PLASTIC DESIGN OF BRACED MULTISTORY STEEL FRAMES

Published by
Committee of Structural Steel Producers
Committee of Steel Plate Producers
AMERICAN IRON AND STEEL INSTITUTE

In cooperation with and editorial collaboration by
AMERICAN INSTITUTE OF STEEL CONSTRUCTION

It is suggested that inquiries for further information on plastic design be directed to: American Institute of Steel Construction, 101 Park Avenue, New York, New York 10017.

AMERICAN IRON AND STEEL INSTITUTE
150 East 42nd Street, New York, New York 10017

Printed in U.S.A. 10000 5/68

Foreword

The basic knowledge for the preparation of this design Manual stemmed from the comprehensive presentation on new developments in the application of plastic design principles to the design of multistory steel building frames at the August, 1965 Summer Conference at Lehigh University. This design concept will provide engineers with a greater insight into the actual behavior of multistory frames and will give them an effective tool for obtaining more economical steel designs.

For the preparation of the Manual, the Committee of Structural Steel Producers and the Committee of Steel Plate Producers of American Iron and Steel Institute retained John L. Rumpf, Professor and Head, Civil Engineering and Mechanics, Drexel Institute of Technology, as principal author and Ira M. Hooper and Professor Joseph A. Yura as co-authors. For their skillful handling of the assignment, the Committees gratefully acknowledge their appreciation.

The Committees also wish to acknowledge the important and valuable contribution made by representatives from the member steel producing companies in writing and reviewing the material for this Manual.

The material contained in the Manual is presented in two parts, basic design information and design examples.

Chapters 1 through 3 present the basic design information and background on the plastic design method for braced frames. Chapters 4 through 8 describe the design of a 24-story, three-bay, braced steel apartment house frame.

Chapter 8 includes all design calculations, arranged in a tabular format, with an explanation for each entry. Chapters 4 and 5 describe the subroutines used to select members, either for strength or drift criteria. Chapter 6 gives design checks, and Chapter 7 discusses connections.

The Appendix presents three design aids, and provides a rapid method for checking lateral-torsional buckling of columns.

The concept of plastic design has been documented through a series of research projects which have been conducted for more than two decades and still continue. These projects have been under the sponsorship of American Iron and Steel Institute, American Institute of Steel Construction, the Navy Department, the Office of Naval Research and the Welding Research Council.

Practical procedures for the plastic design of continuous beams and one and two-story rigid frames are described in the American Institute of Steel Construction Manual, ***Plastic Design in Steel.***

The American Institute of Steel Construction is the non-profit service organization for the fabricated structural steel industry in the United States and is dedicated to presenting the most advanced information available to the technical professions. It is suggested that inquiries for further information on plastic design be directed to that Institute.

The authors and American Iron and Steel Institute wish to express their appreciation to all those who assisted in the preparation of the Manual, reviewed the manuscript and contributed suggestions. In particular, it wishes to thank T. R. Higgins and Professors George C. Driscoll, Jr., Theodore V. Galambos and Le-Wu Lu.

Committee of Structural Steel Producers
Committee of Steel Plate Producers
American Iron and Steel Institute

Table of Contents

TABLE OF CONTENTS (Continued)

Nomenclature

A — Cross-sectional area. Subscripts b, c, g denote bracing, column and girder respectively

A_b — Area of bracing member

A_{bm} — Minimum K-bracing area (Eq. 6.8)

A_f — Area of one flange of girder

A_s — Required area of two stiffeners

B — Distance between exterior columns of a bent

C_c — Column slenderness ratio at transition from inelastic to elastic buckling

C_m — Equivalent moment coefficient

D — Spacing of braced bents, or distance between the braced bents of a building

E — Modulus of elasticity

F — Load factor
Stress

F_a — Axial compressive stress permitted in the absence of bending stress

F_b — Bending stress permitted in the absence of axial stress

F_{cr} — Critical stress for axially loaded compression members

F_e' — Euler buckling stress divided by factor of safety

F_y — Specified minimum yield point for type of steel being used. Subscripts c, g, s denote column, girder and stiffener respectively.

FEM — Fixed end moment

H — Wind load per story

I — Moment of inertia

I_a — Moment of inertia of vertical bracing truss chords

K — Effective length factor

K_b — Ratio of actual K-bracing area to minimum K-bracing area

L — Distance between centerlines of vertical bracing truss chords

L_b — Length of bracing member

L_g — Length of clear girder span

LTB — Lateral-Torsional Buckling

M — Bending moment

M_d — Moment at the ends of girder under factored dead load

M_e — Moment at the ends of a girder under factored gravity load

M_g — Plastic moment required for mechanism in the absence of an axial load

M_m — Uniform moment about major axis causing lateral-torsional buckling of the member in the absence of a concentric load

M_p — Plastic moment ($F_y Z$)

M_{pc} — Plastic moment modified to include the effect of axial compression

M_j Moment about the center of a joint. Subscripts A, B, U and L denote left end of girder, right end of girder, upper end of column, and lower end of column, respectively

M_{je} Moment about center of joint caused by eccentrically framed members

M_y Moment at first yield ($F_y S$)

M_1, M_2 Peak beam-column moment capacity from M-θ curves

M_Δ Moment caused by $P\Delta$ effect

N Number of braced bays

P Axial load

P_H Total shear in a story

P_b Axial force in bracing members. Subscripts H and V denote horizontal and vertical components of force

P_c Axial load in columns due to wind and $P\Delta$ effect

P_{cr} Critical concentric buckling load

P_{ex} Major axis concentric Euler buckling load

P_g Axial force in girder

P_{ox} Major axis concentric buckling load

P_{oy} Minor axis concentric buckling load

P_u Limiting load

P_w Total working gravity load above a story

P_y Plastic axial load, AF_y

Q_Δ Shear caused by $P\Delta$ effect

R Total rotation in a story (sum of chord and web rotation) = Δ/h
Live load reduction factor.

R_b Rotation in a story due to bracing length changes

R_c Chord rotation in a story due to column length changes

R_g Rotation in a story due to girder length changes

R_w Web rotation in a story due to girder and bracing length changes

S Elastic section modulus

T Tensile flange force

V Shear force

V_u Shear force that causes the web to yield in shear

W Working wind shear at a story

Z Plastic section modulus

b Width of flange

b_s Width of stiffener

d Depth of section. Subscripts c and g denote column and girder depth, respectively

e Axial change in length. Subscript b denotes change of length of bracing

f_v Computed shear stress

h Story height

h_t Total height of braced bent

k Distance from outer face of flange to toe of web fillet
Proportionality factor for drift calculation

l Length of member
Length of unbraced beam segment

l_{cr} Critical unbraced length

n Number of level or story (Roof = 1)

q Ratio of column end moments

r Radius of gyration. Subscripts x and y refer to major and minor axis, respectively

t Flange or plate thickness. Subscripts c, g and s denote column, girder and stiffener, respectively

w Working gravity load (dead plus live load)
Web thickness

w_H Total equivalent factored horizontal load, wind plus $P\Delta$ effects (Eq. 5.15)

w_c Column web thickness

w_d Working dead load

w_g Average working gravity load over the entire building

w_l Working live load

w_p Uniformly distributed unit load corresponding to formation of a plastic mechanism in a fixed end beam

w_w Working wind load

w_Δ Equivalent factored horizontal load, $P\Delta$ effects (Eq. 5.16)

α Chord angle change in a story

α_a Chord angle change above a story

Δ Drift. Subscripts b, c, g refer to bracing, column and girder respectively

Δ_c Chord drift in a story

Δ_t Total drift at top of bent

Δ_w Web drift in a story

δ Maximum deflection of a simply supported beam

ϵ Strain

ϵ_{st} Strain at onset of hardening

ϵ_y Strain at yield point

θ End slope
Slope of diagonal stiffeners

$\Sigma\delta P$ Total factored gravity load increment applied at each level

ϕ Curvature

ϕ_{base} Curvature at bottom story

ϕ_y Curvature corresponding to moment at first yielding

CHAPTER 1

Introduction

1.1 OBJECTIVE

The objective of this publication is to acquaint practicing engineers with the present state of the theory for the plastic design of braced multistory steel frames. It is hoped that the information presented will stimulate the use of plastic design methods for frames of this type, and that this in turn will produce an input of useful ideas contributing to the full development of the concept.

1.2 CONTENTS

The information contained herein is mainly a digest of the research material presented to engineering educators at the Lehigh University Conference on Plastic Design of Multistory Frames[1] in August 1965. An effort has been made to include enough theory for the engineer to understand the behavior of the structure but to concentrate principally on design aspects. The engineer who wishes to delve into the background of research should study the references listed. The design example of a braced multistory frame will serve as a guide to the efforts of the practicing engineer as he applies the principles of plastic design to his own work. The grades of steel used in the design example are A36 with F_y = 36 ksi and A441 or A572 with F_y = 50 ksi. Design aids for these values are included. A listing of the notation used is given for ready reference. Sign conventions are discussed as they are developed.

1.3 THE FUTURE OF MULTISTORY FRAMES

Multistory and high-rise buildings have been common in some of our nation's large cities, but recent sociological trends have forced the use of such structures in more numerous locations and have pushed them to even greater heights. As the population increases and tends to concentrate in urban areas, and as land costs skyrocket, the multistory building becomes the economical solution to housing people for living and working. The tall building will be the common structure of the future and economy of the structural frame is of increasing importance. Structural steel frames proportioned by plastic design methods may offer savings over frames of other materials and over steel frames designed by allowable stress methods.

1.4 THE DESIGN TEAM

Regardless of the design method, the building process today demands an integrated team of architects, and electrical, mechanical and structural engineers. Each must understand the other's requirements, for rising costs and increased demand for excellence in construction require the integration of all building components into a compact structure with a minimum of wasted volume. The structural engineer must understand the architect's desire to have the structural frame complement the function and the motif of the building. He must be appreciative of the space needed for the conduits and ducts required by the electrical and mechanical engineers as they attempt to regulate the internal environment of the modern building. Within such constraints he must produce a safe and economical structural frame. The frame must safely support the gravity and wind loads without undue deflection or sway affecting the operation of other building components or producing unpleasant sensations to the occupants.

1.5 NEW STRUCTURAL CONCEPTS

Fortunately, the structural engineer is assisted in fulfilling these requirements by new knowledge of how structures behave, and by the advent of new materials, products and construction techniques. Research on the behavior of steel structures during the last twenty years has led to the development of the plastic design philosophy as contrasted to the more established methods of elastic design, more correctly known as allowable stress design. Composite design uses the integrated strength of steel and concrete. New high strength structural steels of carbon, low alloy and heat treated types permit a reduction in the sizes of members. High strength bolts and new welding techniques produce economical, rigid connections of greater compactness and more direct transfer of stress.

1.6 ALLOWABLE STRESS DESIGN

The current method of designing rigid multistory building frames [2] involves the determination of the internal shears, moments and thrusts caused by working loads using methods of allowable stress analysis for statically indeterminate structures. Because of the high order of redundancy of the multistory rigid frame the analysis is usually reduced to a statical one by making appropriate assumptions as in the "portal" or "cantilever" methods. Using the internal forces and an allowable stress, derived principally by dividing the yield point stress of the steel by a factor of safety, the members are proportioned using ordinary mechanics of materials equations. Inherent in this approach is the philosophy that the limit of usefulness of the structure is reached as soon as the yield point stress is developed at one point in the frame. Other points in the frame will be understressed, and thus uneconomical in the use of material. This method does not recognize that local yielding in a rigidly connected steel structure permits a redistribution of the internal forces to less highly stressed parts of the structure, and consequently it underestimates the load carrying capacity of the structure as a whole. Local yielding is not detrimental to the behavior of the structure provided it is contained by adjacent elastic regions of the frame.

1.7 PLASTIC DESIGN

On the other hand, the plastic design philosophy recognizes the redistribution of internal forces that takes place when complete yielding (plastic hinges) develops at regions of high bending moment. It focuses on the limit of usefulness as the ultimate load that can be carried just before the structure develops a sufficient number of plastic hinges to permit unrestrained deformation of the structure. This ultimate load is an indication of the strength of the whole structure, and it exceeds the working load by a factor F. The quantity F, called the load factor, is selected to be consistent with the factors of safety inherent in the allowable stress design of a simply supported beam. In this publication the following values, adopted from the Lehigh Conference, are used for beams, columns and frames:

Gravity loading	$F = 1.70$
Gravity and wind loading	$F = 1.30$

Uncertainty about stability problems was the chief reason for a somewhat higher load factor specified for frames in the past. [3] New research presented at the Lehigh Conference has led to a better understanding of the behavior of columns and therefore the values of F shown appear justified.

Deflection may also constitute a limit of usefulness for the structure, and whether designing by allowable stress or plastic methods, it is necessary to consider the vertical beam deflections and horizontal frame deflections (drift) under working loads. Deflections rather than strength may actually govern the design.

CHAPTER 2

Dimensions and Loading

2.1 CHOICE OF DIMENSIONS

The overall dimensions of the multistory building are governed by the size and shape of the site available and by set-backs from the property lines required by zoning ordinances. For reasons of architectural layout it is often advantageous for the building to be long and narrow. Within these area limitations it is the responsibility of the architect-engineer design team to determine the required number of floors to fulfill the owner's space needs. Many municipalities have zoning ordinances restricting heights of buildings, but these restrictions are being removed or liberalized as codes are revised.

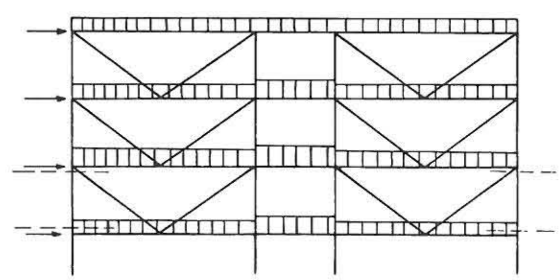
The design team must decide on bay sizes for the structural frame that fit the architectural and mechanical-electrical layouts of the integrated structure. There is a trend toward the use of larger bay dimensions, particularly with composite floor beams. Longer spans increase the depth of the floor system, thereby increasing the height of the building. However, increased floor depth often permits more economical construction even though the building volume is increased.

Regardless of the method of structural design, the items mentioned above must be considered and examined from their technical and economical aspects before bay sizes are established. The bay sizes shown for the apartment house example of Chapter 8 represent a possible, but not necessarily the best, framing plan for that structure. They represent a compromise based on the integrated requirements.

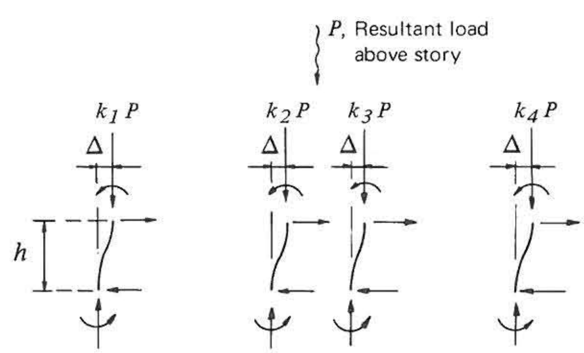
2.2 BRACING METHODS

The multistory building must be designed to provide resistance to horizontal forces applied in any direction. A number of devices may be used, including shear walls or core sections, but in the example in Chapter 8 attention will be directed toward proportioning of the steel bents to provide the necessary strength and limitation to drift. There are two conventional methods of providing the necessary resistance.

One or more bents of a frame may be braced for the full height of the building using diagonal or K-bracing. This creates a vertical cantilever truss to which all wind load is transmitted. In the allowable stress design of this type of framing the girders may have either simple or rigid connections to the columns. Plastic design requires rigid connections. Rigid connections have real advantages in allowable stress design also. For example, rigidly connected members reduce beam deflections, reduce beam depth, and reduce floor cracking.



PORTION OF BENT



ISOLATED ONE-STORY COLUMNS

FIG 2.1 THE PΔ EFFECT DUE TO SWAY

On the other hand, resistance to the horizontal forces may be provided entirely by the bending resistance of rigidly connected girders and columns.

It is desirable to define braced and unbraced bents in terms of the method of resisting secondary moments produced by drift. When a building drifts, each floor moves laterally with respect to the adjacent floors as indicated in Fig. 2.1. The vertical forces kP on the columns at one floor become eccentric with respect to the column axes at the floor beneath by an amount Δ, producing secondary moments totaling $P\Delta$.

In this publication the following definitions and assumptions will be used:

Braced Bent — Has physical brace in at least one bay of a bent on each floor. $P\Delta$ effect is controlled by the shear resistance of the bracing system. Girder connections are rigid.

Unbraced Bent — No physical brace. Strength depends on bending resistance of all members. $P\Delta$ effect must be resisted by the columns in bending. Girder connections are rigid.

Supported Bent — Depends on adjacent braced or unbraced bents for resistance to horizontal forces and $P\Delta$ effects; is designed for gravity loads only. Girder connections are rigid.

2.3 GRAVITY LOADS

Building codes specify the working live loads for floors, the roof load and wind loads. The dead load, floor live loads and roof loads are referred to as gravity loads. Although the dead load is always present many variable patterns of live loading are possible. Codes[4] permit a reduction in the live load for beams or girders supporting large floor areas and for columns supporting several tiers of floors. Such reductions recognize the improbability of having the full live load acting over large areas and on all floors simultaneously.

Partial live loading in a checkerboard pattern may control the column design. Checkerboard loads produce a lower axial force in the columns but may produce a more critical bending effect.

2.4 HORIZONTAL LOADS

Wind loads are usually expressed as a resultant unit pressure applied horizontally against the windward side of the building. Many modern codes require an increase in wind pressure as the height above the ground increases. It is customary to convert the wind pressure to forces applied at each floor level, and to assume that the floors, acting as diaphragms, transfer the wind forces along the building to the periodically spaced braced frames.

The application of plastic design to seismic loading is an area of current study.[5]

CHAPTER 3

Fundamentals of Plastic Design

3.1 MATERIAL PROPERTIES

The successful application of plastic design to structures depends on two desirable properties of structural steel—strength and ductility. These are portrayed by the stress-strain diagram (Fig. 3.1). The level of strength used in plastic design is that of the yield plateau, F_y. The length of

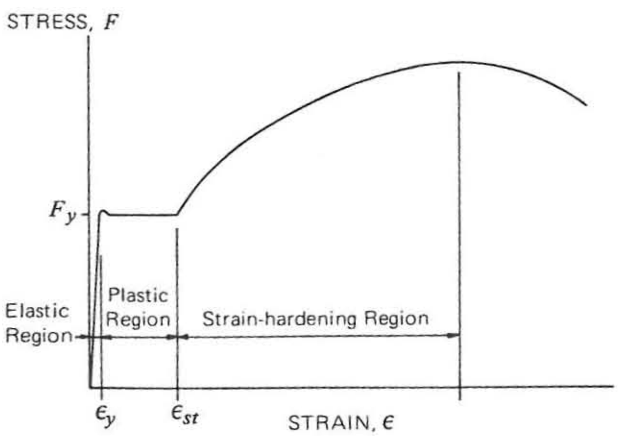


FIG 3.1 STRESS-STRAIN DIAGRAM FOR STRUCTURAL STEEL

that plastic plateau is a measure of the ductility; for A36, A441, and A572 steels the strain at the limit of the plastic region, ϵ_{st}, is approximately 12 times the strain at the initiation of yielding, ϵ_y. In plastic design the actual stress-strain diagram is replaced by an idealized diagram representing steel as an elastic-plastic material (Fig. 3.2).

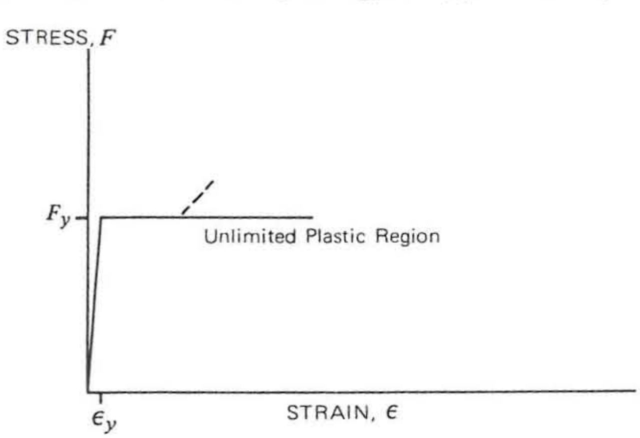


FIG 3.2 IDEALIZED STRESS-STRAIN DIAGRAM

The allowable stress design method defines the limit of usefulness of a cross-section as occurring when the strain in one fiber only reaches ϵ_y, but the plastic design method considers the remaining usefulness after the attainment of ϵ_y in all fibers. That is, the cross-section becomes fully plastic (Fig. 3.3).

3.2 IDEALIZED CONCEPTS FOR BEAMS

Plastic design has its chief utility in the design of structures composed of bending members. In such members the strains are proportional to the distance from the neutral axis under all magnitudes of loading but the stresses are not proportional once the fibers have strained beyond ϵ_y. When the bending moment at a section becomes so great that practically all fibers have strains greater than ϵ_y, the stress distribution diagram approaches a fully yielded condition known as a

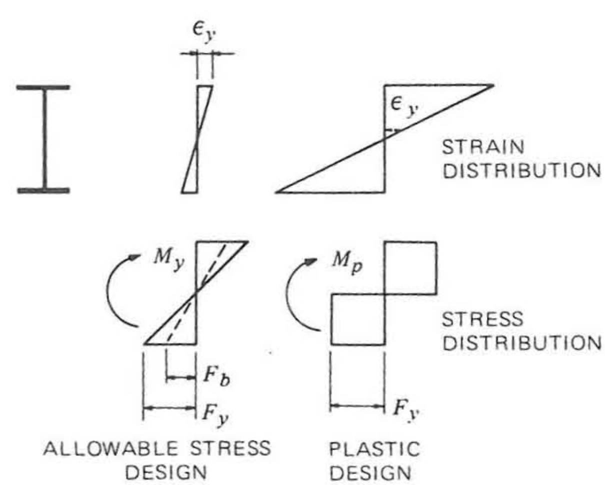


FIG 3.3 LIMIT OF USEFULNESS, BENDING ONLY

the rules for spacing of lateral bracing provide for a variable distance, l_{cr}, depending upon the ratio of the moment, M_p, at the braced hinge and the moment, M, at the other end of the unbraced segment (Fig. 3.7).

Recent analytical work [1] taking into account different kinds of steels and the stress condition of the adjacent segments, justifies the provisions tabulated in Table 3.2 for l_{cr} with the common condition of elastically stressed adjacent segments.

TABLE 3.2

Specified Minimum Yield Point, F_y	$(l_{cr})_1$ Uniform Moment $M/M_p \geqslant 0.7$	$(l_{cr})_2$ Moment Gradient $-1.0 \leqslant \frac{M}{M_p} < 0.7$
36 ksi	$38r_y$	$65r_y$
50 ksi	$28r_y$	$55r_y$

If a braced segment, l, of a beam is bent about its strong axis by equal end moments causing uniform moment the end moments will reach M_p provided $l \leqslant l_{cr}$. However, if $l > l_{cr}$ lateral-torsional buckling will occur at $M_m < M_p$ as

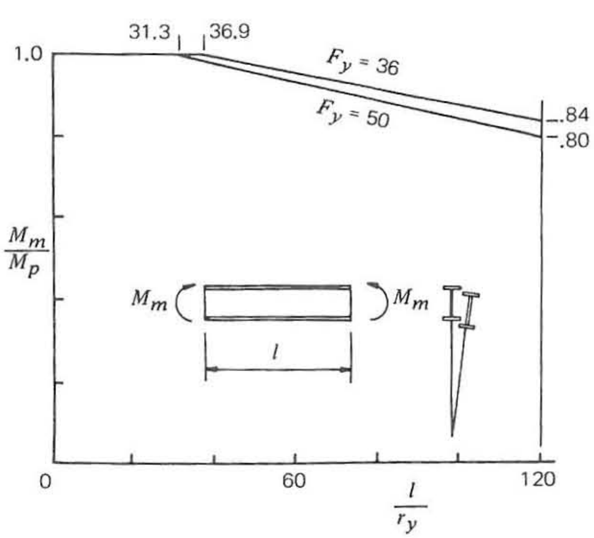


FIG 3.8 LATERAL TORSIONAL BUCKLING UNDER UNIFORM MOMENT

shown by Fig 3.8. This value of M_m is of importance in the lateral-torsional buckling of beam-columns.

In segments where the beam is behaving elastically or at the last hinge of the plastic mechanism the spacing of braces is determined by rules of allowable stress design. Recommendations for sizes of lateral braces are given in Ref 1.

3.3c SHEARING FORCE IN BEAMS

The simplified plastic theory is developed for conditions of pure bending but in practice flexure is usually accompanied by shearing forces. The influence of shear is masked by strain hardening and local and lateral buckling, but, as a design criterion, the limiting shear may be taken as the force that causes the entire web to yield in shear, V_u. Beams and columns should be proportioned according to

$$V \leqslant V_u = 0.55 F_y w d \qquad (3.2)$$

where F_y is in ksi.

If V exceeds the shear carrying capacity of the beam, V_u, a new beam with greater web area may be chosen, or the web may be reinforced with doubler plates.

3.3d AXIAL FORCE IN BEAMS

If a member short enough to preclude failure in a buckling mode is subjected to an axial force in addition to a bending moment, the plastic hinge develops at a reduced plastic moment value designated as M_{pc}. M_{pc} depends on the cross-sectional properties of the member, the yield stress of the steel, and the magnitude of the axial load. The influence of the axial force in reducing the value of M_{pc} is seen in Fig 3.9 where the approximate interaction equation for strong axis bending of WF column sections is also plotted.

$$\frac{M_{pc}}{M_p} = 1.18 \left(1 - \frac{P}{P_y}\right) \qquad (3.3)$$

For values of $P \leqslant .15\,P_y$ it is permissible to take $M_{pc} = M_p$.

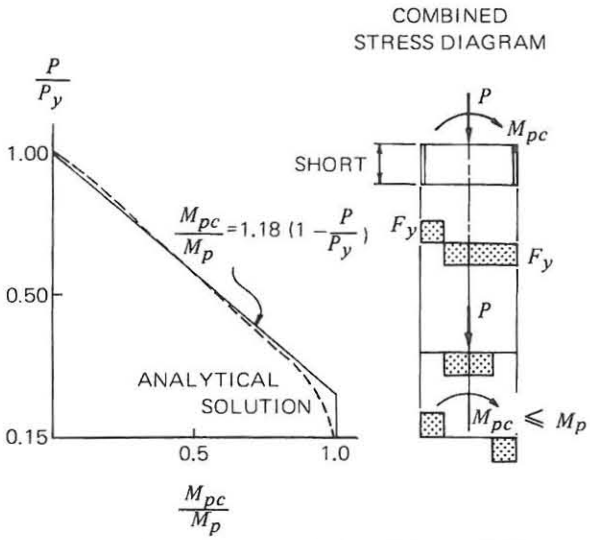


FIG 3.9 INTERACTION OF AXIAL FORCE AND MOMENT FOR STRONG AXIS BENDING OF SHORT WF COLUMN

The quantity M_{pc} is a basic characteristic of the cross-section of a short compression member but it is not necessarily indicative of the carrying capacity of longer columns where the slenderness ratio, $\frac{h}{r}$, may have an appreciable influence on the behavior of the column.

3.4 COLUMNS

Columns in multistory building frames will be loaded by axial forces alone if the shears and end moments from the girders are symmetrical about the column centerline at all floor levels. If these forces are not symmetrical the member becomes a beam-column subjected to axial force and bending moment. When lateral loads are applied to the frame, one floor may move laterally a small distance Δ with respect to the next one below and a moment, $P\Delta$, may have to be considered. (See Fig. 2.1.)

3.4a AXIALLY LOADED COLUMNS

Building columns usually have slenderness ratios less than C_c and failure will occur by inelastic buckling. The critical concentric buckling load is given by

$$P_{cr} = F_{cr} A \qquad (3.4)$$

where F_{cr} is a critical stress expressed as the allowable stress of Formula (1) of the AISC Specification multiplied by a load factor of 1.7. Thus,

$$F_{cr} = 1.7 F_a = \frac{1.7\left[1 - \dfrac{(K\frac{h}{r})^2}{2C_c^2}\right]F_y}{F.S.} \qquad (3.5)$$

for $K\frac{h}{r} \leqslant C_c$ where $C_c = \dfrac{23{,}900}{\sqrt{F_y}}$

The factor of safety, $F.S.$, is a variable quantity ranging from 1.67 to 1.92. C_c is the column slenderness ratio at the transition from inelastic to elastic buckling.

The strength of axially loaded columns is also influenced by the conditions of flexural restraint at the ends of the columns. The restraint of the supports is indicated by the effective length factor K. For columns in plastically designed braced frames $K = 1$ should be used. In a braced frame, translation at the column ends is inhibited, and when hinges form at the ends of girders the end restraint of the columns is reduced and the buckled shape of the individual column approximates the pinned-end condition.

3.4b BEAM-COLUMNS

The ultimate strength of a beam-column depends on:

1. the material properties, expressed by F_y
2. the slenderness ratio, h/r
3. the axial load ratio, P/P_y
4. the magnitude of upper and lower end moments, M_U and M_L, respectively

5. the direction of the end moments expressed by q, the ratio of the numerically smaller to the numerically larger end moment

The ultimate strength of beam-columns may be represented by moment-rotation curves or by interaction curves. Both procedures will be described briefly.

The effect of the magnitude of the axial load on a short column's ability to resist moment has been illustrated in Fig 3.9. Another way of showing this is by M-P-Φ diagrams as plotted in Fig 3.10 for a particular size column. This plot shows the influence of the axial load in reducing the moment carrying capacity of an 8WF31 column, but it is reasonably indicative of the behavior of all other size columns.

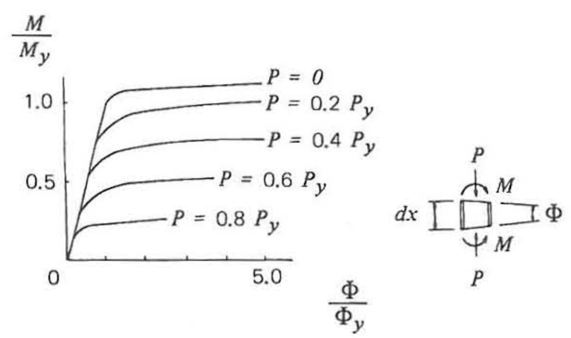


FIG. 3.10 M-P-Φ DIAGRAM FOR 8WF31 WITH RESIDUAL STRESS

Using the M-P-Φ curves it is possible, by numerical integration, to represent the ultimate strength of beam-columns by a series of "end moment—end rotation", M-θ curves. The end moments play an important role in influencing the behavior of the beam-column. Several important cases for strong axis bending are illustrated in Fig 3.11 for beam-columns with $h/r = 30$ and $P/P_y = 0.6$. The charts of Design Aid II show M-θ curves for two end moment conditions and values of P/P_y from 0.3 to 0.9 for beam-columns bent about the strong axis.

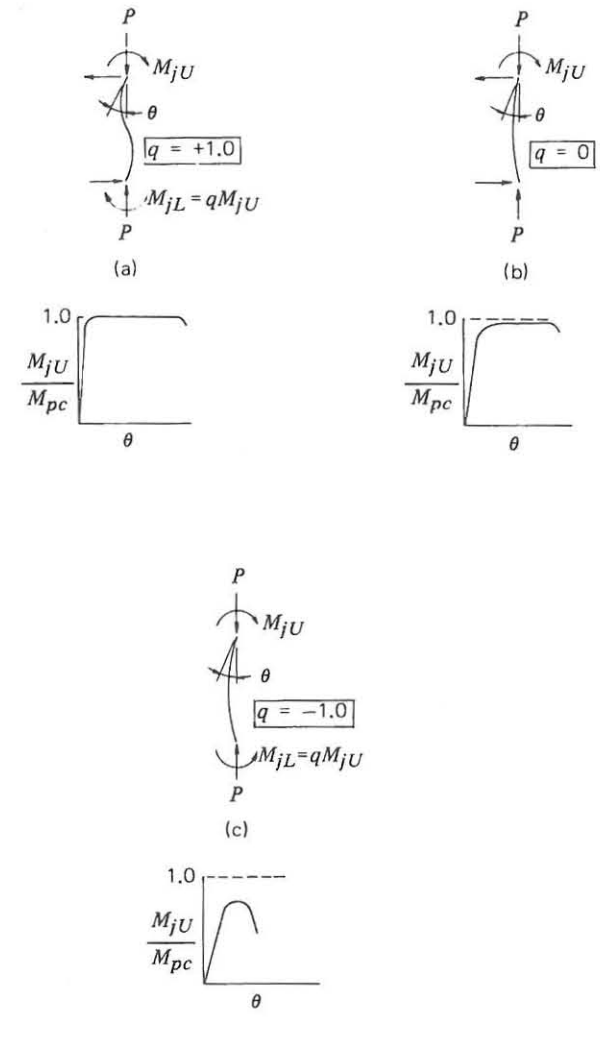


FIG 3.11 MOMENT-ROTATION CURVES (STRONG AXIS BENDING)

In Fig 3.11a a beam-column is bent in double curvature by end moments of equal magnitude acting in the same direction, $q = +1.0$. This is a favorable configuration in which the plastic hinges form at the ends at a value of M_{pc}, and are maintained through a considerable rotation.

In the beam-column of Fig 3.11b bending is produced by a moment at one end only, $q = 0$. Even in this case the maximum moment that can be developed at the end is practically M_{pc}. However, study of Design Aid II will show that for greater slenderness ratios and higher ratios of P/P_y there may be a reduction below M_{pc}.

The Design Aid charts for $q = 0$ may be used for the case of $q = +1.0$ by using an equivalent slenderness ratio equal to one-half of the actual. For A36 steel columns bent in double curvature it is only for $P/P_y > 0.9$ and $h/r > 40$ that there is an appreciable reduction below M_{pc}.

In Fig 3.11c the beam-column is bent in single curvature by equal end moments, $q = -1.0$. The plastic hinge does not occur at the ends, the end moments never reach the value of M_{pc}, rotation capacity is reduced, and unloading occurs after a small rotation.

The charts of Design Aid II may be used for design by assuming a column size, calculating P/P_y and h/r, entering the appropriate chart for P/P_y and q, and reading the maximum value of M/M_{pc}. The latter value multiplied by M_{pc} must equal or exceed the given external moment M for the design to be satisfactory. These charts are most useful when making sub-assemblage checks of the design where joint rotations are of concern. The end points on the moment-rotation curves represent the development of local buckling.

For steel other than A36 the same curves may be used by calculating an equivalent slenderness ratio as follows:

$$\left(\frac{h}{r_x}\right)_{\text{equiv.}} = \left(\frac{h}{r_x}\right)_{\text{actual}}\sqrt{\frac{F_y}{36}} \qquad (3.6)$$

and modifying the rotation obtained by

$$\Theta = \Theta_{\text{chart}}\sqrt{\frac{F_y}{36}} \qquad (3.7)$$

Curves for other values of q are available [1] but those given in Design Aid II are usually sufficient for design purposes.

A second method of designing beam-columns uses strong axis interaction curves obtained by plotting the maximum moments from the moment rotation curves of Design Aid II for various values of P/P_y and h/r. The right hand charts of Design Aid III were obtained in this way. Since the in-plane bending strength of WF sections is insensitive to the actual cross-section dimensions, diagrams such as these will suffice for all members.

If a beam-column has significantly different section properties for the major and minor axes, and if the external moments are applied about the major axis, unbraced beam-columns may experience lateral-torsional buckling before the in-plane bending capacity is reached. The rotation capacity will also be impaired. A conservative estimate of the lateral-torsional buckling strength of beam-columns bent about the major axis by end moments may be made by the following interaction equation.

$$\frac{P}{P_{oy}} + \frac{C_m M}{M_m}\left(\frac{1}{1 - P/P_{ex}}\right) \leqslant 1.0 \qquad (3.8)$$

where:

- P = applied factored axial load
- P_{oy} = minor axis concentric buckling load from Eqs. 3.4 and 3.5
- C_m = $0.6 - 0.4q$ but not less than 0.4
- M = numerically larger end moment
- M_m = uniform moment about major axis causing lateral-torsional buckling of a beam without concentric load. (See Fig 3.8)

The term in parentheses is an amplification factor similar to that in Formula (7a) of the AISC Specifications.[3] P_{ex} is the major axis concentric Euler buckling load given by

$$P_{ex} = 1.92 A F_e' \qquad (3.9)$$

where F_e' is an allowable stress given by Formula (2) of the AISC Specification as

$$F_e' = \frac{149{,}000}{\left(K\frac{h}{r}\right)^2} \quad \text{(ksi)} \qquad (3.10)$$

and 1.92 is a load factor chosen to negate the factor of safety used in Eq. 3.10.

Design Aid III includes three pairs of charts that give the moment capacity of A36 steel WF beam-columns bent about the major axis with a constant end moment ratio q. Charts are provided for double curvature bending ($q = +1.0$), one end pinned ($q = 0$), and single curvature bending ($q = -1.0$).

The first chart of each pair is based on the lateral-torsional buckling (LTB for brevity) moment capacity derived from Eq. 3.8 for specified values of h/r_y.

The LTB charts assume that the beam-column is braced about both axes only at its ends and that $r_x/r_y = 1.7$, which is a common ratio for *WF* columns of width equal to depth; other light sections have higher values of r_x/r_y. These charts give slightly conservative values of Eq. 3.8 for *WF* columns with $r_x/r_y > 1.7$. The intercepts of the h/r_y curves on the load (P/Py) axis are the ratios P_{oy}/P_y where P_{oy} is the minor axis buckling load from Eq. 3.4. Hence, the LTB charts automatically provide a check for minor axis column buckling due to concentric load.

The second chart is based on the maximum in-plane bending moment capacity determined from the peaks of the M-θ curves for specified values of h/r_x in Design Aid II.

The horizontal coordinate axis of the interaction charts indicates the beam-column moment capacity in the form M/M_{pc}. The reduced plastic moment M_{pc} from Eq 3.3 is an upper bound on the moment capacity of *WF* beam-columns bent about the major axis. Note that the axial load ratio P/P_y is used both to enter the interaction charts and to determine M_{pc}.

Design Aids II and III may be used for steels with other values of F_y by entering the curves with an equivalent slenderness ratio from Eq 3.6 and by modifying the end rotation θ using Eq 3.7.

The M-θ curves in Design Aid II are based on in-plane behavior only. If the beam-column moment exceeds the lateral-torsional buckling moment capacity from Design Aid III, lateral bracing must be provided to ensure in-plane behavior. If the beam-column is unbraced between its ends, the M-θ curve is valid only for moments less than the lateral-torsional buckling moment. For an unbraced beam-column in ***single curvature*** bending ($q = -1.0$), lateral-torsional buckling ***always*** limits the maximum moment capacity to a value below the peak of the M-θ curve. In the more usual case of double curvature bending ($q = +1.0$), the maximum in-plane moment capacity of an unbraced beam-column can frequently be attained without lateral-torsional buckling, depending on the minor axis slenderness h/r_y and the axial load ratio P/P_y.

The behavior of beam-columns illustrated by the M-θ curves of Design Aid II will not develop if a local buckle of the flange or web occurs. To prevent an early occurrence of local buckling the width-thickness ratio of the component parts must be limited to certain values as shown in Table 3.3.

TABLE 3.3

Specified Minimum Yield Point, F_y	Flange b/t	Web d/w
36 ksi	17.4	70–100 P/P_y but need not be less than 43
50 ksi	14.8	60–85 P/P_y but need not be less than 36

CHAPTER 4

Design of Supported Bents for Gravity Loads

4.1 INTRODUCTION

This is the first of several chapters which illustrate the plastic design of a braced multistory building. Included in this chapter are a description of the building to be designed and the scope of the design example. This is followed by an explanation of the design of a multistory bent for full gravity loads. The design calculations for the example described here are grouped together in Chapter 8 for easy reference.

4.2 DESCRIPTION OF BUILDING

The plastic design example concerns a 24-story apartment building. Preliminary structural plans are summarized in Fig. 8.1. The main structural elements are 3-bay rigid bents with AISC Type 1 (rigid) girder-to-column connections, spaced 24 ft. apart. The floor framing includes a 2½ in. lightweight concrete slab on a corrugated steel form supported by open web steel joists. Tie beams and spandrels between the rigid bents are framed to the columns using AISC Type 2 (simple) connections. This structural system causes column moments from gravity loads to occur only in the plane of the rigid bents.

Section A-A in Fig. 8.1 indicates an 8 ft. clear ceiling height and a construction depth of 1 ft.-8 in. These give a story height of 9 ft.-8 in. except in the bottom two stories where the height is increased to 12 ft. A depth limitation of 14 in. is set for the rigid frame girders to maintain a flush ceiling in contact with the bottom chord of the steel joists.

The numbering system used to identify members in the design calculations is shown in Fig. 8.1. The column lines are numbered 1 to 4 and the floor levels are numbered from the roof down. The letters A and B designate individual rigid bents.

The lower portion of Fig. 8.1 summarizes the working loads. To simplify the numerical work, the floor loads in the 8 ft. corridor are applied over the full 12 ft. width of the interior bay between column lines 2 and 3.

The intent of this example is to illustrate the application of plastic design concepts to a practical building problem. The framing in Fig. 8.1 is one of several practical structural solutions for this building and should not be regarded as an optimum structural system.

4.3 WIND BRACING

The size and shape of the building in Fig. 8.1 suggest that resistance to wind is an important structural consideration. Vertical bracing is usually the most economical solution when architectural requirements permit its use. It is important to give early consideration to the integration of architectural and structural requirements so that a vertical bracing system can be incorporated into the walls of a building. If possible, the vertical bracing system should be symmetrical in plan to avoid torsional effects.

The dashed lines on the floor plan in Fig. 8.1 indicate the vertical bracing system used in this design example. Vertical bracing is located in the exterior walls on column lines 1 and 4 to carry wind loads acting on the short side of the building. As an alternative, the exterior masonry walls can be used to resist wind on the short side of the building. The stiffness of these walls may resist a portion of the wind shear even if vertical bracing is provided. K-bracing is used in the exterior bays of three rigid bents to resist wind acting on the long sides of the building.

The plastic design example considers the design of the supported Bents A and braced Bent B shown on the floor plan in Fig. 8.1. No vertical bracing is provided in the supported Bents A which are designed to carry only gravity loads. Horizontal forces are transmitted from Bents A by diaphragm action of the floor slab, to Bent B. The K-bracing in the plane of Bent B is assumed to resist the horizontal shears from wind on a 96 ft. length of the building and to provide the stiffness needed to resist in-plane frame instability ($P\Delta$ effects, see Chapter 2 and Art. 5.4) for three Bents A and one Bent B. Interior partitions enclose the K-bracing.

It is assumed that wind forces parallel to column lines 1 to 4 on the ends of the building in Fig. 8.1 are resisted by the exterior walls or by vertical bracing between bents, so that wind in this direction does not influence the design of Bents A and B. Resistance to out-of-plane sidesway buckling of Bents A and B is provided by the same bracing systems. Reference 6 discusses how the stiffness of walls may be used to resist sidesway buckling.

4.4 SCOPE OF DESIGN EXAMPLE

The design example is organized into four parts:

- Part 1: Design of Supported Bent A for Gravity Load—Chapter 4
- Part 2: Design of Braced Bent B for Gravity and Combined Loads—Chapter 5
- Part 3: Design Checks for Bents A and B—Chapter 6
- Part 4: Design of Typical Connections—Chapter 7

The calculations are arranged in a tabular manual subroutine format, for ease of reference and to suggest the potential for computer subroutines. A condensed form of the calculations can be adopted after attaining familiarity with plastic design. The manual subroutines used in each part of the design example are listed in Tab. 8.1.

The emphasis in Parts 1 and 2 of the design example is on the selection of members to satisfy one or more design criteria which are likely to control. Design checks of the trial members for other pertinent design criteria are considered in Part 3.

The manual subroutines used in the design of Bent A include Tables 8.2 to 8.8 and are listed in Tab. 8.1. The major steps in the design are summarized below.

1. Design the roof and floor girders for factored gravity load in Tabs. 8.2 and 8.3.
2. Tabulate column load data and gravity loads in the columns in Tabs. 8.4 and 8.5.
3. Determine the column moments for factored gravity load in Tab. 8.6.
4. Select column sections for factored gravity load and investigate these sections for in-plane bending and lateral-torsional buckling under combined axial load and bending in Tabs. 8.7 and 8.8.

These steps are described in Arts. 4.5 to 4.8. The column design criterion is stated in Art. 4.7 and reviewed in Art. 4.9.

4.5 DESIGN OF GIRDERS IN BENT A

The roof girders for Bent A are selected in Tab. 8.2 and the floor girders in Tab. 8.3. The criterion used in designing these girders is the formation of a 3-hinged beam mechanism (Fig. 3.4) under uniformly distributed factored gravity loading. The end hinges form in the girders, outside of the girder-to-column joints, so the clear span L_g of the girders is used to find the required plastic moment.

$$M_p = \frac{1.7\, w\, L_g^2}{16} \tag{4.1}$$

Here, w is the uniformly distributed working load on the girder which is multiplied by the gravity load factor F = 1.7. The required plastic modulus $Z = M_p/F_y$ is used to select the girder sections.

It is assumed in Tab. 8.2 that the exterior columns below the roof will provide a plastic moment capacity (reduced for axial load) at least equal to that of the exterior roof girders.

