obtained for a particular frame graphically using the following alignment charts:



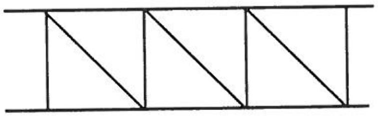
a) braced



b) unbraced

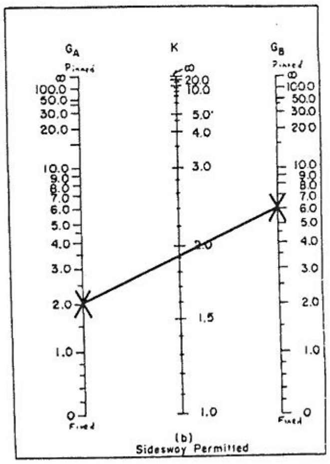
BRACED FRAMES

- Although the AISC Commentary provides the expressions and alignment charts for braced frames, in most design situations K=1.0 is recommended.
- The values of the K-factors for sidesway prevented are often unattainable because the bracing requirements are too severe.
- K<1.0 is possible for systems such as trusses or other structural systems with large lateral stiffness values.



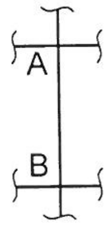
UNBRACED FRAMES

- Finding the K-factor requires an evaluation of the ratio of the I/L of all the columns at a joint to the beams with rigid connections.



(b)
Sidesway Permitted

G is often defined as follows:



$$G = \frac{\Sigma\left(I_c / L_c\right)}{\Sigma\left(I_g / L_g\right)}$$

ALIGNMENT CHART EQUATION

In lieu of a graphical solution, the K-factor can also be determined using the equation that the alignment chart is based upon, which is given in the following expression:

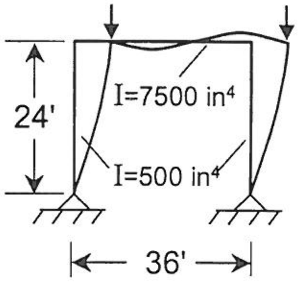
Sway Mode:

$$\frac{\left(\frac{\pi}{K}\right)^2 - 36G_AG_B}{6(G_A+G_B)} - \frac{\left(\frac{\pi}{K}\right)}{\tan\left(\frac{\pi}{K}\right)} = 0$$

The alignment chart provides a relatively simple method of solving for the buckling capacity of frames.

ALIGNMENT CHART EXAMPLE 1A

Consider the frame shown to the right. What is the (in-plane) elastic buckling capacity, P_{cr}, for the frame?



$$G_{Top} = \frac{I_c/L_c}{I_{bA}/L_b} = \frac{500/24}{7500/36} = 0.10$$

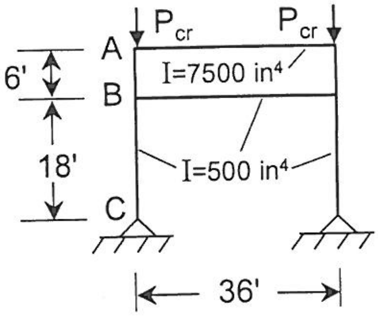
$$G_{Bot} = \infty \qquad \Rightarrow K = 2.03$$

$$\therefore\ P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29000)(500\ )}{(2.03x24x12)^2} = 418\ kips$$

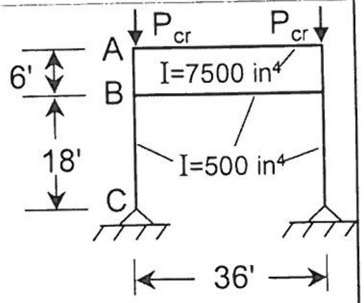
FEA Solution: 416 kips

ALIGNMENT CHART EXAMPLE 1B

Reconsider the previous problem. After the structure has been used for a period of time, the owner decides to change the use of the structure and needs to increase the capacity of the frame. To increase the frame capacity, he permits the clear height to be reduced from 24 ft. to 18 ft. To increase the capacity, a bracing beam is put in with an I=500 in⁴. What is the new capacity of the frame?



ALIGNMENT CHART EXAMPLE 1B



By observation, Column BC will be the critical member due support conditions and the longer unbraced length.

Joint B:

$$G_B = \frac{\Sigma(I_c/L_c)}{\Sigma(I_g/L_g)} = \frac{(500/6)+(500/18)}{(500/36)} = 8$$

$G_C = \infty$

$K_{AB} = 4.07$

$$\therefore\ P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29000)(500\ )}{(4.07x18x12)^2} = 185\ kips\ ????$$ (was 418 k)

FEA Solution: 554 kips

We'll revisit this problem later in the lecture.

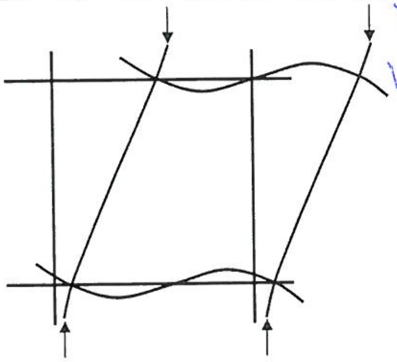
EXAMPLE 1 OBSERVATIONS

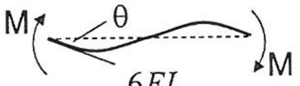
- The alignment chart solution has excellent agreement with the FEA solution for Example 1A. The solution was essentially exact (418 kips versus 416 kips).
- For Example 1B, the alignment chart solution produces silly results that had very poor agreement with the FEA solution (185 kips versus 554 kips).
- The reason for the good and poor agreement with the FEA solutions is primarily due to the basic assumptions that were made in the development of the alignment chart solution. The frame in Example 1A matched the frame that was used in the alignment chart development while the frame in Example 1B did not.

ALIGNMENT CHART DEVELOPMENT

- An important aspect of accurately employing the alignment chart to determine K-factors is to understand the basic assumptions that were made in its development.
- Reviewing the derivation of the basic equation that the alignment chart is based upon will provide insight to the limitations of the alignment chart as well as demonstrating corrections that can be applied when our structures do not satisfy the basic requirements.
- The derivation can be completed relatively simply by employing the differential equation technique.

ALIGNMENT CHART ASSUMPTIONS – SWAY CASE





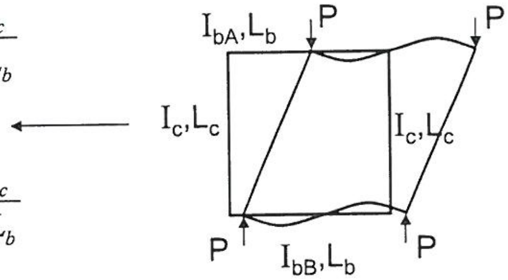
$$M = \frac{6EI}{L}\theta$$

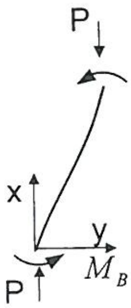
1. Elastic Behavior ✓
2. Reverse curvature bending of beams. ✓ – Equal & opposite
3. Joints are rigid.
4. Joint restraint is distributed to the columns above and below the joint proportional to I/L of the two columns.
5. All columns buckle simultaneously.
6. No axial force in the girders.

ALIGNMENT CHART DERIVATION – SWAY CASE

$$G_A = \frac{I_c/L_c}{I_{bA}/L_b}$$

$$G_B = \frac{I_c/L_c}{I_{bB}/L_b}$$

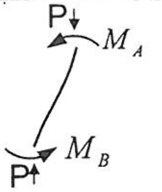




$$M_A = \frac{6EI_{bA}}{L_b}\theta_A = \frac{6EI_c/L_c}{G_A} y'(L_c)$$

$$M_B = \frac{6EI_{bB}}{L_b}\theta_B = \frac{6EI_c/L_c}{G_B} y'(0)$$

ALIGNMENT CHART DERIVATION – SWAY CASE



Define: $k = \sqrt{\dfrac{P_{cr}}{EI_c}}$

Differential Equations of Flexure:

A) Flexure: $y'' = -{M}/{EI}$ Note: M = f(P)

y_H = Asin(kx) + Bcos(kx) (Homogeneous Solution)

