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BRACING FOR STABILITY©

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Bracing for Stability — State-of-the-Art

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Abstract

Design methods for stability bracing of columns, beams and frames are summarized. The importance of initial out-of-straightness on the brace strength and stiffness requirements is shown. Four types of bracing systems are described; relative, discrete, continuous and lean-on. Design examples (AISC - LRFD) illustrate the use of the bracing formulas.

Recommendations for lateral bracing and torsional bracing for beams are presented. Solutions for restrained beams with the top flange braced are given. It is shown that the inflection point cannot be considered a brace point. Stiffener requirements to control cross-section distortion are included in the method for designing cross frame and diaphragm bracing.

Introduction

A general design guide for stability bracing of columns, beams and frames is presented herein. The focus is on simplicity, not exact formulations. The design recommendations cover four general types of bracing systems; namely relative, discrete, continuous and lean-on, as illustrated in Figure 1. A relative brace controls the relative movement of adjacent stories or of points along the length of the column or beam. If a cut everywhere along the braced member passes through the brace, itself, then the brace system is relative as illustrated by diagonal bracing, shear walls, or truss bracing. A discrete brace controls the movement only at that particular brace point. For example, in Figure 1b the column is braced at points 1 by cross beams. A cut at the column midheight does not pass through any brace so the brace system is not relative, but is discrete. Two adjacent beams with diaphragms or cross frames are discretely braced at the cross frame location. Continuous bracing is self evident; the brace is continuously attached along the length of the member such as with siding for columns and metal deck forms for beams

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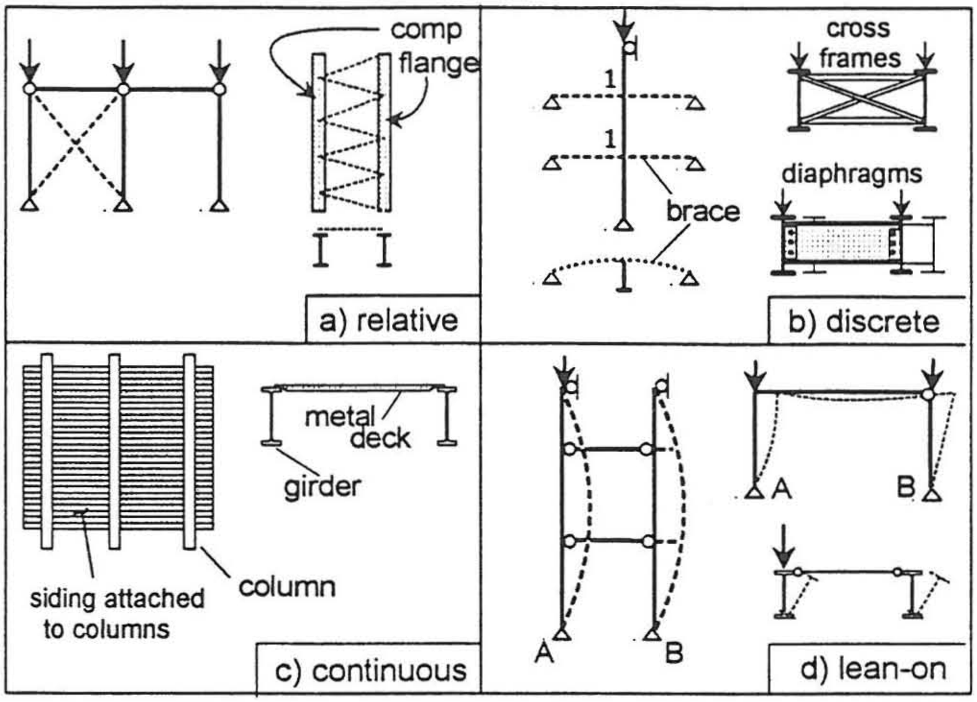


Figure 1 **Types of Bracing Systems**

during construction. A beam or column that relies on adjacent structural members for support is braced in a lean-on system. Structural members that are tied or linked together such that buckling of the member would require adjacent members to buckle with the same lateral displacement characterize lean-on systems as shown in Figure 1d. In the sway mode Member A leans on Member B.

An adequate brace system requires both strength and stiffness. A simple brace design formulation such as designing the brace for 2% of the member compressive force addresses only the strength criterion. Brace connections, if they are flexible, can have a very detrimental effect on stiffness as will be illustrated later. Before presenting the various bracing recommendations, some background material on the importance of initial out-of-straightness and member inelasticity on bracing effects will be discussed.

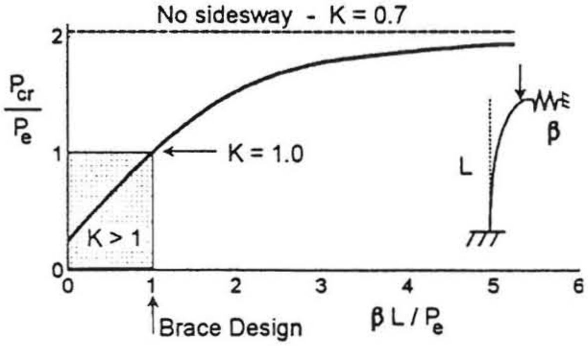


Figure 2 **Braced Cantilever**

Limitations. The brace requirements presented will enable a member to reach the Euler buckling load between the brace points, i.e., use K = 1.0. This is not the same as the no-sway buckling load as illustrated in Figure 2 for the braced cantilever. The ideal brace stiffness is 1.0 P_o/L corresponding to $K = 1.0$. A brace five times this stiffness is necessary to reach 95 percent of the $K = 0.7$ limit. Theoretically, an infinitely stiff brace is required to reach the no-sway limit. In addition, bracing required to reach specified rotation capacity or ductility limits is beyond the scope of this paper.

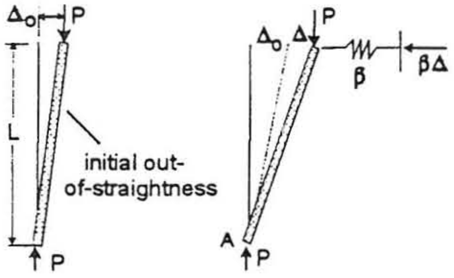


Figure 3 Relative Brace

Background

Member-Out-of-Straightness. Winter (1960) developed the concept of a dual criteria for bracing design, strength and stiffness, and he derived the interrelationship between them using simple models. He showed that the brace force is a function of the initial column out-of-straightness, Δ_o, and the brace stiffness β. The concept is illustrated for the relative brace system shown in Figure 3, where the brace, represented by the spring at the top of the column, controls the movement at the top Δ relative to the column base. Summation of moments about point A gives $P\Delta_T = \beta L(\Delta_T - \Delta_o)$ where $\Delta_T = \Delta + \Delta_o$. If $\Delta_o = 0$ (an initially perfectly plumb member), then $P_{cr} = \beta L$ which indicates that the load increases as the brace stiffness. The brace stiffness required in the sway mode to reach the load corresponding to Euler buckling between brace points, P_o, is called the ideal stiffness, β_i, where $\beta_i = P_o/L$ in this case.