Article 6.3 of Ref. 1 describes a method for redesigning the exterior roof girders when the supporting columns have smaller plastic moment capacities than the girders.

The floor girder design in Tab. 8.3 is similar to that for the roof girders except that the working loads are modified by live load reductions. The live load reduction provisions of the American Standard Building Code (Ref. 4, Section 3.5) are applied in lines 5 to 8 of Tab. 8.3.

The girders selected in Tabs. 8.2 and 8.3 are adequate for factored gravity load. These trial sections will be checked for live load deflection and lateral bracing requirements in Chapter 6.

4.6 COLUMN GRAVITY LOADS AND MOMENTS -BENT *A*

The loading pattern that is likely to control the size of the columns in Bent *A* is full factored gravity load on all girders (F = 1.7). This article is concerned with the determination of the axial loads and moments in the columns for this loading condition. Other gravity loading conditions, consisting of various "checkerboard" live load patterns on alternate floors and bays, will produce different moment and end-restraint conditions in the columns. The effect of checkerboard loading on the columns is considered in Chapter 6. Here, it suffices to comment that checkerboard loading does not govern the column design in this example; it should be investigated when the adjacent girder spans and loads are nearly equal and the ratio of dead load to total load on these spans is less than 0.75.

The column design begins with Tab. 8.4 in which the column loads originating from the roof and from each floor are determined. The first 8 lines in this table are used to record tributary floor areas and unit loads. Lines 9 to 13 include the calculations for the working load in the columns below the roof. Lines 19 to 22 give the total dead load and live load contributed by each floor.

The values are used to find the maximum percent live load reduction, Max. R in line 23 (Ref. 4, Section 3.5). The limiting value of the live load reduction is Max. R or 60 percent. Line 24 gives the percent live load reduction below level 2, based on the tributary floor area. When this rule is applied below level 4, it is found that the permitted live load reduction is controlled by the 60 percent limit from levels 4 to 24. The reduced live loads from the top three floors are entered in lines 27 to 29 of Tab. 8.4. Line 30 gives the constant reduced live load increment from levels 5 to 24. The calculations in this table are independent of the design method since the same working loads are used in plastic design as in allowable stress design.

COLUMN LOADS

The column dead and reduced live loads are tabulated in Tab. 8.5. The first line of numbers in this table is the load increment from one floor which is constant between levels 5 and 22. For example, the dead load increment of 34.6 kips in Col. (1) is obtained from line 19 of Tab. 8.4. The sum of the dead and reduced live loads gives the working loads in Cols. (3) and (8) of Tab. 8.5. Multiplication by F = 1.7 and 1.3 yields the factored loads needed in the plastic design of the columns.

COLUMN MOMENTS

The columns must also resist bending moments which are determined in Tab. 8.6. The sign convention and notation for moments on a joint are indicated below the table. Positive moments act clockwise on the ends of members (or counter-clockwise on joints) and M_j denotes a moment about the center of the joint. The additional subscripts A and B indicate moments at the left and right ends of girders, while U and L denote moments at the upper and lower ends of columns. Equilibrium of moments on a joint is then expressed by the equation

$$\Sigma M_j = 0 \quad \text{or}$$

$$M_{jU} + M_{jL} = -(M_{jA} + M_{jB} + M_{je}) \qquad (4.2)$$

where M_{je} is the moment about the center of the joint caused by eccentrically framed members such as the spandrel beams. The right side of this equation represents the net girder moment on the joint.

Full factored gravity load may be assumed to cause plastic hinges at the ends of all girders in Bent A. Thus the girders apply known moments to the joints. These girder moments do not depend on the joint rotations because the girder plastic hinges eliminate compatibility between the end rotations of the girders and columns. The sum of the column moments, $M_{jU} + M_{jL}$, above and below a joint is statically determined from Eq. 4.2.

The moment at the center of a joint from the girder to the left of the joint is

$$M_{jB} = M_B + V\frac{d_c}{2} \qquad (4.3)$$

where M_B is the girder end-moment (at face of column), V is the girder end-shear, and d_c is the column depth. Under factored gravity load the girder end-moment M_B is taken as the required plastic moment M_p from Eq. 4.1 and the shear V = 1.7 $w\ L_g/2$. Then the girder moments at the center of a joint are conveniently calculated from

$$\begin{aligned} M_{jB} &= M_p\ (1 + 4d_c/L_g) \\ M_{jA} &= -M_p\ (1 + 4d_c/L_g) \end{aligned} \qquad (4.4)$$

These equations are valid for a girder that forms a 3-hinged mechanism under uniformly distributed factored gravity loads.

Equations 4.4 are applied in lines 1 to 6 and lines 9 to 14 of Tab. 8.6. The moments are then summed according to Eq. 4.2 in lines 8 and 16. At the roof, $M_{jL} = 0$ so line 8 gives the column moment M_{jU}. At joints below the roof, half of the net girder moment is distributed to the columns above and below the joint in line 17. This distribution of column moments is a reasonable estimate but may be revised, if convenient, when the columns are designed. See Art. 4.9. The results of the calculations in Tab. 8.6 are summarized in the column moment diagram below the table, with moments plotted on the tension side.

4.7 COLUMN DESIGN ASSUMPTIONS

The assumptions and design criterion for the columns in Bent A are discussed in this article. It is assumed that:

1. The *WF* columns are to be erected in two story lengths with their webs in the plane of the rigid bents.
2. Moments are applied only about the major axis of the columns, with no biaxial bending permitted. For this reason AISC Type 2 (simple) connections are used between the columns and the tie beams and spandrels.
3. Vertical bracing on column lines 1 and 4 at floor levels, or the stiffness of exterior walls, together with diaphragm action of the floor slabs, are considered adequate to prevent out-of-phase sidesway buckling of the rigid bents.
4. No lateral bracing is provided for the columns between floors. (This differs from the assumption of laterally braced columns in Ref. 1).
5. Moment resistance at the column bases is conservatively neglected in the design of the bottom story columns.
6. The columns are limited to 12 and 14*WF* sections to maintain uniform architectural details and to simplify column splices.

The columns resist concurrent axial load and bending moments and are termed beam-columns. Chapter 3 lists the parameters that may influence beam-column behavior. These parameters include the major and minor axis slenderness. The approximation $r_x \approx 0.43d$ (for the lightest rolled *WF* column sections in each nominal size) may be used for a preliminary and conservative estimate of h/r_x. Based on the assumption of 12*WF* columns in the 9.67 ft. stories and 14*WF* columns in the lower 12 ft. stories of Bent A, the major axis slenderness ratio will not exceed 24.

The minor axis slenderness can be estimated from the ratio $r_x/r_y \approx 1.7$ for heavy rolled *WF* column sections. Thus, h/r_y will not exceed 41 in the lower story columns where lateral-torsional buckling may control.

The end-moment ratio q, described in Fig. 3.11, is an important parameter in the design of beam-columns because of its influence on the end moment versus end rotation behavior (M-θ). Full factored gravity load is considered to cause double curvature (q = +1.0) in all columns of Bent A except those in the top and bottom stories where the end moment ratio q = 0 is conservatively assumed.

The sum of the beam-column moment capacities above and below a joint must equal or exceed the net girder moment on the joint from Eqs. 4.2 and 4.4. This is the criterion to be satisfied in the design for full factored gravity load. The range of application of this column design criterion depends on the M-θ behavior of the beam-columns. This criterion will be discussed after the columns have been designed.

It is not necessary to apply the column design criterion for full factored gravity load at every joint in Bent A because of the equal floor loads and because the columns are erected in two story lengths. When the upper and lower segments of one column length have the same unbraced height and end moment ratio, the lower segment will provide the smaller beam-column moment capacity because this segment resists the larger axial load. This lower column segment can be designed to resist half of the net girder moment on the floor above the column splice. The top columns should be checked below the joints on level 2 and at the roof since the segments below the roof are not bent in double curvature.

4.8 DESIGN OF COLUMNS IN BENT A

Trial A36 column sections can be selected using the formula

$$P_y = P + 2.1\ M/d \text{ but not less than } JP \qquad (4.5)$$

where P = required axial load capacity, kips
M = required major axis end moment capacity, kip-ft.
d = estimated column depth, ft.
$P_y = AF_y$, kips
J = 1.12 for F_y = 36 ksi and $h/r_y \leqslant 40$
= 1.18 for F_y = 50 ksi and $h/r_y \leqslant 40$

This formula assumes that the beam-column moment capacity is governed by M_{pc} from Eq. 3.3 and is derived as follows:
Using $M_{pc} = M$ in Eq. 3.3 gives

$$P_y = P + M\ (0.85\ P_y/M_p) \qquad (4.6)$$

The ratio M_p/P_y may be expressed as a function of the depth d in the form

$$\frac{M_p}{P_y} = \frac{ZF_y}{AF_y} = \frac{Z}{S}\frac{2d}{}\left(\frac{r_x}{d}\right)^2 \qquad (4.7)$$

Then Eq. 4.5 follows from the approximations for most *WF* shapes, bent about the major axis

$$Z/S \approx 1.12 \qquad r_x/d \approx 0.43 \qquad (4.8)$$

The term $2.1M/d$ in Eq. 4.5 represents an "axial load equivalent" for the major axis moment. When this term is small compared with P the resulting P/P_y ratio approaches unity and the beam-column moment capacity is controlled by *lateral-torsional buckling*, instead of M_{pc}. See Design Aid III. Assuming the column has a minor axis slenderness of 40 or less (Art. 4.7) and must resist major axis moments of say 0.4 M_{pc} in double curvature bending (q = 1.0), the maximum value of P/P_y = 0.89 for A36 steel, so P_y should exceed 1.12P. This is the basis for the qualification in Eq. 4.5. For A572 steel with F_y = 50 ksi and the same slenderness, the limit on P_y should be increased to 1.18P.

The value of $J = P_y/P$ in Eq. 4.5 may be selected from the *LTB* charts in Design Aid III for other estimated values of h/r_y, q, and M/M_{pc}. For $P/P_y > 0.8$ and $q \geqslant 0$, the *LTB* curves for constant h/r_y are relatively flat so the value of J is not sensitive to the assumption for M/M_{pc}.

Trial column sections for Bent A are selected in Tab. 8.7 using Eq. 4.5. The required axial load and moment are entered in Col. (1). The estimated column depth and the term $2.1M/d$ are recorded in Col. (2). The required P_y and the selected steel grade in Cols. (3) and (4) are used to choose trial sections shown in Col. (5), from Design Aid I. Alternate trial sections in A36 and A572 steel are included in the upper and lower portions of Tab. 8.7.

Cols. (6) to (8) in Tab. 8.7 are not needed in practice but are included to demonstrate the effectiveness of the trial column selection method. The two trial sections at each level in Col. (5) provide P_y values which bracket the required P_y in Col. (3). The P/P_y ratios and M_{pc} values for the lightest column sections are recorded in Cols. (7) and (8).

In the upper stories the lighter columns provide M_{pc} values that are less than the required moment capacity M (where M is based on a 50 percent distribution of net girder moment to the columns above and below each joint). In most cases the difference between M and M_{pc} for these lighter trial sections is substantial. This suggests that an attempt to use the lighter trial sections with a redistribution of column moments is not likely to be valid; redistribution is discussed in Art. 4.9.

In the lower stories, the lighter trial sections in Tab. 8.7 provide M_{pc} larger than M but the larger P/P_y ratios for these sections suggest that their moment capacity may be limited by lateral-torsional buckling. Subsequent calculations indicate that all but one of the lighter trial column sections in Tab. 8.7 are not adequate. The exception is the 14WF142 (A36) interior column below level 20 which provides P_y nearly equal to that estimated from Eq. 4.5.

The point of the preceding comments is that trial columns selected to provide P_y per Eq. 4.5 will usually be lower bound estimates of the required column size for a given nominal depth.

The next step in the design is to investigate the trial beam-column sections for their in-plane bending and lateral-torsional buckling moment capacities in Tab. 8.8. The first three tabular columns are used to record the column data known at the beginning of this investigation. This data includes: the required axial load P and moment M, column height h, end moment ratio q, trial section and steel grade. Cols. (4) and (5) give P_y, M_p, r_x, and r_y for the trial section from Design Aid I. Alternate designs in A36 and A572 (F_y = 50 ksi) steel are included in the upper and lower portions of Tab. 8.8. The exterior and interior columns are grouped on the first and second sheets of this table respectively.

The beam-column calculations begin in Cols. (6) and (7) of Tab. 8.8, which give the P/P_y, M_{pc}/M_p, and slenderness ratios needed to enter the interaction charts in Design Aid III. Note that the slenderness ratios for the A572 steel columns are modified by the coefficient $\sqrt{F_y/36}$ per Eq. 3.6.

Design Aid III is used to find the beam-column moment capacity in the form M/M_{pc}, which is recorded in Tab. 8.8(8). The procedure includes the following steps:

1. Select the correct pair of interaction charts for the end moment ratio q. For values of q between +1.0 and 0, or between 0 and −1.0, conservative estimates of M/M_{pc} can be obtained from the charts for q = 0 or −1.0 respectively.
2. Enter the *lateral-torsional buckling* (LTB) chart with P/P_y, and read M/M_{pc} from the curve for h/r_y.
3. Enter the *in-plane bending* chart with P/P_y and read M/M_{pc} from the curve for h/r_x.

The smaller value of M/M_{pc} from steps (2) and (3) indicates the beam-column capacity and the mode that controls this capacity. Frequently, the interaction charts for columns in double curvature bending give M/M_{pc} = 1.0. This indicates that the moment capacity is governed by M_{pc} from Eq. 3.3 and is not affected by slenderness.

The beam-column check concludes with the calculation of the maximum allowable moment capacity $M = (M/M_{pc}) \times (M_{pc}/M_p) \times M_p$ in Tab. 8.8(8). The trial section is adequate for full factored gravity load if the allowable moment

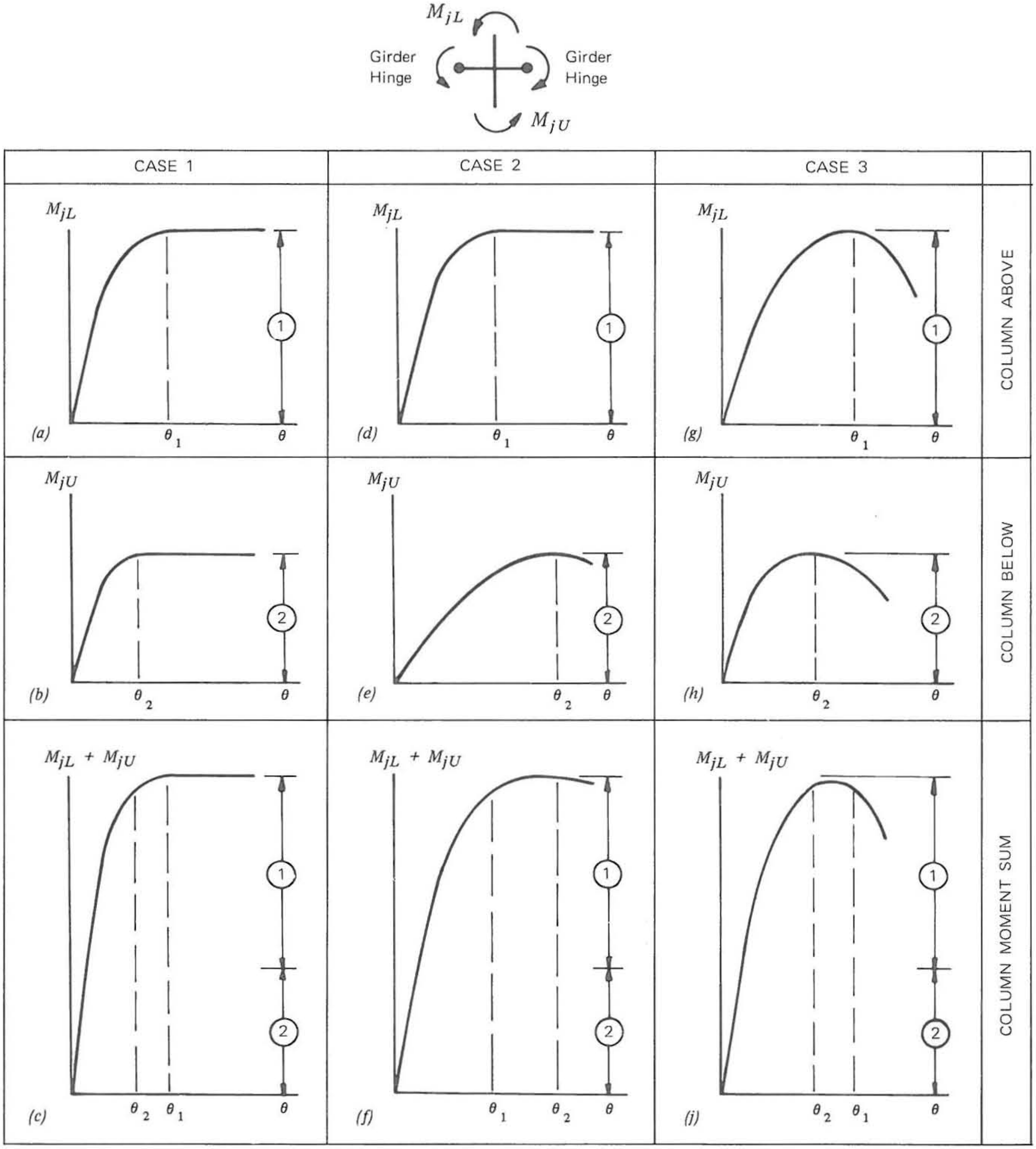


FIG. 4.1 INFLUENCE OF M-θ CURVES ON MAXIMUM COLUMN MOMENT SUM

capacity is at least equal to the required moment.

Separate checks for the top columns are performed below level 2 and below the roof in Tab. 8.8 because these columns resist different combinations of axial load and moment. The P/P_y ratio in Col. (6) for the exterior columns below the roof is less than 0.15, so M_{pc}/M_p = 1.0 for these columns.

The end moment ratio $q = 0$ and P/P_y exceeds 0.9 for the trial 14*WF*167 (A36) exterior and interior columns in the bottom story of Bent A. The result is that LTB limits the moment capacity of these columns to less than 50 percent of the net girder moment at the joints on level 24. When the bottom story columns are increased by one section to a 14*WF*176 (A36), P/P_y is reduced to less than 0.9 with a substantial increase in column moment capacity.

The exterior and interior A572 columns below level 24 in Tab. 8.8 illustrate a second case where LTB requires an increase of one section in the size of columns with an end moment ratio $q = 0$.

Lateral-torsional buckling is less likely for beam-columns in double curvature bending but may still limit their moment capacity if P/P_y and h/r_y are sufficiently large. This is illustrated by the interior A36 steel columns below levels 12, 16 and 20. The LTB reductions in moment capacity are sufficiently small that no increase in column size is needed.

4.9 REVIEW OF COLUMN DESIGN

Several useful observations can be made from a review of the beam-column investigation for full factored gravity load in Tab. 8.8.

1. All columns with axial loads less than $0.8P_y$ had moment capacities controlled by M_{pc}.
2. None of the 12 double-curvature columns ($q = +1.0$) in A36 steel had to be increased in size because of LTB with axial loads up to $0.91P_y$, although 3 of these columns had some moment capacity reduction due to LTB with P/P_y in the range from 0.85 to 0.91.
3. All bottom story column sizes ($q = 0$) were controlled by LTB.
4. All of the column sections selected using Eq. 4.5 in Tab. 8.7 provided adequate moment capacities for full factored gravity load. With one exception, the next lighter column sections were not adequate.

These observations suggest the results to be expected in many plastic designs for columns with similar slenderness.

The range of application of the column design criterion for full factored gravity load (Art. 4.7) depends on the M-θ behavior of the beam-columns above and below a joint. It is important to understand how this M-θ behavior may influence the maximum column moment sum as indicated in the following discussion. It is initially assumed that LTB does not limit the beam-column moment capacity above or below the joint.

Figures 4.1 and 4.2 will be used to describe cases that may determine the maximum value of the column moment sum. The girder moments on the joint at the top of Fig. 4.1 are considered constant in view of the girder plastic hinges that form under full factored gravity load.

The top and middle curves in Fig. 4.1 represent M-θ curves for the columns above and below the joint with peak moments M_1 and M_2. The bottom curves give the sum of the column moments $M_{jL} + M_{jU}$ at any joint rotation θ.

The salient features of each case in Figs. 4.1 and 4.2 are:

Case 1. Both M-θ curves have a plastic plateau.

$$(M_{jL} + M_{jU})_{\text{max.}} = M_1 + M_2$$

This case occurs for most *WF* columns in double curvature bending ($q = +1.0$) about the major axis and for many columns with $q = 0$. Limitations on axial load, slenderness, and yield stress for Case 1 are stated later in this article.

Case 2. One M-θ curve does not have a plastic plateau.

$$(M_{jL} + M_{jU})_{\text{max.}} = M_1 + M_2 \quad \text{for } \theta = \theta_2$$

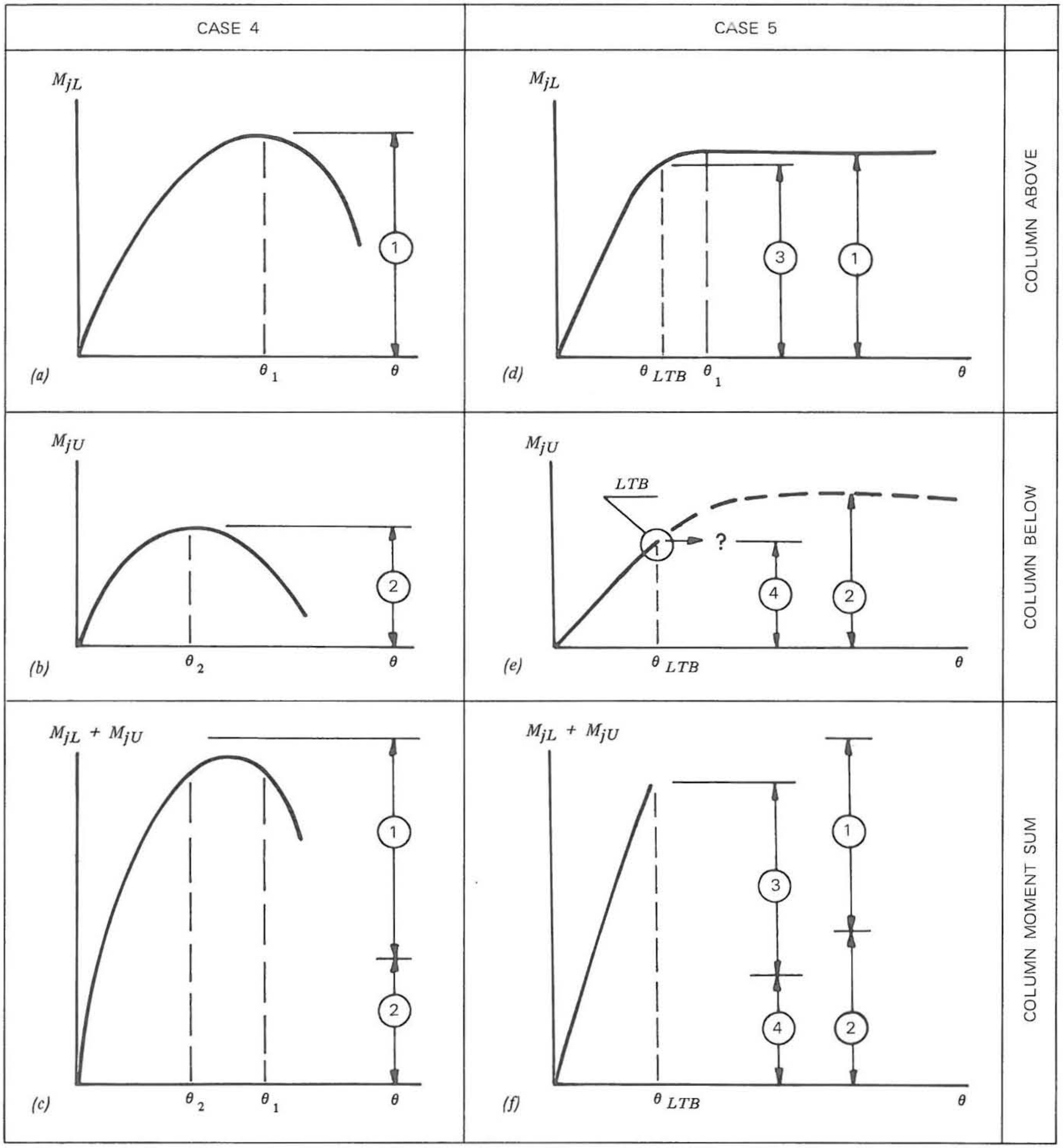


FIG. 4.2 INFLUENCE OF M-θ CURVES ON MAXIMUM COLUMN MOMENT SUM

This case may occur for slender lower story columns with large axial loads and $q = 0$, for example, in the bottom story. The M-θ curves in Design Aid II for $q = 0$ serve as a guide to define the qualifications, slender and large axial load.

Case 3. Both M-θ curves have no plastic plateau but the peak moments occur at nearly equal joint rotations.

$$(M_{jL} + M_{jU})\text{max.} \approx M_1 + M_2 \quad \text{for } \theta_1 \approx \theta_2$$

This case may occur for columns with nearly equal axial loads, slenderness, and end moment ratios above and below the joint if $0 > q > -1.0$. See Design Aid II.

Case 4. Similar to case 3 but the peak moments occur at substantially different joint rotations.

$$(M_{jL} + M_{jU})\text{max.} < M_1 + M_2 \quad \text{for } \theta \text{ between } \theta_1 \text{ and } \theta_2$$

This infrequent case may occur when $q \leqslant 0$ and the axial loads, or slenderness, or end moment ratios above and below the joint are dissimilar. The column moment sum at $\theta = \theta_1$ is a convenient lower bound estimate of $(M_{jL} + M_{jU})$max.

Case 5. LTB limits both the moment capacity of one column and the joint rotation.

$$(M_{jL} + M_{jU})\text{max.} < M_1 + M_2$$

$$(M_{jL} + M_{jU})\text{max.} = M_3 + M_4 \quad \text{for } \theta = \theta_{LTB}$$

where M_4 is the LTB moment for the lower column from Design Aid III, θ_{LTB} is the joint rotation for $M_{jU} = M_4$ from Design Aid II, and M_3 is the end moment for the upper column at $\theta = \theta_{LTB}$ from Design Aid II. This case occurs for heavily loaded, laterally unbraced columns with $q = 0$ (for example, bottom story columns with no base restraint) and for columns in single curvature, $q = -1$. See Design Aid III.

In Case 5, if the joint rotation exceeds θ_{LTB} the column end moment below the joint depends on the post lateral-torsional buckling behavior of the beam-column. This type of beam-column behavior is not well understood at present, which is the reason for the conservative design limitation $\theta \leqslant \theta_{LTB}$.

These five cases do not exhaust all possible combinations but they do clearly indicate how the shape of the M-θ curves governs the maximum column moment sum at a joint. Note that a plastic plateau is desirable but not necessary in plastic design.

Example 4.1—This example describes the combination of M-θ curves to determine the maximum column moment sum at a joint. The three columns, designated by A, B and C in Table 4.1, correspond to Cases 1, 3 and 5 in Figs. 4.1 and 4.2.

Each column is two stories high and uses the same section, a 12WF120 in A36 steel. Each column carries a factored axial load of 890 kips ($P/P_y = 0.70$) in the upper story and 1017 kips ($P/P_y = 0.8$) in the lower story. The M_{pc} values for these axial moment diagrams and minor axis bracing conditions vary as indicated in Table 4.1. The data in this table was selected for ease of reference to the M-θ curves in Design Aid II.

The maximum column moment sum, $M_{jL} + M_{jU}$, that is available to resist the girder moments on the middle floor joint is to be determined. Plastic hinges are assumed at the ends of the girders that apply the major axis column moments. The axial load, slenderness, and end moment ratios that determine the major axis in-plane bending behavior of the columns are recorded in Lines 2 to 4 of Table 4.1.

Columns A and B—These columns, with minor axes braced to prevent LTB, reach their maximum moment capacity in the in-plane bending mode. The peaks of the M-θ curves in Design Aid II give the values of max. M/M_{pc} and end rotation θ in Lines 5 and 6 of Table 4.1. The peak moments M_1 for the upper column and M_2 for the lower column in Line 7 are obtained using the M_{pc} values recorded with the axial loads in Table 4.1. Line 8 gives the moment sum $M_1 + M_2$.

The column moment sum in Line 8 was obtained without regard for the different joint rotations corresponding to M_1 and M_2. The ability of the columns to resist $M_1 + M_2$ depends on the shape of the $M\text{-}\theta$ curves. The top and middle diagrams in Figs. 4.3 and 4.4 are $M\text{-}\theta$ curves from Design Aid II for the upper and lower stories of Columns A and B. The bottom diagram in these figures gives the column moment sum versus the joint rotation at the middle floor.

Fig. 4.3 for Column A shows that both $M\text{-}\theta$ curves have a plastic plateau (Case 1). In this case the individual peak moments M_1 and M_2 can be added without regard for the joint rotation to obtain the maximum column moment sum.

In Fig. 4.4 for Column B, both $M\text{-}\theta$ curves have rounded peaks that occur at different joint rotations θ_1 and θ_2. Hence, $M_1 + M_2$ overestimates the maximum column moment sum that can be resisted. However, θ_1 and θ_2 do not differ by a large amount (Case 3) and the maximum column moment sum of 292 kip ft. is only slightly less than $M_1 + M_2 = 298$ kip ft.

Column C—The lower story of this column, with no lateral bracing between floors, reaches its maximum moment capacity in the LTB mode. The LTB moment, denoted by M_4, is determined from Design Aid III using the P/P_y, q, and h/r_y values in Lines 2, 4, and 9 of Table 4.1. The result is $M_4/M_{pc} = 0.66$ in Line 10. The joint rotation when the lower column attains its LTB moment is $\theta_{LTB} = 0.0021$ radians in Line 11 and is obtained from Design Aid II. The moment M_3 for the upper column at $\theta = \theta_{LTB}$ is also obtained from Design Aid II with the result $M_3/M_{pc} = 0.79$ in Line 12. Multiplication by M_{pc} gives the column moments in Line 13. The maximum column moment sum $M_3 + M_4$ is recorded in Line 14.

The diagrams for Column C in Fig. 4.5 show how LTB reduces the maximum column moment sum. The dashed portion of these graphs represents in-plane behavior in the absence of LTB. The top and middle diagrams are $M\text{-}\theta$ curves from Design Aid II for the upper and lower stories of Column C. The bottom curve gives the column moment sum at the middle floor joint versus the joint rotation.

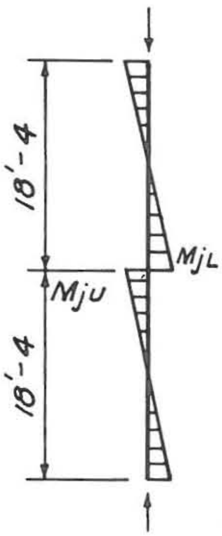
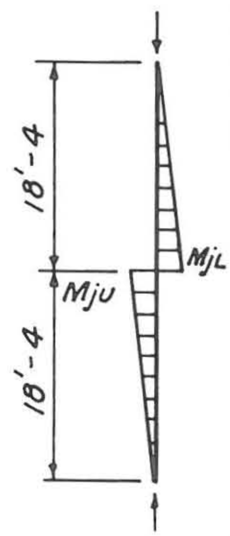
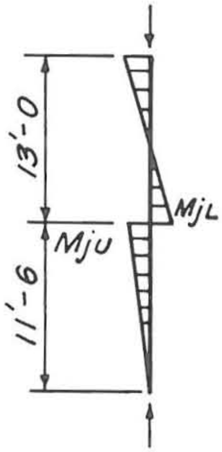
The upper story of Column C, if considered in isolation, could attain its full in-plane moment capacity of M_{pc} with no reduction for LTB according to Design Aid III. However, the LTB moment for the lower story column is conservatively assumed to limit the joint rotation at the middle floor to $\theta \leqslant \theta_{LTB}$. (Current research at Lehigh University suggests that it may be possible in future recommendations to modify this limitation on joint rotation). Thus, LTB in the lower story limits column moments in both the lower and the upper stories.

Example 4.1 illustrates an optional refined procedure for distributing column moments in the inelastic range based on equilibrium and compatibility requirements. This procedure is extended to include elastic girder restraints in Ref. 1. The complete moment-versus-rotation graphs in Figs. 4.3 to 4.5 are included here to illustrate ideas but are not needed in such detail for routine design.

It is helpful to establish limits on beam-column parameters that conserve the plastic plateau portion of the $M\text{-}\theta$ curves. Laterally braced A36 steel *WF* beam-columns bent in double curvature about the major axis with $P/P_y \leqslant 0.9$ and $h/r_x \leqslant 40$ may be considered to provide an adequate plastic plateau. For A572 steel (F_y = 50 ksi) the slenderness limit should be adjusted to $h/r_x \leqslant 34$.

At the upper limits on P/P_y and h/r_x the plastic plateau is modified, but the maximum in-plane moment does not vary significantly with the column end rotation.

For example, see Design Aid Chart II–7 for $P/P_y = 0.9$. The curve for $h/r_x = 20$ and $q = 0$ also represents the $M\text{-}\theta$ curve for $h/r_x = 40$ and double curvature bending ($q = +1.0$). This curve gives moments varying from $M/M_{pc} = 0.93$ to 0.97 in the range $0.0033 \leqslant \theta \leqslant 0.010$. The small variation in moment for a three-fold

Column Data	Column A	Column B	Column C
for columns A,B,C Section 12 WF 120 $F_y = 36$ ksi Major axis bending $r_x = 5.51$ in $P_y = 1271$ kips $r_y = 3.13$ in $M_p = 559$ kft Axial loads Upper story $P = 890$ kips $P/P_y = 0.70$ $M_{pc} = 198$ kft Lower story $P = 1017$ kips $P/P_y = 0.80$ $M_{pc} = 132$ kft			
Lateral bracing	Minor axis braced	Minor axis braced	Minor axis not braced

Line	Item	Units	Source	Column A Upper Story	Column A Lower Story	Column B Upper Story	Column B Lower Story	Column C Upper Story	Column C Lower Story
1	Height h	ft		18'-4	18'-4	18'-4	18'-4	13'-0	11'-6
2	P/P_y			0.70	0.80	0.70	0.80	0.70	0.80
3	h/r_x			40	40	40	40	30	25
4	Moment ratio q			+1.0	+1.0	0	0	+1.0	0
	In-plane bending								
5	Max. M/M_{pc}		DA II	1.00	0.98	0.93	0.86		
6	θ_1 or θ_2	rad.	DA II	0.010	0.008	0.014	0.0095		
7	M_1 or M_2	kft	(5) × M_{pc}	198	129	184	114		
8	$M_1 + M_2$	kft		327		298			
	Lateral-torsional bucking								
9	h/r_y							53	44
10	Lower sty M_4/M_{pc}		DA III						0.66
11	θ_{LTB}	rad.	DA II						0.0021
12	Upper sty M_3/M_{pc}		DA II					0.79	
13	M_3 or M_4	kft	(10 or 12)×Mpc					156	87
14	$M_3 + M_4$	kft						243	

TABLE 4.1—DATA FOR EXAMPLE 4.1

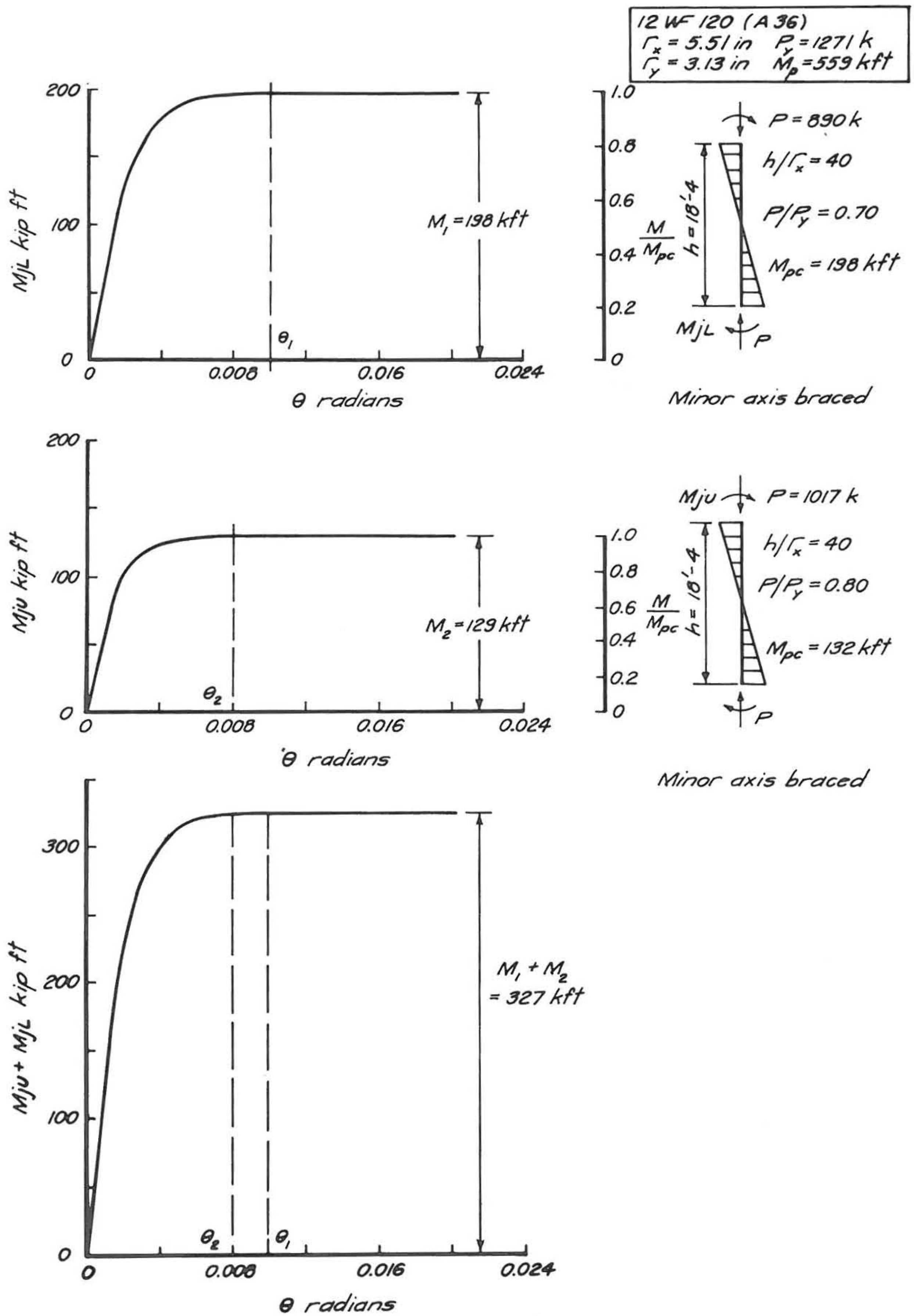


FIG. 4.3—MOMENT SUM FOR COLUMN A IN EXAMPLE 4.1

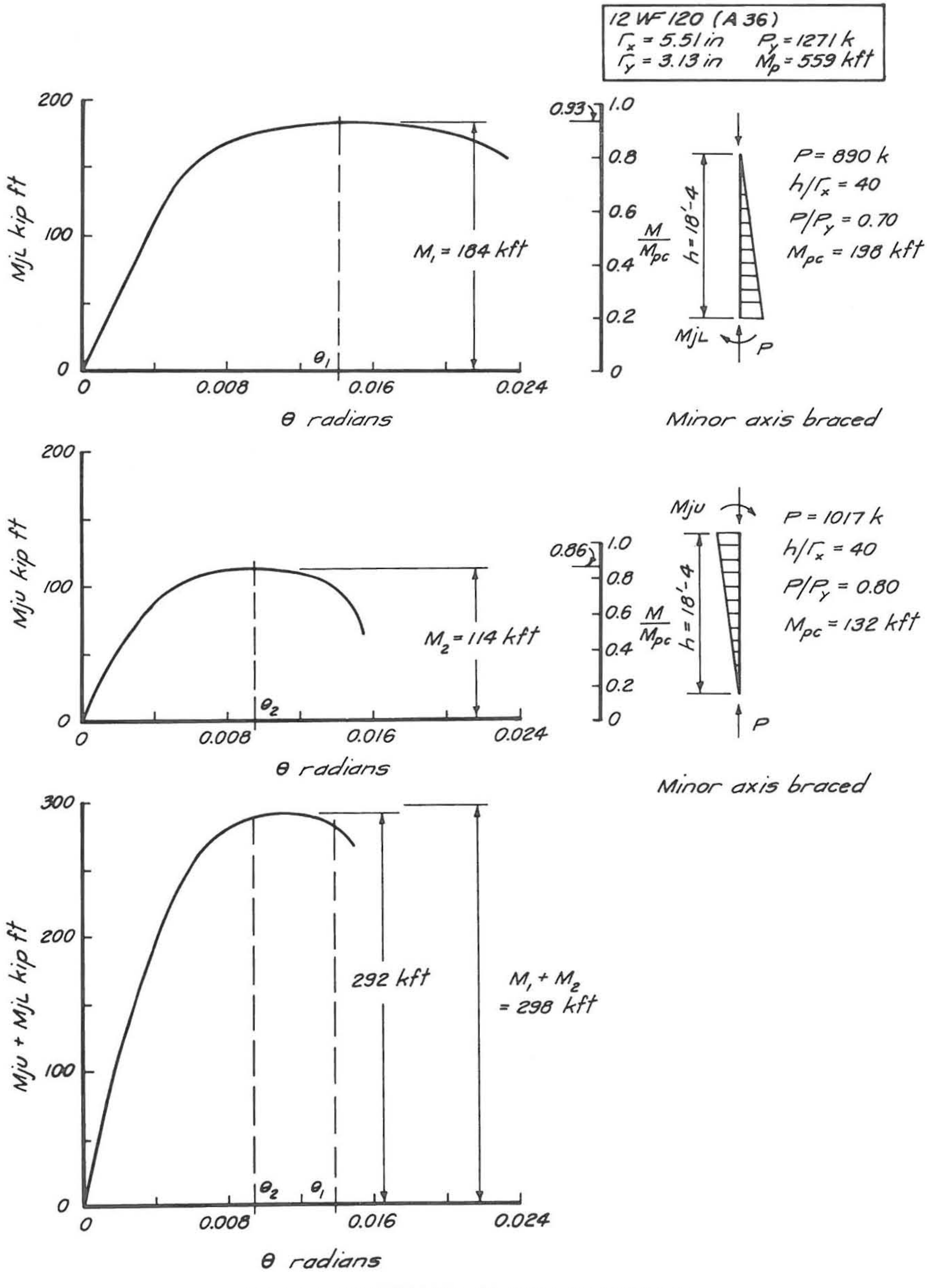


FIG. 4.4—MOMENT SUM FOR COLUMN B IN EXAMPLE 4.1

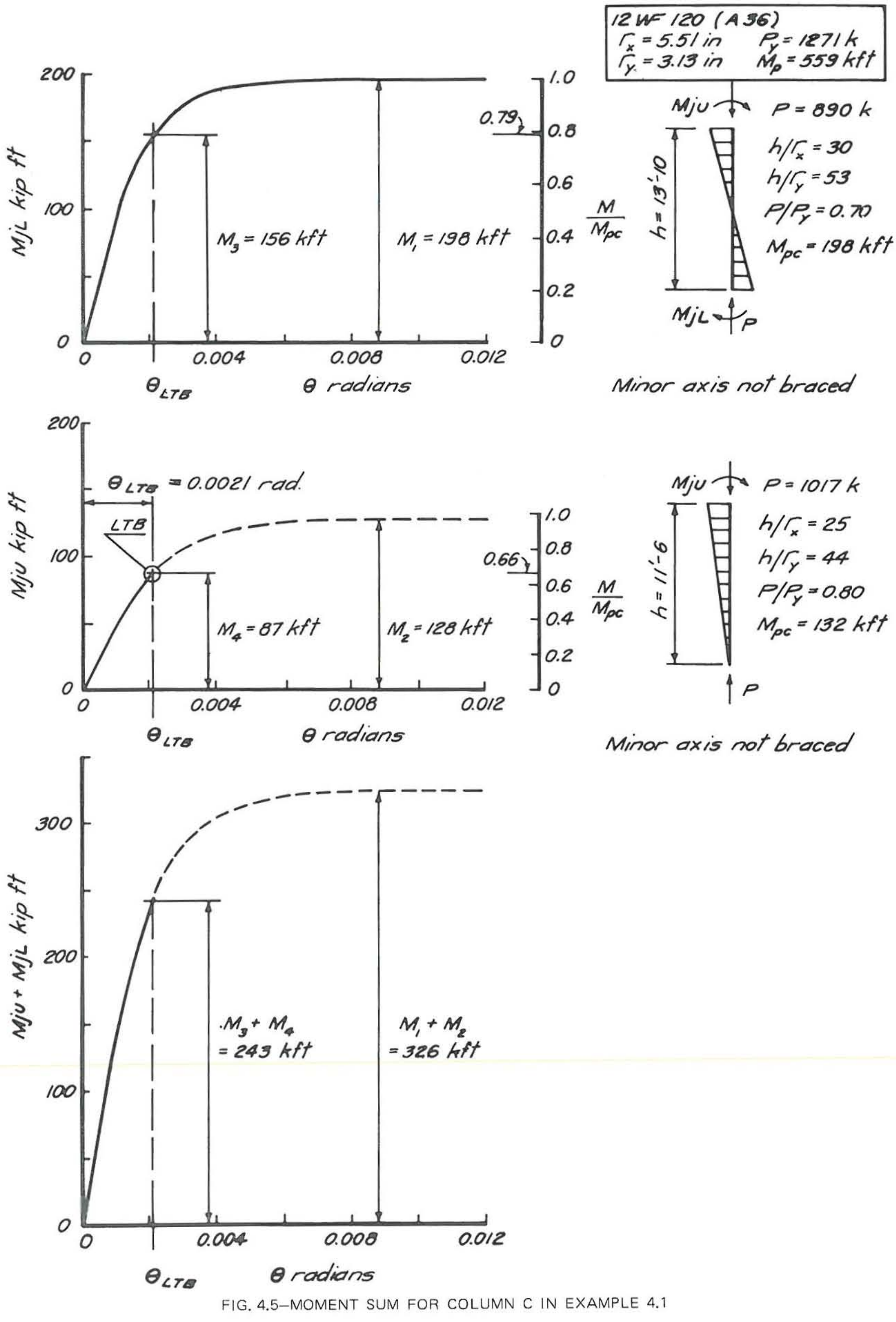


FIG. 4.5—MOMENT SUM FOR COLUMN C IN EXAMPLE 4.1

range in rotation is inconsequential for most practical purposes.