B) Shear: $y''' + k^2 y' = -{V}/{EI}$

y_H = Asin(kx) + Bcos(kx) + C

C) Lateral Load: $y^{IV} + k^2 y'' = 0$

y_H = Asin(kx) + Bcos(kx) + Cx + D

ALIGNMENT CHART DERIVATION – SWAY CASE

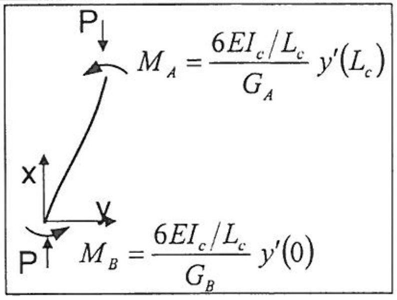
Since we have zero shear the 3rd order DE is efficient:

$$y''' + k^2 y' = 0$$

$y = A\sin(kx) + B\cos(kx) + C$

$y' = Ak\cos(kx) \cancel{+} Bk\sin(kx)$

$y'' = Ak^2\sin(kx) \cancel{+} Bk^2\cos(kx)$



$M_A = \dfrac{6EI_c/L_c}{G_A}\, y'(L_c)$

$M_B = \dfrac{6EI_c/L_c}{G_B}\, y'(0)$

Boundary Conditions (BC):

$$[1]\left(y'' = -{M_B}/{EI_C}\right)_{x=0} \rightarrow -Bk^2(1) = -\left(\frac{-6EI_C/L_C}{EI_C G_B}\right)Ak(1)$$

$$[2]\left(y'' = -{M_A}/{EI_C}\right)_{x=L_C} \rightarrow -Ak^2\sin(kL_C) - Bk^2\cos(kL_C) =$$

$$-\left(\frac{6EI_C/L_C}{EI_C G_A}\right)\left(Ak\cos(kL_c) - Bk\sin(kL_C)\right)$$

ALIGNMENT CHART DERIVATION – SWAY CASE

From BC [1]: $A = -\frac{BkL_c}{6G_B}$

BC [2] becomes:

$$\frac{Bk^3L_C}{6G_B}\sin(kL_C) - Bk^2\cos(kL_C) = \frac{BG_Ak^2}{G_B}\cos(kL_C) + \frac{6BG_Ak}{L_C}\sin(kL_C)$$

Divide both sides of the equation by $(Bk(\sin(kL_C)))/(6G_BL_C)$

$$k^2L_C^2 - \frac{6G_BkL_C}{\tan(kL_C)} = \frac{6G_AkL_C}{\tan(kL_C)} + 36G_AG_B \qquad (*)$$

Defining - $P_{cr} = \frac{\pi^2 EI_C}{(KL_C)^2} \Rightarrow kL_C = \sqrt{\frac{P_{cr}L_c^2}{EI_C}} = \frac{\pi}{K}$ (**)

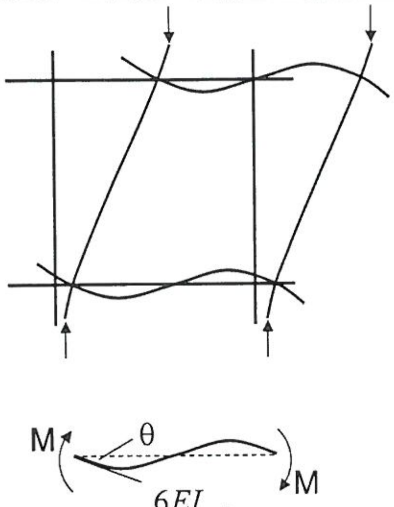
Inserting (**) and Rearranging (*):

$$\boxed{\frac{\left(\frac{\pi}{K}\right)^2 - 36G_AG_B}{6(G_A + G_B)} - \frac{\left(\frac{\pi}{K}\right)}{\tan\left(\frac{\pi}{K}\right)} = 0}$$

Alignment Chart Equation

ALIGNMENT
CHART
CORRECTIONS

ALIGNMENT CHART ASSUMPTIONS – SWAY CASE



$$M = \frac{6EI}{L}\theta$$

1. Elastic Behavior
2. Reverse curvature bending of beams.
3. Joints are rigid.
4. Joint restraint is distributed to the columns above and below the joint proportional to I/L of the two columns.
5. All columns buckle simultaneously.
6. No axial force in the girders.

ASSUMPTION of ELASTIC BEHAVIOR

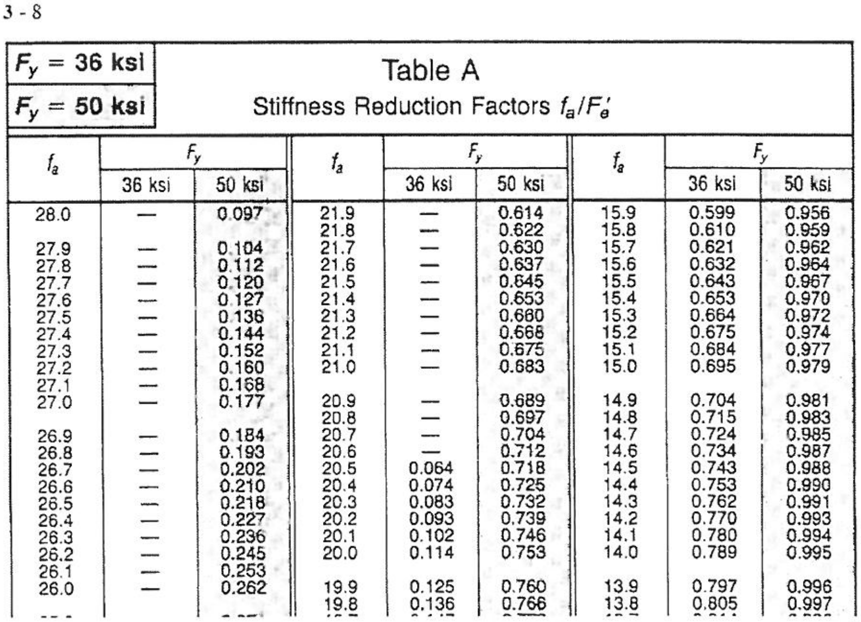
- The condition of elastic behavior in the development of the alignment chart solution assumes that the columns and beams have the same material stiffness.
- If different materials were used in a frame, a relatively simple modification can be applied the definition of G:

$$G = \frac{\sum E_c I_c / L_c}{\sum E_b I_b / L_b}$$

ASSUMPTION of ELASTIC BEHAVIOR

- In a steel frame, even though the elastic stiffness of the columns and beams are the same, a similar modification is necessary if the columns are loaded into the inelastic range.
- The last lecture showed that even though the applied column stress (P/A) may be well below the yield stress, the presence of residual stresses cause yielding on portions of the cross-section thereby reducing the effective material stiffness of the cross-section to a value referred to as the tangent stiffness, $E_T < E$.
- The tangent stiffness, E_T can be obtained from the stiffness reduction factor $\tau = E_T/E$, which is provided in Tables in both the ASD and LRFD Specifications.