For the out-of-plumb column, the relationship between P, β, and Δ_T is plotted in Figure 4a. If $\beta = \beta_i$, P_o can be reached only if the sway deflection gets very large. Unfortunately, such large displacements produce large brace forces, F_{br}, since $F_{br} = \beta\Delta$. For practical design, Δ must be kept small at the maximum expected load level. This can be accomplished by specifying $\beta > \beta_i$. For example if $\beta = 2\beta_i$, then $\Delta = \Delta_o$ at P_o as shown in Figure 4b. The larger the brace stiffness,

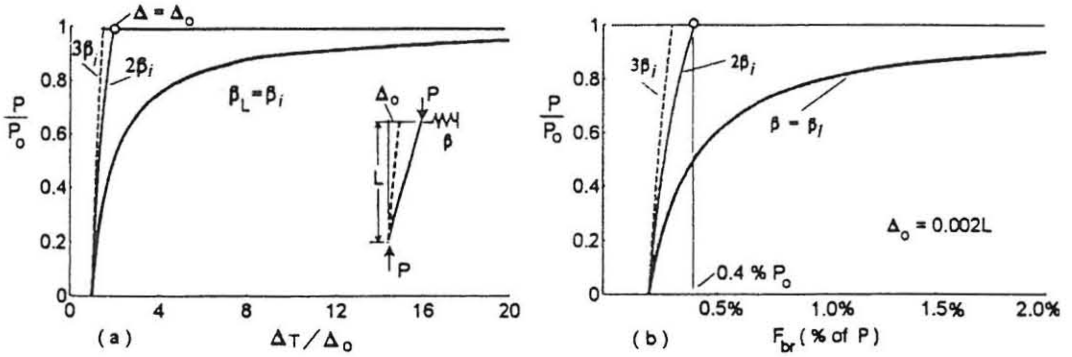


Figure 4 Effect of Initial Out-of-Plumb

the smaller the brace force. The brace force is a linear function of the initial out-of-plumbness. The recommendations given later will assume a particular out-of-straightens and a brace stiffness at least twice the ideal stiffness.

Column Inelasticity. The bracing requirements for relative braces are merely a function of the load on the member, as illustrated above. Other brace system requirements (continuous, lean-on) are based on the elastic or inelastic stiffness of the members to be braced as will be given later. In stability problems the effect of member inelasticity on the buckling solution can be reasonably approximated by using the tangent modulus stiffness E_T instead of the elastic modulus, E. The inelastic stiffness $E_T = \tau E$ where τ is the inelastic stiffness reduction factor. The elastic range is defined by the axial stress in the member, not the slenderness ratio. A member with low slenderness ratio (L/r) will respond elastically if the axial stress is low. In the AISC Specification an axial stress less than 0.3 F_y in ASD or 0.33 F_y in LRFD places the column in the elastic range. The AISC-ASD and LRFD Manuals of Steel Construction tabulate the stiffness reduction factor for P/A stress levels. In LRFD, $\tau = -7.38(P/P_y) \log (1.176\ P/P_y)$ where P_y is the yield load, F_yA. The potential axial buckling capacity of a member is $\tau\ (.877)\ \pi^2\ EI/(KL)^2$ for $P/P_y \geq 1/3$. For $P/P_y < 1/3$, $\tau = 1.0$. The τ factor will be used in some of the example problems.

Safety Factors, ϕ Factors and Definitions

The recommendations presented are based on ultimate strength. Column and beam loads are assumed to be factored loads. For brace stiffness formulations, a $\phi = 0.75$ is recommended in LRFD. If the load calculations are based on service loads as in ASD, a factor of safety of 2.0 can be applied to the factored load stiffness requirements. The strength requirements use the built-in safety factors or ϕ factors within each design specification. In LRFD, the design brace force will be based on factored loads and compared to the design strength of the member and its connections. In ASD, the brace force will be a function of the applied service loads, and this force will be compared to the allowable brace loads and connection capacity.

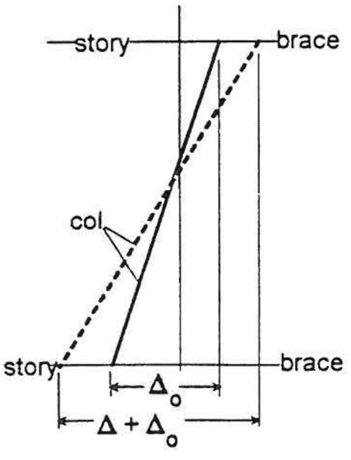


Figure 5 Definitions

The displacement Δ_o for relative and discrete braces is defined with respect to the distance between braces as shown in Figure 5. In frames P is the sum of the column loads in a story to be stabilized by the brace. In the case of a discrete brace for a member, P would be the average load in the compression member above and below the brace point. The initial displacement Δ_o is a small displacement from the straight position at the brace points caused by sources other than the gravity loads or compressive forces. For example Δ_o would be a displacement caused by wind or other lateral forces, erection tolerance (initial out-of-plumb), etc. In all cases, the brace force recommendations are based on an assumed $\Delta_o = 0.002L$. For other Δ_o, use direct proportion. For torsional bracing of columns or beams, an initial twist β_o of 1° is used.

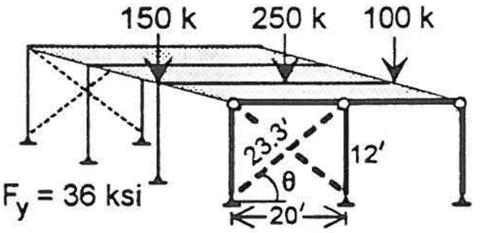
Relative Braces for Columns or Frames

DESIGN RECOMMENDATION
LRFD, $\phi = 0.75$ $\beta_{REQ'D.} = \frac{2P}{\phi L}$ $F_{br} = 0.004P$

The design recommendation is based on an initial out-of-plumbness = 0.002L and a brace stiffness twice the ideal value shown in Figure 4. Example 1 illustrates the bracing design. Each brace must stabilize 1500 kips. The cos functions are necessary to convert the diagonal brace to an equivalent brace perpendicular to the column(s). Stiffness controls the design in this case. If Δ_o is different from 0.002L, change F_{br} in direct proportion to the actual Δ_o, but no change is necessary for β.

EXAMPLE 1 - Relative Brace - Tension System

Typical brace must stabilize three bents
Factored load each bent = 150 + 250 + 100 = 500 kips
Design recommendations assume F_{br} and Δ are perpendicular to the column



Brace Force :

$0.004(3 \times 500)/\cos\theta = 6.99$ k

5/8 threaded rod *OK*

Brace Stiffness : $\frac{A_b E}{L_b}\cos^2\theta = \frac{2(3 \times 500\text{ k})}{0.75(12)}$; $A_{b_{gross}} = 0.364\text{ in}^2$

USE 3/4 ϕ , $A_g = 0.44\text{ in.}^2$

Discrete Bracing Systems for Columns

Discrete bracing systems can be represented by the model shown in Figure 6 for three braces. The exact solution taken from Timoshenko (1961) shows the relationship between P_{cr} and the brace stiffness, β. With no bracing $P_{cr} = \pi^2EI/(4L)^2$. At low brace stiffness the buckling load increases substantially with the buckled shape a single (1st mode) wave. As the brace stiffness is increased,

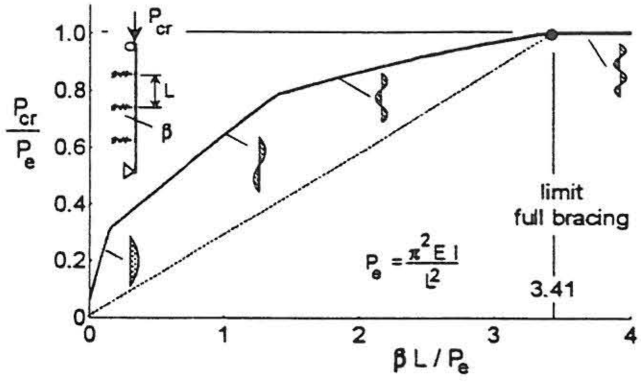


Figure 6 Three Discrete Braces

the buckled shape changes and additional brace stiffness becomes less effective. Full bracing occurs at $\beta L/P_e = 3.41$. This ideal stiffness varies for equally spaced braces between 2.0 for one brace to 4.0 for a large number of braces. Thus 4.0 can be used conservatively for all cases. The design recommendation is based on full bracing assuming the load is at P_e.

