DIRECT ANALYSIS APPROACH FOR THE ASSESSMENT OF FRAME STABILITY: VERIFICATION STUDIES

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INTRODUCTION

In (Maleck and White 2003a and b), the authors have outlined an alternative approach for analysis and design of steel framing systems, termed the *Modified Elastic Approach*. This method accounts for key factors that affect system strength, specifically nominal residual stress and geometric imperfection effects, directly within a second-order elastic analysis. In as such, a more rational analysis-design procedure is obtained that eliminates the need for effective length factors or buckling solutions.

Various forms of the *Modified Elastic* method have been discussed in the recent literature, e.g., (Maleck and White 2001 and Deierlein et al. 2002). (Maleck and White 2003b) presents a summary of benchmark validation studies from (Maleck 2001). (Maleck and White 2003a) gives an overview of the Modified Elastic approach, and presents an example design solution for a representative stability critical industrial-type frame. This frame was originally studied in (Maleck 2001), and example calculations have been discussed previously by Maleck and White (2001) and by Deierlein et al. (2002). However, in these references, resistance factors were applied only to the beam-column strength terms P_n and M_n. Subsequent studies have shown that in general, resistance (ϕ) factors must be applied to the nominal *stiffnesses*

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in addition to the nominal *strengths*. The benchmark validation studies in (Maleck 2001) and (Maleck and White 2003b), which are presented in terms of nominal resistances, are unaffected by this issue. This is because, if both the stiffness and strength terms are factored by an appropriate uniform ϕ prior to conducting the analysis and design calculations, the results are identical to those obtained if the nominal strengths and stiffnesses are used in the calculations but the beam-column interaction curves are subsequently factored along both axes by the above ϕ value.

In addition, the analysis-design solutions illustrated in (Maleck and White 2001) and (Deierlein et al. 2002) used a nominal rigidity of the members of τEI, where τ is a inelastic stiffness reduction factor associated with the column strength. The original developments in (Maleck 2001) were based on nominal rigidities of $0.9\tau EI$ for strong-axis bending and $0.8\tau EI$ for weak-axis bending of I-shapes, with τ taken as the inelastic stiffness reduction factor associated with the AISC-LRFD column curve (AISC 1999). Subsequent studies have shown that these or similar minor reductions in the nominal section rigidities, combined with the use of the ϕ factor on the stiffness, are essential to the accuracy of the method for structures that fail by elastic or near elastic lateral sidesway buckling. Maleck and White (2003a) gives an overall synthesis of the Modified Elastic procedure and presents the results for the example frame using the base nominal $0.9\tau EI$ and $0.8\tau EI$ values from (Maleck 2001).

Recently, it has been suggested in discussions within AISC TC10 that the use of the CRC τ equation is desirable, and that a uniform factored stiffness of $0.8\tau EI$ might be employed both for strong- and weak-axis bending within the Modified Elastic procedure. Also, it has been suggested that this procedure be referred to as the *Direct Analysis* method. This paper summarizes the key concepts associated with this approach, and presents the results of fundamental benchmark validation studies for this procedure.

THE DIRECT ANALYSIS APPROACH

Overview. A general overview of the direct analysis approach is presented here; for detailed discussions of the development of the approach, the reader is referred to (Maleck & White 2003a) and (Deierlein et. al 2002). The Direct Analysis approach is based on a simple principle: if the parameters that affect member and system strength can be directly modeled (in a practical manner) within an elastic analysis, the overall simplicity and rationality of the elastic analysis-design calculations is improved. Based on SSRC Technical Memorandum No. 5 (SSRC 1998), these parameters include (but are not limited to) residual stresses, initial geometric imperfections and boundary conditions. In the Direct Analysis approach, the effects that are not easily captured by simple modifications to elastic frame analysis are addressed within the member resistance equations. The modifications that can be easily made to an elastic analysis to better estimate the strength limit states behavior within the structural system (e.g., to more closely predict the internal forces obtained from a rigorous distributed plasticity or plastic zone analysis) include:

1. Uniform reduction of the section flexural rigidity (EI_e), based on the level of axial force within the beam-column members.
2. Specification of a nominal out-of-plumbness, or lack-of-verticality, of the structural framing.

By proper specification of nominal values for (1) the inelastic stiffness reduction based on the behavior associated with the column strength curve, and (2) column out-of-plumbness or a lack-of-verticality of the structure based on erection tolerances such as specified in (AISC 2000), a simpler, more transparent and more accurate analysis-design approach is achieved.