ASD (9th Ed.) – Stiffness Reduction Factor



3 - 8

F_y = 36 ksi

F_y = 50 ksi

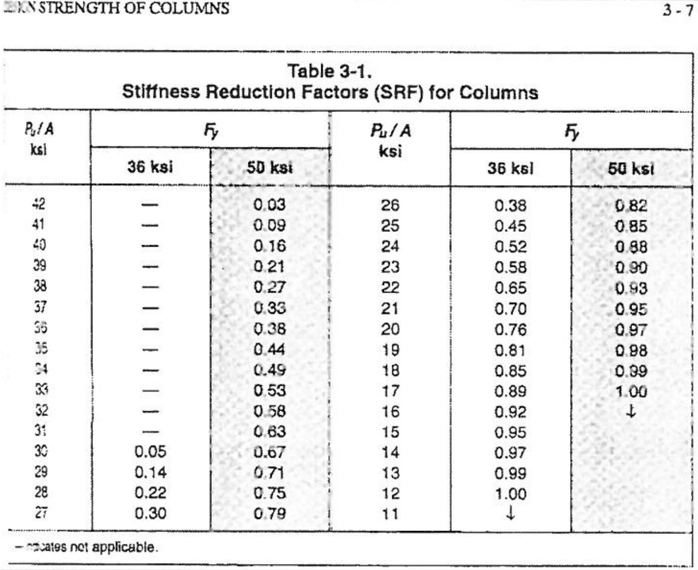
Table A

Stiffness Reduction Factors f_a/F'_e

f_a	F_y		f_a	F_y		f_a	F_y	
	36 ksi	50 ksi		36 ksi	50 ksi		36 ksi	50 ksi
28.0	—	0.097	21.9	—	0.614	15.9	0.599	0.956
			21.8	—	0.622	15.8	0.610	0.959
27.9	—	0.104	21.7	—	0.630	15.7	0.621	0.962
27.8	—	0.112	21.6	—	0.637	15.6	0.632	0.964
27.7	—	0.120	21.5	—	0.645	15.5	0.643	0.967
27.6	—	0.127	21.4	—	0.653	15.4	0.653	0.970
27.5	—	0.136	21.3	—	0.660	15.3	0.664	0.972
27.4	—	0.144	21.2	—	0.668	15.2	0.675	0.974
27.3	—	0.152	21.1	—	0.675	15.1	0.684	0.977
27.2	—	0.160	21.0	—	0.683	15.0	0.695	0.979
27.1	—	0.168						
27.0	—	0.177	20.9	—	0.689	14.9	0.704	0.981
			20.8	—	0.697	14.8	0.715	0.983
26.9	—	0.184	20.7	—	0.704	14.7	0.724	0.985
26.8	—	0.193	20.6	—	0.712	14.6	0.734	0.987
26.7	—	0.202	20.5	0.064	0.718	14.5	0.743	0.988
26.6	—	0.210	20.4	0.074	0.725	14.4	0.753	0.990
26.5	—	0.218	20.3	0.083	0.732	14.3	0.762	0.991
26.4	—	0.227	20.2	0.093	0.739	14.2	0.770	0.993
26.3	—	0.236	20.1	0.102	0.746	14.1	0.780	0.994
26.2	—	0.245	20.0	0.114	0.753	14.0	0.789	0.995
26.1	—	0.253						
26.0	—	0.262	19.9	0.125	0.760	13.9	0.797	0.996
			19.8	0.136	0.766	13.8	0.805	0.997

LRFD (2th Ed.) – Stiffness Reduction Factor

$$\boxed{\mathrm{SRF} = \tau = -7.38\frac{P_u}{P_y}\log\left(\frac{P_u}{0.85P_y}\right) \quad \text{for} \quad \frac{P_u}{P_y} \geq 0.33}$$



N STRENGTH OF COLUMNS 3 - 7

Table 3-1.
Stiffness Reduction Factors (SRF) for Columns

P_u/A ksi	F_y		P_u/A ksi	F_y	
	36 ksi	50 ksi		36 ksi	50 ksi
42	—	0.03	26	0.38	0.82
41	—	0.09	25	0.45	0.85
40	—	0.16	24	0.52	0.88
39	—	0.21	23	0.58	0.90
38	—	0.27	22	0.65	0.93
37	—	0.33	21	0.70	0.95
36	—	0.38	20	0.76	0.97
35	—	0.44	19	0.81	0.98
34	—	0.49	18	0.85	0.99
33	—	0.53	17	0.89	1.00
32	—	0.58	16	0.92	↓
31	—	0.63	15	0.95	
30	0.05	0.67	14	0.97	
29	0.14	0.71	13	0.99	
28	0.22	0.75	12	1.00	
27	0.30	0.79	11	↓	

— indicates not applicable.

EXPRESSION FOR G - Inelastic

$$G = \frac{\sum E_T I_c / L_c}{\sum E I_b / L_b} = \frac{\sum \tau I_c / L_c}{\sum I_b / L_b}$$

When the columns are inelastic, the beams do a better job of restraining the columns than for the case with elastic columns. It's conservative to treat the columns elastic, however it is generally wasteful since the K-factors can be substantially lower than the elastic values.

ALIGNMENT CHART EXAMPLE 2 (LRFD)

The frame shown has P_u= 590 k column loads. The columns are braced out of plane (K=1.0). Design the columns w/ Gr. 50 steel.

Columns: Try a W14x74, A = 21.8 in^2, I_x =795 in^4, I_y = 134 in^4

Beam: W18x50, I_x = 800 in^4

Check out of plane, KL_y = 16': ϕP_n =598 k > 590 k OK

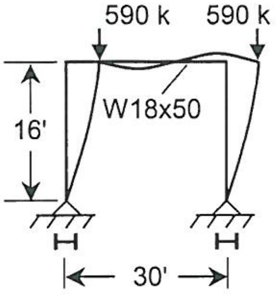
In-Plane: P_u/A = 590/21.8= 27 ksi τ = 0.78

$$G_T = \frac{\tau I_c/L_c}{I_b/L_b} = \frac{0.78(795/16)}{800/30} = 1.45,\ G_B = \infty$$

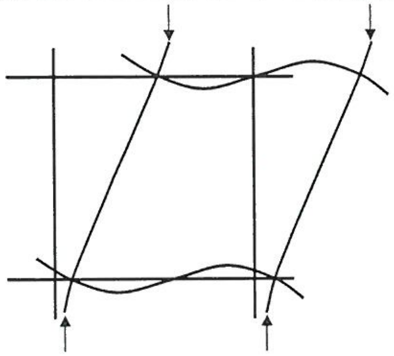
$$\Rightarrow \mathrm{K} = 2.47$$

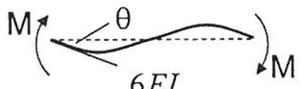
$$\therefore\ P_{cr} = \frac{213000\tau I}{(KL)^2}$$

$$= \frac{213000(0.785)(795\ )}{(2.47x16x12)^2} = 591\,k > 590k$$



ALIGNMENT CHART ASSUMPTIONS – SWAY CASE





$$M = \frac{6EI}{L}\theta$$

1. Elastic Behavior
2. Reverse curvature bending of beams. 
3. Joints are rigid.
4. Joint restraint is distributed to the columns above and below the joint proportional to I/L of the two columns.
5. All columns buckle simultaneously.
6. No axial force in the girders.

2. BENDING STIFFNESS OF RESTRAINING MEMBERS

- The assumption of reverse curvature bending can cause significant errors in the alignment chart solution if the restraining members do not satisfy this condition.
- Applying a simple modification to the stiffness of the restraining members, however can be used to avoid these errors.

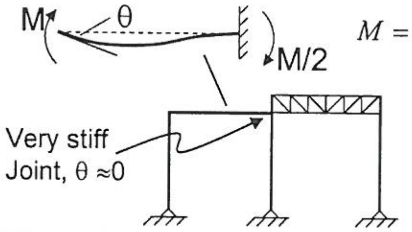
Redefine G:

$$G = \frac{\sum \tau I_c / L_c}{\sum m I_b / L_b} \qquad m = \frac{actual\ stiffness}{assumed\ stiffness}$$

BENDING STIFFNESS OF RESTRAINING MEMBERS

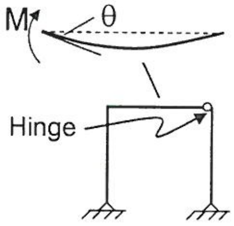
$$M = \frac{6EI}{L}\theta$$

m = 6/6 = 1.0



$$M = \frac{4EI}{L}\theta$$

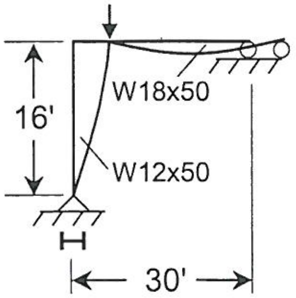
m = 4/6 = 0.67



M=0 $M = \frac{3EI}{L}\theta$

m = 3/6 = 0.5

ALIGNMENT CHART EXAMPLE 3



Considering the frame shown to the right, what is the (in-plane) elastic buckling capacity, P_{cr} for the frame?

Columns: W12x50, I_x =391 in^4

Beam: W18x50, I_x = 800 in^4

Far end of beam is pinned, (3EI/L stiffness). Therefore m=1/2

$$G_{Top} = \frac{I_c/L_c}{m\,I_{bA}/L_b} = \frac{391/16}{(0.5)800/30} = 1.84$$

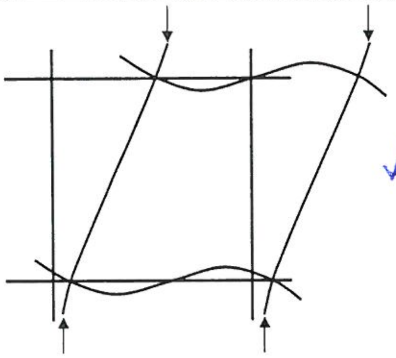
$$G_{Bot} = \infty \qquad \Rightarrow K = 2.59$$

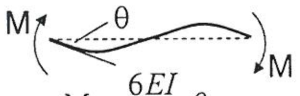
$$\therefore\ P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29000)(391)}{(2.59x16x12)^2} = 452\ kips$$