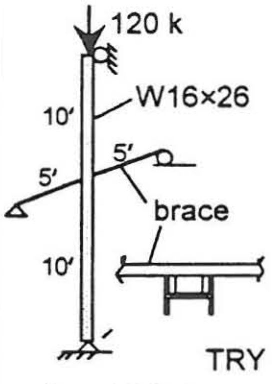
DESIGN RECOMMENDATION
LRFD, $\phi = 0.75$ $\quad \beta_{REQ'D} = \# \frac{2P}{\phi L}$ $\quad F_{br} = 0.01P$

P = factored load, L = required brace spacing, n = number of braces, $\# \approx 4 - (2/n)$

Typically, P may be less than P_e so it is conservative to use the actual column load P to derive the design stiffness represented by the dashed line in Figure 6. Note that the required brace stiffness is inversely proportional to the brace spacing L. In many applications there are more potential brace points than necessary to support the required member forces. Closer spaced braces require more stiffness because the derivations assume that the unbraced length provided is just sufficient to support the column load. For example, say three girts are available to provide weak axis bracing to the columns. Say that the column load is such that only a single full brace at midspan would suffice. Then the required stiffness of the three brace arrangement could be conservatively estimated by using the permissible unbraced length in the brace stiffness equation rather than the actual unbraced length. The continuous bracing formula given in the next section more accurately represents the true response of Figure 6 for less than full bracing.

The design recommendation is based on twice the ideal stiffness to account for initial out-of-straightness. The recommended brace force is 1% of P (See Design Example 2). The value of # is based on equal brace spacing and is unconservative for unequal spacing. For unequal spacing, # can be simply derived using a rigid bar model between braces (Yura, 1994).

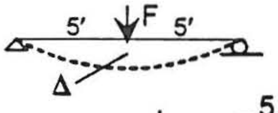
EXAMPLE 2 - Discrete Brace at Midheight



F_y = 36 ksi

A cross member braces the weak axis of the W16×26 at midheight. Factored loads shown.

$$n = 1,\ \# = 2;\ \beta_{req'd} = 2\frac{2(120)}{0.75(120)} = 5.33 \text{ k/in}$$



$$\beta = \frac{F}{\Delta} = \frac{48\,E\,I}{(10 \times 12)^3}$$

$$I_{req'd} = \frac{5.33(120)^3}{48(29000)} = 6.6 \text{ in.}^4$$

TRY C5×6.7, $I_x = 7.5$, $S_x = 3.5$, $F_{br} = 0.01(120) = 1.2$ k

$f_b = 1.2(120)/4(3.5) = 10.3$ ksi *OK*

Continuous Bracing

For a column braced continuously, Timoshenko (1961) gives

$$P_{cr} = P_e\left(n^2 + \frac{\overline{\beta}L^2}{n^2\pi^2 P_e}\right) \qquad (1)$$

where n = number of half sine waves in the buckled shape as shown by the solid line in Figure 7. As the brace stiffness per unit length $\overline{\beta}$ increases, the buckling load and n also increase. The switch in buckling modes for each n occurs when $\overline{\beta}\,L^2 / \pi^2 P_e = n^2\,(n+1)^2$. Substituting this expression for n into Eq. (1) gives

$$P_{cr} = P_e + \frac{2L}{\pi}\sqrt{\beta\,P_e} \qquad (2)$$

Eq. (2) is an approximate solution, shown dashed in Figure 7, which gives the critical load for any value of $\overline{\beta}$ without the need to determine n. In the inelastic range use τP_e for P_e in Eq. (2).

Eq. (2) can also be used for discrete braces by defining $\overline{\beta} \equiv \beta \times$ number of braces / L and by limiting $P_{cr} \leq \pi^2\,EI/\ell^2$ where ℓ is the distance between braces. This approach is accurate for two or more braces. For example, if there are two discrete braces, the ideal discrete brace stiffness is $\beta = 3P_e/\ell$ where ℓ = L/3 and $P_{cr} = \pi^2 EI/\ell^2$. Using Eq. (2) with $\overline{\beta} = 2(3P_e/\ell)/L$ gives $P_{cr} = 1.01\,(\pi^2 EI/\ell^2)$.

The bracing design recommendation given below is based on Eq. (2) with $\overline{\beta}$ adjusted by a factor of two to limit the brace forces, adding a ϕ_{br} = 0.75, and

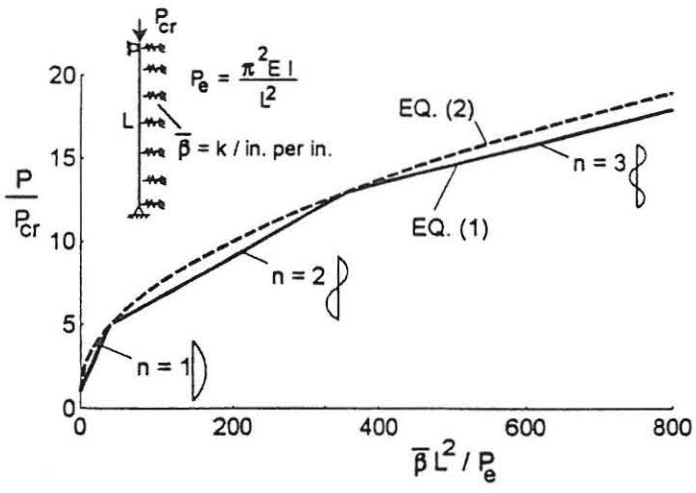


Figure 7 Continuous Bracing

using $P_o = 0.85\ (.877)\ \tau P_E$ which is the AISC-LRFD column design strength. formula. The brace strength requirement $\bar{F}_{br} = \pi^2 P\, \Delta_T / L_o^2$ was developed by Zuk (1956) where L_o is the max theoretical unbraced length that can support the column load. Taking $\Delta_T = 2\Delta_o$ and $\Delta_o = 0.002\ L_o$ gives $\bar{F}_{br} = 0.04\ P / L_o$.

DESIGN RECOMMENDATION LRFD	$\phi_c P_{cr} = P_o + (L/\pi)\ \sqrt{2\ \phi_{br}\ \bar{\beta}\ P_o}$; $F_{br} = 0.04\ P/L_o$ where $P_o = \phi_c\ (.877)\ \tau P_e$, $\phi_c = 0.85$, $\phi_{br} = 0.75$

Lean-On Systems

When some members lean-on adjacent members for stability support (bracing), the ΣP concept (Yura, 1971) can be used to design the members. The approach will be explained using the problem shown in Fig. 8, in which Col. A has a load P with three connecting beams attached between Cols. A and B. There are two principal buckling modes for this structure, the no sway and the sway modes. If Col. B is sufficiently slender, the system will buckle in the sway mode, shown by the dot-dash line in Figure 8a. In the sway mode the buckling strength involves the sum (ΣP_{cr}) of the buckling capacity of each column that sways. The system is stable in the sway mode if the sum of the applied loads (ΣP) is less than the ΣP_{cr}. This assumes all the columns have the same height. If Col. B is sufficiently stiff, the buckling capacity may be controlled by the no sway mode shown dashed. Both modes must be checked.