Based on a wide range of studies, including the verification studies presented in this paper, the following basic rules are suggested for application of the Direct Analysis approach to tiered-type structural framing:

1) The nominal stiffnesses of all the components within the structural system are factored by a uniform value of 0.8. This factor is applied to all the member rigidities, regardless of orientation, and also to connection stiffnesses, column base restraint stiffnesses, etc. (if a finite flexibility of these components is considered in the design). There are two contributors to the 0.8 factor on the stiffness. The first contributor is a nominal reduction of 0.9. This reduction factor accounts for the influence of distributed plasticity effects on the nominal stiffnesses as the strength limit state associated with the most critical component in the structural system is approached. The second contributor is a resistance factor of $\phi = 0.9$. The product of these to factors is rounded from 0.81 to 0.8.

2) For members in which the applied load P_u exceeds $0.5P_y$, an additional reduction factor

$$\tau = [P_u / P_y(1 - P_u / P_y)] \qquad (1)$$

is applied, and therefore effective member rigidities of

$$EI_e = 0.8\tau EI \qquad (2)$$

are required for these types of members. In many types of frames, the $0.5P_y$ limit is not exceeded by the required strengths by any of the design load combinations. In cases where this limit is exceeded, it is usually violated by only a few columns within the structure. Furthermore, the value of P_u remains reasonably constant with design iterations in most frames, and therefore an accurate but conservative value for τ can be selected rather easily.

3) A nominal frame nonverticality of H/500 is included in the analysis by applying a notional load at each story level equal to

$$N_i = 0.002Y_i, \qquad (3)$$

where Y_i is the factored design gravity load acting on the i^{th} story. Alternatively, the frame nonverticality may be directly modeled by alteration of the perfect frame geometry.

If these minor modifications are performed in the context of a second-order elastic analysis, the beam-column member strength checks can be performed using the (AISC 1999) beam-column interaction equations but with the P_n term based on the actual member length. In short, *the need to calculate effective length factors is eliminated,* and thus the member strength checks are greatly simplified.

While out-of-straightness can have an important influence on the maximum strength of members in which the strength limit involves a non-sway failure mode, the modeling of member out-of-straightness within an analysis of the overall structural system is more cumbersome than the modeling of a uniform frame nonverticality. In lieu of direct modeling, the effect of out-of-straightness on the strength is accounted for in the axial strength term of the interaction equation.

One should note that the notional loads described by Eq. 3 are included to model a physical attribute of the structure. They are not meant to be a minimum horizontal load that can be neglected in the presence of a larger applied lateral load; consequently they are additive to applied lateral loads. For frames that have significant sidesway flexibility or are subjected to large vertical loads, the influence of potential nominal out-of-plumb imperfections on the internal second-order moments within the structural system can be significant, even in the presence of an applied lateral load (Maleck and White 2003a).

It should also be noted that the above reduction in the stiffness is intended for use in the assessment of the strength only. Serviceability limits should be checked using nominal stiffness values.

Special Considerations. For frames that are loaded near their vertical load capacity and in which the structural system would tend to fail in elastic sidesway buckling, the Direct Analysis approach behaves in a fashion that represents the true stability behavior more faithfully than traditional buckling solution or effective length based approaches. In these type of structures, the strength of the system is reached due to significant amplification of the sidesway deflections and the associated internal moments as the limit of the structural resistance is approached. This can be problematic if the Engineer does not anticipate this characteristic of the behavior. In traditional buckling solution or effective length approaches, the internal forces and component resistance ratios (i.e., the ratio of the required strengths to the design resistances) tend to increase in only a mildly nonlinear fashion (due to second-order elastic effects) as the design loads increase or if say some of the components are reduced in size. However, when the Direct Analysis Approach is applied to structures in which there are truly large second-order amplifications of the lateral displacements based on the reduced (or the actual inelastic) stiffnesses as the limit of resistance is approached, the component resistance ratios can change in a highly nonlinear fashion. For example, the engineer may find that the interaction equation value for a beam-column member is 0.4, but if the structure is checked for a slightly larger load, the interaction equation value could rapidly increase beyond a value of 1.0 with only a small increase in the required strength. These problems are significant only for structures in which the sidesway amplification is excessive at the required strength level. Therefore, they are likely to be important only for certain special types of structures.