The beam-column investigation in Tab. 8.8 indicates that the above limits on P/P_y and h/r_x are satisfied for all of the double curvature columns. The M-θ curves for these columns may be assumed to provide an adequate plastic plateau such that the maximum column moment sum at each joint is:

1. the sum of the individual beam-column moment capacities
2. independent of the joint rotation.

Hence, no detailed consideration of the M-θ curves is needed in the design of the columns in Bent A for full factored gravity load. The effect of checkerboard live loads will be checked in Chapter 6.

An assumption used in the column design for Bent A is that 50 percent of the net girder moment on a joint is distributed to the column below. This assumption is conservative if the columns above and below the joint have similar P_y, slenderness, and end moment ratios. Under these conditions, the peak moment M_2 is less than M_1 in Fig. 4.1 because the lower column resists a larger axial load.

In the linear elastic portion of the M-θ curves, the M-θ slope determines the column moment distribution. The elastic M-θ slope depends on the column stiffness I/h and the end moment ratio q. The effect of a plastic plateau in the column M-θ curves is to redistribute the column moments in proportion to M_{pc} as shown in Fig. 4.1(c). The column design can be modified to take advantage of this plastic behavior by assuming that the column below a joint resists less than 50 percent of the net girder moment. If the *sum* of the beam-column moment capacities above and below the joint is at least equal to the net girder moment, the design is adequate.

A comparison of the M_{pc} values in Tab. 8.7(8) with the required moment M (based on the 50 percent distribution assumption) indicates whether it is worthwhile to consider a redistribution of the column moments. If $M_{pc} > 0.8M$ it may be possible to use the lighter sections in Col. (5). This condition does not occur in the column design for Bent A.

The procedure for determining the maximum column moment sum when LTB limits the joint rotation and moment capacity of one column (Case 5, Fig. 4.2), can also be applied when the LTB moment for one column is less than 50 percent of the net girder moment on the joint. Redistribution of column moments is a design refinement in the direction of economy but it is not a mandatory design requirement.

The results of the tentative design of supported Bents A are summarized in Fig. 8.2. These are checked in Chapter 6.

CHAPTER 5

Design of Braced Bents for Gravity and Combined Loads

5.1 INTRODUCTION

This chapter illustrates the design of braced Bent B in the multistory building described in Chapter 4 and Fig. 8.1. This is Part 2 of the design example referred to in Art. 4.4.

Bent B must provide adequate strength to resist both factored gravity and factored combined (gravity plus wind) loads. This bent must also be stiff enough to keep the wind drift within acceptable limits.

It is not immediately evident whether strength or stiffness requirements will govern the design of a braced bent. The designer can choose one of these requirements as the basis for selecting member sizes and check the remaining requirements.

The slenderness of the vertical bracing system is an important parameter in indicating whether strength or stiffness requirements will govern the design of a braced bent. The term slenderness refers to the ratio of the total height of the bent h_t to the distance L between the columns in the braced bays. For Bent B, h_t = 236.7 ft. and L = 27.0 ft. in the exterior braced bays so h_t/L = 8.8. This ratio suggests that the vertical bracing system in Bent B is relatively slender and that stiffness is an important design consideration. If the slenderness were 5, strength requirements would be more likely to control the design.

The design of Bent A, described in Chapter 4, illustrates many features of the plastic design method when strength is the controlling criterion. The design of Bent B in this chapter indicates how stiffness criteria can be used to select member sizes. Chapter 6 describes methods for checking design requirements not considered in selecting member sizes.

The manual subroutines used in the design of Bent B include Tabs. 8.9 to 8.20 and are listed in Tab. 8.1. The major steps in the design are summarized below.

1. Design the braced floor girders in the exterior bays for factored gravity load in Tab. 8.9.
2. Tabulate gravity loads for the columns in Tabs. 8.10 and 8.11.
3. Determine the axial forces in the girders, columns and K-bracing of the vertical bracing system caused by factored combined load. This step is referred to as the combined load statics calculation and is performed in Tabs. 8.12 to 8.15.
4. Select columns for chord drift control under factored combined load in Tab. 8.16.
5. Select girders for plastic strength under factored combined load in Tab. 8.17.
6. Select the K-bracing for web drift control and strength criteria under factored combined load in Tabs. 8.18 and 8.19.
7. Check the story rotation and drift in Tab. 8.20.

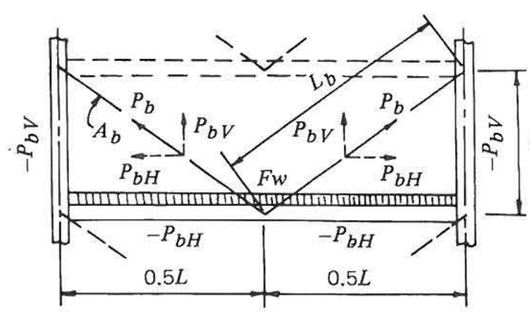
Items 1, 2, 3 and 5 are necessary steps in the design of a braced bent regardless of whether strength or stiffness is the controlling requirement. Items 4 and 6 apply stiffness criteria to select member sizes. If strength requirements are chosen as the basis for selecting columns and bracing, portions of steps 4 and 6 would be applied in the design check phase.

An important feature of the plastic design method for braced multistory buildings is the influence of drift on the stability of the building under factored combined load. This topic is treated in Arts. 5.4, 5.5 and 5.11. The stiffness criteria applied in Steps 4 and 6 above are described in Arts. 5.6 to 5.10.

5.2 DESIGN OF BRACED FLOOR GIRDERS FOR GRAVITY LOADS

Gravity load causes axial forces in the K-bracing and girders in Bent *B*. The K-bracing affects the gravity load mechanism behavior of the girders in two ways:

1. The K-bracing supports the girders at mid-span.
2. The girder axial force and vertical deflection produce secondary girder moments in addition to those caused by gravity loads.

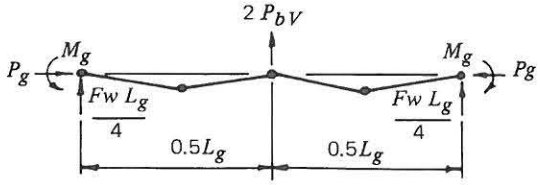


Bracing force components

(a)

$$P_{bH} = P_{bV}\frac{0.5L}{h}$$

$$P_b = P_{bV}\frac{L_b}{h}$$



Mechanism—for elastic brace

(b)

$$M_g = \frac{Fw(0.5L_g)^2}{16}$$

$$P_{bV} = \frac{FwL_g}{4}$$

$$P_g = -P_{bH}$$

FIG. 5.1 GRAVITY LOAD MECHANISMS FOR GIRDERS WITH K-BRACING

A complete analysis of the interaction between K-bracing and girders is not required for design practice. The design can be simplified by:

1. Determining the mechanism behavior of the girder and K-bracing system disregarding secondary deflection effects.
2. Designing the girders as laterally loaded beam-columns to resist the moments and forces from Step 1.

Figure 5.1(a) shows the axial bracing force P_b with vertical and horizontal components P_{bV} and P_{bH}. The bracing forces applied to the girder include a vertical force $2P_{bV}$ at midspan and an axial force $P_g = -P_{bH}$. The girder also supports the uniformly distributed factored gravity load Fw. Fig. 5.1(b) shows the gravity load mechanism assuming that the K-braces are elastic under factored gravity load. This assumption is satisfied if the braces do not yield under a vertical load

$$P_{bV} = \frac{FwL_g}{4} \tag{5.1}$$

where L_g is the clear girder span. The girder moments are

$$M_g = \frac{Fw\,(0.5\,L_g)^2}{16} \tag{5.2}$$

If each brace has an area A_b and yield stress F_y the maximum vertical bracing force is limited to

$$P_{bV} = A_bF_y\frac{h}{L_b} \tag{5.3}$$

The minimum bracing area needed to prevent yielding of the brace is found from Eqs. 5.1 and 5.3 in the form

$$\text{Min. } A_b = \frac{FwL_gL_b}{4F_yh} \tag{5.4}$$

The floor girders and K-bracing required for factored gravity load are designed in Tab. 8.9. The K-bracing geometry calculations shown with this table are based on centerline dimensions to avoid cumulative column load errors in subsequent statics calculations. The table includes separate tabular columns for the girders on levels 23 and 24; the story height and K-bracing arrangement change at these levels. The midspan girder reaction at level 24 is assumed to be distributed equally to the K-bracing above and below this level. The bracing design is not sensitive to this assumption because of the larger bracing forces at level 24 under combined load.

The first 6 lines in Tab. 8.9 give the gravity loads on the girders. The live load reduction in line 4 considers one-half of the floor area supported by the girder, based on the mechanism in Fig. 5.1. The bracing forces due to factored gravity load are determined in lines 7 to 9. The minimum bracing area A_b = 1.34 sq.in. for incipient yielding in line 10 is likely to be exceeded by the bracing supplied.

The next step is to design the girders as laterally loaded beam-columns. Two methods are available for checking the capacity of trial girder sections as laterally loaded beam-columns.[1,7] These methods include interaction equations and interaction charts. The interaction equation is

$$\frac{P}{P_{ox}} + \frac{C_m}{(1 - P/P_{ex})} \frac{Fw}{w_p} = 1.0 \qquad (5.5)$$

where P = factored axial load
P_{ox} = major axis concentric buckling load from Eq. 3.4
P_{ex} = major axis Euler buckling load from Eq. 3.9
Fw = uniformly distributed factored gravity load
w_p = $16 M_p / L_g^2$
C_m = $1 - 0.4P/P_{ex}$ (see Table C 1.6.1.2 in Ref. 3)

The values of w_p and C_m apply for a member with span L_g between fixed supports and plastic moment M_p. For practical purposes, the girders in Bent B are considered to have fixed supports and a span of $0.5L_g$.

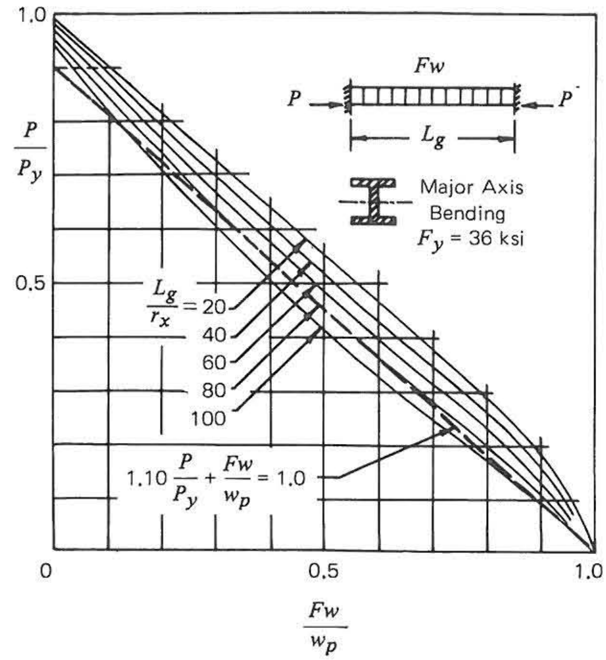


FIG. 5.2 ULTIMATE STRENGTH INTERACTION CURVES FOR FIXED-END COLUMNS SUBJECTED TO UNIFORMLY DISTRIBUTED LOAD. (from Ref. 7)

The interaction chart[7] in Fig. 5.2 may be used in place of Eq. 5.5. This chart is based on numerical integration of M-P-Φ curves (similar to Fig. 3.10) including residual stress effects and secondary moments due to axial load and deflection. The curves in Fig. 5.2 indicate concurrent ultimate load values of axial load (P/P_y) and lateral load (Fw/w_p) for beam-columns with specified major axis slenderness L_g/r_x.

Similar values of Fw/w_p are obtained from Eq. 5.5 and Fig. 5.2 for a given beam-column problem. The interaction equation tends to be more conservative than the theoretically derived interaction curves according to Ref. 7.

The design problem is to select (rather than check) a trial girder section with adequate beam-column capacity. This problem can be simplified for A36 steel beam-columns with $L_g/r_x \leqslant 40$ or for F_y = 50ksi with $L_g/r_x \leqslant 34$. In this slenderness range, the coefficient of Fw/w_p in Eq. 5.5 can be taken as unity and P_{ox}/P_y = 0.91 from Eq. 3.5. These substitutions reduce Eq. 5.5 to the linear form

$$1.10 \frac{P}{P_y} + \frac{Fw}{w_p} = 1.0 \qquad (5.6)$$

This formula closely approximates Eq. 5.5 for A36 steel with L_g/r_x = 40 and is represented by the dashed line in Fig. 5.2.

The terms Fw and w_p are conveniently converted to moments by the equations

$$M_g = \frac{FwL_g{}^2}{16} \qquad (5.7)$$

and,

$$M_p = \frac{w_pL_g{}^2}{16} \qquad (5.8)$$

Then Eq. 5.6 can be rearranged to

$$M_p = M_g + P \times 1.10 \frac{M_p}{P_y} \qquad (5.9)$$

Using Eqs. 4.7 and 4.8 to approximate M_pP_y for *WF* shapes bent about the major axis gives the design equation

$$M_p = M_g + 0.46\,Pd \qquad (5.10)$$

where M_p = required plastic moment capacity for the laterally loaded beam-column, kip–ft.

M_g = plastic moment for a mechanism in the absence of axial load per Eq. 5.7, kip–ft.

P = factored axial load, kips

d = estimated depth, ft.

The second term in Eq. 5.10 represents a "moment equivalent" for the axial load that depends on the estimated depth d. If the estimate for d is appropriate, a *WF* section with plastic moment M_p and depth d can be selected from a plastic section modulus table. The section will satisfy Eq. 5.5 if $L_g/r_x \leqslant 40$ for A36 steel beam-columns with fixed ends, so no check is needed. If L_g/r_x exceeds this bound, it is advisable to select a trial section with M_p somewhat larger than the value from Eq. 5.10 and to check the trial section using Eq. 5.5. Some decrease in M_p for the trial section below the value given by Eq. 5.10 can be accepted for decreasing values of L_g/r_x.

Equation 5.10 is applied in Lines 11 to 15 of Tab. 8.9 to select girders for the K-braced bays in Bent *B*. The moment M_g in line 11 is based on the span 0.5 L_g for the mechanism in Fig. 5.1. The major axis slenderness in Line 18 is less than 40 so no further beam-column check is needed. The web d/w in Line 19 is less than 43; hence, the trial A36 10B19 section is adequate for web buckling under axial load. A 12B16.5 might also be selected to provide M_p = 62.7 kip-ft. from Eq. 5.10 with d = 1.0 ft. However, the web of this section is not adequate for web buckling under axial load according to Table 3.3. Web buckling should be investigated for A36 shapes with $d/w > 43$ that must resist axial load.

5.3 COLUMN GRAVITY LOADS

The K-bracing in Bent *B* modifies the tributary areas for the columns as shown in Fig. 5.3. This figure indicates the portions of full floor and roof loads that produce axial load in selected columns. Figures 5.3(a) and (b) show that the columns below the roof must resist floor loads from one quarter of the exterior bay on Level 2. The floor areas that are the source of the column load increment below Level 2 are shown in Figs. 5.3(c) and (d). The increment of column live load from the floors is the same in

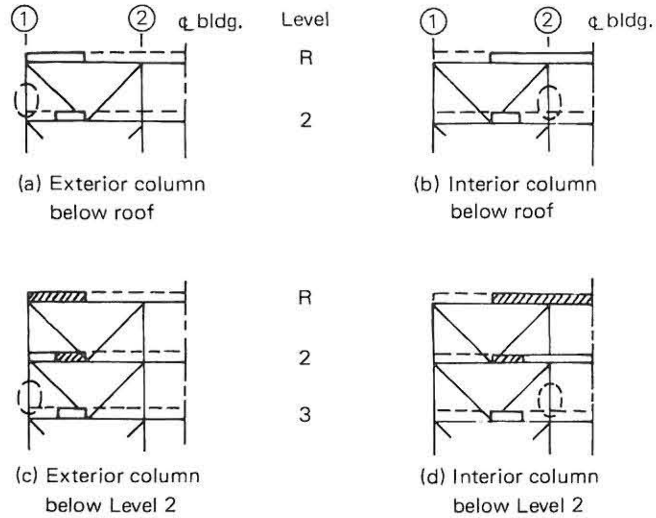


FIG. 5.3 TRIBUTARY AREAS FOR COLUMNS IN BENT *B*

Bents *A* and *B* except for the columns below the roof and below Level 24.

The column load calculations for Bent *B* in Tab. 8.10 are arranged to consider the tributary areas of Fig. 5.3. Lines 9 and 10 give the vertical floor loads resisted by a pair of K-braces. These loads are used in lines 14 and 15 to determine the column loads below the roof, per Fig. 5.3(a) and (b). A small live load reduction could be applied to the loads in line 15 but this is neglected to simplify the design example.

The gravity loads for the columns in Bent *B* are determined in Tab. 8.11 using the same format as for Bent *A*. Calculations for the load increment below Levels 23 and 24 are included below Tab. 8.11. These calculations account for the change in story height and K-bracing arrangement at the bottom of Bent *B*.

5.4 DRIFT CONSIDERATIONS

Horizontal deflection or drift under combined load is a primary consideration in the design of slender braced multistory bents. Working load drift limits of about $\Delta_t = 0.0025h_t$, based on the bare frame deflection have been used in the past and can provide acceptable horizontal frame stiffness. The deflection of the completed building will be less because cladding contributions to stiffness are neglected. This drift limit is an empirical, approximate method of controlling the dynamic response to wind and of limiting partition cracks.

The drift under ***factored*** combined load may influence the strength and stability of the building. Recognition of this feature of structural behavior is an important conceptual contribution of plastic design.

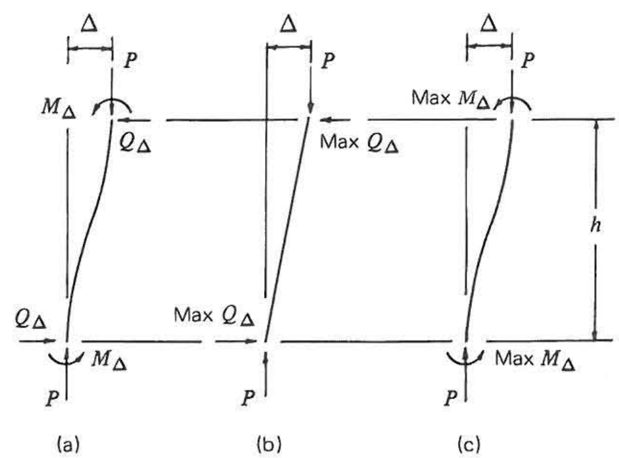


FIG. 5.4 EFFECT OF DRIFT ON COLUMNS

The effect of drift on the columns in a braced bent will be discussed with the aid of Fig. 5.4. In this figure, P is the axial load and Δ is the drift under factored combined load in one story of height h. Figure 5.4(a) shows column moments M_Δ and shears Q_Δ that are needed to hold the column in equilibrium under load P and drift Δ. These moments and shears carry the subscript Δ to emphasize that they act in addition to the column moments and shears caused by gravity load and wind. The combined effect of M_Δ and Q_Δ is to balance the overturning moment

$$P\Delta = Q_\Delta h + 2M_\Delta \tag{5.11}$$

The M_Δ moments must be resisted by adjacent girders and the Q_Δ shears must be resisted by vertical or inclined structural members in the story.

The portions of $P\Delta$ resisted by Q_Δ and M_Δ may vary. However it is possible to establish useful bounds, Max Q_Δ and Max M_Δ, on these shears and moments by assuming that only one of them is active as in Fig. 5.4(b) and (c). Alternate paths for resisting the $P\Delta$ moment are provided by

$$\text{Max. } Q_\Delta = P \frac{\Delta}{h} \qquad (5.12)$$

and,

$$\text{Max. } M_\Delta = \frac{1}{2} P\Delta \qquad (5.13)$$

At this point, it is helpful to describe a major difference between a braced and an unbraced building.

In a *braced* building, the floor slab and the bracing in vertical planes provide an effective path for resisting shear in each story of each bent. This is true both for the shear due to wind and for the Q_Δ shears caused by the $P\Delta$ moments. Hence the $Q_\Delta h$ term in Eq. 5.11 tends to balance most of the $P\Delta$ moment. To simplify the design of braced buildings, it is conservatively assumed that all of the $P\Delta$ moments are balanced by Q_Δ shears per Fig. 5.4(b) and Eq. 5.12. These Q_Δ shears are transmitted to the braced bents by the floor slab and resisted by the vertical bracing system.

In an *unbraced* building, all resistance to shear is considered to be provided by bending of the columns and girders. The sum of the column shears in a story equals the wind shear; there are no other structural members available to resist shear. If the $P\Delta$ moment in one column is balanced by a shear Max. Q_Δ as in Fig. 5.4(b), the remaining columns must resist this Max. Q_Δ in addition to their own $P\Delta$ moments. Hence, the M_Δ term in Eq. 5.11 functions alone in balancing the $P\Delta$ moments. This is the condition considered in Fig. 5.4(c) and Eq. 5.13.

In summary, the $P\Delta$ moments in the columns of a braced building are considered to be balanced by Q_Δ shears in the vertical bracing system. The $P\Delta$ moments in an unbraced building are balanced by column and girder moments.

The combined load statics calculations for Bent B, described in Art. 5.5, use Eq. 5.12 to estimate the $P\Delta$ shear in each story. The story drift under factored combined load is not known when the design commences so it is necessary to estimate a value for the drift index Δ/h. If the acceptable working load drift index of 0.0025 is factored by 1.3 and allowance is made for additional drift due to $P\Delta$ effects, the drift index under factored combined load is assumed to be 0.004. The design example based on a drift index of 0.004 at factored load will be shown to have a drift index close to 0.0025 at working load.

5.5 COMBINED LOAD STATICS CALCULATIONS

The next design step is to determine the axial forces in the vertical bracing system including the girders, columns and K-bracing in Bent B under factored combined load. To simplify this step, Bent B is considered to consist of two pin-connected, vertical, cantilever trusses, which are pin-connected by the girders in the interior bay. More refined structural assumptions, such as including the effects of interior girder restraints, may be advantageous but are not applied here in the interest of brevity. Each truss is assumed to resist one-half of the total shear in each story from three Bents A and one Bent B.

The horizontal shears resisted by Bent B are determined in Tab. 8.12. Cols. (1) and (2) give the wind forces and wind shears at working load (F = 1.0) from the 96 ft. length of the building braced by Bent B (see Floor Framing Plan in Fig. 8.1). The combined load factor F = 1.3 is applied in Col. (3) to find the factored wind shears ΣH below each level. The bracing system in Bent B must also resist the $P\Delta$ shears (Eq. 5.12) from three Bents A and one Bent B. The $P\Delta$ shears are proportional to the total factored gravity loads ΣP on these four bents. Cols. (6) and (7) list ΣP and the $P\Delta$ shears below each level. The $P\Delta$ shears are based on an assumed drift index Δ/h = 0.004 in every story. (See Art. 5.4 and 5.11) Tab. 8.12(8) gives the total factored shears due to wind plus $P\Delta$.

$$P_H = \Sigma H + 0.004\Sigma P \qquad (5.14)$$

These are the shears to be resisted below each level of Bent B.

The axial forces in the girders and columns of Bent B under factored combined load are determined in Tab. 8.13. Col. (1) gives the total shear P_H below each level from the preceding table. Most of the horizontal forces resisted by the vertical bracing system in Bent B are transmitted to this bent through the floor slab. To simplify the statics calculations it is assumed that the horizontal forces are applied to the pin-connected trusses at the midspan K-brace joints in the braced bays. The horizontal component of the K-brace force due to wind plus $P\Delta$ is recorded in Col. (2). Each K-brace resists one-fourth of P_H. Gravity load on the girders also causes bracing forces with horizontal components in Col. (3). The forces in Col. (3) are obtained from the gravity load analysis in Tab. 8.9(8) with a load factor adjustment. The axial girder loads P_g in Tab. 8.13(4) are the sum of the horizontal bracing forces due to gravity and wind plus $P\Delta$.

Wind load causes equal and opposite vertical force components in each pair of K-braces. Hence, the girders do not resist any vertical force from the K-bracing due to wind. This assumes that the K-bracing does not yield in tension or buckle in compression under factored combined load.

The vertical component of the K-brace force due to wind plus $P\Delta$ below each level is listed in Tab. 8.13(6). The vertical components are summed from the roof down in Tab. 8.13(7) to give the axial loads in the columns (truss chord forces) due to wind plus $P\Delta$. These column loads cause tension in windward chords and compression in leeward chords of the two cantilever trusses in Bent B. The compression in the leeward chords is added to the factored column gravity loads from Tab. 8.11 to determine the total compressive column loads under factored combined load in Tab. 8.13(8) and (9).

The calculations in Tab. 8.14 provide a check on the axial column loads in Bent B caused by wind plus $P\Delta$. The first three steps determine the uniformly distributed horizontal load

$$w_H = Fw_w + w_\Delta \qquad (5.15)$$

resisted by the two cantilever trusses. To approximate the overturning moment due to $P\Delta$ an equivalent horizontal load w_Δ is added to the factored wind load Fw_w, where

$$w_\Delta = \frac{\Delta}{h}\frac{\Sigma\delta P}{h} \qquad (5.16)$$

The term $\Sigma\delta P$ is the total factored gravity load increment applied at each level including column live load reductions. With Δ/h = 0.004, Eq. 5.16 gives w_Δ = 0.36 kips per ft., which is 14.4 percent of the factored wind load Fw_w = 2.50 kips per ft. in Tab. 8.14. This is a measure of the influence of estimated $P\Delta$ effects in this design example. The portion of the story shear caused by $P\Delta$ is a significant but not a dominant contribution.

Steps 4 to 6 in Tab. 8.14 show calculations for the cantilever forces and moments at Level 24. The horizontal shear checks with the value in Tab. 8.13(8) below Level 24 and the axial column loads are in reasonable agreement with the values obtained from summing the vertical bracing force components in Tab. 8.13(7). The K-brace slope ratio $h/0.5L$ in Tab. 8.13(5) is a factor in the column load calculations. This ratio should be based on centerline dimensions of the braced bay to avoid cumulative errors in the column loads. This is verified by the check in Step 6 of Tab. 8.14.

The axial forces in the K-bracing under factored combined load are determined in Tab. 8.15. The axial forces due to wind plus $P\Delta$ in

Col. (2) may act in tension or compression, depending on the wind direction. The axial tension in the K-braces due to factored dead plus live gravity load in Col. (3) is obtained from the gravity load analysis in Tab. 8.9(9) with a load factor adjustment. The combination of tension due to dead plus live gravity load and wind plus $P\Delta$ gives the maximum K-brace tension in Col. (5). The minimum K-brace tension due to factored dead gravity loading is obtained in Col. (4) as the ratio $D/(D+L)$ times Col. (3). D and L are the girder dead and reduced live loads from Tab. 8.9. The combination of minimum tension due to gravity dead load and compression due to wind plus $P\Delta$ gives the maximum K-brace compression in Col. (6).

It is pertinent to note that the maximum K-brace tension in Tab. 8.15(5) is at least 40 percent larger than the maximum K-brace compression in Tab. 8.15(6), above Level 23. This is because gravity load tension tends to offset the compression due to wind plus $P\Delta$ when the K-brace "V" points downward. If the K-brace design is controlled by strength (rather than stiffness) considerations, this effect will tend to reduce the required K-brace size because axial forces are more efficiently resisted in tension than in compression. Several factors favor K-bracing over diagonal bracing in Bent B:

1. The total length of K-bracing is smaller. In one 9.67 x 27.0 ft. panel of Bent B, the K-bracing has a length of 2 x 16.6 ft., while diagonal bracing would have a length of 2 x 28.7 ft.
2. The maximum tension in K-bracing is smaller. Tab. 8.15(5) gives a K-brace tension of 223 kips below Level 22. A diagonal brace at this location would have to resist

$$P_b = \frac{604}{2} \times \frac{28.7}{27.0} = 321 \text{ kips, tension}$$

3. K-bracing reduces the axial force in the braced bay girders. Tab. 8.13(4) indicates an axial load of 181 kips in the girder on Level 22. Diagonal bracing would cause an axial load of 302 kips in this girder.
4. K-bracing reduces the effective span of the girders as indicated in Art. 5.2. Diagonal bracing in Bent B would increase the girder weight by a factor of about 2.

Other possible advantages of K-bracing are related to architectural access in braced bays and superior drift characteristics. Further experience with plastic design should indicate how either K-bracing or diagonal bracing can be used to the best advantage.

5.6 DRIFT EQUATIONS

The next step in the design of Bent B involves consideration of drift. This article describes drift equations for the pin-connected vertical bracing system.

Drift of a braced bent under combined load is conveniently considered in two parts: web drift due to axial force in the bracing and girders, and chord drift resulting from axial force in the columns. Only that part of the axial force caused by wind plus $P\Delta$ is included in the drift analysis.

The axial change in length, e, for a member with a length l, and area A caused by an axial load P is

$$e = \frac{Pl}{AE} \text{ for } \frac{P}{P_y} \leqslant 0.7 \qquad (5.17)$$

This form of Hooke's Law is a reasonable approximation for members that form plastic hinges if the plastic zones are confined by adjacent elastic regions. The limit on P/P_y in Eq. 5.17 insures that residual stresses up to $0.3\,F_y$ will not increase the axial displacement.

Drift equations for the K-bracing, girders and columns of Bent B are developed in Fig. 5.5 (a, b, c). In each figure one pair of members is considered to change length by an amount e from Eq. 5.17 while the other members retain their original length. This is done to simplify the geometric relationships between e and drift Δ. The subscripts b, g and c denote bracing, girder and column respectively. The axial forces P_b, P_g and P_c due to wind plus $P\Delta$ are of equal value and opposite sign in each pair of members.

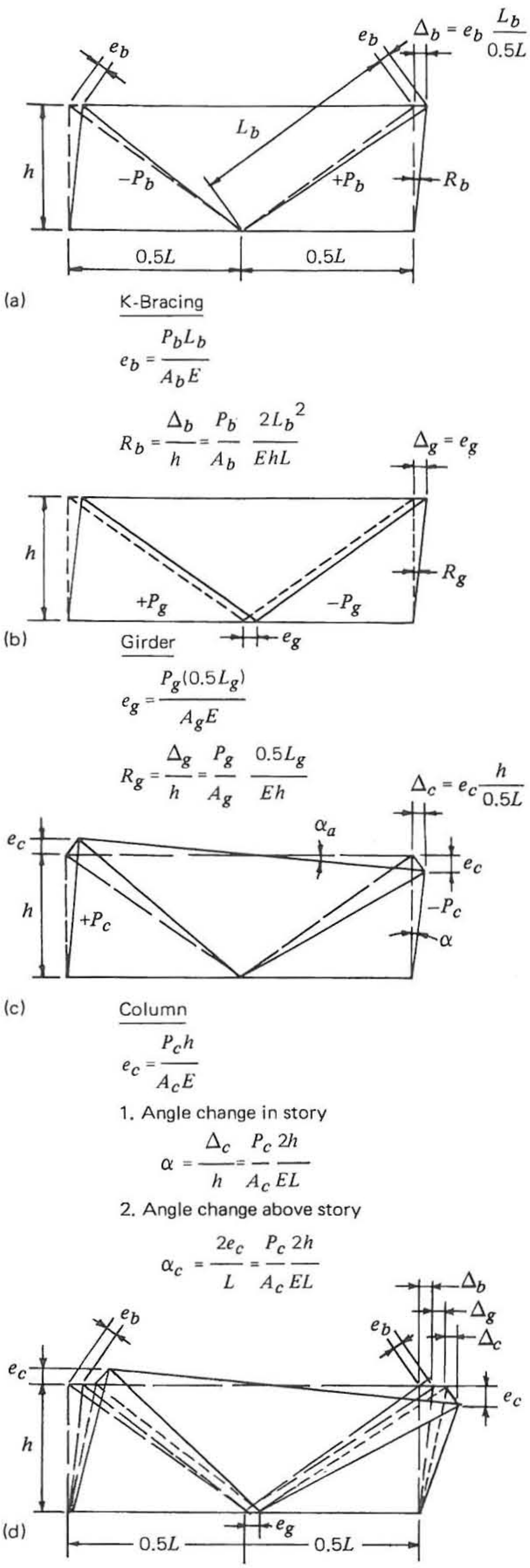


FIG. 5.5 DRIFT EQUATIONS FOR K-BRACED PANELS

The three drift components are combined in Fig. 5.5(d). In the deflected shape shown by solid lines, the length of each member is its original length plus or minus e from Eq. 5.17. This figure geometrically demonstrates that the drift components can be added without violating compatibility for the truss panel.

The drift equations in Fig. 5.5 are based on Hooke's Law and geometric compatibility. These same equations can also be obtained from a virtual work analysis. The only approximations inherent in these equations are those normally made in the deflection analysis of pin-connected trusses.

Rather than determine the drift, it is convenient to work with the rotation $R = \Delta/h$ in radians. From Fig. 5.5 the rotations due to web drift are:

K-bracing $$R_b = \frac{P_b}{A_b}\frac{2L_b^2}{EhL} \qquad (5.18)$$

Girders $$R_g = \frac{P_g}{A_g}\frac{0.5L_g}{Eh} \qquad (5.19)$$

These equations apply to K-braced panels. They can be used for diagonal bracing by replacing the coefficients 2 and 0.5 by unity.

Note that web rotation, Fig. 5.5(a, b), is limited to the story under consideration. However, chord angle change, Fig. 5.5(c), affects both the story under analysis and every story above. If the columns have equal areas A_c, the chord angle changes are:

Chord angle change in story $$\alpha = \frac{P_c}{A_c}\frac{2h}{EL} \qquad (5.20)$$

Chord angle change above story $$\alpha_a = \frac{P_c}{A_c}\frac{2h}{EL} \qquad (5.21)$$

Although these formulas give identical values, it is useful to consider them as fundamentally different. The chord force P_c and angle change α_a above a story are independent of the web bracing system so, Eq. 5.21 is valid for K-bracing or diagonal bracing. Equation 5.20 applies to K-bracing with the brace "V" pointing downward, but can be used for diagonal bracing by substituting unity in place of the factor 2.

The K-brace "V" points upward in the bottom story of the bent in Fig. 5.6, so Eq. 5.20 does not apply. Chord drift does not occur in the bottom story but every story above rotates through an angle α_a from Eq. 5.21. Above a given level, the chord rotation is

$$R_c = \alpha + \Sigma\alpha_a = \frac{\Delta_c}{h} \qquad (5.22)$$

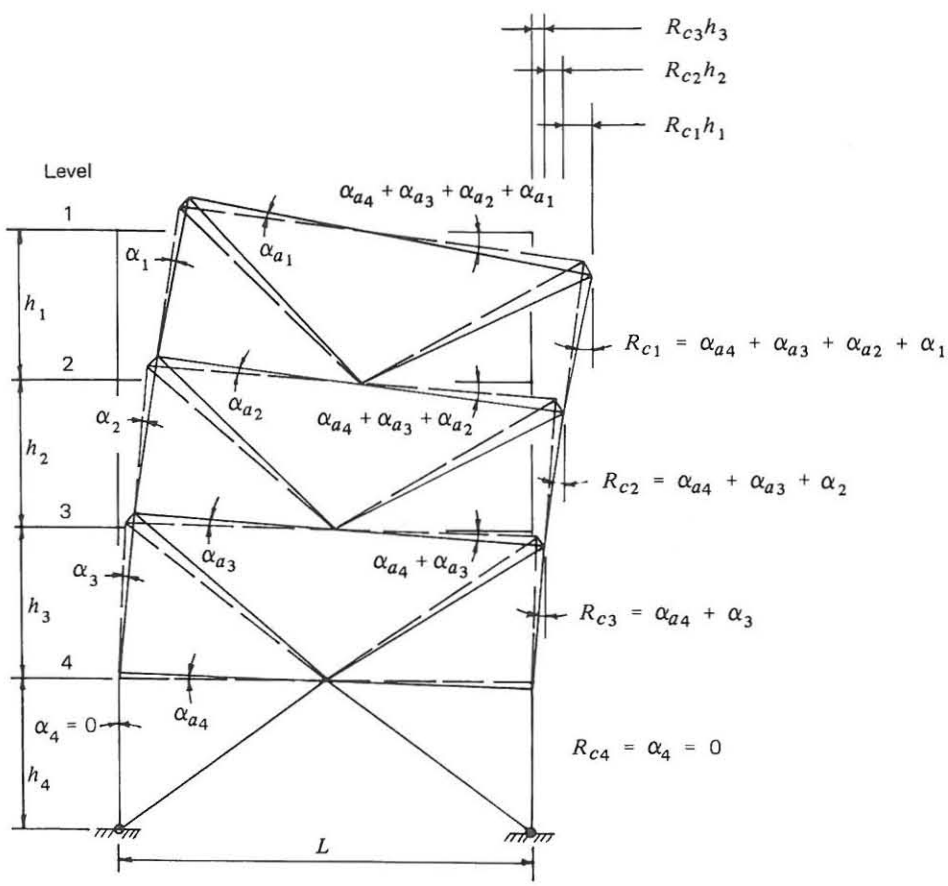


FIG. 5.6 CUMULATIVE EFFECTS OF CHORD ANGLE CHANGE

The cumulative effect of the chord angle change α_a is illustrated in Fig. 5.6. The dashed lines represent the members neglecting chord drift in each story but including chord drift in stories below. The solid lines show the chord rotation R_c due to chord drift. The transition from the dashed to the solid positions is the same as that shown in Fig. 5.5(c).

where α is the chord angle change (Eq. 5.20) in the story above the level, α_a is the sum of the chord angle changes (Eq. 5.21) for all stories below the level, and Δ_c is the chord drift in the story. Equation 5.22 applies to both K-braced and diagonally braced bents if the numerical factors in Eqs. 5.20 and 5.21 are chosen as previously indicated.

Note that chord drift for a K-braced bent depends upon the direction of the K-brace "V's". The difference appears to be marginal, however, because only the first term in Eq. 5.22 drops out when the "V's" point upward. The second term, which dominates the chord rotation, is independent of the web bracing system.

Equation 5.22 can be specialized for Bent B by absorbing the first term into the sum because Eqs. 5.20 and 5.21 give identical values for a K-braced bent.

The total rotation in each story is the sum of the chord and web contributions:

$$R = R_c + (R_b + R_g) = \frac{\Delta}{h} \qquad (5.23)$$

where Δ is the drift in the story. The relative rotation contributions of the chord and web depend upon the relative axial stiffness (area) of the columns and web system members, and upon the story under consideration. The web contribution tends to dominate in the lower stories while the chords contribute more in the upper stories of a braced bent. In spite of these complexities, a relatively simple design method for drift control of braced bents is described in the following articles.

The total rotation R from Eq. 5.23, under factored combined load (F = 1.3), should be limited to about 0.004 radians in each story of Bent B for two reasons:

1. To limit the $P\Delta$ shears resisted by Bent B to the value 0.004 ΣP assumed in the statics calculations in Tabs. 8.12 to 8.15.
2. To limit the drift to $0.0025h_t$ under working loads (F = 1.0). These two factors will be reviewed after the design is completed.

5.7 BEHAVIOR OF BRACED BENTS

To develop a method for selecting trial members for drift control, it is useful to consider the overall behavior of a braced bent. The bent behaves like a vertical cantilever beam under horizontal load. This analogy is used in Tab. 8.14 to check the cantilever shear and bending forces at Level 24 of Bent B.

The braced bent cantilever response is sketched through the sequence from load to drift in Fig. 5.7. The top and middle rows in this figure are concerned with chord and web drift. The chord and web drift contributions are summed to give the total rotation and total drift in the bottom row.

The chord drift sequence in Fig. 5.7 follows the familiar relations for bending of a cantilever beam:

Cantilever Beam	*Braced Bent*
Load	Load
Shear	Shear
Moment	Moment
Bending stiffness	Chord stiffness
Curvature	Chord angle change above story, α_a
Slope	Chord rotation in story, R_c
Deflection	Chord drift from base, $\Sigma\Delta_c$

Except for the relation between moment and curvature, each item in this sequence is obtained by integration or summation of the previous item.

Curvature of a beam represents the angle change per unit length. The elastic relation between curvature ϕ and moment M for a beam is $\phi = M/EI$. A similar relation holds for a braced bent. The cantilever moment resisted by a braced bent is

$$M = P_c L \qquad (5.24)$$

where P_c = axial load in the columns due to wind plus $P\Delta$, and L = distance between columns (chords). If the columns have equal areas A_c, the moment of inertia of the column chords is

$$I_a = 2A_c \left(\frac{L}{2}\right)^2 \qquad (5.25)$$

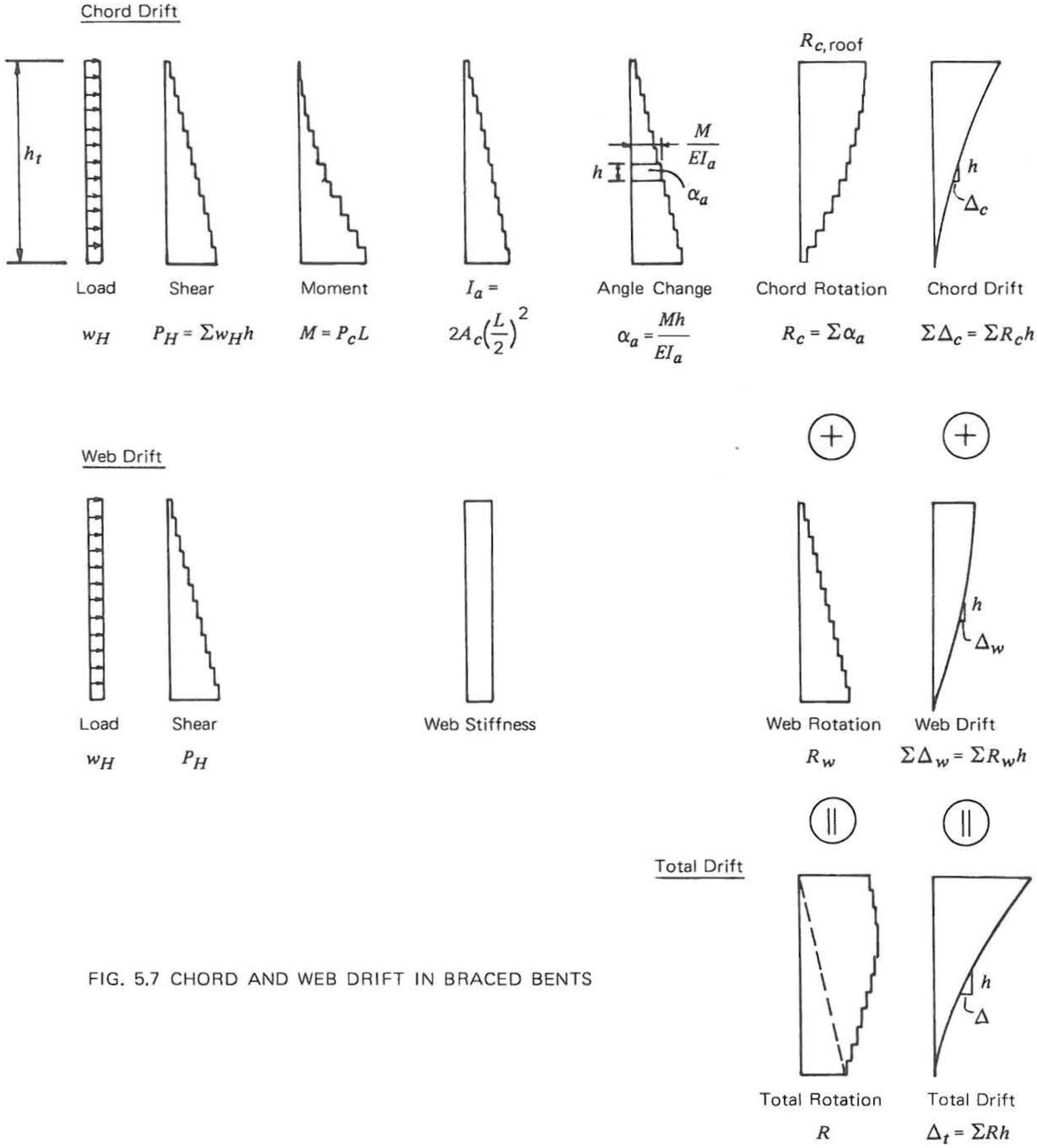


FIG. 5.7 CHORD AND WEB DRIFT IN BRACED BENTS

These relations give the angle change above a story of height h in the form

$$\alpha_a = \frac{Mh}{EI_a} = \frac{P_c}{A_c}\frac{2h}{EL} \qquad (5.26)$$

This is simply an alternate version of Eq. 5.21. The angle change in a beam for a segment of length h is Mh/EI as in Eq. 5.26. This demonstrates the analogy between flexural deflection of a cantilever beam and chord drift of a braced bent.