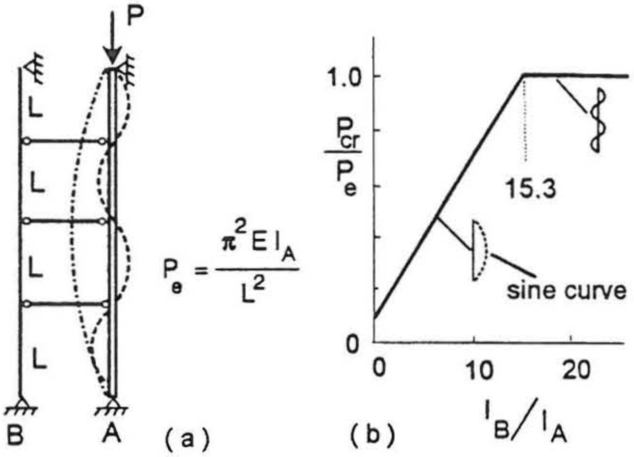


Figure 8 Lean-On Bracing

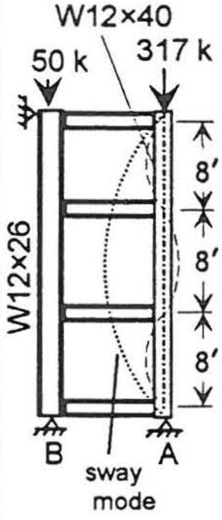
An exact elastic solution, developed with the ANSYS computer program, shows that as I_B increases, the Pcr increases linearly in the sway mode. At $I_B/I_A \geq 15.3$, Col. A buckles in the no sway mode. The I_b required to develop full bracing can be approximated using the ΣP concept. In the sway mode, the elastic capacities of Cols. A and B are $\pi^2 EI_A/(4L)^2$ and $\pi^2 EI_B/(4L)^2$, respectively. The desired P_{cr} corresponding to the no sway mode is $\pi^2 EI_A/L^2$. Equating the sum of the sway capacities to the P_{cr} in the no sway mode,

$$\pi^2 E(I_A + I_B / (4L)^2 = \pi^2 EI_A/L^2$$

gives $I_B = 15I_A$ which is close to the exact solution of $I_B = 15.3\ I_A$. In the inelastic range, τ_i is used where τ_i is based on the axial load in each column, P_i. There can be axial load on all the columns.

Example 3, which is similar to a problem solved by Lutz (1985), shows a W12 × 40 with its weak axis in plane supported by an adjacent column W12 × 26 with the strong axis in-plane. The tie beams have shear only end connections so it is assumed that the tie beams do not contribute to the sway-stiffness of the system. Sway is prevented at the top of the columns. The W12 × 40 has been sized based on buckling between the supports, L = 8 ft. The calculations show that the elastic W12 × 26 adjacent column can brace the weak axis column which is in the inelastic range. A W12 × 19 section would also be satisfactory.

EXAMPLE 3 - Lean-On System



AISC-LRFD Spec., F_y = 36 ksi, Factored loads

Is the W12×26 capable of bracing the W12×40 ?

From th AISC Manual, $\phi P_n = 317$ k for L = 8'

ΣP concept: W12×40, A = 11.6 in.², I_y = 44.1 in.⁴

W12×26, A = 7.65 in.², I_x = 204 in.⁴

Col A : $P_A / F_y A = 317/(36\times 11.8) = 0.746 > 1/3 \therefore$ inelastic

$$\tau = -7.38(0.746)\log(1.176\times 0.746) = 0.313$$

$$\phi P_A = 0.85(0.313)(0.877)\pi^2(29000)(44.1)/(288)^2$$

$$\underline{= 35.5\text{ kips}}$$

Col B : $P_B / F_y A = 50/(36 \times 7.65) = 0.181 < 1/3 \therefore \tau = 1.0$

$$\phi P_B = 0.85(0.877)\pi^2(29000)(204)/(288)^2 \underline{= 524\text{ kips}}$$

$\Sigma P = 35 + 524 = 559 > \Sigma P = 317 + 50 = 367$ k *OK*

Strength of Beams with Bracing

Before beam bracing design requirements are presented, some background material on flexural-torsional buckling of beams (usually just called lateral buckling) will be summarized. A beam with an unbraced length L_b will bend laterally and twist at a critical moment given by $M_{cr} = C_b(\pi/L_b)\ \sqrt{EI_yGJ + \pi^2E^2I_yC_w / L_b^2}$ as given in the AISC-LRFD Specifications. C_b is modification factor that accounts for variations in the moment diagram and support conditions. The unbraced length is defined as the distance between points braced against lateral displacement of the compression flange or between points braced to prevent twist of the cross section. Bracing systems for beams must prevent the relative displacement of the top and bottom flanges, i.e. twist of the section. Lateral bracing (joists attached to the top flange of a simply supported beam) and torsional bracing (cross frame or diaphragm between adjacent girders) can effectively control twist. M_{cr} is also affected by the load position (top flange loading is more detrimental) and end restraints (Galambos, 1988).