In general, the second-order analysis used in the direct analysis approach must be rigorous; that is, the analysis should include both P-Δ and P-δ effects. Approximate P-Δ analysis methods are permitted only if the applied axial loads on all columns satisfy the following limit:

$$P_u < 0.15\,P_{eL} \quad (4)$$

where

$$P_{eL} = \pi^2 EI_e / L^2 \quad (5)$$

and P_{eL} is determined in the plane of bending. Equation 4 is a conservative limit for which the influence of P-δ moments on the *sidesway* displacements can be neglected. In lieu of a direct second-order analysis, first-order analysis results may be modified by B1 and B2 amplification factors determined using the reduced stiffness. However, the B2 amplification factor in (AISC 1999) is in effect a P-Δ analysis solution. If Eq. 4 is not satisfied, the form of this equation given by LeMessurier (1977) including the C_L term may be used to account for the P-δ effects on the sidesway deflections and internal moments. The B1 amplification factor must be included in general even when the axial loads satisfy Eq. 4.

VERIFICATION STUDIES

Background. In (Maleck 2001), four small sensitive benchmark frame configurations and two braced beam-column configurations were studied to assess the accuracy of the proposed "Modified Elastic" analysis-design approach. The complete set of frames is similar to that originally studied by Kanchanalai (1977) with the exception that the effects of initial geometric imperfections were included in the (Maleck 2001) studies. Parameters considered in this study included slenderness, member orientation (strong or weak-axis), beam-column end restraint (G), and leaning column load (α). The results of these studies are summarized in (Maleck and White 2003b).

In these studies, interaction curves were developed based on analysis results from the "Modified Elastic" approach and the current LRFD method for the studied frames. Both first-order (P versus M1) and second-order (P versus M2) interaction curves were considered in these studies, where

- M1 is the maximum primary bending moment in the member due to the applied loading, and
- M2 is the maximum internal second-order bending moment

The P versus M1 interaction curves represented the maximum loadings that can be *applied* to the benchmark structures; therefore, the P versus M1 interaction curves are referred to as both "applied loading" as well as "first-order" strength curves. These curves were compared to interaction curves established by rigorous spread-of-plasticity analyses

As previously stated, these benchmark studies focused only on *nominal strengths*. If only P_n and M_n are factored, *without factoring the stiffness*, then in the limit of structures that fail by elastic sidesway buckling, factoring of P_n and M_n has a negligible influence on the calculated design resistance. The reason for this behavior is as follows. In the limit of elastic sidesway buckling, the strength of the structure is effectively controlled entirely by its *elastic stiffness*. As the elastic stability limit is approached, the internal moments tend to increase dramatically such that large changes in internal moments are obtained with only a small change in the externally applied loadings. As a result, if the stiffness is not reduced (e.g., by 0.8) the design load capacity in these types of structures is essentially predicted as the nominal elastic sidesway buckling load, regardless of the fact that the factored strengths $\phi_c P_n$ (based on the actual unsupported length) and $\phi_b M_n$ are used.

Design of Current Study. From the reasonably comprehensive set of nominal strength studies performed in (Maleck 2001), a subset of ten strong-axis frame configurations and seven weak-axis frame configurations are selected for additional study using factored strength and stiffness values and the CRC tau equation (when required). The subset of frames taken from (Maleck 2001) that is used in this study are shown in Fig. 1. The chosen frames represent cases that exhibit the largest unconservative and conservative errors in the initial nominal study for either the direct analysis or current LRFD approach, or both. Certain frames are also included because they exhibited significant distributed plasticity effects (SP_S60_G0, SP_S80_G0) or failed by elastic sidesway buckling (UP_S40_G1_α2, SP_80_G3).

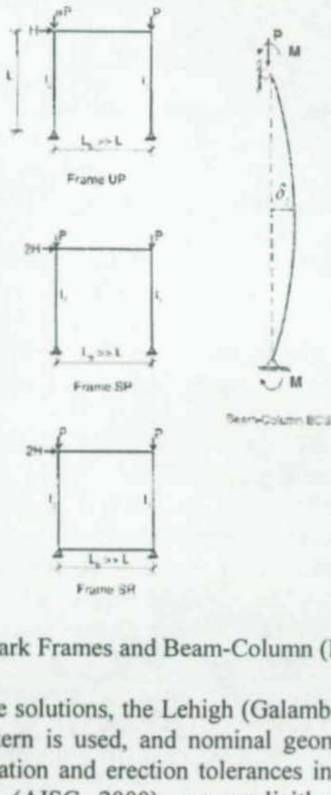


Figure 1. Benchmark Frames and Beam-Column (Maleck 2001).