A similar analogy exists between shear deflection of a cantilever and web drift of a braced bent. Shear deflection of a solid web beam is usually negligible but this is not true for braced bents. The axial forces P_g and P_b in the web members of the bent are statically related to the story shear. The web stiffness depends on the geometry and areas of the web members as indicated in Eqs. 5.18 and 5.19. The ratio of story shear to web stiffness yields the rotation $R_w = R_b + R_g$.

The chord rotation diagram in Fig. 5.7 is a second degree parabola if the moment diagram is parabolic and the I_a diagram is linear.

The web rotation diagram is linear if the shear diagram is linear and the web stiffness is constant. Under these conditions the total rotation diagram includes a linear and a parabolic component and reaches a peak value several stories below the roof. The I_a and web stiffness diagrams in Fig. 5.7 are useful approximations but some variation from these idealized diagrams may be expected in practice.

For a slender braced bent, rotation due to chord drift tends to dominate the total rotation, particularly in the upper stories. The column areas in the braced bays should then be selected to limit the chord rotation in the top story by reducing angle changes near the base.

An initial estimate of the bottom story column area $A_{c,\text{base}}$ in sq. in., to control chord drift can be obtained from the equation

$$A_{c,\text{base}} = \frac{P_{H,\text{base}}}{200}\left(\frac{h_t}{L}\right)^2 \qquad (5.27)$$

where $P_{H,\text{base}}$ is the total factored shear in kips due to wind plus $P\Delta$ at the base of one truss and h_t/L is the truss slenderness. This equation applies to single bay trusses like the exterior bays in Bent B.

Equation 5.27 is based on the chord drift sequence in the top row of Fig. 5.7. If the angle change diagram is approximated as a triangle, the chord rotation in the story below the roof is the area under the $\alpha_a/h = M/EI_a$ diagram.

$$R_{c,\text{roof}} = \frac{M_{,\text{base}}\, h_t}{2EI_{a,\text{base}}} \qquad (5.28)$$

The drift index (ratio of drift at roof to total height of bent) due to chord drift is the moment about the roof of the area under the M/EI_a diagram divided by h_t.

$$\frac{\Sigma\Delta_c}{h_t} = \frac{M_{,\text{base}}\, h_t}{3EI_{a,\text{base}}} \qquad (5.29)$$

In these formulas, $I_{a,\text{base}}$ is the moment of inertia of the column chords at the base of one truss from Eq. 5.25, and $M_{,\text{base}}$ is the factored overturning moment due to wind plus $P\Delta$ at the base of the truss. This overturning moment can be expressed in terms of the factored base shear $P_{H,\text{base}}$, including $P\Delta$ effects per Eq. 5.14, in the form

$$M_{,\text{base}} = \tfrac{1}{2} P_{H,\text{base}}\, h_t \qquad (5.30)$$

for the uniformly distributed load in Fig. 5.7.

From Eqs. 5.28 and 5.29, the relation between the chord rotation in the story below the roof and the drift index is

$$\frac{\Sigma\Delta_c}{h_t} = \frac{2}{3} R_{c,\text{roof}} \qquad (5.31)$$

This indicates that a limitation on the chord rotation $R_{c,\text{roof}}$ also serves to limit the drift index due to chord drift.

After substitutions for;

$I_{a,\text{base}} = 2A_{c,\text{base}}\left(\frac{L}{2}\right)^2$ and $M_{,\text{base}}$ in Eq. 5.28

$$A_{c,\text{base}} = \frac{P_{H,\text{base}}}{2ER_{c,\text{roof}}}\left(\frac{h_t}{L}\right)^2 \qquad (5.32)$$

This estimate for the bottom story column area $A_{c,\text{base}}$ reduces to Eq. 5.27 with the substitutions E = 29,000 ksi and $R_{c,\text{roof}}$ = 0.0035 radians. It is assumed that the total rotation in the story below the roof will be approximately 0.004 radians under factored combined load. Of this total rotation, 0.0035 radians is assumed for chord drift, leaving 0.0005 radians for web drift. Other values of $R_{c,\text{roof}}$ can be used in Eq. 5.32 to alter the chord stiffness of the vertical bracing system. This equation is independent of the column yield stress and the truss web system.

In applying Eq. 5.27 to Bent B, note that two identical trusses resist the wind plus $P\Delta$ shears, so $P_{H,\text{base}}$ is one-half of the base shear from Tab. 8.12(8) below level 24. With $P_{H,\text{base}}$ = 652/2 = 326 kips under factored combined load and a truss slenderness h_t/L = 8.8 for Bent B, Eq. 5.27 gives $A_{c,\text{base}}$ = 126 in.^{2} below level 24 for chord drift control.

This column area may be compared with that needed to resist the critical combination of factored axial load and bending. The factored axial loads are obtained from Tab. 8.11(4) and (9) for gravity load and Tab. 8.13(8) and (9) for combined load. Using the largest value, P = 2851 kips for the interior columns under factored combined load, and assuming P/P_y = 0.7 and A36 steel, a column area of 2851/(0.7 x 36) = 114 in.^{2} is estimated below Level 24 to resist the factored axial load. The value P/P_y = 0.7 is the axial load limit in Eq. 5.17. The column area for chord drift control exceeds that estimated for factored axial load. Hence, the design example in Chapter 8, beginning with Tab. 8.16, assumes that stiffness requirements will control the design of braced Bent B. The calculations in Tabs. 8.10 to 8.15 are all based on statics and are needed regardless of what criterion is used to select member sizes.

The following articles describe a braced bent design method for selecting columns to control chord drift, and web members to control web drift.

5.8 CHORD DRIFT CONTROL

The rotation at the roof due to chord drift is the area under the angle change (α_a/h) diagram

$$R_{c,\text{roof}} = \tfrac{1}{2} k h_t \left(\frac{\alpha_a}{h}\right)_{\text{base}} \qquad (5.33)$$

where h_t is the total height of the braced bent, k is a dimensionless coefficient, and (α_a/h)$_{\text{base}}$ denotes the "curvature" in the bottom story. Rearranging Eq. 5.21 gives,

$$\left(\frac{\alpha_a}{h}\right)_{\text{base}} = \frac{P_{c,\text{base}}}{A_{c,\text{base}}}\left(\frac{2}{EL}\right) \qquad (5.34)$$

From Eq. 5.33 and 5.34

$$A_{c,\text{base}} = k\,\frac{P_{c,\text{base}}}{R_{c,\text{roof}}}\left(\frac{h_t}{EL}\right) \qquad (5.35)$$

This result indicates that the column area at the base $A_{c,\text{base}}$ is inversely proportional to the rotation at the roof due to chord drift, and is directly proportional to the truss slenderness h_t/L and to the axial load in the bottom story columns $P_{c,\text{base}}$ due to wind plus $P\Delta$. Equation 5.35 is an alternate form of Eq. 5.32 using axial force in the bottom story columns in place of base shear.

The dimensionless coefficient k in Eqs. 5.33 and 5.35 depends upon the shape of the moment and I_a diagrams in Fig. 5.7. The value k = 1.0 is a reasonable first approximation and corresponds to:

1. A second degree parabola for the moment diagram, and
2. A linear variation of column area A_x with distance h_x above the base

$$A_x = \left(1 - \frac{h_x}{h_t}\right) A_{c,\text{base}} \qquad (5.36)$$

Regardless of the shape of the moment and I_a diagrams in Fig. 5.7, an improved estimate for k can be obtained as follows:

1. Determine $A_{c,\text{base}}$ from Eq. 5.35 for k = 1.0 and an assigned value $R_{c,\text{roof}}$ (discussed later). Select a trial section for the bottom story columns in the braced bays.

2. Select trial sections for the columns above using Eq. 5.36 or any other appropriate variation of column areas.
3. Determine the angle changes α and α_a above each level using Eqs. 5.20 and 5.21.
4. Determine the chord rotation R_c below the roof from Eq. 5.22.
5. An improved estimate for k is $kR_c/R_{c,\text{roof}}$

These five steps represent one cycle of an iterative loop, and can be repeated using the improved estimate for k in Step 1. The loop converges when R_c in Step 4 is approximately equal to $R_{c,\text{roof}}$ in Step 1. Convergence is obtained in the second cycle for the example described subsequently.

The column areas in the braced bays of Bent B decrease with increasing values of $R_{c,\text{roof}}$ in Eq. 5.35. However, $R_{c,\text{roof}}$ should be taken as less than 0.004 radians for the reasons cited at the end of Art. 5.6, and to allow for rotation due to web drift in the top story of Bent B. The assumption

$$R_{c,\text{roof}} = 0.0035 \text{ radians} \qquad (5.37)$$

is used in the design example. If larger values of $R_{c,\text{roof}}$ are assumed, the story rotations will tend to exceed 0.0025 under working wind load and 0.004 under factored combined load. If smaller values of $R_{c,\text{roof}}$ are used, the column areas become excessive in this example.

Columns for Bent B are selected for chord drift control in Tab. 8.16. Step 1 shows the application of Eq. 5.35 to determine the area of the bottom story columns, using $k = 1.2$. This k value is the result of calculations described later.

Step 2 uses Eq. 5.36 to estimate column areas for the stories above the base level as shown in Tab. 8.16(3). The next three tabular columns list trial sections, areas, and axial loads P_c due to wind plus $P\Delta$. Note that 14*WF*426 in the bottom story does not meet the estimated area requirement. A coverplated or "jumbo" column section could be selected but this is not done in order to demonstrate the flexibility of the method.

The angle changes α_a due to chord drift (Fig. 5.6c) are recorded in Tab. 8.16(7). Note (1) indicates how Eq. 5.21 is applied in this calculation. The angle changes α_a are summed from the base in Tab. 8.16(8). This gives the rotation due to chord drift R_c according to Eq. 5.22. The value R_c = 0.00353 radians below the roof agrees with the assumption $R_{c,\text{roof}}$ = 0.0035 radians in step 1. This verifies the choice $k = 1.2$.

The columns in Tab. 8.16 satisfy the tentative rotation criterion for chord drift. These sections are checked for beam-column capacity in Art. 6.2.

The success of this design method for chord drift control depends in part on the choice of k in Eq. 5.35. Note that the method provides a check on the value assumed for k. The value k = 1.2 in Tab. 8.16 was obtained from one cycle of the five steps outlined previously. The calculations used the same format as Tab. 8.16 with the results summarized below:

Step 1. $A_{c,\text{base}} = 120 \text{ in.}^2$
for $R_{c,\text{roof}} = 0.0035$ radians and $k = 1.0$

Step 2.

Level	Trial Section	Level	Trial Section	Level	Trial Section
2	12*WF*40	10	14*WF*150	18	14*WF*287
4	12*WF*50	12	14*WF*184	20	14*WF*320
6	12*WF*85	14	14*WF*219	22	14*WF*370
9	12*WF*120	16	14*WF*264	24	14*WF*426

Step 3.

Level	$\alpha_a \times 10^5$ (radians)	Level	$\alpha_a \times 10^5$ (radians)	Level	$\alpha_a \times 10^5$ (radians)
R		9		17	
	1		12		22
2		10		18	
	3		15		25
3		11		19	
	5		15		24
4		12		20	
	8		17		27
5		13		21	
	7		17		26
6		14		22	
	10		20		28
7		15		23	
	9		19		34
8		16		24	
	12		21		34
9		17		25	

Step 4. R_c = 0.00411 radians below roof.

Step 5. Estimate k = 0.00411/0.0035 = 1.18. Say 1.20.

Note that if R_c in Step 4 had been less than 0.004 radians, it might have been possible to continue the design with the trial column sections from Step 2, with some increase in web stiffness requirements for Bent B.

Future braced bent design studies by practicing engineers will be of value in suggesting appropriate assumptions for k and $R_{c,\text{roof}}$ in Eq. 5.35. These design studies will also help to define the braced bent geometry and load parameters that combine to make chord drift a significant design criterion.

5.9 WEB DRIFT CONTROL

The web members should be designed to limit the total story rotation (Eq. 5.23) under factored combined load to about 0.004 radians in each story of Bent B. Once the columns have been selected in Tab. 8.16, the rotation due to chord drift R_c (Eq. 5.22) is known in each story. This gives the allowable web rotation

$$R_w = R_g + R_b = 0.004 - R_c \qquad (5.38)$$

listed in Tab. 8.16(9).

At this point several alternatives are open to the engineer for web drift control:

1. Design the girders for strength as laterally loaded beam-columns (see Art. 5.2). This gives the minimum required girder size and establishes the rotations R_g in Eq. 5.38. The minimum bracing area, Min A_b, needed to limit the total rotations can be determined from Eqs. 5.38 and 5.18. The bracing is then designed for stiffness (area) requirements and checked for strength criteria (Art. 5.10).
2. Design the bracing for strength requirements. This determines the minimum required bracing size and establishes the rotations R_b in Eq. 5.38. The minimum girder area, Min A_g, needed to limit the total rotations can be obtained from Eqs. 5.38 and 5.19. The girders are then designed for stiffness (area) requirements, and checked for strength and web buckling criteria as laterally loaded beam-columns.
3. Use some combination of Alternatives 1 and 2. For example, determine the minimum girder sizes as in Alternative 1 but use girders of larger area. This approach serves to increase the girder stiffness and to decrease stiffness (area) requirements for bracing.

Alternatives 1 and 2 could be used to establish upper and lower bounds on the girder and bracing sizes. The web system may then be designed within these bounds to optimize a weight or cost function. The cost analysis should include an allowance for connections. This optimization approach should also be extended to include the columns by varying the rotation allowance for chord drift (Art. 5.8). These ideas suggest the potential for future developments in the field of braced multistory frames.

Alternative 1 is illustrated in the design example. The girders in Bent B are selected for factored combined load in Tab. 8.17. The girders are designed as laterally loaded beam-columns using Eq. 5.10 as explained in Art. 5.2. On Levels 2 to 8 in the exterior bays, the 10B19 girders selected for factored gravity load in Tab. 8.9(17) are also adequate for factored combined load. Below Level 8, combined load requires an increase in the girder sizes as shown in Tab. 8.17(5).

Web drift calculations for the girders are performed in Tab. 8.18. Step 1 shows how Eq. 5.19 is applied to give the rotation R_g in Col. (4).

The quantity $R_W - R_g$ in Tab. 8.18(5) represents the allowable rotation for the K-bracing. Equation 5.18 is used in Step 2 to determine the minimum K-brace area, Min A_b, needed to limit rotation. The values of Min A_b in Col. (7) increase from the roof to Level 7, and then decrease from Level 7 to 24. If the minimum required bracing area exceeds that for bracing members of practical size, the girders can be increased as in Alternative 3.

5.10 DESIGN OF K-BRACING

There are three design criteria that are likely to control the K-bracing areas A_b in Bent B:

1. The tension capacity A_bF_y must exceed the maximum K-brace tension in Tab. 8.15(5).
2. The compression capacity A_bF_{cr} must exceed the maximum K-brace compression in Tab. 8.15(6). Eq. 3.5 gives the buckling stress F_{cr}.
3. The bracing area should satisfy the minimum stiffness requirement in Tab. 8.18(7).

The bracing stiffness requirement stems from the limitation R = 0.004 radians for the story rotation under factored combined load. No serious consequences should result from exceeding this limitation by several percent. See Art. 5.11. Thus, the values of Min A_b in Tab. 8.18(7) may be regarded as a guide to acceptable bracing stiffness rather than an absolute minimum design requirement.

Two additional design criteria should be considered for the bracing. The AISC Specification limits the slenderness ratio for compression members to 200 and for tension members to 300. The bracing stiffness must also be adequate to prevent sidesway buckling of the building under factored gravity load as discussed in Chapter 6.

The K-bracing for Bent B is designed in Tab. 8.19 using weldable pipe with F_y = 36 ksi. Pipe is an efficient section for bracing members that must resist compression. The information known when the bracing design commences is recorded in the lower half of each line in Tab. 8.19 and includes: the minimum area requirement in Col. (3), the length, Net L_b, in Col. (4), and the maximum K-brace compression and tension in Cols. (7) and (8). The net buckling length of the bracing is less than the total length as suggested in Note (1).

The trial pipe size for the K-bracing in Tab. 8.19(2) is selected using the minimum area requirement as a guide. The brace area, radius of gyration, slenderness, and buckling stress are entered in Cols. (3) to (6). The trial brace is adequate for strength criteria if the allowable (ultimate) compression and tension capacity in Cols. (7) and (8) exceeds the maximum values recorded previously.

Above Level 23, the K-bracing for Bent B is controlled by stiffness (area) requirements in Tab. 8.19. The maximum K-brace forces are less than 75 percent of the tension and compression capacity above level 23. The K-bracing is stable and elastic (including a residual stress allowance) under factored combined load. Note that these bracing conditions are assumed in the design of the K-braced girders (Art. 5.2) and in the drift equations (Art. 5.6). In the bottom story of Bent B, the inverted K-brace, the absence of chord drift and the 12 ft. story height combine to make compression strength the controlling design criterion for the K-bracing.

5.11 STORY ROTATION AND DRIFT

Previous articles have described methods for selecting columns to limit chord drift and girders and bracing to limit web drift in braced bents. This article compares these drift contributions for Bent B and reviews the reasons stated at the end of Art. 5.6 for limiting the total rotation under factored combined load.

Rotation and drift calculations are performed in Tab. 8.20. The first six columns are concerned with the rotation under factored combined load. The last three columns deal with the drift under working load.

The previously determined rotations due to axial load in the columns and girders are listed in Tab. 8.20(1) and (2). The rotations for the K-bracing are obtained in Cols. (3) to (5) using Eq. 5.18, with the substitutions shown in Item 2 of Tab. 8.18. The rotations in Tab. 8.18(5) differ from those in Tab. 8.20(5) because the latter values are based on the actual brace area provided. The column and web contributions are summed to give the total rotation in Tab. 8.20(6).

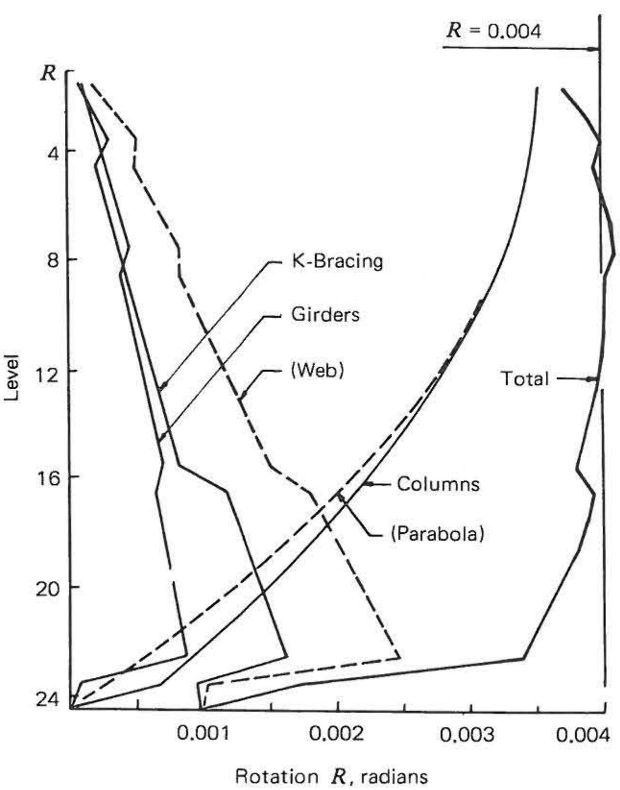


FIG. 5.8 ROTATION FOR BENT B AT FACTORED COMBINED LOAD (F = 1.3)

Figure 5.8 is a graphical summary of the story rotations for Bent B under factored combined load, from Tab. 8.20. The dashed "web" curve in this figure gives the sum of the K-brace and girder contributions to rotation. The kinks in the web curve result from changes in K-brace and girder sizes.

The web rotation curve in Fig. 5.8 shows a nearly linear variation with distance from the roof except in the bottom stories. The column chord rotation curve is approximately parabolic. The coefficient k = 1.2 for Bent B, discussed in Art. 5.8, differs from unity because of the difference between the computed column chord rotation curve and the dashed parabola in Fig. 5.8. Note the similarity between the column chord rotation and web rotation curves in Figs. 5.7 and 5.8.

The relative magnitudes of the chord and web contributions to the total rotation are evident in Fig. 5.8. The chord contribution dominates in the upper stories and the web contribution is larger in the lower stories. In spite of the varying chord and web contributions, the total story rotation or drift index is nearly constant in the upper three-quarters of Bent B. Note that the statics calculations for Bent B under factored combined load, beginning with Eq. 5.14 in Art. 5.5, are based on a constant assumed drift index. The total story rotation curve in Fig. 5.8 indicates that this simple design approximation is reasonably satisfied. In the bottom stories, the initially assumed drift index is conservative and could be reduced.

The preceding discussion is concerned with the total rotation $R = \Delta/h$. This variable, together with the total factored gravity load ΣP, is used to account for the frame stability effects under factored combined load in the plastic design of braced multistory buildings. A second variable describing the combined load behavior of braced bents is the total drift $\Delta_t = \Sigma Rh$, where the sum is taken from the base.

Drift curves for Bent B under factored combined load are shown in Fig. 5.9. The "web" curve in this figure gives the sum of the K-brace and girder contributions to drift. The column chord contribution exceeds the web contribution to drift in the upper half of Bent B. At the roof, approximately two-thirds of the total drift is contributed by the columns and one-third by the web members. The total drift curve is nearly linear and parallel to the dashed line for Δ_t = $0.004h_t$, except in the bottom stories.

The working load drift calculations for Bent B in Tabs. 8.20(7) to (9) are based on the following simplifying assumption: the working load rotation in Col. (7) can be estimated as 1/1.3 times the rotation under factored combined load. Because of this assumption $P\Delta$ effects corresponding to a drift index of Δ/h = 0.004/1.3 = 0.003 are included in the working load drift estimates. The result is that the calculated drift is about 14 percent larger than that which would be obtained by neglecting $P\Delta$ effects.

The total working load drift at the roof of Bent B in Tab. 8.20(9) gives an overall drift index of 0.0028 as indicated in Note (2) below the table. For comparison with conventional drift criteria, neglecting $P\Delta$ effects, the drift index should be adjusted to 0.0028/1.14 = 0.0024. Bent B provides ample stiffness for limiting drift under the 20 psf wind load.

The method described in Arts. 5.8 and 5.9 for chord drift and web drift control can be applied in an allowable stress design of a braced bent. In this application, the total wind shear in Tab. 8.12(8) is replaced by the working load wind shear in Tab. 8.12(2).

The two reasons listed at the end of Art. 5.6 for limiting the total rotation under factored combined load to about 0.004 radians in each story are reviewed below.

The first reason for limiting the total rotation in each story was to avoid $P\Delta$ shears larger than the value 0.004 ΣP assumed in the combined load statics calculations for Bent B.

If the total rotation, computed using 0.004 ΣP for the $P\Delta$ shear, is significantly larger than 0.004 radians, both the rotation and the $P\Delta$ shear have been underestimated. In such a situation, it is possible to begin the second cycle of an iterative frame stability check. This check is based on an adaptation of Ref. 8 and uses the following steps:

1. Determine the $P\Delta$ shear in each story as $R\Sigma P$ where R is the total rotation from the previous drift calculation under factored combined load. For example, use R from Tab. 8.20(6).
2. Repeat the statics analysis in Tab. 8.12 to 8.15 using the new $P\Delta$ shears from Step 1 in Tab. 8.12(7).
3. Repeat the chord, web and total rotation calculations in Tabs. 8.16, 8.18, and 8.20. In this step, note that Eq. 5.17 and the rotation formulas in Fig. 5.5 should be modified for inelastic behavior under axial load if P/P_y is significantly larger than 0.7.

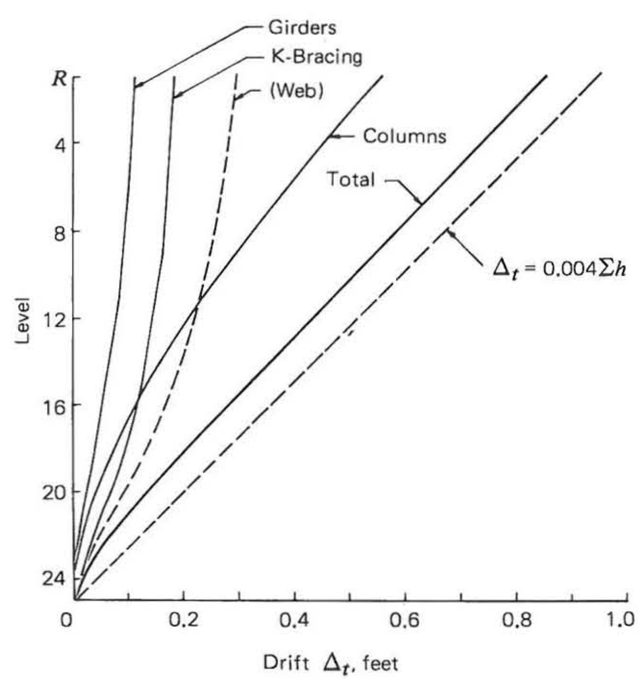


FIG. 5.9 DRIFT FOR BENT B AT FACTORED COMBINED LOAD (F = 1.3)

Alternatively, member sizes may be increased to satisfy this limitation on P/P_y.

4. If the total rotations obtained in Step 3 are equal to or smaller than the corresponding values from Step 1, the iteration is said to have converged and the bent is stable under the factored combined load. If this is not the case, two alternatives are open.
   a. Repeat the cycle using the total rotations from Step 3 (or larger values to speed convergence) in place of the previous rotations in Step 1.
   b. If the rotations in Step 3 are large (say on the order of two times the working load drift criterion), the members that make the larger contributions to the total rotation should be increased in size.

This iterative stability check for braced multistory bents under factored combined load follows well defined steps, but can become protracted for a tall bent. It is not expected that this stability check method will find frequent application in practice. It is included here to indicate the possible significance of calculated total rotations that exceed the rotation used to estimate the $P\Delta$ shears by a substantial margin.

In several stories, the calculated total rotations for Bent B in Tab. 8.20(6) exceed, by a small margin, the value 0.004 radians used to estimate $P\Delta$ shears in Tab. 8.12(7). This does not necessarily indicate the need for a combined load stability check. For example, consider the factored shears below Level 7 of Bent B.

Wind shear	=	165 kips	Tab. 8.12(3)
$P\Delta$ shear	=	24.2 kips based on R = 0.004 radians	Tab. 8.12(3)
$P\Delta$ shear	=	24.8 kips based on R = 0.00409 radians	

The increase in total rotation causes only a small change in the $P\Delta$ shear. From another viewpoint, a 3 percent or 4.9 kip change in the factored wind shear corresponds to a 20 percent change in the initially assumed $P\Delta$ shear for Bent B. Unless the $P\Delta$ shear is a substantial portion of the total shear in the story, small differences between the initially assumed and calculated total rotation do not produce significant changes in the total story shear.

The second reason for limiting the total rotation under factored combined load was to satisfy the drift criterion under working wind load. The adjusted working load drift index of 0.0024 for Bent B is within acceptable drift limits.

For a working load drift index of 0.003 many of the columns in Bent B can be reduced by about two sizes. The drift criterion is significant in determining the weight of steel in a braced bent. In present practice, the choice of a drift index depends on the engineer's judgment. Research is needed to assist the engineer in making his choice.

The results of the tentative design of braced Bent B are summarized in Fig. 8.2; these are to be checked in Chapter 6.

CHAPTER 6

Design Checks and Secondary Considerations

6.1 INTRODUCTION

The primary stage of a structural design is usually concerned with the proportioning of members for strength and/or stiffness. The design conditions considered are full factored gravity loading, factored combined loading, and wind drift. In Chapter 5 drift was chosen as the governing design condition for the columns in Bent B. In this chapter the strength of these columns will be checked for gravity load (F = 1.7) and combined load (F = 1.3).

In addition there are secondary conditions (secondary meaning that they are not usually considered in the initial design) that may govern the design of individual elements in the structure. These conditions are:

1. partial live or "checkerboard" loading
2. deflections at working load
3. sidesway under factored gravity load
4. spacing of lateral bracing
5. effect of shear on bending capacity
6. uplift at footings

The approach used for these design checks is to make conservative assumptions and approximations in order to find out if there is a problem in the first place. If the preliminary conservative calculations do not satisfy the particular requirement, then more careful analysis is performed. The idea is that usually the secondary design situations are not critical, so they do not warrant undue design time.

6.2 DESIGN CHECKS, BENT B—FACTORED GRAVITY AND COMBINED LOAD

The girders in Bent B have been designed for factored gravity and combined loading in Tabs. 8.9 and 8.17. The columns in Bent B, however, have been selected for chord drift control in Tab. 8.16, so they must be checked for adequate strength to support the factored gravity load (F = 1.7) and factored combined load (F = 1.3). It will be necessary to check both loading conditions for beam-column strength.

The calculations for the column check are similar to those for the design of the Bent A columns given in Tabs. 8.6 and 8.8. First the moments applied to the columns through the girders are evaluated in Tab. 8.21. In determining the net girder moments that are applied to the columns, it is assumed that M_p occurs at the ends of the girders only for the particular loading condition that controlled the girder size. For other loading conditions it is assumed the girder is elastic, and the end moment is, conservatively, the fixed-end moment FEM. An upper limit is placed on the fixed-end moment equal to the bending capacity of the girder. Factored combined load (F = 1.3) governed the size of the girders in the exterior bay, whereas the girders in the interior bay were selected on the basis of factored gravity load (F = 1.7). Consequently, it is assumed that the exterior-bay girders are elastic at F = 1.7 and the interior-bay girders are elastic at F = 1.3. The girder moments at the column centerline M_j are calculated by increasing the clear span moments M_e by the quantity ¼ FwL_gd_c (end shear x one-half the column depth). The net girder moment is assumed to be equally divided between the two columns at a joint.

In Tab. 8.22, the axial loads and moments in the columns are compared for the two loading cases in order to determine the controlling conditions. Only a few representative columns are compared in this check, and an asterisk (*) indicates the more critical loading condition. From Tab. 8.22, the check of the top story columns is governed by factored gravity load, whereas factored combined loading controls for the lower stories.

Selected columns in Bent B are checked for in-plane bending and lateral-torsional buckling in Tab. 8.23 using the procedure described in Art. 4.9. The axial load (P/P_y) and slenderness ratios for the columns are in the range where the beam-column moment capacity is limited by M_{pc} so the allowable M/M_{pc} = 1.0 in Tab. 8.23(8). The M_{pc} values exceed the required moments in Tab. 8.23(1) by a substantial margin. Hence, the columns previously selected for chord drift control provide adequate beam-column capacities for full factored gravity and combined load.

6.3 CHECKERBOARD LOADING

The columns of Bents A and B can safely carry the full factored dead and live loads on all the stories. Full loading usually causes the columns to bend in double curvature (q = + 1.0),

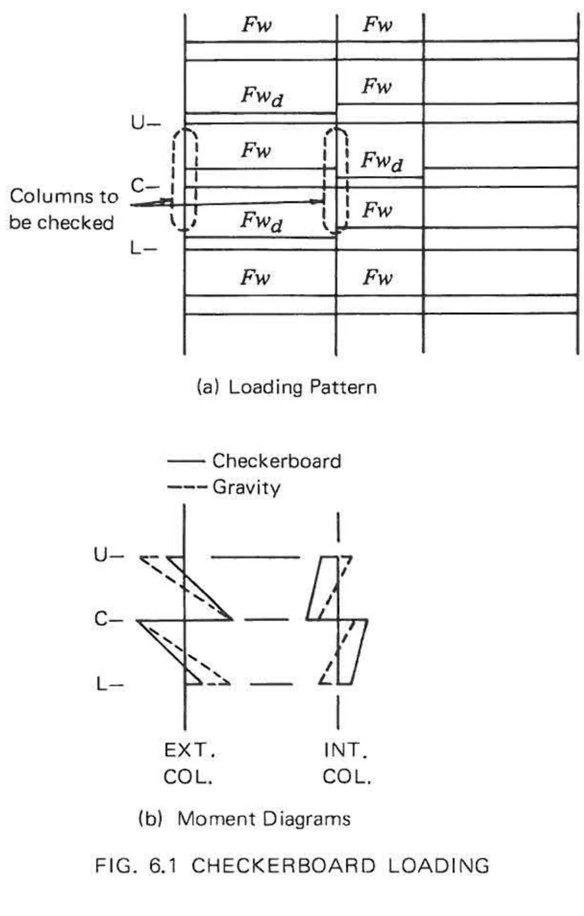


FIG. 6.1 CHECKERBOARD LOADING

and the strength of a column is highest when bent in this configuration. If the ratio of the end moments q is reduced ($q < +1.0$), the column strength can be adversely affected. See Design Aid III.

A situation more critical than full loading can develop if the factored live load is removed at a few locations. The typical loading arrangement that should be considered is shown in Fig. 6.1(a). The factored live loads are removed in alternate bays at levels U,C and L only. The column moments caused by this localized checkerboard arrangement tend to approach the most critical single curvature case ($q = -1$) while keeping the axial load relatively unchanged. The possibility of having a complete checkerboard pattern is extremely remote and not even as critical a condition, since axial load in the column would be substantially reduced. In the "localized" checkerboard loading, the axial load will be reduced slightly, but at the lower stories the reduction is usually insignificant. A comparison between the moment diagrams for full gravity and checkerboard loadings is shown in Fig. 6.1(b). Not only can q be reduced from the double curvature case, but the moment applied to the columns at Level C can be increased.

To evaluate the strength of a column under checkerboard loading, the end moments and q must be determined. In the full loading case, Eq. 4.3 was used to calculate the net girder moment where M_B was taken as the required M_p from Eq. 4.1. The net girder moment at the column centerline for checkerboard loading can be determined from Eq. 6.1 .

Net girder moment =

$$\pm\left[M_p + \frac{FwL_g d_c}{4}\right]_{\substack{\text{FULL}\\ \text{LOAD}}} \mp \left[M_d + \frac{Fw_d L_g d_c}{4}\right]_{\substack{\text{DEAD}\\ \text{LOAD}}} \qquad (6.1)$$

where M_d is the moment at the ends of the girder under factored dead load alone and is assumed as

$$M_d = \frac{F w_d L_g^{\,2}}{12} \qquad (6.2)$$

but may not exceed M_p. The sign convention is shown in Tab. 8.21. If $M_d = M_p$, plastic hinges form at the ends of the girders under factored dead load alone, so there is no significant difference between checkerboard loading and full loading. In the design check, it is conservative to assume that only the columns resist the net girder moment and that this moment is equally divided between the columns framing into the joint. Once the column end moments are evaluated, q is calculated, and the column strength determined.

In the *exterior* columns, checkerboard loading only affects q since the column moments at the floors above and below the Level C under consideration are reduced while the moment at Level C remains constant as shown in Fig. 6.1(b).

However, q must be greater than zero because of the restraining effect of the members at the levels above and below. Therefore, it is conservative to use $q = 0$ for the exterior columns. A comparison of Design Aids III–1b and III–2b shows that for $h/r_y < 25$ there is no difference between the major axis bending strengths for $q = +1.0$ (double curvature) and $q = 0$ (one end pinned) unless $P/P_y > 0.6$. In fact, the difference does not become significant (∿ 5%) until $P/P_y > 0.9$.

A reduction in q from +1.0 to 0 can also affect the lateral-torsional buckling strength (LTB). A comparison of Design Aids III–1a and III–2a shows when this change in q has an effect, and the results are given in Fig. 6.2. Combinations of P/P_y and h/r_y that fall below the curve indicate that when $+1.0 < q < 0$, there will be no change in LTB strength (actually there will be no lateral torsional buckling). If values fall above the line, further analysis is indicated; Design Aid III–2a must be used to check for the actual LTB strength.

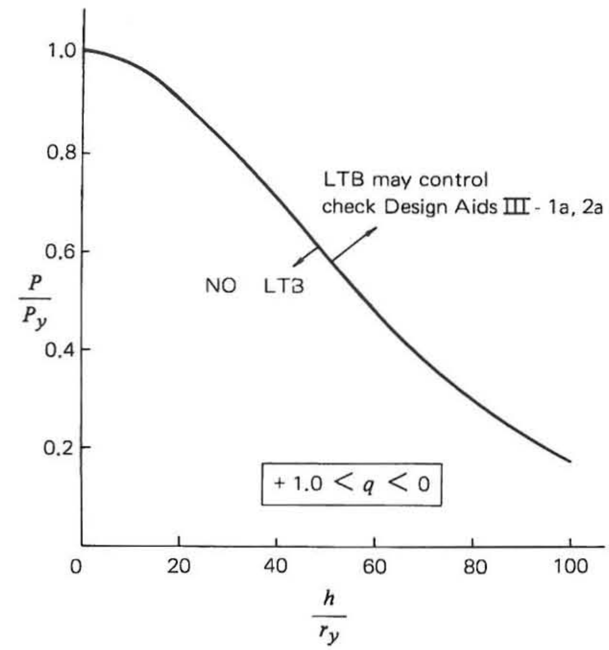


FIG. 6.2 EFFECT OF LATERAL-TORSIONAL BUCKLING ON BEAM-COLUMN STRENGTH

In the *interior* columns, checkerboard loading affects both q and the maximum bending moment, and q can vary over the full range of +1.0 to −1.0. The curves in Design Aid II indicate that most columns with $q = +1.0$ or 0 maintain their maximum bending strength over a reasonably large range of end rotation. Consequently a good estimate of the total available column strength at a joint is achieved by adding the maximum moments for the two columns as shown for Cases 1 and 2 of Fig. 4.1. When $q = -1.0$ however, the strength varies continuously with end rotation, so the rotations must be considered when evaluating the total available column strength at a joint as shown by Cases 3 and 4 in Art. 4.9.

In most instances, it will not be necessary to consider the interior column rotations because q will be between +1.0 and 0, or in many cases where q is between 0 and −1.0, the column end moments are so small they can be neglected.

This is especially true where girder live-load reductions have been used.

In summary, the following procedure is recommended for checking column strength under checkerboard loading:

1. Evaluate the net girder moment at Levels U, C, and L using Eqs. 6.1 and 6.2; distribute one-half of the moment to the columns above and below each joint; and calculate q. For a bent with repetitive girder framing and loading, one set of calculations will suffice.
2. In some cases it will be sufficient to observe that plastic hinges form at the ends of the beams under factored dead load alone, that is $M_d = M_p$. Then checkerboard loading causes no significant difference from full loading and no further check is required.
3. For q between +1.0 and 0 (all exterior columns and most interior columns):
   a. If $h/r_y < 25$ and $P/P_y < 0.9$, column strength is the same as full loading; if values of h/r_y and P/P_y fall below the curve in Fig. 6.2, lateral torsional buckling does not govern. Compare the maximum column moment with the allowable moment determined for full loading.
   b. Step (a) eliminates most columns from further checks. When the conditions of Step (a) do not apply, the column strength for $q = 0$ may be determined from Design Aids II and III and compared with the applied loads as outlined in Art. 4.9.
4. For q between 0 and -1:
   a. If the column moments do not exceed $0.05M_{pc}$, the column will be adequate for major-axis bending since under these conditions a small redistribution of the column moments can be accommodated.
   b. If the column moments are in excess of $0.05M_{pc}$, use Design Aids II and III as described in Art. 4.9 to determine the column strength.

The calculations for column end moment and q for Bents A and B are given in Tab. 8.24. Since plastic hinges form under factored dead load in Bent A, the net girder moment and q for checkerboard loading will be the same as those for the gravity loading. Full gravity loading was checked in Tab. 8.8; all columns of Bent A are satisfactory. Since $q = 0$ for Bent B, Step 3(a) is used to check the columns in Tab. 8.24; all the columns are satisfactory.

6.4 DEFLECTIONS AT WORKING LOAD

The deflection requirements in Section 1.13 of the AISC Specification[3] will be used as a guide. The live load deflection of the floor girders must be less than 1/360 span.

As a first step in checking deflections, all girders will be assumed simply supported. If the deflection guides are satisfactory for simple supports, then they must also be satisfied for the real girders that have restrained ends. The midspan deflection ratio of a simply supported girder is:

$$\frac{\delta}{L_g} = \frac{5\, w_l L_g^{\,3}}{384\, EI} \qquad (6.3)$$

Reduced live loads w_l are used in the calculations, and deflections are calculated only at working load. In Tab. 8.25, the live-load deflections at service loads are calculated and compared with 1/360 L_g All girders satisfy this requirement. In the check on Bent B, only the lightest girder is considered since it is the most critical.

6.5 SIDESWAY UNDER GRAVITY LOAD

When a structure is loaded with factored gravity load alone, there is a possibility that the frame may move laterally under a slight disturbing action. Any sway deflection causes $P\Delta$ moments that tend to overturn the structure as shown in Fig. 6.3.

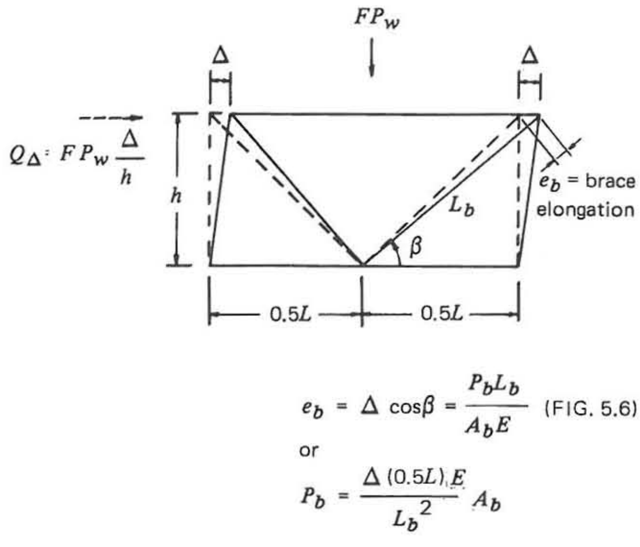


FIG. 6.3 DRIFT UNDER GRAVITY LOAD

If the braced Bent B is assumed to behave like a pin-connected truss, the $P\Delta$ moments are resisted by shears Q_Δ in the K-bracing system given by Eq. 5.12.

The sway deflection Δ is geometrically related to the deformation in the K-braces, and, in an elastic system directly related to the force P_b in the K-braces,where

$$P_b = \frac{\Delta(0.5L)\ EA_b}{L_b^2} \tag{6.4}$$

as shown in Fig. 6.3. The P_b given by Eq. 6.4 is the force in the brace required to produce a sway Δ. If the horizontal component of the bracing forces P_{bH} is greater than the shears Q_Δ caused by $P\Delta$ then the structure will not sway under gravity load, or using Eq. 5.12,

$$P_{bH} > FP_w \frac{\Delta}{h} \tag{6.5}$$

where P_w is the total working gravity load above the level under consideration and $F = 1.7$. Since $P_{bH} = 0.5LP_b/L_b$ and there are two braces in a bay, Eq. 6.5 becomes

$$\sum^{n=N} \frac{2\Delta\,(0.5L)^2\ EA_b}{L_b^{\,3}} > 1.7\,P_w \frac{\Delta}{h} \tag{6.6}$$

where N is the number of braced bays. If the K-bracing sizes and geometry are the same at a given story, Eq. 6.6 can be simplified to

$$\frac{N}{2}\ \frac{L^2}{L_b^{\,3}}\ EA_b > \frac{1.7\,P_w}{h} \tag{6.7}$$

At a given story the minimum K-bracing area A_{bm} required in a braced bent is the factored wind shear (F = 1.3) in the brace divided by the yield stress, or

$$A_{bm} = \frac{1.3\,WL_b}{2N(0.5L)\,F_y} \tag{6.8}$$

where W is the total working wind shear at this level. Defining

$$K_b = \frac{A_b}{A_{bm}} \tag{6.9}$$

and substituting Eq. 6.8 into Eq. 6.7 gives

$$\frac{L}{L_b^2}\ \frac{E}{F_y}\ K_b W > 2.62 \frac{P_w}{h} \tag{6.10}$$

If the geometry and loading of each story are fairly similar, then $W = Dnhw_w$ and $P_w = DnBw_g$, where D is the spacing of the braced bents, n is the level number (Roof = 1), w_w is the working wind load (psf), B is the distance between the exterior columns of the bents, and w_g is the average working gravity load (psf) over the structure. For bracing using A36 steel, Eq. 6.10 becomes

$$K_b\ sin^2\,\beta > 0.00325 \frac{Bw_g}{Lw_w} \tag{6.11}$$

where β is the angle between the brace and the girder.