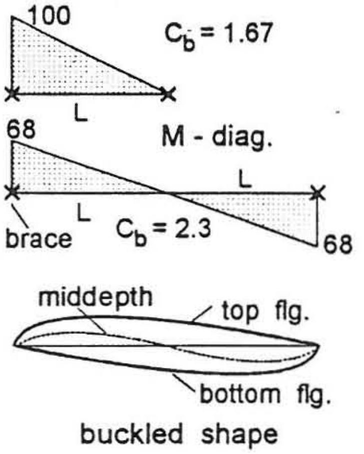
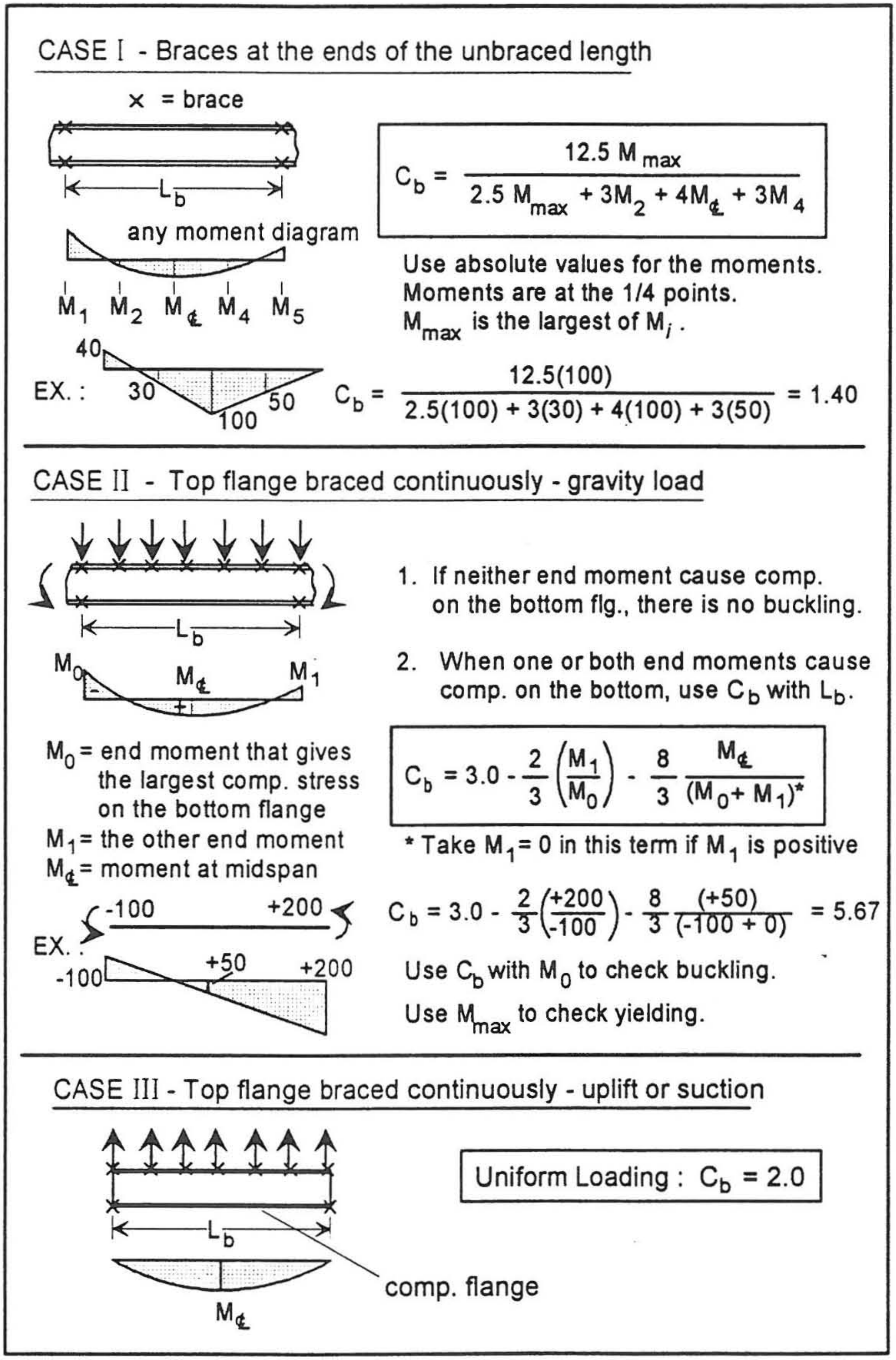


Figure 9 Beam with Inflection Point

The suitability of assuming the inflection point as a brace point in restrained beams to define L_b is frequently raised. In many cases the top flange is laterally braced by the slab or joists all along the span while the bottom flange is unbraced. An inflection points cannot be considered a brace point as illustrated by the example shown in Figure 9. One beam has a moment at one end (C_b = 1.67) with L_b = L and the other beam has an inflection point at midspan (C_b = 2.3) with L_b = 2L. The 2L span with the inflection point will buckle at a load that is 68% of the beam with span L. If the inflection point is a brace point, the critical moment of both beams would be the same. The buckled shape of the 2L beam shows that the top flange and bottom flange move laterally in opposite directions at midspan. Even an actual brace on one flange at the inflection point does not provide effective bracing at midspan.

The cases discussed above were solved using a finite element computer program and approximate C_b formulas developed as given in Figure 10. These C_b values can be used in design with L_b = span length if twist is positively controlled only at the supports. Three general cases are derived: bracing only at the ends, top flange laterally braced with top flange gravity loading and top flange braced with uplift loading. The C_b formula for Case I was adapted from Kirby and Nethercot (1979).



CASE I - Braces at the ends of the unbraced length

$$C_b = \frac{12.5\ M_{max}}{2.5\ M_{max} + 3M_2 + 4M_{℄} + 3M_4}$$

Use absolute values for the moments.
Moments are at the 1/4 points.
M_{max} is the largest of M_i .

$$C_b = \frac{12.5(100)}{2.5(100) + 3(30) + 4(100) + 3(50)} = 1.40$$

CASE II - Top flange braced continuously - gravity load

1. If neither end moment cause comp. on the bottom flg., there is no buckling.
2. When one or both end moments cause comp. on the bottom, use C_b with L_b.

$$C_b = 3.0 - \frac{2}{3}\left(\frac{M_1}{M_0}\right) - \frac{8}{3}\,\frac{M_{℄}}{(M_0 + M_1)^*}$$

* Take $M_1 = 0$ in this term if M_1 is positive

M_0 = end moment that gives the largest comp. stress on the bottom flange
M_1 = the other end moment
$M_{℄}$ = moment at midspan

$$C_b = 3.0 - \frac{2}{3}\left(\frac{+200}{-100}\right) - \frac{8}{3}\,\frac{(+50)}{(-100 + 0)} = 5.67$$

Use C_b with M_0 to check buckling.
Use M_{max} to check yielding.

CASE III - Top flange braced continuously - uplift or suction

Uniform Loading : $C_b = 2.0$

Figure 10 C_b for Braced Beams

Beam Bracing

Two types of bracing systems are available: lateral bracing and torsional bracing. A general discussion of beam bracing and the development of the design recommendations herein are presented elsewhere (Yura, 1993). Lateral bracing can be relative, discrete, continuous or lean-on. Only relative and discrete lateral bracing requirements are presented here. Continuous bracing is addressed by Trahair (1982) and Yura (1992). Beams that are linked together lean-on each other and the lateral buckling cannot occur at the links unless all the members buckle. Buckling of an individual beam can occur only between the cross members in a lean-on system. No additional bracing requirements are necessary in lean-on systems.

If two adjacent beams are interconnected by a properly design cross frame or diaphragm at midspan, that point can be considered a brace point when evaluating the beam buckling strength. Since the beams can move laterally at midspan, the effectiveness of such a bracing system is sometimes questioned. As long as the two flanges move laterally the same amount, there will be no twist. If twist is prevented, the beam can be treated as braced. Tests and theory confirm this approach (Yura, 1992)

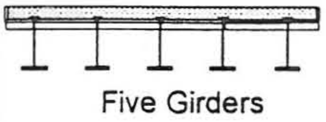
Lateral Bracing. The effectiveness and size of a lateral brace depends on its location on the cross section, the moment diagram, the number of discrete braces in the span, and location of load on the cross section. Lateral bracing is most effective when it is attached to the compression flange. The exception to this is for cantilevers where top (tension) flange bracing is effective. The design provisions herein are applicable only for bracing attached near the compression flange. The provisions also assume top flange loading which is a worse case. When the beam has an inflection point lateral bracing must be attached to both flanges and the stiffness requirements are greater as given by the C_d factor in the brace requirements. For example, for a beam in reverse curvature as shown in Figure 8, a brace on both the top and bottom flange at midspan will require twice as much stiffness as a similar length beam with compression on only one flange.