For the plastic zone solutions, the Lehigh (Galambos and Ketter 1959) residual stress pattern is used, and nominal geometric imperfections equal to the fabrication and erection tolerances in the AISC Code of Standard Practice (AISC 2000) are explicitly modeled. These imperfections are:

$$\Delta_0 = \ell/500 \tag{6}$$

and

$$\delta_0 = \ell/1000 \tag{7}$$

where

Δ_0 = overall frame nonverticality

δ_0 = maximum amplitude of a half sine wave member out-of-straightness

ℓ = member length

When these parameters are used in a rigorous distributed plasticity analysis, the resulting beam-column strength solutions closely match the current AISC LRFD beam-column strength procedures (AISC 1999) for the base case of W8x31 members in strong-axis bending (Maleck 2001, White and Clarke 1997, ASCE 1997). This should be expected since the AISC LRFD beam-column strength equations were calibrated in part to the results from rigorous plastic zone solutions of this type (ASCE 1997, Maleck and White 2003b). The analysis models typically used for these distributed plasticity solutions have been based on Euler-Bernoulli beam kinematics and in-plane response only.

LeMessurier's (1977) approach is used for the second-order elastic design-analysis procedure in all of the sidesway-uninhibited benchmarks considered within this study. LeMessurier's method accounts for both P-large delta and P-small delta effects and is, for all practical purposes, exact for the sidesway problems studied in this work. The "exact" closed-form analytical solutions are used to determine the second-order elastic internal moments within the sidesway-inhibited problem considered (beam-column BCS_, see Fig. 1). For the approximate (P–Δ) sidesway inhibited solutions, a B1 amplification factor is used in lieu of the "exact" closed-form solution. For the direct analysis approach, the above solutions are based on the effective stiffness EI_e (see Eq. 3). The approximate first-order solutions are also determined using LeMessurier's approach; in these solutions the amplification term associated with P-δ effects is simply omitted.

Results and Discussion. A comparison of results for the studied frames is presented in Tables 1 (strong-axis) and Table 2 (weak-axis). The focus in this report is the error in the applied load curves (P vs. M1); for a general discussion of the error associated with second-order curves (P vs. M2), the reader is referred to (Maleck and White 2003b).

Results of the factored solutions are presented for both the proposed Direct Analysis approach and the current LRFD (1999) approach. For both methods, errors from rigorous second-order analyses (P-δ) and approximate second-order analyses (P-Δ) are presented. The error is defined as

$$e = \frac{r_{PZ} - r}{r_{PZ}} \quad (8)$$

where r_{PZ} is the distance to the plastic zone strength *along a radial line* from the origin of the interaction curve plots, normalized in terms of M/M_p and P/P_y, and r is the corresponding distance along the same radial line to the predicted design strength. Negative errors are unconservative.

In general, it can be seen from Tables 1 and 2 that the Direct Analysis approach produces results with reasonable levels of unconservative error in the applied load interaction curves, and improved accuracy of these curves relative to that of the LRFD solutions, even for the cases of maximum unconservative error. The error for the Direct Analysis approach using a rigorous second-order analysis range from –6% to +13 for strong axis bending and –13 to +15 for weak-axis bending. The corresponding LRFD errors range from –8 to +17 for strong-axis and –17 to +17 for strong axis bending. The large unconservative errors in the methods for the "BCS" problems in weak-axis bending relate to the well known fact that the single AISC column curve equation tends to give capacities that are somewhat liberal relative to theoretical strengths for weak-axis buckling of I-shaped members (Salmon and Johnson 1996). The maximum and minimum errors are strongly dependent on the type of column end restraint, or more specifically on the moment gradient within the members. The conservative errors tend to be highest for the symmetric restrained-base (SR_) frames, and these errors tend to be concentrated within the high axial load regions for this case. Conversely, the highest unconservative errors are found in the braced, single curvature (BSC_) cases.