The structure will not sway under gravity load if Eq. 6.11 is satisfied at every level. However, if Eq. 6.11 is satisfied using K_b = 1.0 (the minimum possible value which corresponds to actually using the minimum bracing area), no further check is necessary. For the unusual case where using K_b = 1.0 does not satisfy this equation, the actual K_b must be used and the equation checked at every level.

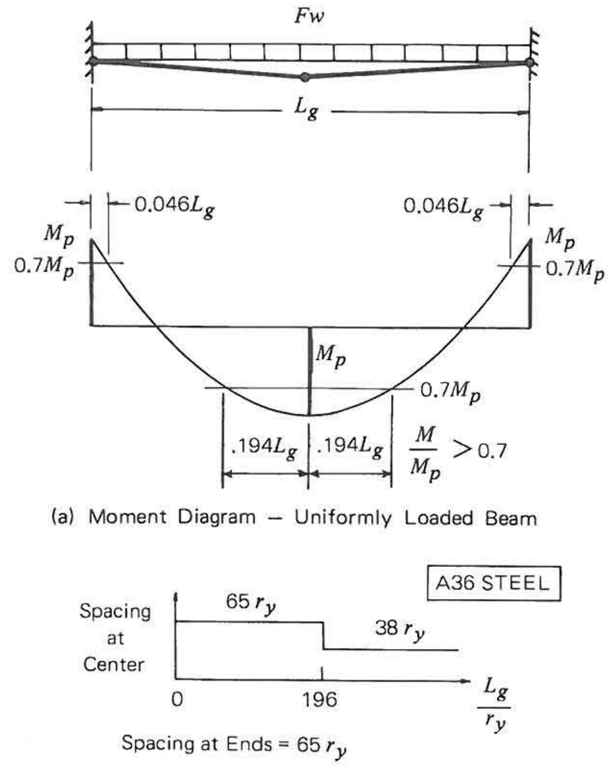
For the bent shown in Fig. 8.1 and assuming K_b = 1.0, Eq. 6.11 becomes

$$1.0 \left(\frac{9.67}{13.50}\right)^2 > 0.00325 \left(\frac{27}{66}\right) \frac{103 \text{ psf*}}{20 \text{ psf}}$$

or

$$0.51 > 0.0069$$

therefore the structure will not sway under gravity load.



(a) Moment Diagram – Uniformly Loaded Beam

(b) Design Aid for Bracing Spacing on A36 Beams

FIG. 6.4 SPACING OF BRACING FOR UNIFORMLY LOADED BEAM

6.6 SPACING OF LATERAL BRACING

In a braced multistory frame the moment diagram at the girder design condition is shown in Fig. 6.4(a). Lateral bracing of the compression flange is required in the vicinity of the plastic hinges to ensure that M_p can be reached and a mechanism can form. The lateral bracing requirements are given in Tab. 3.1. These requirements can be given in a more convenient design form for a uniformly loaded girder by combining them with the moment diagram of Fig. 6.4(a).

The required bracing spacing at the plastic hinge locations (center and both ends) is given in Fig. 6.4(b) for A36 steel. These rules were derived by determining the range over which a bracing rule is applicable. For example, from Fig. 6.4(a), at the center $M/M_{pc} > 0.7$ if $l_{cr} < 0.194L_g$, so $l_{cr} = 38r_y$. Eliminating l_{cr} gives $L_g/r_y > 38/0.194$ = 196. At the center if $L_g/r_y > 196$, then braces must be spaced at $38r_y$. For $L_g/r_y < 196$, a spacing of $65r_y$ is permissible. Thus it is only necessary to calculate L_g/r_y to determine the bracing spacing at the center.

Bracing from the ends can be placed at $65r_y$. For the $38r_y$ rule to govern, it would be necessary to have a girder with a $L_g/r_y > 825$. For rolled sections, such a girder cannot exist because deflection limitations would restrict the girder length to a much smaller value.

The maximum bracing spacing for the compression flange for the girders of Bents A and B

* From the loads in Fig. 8.1, $w_g = \dfrac{(2)95 \times 27 + 140 \times 12}{66} = 103$ psf

Using live-load reductions could further reduce this average gravity load.

are given in Tab. 8.26. As a practical consideration, bracing can be provided only at the joist locations. A tentative floor system design has established a 3-ft. joist spacing for the exterior bays and a 2-ft. spacing for the interior bay. The joists will be positively attached to the top flange of the girders. Tab. 8.26 shows that in all cases, the allowable bracing spacing is greater than the joist spacing so the top flange is adequately braced. Bracing may also be required in the compression regions of the bottom flange. In Bent A, a short length of the bottom flange is in compression at the ends of the girder. Since the girder is rigidly attached to the column and the length of the negative moment region is less than $65r_y$, no bracing is necessary. In Bent B, however, the exterior bay girders have a compression region at the bottom flange at midspan where the K-brace connection is located. At this point, bottom flange bracing must be provided. This can be accomplished by welding joist chord extensions to the bottom flange of the girder. In summary, the joists will provide adequate top-flange bracing for the girders. No other bracing is required for Bent A but the exterior bay girders of Bent B require two bottom flange braces near midspan as shown in Tab. 8.26.

6.7 EFFECT OF SHEAR ON BENDING CAPACITY

Eq. 3.3 gives the maximum allowable shear force which a member can resist. If the actual shear is greater, then the web of the section must be strengthened or the member size increased. The shear in the girders is checked in Tab. 8.27. The maximum applied shear is given by

$$V_{\text{max}} = \frac{FwL_g}{2} \tag{6.12}$$

from equilibrium or symmetry. The largest shear occurs when $F = 1.7$. All girders for Bents A and B are satisfactory as shown in Tab. 8.27.

6.8 UPLIFT AT FOOTINGS—BENT B

The engineer must provide for possible uplift forces at the footings of Bent B under combined load. An estimate of these uplift forces is given in Tab. 8.28. At working load wind can cause 203 kips uplift at the exterior footing and 268 kips uplift at the interior footing. The exterior column uplift can be resisted by the exterior foundation wall carrying shears to the adjacent Bents A. Interior column uplift could be accommodated by bracing in the interior bay at the bottom level.

CHAPTER 7

Connections

7.1 INTRODUCTION

The successful performance of every structure depends upon the connections as well as upon the main members. Connections that are not capable of achieving the assumed degree of end fixity cause the girders to carry higher mid-span moments than allowed for in design. Thus, the behavior of the structure as a whole is changed and its ultimate strength may be quite different from that computed by the designer.

Design of a connection must consider not only angles, plates, welds and bolts but also the webs and flanges of girders and columns near the juncture.

The requirements for connections are:

1. strength
2. rigidity
3. lack of interference with architectural features
4. economical fabrication
5. ease of erection

These are requirements for allowable stress design as well as for plastic design. The performance of connections depends on the ductility of the steel to produce a redistribution of localized stress peaks, and it is the ultimate strength, substantiated by physical tests, that provides the basis for design of connections by either method.

For plastically designed structures, strength and rigidity are important requirements. Connections located at points of maximum moment must not only develop the plastic moment M_p in the connected members, but must maintain these members in their relative positions while plastic hinges develop at other locations.

Phenomena that may affect the development of strength and adequate rotation are:

1. excessive column web shear deformation causing loss of strength
2. column web crippling influencing strength and rotation
3. excessive column flange distortion leading to weld and fastener failures
4. poor welding and poor welding details
5. improper bolt tension

7.2 TYPES OF CONNECTIONS

In multistory building frames the important connections to be considered are: beams to girders, interior tie beams and spandrel beams to columns, girders to columns, column splices, and bracing to girders and columns. Connections are classified according to the AISC designation as:

Type 1. "Rigid frame"—girder-to-column connections have sufficient rigidity to hold virtually unchanged the original angles between intersecting members until M_p develops in a region immediately adjacent to the connection.

Type 2. "Simple"—assumes ends of beams and girders are connected for shear only and are free to rotate from the beginning of loading.

As noted in Art. 4.2 the application of plastic design principles to multistory braced bents requires the use of Type 1 connections between the girders and columns of the Supported Bents A and the Braced Bent B. The connections for the tie beams and spandrels between these bents are Type 2 to avoid introducing biaxial bending into the columns. Beam-to-girder connections may be Type 1 or 2.

Type 1 connections are achieved most simply by welding, although connections combining shop welding and field high-strength bolting also provide good strength characteristics and economy. Usually the all welded connection requires simpler details with less likelihood of interfering with architectural features. Most of the information about the behavior of Type 1 connections has been obtained from tests of welded connections.

7.3 GIRDER-TO-COLUMN CONNECTIONS

Girder-to-column connections may be classified as corner, exterior, or interior type. In all of these, the items tabulated at the end of Art 7.1 must be prevented by proper proportioning of the connection material, the welds and bolts, and the column flanges and web with or without reinforcement by stiffeners.

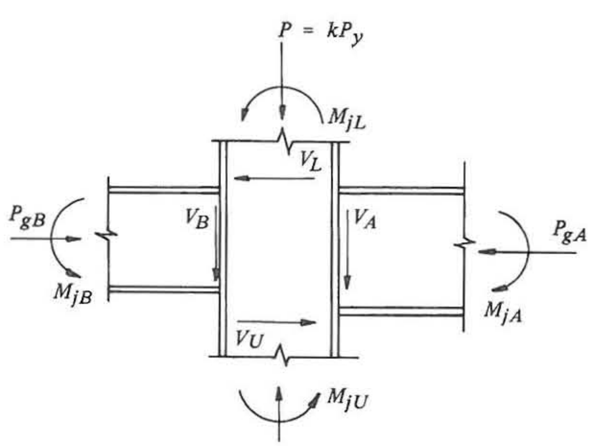
The loads on a girder-to-column connection are combinations of negative or positive girder bending moments, girder shears, axial girder forces, and column axial force as shown in Fig. 7.1a. These loads do not necessarily act at their maximum values simultaneously. It is customary to assume that the girder shear V is carried by a web connection or a seat, while the moment is converted to an equivalent couple of flange forces C and T as shown in Fig. 7.1b.

The compression flange force C fans out as it is transmitted through the column flange to the toe of the fillet where it may cripple the column web w_c. Research has shown that crippling will not occur if the following inequality is satisfied:

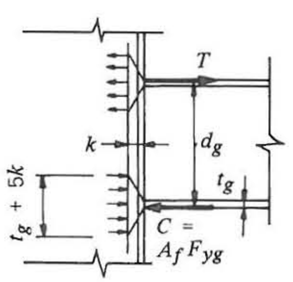
$$w_c(t_g + 5k)F_{yc} \geqslant A_f F_{yg} \qquad (7.1)$$

The tensile flange force T has a different effect on the column. It bends the column flange as shown in Fig. 7.2 and in the process the ductility of the weld joining the girder flange to the column may be exceeded, causing weld fracture. Research has shown that this is not likely to occur if the column flange thickness satisfies the following inequality:

$$t_c \geqslant 0.4 \sqrt{A_f \frac{F_{yg}}{F_{yc}}} \qquad (7.2)$$



(a)



(b)

FIG. 7.1 FORCES ON INTERIOR GIRDER TO COLUMN CONNECTION

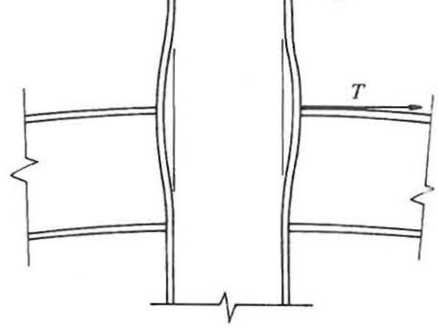


FIG. 7.2 BENDING OF COLUMN FLANGES DUE TO TENSILE FLANGE FORCE

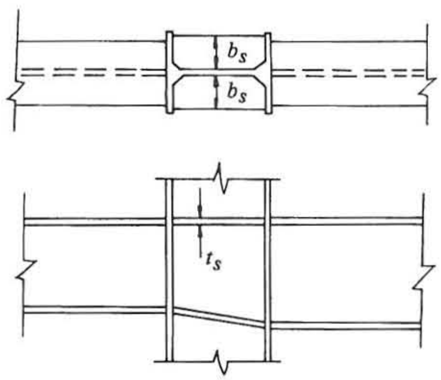
If the requirements of Eqs. 7.1 and 7.2 are not satisfied, additional resistance must be provided by stiffeners welded between the column flanges, either horizontally in line with the girder flanges or vertically between the column flange tips as shown in Fig. 7.3. Vertical stiffeners are considered to be only 50% as effective as horizontal stiffeners. The following equations are used to proportion stiffeners arranged in symmetrical pairs.

Horizontal stiffeners:

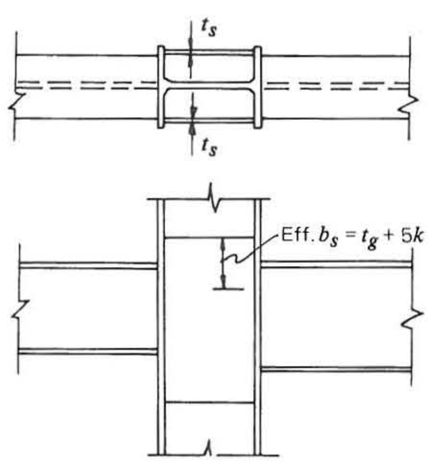
$$A_f F_{yg} - w_c(t_g + 5k)\, F_{yc} - 2b_s\, t_s\, F_{ys} = 0 \quad (7.3)$$

Vertical stiffeners:

$$A_f F_{yg} - w_c(t_g + 5k) F_{yc} - \tfrac{1}{2} \times 2(t_g + 5k)\, t_s F_{ys} = 0 \quad (7.4)$$



(a) HORIZONTAL STIFFENERS



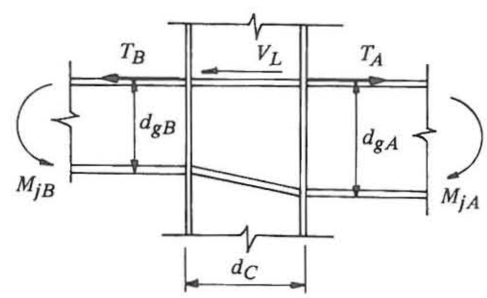
(b) VERTICAL STIFFENERS

FIG. 7.3 TYPES OF COLUMN STIFFENERS

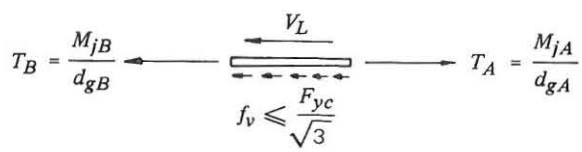
An unbalance of girder moments at a girder-to-column connection produces shear in the column web. If the shear stress in the web is excessive, diagonal stiffeners or a doubler plate must be used. The forces on an interior connection are shown in Fig. 7.4a, where M_{jA} is greater than M_{jB} and V_L is the shear in the column just above the top stiffener. Fig. 7.4b shows a freebody diagram of the top stiffener. Column web shearing stresses are required for equilibrium. Assuming that the shearing yield stress is $\frac{F_y}{\sqrt{3}}$ the following inequality must be satisfied:

$$w_c\, d_c \frac{F_{yc}}{\sqrt{3}} \geqslant T_A - T_B - V_L \quad (7.5)$$

If the thickness of the column web is less than that required by Eq. 7.5, diagonal stiffeners or doubler plates must carry the excess shear.



(a)



(b)

FIG. 7.4 SHEAR STRESS IN COLUMN WEB

The design of diagonal stiffeners is based on the stiffener carrying the excess shear. Thus, from Fig. 7.5, the required area of two stiffeners symmetrically arranged is given by:

$$A_s F_{ys} \cos\theta \geqslant T_A - T_B - V_L - w_c\, d_c \frac{F_{yc}}{\sqrt{3}} \quad (7.6)$$

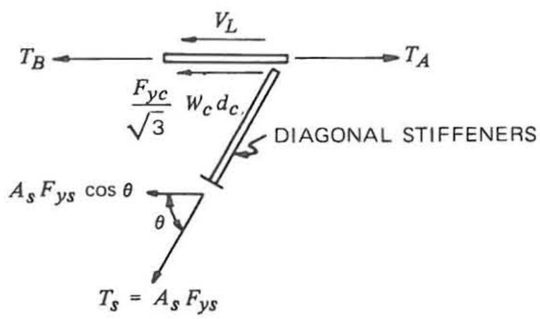


FIG. 7.5 FORCES ON DIAGONAL STIFFENERS

Corner and exterior connections are special cases of the condition described above and similar analyses hold.

The effect of high axial stress on the shearing resistance of the column web is a subject of continuing research, but it is believed to be of only academic interest; beam-columns with high axial load allow only a small percentage of the strength to carry moments that produce column web shear.

7.4 WELDED CONNECTIONS

The welding of girders to columns and of column stiffeners requires welds proportioned by plastic design stress values. Butt welds may be assumed capable of developing on their minimum throat section the tensile yield stress F_y of the base material. Fillet welds may be designed for the shearing yield stress of the weld metal on the minimum throat section. A safe value for design may be obtained by multiplying the allowable working stress value by 1.67. Thus for E60 electrodes,

$$F_y = 1.67 \times 13.6 = 22.7 \text{ ksi}$$

7.5 BOLTED CONNECTIONS

It is economical to shop weld as many parts of a connection as possible. However, the field connection may be accomplished most economically by welding or high strength bolting, depending on such factors as local codes, availability of labor, or the inspection procedures required.

Since the allowable stress design of bolted connections is based upon their behavior at ultimate load, the design of bolted connections for a plastically designed structure involves similar procedures, except that the ultimate strength of the bolts must be used instead of allowable stress.

In plastic design, as in allowable stress design, the designer should be free to decide which bolted connections must be friction-type and which may be bearing-type. Connections subjected to stress reversal or where slippage would be undesirable must be friction-type. Thus, girder moment connections and bracing connections subjected to wind reversal should be designed as friction-type, but girder shear connections might be bearing-type. However, the AISC Specification states in Section 2.7, "when used to transmit shear produced by the ultimate loading, one bolt may be substituted for a rivet of the same nominal diameter". This amounts to recognition of only friction-type connections in plastically designed structures.

The allowable "shear" stresses prescribed for high strength bolts in friction-type connections give a factor of safety against slip of about 1.4 under working gravity loads. When the shear stress is increased one-third for wind, the factor of safety approaches unity. Thus, when the allowable stresses are multiplied by 1.67 to obtain an ultimate shear stress, slip will occur under all factored loading conditions. Of course, it is not expected that factored loading will actually act on the structure.

High strength bolts that resist tension resulting from factored loading may be designed for resisting a tensile force equal to the guaranteed minimum proof load. Thus, even under factored loading it is unlikely that the initial installation tension will be exceeded. In calculating the applied tensile force on a bolt, allowance should be made for tension caused by prying action.

7.6 COLUMN SPLICES

Column sections change and are spliced every second or third story. The splice is usually placed about two feet above the floor level. The splice must be designed for:

1. An axial compressive force resulting from the factored dead and live load. ($F.=1.7$)
2. Axial compression force plus shear and moment caused by wind acting in conjunction with dead and live load. ($F. = 1.3$)
3. Axial tensile force plus shear and moment when tension occurs under a condition of full factored wind load combined with 75% of the factored dead load, and no live load. ($F. = 1.3$)

According to the AISC Specification, in tier buildings 100% of the axial compression force may be transmitted from one column section to the next by bearing, provided that both sections are milled. Partial penetration groove welds having no root opening may be used to join column flanges when the stress to be transferred will permit them.

When columns of the same nominal depth are spliced, full bearing is possible because the inside-of-flange dimension is the same for all weights. The weld or bolts and the splice material serve only to hold all parts securely in place. If the lower column is much deeper than the upper one, it is necessary to weld stiffeners on the inside of the lower column flange to provide an adequate bearing surface. Alternative solutions are to provide a bearing butt plate on the lower column or to develop the strength of fills fastened on the outside of the flanges of the upper column.

Horizontal shear forces are resisted by plates on both sides of the column webs extending across the joint of the upper and lower column sections. If a butt plate is used, shear is resisted by bolts connecting web angles to the butt plate. Web plates or angles also aid erection by holding the column sections in line during field welding.

Tension resulting from significant moments at column splices is transmitted by full penetration flange welds or by splice plates fillet welded or bolted to the flanges. For typical details see Ref. 9.

7.7 BRACING CONNECTIONS

Diagonal bracing is often laid out with its centerline intersecting the centerlines of girders and columns as for a pin connected truss. This arrangement usually permits the horizontal component of the bracing force to be transmitted into the girder flange and the vertical component into the column flange—a direct transfer into the logical resisting member without introducing a shear into the other. However, other considerations often cause deviations from this ideal arrangement. Welded girder-to-column connections, because of their simplicity of detail, facilitate the connecting of bracing.

Bracing connection details depend upon the type of member used for the bracing, i.e., rods, pairs of angles, H-section, or tubes like the pipe used in the design example. Gusseted connections consisting of plates and angles, or tees shop welded to the brace may be used. The high strength bolt is ideally suited for making the field connection because of its ability to pull up the draw in the brace. Tubes may be connected to gusset plates by slotting the tube, and fillet welding the tube to the plate, or full penetration butt welds joining tubes to end plates provide an excellent and simple connection.

In K-bracing two diagonals join one another at midspan of the girder. Research [10] has shown that a stronger connection is developed if the centerlines of the pipe braces intersect before reaching the girder centerline, i.e., have a negative eccentricity. This geometric arrangement causes a partial intersection of the pipes and a more direct balancing of the vertical components of the bracing forces.

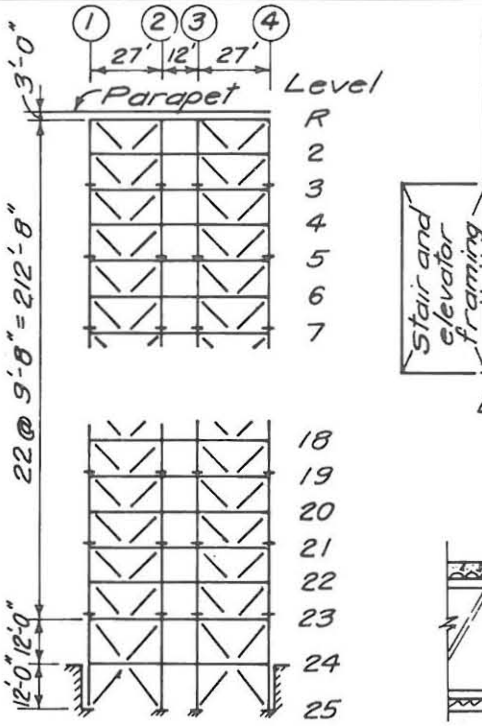
CHAPTER 8

Design Example

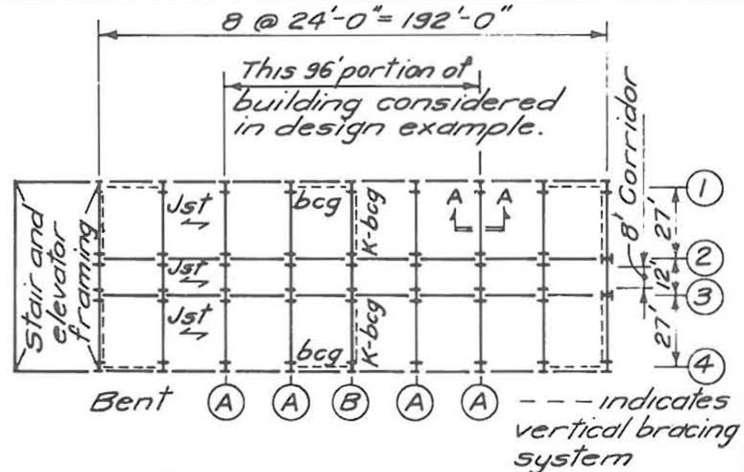
This chapter includes plastic design calculations for the braced multi-story building in Fig. 8.1. Chapters 4 to 7 describe the design steps which are indexed in Table 8.1. A design summary of main member sizes is given in Fig. 8.2.

DESIGN EXAMPLE APARTMENT HOUSE TABLE OF CONTENTS	TABLE 8.1

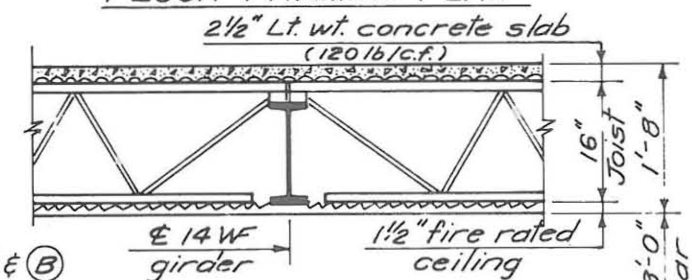
DESIGN EXAMPLE APARTMENT HOUSE PRELIMINARY DESIGN DATA	FIGURE 8.1



ELEVATION-BENTS (A) & (B)
K-bracing for Bent B only



FLOOR FRAMING PLAN



SECTION A-A at Exterior Girders

LOADS

Live load reduction per American Std. Bldg. Code A58.1-1955, Sect. 3.5

Floor loads

	Ext. bay	Int. bay
2½" Lt. wt. slab	25	25
Floor finish	1	1
Ceiling	5	5
Partitions	20	40 (1)
Joist	3	4
Mechanical	1	5
Dead load	55 psf	80 psf
Live load	40 psf	60 psf
Total load	95 psf	140 psf

Roof loads

Metal deck	4
Lt. wt. fill	22
Roofing	5
Insulation	2
Ceiling	5
Joist	2
Mechanical	5
Dead load	45 psf
Live load	30 psf
Total load	75 psf

(1) 2 corridor walls at 30 psf × 8'/12' = 40 psf

Exterior walls (average) 62 psf × 9.67' =	600 lb/ft
Parapet	250 lb/ft
2 interior partitions at K-braced bays	50 psf
Wind - full height	20 psf
DL - Column steel + fireproofing 210 plf × 9.67' =	2.0 kips

Load Factors -
Gravity F = 1.70
Combined F = 1.30

DESIGN EXAMPLE - PART I
SUPPORTED BENT A
ROOF GIRDERS

TABLE 8.2

Line	Item	Units	Operation	Bay Exterior	Bay Interior
1	Bay span	ft		27.0	12.0
2	Bent spacing	ft		24.0	24.0
3	Unit DL+LL on roof	psf		75	75
4	Est. column depth	ft		1.0	1.0
5	Clear span	ft	(1) - (4)	26.0	11.0
6	Roof load on girder	klf	(2) × (3)	1.80	1.80
7	Est. DL of girder	klf		0.03	0.02
8	Working load	klf	(6)+(7)	1.83	1.82
9	Factored load (F = 1.7)	klf	(8) × 1.7	3.11	3.09
10	Req'd. M_p	k-ft	$(9)\times(5)^2/16$	131.4	23.4
11	Req'd. Z (A36 steel)	in^3	(10) × 12/36	43.8	7.8
12	Section			14 WF 30 (1)	8B13
13	Provide Z	in^3		47.1	11.4

Note (1) Select 14 WF girder to maintain flush ceiling per Section A-A in Fig. 8.1

DESIGN EXAMPLE - PART I SUPPORTED BENT A FLOOR GIRDERS	TABLE 8.3

Line	Item	Units	Operation	Bay	
				Exterior	Interior
1	Bay span	ft		27.0	12.0
2	Bent spacing	ft		24.0	24.0
3	Unit DL on floor	psf		55	80
4	Unit LL on floor	psf		40	60
	<u>Live Load Reduction</u>				
5	Floor Area	sf	(1) × (2)	648	288
6	0.08 × (floor area)	pct		51.8	23.0
7	100 (D+L)/4.33 L	pct		54.8	53.8
8	Percent LL Reduction	pct	Min. (6) or (7)	51.8	23.0
9	Est. column depth	ft		1.0	1.0
10	Clear span	ft	(1) - (9)	26.0	11.0
11	Floor DL on girder	klf	(2) × (3)	1.32	1.92
12	Est. DL of girder	klf		0.04	0.02
13	Reduced LL on girder	klf	$(2)\times(4)\times\left[1-\frac{(8)}{100}\right]$	0.46	1.11
14	Working load	klf	(11)+(12)+(13)	1.82	3.05
15	Factored load (F = 1.7)	klf	(14 × 1.7)	3.09	5.19
16	Req'd. M_p	k-ft	$(15)\times(10)^2/16$	130.5	39.2
17	Req'd. Z (A36 steel)	in^3	(16) × 12/36	43.5	13.1
18	Section			14 WF 30 (1)	10B15
19	Provide Z	in^3		47.1	16.0

Note (1) Select 14 WF girder to maintain flush ceiling per Section A-A in Fig. 8.1

DESIGN EXAMPLE - PART I SUPPORTED BENT A COLUMN LOAD DATA, Working Loads (F = 1.0)	TABLE 8.4

Line	Item	Units	Operation	Column	
				Exterior	Interior
	<u>Tributary area per floor</u>				
1	From exterior bay (13.5 × 24)	sf		324	324
2	From interior bay (6.0 × 24)	sf		—	144
3	Total	sf	(1) + (2)	324	468
4	Unit roof load (DL + LL)	psf		75	75
	<u>Unit floor loads</u>				
5	Exterior bay - dead	psf		55	55
6	- live	↓		40	40
7	Interior bay - dead	↓		—	80
8	- live	↓		—	60
	<u>Loads below roof</u>				
9	DL + LL from roof	kips	(3) × (4)	24.3	35.1
10	Est. DL girder (@ 0.03 klf)	↓		0.4	0.6
11	Est. DL column	↓		2.0	2.0
12	DL parapet (@ 0.25 klf)	↓	0.25 × 24.0	6.0	—
13	Working load below roof	↓	Sum (9 to 12)	32.7	37.7
	<u>Loads per floor</u>				
14	DL from floor - Ext. bay	kips	(1) × (5)	17.8	17.8
15	- Int. bay	↓	(2) × (7)	—	11.5
16	DL girder (@ 0.03 klf)	↓		0.4	0.6
17	DL ext. wall (@ 0.60 klf)	↓	0.60 × 24.0	14.4	—
18	DL column	↓		2.0	2.0
19	Total DL per floor	↓	Sum (14 to 18)	34.6	31.9
20	LL from floor - Ext. bay	kips	(1) × (6)	13.0	13.0
21	- Int. bay	↓	(2) × (8)	—	8.6
22	Total LL per floor	↓	(20) + (21)	13.0	21.6
	<u>Live Load Reduction</u>				
23	Max. R = 100 (D + L)/4.33L < 60	pct	D = (19) L = (22)	~~84.6~~	(1) 57.2
		↓	Limit	60.0	60.0
24	0.08 (trib area) - Level 2	↓	0.08 × (3)	25.9	37.4
25	- Level 3	↓	2 × (24)	51.8	~~74.8~~
		↓	Limit Max. R		60.0
26	- Level 4 & below	↓	3 × (24)	~~77.7~~	
		↓	Limit Max. R	60.0	60.0
27	<u>Red. LL from floors</u> - below Level 2	kips	(22) × [1 − R/100]	9.6	13.5
28	- below Level 3	↓	2 × (22) × [1 − R/100]	12.5	17.3
29	- below Level 4	↓	3 × (22) × [1 − R/100]	15.6	25.9
30	<u>Red. LL increment</u> - Levels 5 to 24	↓	(22) × [1 − 0.60]	5.2	8.6

Note (1) Use 60.0, roof contributes dead load.

DESIGN EXAMPLE - PART I SUPPORTED BENT A COLUMN GRAVITY LOADS	TABLE 8.5

Level	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Exterior Columns					Interior Columns				
	DL kips	Red. LL kips	Working Load kips	WL×1.7 kips	WL×1.3 kips	DL kips	Red. LL kips	Working Load kips	WL×1.7 kips	WL×1.3 kips
Load Increment	34.6	5.2	39.8	67.7	51.7	31.9	8.6	40.5	68.9	52.7
R	23	10	33	56	43	24	14	38	65	49
2	58	20	78	133	101	56	28	84	143	109
3	92	23	115	196	150	88	31	119	202	155
4	127	26	153	260	199	120	40	160	272	208
5			193	328				201	341	
6			233	395				241	410	
7			272	463				282	479	
8			312	531				322	548	
9			352	599				363	617	
10			392	666				403	685	
11			432	734				444	754	
12			471	802				484	823	
13	Add load increment	Add load increment	511	869	Add load increment	Add load increment	Add load increment	525	892	Add load increment
14			551	937				565	961	
15			591	1005				606	1030	
16			631	1072				646	1099	
17			670	1140				687	1168	
18			710	1208				727	1237	
19			750	1276				768	1306	
20			790	1343				808	1374	
21			830	1411				849	1443	
22	750	120	870	1479	1130	694	195	889	1512	1157
23	(1) 785	125	910	1547	1183	726	204	930	1581	1209
24	(2) 824	130	954	1622	1240	759	212	971	1651	1262

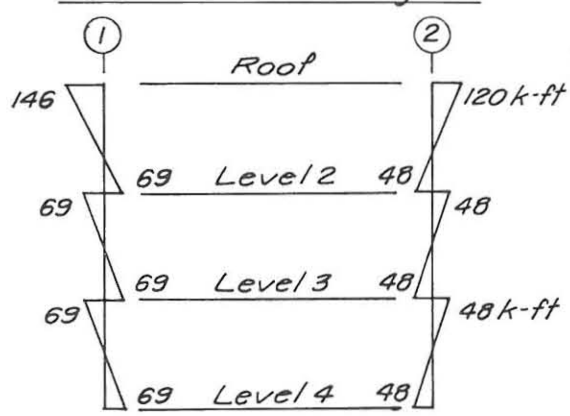
Note (1) DL increment below Level 23
Add DL column 0.21 klf × (12.0 − 9.67) = 0.5 kip

Note (2) DL increment below Level 24
Add DL column 0.5
Add DL exterior wall 14.4 (Tab. 8.4(17)) × $\frac{12.0 - 9.67}{9.67}$ = 3.5
Add 4.0 kips

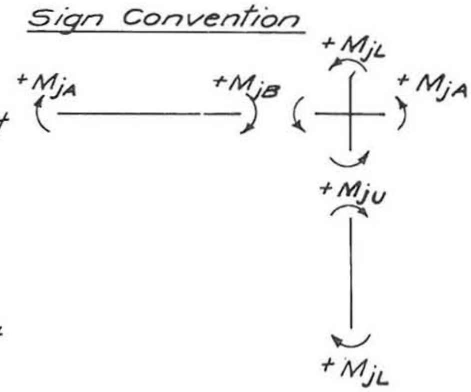
DESIGN EXAMPLE - PART I SUPPORTED BENT A COLUMN MOMENTS, Factored Gravity Load ($F = 1.7$)	TABLE 8.6

Line	Item	Units	Operation	Column Exterior	Column Interior
	Moments at roof				
1	Girder left: Req'd M_p	k-ft	Tab 8.2 (10)	—	131.4
2	Est. d_c/L_g		Tab 8.2 (4)/(5)	—	0.038
3	At ℄ col. $M_{jB} = M_p\,(1+4d_c/L_g)$	k-ft		—	+151.4
4	Girder right: Req'd M_p	k-ft	Tab 8.2 (10)	131.4	23.4
5	Est d_c/L_g		Tab 8.2 (4)/(5)	0.038	0.091
6	At ℄ col. $M_{jA} = M_p\,(1+4d_c/L_g)$	k-ft		−151.4	−31.9
7	Spandrel (6.0 k × 0.5 ft × 1.7)	k-ft	Tab 8.4 (12) × $\frac{d_c}{2}$ × 1.7	+ 5.1	—
8	Column moment at roof	k-ft	−1 [(3)+(6)+(7)]	+146.3	−119.5
	Moments at Levels 2 to 24				
9	Girder left: Req'd M_p	k-ft	Tab 8.3 (16)	—	130.5
10	Est. d_c/L_g		Tab 8.3 (9)/(10)	—	0.038
11	At ℄ col. $M_{jB} = M_p\,(1+4d_c/L_g)$	k-ft		—	+ 150.3
12	Girder right: Req'd M_p	k-ft	Tab 8.3 (16)	130.5	39.2
13	Est. d_c/L_g		Tab 8.3 (9)/(10)	0.038	0.091
14	At ℄ col. $M_{jA} = M_p\,(1+4d_c/L_g)$	k-ft		−150.3	− 53.5
15	Spandrel (14.4 k × 0.5 ft × 1.7)	k-ft	Tab 8.4 (17) × $\frac{d_c}{2}$ × 1.7	+ 12.2	—
16	Net girder moment on joint	k-ft	−1 [(11)+(14)+(15)]	+138.1	− 96.8
17	Column moment	k-ft	0.5 × (16)	+ 69.1	− 48.4

Column Moment Diagram



Sign Convention



DESIGN EXAMPLE - PART 1 SUPPORTED BENT A SELECT COLUMNS, Factored Gravity Load ($F = 1.7$)	TABLE 8.7

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Col Below Level	Req'd P kips Req'd M k-ft	2.1 M/d kips Est d ft	Req'd P_y kips	Steel	Trial Section	Prov P_y kips	P/P_y	Remarks $M_{pc} = 1.18(1 - \frac{P}{P_y}) \times M_p$
	Tab 8.5 (4) or (9) Tab 8.6 (17)	$2.1 \times \frac{(1)}{(2)}$	(1) + (2) or (1)×1.12 for F_y = 36ksi (1)×1.18 for F_y = 50ksi		DA-I	DA-I	(1)/(6)	Source or Operation
1 & 4 Lev 4	260 69	145 1.0	405	A36	12WF40	424	0.61	
1 & 4 Lev 8	531 69	145 1.0	676	A36	12WF79 ~~12WF58~~	836 614	0.64 0.87	M_{pc} = 40k-ft < 69 NG
1 & 4 Lev 12	802 69	145 1.0	947	A36	12WF92 ~~12WF85~~	974 899	0.82 0.89	M_{pc} = 50k-ft < 69 NG
1 & 4 Lev 16	1072 69	145 1.0	1217	A36	12WF120 ~~12WF106~~	1271 1123	0.84 0.95	M_{pc} = 29k-ft < 69 NG
1 & 4 Lev 20	1343 69	124 1.17	1504	A36	14WF142 ~~14WF136~~	1507 1439	0.89 0.94	M_{pc} = 52k-ft < 69 NG
1 & 4 Lev 24	1622 69	124 1.17	1817	A36	14WF176 14WF167	1862 1767	0.87 0.92	M_{pc} = 86k-ft > 69 (1)
2 & 3 Lev. 4	272 48	101 1.0	373	A36	12WF40	424	0.64	M_{pc} = 73k-ft > 48 OK
2 & 3 Lev 8	548 48	101 1.0	649	A36	12WF79 ~~12WF58~~	836 614	0.66 0.89	M_{pc} = 34k-ft < 48 NG
2 & 3 Lev 12	823 48	101 1.0	924	A36	12WF92 ~~12WF85~~	974 899	0.85 0.92	M_{pc} = 37k-ft < 48 NG
2 & 3 Lev 16	1099 48	101 1.0	1231	A36	12WF120 ~~12WF106~~	1271 1123	0.86 0.98	NG
2 & 3 Lev 20	1374 48	86 1.17	1539	A36	14WF150 14WF142	1587 1507	0.87 0.91	M_{pc} = 114k-ft > 48 (1)
2 & 3 Lev 24	1651 48	86 1.17	1849	A36	14WF176 14WF167	1862 1767	0.89 0.93	M_{pc} = 75k-ft > 48 (1)
1 & 4 Lev 16	1072 69	145 1.0	1265	A572	12WF92	1353	0.79	
1 & 4 Lev 20	1343 69	145 1.0	1585	A572	12WF120 12WF106	1766 1560	0.76 0.86	M_{pc} = 112k-ft > 69 (1)
1 & 4 Lev 24	1622 69	124 1.17	1914	A572	14WF136 14WF127	1999 1867	0.81 0.87	M_{pc} = 144k-ft > 69 (1)
2 & 3 Lev 16	1099 48	101 1.0	1297	A572	12WF92	1353	0.81	
2 & 3 Lev 20	1374 48	101 1.0	1621	A572	12WF120 12WF106	1766 1560	0.78 0.88	M_{pc} = 96k-ft > 48 (1)
2 & 3 Lev 24	1651 48	86 1.17	1948	A572	14WF136 14WF127	1999 1867	0.83 0.88	M_{pc} = 133k-ft > 48 (1)

Note (1) Check LTB
See Table 8.8

DESIGN EXAMPLE - PART I
SUPPORTED BENT A
EXTERIOR COLUMNS, Factored Gravity Load (F = 1.7)

TABLE 8.8

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Col	Req'd P kips	h ft	Trial Section	P_y kips	r_x in	P/P_y	h/r_x	Allow M/M_{pc}	Remarks Note (1)
Below Level	Req'd M k-ft	Ratio q	Steel	M_p k-ft	r_y in.	M_{pc}/M_p	h/r_y	Allow M k-ft	
	Tab 8.5(4)		DA-I	DA-I	DA-I	(1)/(4)	12×(2)/(5)	DA-III	} Source or
	Tab 8.6(17)			DA-I	DA-I	1.18 × (1−P/P_y)	12×(2)/(5)	(8)×(6)×(4)	} Operation
1 & 4	133	9.67	12WF40	424	5.13	0.314	23	1.00	
Lev 2	69	+1.0	A36	173	1.94	0.809	60	140	> 69 ok
1 & 4	56	9.67	A36	173		0.13 < 0.15	60	1.00	Say q = 0
Roof	146	+0.47	A36	173		1.0	60	173	> 146 ok
1 & 4	260	9.67	12WF40	424	5.13	0.613	23	1.00	
Lev 4	69	+1.0	A36	173	1.94	0.457	60	79	> 69 ok
1 & 4	531	9.67	12WF79	836	5.34	0.635	22	1.00	
Lev 8	69	+1.0	A36	358	3.05	0.431	38	154	> 69 ok
1 & 4	802	9.67	12WF92	974	5.40	0.823	21	1.00	
Lev 12	69	+1.0	A36	421	3.08	0.209	38	88	> 69 ok
1 & 4	1072	9.67	12WF120	1271	5.51	0.843	21	1.00	
Lev 16	69	+1.0	A36	560	3.13	0.185	37	104	> 69 ok
1 & 4	1343	9.67	14WF142	1507	6.32	0.891	18	1.00	
Lev 20	69	+1.0	A36	764	3.97	0.129	29	99	> 69 ok
1 & 4	1622	12.0	~~14WF167~~	1767	6.42	0.918	22	0.09	(LTB)
Lev 24	69	0	A36	909	4.01	0.097	36	8	< 69 NG
			14WF176	1862	6.45	0.871	22	0.56	(LTB)
			A36	964	4.02	0.152	36	82	> 69 OK
Alternate design using A572 steel						F_y = 50 ksi			$\sqrt{50/36} = 1.18$
1 & 4	1072	9.67	12WF92	1353	5.40	0.793	21×1.18 = 25	1.00	
Lev 16	69	+1.0	A572	584	3.08	0.244	38×1.18 = 45	142	> 69 OK
1 & 4	1343	9.67	~~12WF106~~	1560	5.46	0.861	21×1.18 = 25	0.48	(LTB)
Lev 20	69	+1.0	A572	681	3.11	0.164	37×1.18 = 44	54	< 69 NG
			12WF120	1766	5.51	0.760	21×1.18 = 25	1.0	
			A572	777	3.13	0.283	37×1.18 = 44	220	> 69 OK
1 & 4	1622	12.0	~~14WF127~~	1867	6.29	0.870	23×1.18 = 27	0.19	(LTB)
Lev 24	69	0	A572	941	3.76	0.153	38×1.18 = 45	27	< 69 NG
			14WF136	1999	6.31	0.811	23×1.18 = 27	0.59	(LTB)
			A572	1011	3.77	0.223	38×1.18 = 45	133	> 69 OK

Note (1) LTB indicates that allowable M/M_{pc} in col. (8) is controlled by lateral torsional buckling. DA indicates design aid.