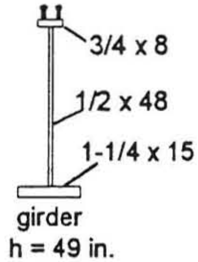
LATERAL BRACE DESIGN RECOMMENDATIONS, LRFD, $\phi = 0.75$

	Relative	Discrete
Stiffness:	$\beta_L = 2.5M_f C_d / \phi L_b h$	$\beta_L = 10M_f C_d / \phi L_b h$
Strength:	$F_{br} = 0.004 M_f C_d / h$	$F_{br} = 0.01 M_f C_d /h$

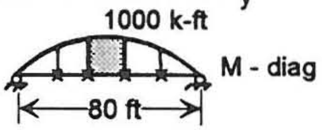
where M_f = max. moment, h = beam depth, L_b = unbraced length
C_d = 1.0 single curvature, = 2.0 reverse curvature

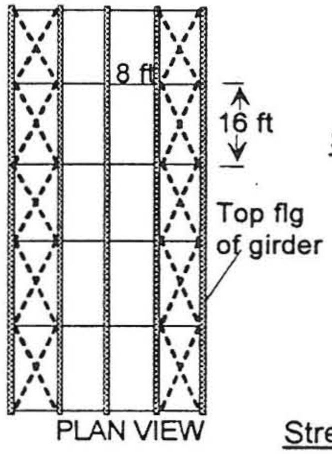
EXAMPLE 4 - Relative Lateral Brace System





Design the diagonals of the top flange horiz. truss to stabilize the five 80 ft girders with the factored moments shown. $F_y = 36$ ksi





Stiffness : $\beta_L = \dfrac{2.5(1000 \times 12)}{0.75(49)(16 \times 12)} = 4.25$ k/in ea. girder

$\times$ 2.5 girders = 10.6 k/in

$$\left(\frac{AE}{L}\right)_b \cos^2\theta = \frac{A_b(29000)}{8 \times 12 \times \sqrt{5}}\left(\frac{1}{\sqrt{5}}\right)^2 = 10.6$$

$A_b = \underline{0.393\ \text{in}^2}$ ⟵ CONTROLS

Strength : $F_{br} = 0.004(2.5)(1000 \times 12) / 49 = 2.45$ k

$A_b = 2.45\sqrt{5} / (0.9 \times 36) = \underline{0.17\ \text{in}^2}$

USE L 2 × 2 × 1/8 - A = 0.484 in^2

The lateral bracing provisions are illustrated in Example 4 where a top flange relative brace truss system is used to stabilize the compression flange during construction of the composite plate girders. Each truss system must stabilize 2.5 girders.

Torsional Bracing. Cross frames or diaphragms at discrete locations or continuous bracing provided by the floor system in through girders or Pony trusses, or by metal decks and slabs represent torsional bracing systems. In the development of the design recommendations (Yura, 1993), it was determined that factors that had a significant effect on lateral bracing had a substantially reduced effect on torsional bracing. The number of braces, top flange loading and brace location on the cross section are relatively unimportant when sizing a torsional brace. A torsional brace is equally effective if it is attached to the tension flange or the compression flange. A moment diagram with compression in both flanges (reverse curvature) does not alter the torsional brace requirements.

On the other hand, the effectiveness of a torsional brace is greatly affected by cross section distortion at the brace point as illustrated in Figure 11. The top flange is prevented from twisting by the torsional brace but the web distortion permits a relative displacement between the two flanges. A stiffener at the brace location can be used to prevent the distortion. The design method considers web

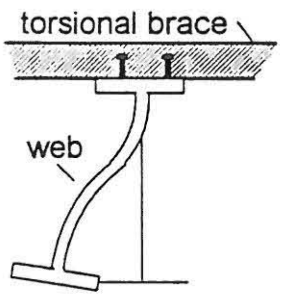


Figure 11

Web Distortion

distortion and any required stiffeners. Discrete braces and continuous bracing use the same basic design formula.

The continuous bracing stiffness $\overline{\beta}_T = \beta_T \, n / L$ where β_T = discrete brace stiffness, n = number of braces and L = span length. β_T and $\overline{\beta}_T$ are defined as the torsional stiffnesses of the bracing system. The system stiffness β_T is primarily related to the stiffness of the brace, β_b, and the stiffness of the web plus any stiffeners, β_{sec}, by

$$\frac{1}{\beta_T} = \frac{1}{\beta_{sec}} + \frac{1}{\beta_b} \qquad (3)$$

The β_b for diaphragm systems is given in Figure 12. The discrete web-stiffener detail can vary over the web as shown in Figure 13. The stiffness of each portion of the web is given by

$$\beta_c\,,\beta_s\,,\beta_t = \frac{3.3E}{h_i}\left(\frac{h}{h_i}\right)^2\left(\frac{(1.5h_i)t_w^3}{12} + \frac{t_s\,b_s^3}{12}\right) \qquad (4)$$

where $1/\beta_{sec} = \Sigma(1/\beta_i)$ and t_s is the thickness of the stiffener. For continuous bracing, replace 1.5h with 1 in. and neglect the t_s term if there is no stiffener. The design recommendations were developed for singly and doubly symmetric sections. The portion of the web within h_b can be considered inifintely stiff. For rolled sections ($h/t_w < 60$) cross-section distortion will not be significant if the diaphragm connection extends at least one-half the web depth. An initial twist of 1° (0.0175 radians) was used to develop the strength requirement, M_{br}.

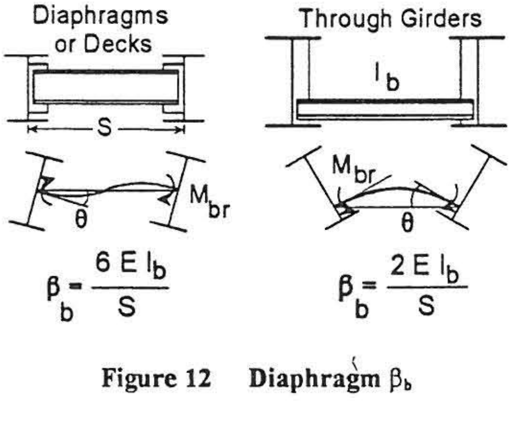


Figure 12 Diaphragm β_b

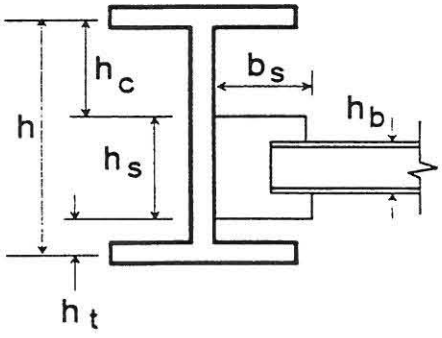
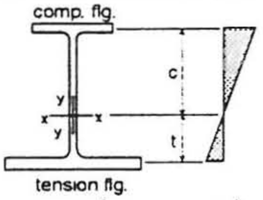


Figure 13 Partially Stiffened Webs

TORSIONAL BRACE DESIGN RECOMMENDATIONS, LRFD, $\phi = 0.75$

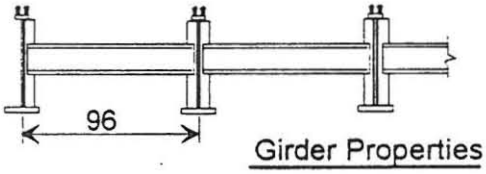


Stiffness: $\beta_T = \overline{\beta}_T L / n = 2.4 L M_f^2 / \left(\phi n E I_{eff} C_{bb}^2\right)$

Strength: $M_{br} = F_{br} h_b = 0.04 L M_f^2 / \left(n E I_{eff} C_{bb}^2\right)$

where M_f = max. moment, $I_{eff} = I_{yc} + (t/c) I_{yt}$, L = span length, n = number of span braces, and C_b = moment modification factor for the full bracing condition.