Uncorrected solution produces P_{cr} = 574 kips

FEA Solution: 454 kips

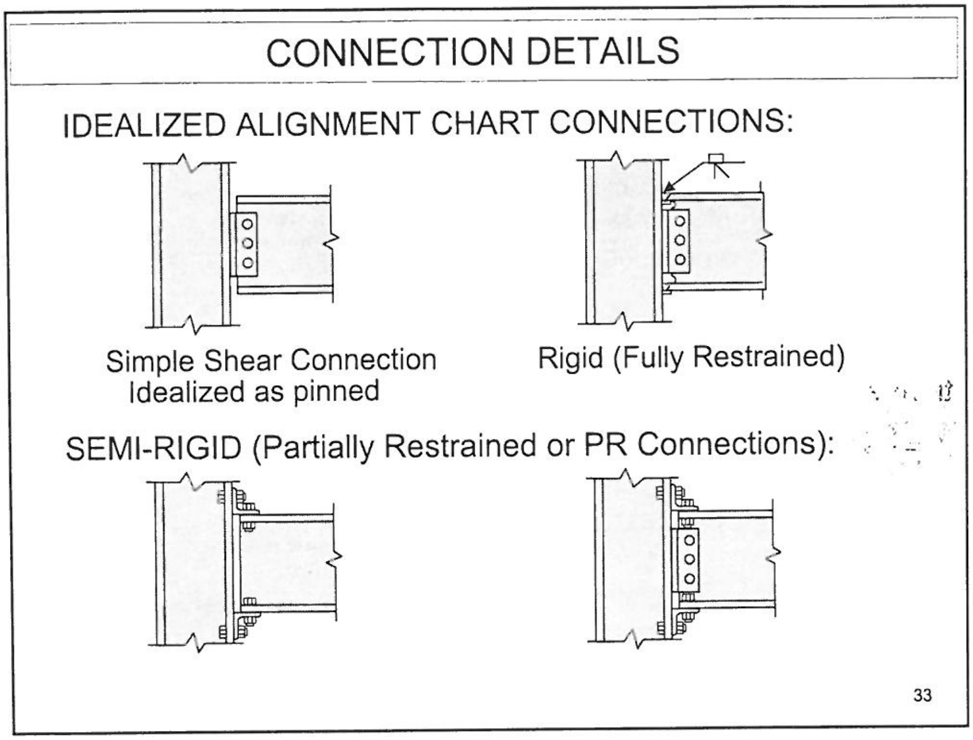
ALIGNMENT CHART ASSUMPTIONS – SWAY CASE





$$M = \frac{6EI}{L}\theta$$

1. Elastic Behavior
2. Reverse curvature bending of beams.
3. ✓ Joints are rigid.
4. Joint restraint is distributed to the columns above and below the joint proportional to I/L of the two columns.
5. All columns buckle simultaneously.
6. No axial force in the girders.



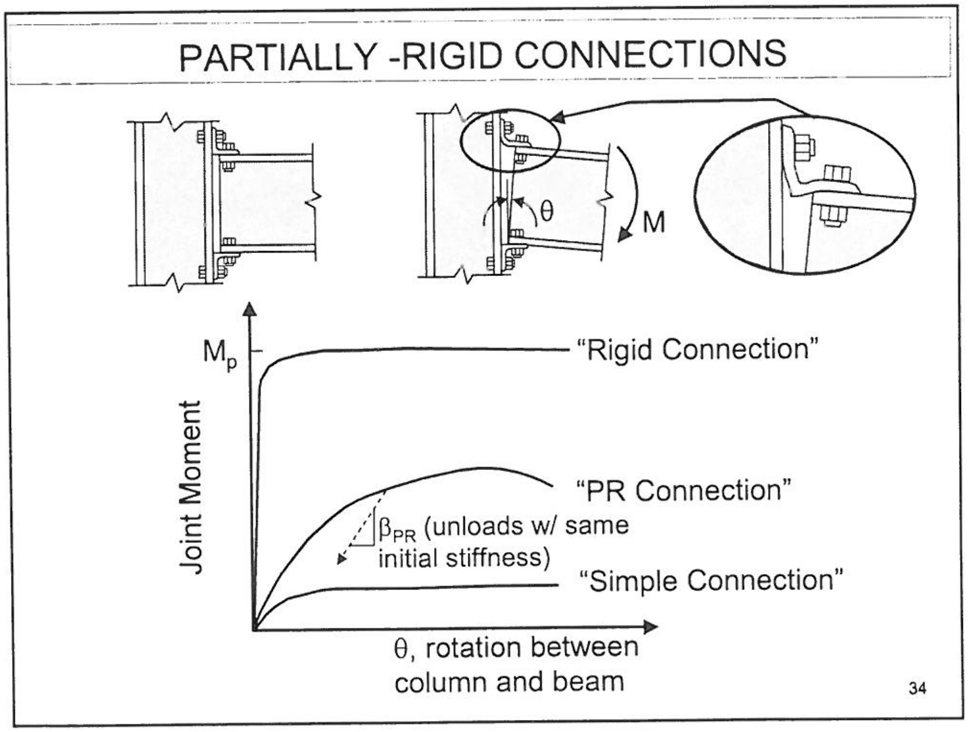
CONNECTION DETAILS

IDEALIZED ALIGNMENT CHART CONNECTIONS:

Simple Shear Connection
Idealized as pinned

Rigid (Fully Restrained)

SEMI-RIGID (Partially Restrained or PR Connections):



PARTIALLY -RIGID CONNECTIONS

β_{PR} (unloads w/ same initial stiffness)

θ, rotation between column and beam

PARTIALLY -RIGID CONNECTIONS – Past Studies

- The use of PR connections requires determining the stiffness and strength of the connections. There have been several previous studies on the stiffness and strength of a variety of these connections. These studies have included experimental and analytical studies.

1. Ackroyd, M. H. "Simplified Frame Design of Type PR Construction", AISC Eng. Journal, Vol. 24, No. 4, pp. 141-146.
2. Deierlein, G.G., Hsieh, S. H., and Shen, Y. J., "Computer-Aided Design of Steel Structures with Flexible Connections," Proceedings of 1990 NASCC, pp. 9.1-9.21.
3. Disque, R. O., "Directional Moment Connections – Proposed Design Method for Unbraced Steel Frames," Engineering Journal, Vol. 12, No. 1, pp. 14-18.
4. Gerstle, K. H., and Ackroyd, M. H., "Behavior and Design of Flexibly Connected Building Frames", Proceedings of the 1989 NASCC, pp 1.1-1.28.

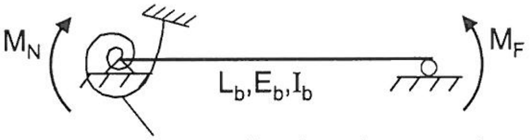
PARTIALLY -RIGID CONNECTIONS – Past Studies

5. Geschwindner, L. F., AISC Engineering Journal Vol 28, No. 2 (2nd Quar.) 1991.
6. Goverdhan, A. V., "A Collection of Experimental Moment Rotation Curves and Evaluation of Prediction Equations for Semi-Rigid Connections, MS Thesis, Vanderbilt Univ. 1984.
7. Kishi, N. and Chen, W.F., ASCE Journal of Structural Engineering, Vol. 116 No. 7 July 1990, pp. 1813-1834.
8. Nethercot, D. A. and Chen, W. F., "Effects of Connections on Columns," Journal of Constructional Steel Research, pp. 201-239. Elsevier Applied Science Publishers.

PARTIALLY -RIGID CONNECTIONS

If we know the connection stiffness, β, we can modify the alignment chart solution to account for the PR connection.

Consider the moment rotational behavior of a beam with PR connections:



Moments shown in a positive sense

Connection to column under consideration with stiffness, β.