Table 1. Error in strong-axis benchmark cases for rigorous (P-δ) and approximate (P-Δ) second-order analyses

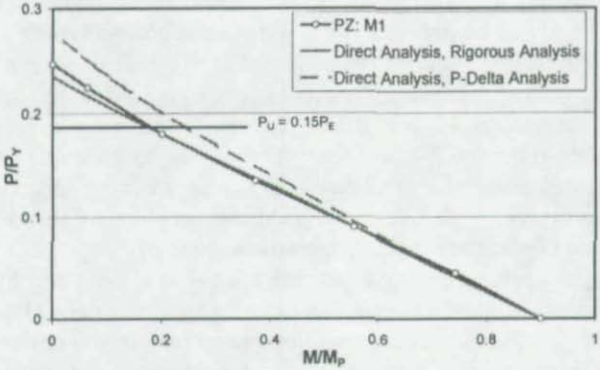
	DIRECT ANALYSIS				LRFD			
	P-δ		P-Δ		P-δ		P-Δ	
Designation	ε^-	ε^+	ε^-	ε^+	ε^-	ε^+	ε^-	ε^+
UP_S40_G1_α2	0	8	0	6	0	15	0	14
SP_S20_G0	-2	3	-2	3	-1	3	-1	3
SP_S40_G0	-4	3	-7	3	0	2	0	2
SP_S60_G0	-3	0	-14	0	-5	4	-5	3
SP_S80_G0	-1	5	-12	0	0	10	0	8
SP_S80_G3	0	9	0	6	0	18	0	17
SR_S40_G3	0	4	0	4	0	13	0	13
SR_S80_G0	0	13	0	12	0	3	0	3
BCS_S80	-6	5	-7	4	-8	2	-11	2
BCS_S120	-2	10	-4	8	-6	4	-10	1
average	-2	6	-5	5	-2	7	-3	7

Table 2. Error in weak-axis benchmark cases for rigorous (P-δ) and approximate (P-Δ) second-order analyses

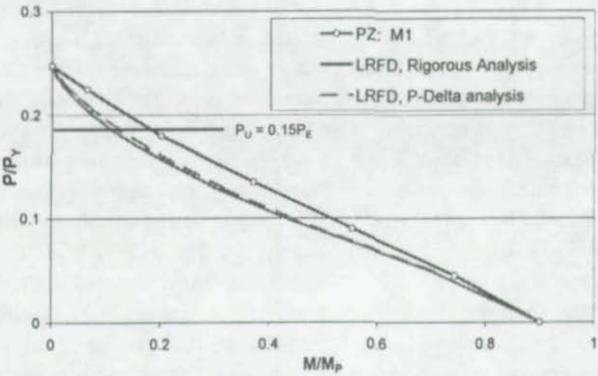
	DIRECT ANALYSIS				LRFD			
	P-δ		P-Δ		P-δ		P-Δ	
Designation	ε^-	ε^+	ε^-	ε^+	ε^-	ε^+	ε^-	ε^+
UP_W40_G1_α2	-2	3	-3	1	0	16	0	15
SP_W60_G0	-11	0	-23	0	-17	0	-17	0
SP_W80_G3	0	7	-2	3	0	14	0	13
SR_W40_G3	0	15	0	15	0	17	0	17
SR_W80_G0	0	8	0	9	-8	3	-8	3
BCS_W80	-9	0	-9	0	-10	0	-13	0
BCS_W120	-13	0	-13	0	-13	0	-14	0
average	-5	5	-7	4	-7	7	-7	7

Applied load interaction curves are presented for the case of SP_S80_G0 in Fig. 2. This frame is representative of high error associated with use of an approximate (P-Δ) second-order analysis in lieu of a rigorous (P-δ) second-order analysis. In the absence of applied moment, the maximum factored strength predicted by the plastic zone analysis is $P/P_y = 0.248$; the conservative maximum factored strength predicted by the Direct Analysis approach is $P/P_y = 0.236$; and the unconservative prediction using an approximate second-order analysis is $P/P_y = 0.278$. The maximum applied axial load predicted by the plastic zone analysis exceeds the limit of $P_u/P_{eL} < 0.15$ (see Eq. 4) by approximately 25%, and the critical load predicted by the Direct Analysis method exceeds this limit by approximately 21%. There is an 18% difference in the resulting predictions of the critical applied load between the rigorous and approximate second-order analyses.

As can be seen in Fig. 2a, the Direct Analysis approach provides an accurate estimate of the allowable applied loads when a rigorous second-order analysis is used. Conservative error is present when the axial load dominates interaction check; however, up to the limit of $P_u < 0.15P_{eL}$, there is little to no error. For the SP_S80_G0 frame shown, the maximum unconservative error is −12% when an approximate analysis is performed; this value is −8% at the limit where the axial capacity predicted by the approximate analysis is equal to the recommended limit of $0.15P_{eL}$. Similarly, for the SP_S60_G0 the unconservative error at $P = 0.15P_{eL}$ is −8%; this frame has the highest increase in error when the P-δ amplification effects are neglected (with a maximum unconservative error of −14% at the predicted axial capacity in the absence of applied moment.) The unconservative error associated with the approximate analysis for these critical cases at the proposed applied axial force limit is somewhat high, but no larger the maximum unconservative error predicted by the current LRFD approach when a rigorous second-order analysis is used (see Table 1).



a. Direct Analysis



b. LRFD (1999)

Figure 2. Frame SP_S80_G0 factored applied load curves.