EXAMPLE 5 - Torsional Beam Bracing



Same as Ex. 4 except use diaphragm system shown.
M_{max} = 1000 k-ft, C_b = 1.0
4 braces, F_y = 36 ksi, L = 80 ft

Girder Properties

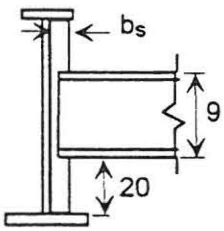
h = 49.0 , c = 30.85 , t = 18.15 in

$I_x = 17500$, $I_{yc} = 32.0$, $I_{yt} = 352$ in^4 $\quad I_{eff} = 32 + \dfrac{18.15}{30.85} 352 = 239$ in ^{4}

Strength : $$M_{br} = \frac{0.04(80 \times 12)(1000 \times 12)^2}{4(29000)239(1.0)^2} = 199 \text{ in-k}$$

$$S_{x\,req'd} = 199 / (0.9 \times 36) = 6.16 \text{ in}^3$$

Stiffness : The stiffness of the diaphragms on the exterior girders is $6EI_{br}/S$. Since there are diaphragms on both sides of each interior girder, the stiffness is $2 \times 6EI_{br}/S$. The average stiffness available to each girder is $(2 \times 6 + 3 \times 12)/5 = 9.6\ EI_{br}/S$.



$$\beta_{T\,req'd} = \frac{2.4\,(80 \times 12)(1000 \times 12)^2}{(0.75)4(29000)239(1.0)^2} = 15960 \text{ in-k / rad}$$

$$I_{br_{min}} = 15960(96) / (9.6 \times 29000) = 5.50 \text{ in}^4$$

Try C9×13.4 : $S_x = 12.5$ in$^3 > 6.16$, $I_x = 47.9$ in^4

$$\beta_b = 9.6(29000)47.9 / 96 = 138{,}900 \text{ in-k / radian}$$

$$\frac{1}{15960} = \frac{1}{138{,}900} + \frac{1}{\beta_{sec}} \; ; \; \beta_{sec} = 17900 \text{ in-k/rad}$$

$$\frac{1}{17900} = \frac{2}{\beta_c} \; ; \; \beta_c = 2(179000) = \frac{3.3(29000)}{20}\left(\frac{49}{20}\right)^2\left(\frac{1.5(20)(0.5)^3}{12} + \frac{0.375\, b_s^3}{12}\right)$$

b_s =3.10 - USE 3/8 × 3-1/2 stiffener

In EXAMPLE 5 a diaphragm torsional bracing system is used for the problem given in EXAMPLE 4. The C9 × 13.4 diaphragm will not brace the girders if a stiffener is not used. Even a much larger diaphragm cannot work without web stiffeners because of the web distortion. Similar example problems using cross frames are given elsewhere (Yura, 1993).

Summary

Brace design requirements involve both stiffness and strength. Care should be exercised when using published solutions that do not consider initial out-of-straightness. The recommendations contained here cover many practical situations. Work is underway to incorporate bracing recommendations in various steel design specifications which are currently lacking on the topic of bracing.

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(B)

SUMMARY OF BRACING RECOMMENDATIONS

1/15

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Austin, TX

March 1995

Relative Brace Systems

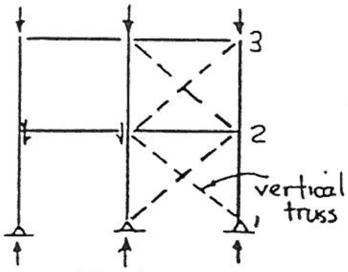


Fig. 1

A relative brace is one which controls the relative movement of adjacent stories or points along the length of the column, typical of vertical truss bracing, diagonal bracing or shear walls. An example of a relative brace system is shown in Fig. 1. Relative brace systems can be represented by the model shown in Fig. 2.

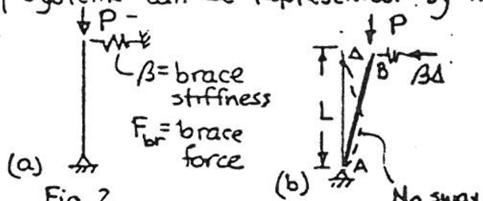


Fig. 2

$\Sigma M_A = 0$

$P\Delta = \beta \Delta L$

$\boxed{\beta = P/L}$

No sway if $\beta > P/L$ or $\beta > \beta_i = P_e/L = \frac{\pi^2 EI}{L^3}$

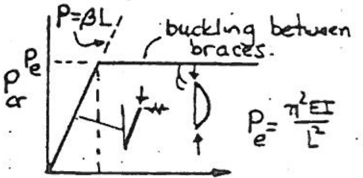


Fig 3

To prevent sway buckling, $\beta > P/L$. The sway buckling load increases linearly with brace stiffness until the brace reaches an ideal value, β_i. For $\beta > \beta_i$, there is no increase in buckling load; the column buckles between the braces. It was assumed above that the column is perfectly straight.

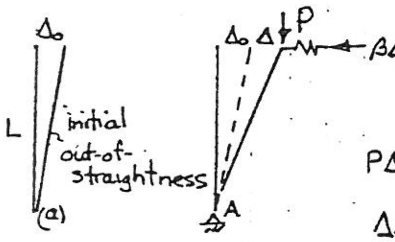


Fig. 4

$\Sigma M_A = 0$

$P(\Delta + \Delta_o) = \beta \Delta L$

$\Delta_T = \Delta_o + \Delta$

$P\Delta_T = \beta L(\Delta_T - \Delta_o)$

$$\Delta_T = \frac{\Delta_o}{1 - P/\beta L} = \frac{\Delta_o}{1 - P/P_{cr}}$$

$$\boxed{\beta = \frac{P}{L}\left(1 + \frac{\Delta_o}{\Delta}\right)}$$

$$\boxed{F_{br} = \beta\Delta = \frac{P}{L}(\Delta + \Delta_o)} = \beta L\left(\frac{\Delta_o}{L}\right)\left(\frac{1}{\frac{\beta L}{P} - 1}\right)$$

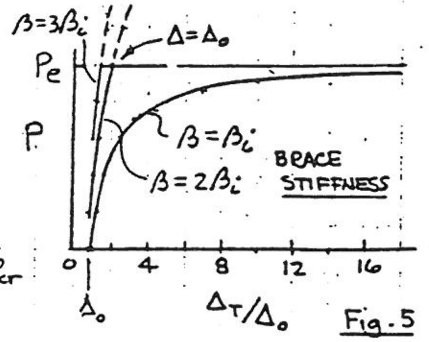


Fig. 5

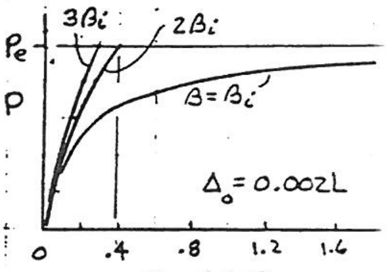


Fig. 6

Fig. 5 shows that P_e can be reached for $\beta = \beta_i$ only if the deflection get very large. Unfortunately such large displacements produce large brace forces. At $P = 0.9P_e$, $\Delta_T = 10\Delta_o$ and $F_{br} = 1.8\% P_e$ as shown in Fig. 6. For practical design the brace stiffness must be larger than β_i so brace forces will be more reasonable. If $\beta = 2\beta_i$ is chosen for design, $\Delta = \Delta_o$ at P_e and $F_{br} = 0.4\%$. Usually, the brace stiffness provided is much larger than $2\beta_i$ and this will reduce the brace force required at P_e even further because Δ is reduced. For example, if $\beta = 10\beta_i$, $F_{br} = 0.22\% P_e$. At $\beta = 1.25\beta_i$, $F_{br} = 2\% P_e$