Angle of rotation at the column due to moments:

$$\theta_A = \frac{M_N L}{3EI} + \frac{M_F L}{6EI} + \frac{M_N}{\beta} = \frac{2M_N L}{6EI} + \frac{M_N}{M_N}\frac{M_F L}{6EI} + \frac{M_N}{\beta}\frac{6EI/L}{6EI/L}$$

$$\theta_A = \frac{M_N}{6EI/L}\left[2 + \frac{M_F}{M_N} + \frac{6EI/L}{\beta}\right]$$

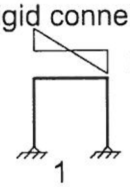
PARTIALLY -RIGID CONNECTIONS

Beam Stiffness: $\frac{M_N}{\theta_A} = \frac{6EI/L}{\left[2 + \frac{M_F}{M_N} + \frac{6EI/L}{\beta}\right]}$

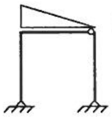
Recall from the last section:

$$G = \frac{\sum \tau I_c/L_c}{\sum m I_b/L_b} \qquad m = \frac{actual\ stiffness}{assumed\ stiffness} = \frac{1}{\left[2 + \frac{M_F}{M_N} + \frac{6EI/L}{\beta}\right]}$$

Check for rigid connections $(\beta = \infty)$:



$M_N = -M_F$



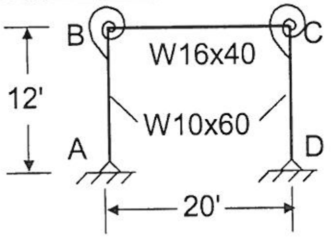
$M_F = 0$

$$m = \frac{1}{\left[2 + \frac{-M}{M} + 0\right]} = \frac{1}{1} = 1$$

Okay

$$m = \frac{1}{\left[2 + \frac{0}{M} + 0\right]} = \frac{1}{2}$$

ALIGNMENT CHART EXAMPLE 4



Consider the frame shown that has a PR connection with β=37100 k"/rad between the W10x60 columns and the W16x40 beams. Determine the elastic buckling capacity of the frame.

W16x40 I = 518, W10x60 I=116

Due to the symmetry, $M_N=M_F$:

$$G_T = \frac{\tau I_c/L_c}{mI_b/L_b} = \frac{1.0(116/12)}{518/20}\underbrace{\left[2-1+\frac{6x29000x518}{240(37100)}\right]}_{1/m} = 4.15$$

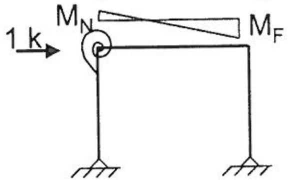
$$G_B = \infty \quad \Rightarrow \quad K = 3.22$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29000x116)}{(3.22x12x12)^2} = 155\ k$$

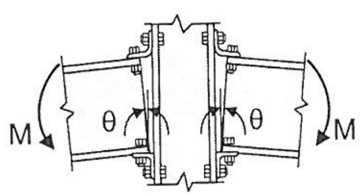
FEA: P_{cr} = 155 k

PARTIALLY -RIGID CONNECTIONS

- If the frame is unsymmetric, a lateral load needs to be applied to determine the magnitude of M_N and M_F relative to one another.

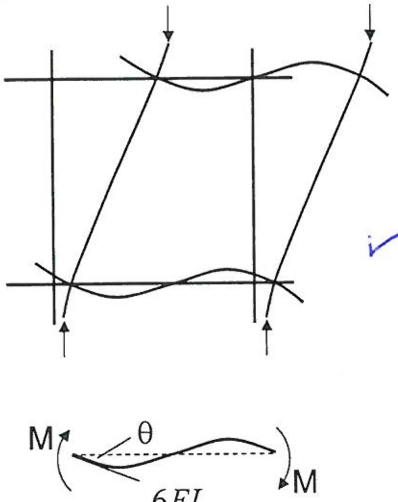


- For columns with PR connections framing in on both sides, only rely on one of the joints for restraint against buckling.



Rotation of the joint during buckling will require one of the joints to "unload" or relax before the connection can deliver restraint to the column.

ALIGNMENT CHART ASSUMPTIONS – SWAY CASE



$$M = \frac{6EI}{L}\theta$$

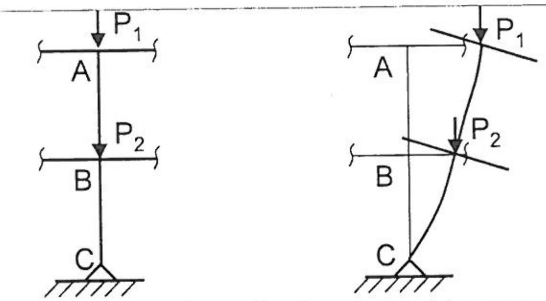
1. Elastic Behavior
2. Reverse curvature bending of beams.
3. Joints are rigid.
4. ✓ Joint restraint is distributed to the columns above and below the joint proportional to I/L of the two columns.
5. All columns buckle simultaneously.
6. No axial force in the girders.

COLUMNS SHARE JOINT RESTRAINT EQUALLY

$$G = \frac{\sum \tau I_c / L_c}{\sum m I_b / L_b}$$

- Part of the reason that direct application of the alignment chart solution for Example 1B with the bracing beam produced poor results is due to the assumption that the joint restraint is shared equally between all the columns framing into the joint.
- In reality, the joint restraint is distributed to the columns based upon the need or demand from the column.

DISTRIBUTION of JOINT RESTRAINT



Consider the above system of columns and beams:

- Columns AB and BC have the same moment of inertia, I. This is not a requirement for this principle, however it helps demonstrate the behavior.
- If the two columns are both buckling members, they must buckle simultaneously.
- Since Column BC has a higher load, it will therefore demand more of the joint restraint than Column AB.

DISTRIBUTION of JOINT RESTRAINT

We can account for the distribution of the joint restraint by modifying our expression for G as reported in Stockwell (AISC Eng. Journal 1976 – 3rd Quarter):

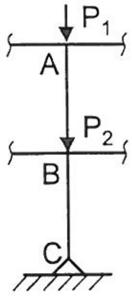
$$G_i = \frac{\left(\tau I_c / L_c\right)_i}{X_i \sum m I_b / L_b}$$

Properties for column i

Proportion of the beam restraint required by column i to support applied loads

The above expression can be used in a frame to evaluate the buckling capacity. The problem begins by starting at a joint at the end of the column where there is only one column framing into the joint. Based upon the applied loading and the alignment chart we can successively move through the joints in the structure until we get to the other end of the column where we can check the required G with the restraint provided in the structural boundary conditions.

SOLUTION STEPS for DISTRIBUTING JOINT RESTRAINT

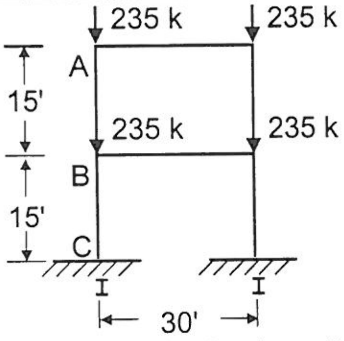


1. Start at one end of the column where there is only one column framing into the joint (say joint A) and obtain G_A.
2. Based upon G_A and applied column load solve for the required K_{BA}. Using G_A and K_{BA} solve for the required G_{BA} at the bottom of column AB.
3. Solve for the proportion of the restraint at joint B to support the applied load using the expression:

$$X_{BA} = \frac{(\tau I_c/L_c)_{BA}}{G_{BA}\sum m I_b/L_b}$$

4. The remainder of the proportion of the restraint $X_{BC} = 1 - X_{BA}$ can then be used to calculated G_{BC}: $$G_{BC} = \frac{(\tau I_c/L_c)_{BC}}{X_{BC}\sum m I_b/L_b}$$
5. Using G_{BC} and the applied load solve for the required K_{BC}. Using G_{BC} and K_{BC}, solve for the required G_C and compare to the actual support conditions. If required G_C > actual G_C OK.

EXAMPLE 5: DISTRIBUTING JOINT RESTRAINT



Consider the frame shown in which the upper story columns have loads of 235 k while the bottom story columns have loads of 470 k. Evaluate the elastic buckling capacity of the frame.

Columns: W12x50 – I_y = 56.3 in^4

Beams: W18x50 – I_x = 800 in^4

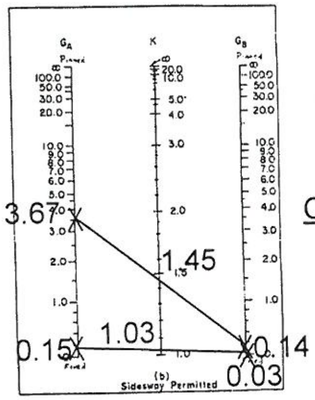
Start at Joint A, where there is only one column:

$$G_A = \frac{(I_c/L_c)_{AB}}{m I_b/L_b} = \frac{(56.3/15)}{1.0(800/30)} = 0.14$$

Based upon the 235 k load in the upper story, the required K can be solved for:

$$K^2 = \frac{\pi^2 EI}{PL^2} = \frac{\pi^2(29000)56.3}{235(15x12)^2} = 2.12 \quad \rightarrow \quad K_{BA} = 1.45$$

EXAMPLE 5: DISTRIBUTING JOINT RESTRAINT



Using G_{BA} required of 3.67:

$$X_{BA} = \frac{(I_c/L_c)_{BA}}{G_{BA}\sum mI_b/L_b} = \frac{(56.3/15)}{3.67(1.0x800/30)} = 0.04$$

$$\therefore X_{BC} = 1 - X_{BA} = 1 - 0.04 = 0.96$$

Column BC:

Therefore, at the top of column BC, the value of G_{BC} is given by:

$$G_{BC} = \frac{(I_c/L_c)_{BC}}{X_{BC}(mI_b/L_b)} = \frac{(56.3/15)}{0.96(1.0x800/30)} = 0.15$$

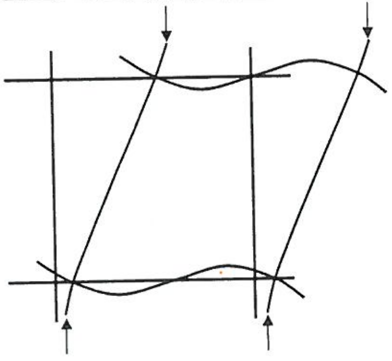
Based upon the 470 k load in the ~~upper~~ lower story, the required K can be solved for:

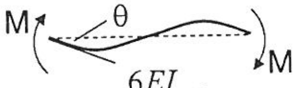
$$K^2 = \frac{\pi^2 EI}{PL^2} = \frac{\pi^2(29000)56.3}{470(15x12)^2} = 1.06 \rightarrow K_{BA} = 1.03$$

We had loads of 235 k, the critical load from an FEA solution is 240 k

The required G_C =0.03, which is less restraint than actually supplied by the fixed end. Therefore the frame is stable.

ALIGNMENT CHART ASSUMPTIONS – SWAY CASE

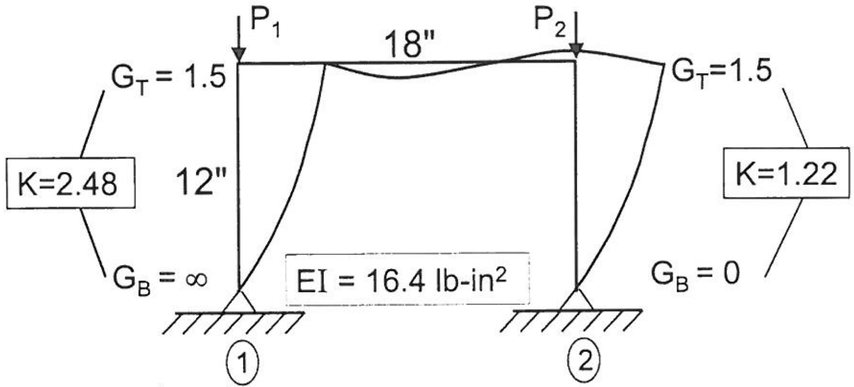




$$M = \frac{6EI}{L}\theta$$

1. Elastic Behavior
2. Reverse curvature bending of beams.
3. Joints are rigid.
4. Joint restraint is distributed to the columns above and below the joint proportional to I/L of the two columns.
5. All columns buckle simultaneously.
6. No axial force in the girders.

FRAME BUCKLING MODEL

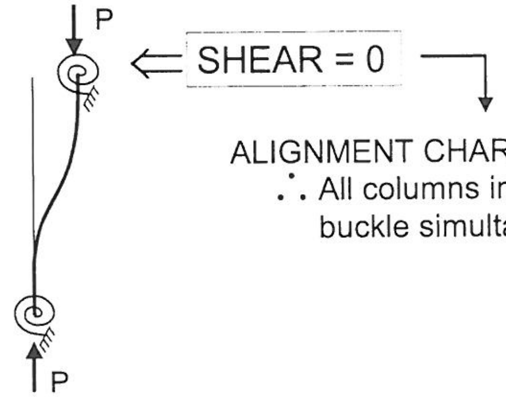


$$P_{cr1} = \frac{\pi^2(16.4)}{(2.48x12)^2} = 0.18 \qquad P_{cr2} = 0.76$$

$$\Sigma P_{cr} = 0.18 + 0.76 = 0.94 \text{ lb.}$$

ALIGNMENT CHART CAPACITY

IN UNBRACED FRAMES, THE INDIVIDUAL ALIGNMENT CHART “K” MAY GIVE A POOR INDICATION OF THE BUCKLING LOAD ON THAT PARTICULAR COLUMN.



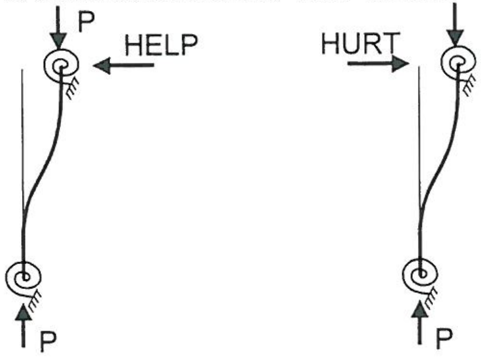
SHEAR = 0

ALIGNMENT CHART ASSUMPTION
∴ All columns in a story
buckle simultaneously

ALIGNMENT CHART CAPACITY

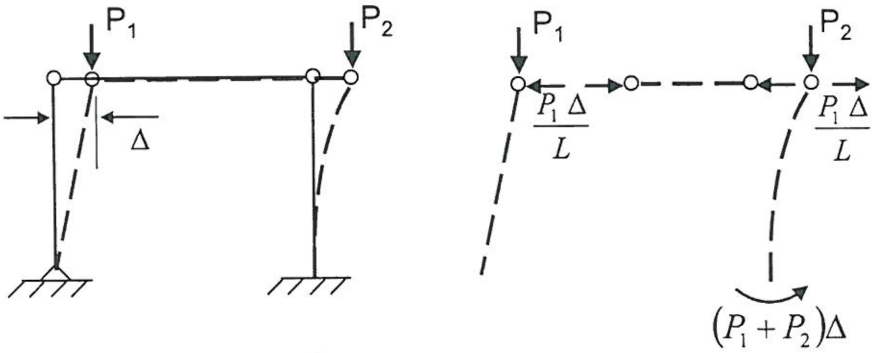
ACTUAL COLUMNS

Shears are present that generally "help" the load carrying capacity of some columns and "hurt" others.

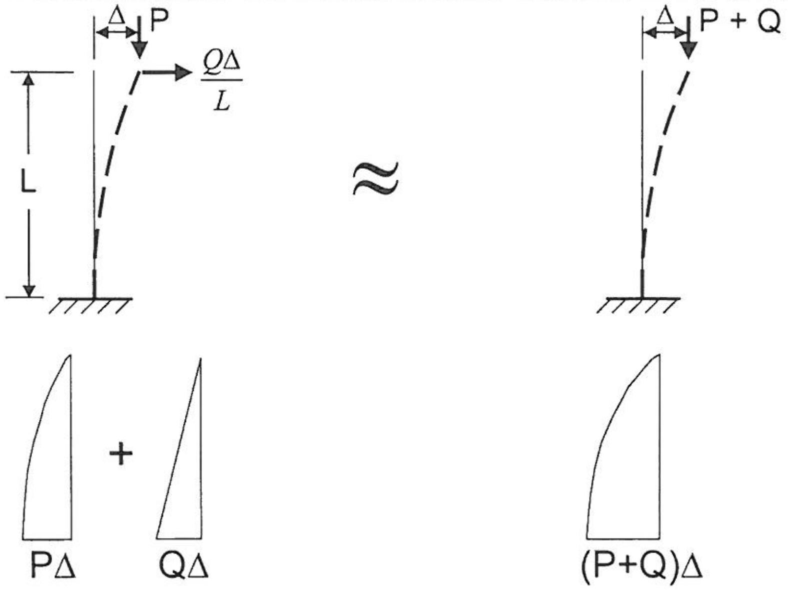


For gravity loading: Σ Shear = 0

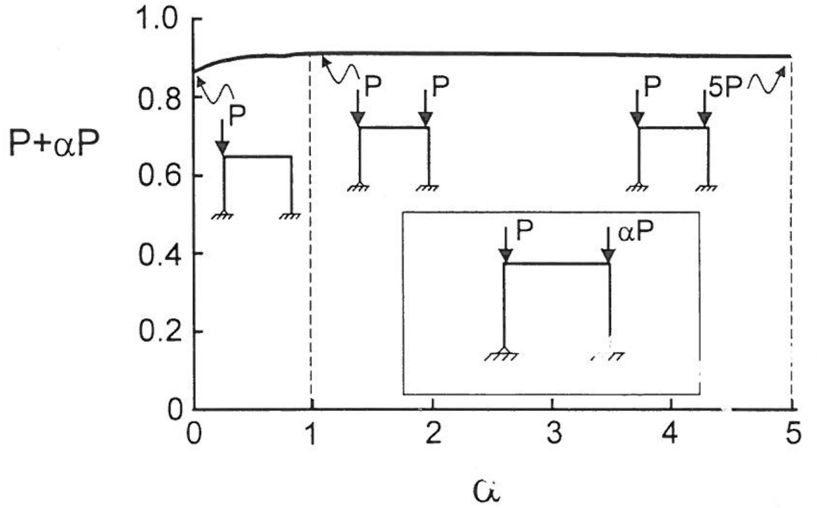
LEAN-ON COLUMNS



WHY THE ΣP CONCEPT WORKS



EXACT SOLUTIONS



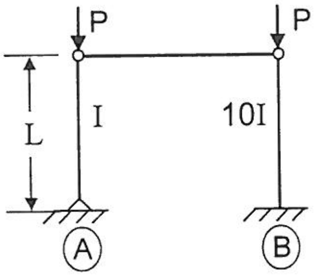
ΣP CONCEPT ELASTIC

- FOR SWAY BUCKLING OF A STORY:

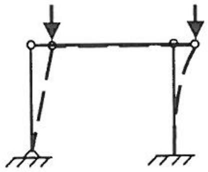
$$\Sigma P_{\substack{Story\ Column \\ Loads}} \leq \Sigma P_{cr\,i}$$

Sway Buckling Load of each column using Alignment Chart K-Factor

- EACH COLUMN MUST SUPPORT ITS OWN LOAD IN THE NO-SWAY MODE (ie. WITH K=1.0)



IN UNBRACED FRAMES, SWAY AND NO-SWAY BUCKLING MODES MUST BE CHECKED



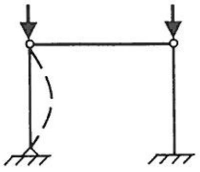
SWAY MODE

Col. A: $P_{cr} = 0$

Col. B: $P_{cr} = \dfrac{\pi^2 E(10I)}{(2L)^2}$

$$\Sigma P = 2P = \frac{10}{4}\frac{\pi^2 EI}{L^2}$$

$$P_{cr} = 1.25\frac{\pi^2 EI}{L^2}$$

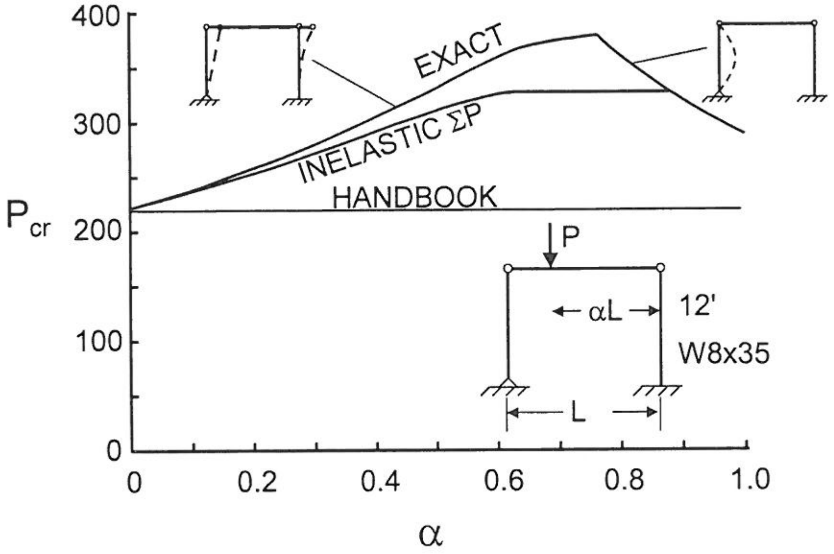


NO SWAY MODE

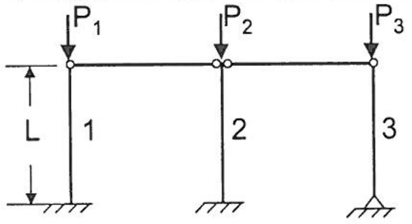
$$\boxed{P_{cr} = \frac{\pi^2 EI}{L^2}}$$

CONTROLS

INELASTIC ΣP CONCEPT



INELASTIC ΣP CONCEPT



Sway Buckling Capacities

$$P_{cr1} = A\frac{\pi^2 EI_1\enclose{circle}{\tau_1}}{(K_1L)^2}$$ Based upon P_1

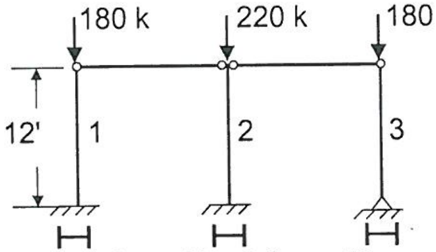
$$P_{cr2} = A\frac{\pi^2 EI_2\enclose{circle}{\tau_2}}{(K_2L)^2}$$ Based upon P_2

- $\frac{12}{23}$ *in ASD*
- 0.877ϕ *in LRFD*

$$P_{cr3} = 0$$

$$\Sigma P_{cr} \geq \Sigma P$$

- Service Loads in ASD
- Factored Loads in LRFD



— Service Loads

All Columns are W10x45

F_y = 36 ksi ASD

A = 13.3 in², I_x = ~~238~~ in⁴ 248

$r_x / r_y = 2.15$

$K = 2.0$ $K = 2.0$ $K = \infty$

$\frac{180}{13.3} = 13.5 ksi$ $16.5 ksi$

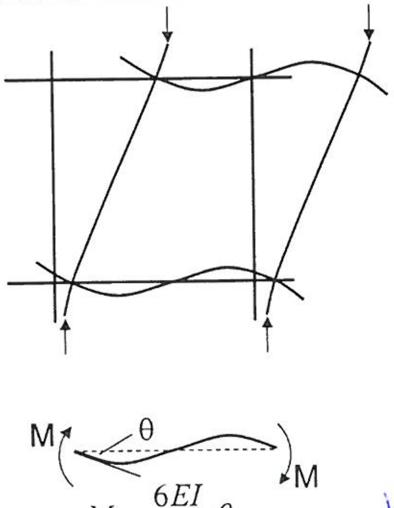
$\tau = 0.830$ $\tau = 0.531$ — From Table A, pg. 3-8, 9th Ed. – AISC Manual

$$\frac{12}{23}\frac{\pi^2(29000)(\cancel{248})(0.830)}{(2.0x144)^2} = 370 \cancel{+ 237 + 0 = 607k > 580k} \text{ OK}$$

$370 + 237 = 607 \not> 580K$

Using Col. Load Tables: 2x12/2.15 = 11.2 ft. ΣP_a = 2x224k = 448 k
No Good

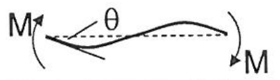
ALIGNMENT CHART ASSUMPTIONS – SWAY CASE



$$M = \frac{6EI}{L}\theta$$

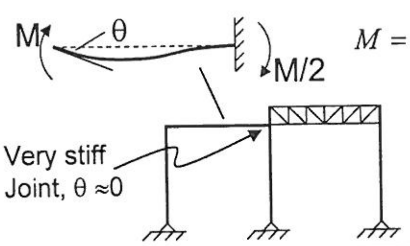
1. Elastic Behavior
2. Reverse curvature bending of beams.
3. Joints are rigid.
4. Joint restraint is distributed to the columns above and below the joint proportional to I/L of the two columns.
5. All columns buckle simultaneously.
6. ✓ No axial force in the girders.