The LRFD results, presented in Fig. 2b, do not exhibit the sensitivity to the second-order analysis method (P-δ vs. P-Δ) seen in the Direct Analysis results (see Fig. 2a). As can been seen in Table 1, the maximum error in the current LRFD approach is, in general, not particularly sensitive to the accuracy of the second-order analysis method used.

Figure 3 shows the applied load curves for SP_W60_G0 predicted by the Direct Analysis approach. This frame exhibits the highest unconservative error of the sway frames in weak-axis bending, and the highest increase in error when P-δ effects are not directly included in the analysis (see Table 2). At the limit that Pcr = 0.15PeL for the approximate second-order analysis, the error is –17%; this is equivalent to the maximum unconservative error predicted by the LRFD approach using a *rigorous* analysis.

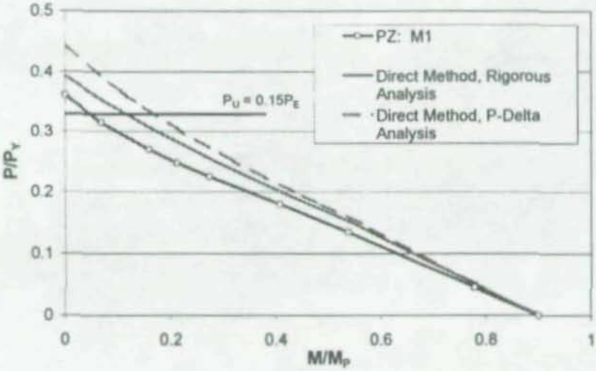


Figure 3. Frame SP_W60_G0 factored applied load curves, Direct Analysis.

Results for the Direct Analysis method for frame SR_S80_G0 are shown in Fig. 4. The conservative error is typically highest in cases where the member is subjected to reverse curvature bending. It can also be seen from the plot that there is low sensitivity to the accuracy of the second-order analysis; as would be expected since there is little P-δ amplification of the moments in the case of reverse-curvature bending. The above results suggest that the use of an appropriate equivalent uniform rigidity (EI_e) accounting for the effect of moment gradient within the beam-column could produce improved accuracy over the simpler approach selected in this research. However, making EI_e a function of the moment gradient would make its value more sensitive to changes in the structure during design iterations and for different design load combinations and add complexity to what is meant to be a simple analytical procedure.

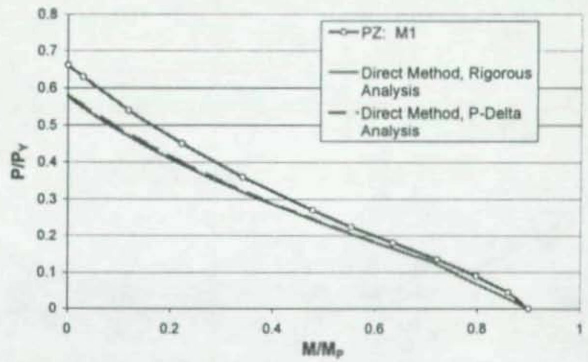


Figure 4. Frame SR_S80_G0 factored applied load curves, Direct Analysis.

CONCLUSIONS

This paper summarizes the results of factored benchmark studies designed to study the validity and accuracy of a variation on the Modified Elastic method proposed by (Maleck and White 2002a), termed the Direct Analysis approach The Direct Analysis approach is generally more accurate than the current AISC-LRFD (1999) method in predicting allowable applied forces for stability critical benchmark cases. Since the Direct Analysis method eliminates the need to calculate column buckling loads or effective length factors, it is simpler to use than the current AISC-LRFD approach for many types of problems. However, care must be taken when using approximate (P-Δ) second-order analysis algorithms in conjunction with the Direct Analysis approach, as unconservative errors as high as -14% for strong-axis bending and –23% for weak-axis bending are achieved for small, stability-critical benchmark frames. A limit on the applied axial load of $P_u < 0.15\ P_{eL}$ for every column is suggested when an approximate second-order analysis is used. A rigorous analysis that includes P-δ effects is preferred for improved accuracy and reduced unconservative error.

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