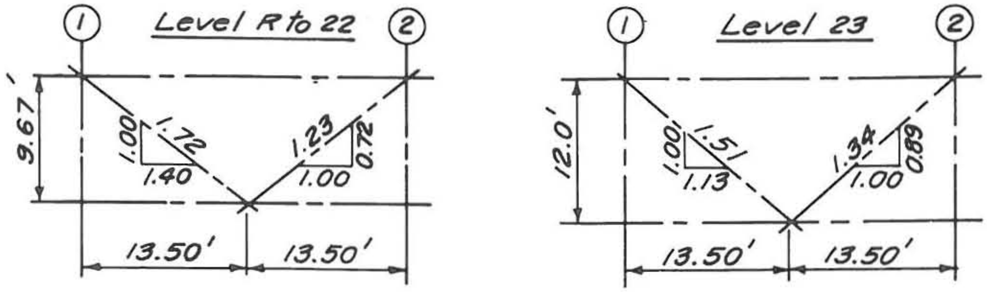
DESIGN EXAMPLE - PART I SUPPORTED BENT A INTERIOR COLUMNS, Factored Gravity Load (F = 1.7)	TABLE 8.8 CONT.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Col	Req'd P kips	h ft	Trial Section	P_y kips	r_x in.	P/P_y	h/r_x	Allow M/M_{pc}	Remarks Note (1)
Below Level	Req'd M k-ft	Ratio q	Steel	M_p k-ft	r_y in.	M_{pc}/M_p	h/r_y	Allow M k-ft	
	Tab 8.5 (9) Tab 8.6 (17)		DA-I	DA-I DA-I	DA-I DA-I	(1)/(4) $1.18 \times (1-P/P_y)$	$\frac{12\times(2)}{(5)}$ $\frac{12\times(2)}{(5)}$	DA-III (8)×(6)×(4)	} Source or } Operation
2&3 Lev 2 2&3 Roof	143 48 65 120	9.67 +1.0 9.67 +0.54	12WF40 A36 A36 A36	424 173 173 173	5.13 1.94 1.94 1.94	0.338 0.781 0.153 1.00	23 60 60 60	1.00 135 1.00 173	 > 48 ok Say q = 0 > 120 ok
2&3 Lev 4	272 48	9.67 +1.0	12WF40 A36	424 173	5.13 1.94	0.642 0.423	23 60	1.00 73	 > 48 ok
2&3 Lev 8	548 48	9.67 +1.0	12WF79 A36	836 358	5.34 3.05	0.655 0.407	22 38	1.00 146	 > 48 OK
2&3 Lev 12	823 48	9.67 +1.0	12WF92 A36	974 421	5.40 3.08	0.845 0.183	21 38	0.98 76	(LTB) > 48 OK
2&3 Lev 16	1099 48	9.67 +1.0	12WF120 A36	1271 559	5.51 3.13	0.862 0.163	21 37	0.87 79	(LTB) > 48 OK
2&3 Lev 20	1374 48	9.67 +1.0	14WF142 A36	1507 764	6.32 3.97	0.912 0.104	18 29	0.81 64	(LTB) > 48 OK
2&3 Lev 24	1651 48	12.0 0	~~14WF167~~ A36 14WF176 A36	1767 909 1862 964	6.42 4.01 6.45 4.02	0.935 0.077 0.887 0.133	22 36 22 36	0 0 0.44 56	(LTB) NG (LTB) > 48 OK
Alternate design using A572 steel				$F_y = 50$ ksi				$\sqrt{50/36} = 1.18$	
2&3 Lev 16	1099 48	9.67 +1.0	12WF92 A572	1353 584	5.40 3.08	0.812 0.222	21×1.18= 25 38×1.18= 45	1.00 130	 > 48 OK
2&3 Lev 20	1374 48	9.67 +1.0	~~12WF106~~ A572 12WF120 A572	1560 681 1624 715	5.46 3.11 5.51 3.13	0.881 0.140 0.778 0.262	21×1.18 = 25 37×1.18 = 44 21×1.18 = 25 37×1.18 = 44	0.18 17 1.0 204	(LTB) < 48 NG > 48 OK
2&3 Lev 24	1651 48	12.0 0	~~14WF127~~ A572 14WF136 A572	1867 941 1999 1011	6.29 3.76 6.31 3.77	0.884 0.137 0.826 0.205	23×1.18 = 27 38×1.18 = 45 23×1.18 = 27 38×1.18 = 45	0.04 5 0.51 106	(LTB) < 48 NG (LTB) > 48 OK

Note (1) LTB indicates that allowable M/M_{pc} in col. (8) is controlled by lateral torsional buckling. DA indicates design aid.

DESIGN EXAMPLE – PART 2 BRACED BENT B FLOOR GIRDERS, Factored Gravity Load (F=1.7)	TABLE 8.9

K-Bracing Geometry - Bent B



$h = 9.67'$ $0.5L = 13.50'$ $L_b = 16.60'$ $h = 12.00'$ $0.5L = 13.50'$ $L_b = 18.06'$

Line	Item	Units	Operation	Levels 2 to 22	Level 23	Level 24
	Loads					
1	Floor DL 55psf × 24ft	klf		1.32		1.32
2	Portion DL 50psf × h	↓		0.48		0.60
3	Girder DL			0.02		0.04
4	Red. floor LL 40psf × 24 × (1−0.259)		Note (3)	0.71		0.71
5	Working load			2.53		2.67
6	Factored load (F=1.7)		(5) × 1.7	4.30	4.30	4.54
					Note (1)	
	K-Brace Forces		$L_g = 26.0'$			
7	Vert. force in brace P_{bV}	kips	$(6) \times L_g/4$	28.0	14.8	−14.8
8	Horiz. " " " P_{bH}	↓	$(7) \times 0.5L/h$	39.2	16.7	−16.7
9	Resultant brace force P_b		$(7) \times L_b/h$	48.2	22.3	−22.3
10	Min. A_b for elastic brace	in^2	(9)/36	1.34	—	—
	Braced Girder					
11	Moment $M_g = Fw(0.5L_g)^2/16$	k-ft	$(6) \times [13.0]^2/16$	45.4	45.4	48.0
12	Axial force $P_g = P_{bH}$	kips	(8)	39.2		0
13	Est. depth d	ft		0.83		
14	$0.46\ P_g d$	k-ft	0.46×(12)×(13)	14.9		
15	Req'd $M_p = M_g + 0.46\ P_g d$	k-ft	(11) + (14)	60.3		
16	Req'd Z (A36 Steel)	in^3	(15) × 12/36	20.1		
17	Section		Note (2)	10B19	10B19	10B19
18	$0.5\ L_g/r_x$			38 < 40		
19	Web d/W			41.0 < 43		

Note (1) Split vertical reaction from girder between K-braces above and below Level 24.
Note (2) See Tab 8.17 for girders required for combined load.
Note (3) Percent LL Reduction = 0.08 × 13.5 ft × 24 ft = 25.9

DESIGN EXAMPLE - PART 2 BRACED BENT B COLUMN LOAD DATA	TABLE 8.10

Line	Item	Units	Operation	Column	
				Exterior	Interior
	Loads per Floor				
1	DL from floor - Ext. bay	kips	Tab 8.4 (14)	17.8	17.8
2	- Int. bay	↓	Tab 8.4 (15)	—	11.5
3	DL Girder (@ 0.03 klf)	↓		0.4	0.6
4	DL Column	↓		2.0	2.0
5	DL Ext. wall (@ 0.60 klf)	↓	0.60 × 24.0	14.4	—
6	DL K-brace partition (50psf × 9.67′)	↓	0.48 × 13.0	6.2	6.2
7	DL K-brace (Est. 0.02 klf × 16.6′)	↓		0.3	0.3
8	Total DL per floor	↓	Sum (1 to 7)	41.1	38.4
9	DL to pair of K-braces	↓	(1)+(3)+(6)+(7)	24.7	—
10	LL from floor - Ext. bay	↓	Tab 8.4 (20)	13.0	13.0
11	- Int. bay	↓	Tab 8.4 (21)	—	8.6
12	Total LL per floor	↓	(10) + (11)	13.0	21.6
	Loads below roof				
13	DL + LL below roof	↓	Tab 8.4 (13)	32.7	37.7
14	DL from K-brace	↓	0.5 × (9)	12.4	12.4
15	LL from K-brace	↓	0.5 × (10)	6.5	6.5
16	Working load below roof	↓	(13)+(14)+(15)	51.6	56.6
			Note (1)		
17	Red. LL from floors - below Level 2	↓	Tab 8.4 (27)	9.6	13.5
18	- below Level 3	↓	Tab 8.4 (28)	12.5	17.3
19	- below Level 4	↓	Tab 8.4 (29)	15.6	25.9
20	Red. LL increment - Levels 5 to 23	↓	Tab 8.4 (30)	5.2	8.6

Note (1) Use live load reduction as for Bent A

DESIGN EXAMPLE - PART 2 BRACED BENT B COLUMN GRAVITY LOAD	TABLE 8.11

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	
Level	Exterior Columns					Interior Columns					
	DL kips	Red LL kips	Working Load kips	WL×1.7 kips	WL×1.3 kips	DL kips	Red LL kips	Working Load kips	WL×1.7 kips	WL×1.3 kips	
	41.1	5.2	46.3	78.7	60.2	38.4	8.6	47.0	79.9	61.1	← Load Increment
R	36	16	52	88	68	37	20	57	97	74	
2	77	26	103	175	134	75	34	109	185	142	
3	118	29	147	250	191	114	37	151	257	196	
4	159	32	191	325	248	152	46	198	337	257	
5	Add load increment	Add load increment	237	404	308	Add load increment	Add load increment	245	417	318	
6			284	482	368			292	497	379	
7			330	561	429			339	577	440	
8			376	640	489			386	657	501	
9			423	719	549			433	737	563	
10			469	797	609			480	816	624	
11			515	876	669			527	896	685	
12			561	955	730			574	976	746	
13			608	1033	790			621	1056	807	
14			654	1112	850			668	1136	868	
15			700	1191	910			715	1216	929	
16			747	1269	970			762	1296	990	
17			793	1348	1031			809	1376	1051	
18			839	1427	1091			856	1456	1112	
19			886	1506	1151			903	1536	1174	
20			932	1584	1211			950	1615	1235	
21			978	1663	1271			997	1695	1296	
22	899	126	1025	1742	1332	843	201	1044	1775	1357	
23	(1)941	131	1072	1822	1394	(1)883	210	1093	1858	1421	
24	(2)975	134	1109	1885	1442	(2)910	216	1126	1914	1464	

Note (1) DL increment below Level 23

Add DL column 0.21 klf (12.0 − 9.67) = 0.5 kip

Add DL K-brace part'n. 0.05 ksf × $\frac{26.0}{4}$ × 2.33 = 0.8

Add 1.3 kip

Note (2) Load increment below Level 24

Line	Item	Unit	Ext. col.	Int. col.
1	DL floor	kips	8.9	20.4
2	DL girder	↓	0.2	0.4
3	DL column		2.5	2.5
4	DL ext. wall		17.9	—
5	DL k-brace partition		3.9	3.9
6	Total DL increment		33.4	27.2
7	Total LL increment		2.6	6.0

From Tab 8.4 (14,15)
17.8 × 0.5 = 8.9
11.5
20.4 kips (→ Line 1, Int. col.)

From Tab 8.4 (17)
14.4 × 12.0/9.67 = 17.9 kips (→ Line 4, Ext. col.)

From Tab. 8.4 (20, 21)
13.0 × 0.5 = 6.5
8.6
0.40 × 15.1 kips (→ Line 7, Int. col.)

DESIGN EXAMPLE - PART 2
BRACED BENT B
HORIZONTAL FORCES, Combined Load (F = 1.3)

TABLE 8.12

Level	(1) Wind Load to Bent B (F=1.0) kips	(2) Wind Shear Bent B (F=1.0) kips	(3) Factored Wind Shear ΣH (F=1.3) kips	(4) Factored Gravity Loads Bent A (F=1.3) kips	(5) Factored Gravity Loads Bent B (F=1.3) kips	(6) Total Gravity Load ΣP (F=1.3) kips	(7) PΔ Shear =0.004ΣP (F=1.3) kips	(8) Total Shear P_H=ΣH+ 0.004ΣP (F=1.3) kips
	20 psf × 96 ft × Avg h	Sum (1)	(2)×1.3	Tab 8.5 [(5)+(10)] ×2	Tab 8.11 [(5)+(10)] ×2	3×(4) +(5)	0.004×(6)	(3)+(7)
Load Increment		18.6	24.2	208.8	242.6	869.0	3.48	27.7
R	15.0	15.0	19.5	184	284	836	3.3	22.8
2	18.6	33.6	43.7	420	552	1812	7.2	50.9
3	↓	52.2	67.9	610	774	2604	10.4	78.3
4		70.8	92.1	814	1010	3452	13.8	106
5		89.4	116					
6		108	141	Add load increment	Add load increment	Add load increment	Add load increment	Add load increment
7		127	165					
8		145	189					
9		164	213					
10		182	237					
11		201	262					
12		220	286					
13		238	310					
14		257	334					
15		275	358					
16		294	383					
17		313	407					
18		331	431					
19		350	455					
20		368	479					
21	↓	387	504					
22	18.6	406	528	4574	5377	19094	76.4	604
23	20.8	426	554	4784	5630	19982	79.9	634
24	11.5	438	569	5004	5812	20824	83.3	652

DESIGN EXAMPLE - PART 2
BRACED BENT B
GIRDER AND COLUMN AXIAL FORCES, Combined Load (F=1.3)

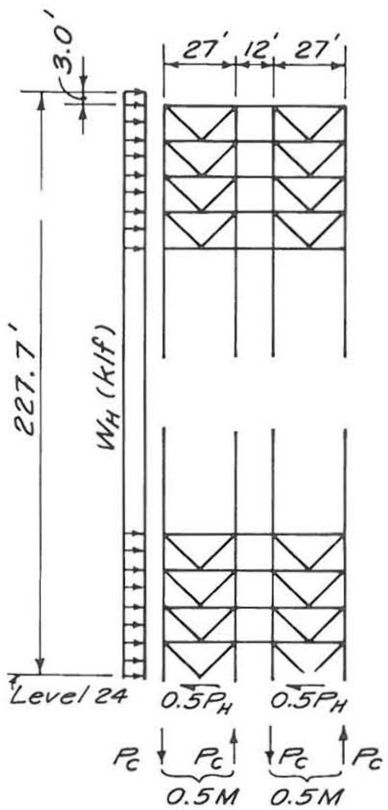
TABLE 8.13

Level	(1) P_H Wind + PΔ kips	(2) P_{bH} Wind + PΔ kips	(3) P_{bH} Gravity kips	(4) Girder Ax. Load P_g kips	(5) h/0.5L	(6) P_{bV} Wind + PΔ kips	(7) Column Ax. Load Wind + PΔ kips	(8) Total Column Load: Exterior Col. kips	(9) Total Column Load: Interior Col. kips
	Tab 8.12 (8)	0.25×(1)	Tab 8.9(8) x 1.3/1.7	(2)+(3)	k-brace geom.	(2)×(5)	Sum (6)	(7) + Tab 8.11(5)	(7) + Tab 8.11(10)
Load increment	27.7	6.9		6.9		5.0			
R	22.8	5.7	30.0 →	35.7	0.72	4.1	4.1	72	78
2	50.9	12.7	↓	42.7	↓	9.1	13.2	147	155
3	78.3	19.6		49.6		14.1	27.3	218	223
4	106	26.5		56.5		19.1	46.4	294	303
5	Add load increment	Add load increment		63.4		24.1	70.5	379	389
6				70.3		29.1	99.6	468	479
7				77.2		34.1	134	563	574
8				84.1		39.1	173	662	674
9				91.0		44.1	217	766	780
10				97.9		49.1	266	875	890
11				105		54.1	320	989	1005
12				112		59.1	379	1109	1125
13				119		64.1	443	1233	1250
14				126		69.1	512	1362	1380
15				132		74.1	587	1497	1516
16				139		79.1	666	1636	1656
17				146		84.1	750	1781	1801
18				153		89.1	839	1930	1951
19				160		94.1	933	2084	2107
20				167		99.1	1032	2243	2267
21				174		104	1136	2407	2432
22	604	151	30.0	181	0.72	109	1245	2577	2602
23	634	159	12.8	172	0.89	142	1387	2781	2808
24	652	163	12.8	18 (1)	0.89	145	(2)	2829	2851

Note (1) Axial force in girder at Level 24 taken as the increase in horizontal shear (652 - 634 = 18 kips) at Level 24. Loads above Level 24 do not cause axial load in this girder.

Note (2) Axial load in columns below Level 24, due to wind + PΔ, is same as above Level 24. Base of column carries bracing force P_{bV} = 145 kips to foundation.

DESIGN EXAMPLE – PART 2 BRACED BENT B COLUMN AXIAL FORCES, WIND AND PΔ	TABLE 8.14



Horizontal Load EW - Bent B

① For factored wind
$F_{W_w} = 20\,psf \times 96\,ft \times 1.3 = 2.50\ klf$

② For PΔ: Approximate overturning moment due to PΔ by applying horizontal force of
$0.004\ \Sigma\delta P = 3.48$ kips [Tab 8.12 (7)]
at each level. For convenience, replace this force by
$W_\Delta = 3.48/9.67 = 0.36\ klf$

③ For wind + PΔ
$W_H = 2.50 + 0.36 = 2.86\ klf$

Forces at Level 24

④ Horizontal shear
$P_H = 2.86 \times 227.7 = 651$ kips   Check OK
vs. 652 kips in Tab 8.12 (8)

⑤ Overturning moment
$M = \frac{1}{2} \times 2.86 \times (227.7)^2 = 74,142$ kip-ft

⑥ Axial load in columns (truss chords)
$P_C = \frac{74,142}{2 \times 27.0} = 1373$ kips   Check OK
vs. 1387 kips in Tab 8.13 (7)

DESIGN EXAMPLE - PART 2 BRACED BENT B K-BRACING FORCES, Combined Load (F = 1.3)	TABLE 8.15

	(1)	(2)	(3)	(4)	(5)	(6)	
Level	$L_b/0.5L$	P_b Wind+PΔ kips	P_b Gravity (D+L) kips	P_b Gravity (D) kips	P_b Max Tension kips	P_b Max Compres kips	
	k-brace geom.	(1) × Tab. 8.13(2)	Tab 8.9 (9) × 1.3/1.7	$(3)\times\left(\frac{D}{D+L}\right)$	(3) + (2)	(4) − (2)	Source or operation
	—	8.5	—	Tab. 8.9 $\frac{(1)+(2)+(3)}{(5)}$	8.5	−8.5	Load Increment
R	1.23	7.0	36.9	26.5	+ 43.9	+ 19.5	
2	↓	15.6	↓	↓	+ 52.5	+ 10.9	
3	↓	24.1	↓	↓	+ 61.0	+ 2.4	
4	↓	32.6	↓	↓	+ 69.5	− 6.1	
5	↓	41.1	↓	↓	+ 78.0	− 14.6	
6	↓	49.6	↓	↓	+ 86.5	− 23.1	
7	↓	58.1	↓	↓	+ 95.0	− 31.6	
8	↓	66.6	↓	↓	+104	− 40.1	
9	↓	75.1	↓	↓	+112	− 48.6	
10	↓	83.6	↓	↓	+121	− 57.1	
11	↓	92.1	↓	↓	+129	− 65.6	
12	↓	101	↓	↓	+138	− 74.1	
13	↓	109	↓	↓	+146	− 82.6	
14	↓	118	↓	↓	+155	− 91.1	
15	↓	126	↓	↓	+163	− 99.6	
16	↓	135	↓	↓	+172	−108	
17	↓	143	↓	↓	+180	−117	
18	↓	152	↓	↓	+189	−125	
19	↓	160	↓	↓	+197	−134	
20	↓	169	↓	↓	+206	−142	
21	↓	177	↓	↓	+214	−151	
22	1.23	186	36.9	26.5	+223	−159	
23	1.34	213	17.1	12.6	+230	−200	
24	1.34	218	−17.1	−12.6	+205 (1)	−235 (1)	

Note (1) Below Level 24, for inverted K-brace
P_b (Max tension) = (4) + (2)
P_b (Max compres) = (3) − (2)

DESIGN EXAMPLE – PART 2 BRACED BENT B COLUMNS AND CHORD ROTATION	TABLE 8.16

① **Estimate column area at base** $A_{c,\,base} = 1.2 \times \frac{P_{c,\,base}}{R_{c,\,roof}} \times \frac{h_t}{EL}$

Assume chord rotation below roof	$R_{c,\,roof}$	= 0.0035 radians
Axial load at base (wind + PΔ)	$P_{c,\,base}$	= 1387 kips [Tab 8.13(7)]
Total height of frame	h_t	= 236.7 ft
Distance between columns	L	= 27.0 ft
Modulus	E	= 29,000 ksi
Column area at base	$A_{c,\,base}$	= 144 in²

② **Select columns above**

$$A_x = \left(1 - \frac{h_x}{h_t}\right) A_{c,\,base} \qquad h_x = \text{height above base}$$

Level	(1) h_x ft	(2) h_x/h_t	(3) A_x in²	(4) Section	(5) A_c in²	(6) P_c kips	(7) $\alpha_a \times 10^5$	(8) $R_c \times 10^5 = \Sigma \alpha_a$	(9) $R_w \times 10^5$ Allow.
		(1)/236.7	[1.0-(2)] × 144	A36 steel		Tab 8.13 (7)	Note (1)	Sum (7) from base	400 - (8)
R									
2	227.1	0.959	5.9	12WF40	11.77	4.1	1	353	47
						13.2	3	352	48
3									
4	207.8	0.878	17.6	12WF79	23.22	27.3	3	349	51
5						46.4	5	346	54
6	188.4	0.796	29.4	12WF106	31.19	70.5	6	341	59
7						99.6	8	335	65
8	169.1	0.714	41.2	14WF142	41.85	134	8	327	73
9						173	10	319	81
10	149.7	0.632	53.0	14WF184	54.07	217	10	309	91
11						266	12	299	101
12	130.4	0.551	64.7	14WF219	64.36	320	12	287	113
13						379	15	275	125
14	111.0	0.469	76.5	14WF264	77.63	443	14	260	140
15						512	16	246	154
16	91.7	0.387	88.3	14WF314	92.30	587	16	230	170
17						666	18	214	186
18	72.4	0.306	99.9	14WF342	100.6	750	18	196	204
19						839	21	178	222
20	53.0	0.224	111.7	14WF398	117.0	933	20	157	243
21						1032	22	137	263
22	33.7	0.142	123.6	14WF426	125.3	1136	22	115	285
23						1245	25	93	307
24			144	14WF426	125.3	1387	34	68	332
						1387	34	0	400

← Source or operation

Note:

$$\alpha_a = \frac{P_c}{A_c} \times \frac{2h}{EL} = \frac{P_c}{A_c} \times \begin{cases} 2.47 \times 10^{-5} & \text{for } h = 9.67 \text{ ft above Levels 2 to 23} \\ 3.06 \times 10^{-5} & \text{for } h = 12.0 \text{ ft above Levels 24 \& 25} \end{cases}$$

DESIGN EXAMPLE - PART 2 BRACED BENT B GIRDERS, Combined Load ($F = 1.3$)	TABLE 8.17

A 36 Steel

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Level	1.3W klf	M_g k-ft	$0.46 P_g d$ k-ft	Req'd M_p k-ft	Section	$0.5 L_g / r_x$	Web d/w	Remarks
	$0.5 L_g$ ft	P_g kips	Est. d ft	Req'd Z in^3	P_y kips	P_g / P_y	Allow. d/w	
	1.3 × Tab 8.9(5)	$(1a)\times(1b)^2 / 16$	0.46 × (2b) × (3b)	(2a) + (3a)		$(1b)\times 12 / r_x$	Note (3)	Source or Operation
		Tab 8.13 (4)		$(4a)\times \frac{12}{36}$	A × 36	(2b)/(5b)	70-100 (6b) Min 43	
4	3.29	34.8	21.6	56.4	(1) 10B19	35<40	OK	OK
	13.0	56.5	0.83	18.8				
8	3.29	34.8	32.1	66.9	10B19	35<40	OK	OK
	13.0	84.1	0.83	22.3				
12	3.29	34.8	42.8	77.6	10WF25	37<40	OK	OK
	13.0	112	0.83	25.9				
16	3.29	34.8	53.1	87.9	10WF25	37<40	OK	OK
	13.0	139	0.83	29.3				
20	3.29	34.8	63.8	98.6	10WF29	36<40	OK	OK
	13.0	167	0.83	32.9				
22	3.29	34.8	69.1	103.9	10WF29	36<40	OK	OK
	13.0	181	0.83	34.6				
24	3.47	36.7	6.9	43.6	10WF29	36<40	OK	OK
	13.0	18	0.83	14.5				

Roof Girders - Bent B - Exterior Bays

	1.3 × Tab 8.2(8)							
R	2.38	100.6	19.2	119.8	(1) 14WF30	54.5>40	51.3	Check bm-col
	26.0	35.7	1.17	39.9	317	0.11	59	for L/r > 40
								Note (4)

Floor Girders - Bent B - Interior Bay

	1.3 × Tab 8.3(14)							
2 to	3.97	30.0	5.3	35.3	(1) 10B15	33<40	43.5	OK
22	11.0	(2) 13.9	0.83	11.8	158	0.09	61	

Note (1) Girder required for gravity load with $F = 1.7$

Note (2) Axial load taken as 1/2 of the increment of horizontal shear on Bent B. From Tab. 8.12 (8) $P_g = 0.5 \times 27.7 = 13.9$ kips.

Note (3) OK indicates web $d/w < 43$

Note (4) Check laterally loaded beam - column using Eq. 5.5

$P = 35.7$ kips $M_g = 100.6$ kip ft. $\frac{P}{P_{ox}} + \frac{(1 - 0.4 P/P_{ex})}{(1 - P/P_{ex})} \times \frac{M_g}{M_p} \leq 1.0$

$P_{ox} = 269$ kips $M_p = 141.3$ kip ft.

$P_{ex} = 849$ kips

$0.132 + 1.026 \times 0.712 = 0.862 < 1.0$ OK

DESIGN EXAMPLE - PART 2 BRACED BENT B WEB ROTATION, Combined Load (F = 1.3)	TABLE 8.18

① For girders $L_g = 26.0$ ft $E = 29{,}000$ ksi

$$R_g = \frac{P_g}{A_g} \times \frac{0.5\,L_g}{Eh} = \frac{P_g}{A_g} \times \begin{cases} 4.64 \times 10^{-5} \text{ for } h = 9.67 \text{ ft. above Levels 2 to 23} \\ 3.74 \times 10^{-5} \text{ for } h = 12.0 \text{ ft. above Level 24} \end{cases}$$

② For K-bracing $L = 27.0$ ft

K-brace geometry

$$R_b = \frac{P_b}{A_b} \times \frac{2L_b^2}{EhL} = \frac{P_b}{A_b} \times \begin{cases} 7.28 \times 10^{-5} \text{ for } L_b = 16.60 \text{ ft. above Levels 2 to 23} \\ 6.94 \times 10^{-5} \text{ for } L_b = 18.06 \text{ ft. above Levels 24 \& 25} \end{cases}$$

Assume $R_b = R_w - R_g$ and find Min. A_b required to limit rotation.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Level	Exterior Girder	A_g in^2	P_g Wind + PΔ kips	$R_g \times 10^5$	$R_w - R_g$ $\times 10^5$	P_b Wind + PΔ kips	Min A_b to limit rotation in^2
Source or Operation	Tab 8.17 (5)		Tab 8.13 (2)	[(3)/(2)] × const.	Tab 8.16 (9) −(4)	Tab 8.15 (2)	[(6)/(5)] × const.
R	14 WF 30			11	36	7.0	1.42
2	10 B 19	5.61	12.7	16	32	15.6	3.55
3	↓	↓	19.6	22	29	24.1	6.05
4			26.5	28	26	32.6	9.13
5			33.4	33	26	41.1	11.5
6			40.3	39	26	49.6	13.9
7			47.2	45	28	58.1	15.1
8	10 B 19	5.61	54.1	39	42	66.6	11.5
9	10 WF 25	7.35	61.0	43	48	75.1	11.4
10	↓	↓	67.9	47	54	83.6	11.3
11			74.8	52	61	92.1	11.0
12			81.7	56	69	101	10.7
13			88.6	60	80	109	9.92
14			95.5	64	90	118	9.54
15			102	69	101	126	9.08
16	10 WF 25	7.35	109	63	123	135	7.99
17	10 WF 29	8.53	116	67	137	143	7.60
18	↓	↓	123	71	151	152	7.33
19			130	75	168	160	6.93
20			137	78	185	169	6.65
21			144	82	203	177	6.35
22			151	86	221	186	6.13
23			158	8	324	213	4.56
24	10 WF 29	8.53	18 (1)	0	400	218	3.78

Note (1) See note (1) in Tab. 8.13

DESIGN EXAMPLE - PART 2 BRACED BENT B K-BRACING, Combined Load (F = 1.3)	TABLE 8.19

Use weldable pipe with $F_y = 36$ ksi

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Line	Brace below Levels	Pipe Size in	Area A_b in^2	r_b in	$\frac{L_b}{r_b}$	Buckling Stress F_{cr} ksi	Allow. Compress. $A_b F_{cr}$ kips	Allow. Tension $A_b F_y$ kips	Remarks
			Min A_b in^2	Net L_b ft			Max. Compress. kips	Max. Tension kips	
		AISC Manual pg. 1-72			12×(4b) / (4a)	1.7×F_a AISC Tab. 1-36	(3) × (6)	(3) × 36	
		—	Tab 8.18(7)	Note (1)	—	—	Tab 8.15 (6)	Tab 8.15 (5)	
1	R to 3	5φ E.S.	6.11 6.05	1.84 15.3	100	22.1	-135 (+2.4)	+220 +61.0	OK
2	4 to 7	5φ D.E.S.	11.34 15.1	1.72 15.3	107	20.5	-232 -31.6	+408 +95.0	OK for Max. T & C A_b < Min. A_b Say OK
3	8 to 11	5φ D.E.S.	11.34 11.5	1.72 15.3	107	20.5	-232 -65.6	+408 +129	OK for Max. T & C $A_b \approx$ Min. A_b OK
4	12 to 15	5φ D.E.S.	11.34 10.7	1.72 15.3	107	20.5	-232 -99.6	+408 +163	OK
5	16 to 19	6φ E.S.	8.40 7.99	2.20 15.3	84	25.3	-213 -134	+302 +197	OK
6	20 to 22	6φ E.S.	8.40 6.65	2.20 15.3	84	25.3	-213 -159	+302 +223	OK
7	23 & 24	~~6φ E.S.~~	8.40 4.56	2.20 16.8	92	23.7	-199 -235	+302 +230	NG for Max. C
8	do	6φ D.E.S.	15.64 4.56	2.06 16.8	98	22.5	-352 -235	+563 +230	OK

Note (1) To find buckling stress for compression brace use Net L_b = Total L_b − 1.3 ft. to allow for depth of 10 WF girder and 12 WF column. From K-brace geometry; below Levels R to 22, Net L_b = 16.6 − 1.3 = 15.3 ft, below Levels 23 & 24, Net L_b = 18.1 − 1.3 = 16.8 ft.

DESIGN EXAMPLE - PART 2
BRACED BENT B
STORY ROTATION AND DRIFT

TABLE 8.20

Level	(1) Column $R_c \times 10^5$ F=1.3	(2) Girder $R_g \times 10^5$ F=1.3	(3) Bracing P_b Wind+PΔ kips F=1.3	(4) Bracing A_b in²	(5) Bracing $R_b \times 10^5$ F=1.3	(6) Total Rotn. $R \times 10^5$ F=1.3	(7) WL Rotn. $R \times 10^5$ F=1.0	(8) Story Drift $\Delta = Ph$ ft. F=1.0	(9) Total Drift Δ_t ft. F=1.0
	Tab 8.16 (8)	Tab 8.18 (4)	Tab 8.15 (2)	Tab 8.19 (3)	(3)/(4) × const. Note (1)	(1)+(2)+(5)	(6)/1.3	(7)×h	Sum(8) from base (2)
R	353	11	7.0	6.11	8	372	286	0.028	0.658
2	352	16	15.6	↓	19	387	298	0.029	0.630
3	349	22	24.1	6.11	29	400	308	0.030	0.601
4	346	28	32.6	11.34	21	395	304	0.029	0.571
5	341	33	41.1	↓	26	400	308	0.030	0.542
6	335	39	49.6		32	406	312	0.030	0.512
7	327	45	58.1		37	409	315	0.030	0.482
8	319	39	66.6		43	401	308	0.030	0.452
9	309	43	75.1		48	400	308	0.030	0.422
10	299	47	83.6		54	400	308	0.030	0.392
11	287	52	92.1		59	398	306	0.030	0.362
12	275	56	101		65	396	305	0.029	0.332
13	260	60	109		70	390	300	0.029	0.303
14	246	64	118		76	386	297	0.029	0.274
15	230	69	126	11.34	81	380	292	0.028	0.245
16	214	63	135	8.40	117	394	303	0.029	0.217
17	196	67	143	↓	124	387	298	0.029	0.188
18	178	71	152		132	381	293	0.028	0.159
19	157	75	160		139	371	285	0.028	0.131
20	137	78	169		146	361	278	0.027	0.103
21	115	82	177		153	350	269	0.026	0.076
22	93	86	186	8.40	161	340	262	0.025	0.050
23	68	8	213	15.64	95	171	132	0.016	0.025
24	0	0	218	15.64	97	97	75	0.009	0.009

Note (1) See item ② in Tab. 8.18 for "constant."

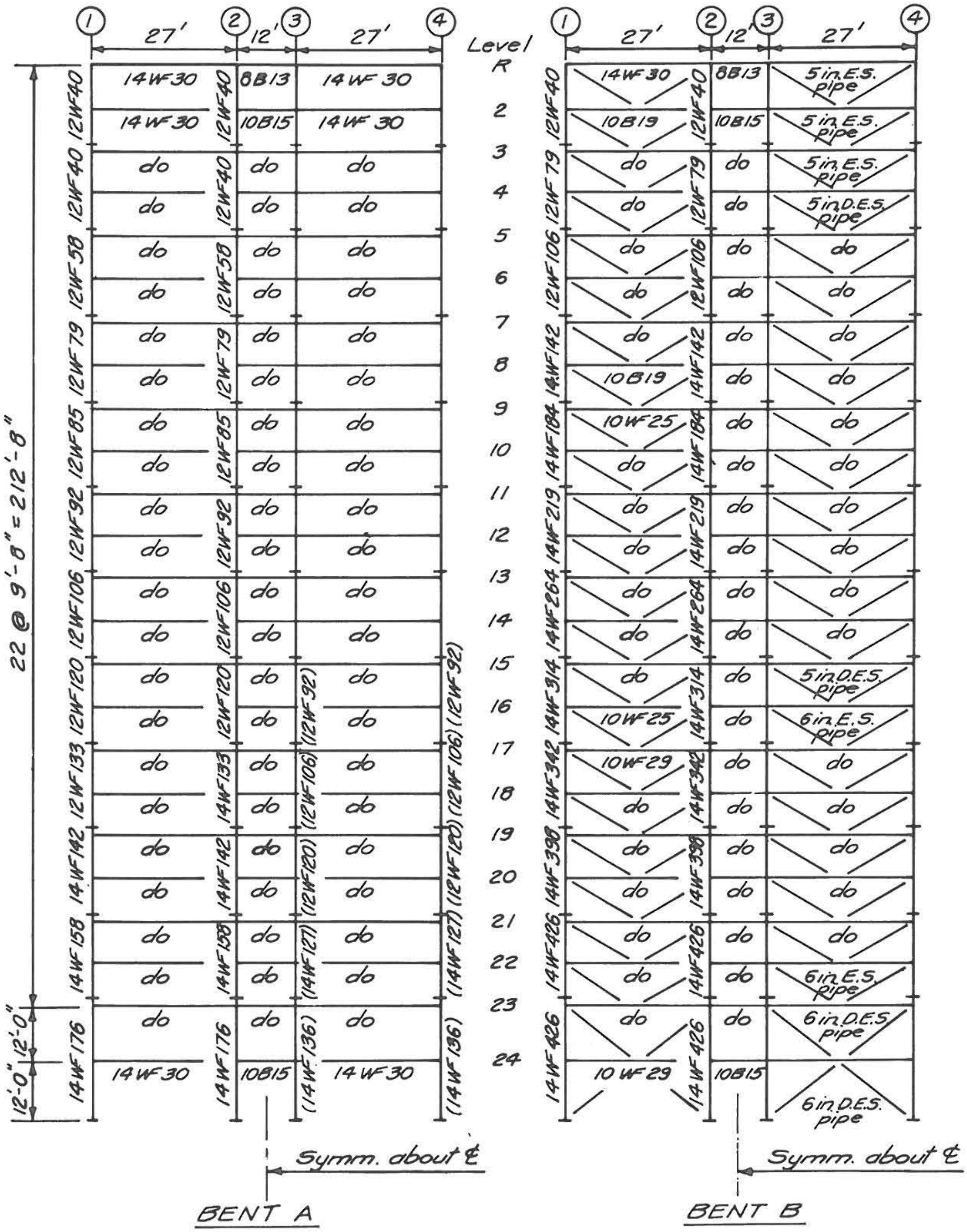
Note (2) Working load drift index $\frac{\Delta_t}{h_t} = \frac{0.658}{236.7} = 0.0028$ incl. PΔ effects.

Adjust drift index to eliminate PΔ effects.

From Tab. 8.14 $\frac{w_H}{F w_w} = 1.14$

Adjusted drift index $= \frac{0.658}{236.7 \times 1.14} = \underline{0.0024} < 0.0025$ OK

DESIGN EXAMPLE - SUMMARY BENTS A AND B SIZES OF MEMBERS	FIGURE 8.2



All steel A36 except columns in Bent A shown thus (12WF120) which are alternate sizes in A572 (F_y = 50 ksi) steel.

DESIGN EXAMPLE - PART 3 BRACED BENT B COLUMN MOMENTS	TABLE 8.21

Girder End Moments, M_e

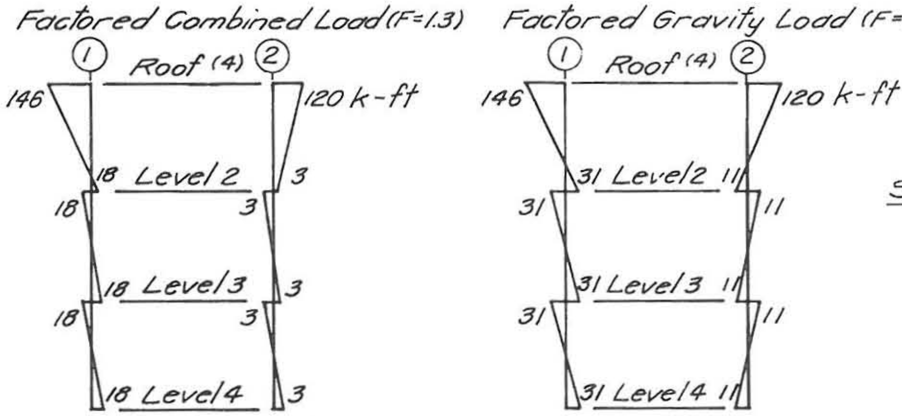
$$M_e = \frac{Fw}{12 \text{ or } 16} L_g^2 \; ; \; \text{use} \begin{cases} 12 - \text{Elastic moment} \\ 16 - \text{Plastic moment} \end{cases}$$

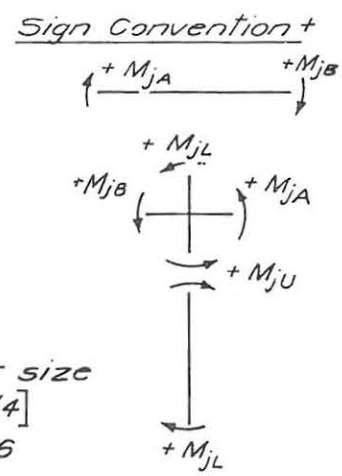
Line	Item	Units	Exterior Bay	Interior Bay
1	L_g	ft	13.0 [(1)]	11.0
2	w	k/f	2.53 [Tab. 8.9(5)]	3.05 [Tab. 8.3(14)]
3	M_e - factored gravity load (F=1.7)	k-ft	$\frac{FwL_g^2}{12}$ = 60.5	$\frac{FwL_g^2}{16}$ = 39.2 [(2)]
4	M_e - factored combined load (F=1.3)	k-ft	$\frac{FwL_g^2}{16}$ = 34.7 [(2)]	$\frac{FwL_g^2}{12}$ = 40.0

Line	Item	Units	Operation	F = 1.3		F = 1.7	
				Ext. Col.	Int. Col.	Ext. Col.	Int. Col.
	Column Moments Levels 2 to 22						
5	Girder end moment M_e	k-ft	Same as (3) or (4)	34.7	40.0	60.5	39.2
6	Moment from shear (Note 3)	k-ft	$\frac{FwL_g d_c}{4}$; d_c = 1.0	10.7	10.9	14.0	14.3
7	Moment at Col. ℄	k-ft	(5) + (6)	45.4	50.9	74.5	53.5
8	Girder left at Col. ℄, M_{jB}	k-ft		—	45.4	—	74.5
9	Girder right at Col. ℄, M_{jA}	k-ft		-45.4	-50.9	-74.5	-53.5
10	Spandrel Moments (12.2 × F/1.7)	k-ft	Tab. 8.6(15) × F/1.7	9.4	—	12.2	—
11	Net girder moment on joint	k-ft	-1[(8)+(9)+(10)]	36.0	5.4	62.3	-21.0
12	Column moment	k-ft	(11) × 0.5	18.0	2.7	31.2	-10.5

Column Moment Diagrams

Factored Combined Load (F=1.3) — Factored Gravity Load (F=1.7)





Notes:

(1) 0.5 L_g because of K-brace - see Tab. 8.9

(2) Loading condition that controlled the girder size

(3) Girder moment at column ℄ = $[M_e + FwL_g d_c/4]$

(4) Roof moments same as Bent A - see Tab. 8.6

DESIGN EXAMPLE – PART 3 BRACED BENT B COLUMN CHECK	TABLES 8.22 8.23

Below Level	(1) Exterior Columns, F = 1.7, P kips Tab. 8.11(4)	(2) Exterior Columns, F = 1.7, M kip-ft. Tab. 8.21	(3) Exterior Columns, F = 1.3, P kips Tab. 8.13(8)	(4) Exterior Columns, F = 1.3, M kip-ft. Tab. 8.21	(5) Interior Columns, F = 1.7, P kips Tab. 8.11(9)	(6) Interior Columns, F = 1.7, M kip-ft. Tab. 8.21	(7) Interior Columns, F = 1.3, P kips Tab. 8.13(5)	(8) Interior Columns, F = 1.3, M kip-ft. Tab. 8.21
Roof	88 *	146	72	146	97 *	120	78	120
2	175 *	31	147	18	185 *	11	155	3
8	640	31	662 *	18	657	11	674 *	3
16	1269	31	1636 *	18	1296	11	1656 *	3
24	1885	31	2829 *	18	1914	11	2851 *	3

* Indicates controlling condition used in Tab. 8.23

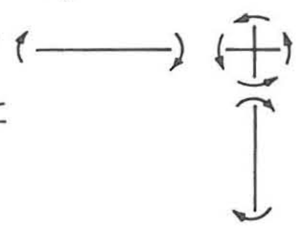
Table 8.23 Check Columns – Bent B

Col. / Below Level	(1) Req'd P kips / Req'd M k-ft.	(2) h ft. / Ratio q	(3) Trial Section / Steel	(4) P_y kips / M_p k-ft.	(5) r_x in. / r_y in.	(6) P/P_y / M_{pc}/M_p	(7) h/r_x / h/r_y	(8) Allow M/M_{pc} / Allow M k-ft.	(9) Remarks
	* Value Tab. 8.22		Tab. 8.16(4)	DA-I	DA-I	(1)/(4)	12×(2) / (5)	DA-III	Source or Operation
	* Value Tab. 8.22			DA-I	DA-I	1.18 × (1−P/P_y)	12×(2) / (5)	(8)×(6)×(4)	
Exterior Columns									
1 & 4	175	9.67	12WF40	424	5.13	0.413	23	1.00	
Lev. 2	31	+1.0	A36	173	1.94	0.693	60	120	> 31 OK
1 & 4	88	9.67	A36	173	1.94	0.208	60	1.00	Say q = 0
Roof	146	+0.21	A36	173	1.94	0.935	60	162	> 146 OK
1 & 4	662	9.67	14WF142	1507	6.32	0.439	18	1.00	
Lev. 8	18	+1.0	A36	765	3.97	0.662	29	506	> 18 OK
1 & 4	1636	9.67	14WF314	3323	6.90	0.492	16	1.00	
Lev. 16	18	+1.0	A36	1835	4.20	0.599	28	1099	> 18 OK
1 & 4	2829	12.0	14WF426	4509	7.26	0.627	20	1.00	
Lev. 24	18	0	A36	2608	4.34	0.440	33	1148	> 18 OK
Interior Columns									
2 & 3	185	9.67	12WF40	424	5.13	0.436	23	1.00	
Lev. 2	11	+1.0	A36	173	1.94	0.666	60	115	> 11 OK
2 & 3	97	9.67	A36	173	1.94	0.229	60	1.00	Say q = 0
Roof	120	+0.09	A36	173	1.94	0.910	60	157	> 120 OK
2 & 3	674	9.67	14WF142	1507	6.32	0.447	18	1.00	
Lev. 8	3	+1.0	A36	765	3.97	0.653	29	499	> 3 OK
2 & 3	1656	9.67	14WF314	3323	6.90	0.498	16	1.00	
Lev. 16	3	+1.0	A36	1835	4.20	0.592	28	1086	> 3 OK
2 & 3	2851	12.0	14WF426	4509	7.26	0.632	20	1.00	
Lev 24	3	0	A36	2608	4.34	0.434	33	1132	> 3 OK

DESIGN EXAMPLE - PART 3 BENTS A AND B COLUMN CHECK, Checkerboard Load	TABLE 8.24

Line	Item	Units	Operation	Bent A				Bent B			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
			Bay →	Levels U,L Int.	Levels U,L Ext.	Level C Int.	Level C Ext.	Levels U,L Int.	Levels U,L Ext.	Level C Int.	Level C Ext.
			Loading →	Full	Dead	Dead	Full	Full	Dead	Dead	Full
1	Fw	klf	Tab. 8.3(15) or 8.9(6); F=1.7	5.19			3.09	5.19			4.30
2	Fw_d	klf	F = 1.7 × Tab. 8.3 or 8.9		2.31	3.30			3.09	3.30	
3	L_g	ft	Tab. 8.3(10) or 8.21(1)	11.0	26.0	11.0	26.0	11.0	13.0	11.0	13.0
4	d_c	ft	Tab. 8.3(9) or 8.21(6)	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
5	M_p or M_e	k-ft	Tab. 8.3(16) or Tab. 8.21(3)	-39.2	+130.5	-39.2	+130.5	-39.2	+60.5	-39.2	+60.5
6	M_d	k-ft	Eq. 6.2; $\frac{1}{12}(2)\times(3)^2$		+130.2	-33.3			+43.5	-33.3	
7	M_j Full	k-ft	Eq. 6.1; $(5)+\frac{1}{4}(1)(3)(4)$					-53.4			+74.5
8	M_j Dead	k-ft	Eq. 6.1; $(6)+\frac{1}{4}(2)(3)(4)$	Compare (5) and (6)					+53.6	-42.4	
9	Net Gird. Mom.	k-ft	Eq. 6.1; (7) + (8)	$M_d \sim M_p$ ∴ plastic				+0.2		+32.1	
10	Col. Mom.	k-ft	− 0.5 × (9)	hinges form under				−0.1		−16.1	
11	q	—	(10) Level U ÷ (10) Level C	factored dead load				~0			

Sign Convention +



Bent A - Plastic hinges form under factored dead load ∴ checkerboard loading is the same as full gravity loading. All columns OK - See Tab. 8.8

Bent B - Exterior Columns

From Tab. 8.23 (6), all $P/P_y \leq 0.90$
Tab. 8.23 (7), all $h/r_x \leq 25$ } ∴ Bending strength OK

All P/P_y and h/r_y (Tab. 8.23) fall below the curve in Fig. 6.2 ∴ LTB OK

All exterior columns OK for checkerboard loading

Interior Columns $q = 0$

From Tab. 8.23 (6), all $P/P_y < 0.90$
Tab. 8.23 (7), all $h/r_x < 25$ } ∴ Bending strength same as full loading Tab 8.23 (8)

All P/P_y and h/r_y (Tab. 8.23) fall below the curve in Fig. 6.2 } ∴ LTB OK

All Allow M, Tab. 8.23 (8) > 16.1 k-ft

All interior columns OK for checkerboard loading

DESIGN EXAMPLE - PART 3 BENTS A AND B GIRDER DEFLECTION, Working Live Load	TABLE 8.25

Satisfy Eq. 6.8 $\delta/L_g = \frac{5}{384}\frac{W_\ell L_g^3}{EI} \leq \frac{1}{360}$

For E = 29,000 ksi, $\delta/L_g \leq 6.47 \times 10^{-5} \frac{W_\ell L_g^3}{I}$ where $I = in^4$, W_ℓ = klf, L_g = ft

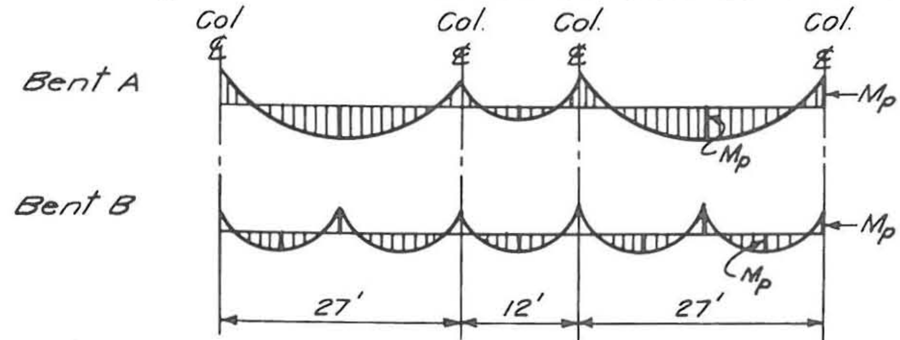
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Member	Location	RED. W_ℓ	L_g	I	L_g^3	$\delta/L_g \times 10^5$	Remarks
		klf	ft	in^4	ft^3	—	
		Tab 8.3(13) or 8.9(5)	Tab 8.3(10)	Handbook	$(4)^3$	$6.47 \times \frac{(3)(6)}{(5)}$	
Bent A							
14WF30	Floor/Ext.	0.46	26.0	289.6	17580	181	< 278 OK
10B15	Floor/Int.	1.11	11.0	68.8	1331	139	< 278 OK
14WF30	Roof/Ext.	0.72	26.0	289.6	17580	282	≈ 278 Say OK
8B13	Roof/Int.	0.72	11.0	39.5	1331	157	< 278 OK
Bent B (1)							
10B19	Floor/Ext.	0.71	13.0	96.2	2197	105	< 278 OK Note (2)

Notes:

(1) Roof and interior bay girders same as Bent A
(2) Lightest floor girder ∴ other members will be OK

DESIGN EXAMPLE - PART 3 BENTS A AND B GIRDER LATERAL BRACING	TABLE 8.26

Moment Diagrams - drawn on tension side - at mechanism load



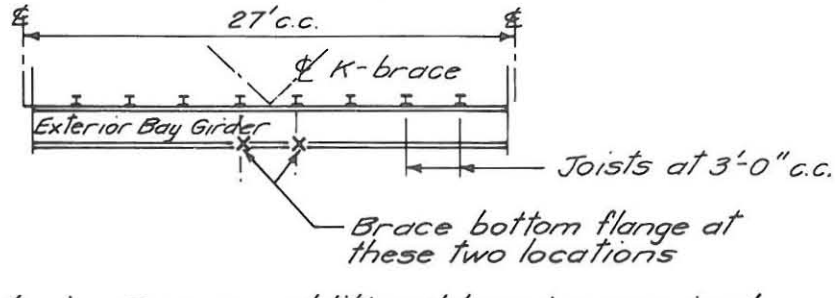
Tentative floor joist spacing : Ext. Bay - 3 ft. o.c.
Int. Bay - 2 ft. o.c.