BENDING STIFFNESS OF RESTRAINING MEMBERS



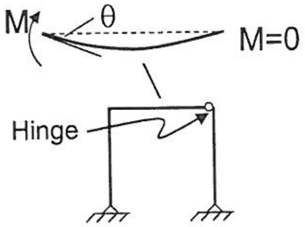
$$M = \frac{6EI}{L}\theta$$

m = 6/6 = 1.0



$$M = \frac{4EI}{L}\theta$$

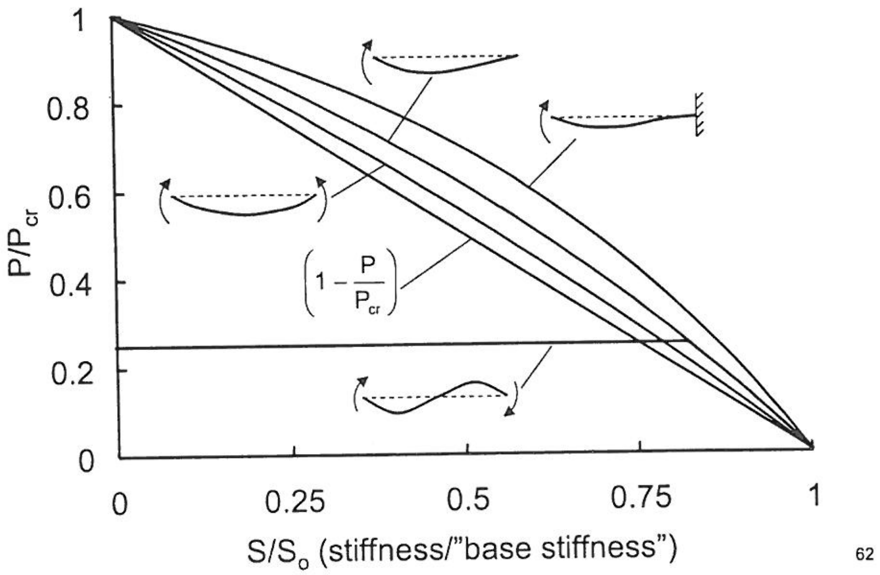
m = 4/6 = 0.67



$$M = \frac{3EI}{L}\theta$$

m = 3/6 = 0.5

EFFECT OF AXIAL FORCE ON BENDING STIFFNESS



EFFECT OF AXIAL FORCE ON BENDING STIFFNESS

- We can therefore apply a relatively simple modification to the expression for G to account for the effect of axial force on the bending stiffness of the restraining members:

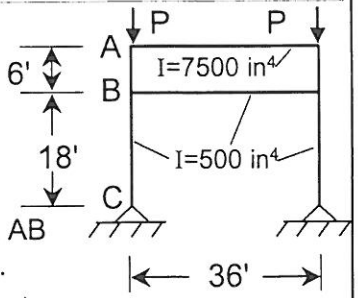
$$G_i = \frac{(\tau I_c/L_c)_i}{X_i \sum \left[m \frac{I_b}{L_b}\left(1 - \frac{P}{P_{cr}}\right)\right]}$$

Axial Force in restraining member

Buckling capacity of restraining member

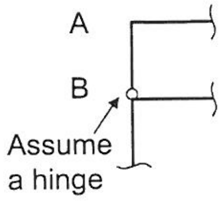
- Based upon the above definition of G, any member with an axial force less than its buckling capacity can provide restraint to another member.

RECONSIDER EXAMPLE 1B



Direct application of the alignment chart earlier gave very poor results and estimated P_{cr} = 185 k. Use some of the corrections outlined to estimate the buckling capacity of the frame.

Consider Column AB: Assume that Column AB receives no restraint from the 500 in^4 beam.



$$G_A = \frac{\Sigma(I_c/L_c)}{\Sigma(I_g/L_g)} = \frac{(500/6)}{(7500/36)} = 0.4$$

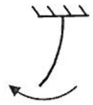
$G_B = \infty$ (With no help from 500 in^4 bracing beam)

$$K_{AB} = 2.13$$

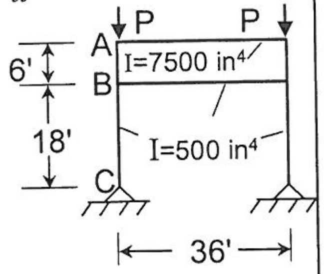
$$\Rightarrow \quad P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29000)x500}{(2.13x6x12)^2} = \underline{\underline{6085\ kips}}$$

RECONSIDER EXAMPLE 1B - continued

Clearly Column AB does not require any restraint at joint B and this member will actually act as a restraining member for Column BC. Treat Column AB as a cantilever member with EI/L stiffness.



M =(EI/L)θ $\therefore \quad m = \dfrac{actual\ stiff.}{assumed\ stiff.} = \dfrac{EI/L}{6EI/L} = 0.167$



Column BC:

$$G_i = \frac{(\tau I_c/L_c)_i}{X_i \sum \left[m \frac{I_b}{L_b}\left(1 - \frac{P}{P_{cr}}\right)\right]}$$

Elastic Buckling

$$G_{BC} = \frac{(1.0x500/18)}{1.0\left[1.0\frac{500}{36} + 0.167\frac{500}{6}\left(1 - \frac{P}{6085}\right)\right]}$$

X_{BC} $m_{\text{bracing beam}}$ m_{BC} $P_{cr\ AB}$

RECONSIDER EXAMPLE 1B - continued

$$G_{BC} = \frac{27.8}{\left[13.9 + 13.9\left(1 - \frac{P}{6085}\right)\right]}$$

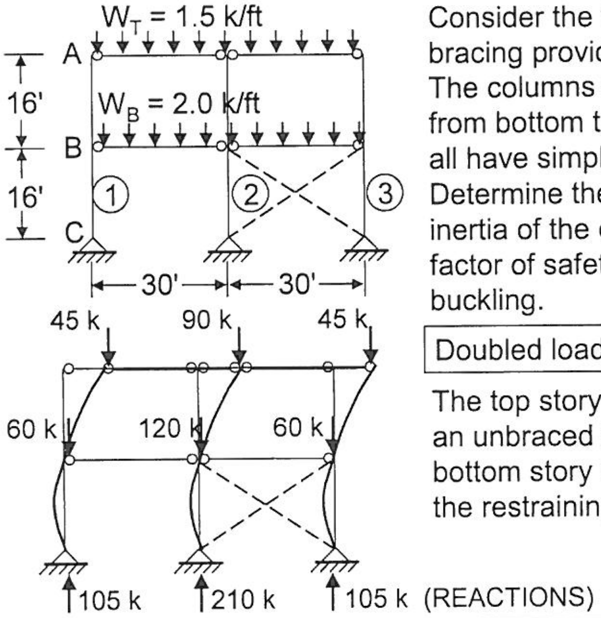
Guess P = 560 k (educated guess to save space):

$$G_{BC} = 1.05 \qquad \Rightarrow K_{BC} = 2.34$$
$$G_C = \infty$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29000x500)}{(2.34x18x12)^2} = 560\ k$$

FEA Solution: P_{cr} = 555k

EXAMPLE 6



Consider the frame shown with bracing provided in the first story. The columns are all continuous from bottom to top while the beams all have simple end connections. Determine the required moment of inertia of the columns to have a factor of safety of 2 against elastic buckling.

Doubled loads for F.S. = 2

The top story will buckle as an unbraced frame while the bottom story columns act as the restraining members.

EXAMPLE 6

Apply the ΣP concept to the top story:

$$\Sigma P = 2(45k)+90k = 180k \le 2P_{cr1,3} + P_{cr2} = \Sigma P_{cr}$$

$$180k = 2\left(\frac{\pi^2(29000)I}{K_1^2}\right)+\frac{\pi^2(29000)I}{K_2^2} = 15.53\frac{I}{K_1^2}+7.76\frac{I}{K_2^2}$$

Consider expressions for G for top story columns:

Forces in bottom story restraining columns:
Columns 1 & 3: P=105 k; Column 2: P=210 k

Buckling capacity of lower story columns:

$$P_{cr} = \frac{\pi^2(29000)I}{(16x12)^2} = 7.76I$$

EXAMPLE 6

Columns 1&3:

$$G_B = \frac{(I/16)}{\frac{3}{6}\frac{I}{16}\left[1-\frac{105}{7.76I}\right]} = \frac{1}{\frac{1}{2}\left(1-\frac{13.5}{I}\right)} \qquad \text{Eq. 1}$$

Column 2:

$$G_B = \frac{(I/16)}{\frac{3}{6}\frac{I}{16}\left[1-\frac{210}{7.76I}\right]} = \frac{1}{\frac{1}{2}\left(1-\frac{27.1}{I}\right)} \qquad \text{Eq. 2}$$

From ΣP Concept:

$$180k = 15.53\frac{I}{K_1^2} + 7.76\frac{I}{K_2^2} \qquad \text{Eq. 3}$$

Using Trial and Error:

I	$G_{T1,2,3}$	$G_{B1,3}$	$K_{1,3}$	G_{B2}	K_2	Equation 3
50	∞	2.74	2.85	4.37	3.27	$180k \neq 132k$ (Too Small)
65	∞	2.52	2.78	3.43	3.03	$180k \neq 185k$ (Too Big)
64	∞	2.53	2.79	3.47	3.04	$180k = \approx 181k$ (Okay)

FEA $I_{req'd}$ = 60.5 in^4

BASIC DESIGN FOR STABILITY
Part I - Columns and Frames©