Joists are attached to the top flange of girders

(1) Section	(2) L_g ft	(3) r_y in	(4) $\frac{L_g \times 12}{r_y}$	(5) Max. Bracing Spacing[(1)] Center	(6) Ends	(7) Remarks
Bent A						
8B13	11.0	0.83	159	$65 r_y$ = 54 in.	$65 r_y$ = 54 in.	> Joist spac.
10B15	11.0	0.80	165	$65 r_y$ = 52 in.	$65 r_y$ = 52 in	> ∴ top flg.
14WF30	26.0	1.40	223	$38 r_y$ = 53 in.	$65 r_y$ = 91 in.	> OK
Bent B.[(2)]						
10B19	13.0	0.86	181	$65 r_y$ = 56 in.	$65 r_y$ = 56 in.	> Joist spac.
10WF25	13.0	1.31	119	$65 r_y$ = 85 in.	$65 r_y$ = 85 in.	> ∴ top flg.
10WF29	13.0	1.34	116	$65 r_y$ = 87 in.	$65 r_y$ = 87 in.	> OK

Bent A - No additional bracing required

Bent B - Compression in bottom flange at midspan of exterior bay. Provide bottom chord joist extensions at the two locations shown.



Interior Bay - no additional bracing required

Note :

(1) From Fig. 6.4.
(2) Interior bay same as Bent A

DESIGN EXAMPLE - PART 3 BENTS A AND B GIRDER SHEAR, UPLIFT AT FOOTINGS	TABLES 8.27 8.28

Eq. 6.12 ≤ Eq. 3.3

$V_{max} \leq V_{u\,allow}$

$1.7 \frac{wL_g}{2} \leq 0.55 F_y wd = 20 wd$ for A 36 steel

(1) Section	(2) d in	(3) w in	(4) Vu kips	(5) 1.7w klf	(6) L_g ft.	(7) Vmax kips	(8) Remarks
Bent A							
8B13	8.00	0.230	36.4	3.09	11.0	17.0	< 36.4 OK
10B15	10.00	0.230	45.5	5.19	11.0	28.6	< 45.5 OK
14WF30	13.86	0.270	74.1	3.11	26.0	40.5	< 74.1 OK
Bent B (1)							
10B19 (2)	10.25	0.250	50.8	4.30	13.0	28.0	< 50.8 OK

Notes: (1) Interior bay same as Bent A
(2) Lightest section ∴ others are OK also

Table 8.28 - Uplift at Footings - Bent B - Working Load F=1.0

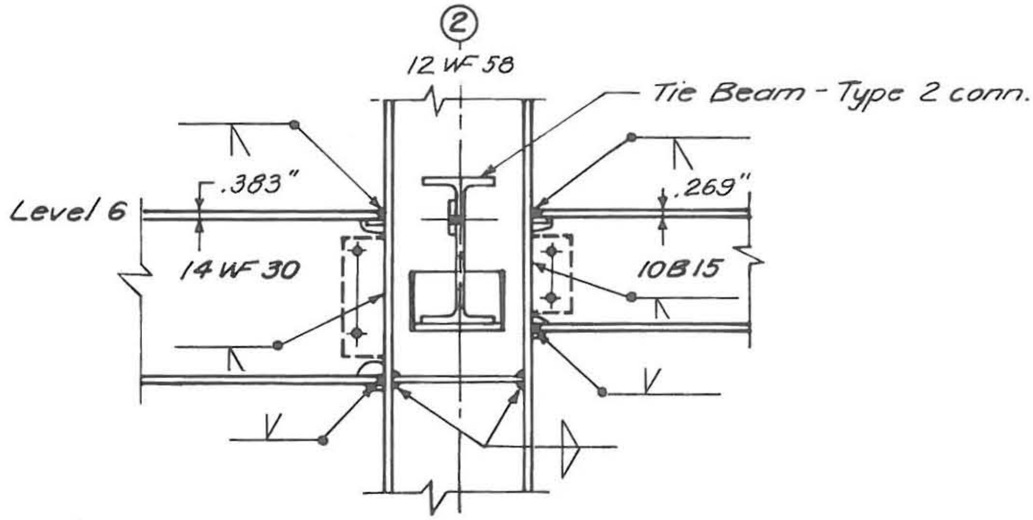
1 Col. load, wind + PΔ (Tab. 8.13 w/ F = 1.3) = 1387 kips
2 P_{bv} from k-brace [Tab. 8.13 (6) w/F = 1.3] = 145
3 Upward Force at F = 1.3 = 1532
at F = 1.0 (3)÷1.3 = 1178 kips

Uplift = Upward force - Dead gravity load

Exterior columns = 1178 - 975 = 203 kips uplift
Interior columns = 1178 - 910 = 268 kips uplift

Dead gravity load Tab. 8.11 (1,6)

DESIGN EXAMPLE - PART 4 TYPICAL CONNECTIONS GIRDER TO COLUMN - BENT A All welded	EXAMPLE 1



a. A.I.S.C. Type 1 connection

b. Weld girder flanges and web for full depth to develop the full plastic moment and the factored shear. Research shows M_p developed despite loss of section at cope holes.

c. Shear plate shop welded to column carries erection bolts and serves as backup for web weld.

d. Check column web crippling at 14 WF 30 compression flange

$$W_c\,(t_g + 5k)\,F_{yc} \geqq A_f\,F_{yg}$$

$$0.36\,(0.38 + 5 \times 1.25)\,36 \text{ vs. } 6.73 \times 0.38 \times 36$$

$$86 \text{ kips} < 92 \text{ kips}$$

Design force for determining stiffener area and welds is -

$$92 - 86 = 6 \text{ kips}$$

In this case use a nominal thickness for stiffener equal to 14 WF 30 flange. Say 3/8". Web crippling at compression flange of 10B15 not critical by inspection.

e. Check column flange bending at 14 WF 30 tension flange

$$t_c \geqq 0.4\sqrt{A_f \frac{F_{yg}}{F_{yc}}}$$

$$0.64 \text{ Vs. } 0.4\sqrt{6.73 \times 0.38}$$

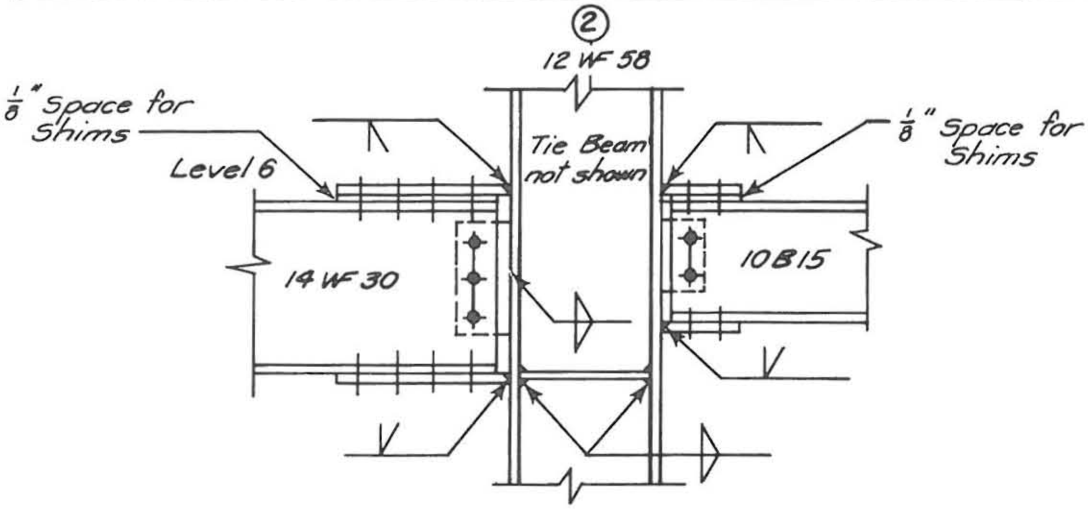
$$0.64 = 0.64 \quad \text{OK}$$

No stiffener req'd.

f. E 70 electrodes. Ultimate strength of fillet weld is $1.67 \times 15.8 \times .707 \times \frac{1}{16} = 1.16 = 1.16$ kips per inch per $\frac{1}{16}$ weld size.

Weld size	Stress k/in
3/16	3.5
1/4	4.6
5/16	5.8
3/8	7.0
1/2	9.3

DESIGN EXAMPLE - PART 4 TYPICAL CONNECTIONS GIRDER TO COLUMN - BENT A Field bolted	EXAMPLE 2



a. A.I.S.C. Type I connection.

b. Assume M_p of girder developed by flange plates and factored shear carried by web plate. Research shows M_p of beam can be developed at net section through first row of holes; friction-type bolted connection effectively reinforces net section.

c. E 70 electrodes - See Example I

d. Friction-type bolted connection. Ultimate single shear strength of 7/8" dia. A325 bolt is

$$1.67 \times 15 \times 0.6 = 15 \text{ kips}$$

e. Determine flange plate and bolts for 14 WF 30

$$M_p \text{ furnished} = 141.3 \text{ k-ft}$$

$$C = T = \frac{M_p}{d_g} = \frac{141.3 \times 12}{13.86} = 122 \text{ kips}$$

Plate size - assume 7" width

$$t(7-2\times1)36 = 122$$

$$t = 0.68" \quad \text{Use } \tfrac{11}{16}"$$

$$\text{No. of bolts} = \frac{122}{15} = 8.1$$

Use 8 - 7/8" A 325

f. Determine web connection

$$V = \frac{1.7 \times 1.82 \times 26}{2} = 40.2 \text{ kips} \qquad \text{See Tab. 8.3 Line 14}$$

$$\text{No. of bolts} = \frac{40.2}{15} = 2.7 \quad \text{Use } 3 - \tfrac{7}{8}" \text{ A325}$$

Plate size - assume 8½" long

$$t \times 8.5 \times \frac{36}{\sqrt{3}} = 40.2$$

$$t = .23 \quad \text{Use } \tfrac{1}{4}"$$

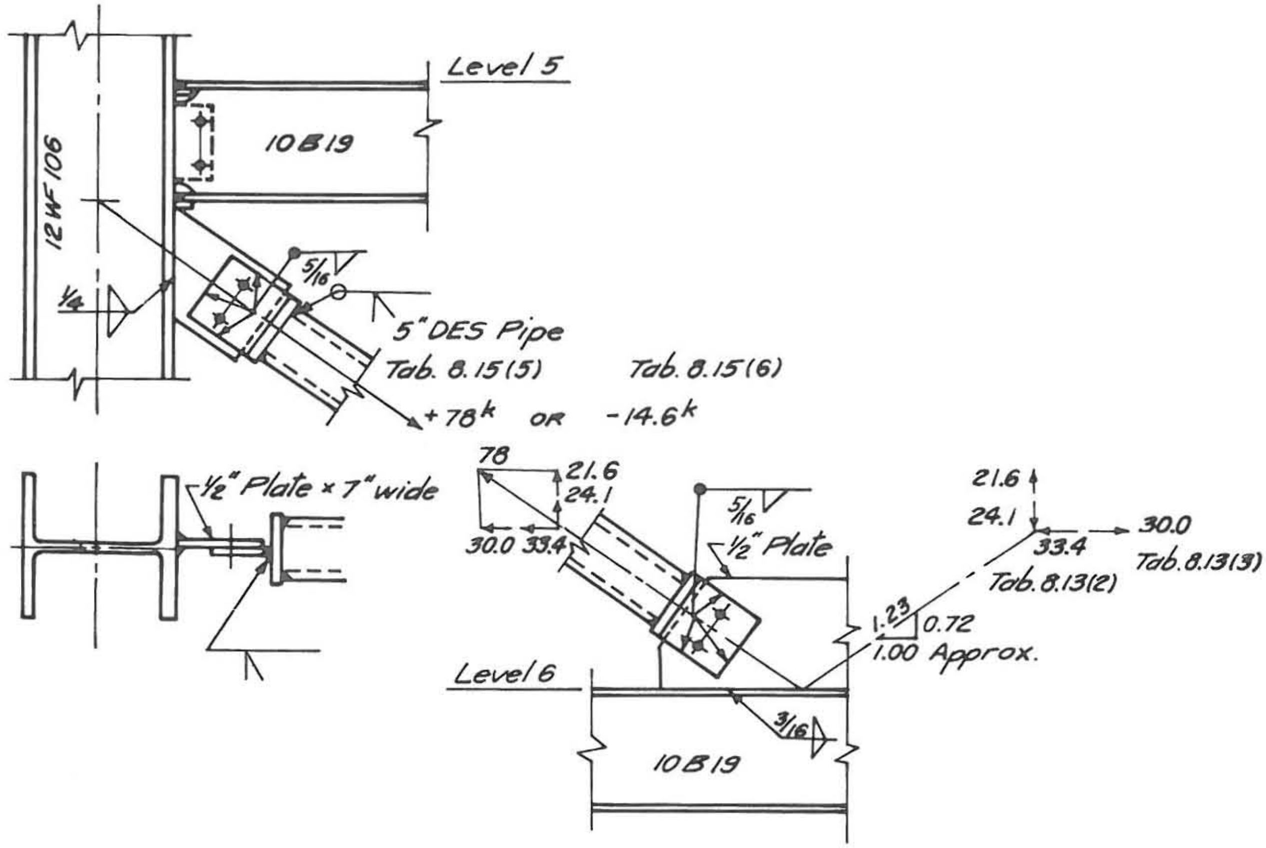
Shop fillet weld to column

$$\frac{40.2}{2 \times 8.5} = 2.4 \text{ k/in.}$$

Use ¼" fillet weld both sides
Min. size for column flange

g. Stiffeners from previous example OK

DESIGN EXAMPLE - PART 4 TYPICAL CONNECTIONS BRACING CONNECTIONS - BENT B	EXAMPLE 3



a. Welded T Flange 6 × 6 × ¾" (same as pipe thickness)

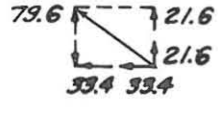
Web $A = \frac{P}{F_y} = \frac{78}{36} = 2.16$ - Use 6" × ½" = 3.0 sq. in. for stiffness

b. E 70 electrodes (See Example 1)

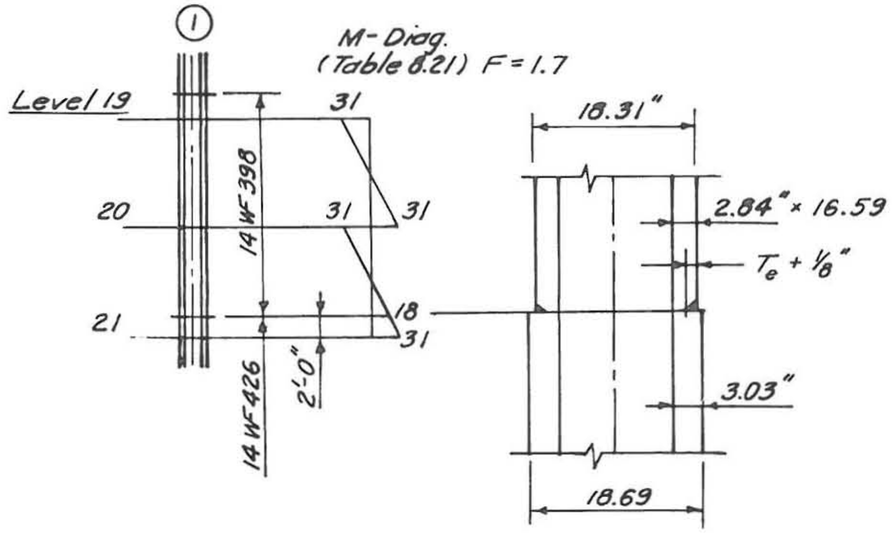
c. Field weld T to gusset plate
(5+6+5) × 5.8 = 93 kips > 78 kips Use 5/16"

d. Shop weld gusset plate to column
2 × 8.6 × 4.6 = 75 kips > 78 kips Use ¼"

e. Shop weld gusset plate to 10B19 at midspan
2 × 24 × 3.5 = 168 kips > 79.6 Use 3/16"



DESIGN EXAMPLE – PART 4 TYPICAL CONNECTIONS COLUMN SPLICE – BENT B	EXAMPLE 4



Condition 1.

Axial compression – D.L. + L.L.; F = 1.7	– 1584 k	Tab. 8.11(4)
Moment in column at splice	18 k-ft	M-Diag. above

Condition 2

Axial compression – D.L. + L.L.; F = 1.3	– 1211 k	Tab. 8.11(5)
Wind + PΔ; F = 1.3	– 1032 k	Tab. 8.13(7)
	– 2243 k	Tab. 8.13(8)
Moment in column at splice	10.5 k-ft	Tab. 8.21 F = 1.3
Shear and moment due to wind	0	Braced frame

Condition 3

Axial tension – Wind; F = 1.3	+ 1032	Tab. 8.13(7)
75% factored D.L. .75 × 1.3 × 816.8	– 796	Tab. 8.11(1)
	+ 236	

Design for – 2243 k, + 236 k, moment negl.

Use partial penetration bevel groove weld.

$$\text{Min. } T_e = \sqrt{\frac{T}{6}} = \sqrt{\frac{2.84}{6}} = 0.685$$

T_e to carry + 236 k below

$$\frac{236}{36} = 2 \times 16.59 \times T_e$$

$T_e = 0.20$ Use 7/8" weld $T_e + 1/8$"

References

1. Lehigh University
"Plastic Design of Multistory Frames." Lecture Notes Vols. 1, 2. Bethlehem, Pa., Aug. 1965.

2. American Iron and Steel Institute
"Design of High Rise Steel Framed Buildings." Continuing Education Program, New York, N.Y.

3. American Institute of Steel Construction
Steel Construction Manual 6th Ed. AISC New York, N.Y., 1963.

4. U.S.A. Standard Institute
"Minimum Design Loads in Buildings and Other Structures." American Standard Building Code A58.1-1955.

5. S.C. Goel and G.V. Berg
"Inelastic Earthquake Response of Tall Steel Frames." ASCE Conference, Seattle Wash. 1967. Conference Preprint 503.

6. T.V. Galambos
"Lateral Support for Tier Building Frames," AISC Eng. Journal, Vol. 1, p. 16, Jan. 1964.

7. Le-Wu Lu and H. Kamalvand
"Ultimate Strength of Laterally Loaded Columns." Fritz Engineering Laboratory Report 273.52, Lehigh University, November 1966.

8. N. M. Newmark
"Numerical Procedures for Computing Deflections, Moments & Buckling Loads." Trans. ASCE Vol. 108, p. 1161, 1943.

9. O. W. Blodgett
"Design of Welded Structures." The James F. Lincoln Arc Welding Foundation, Cleveland, Ohio, 1966.

10. J.G. Bouwkamp
"Concept of Tubular–Joint Design." Proceedings ASCE, Vol. 90, ST2, p. 3864, 1964.

Design Aid I

PROPERTIES OF BEAM-COLUMNS

Section	A	r_x	r_y	Z	A36		A572 $F_y = 50$		A441		
					P_y	M_p	P_y	M_p	F_y	P_y	M_p
	in^2	in	in	in^3	kips	k-ft	kips	k-ft	ksi	kips	k-ft
14WF426	125.3	7.26	4.34	863.3	4509	2608	6265	3622	42	5263	3042
14WF398	117.0	7.17	4.31	803.0	4211	2409	5850	3346	42	4914	2811
14WF370	108.8	7.08	4.27	737.3	3916	2212	5440	3072	42	4570	2580
14WF342	100.6	6.99	4.24	637.0	3621	2019	5030	2804	42	4225	2355
14WF320	94.12	6.63	4.17	592.2	3388	1777	4706	2468	42	3953	2073
14WF314	92.30	6.90	4.20	611.5	3323	1835	4625	2547	42	3885	2139
14WF287	84.37	6.81	4.17	551.6	3037	1655	4218	2298	42	3543	1930
14WF264	77.63	6.74	4.14	502.4	2795	1507	3882	2093	42	3261	1758
14WF246	72.33	6.68	4.12	464.5	2604	1394	3616	1935	42	3037	1625
14WF237	69.69	6.65	4.11	445.4	2509	1336	3484	1856	42	2926	1559
14WF228	67.06	6.62	4.10	427.2	2414	1282	3353	1780	42	2816	1495
14WF219	64.36	6.59	4.08	408.0	2317	1224	3218	1700	42	2703	1428
14WF211	62.07	6.56	4.07	391.7	2235	1175	3104	1632	46	2855	1502
14WF202	59.39	6.54	4.06	373.6	2138	1121	2970	1556	46	2732	1432
14WF193	56.73	6.51	4.05	355.1	2042	1065	2836	1480	46	2609	1362
14WF184	54.07	6.49	4.04	337.5	1947	1013	2704	1406	46	2488	1294
14WF176	51.73	6.45	4.02	321.3	1862	964	2586	1339	46	2379	1232
14WF167	49.09	6.42	4.01	302.9	1767	909	2454	1262	46	2258	1161
14WF158	46.47	6.40	4.00	286.3	1673	859	2324	1193	46	2138	1098
14WF150	44.08	6.37	3.99	270.2	1587	811	2204	1125	46	2028	1035
14WF142	41.85	6.32	3.97	254.8	1507	765			46		
14WF136	39.98	6.31	3.77	242.7	1439	728	1999	1011	50	1999	1011
14WF127	37.33	6.29	3.76	225.9	1344	678	1867	941	50	1867	941
14WF119	34.99	6.26	3.75	210.9	1260	633			50		
14WF111	32.65	6.23	3.73	196.0	1175	588			50		
14WF84	24.71	6.13	3.02	145.4	890	436			50		
14WF78	22.94	6.09	3.00	134.0	826	402			50		
14WF74	21.76	6.05	2.48	125.6	783	377	1088	523	50	1088	523
14WF68	20.00	6.02	2.46	114.8	720	344	1000	478	50	1000	478
14WF61	17.94	5.98	2.45	102.4	646	307			50		
14WF53	15.59	5.90	1.92	87.1	561	261	780*	363	50	780*	363
14WF48	14.11	5.86	1.91	78.5	508	236	706*	327	50	706*	327
14WF43	12.65	5.82	1.89	69.7	455*	209			50		

Note: Values of P_y and M_p are shown for compact sections only.

F_y = 36 ksi; $\frac{b}{t} \leqslant 17.4$, $\frac{d}{w} \leqslant 43$ $\qquad$ F_y = 50 ksi; $\frac{b}{t} \leqslant 14.8$, $\frac{d}{w} \leqslant 36$

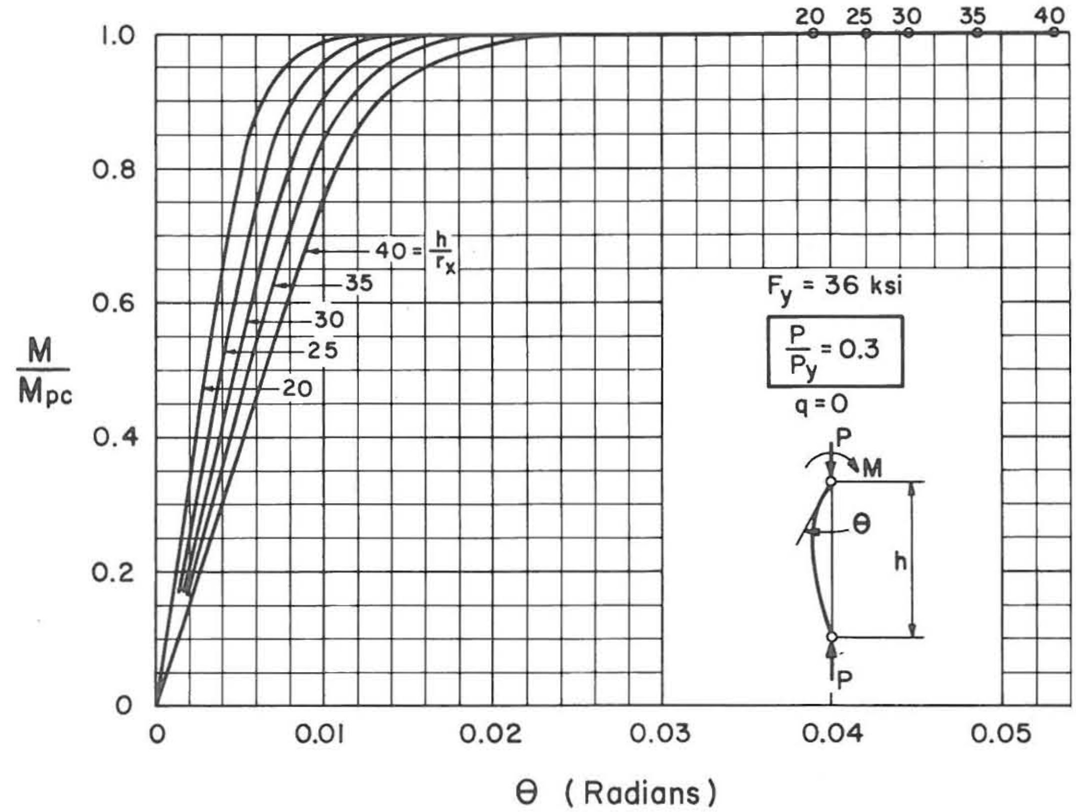
*Section satisfies $\frac{b}{t}$ requirement, but may exceed $\frac{d}{w}$ limitations shown below.

F_y = 36 ksi; $\frac{d}{w} \leqslant 70-100\, P/P_y$ but need not be less than 43 $\qquad$ F_y = 50 ksi; $\frac{d}{w} \leqslant 60-85\, P/P_y$ but need not be less than 36

Design Aid I—(Cont.)

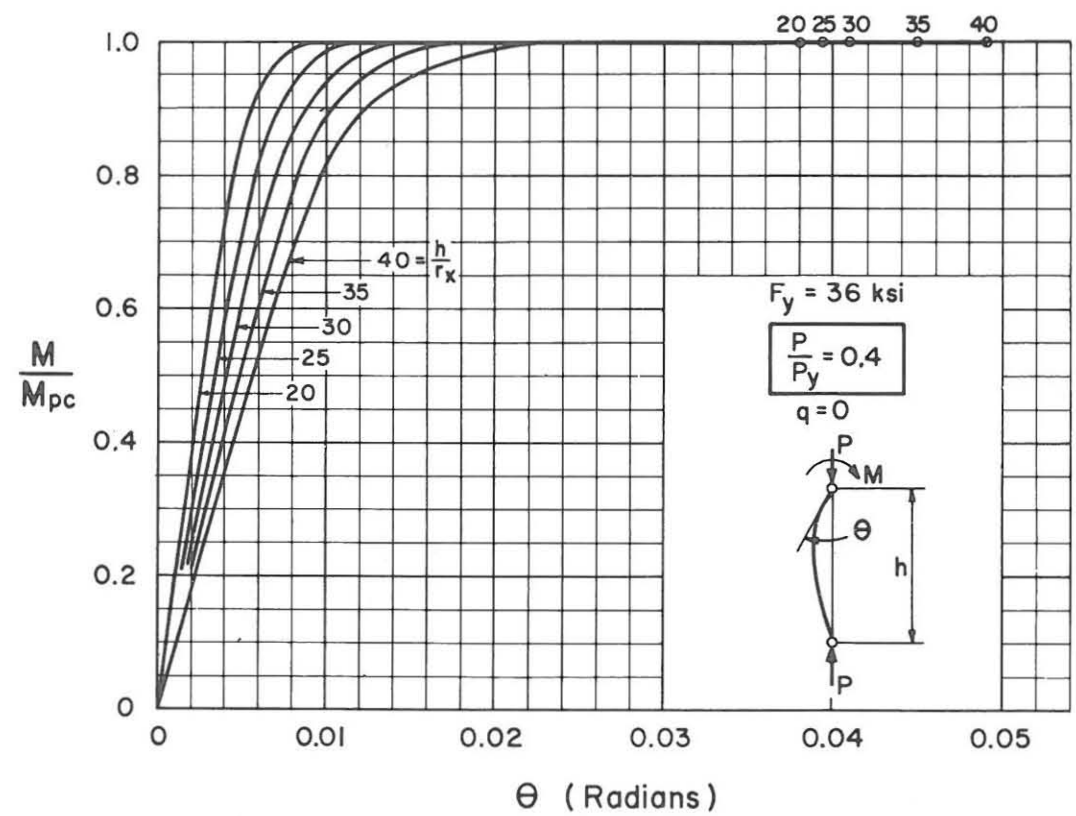
PROPERTIES OF BEAM-COLUMNS

Section	A	r_x	r_y	Z	A36		A572 $F_y = 50$		A441		
					P_y	M_p	P_y	M_p	F_y	P_y	M_p
	in²	in	in	in³	kips	k-ft	kips	k-ft	ksi	kips	k-ft
12WF190	55.86	5.82	3.25	311.5	2011	935	2793	1298	46	2570	1194
12WF161	47.38	5.70	3.20	259.7	1706	778	2369	1082	46	2179	996
12WF133	39.11	5.59	3.16	209.7	1408	629	1956	874	46	1800	804
12WF120	35.31	5.51	3.13	186.4	1271	559	1766	777	46	1625	715
12WF106	31.19	5.46	3.11	163.4	1123	490	1560	681	50	1560	681
12WF99	29.09	5.43	3.09	151.8	1047	455	1454	632	50	1454	632
12WF92	27.06	5.40	3.08	140.2	974	421	1353	584	50		
12WF85	24.98	5.38	3.07	129.1	899	387			50		
12WF79	23.22	5.34	3.05	119.3	836	358			50		
12WF58	17.06	5.28	2.51	86.5	614	260			50		
12WF53	15.59	5.23	2.48	78.2	561	235			50		
12WF50	14.71	5.18	1.96	72.6	530	218	736	302	50	736	302
12WF45	13.24	5.15	1.94	64.9	477	195	662	270	50	662	270
12WF40	11.77	5.13	1.94	57.6	424	173			50		
10WF112	32.92	4.67	2.67	147.5	1184	443	1646	615	50	1646	615
10WF100	29.43	4.61	2.65	130.1	1058	390	1472	542	50	1472	542
10WF89	26.19	4.55	2.63	114.4	943	343	1310	477	50	1310	477
10WF77	22.67	4.49	2.60	97.7	816	293	1134	407	50	1134	407
10WF72	21.18	4.46	2.59	90.7	762	272	1059	378	50	1059	378
10WF66	19.41	4.44	2.58	82.8	699	248	970	345	50	970	345
10WF60	17.66	4.41	2.57	75.1	636	225	883	313	50	883	313
10WF54	15.88	4.39	2.56	67.0	572	201			50		
10WF45	13.24	4.33	2.00	55.0	477	165	662	229	50	662	229
10WF39	11.48	4.27	1.98	47.0	413	141			50		
8WF67	19.70	3.71	2.12	70.1	709	210	985	292	50	985	292
8WF58	17.06	3.65	2.10	59.9	614	180	853	250	50	853	250
8WF48	14.11	3.61	2.08	49.0	508	147	706	204	50	706	204
8WF40	11.76	3.53	2.04	39.9	423	120	588	166	50	588	166
8WF35	10.30	3.50	2.03	34.7	371	104			50		
8WF28	8.23	3.45	1.62	27.1	296	81.3	412	113	50	412	113
8WF24	7.06	3.42	1.61	23.1	254	69.3			50		
8WF20	5.88	3.43	1.20	19.1	212	57.3	294	79.6	50	294	79.6
8WF17	5.00	3.36	1.16	15.8	180	47.4			50		



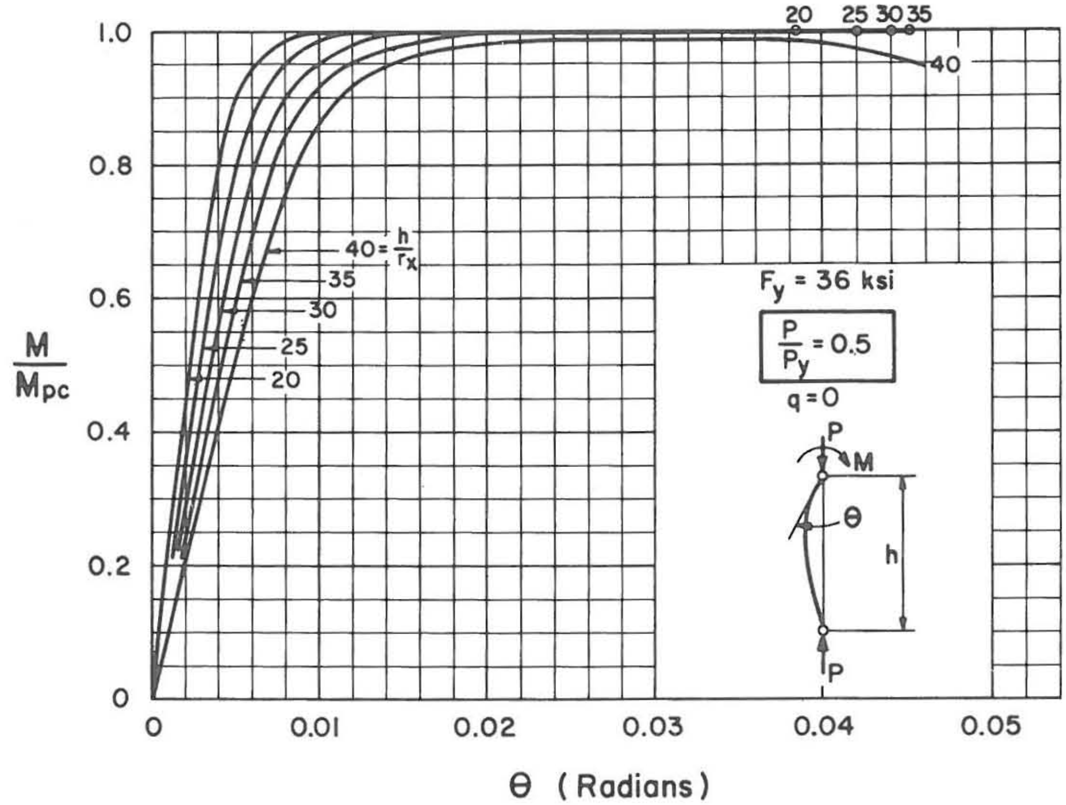
1.0
0.8
0.6
0.4
0.2
0
M/Mpc
20 25 30 35 40
40 = h/rx
35
30
25
20
Fy = 36 ksi
P/Py = 0.3
q = 0
P
M
θ
h
P
0 0.01 0.02 0.03 0.04 0.05
θ (Radians)

D.A.II-1

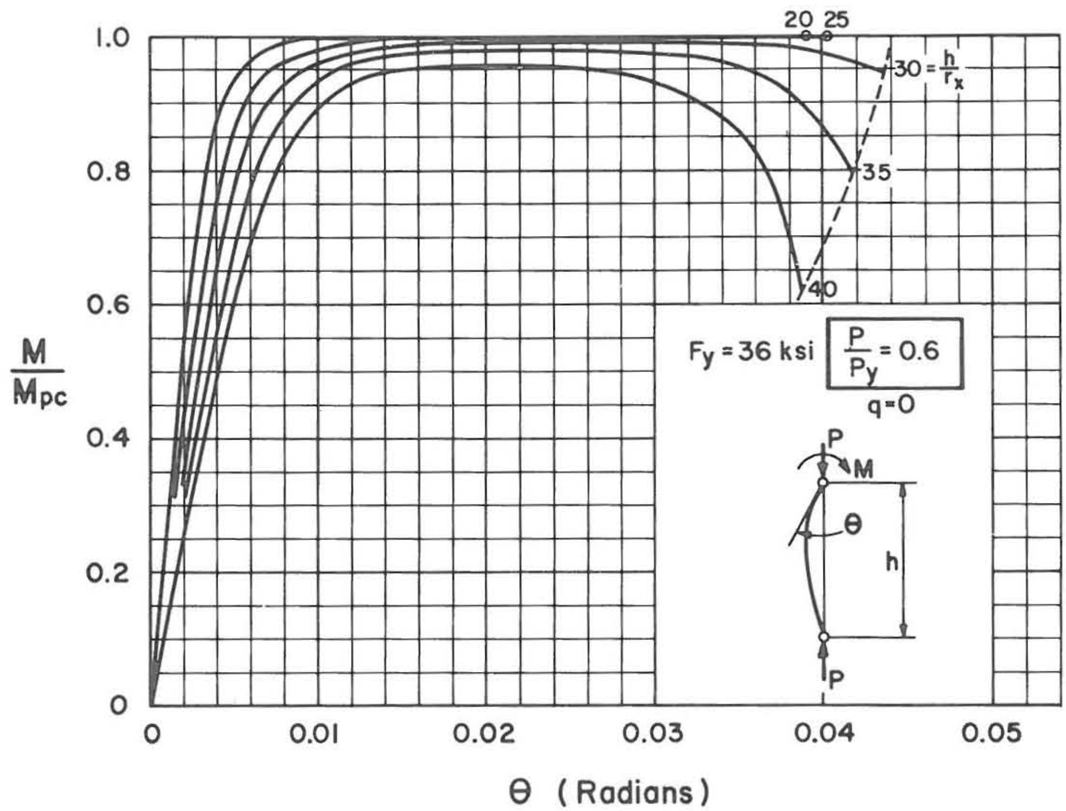


1.0
0.8
0.6
0.4
0.2
0
M/Mpc
20 25 30 35 40
40 = h/rx
35
30
25
20
Fy = 36 ksi
P/Py = 0.4
q = 0
P
M
θ
h
P
0 0.01 0.02 0.03 0.04 0.05
θ (Radians)

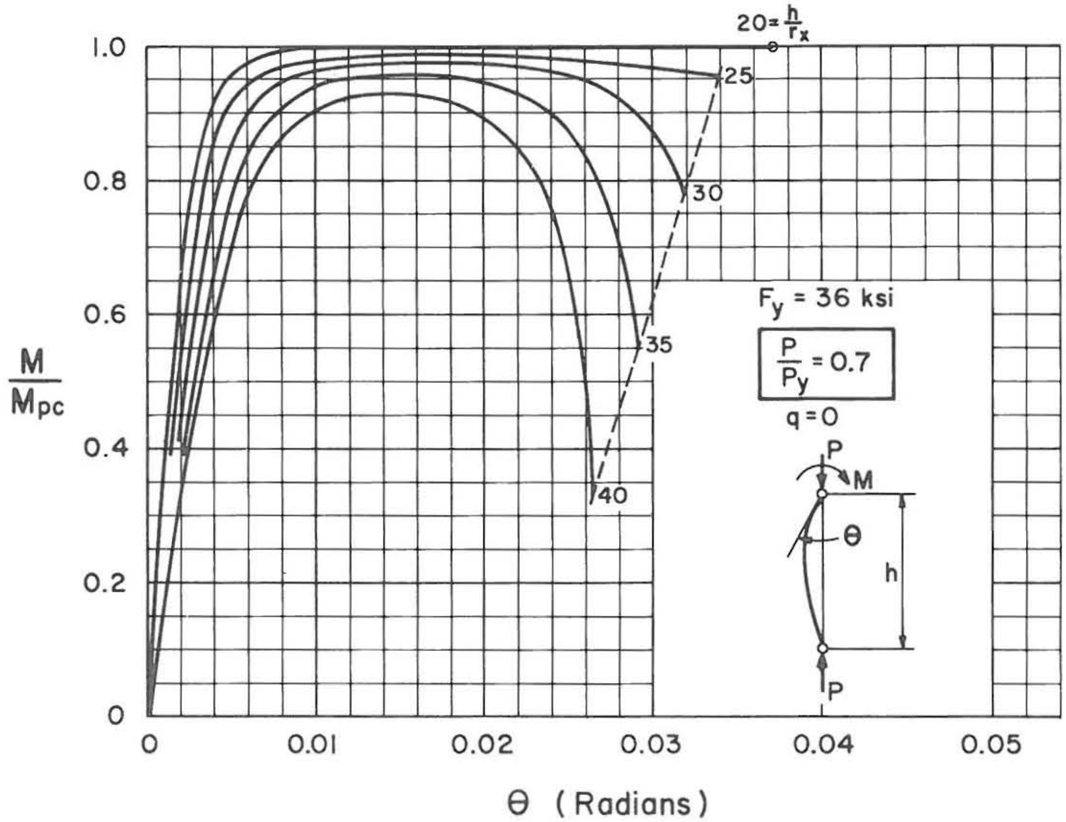
D.A.II-2


20
25
30
35
40
$40 = \frac{h}{r_x}$
35
30
25
20
F_y = 36 ksi
$\frac{P}{P_y}$ = 0.5
q = 0
P
M
θ
h
P
$\frac{M}{M_{pc}}$
1.0
0.8
0.6
0.4
0.2
0
0.01
0.02
0.03
0.04
0.05
θ (Radians)

D.A.II-3

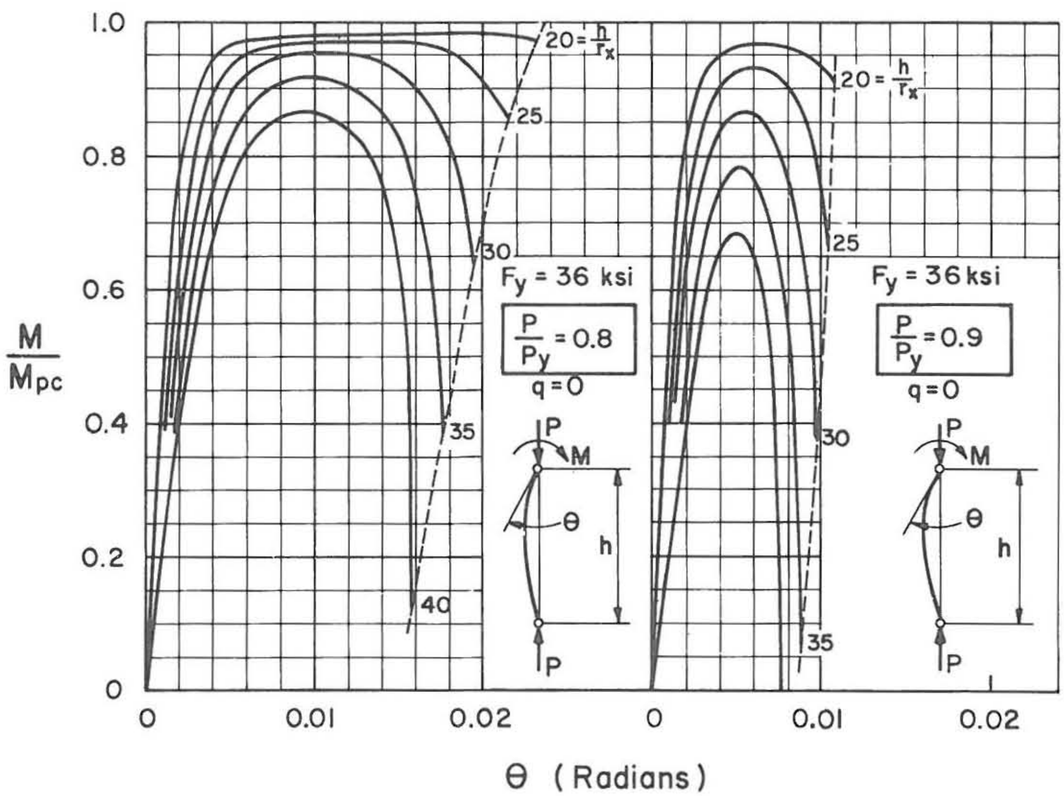

20
25
$30 = \frac{h}{r_x}$
35
40
F_y = 36 ksi
$\frac{P}{P_y}$ = 0.6
q = 0
P
M
θ
h
P
$\frac{M}{M_{pc}}$
1.0
0.8
0.6
0.4
0.2
0
0.01
0.02
0.03
0.04
0.05
θ (Radians)

D.A.II-4



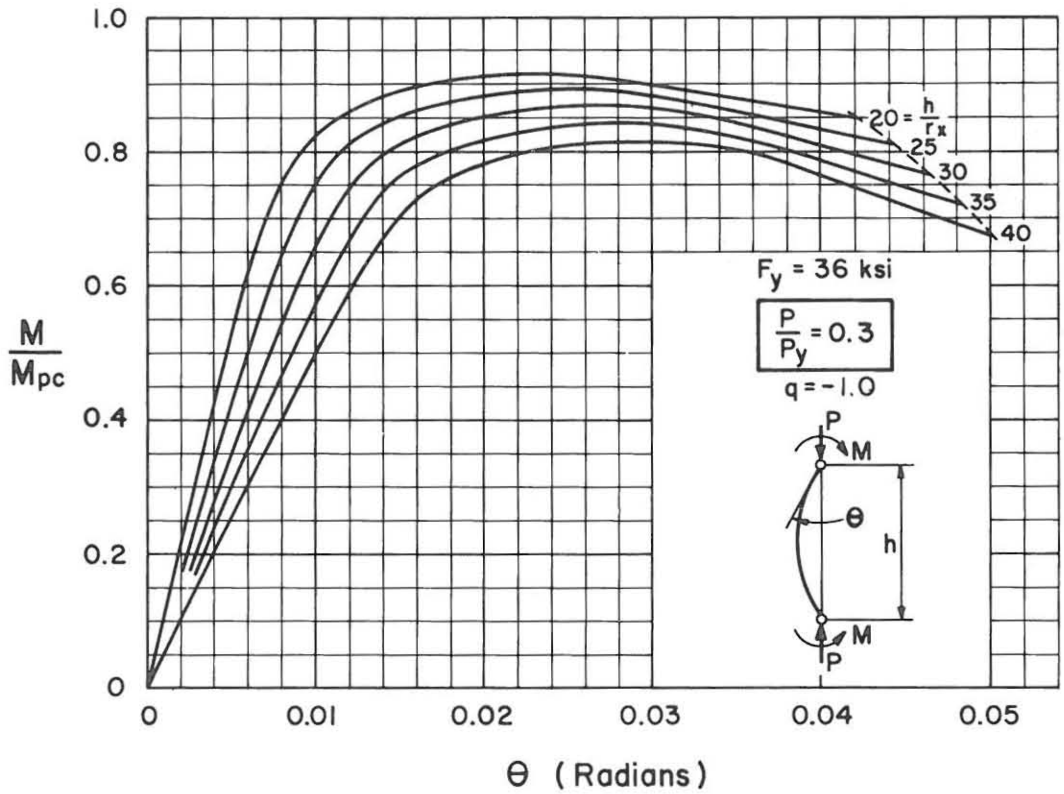
20 = h/r_x
25
30
35
40
1.0
0.8
0.6
0.4
0.2
0
M/M_pc
0
0.01
0.02
0.03
0.04
0.05
Θ (Radians)
F_y = 36 ksi
P/P_y = 0.7
q = 0
P
M
Θ
h
P

D.A. II-5

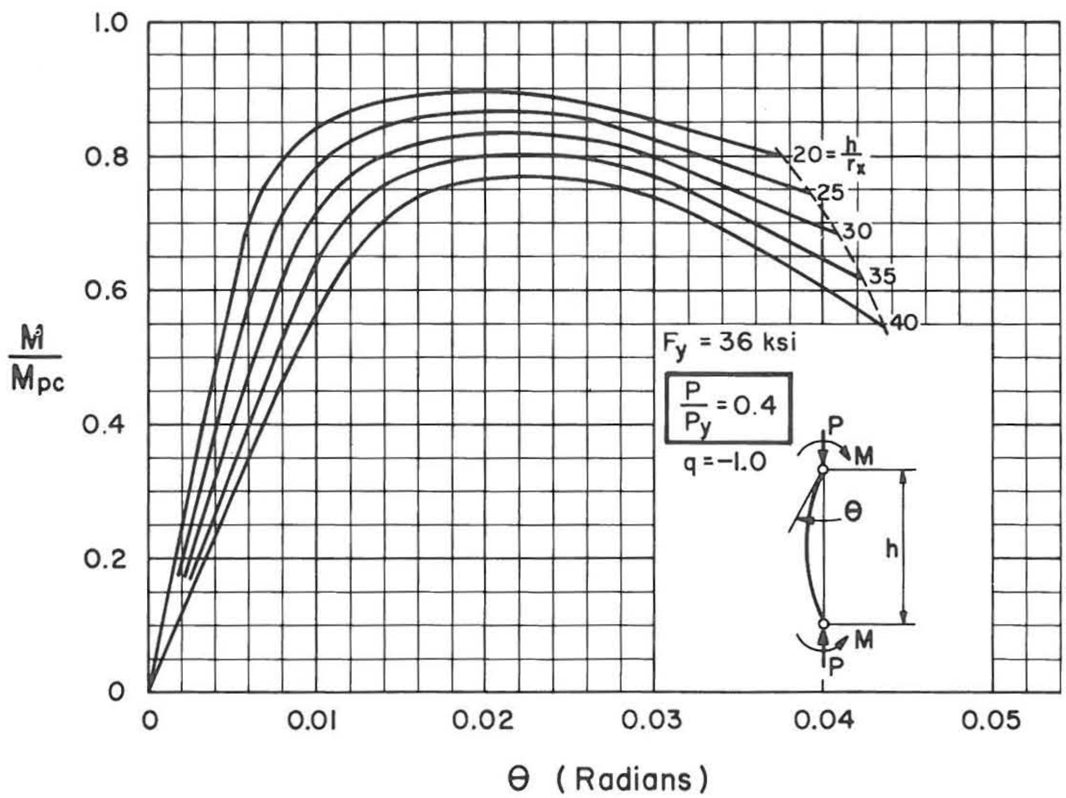


20 = h/r_x
25
30
35
40
20 = h/r_x
25
30
35
1.0
0.8
0.6
0.4
0.2
0
M/M_pc
0
0.01
0.02
0
0.01
0.02
Θ (Radians)
F_y = 36 ksi
P/P_y = 0.8
q = 0
F_y = 36 ksi
P/P_y = 0.9
q = 0
P
M
Θ
h
P

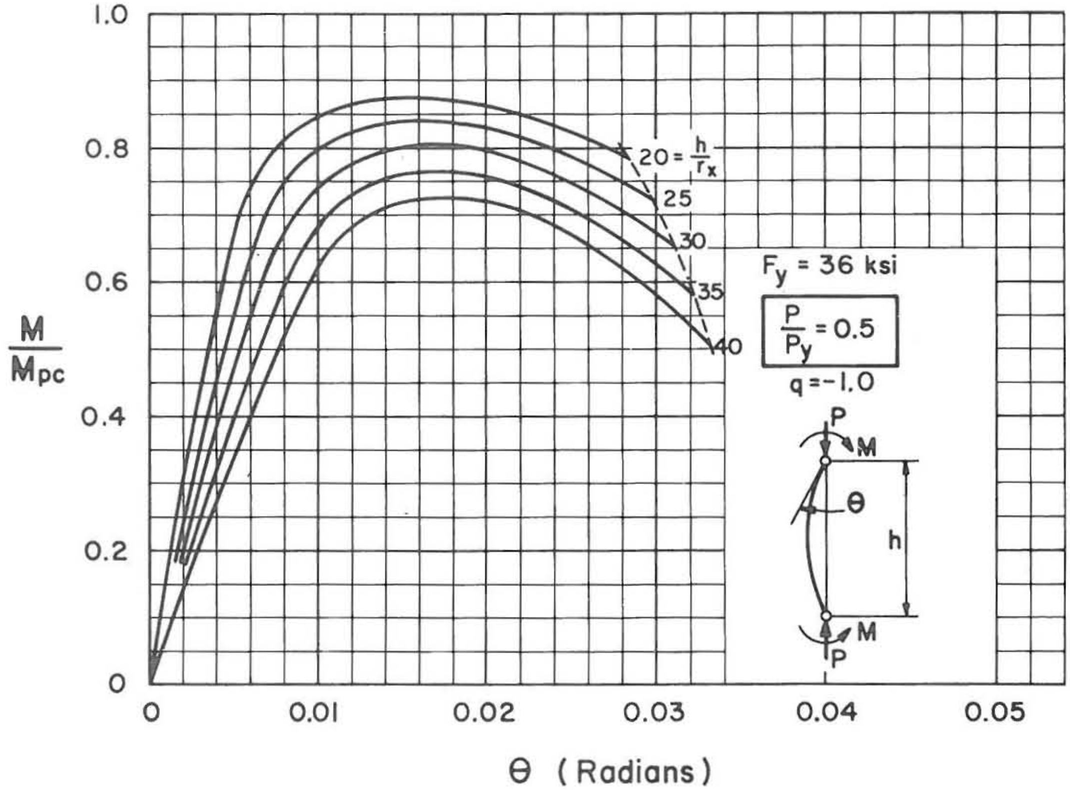
D.A. II-6


1.0
0.8
0.6
0.4
0.2
0
M/Mpc
0
0.01
0.02
0.03
0.04
0.05
θ (Radians)
20 = h/rx
25
30
35
40
Fy = 36 ksi
P/Py = 0.3
q = -1.0
P
M
θ
h
M
P

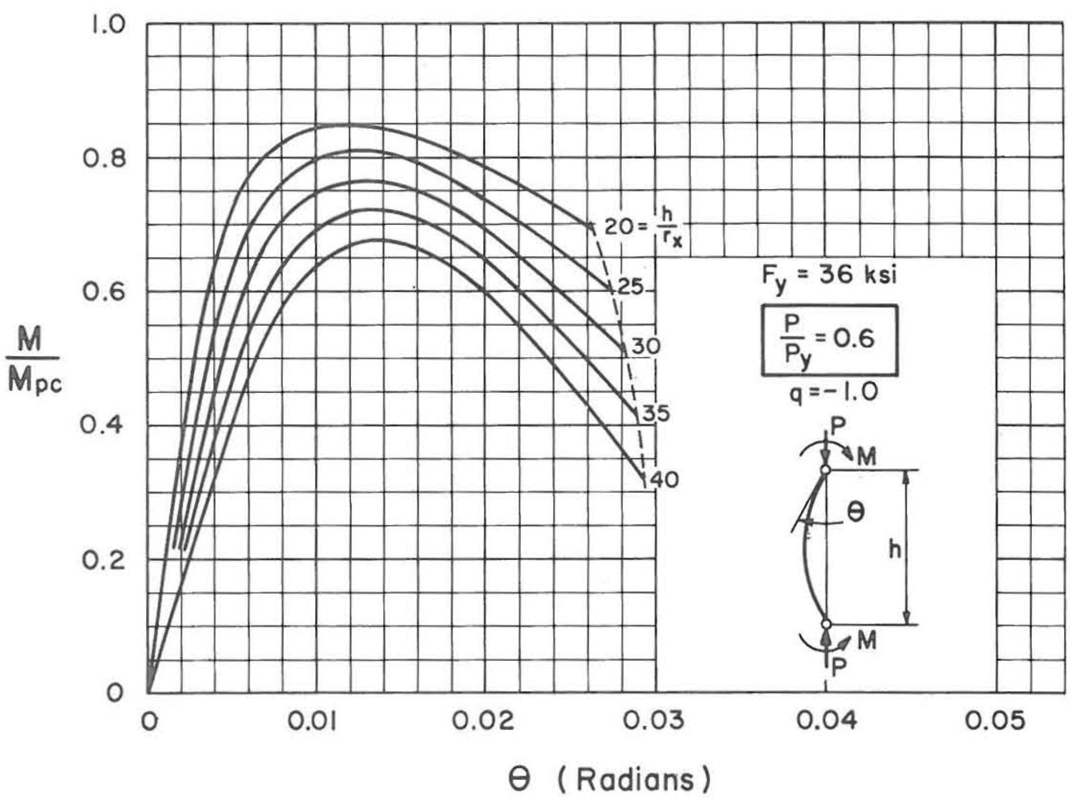
D.A. II-7


1.0
0.8
0.6
0.4
0.2
0
M/Mpc
0
0.01
0.02
0.03
0.04
0.05
θ (Radians)
20 = h/rx
25
30
35
40
Fy = 36 ksi
P/Py = 0.4
q = -1.0
P
M
θ
h
M
P

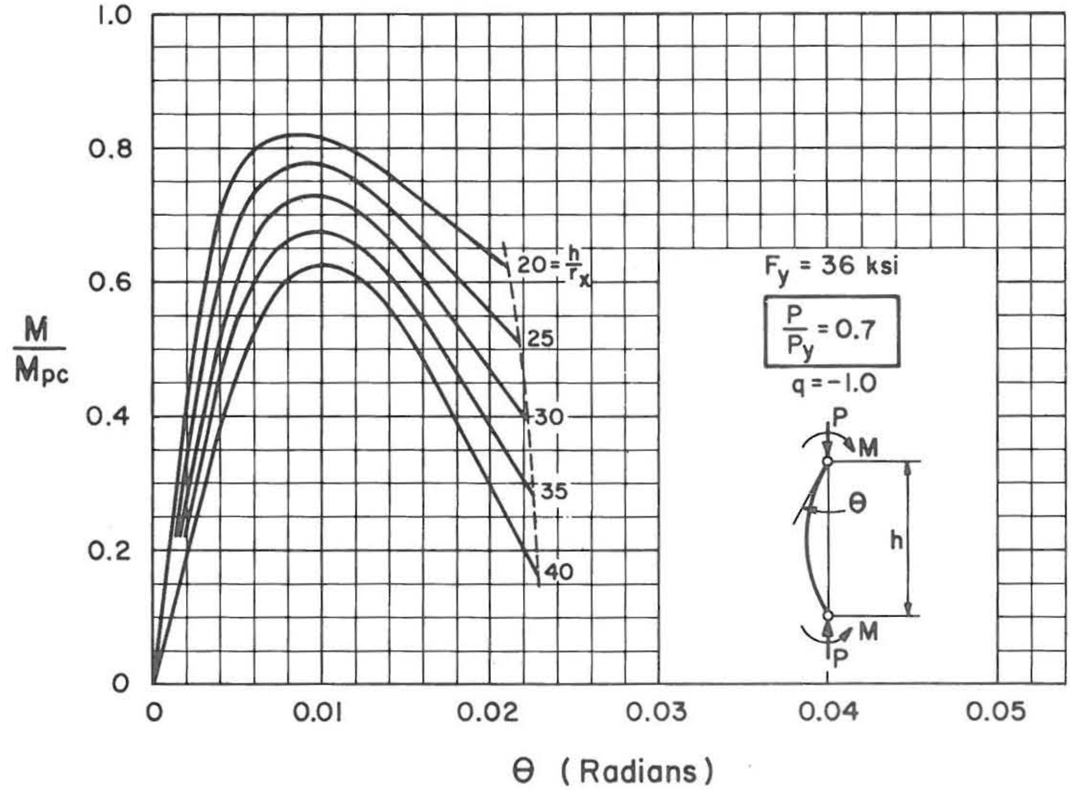
D.A. II-8



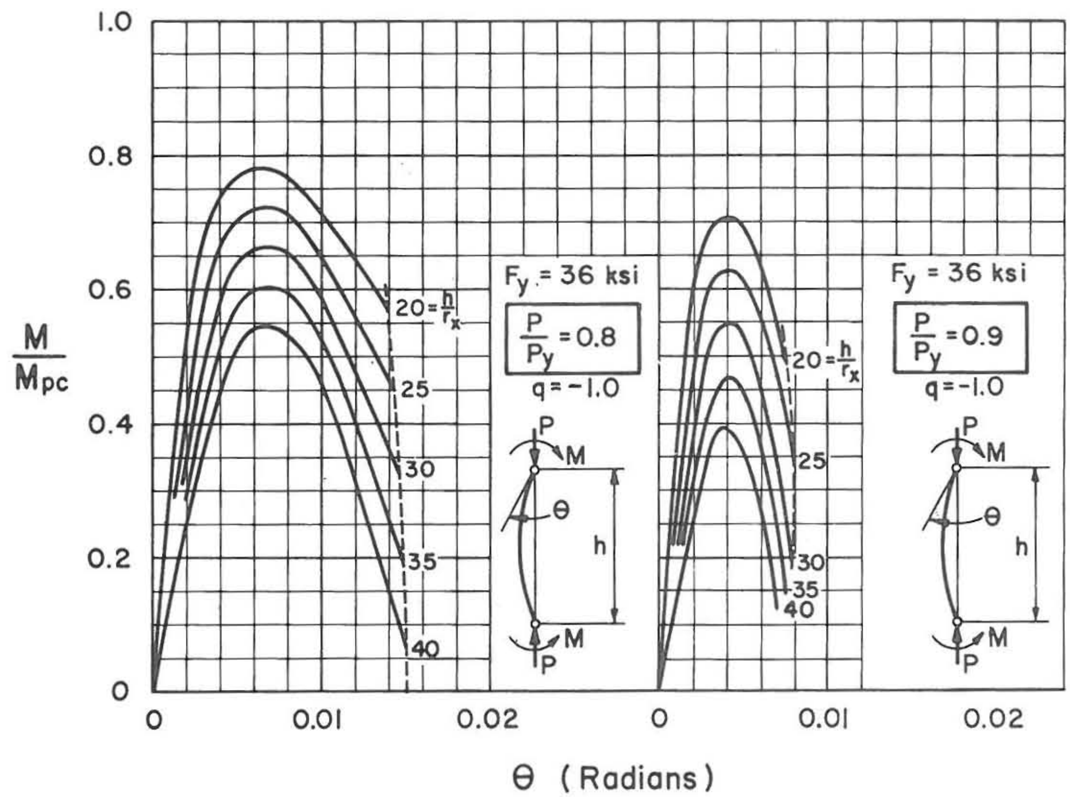
D.A.II-9



D.A.II-10



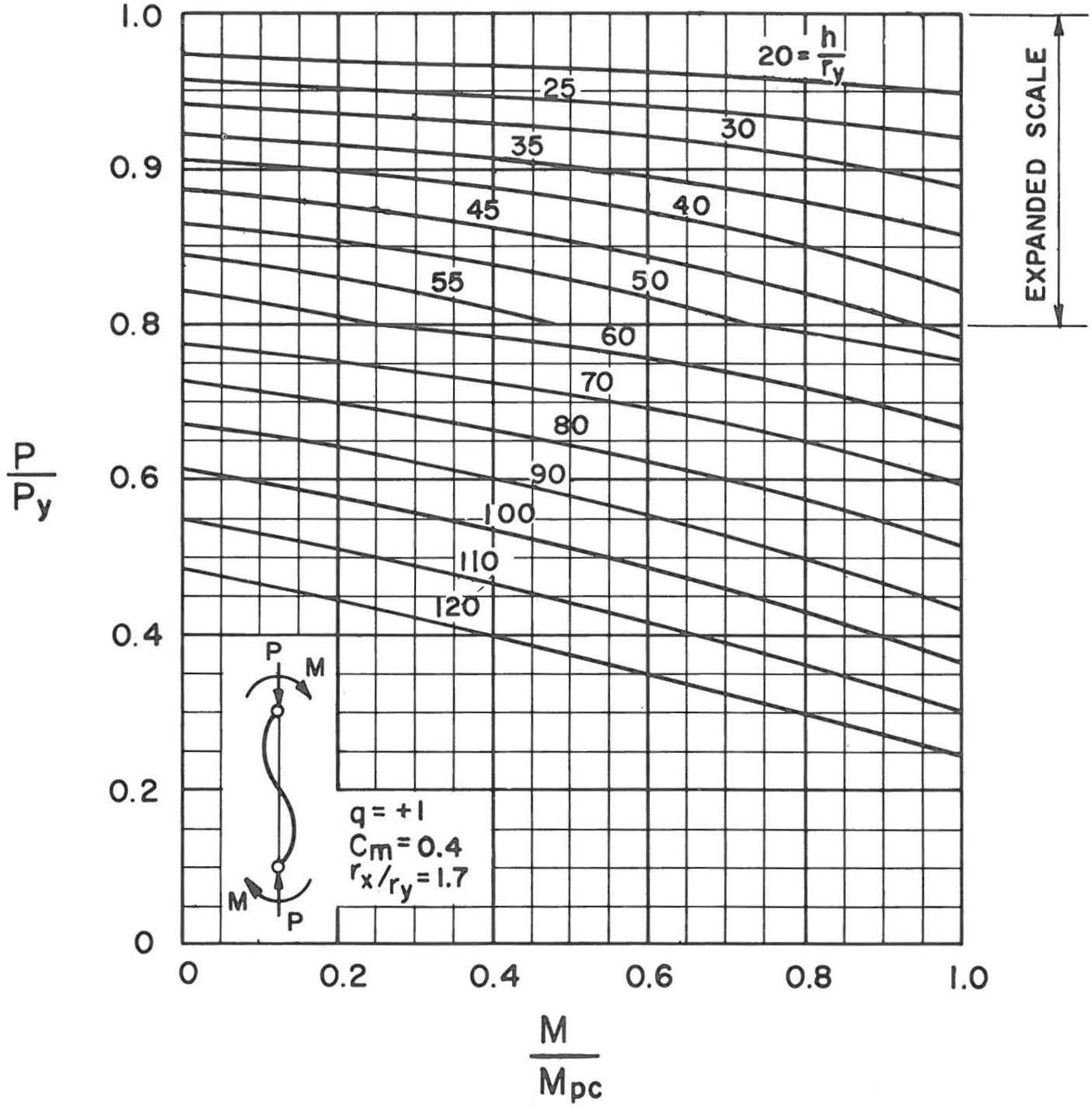
D.A.II-11



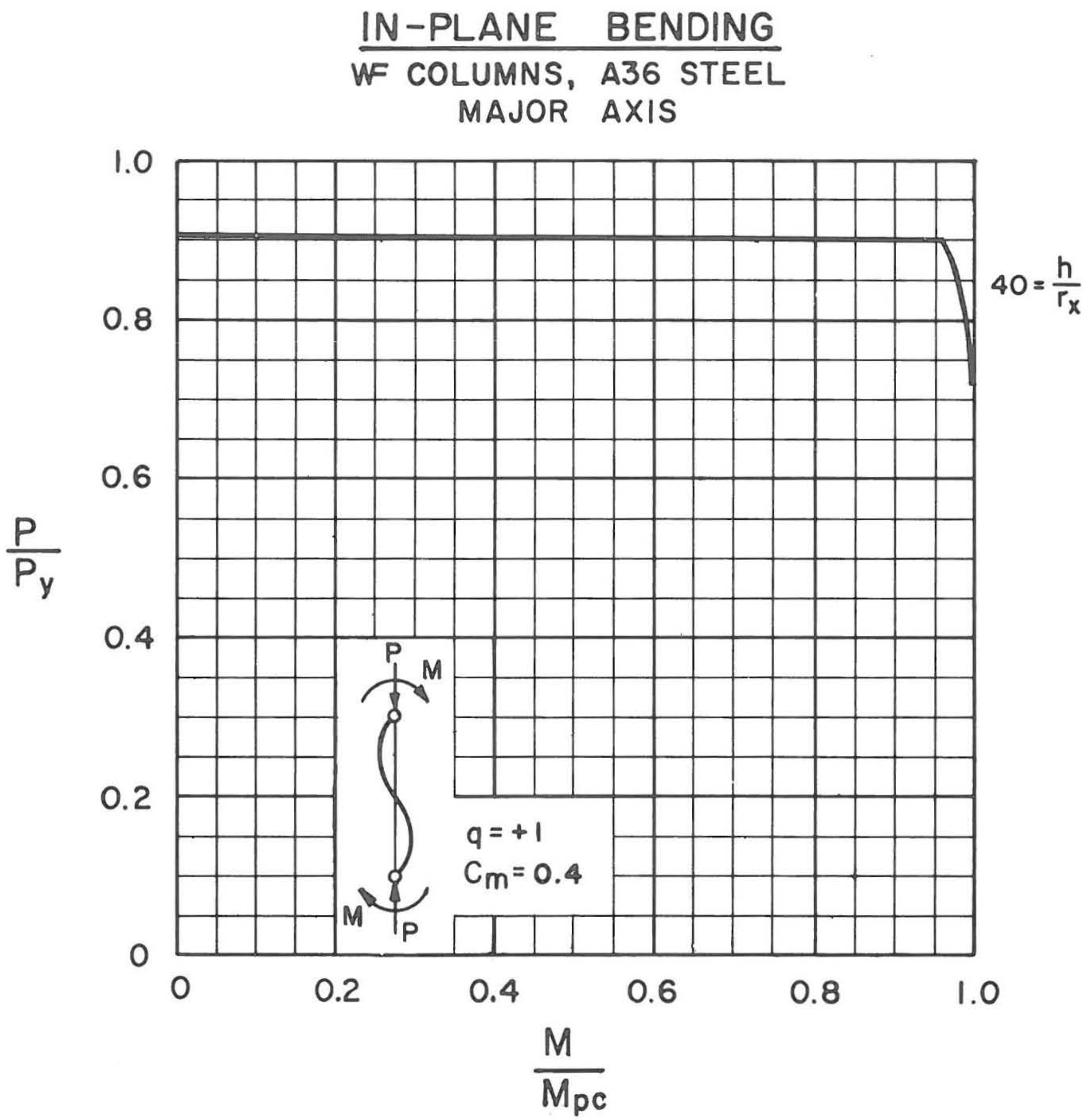
D.A.II-12

LATERAL TORSIONAL BUCKLING

WF COLUMNS, A36 STEEL
MAJOR AXIS BENDING



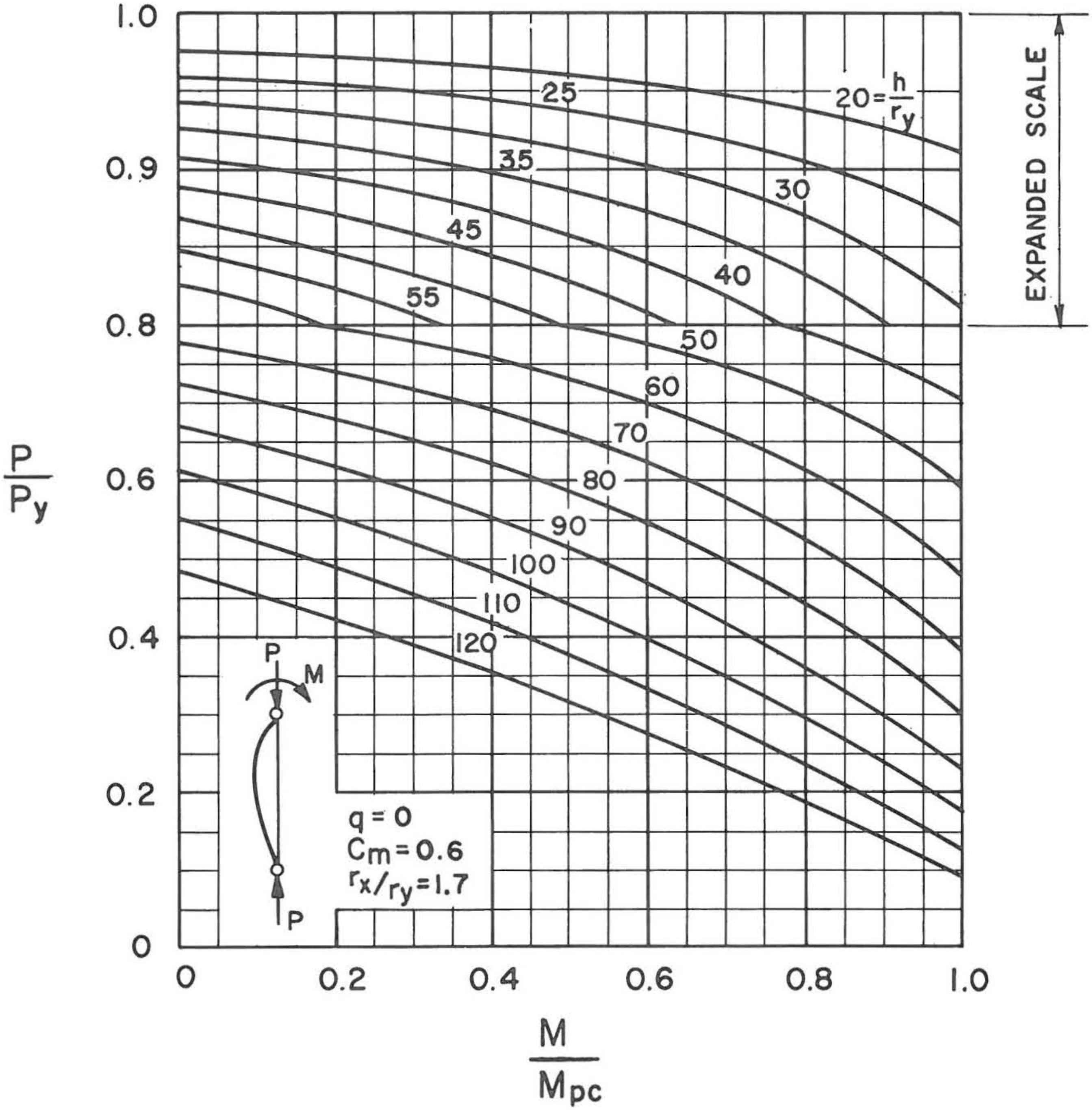
D.A. III - 1a

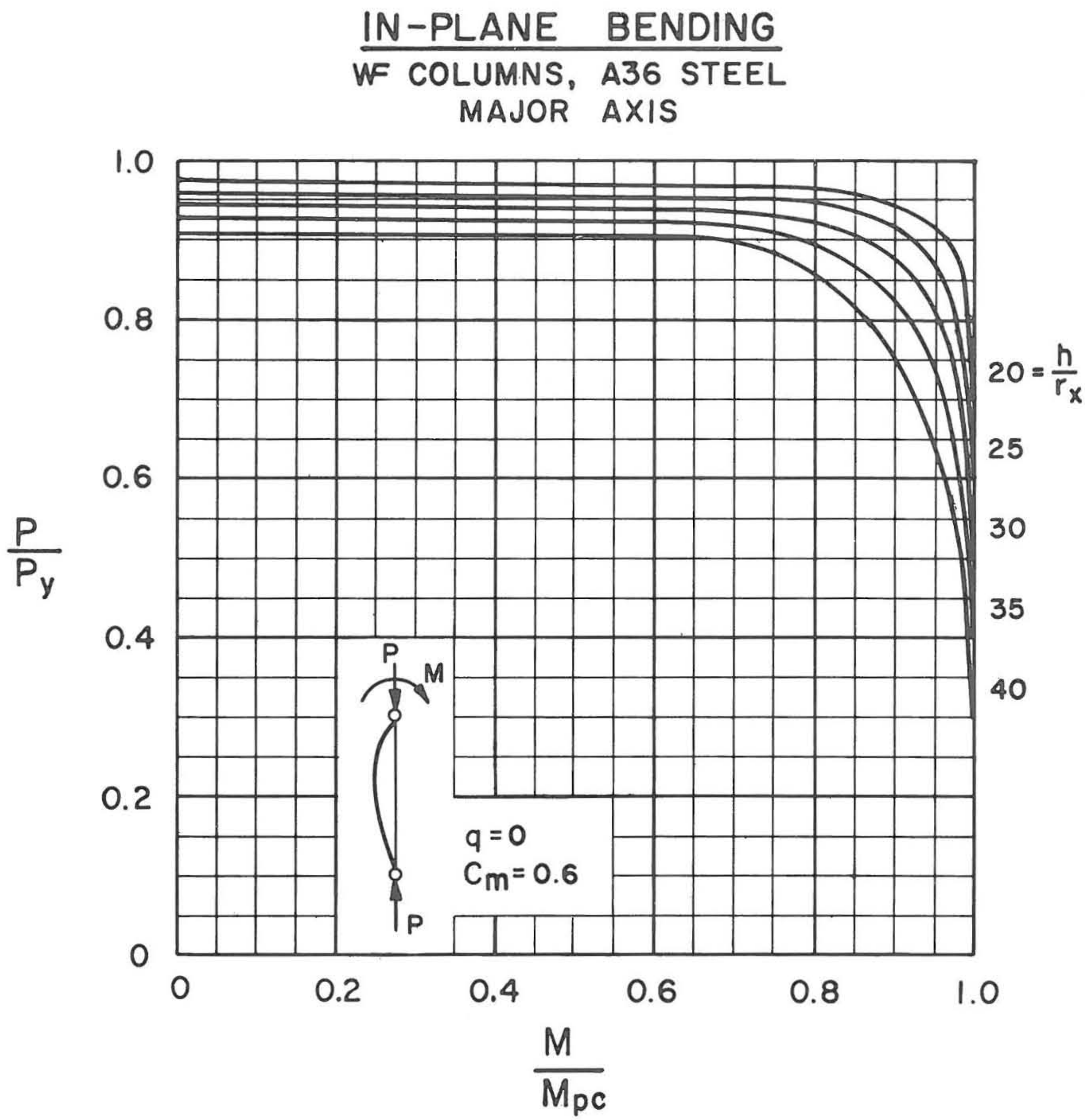

IN-PLANE BENDING
WF COLUMNS, A36 STEEL
MAJOR AXIS
1.0
0.8
0.6
0.4
0.2
0
P/Py
40 = h/rx
P
M
M
P
q = +1
Cm = 0.4
0
0.2
0.4
0.6
0.8
1.0
M/Mpc

D.A.III-1b

LATERAL TORSIONAL BUCKLING

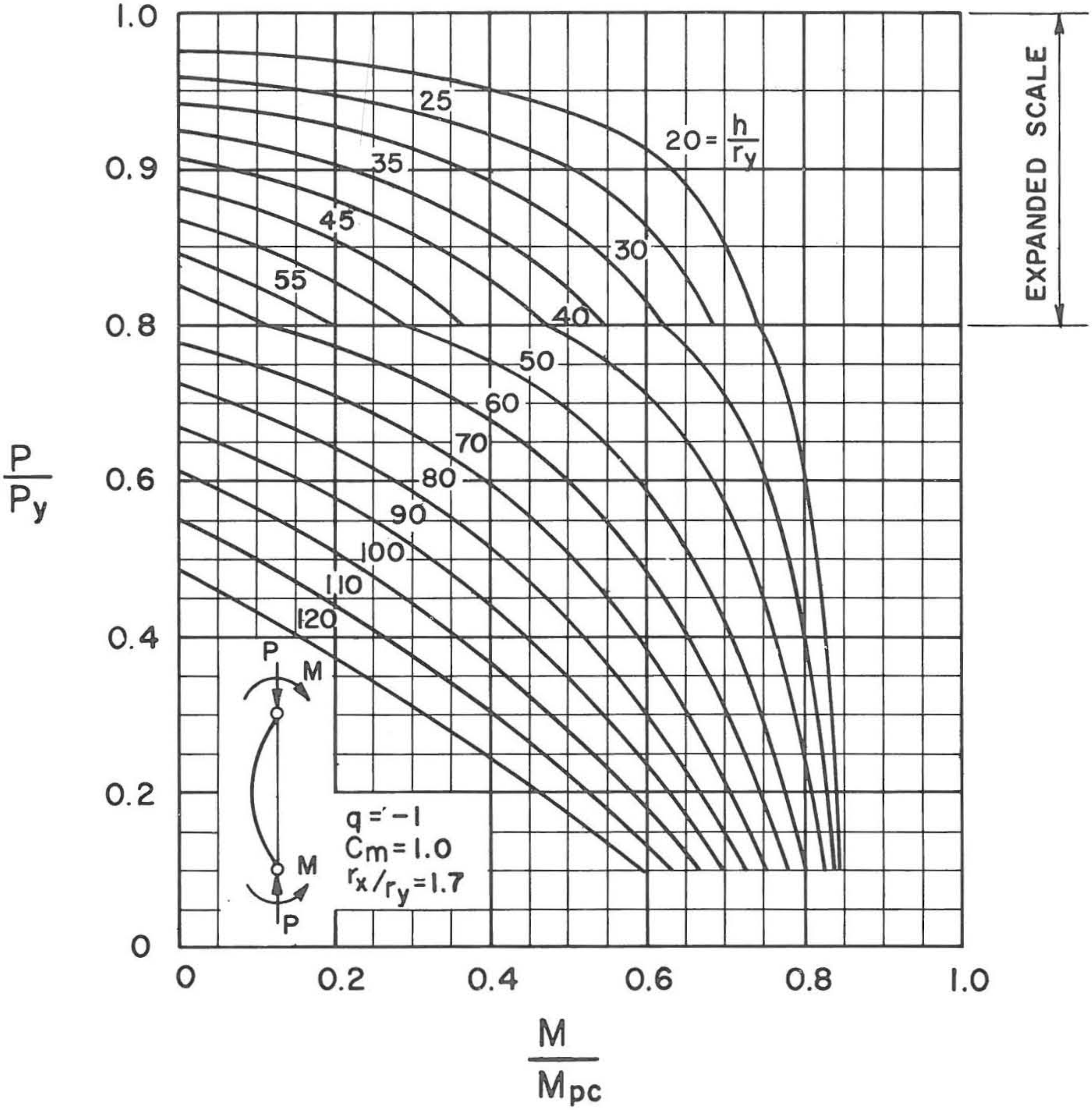
WF COLUMNS, A36 STEEL
MAJOR AXIS BENDING



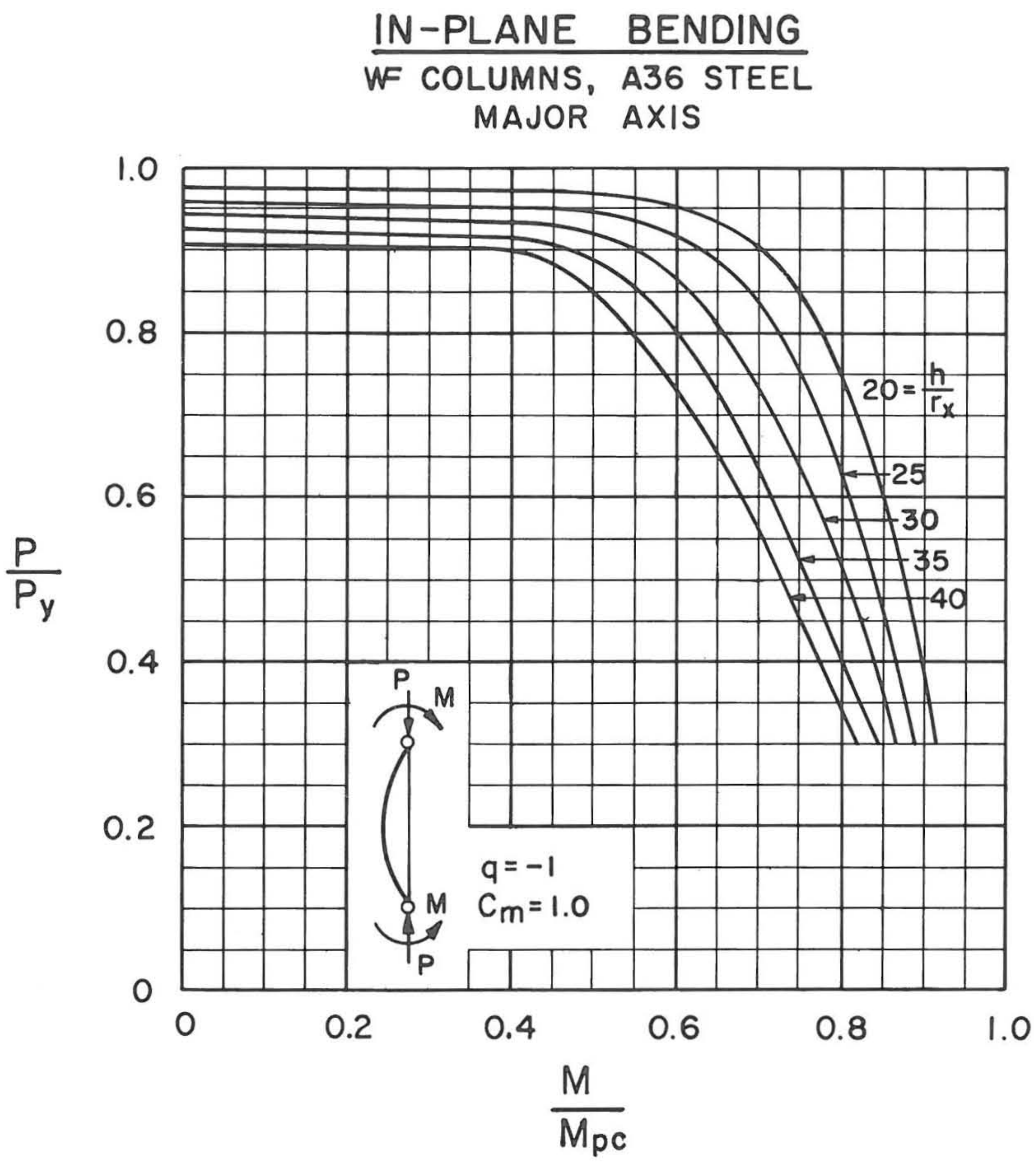

IN-PLANE BENDING
WF COLUMNS, A36 STEEL
MAJOR AXIS
1.0
0.8
0.6
0.4
0.2
0
P/Py
0
0.2
0.4
0.6
0.8
1.0
M/Mpc
20 = h/rx
25
30
35
40
P
M
P
q = 0
Cm = 0.6

LATERAL TORSIONAL BUCKLING

WF COLUMNS, A36 STEEL
MAJOR AXIS BENDING



D.A. III-3a


IN-PLANE BENDING
WF COLUMNS, A36 STEEL
MAJOR AXIS
1.0
0.8
0.6
0.4
0.2
0
P/Py
20 = h/rx
25
30
35
40
P
M
M
P
q = -1
Cm = 1.0
0
0.2
0.4
0.6
0.8
1.0
M/Mpc

NOTES