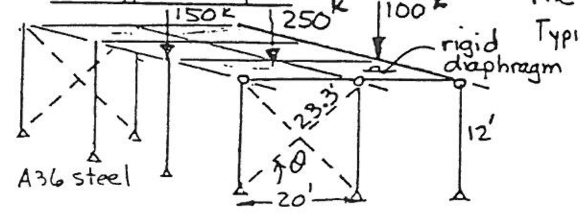
DESIGN RECOMMENDATION
LRFD

$\beta_{REQ'D} = \frac{2P}{\phi L}$; $F_{br} = 0.004P$

where P = factored load L = distance between braces, $\phi = 0.75$

The brace force recommendation is based on the assumption that $\Delta_o = 0.002L$. Δ_o is the displacement at the brace point caused by wind or other lateral forces, erection tolerance (initial out-of-plumb), bolt hole oversize, etc. If Δ_o is different from 0.002L, change the F_{br} in direct proportion to the actual Δ_o. No change is necessary for β.

Design Example



The factored load on each bent = $(150+250+100^k) = 500^k$

Typical brace must stabilize 3 bents so $P = 3 \times 500 = 1500$ k

Design recommendations assume F_{br} and Δ are perpendicular to the column.

Brace Force: $0.004(1500)/\cos\theta = 6.99^k$ ($\frac{5}{8}$ threaded rod$=10.0^k$,

Stiffness: $\frac{A_b E}{L_b}\cos^2\theta = \frac{2(1500)}{0.75(12)}$; $A_{b_{gross}} = 0.364\text{ in}^2$ ($\frac{3}{4}\phi$, $A_g = .44$)

Single Point Bracing Systems

-also called discrete braces

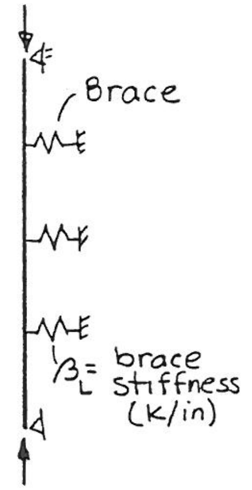


FIG.1

A single point brace controls the deflection of the column at the point of attachment, only. The brace stiffness is not affected by deflection at other points along the column. An example of a single point brace is shown below.

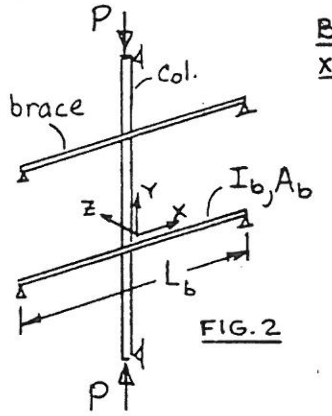
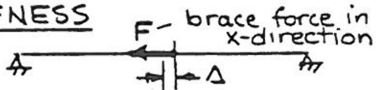


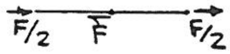
FIG. 2

BRACE STIFFNESS

x-direction:



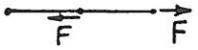
If the brace can support compression,



$$\Delta = \frac{\left(\frac{F}{2}\right)\left(L_b/2\right)}{A_b E}$$

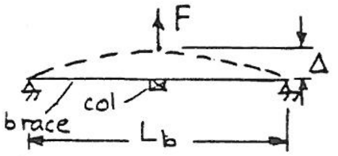
$$\beta_{Lx} = \frac{F}{\Delta} = \frac{4A_bE}{L_b}$$

If the brace system is a tension system,



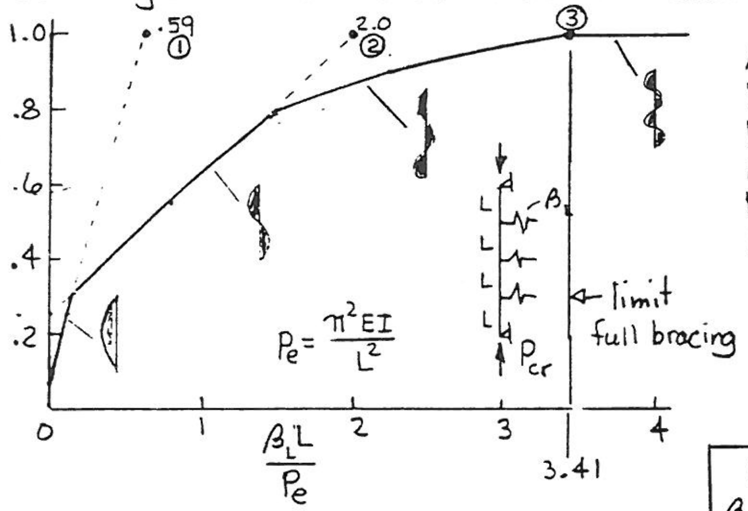
$$\Delta = \frac{F\frac{L_b}{2}}{A_b E} \; ; \; \beta_{Lx} = \frac{2A_bE}{L_b}$$

Z-direction:



$$\Delta = \frac{FL_b^3}{48EI_b} \quad ; \quad \beta_{Lz} = \frac{48EI_b}{L_b^3}$$

Single point bracing systems can be represented as shown in Fig.1. The exact solutio is complex; a plot of the relationship between P_{cr} and β_L taken from Timoshenko & Gere i given in Fig 3 for the case of three intermediate braces. With no bracing $P_{cr} = \frac{\pi^2 E}{(4L}$



At low brace stiffness, the buckling load increases substantially with the buckled shape a single (1st Mode) wave. As the brace stiffness is increased, the buckled shape changes and additional brace stiffness becomes less effective. Full bracing occurs at $\beta_L L/P_e = 3.41$. The ideal full bracing stiffness for various numbers of intermediate braces is given in Timo. & Gere and summarized below. The maximum stiffness = $4\beta_L L/$

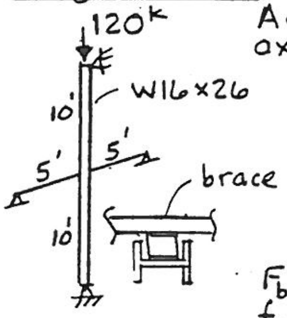
# of braces	1	2	3	4	5	large
$\beta_L L/P_e$	2	3	3.41	3.63	3.73	4.0

DESIGN
RECOMMENDATION
LRFD

$$\beta_{REQ'D} = \# \frac{2P}{\phi L} \; ; \; F_{br} = 0.01P$$

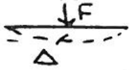
where
P = factored load, L = req'd bracing spacing
$\phi = 0.75$, # between 2-4 from table
or # ≈ 4 - (2/n), n = no. braces

Design Example



A cross member braces the wea axis of the W16×26 at midheig

$$\beta_{REQ'D} = 2\frac{2(120)}{.75(120)} = 5.33 \text{ k/in}$$



$$\beta = \frac{F}{\Delta} = \frac{48EI}{L^3}$$

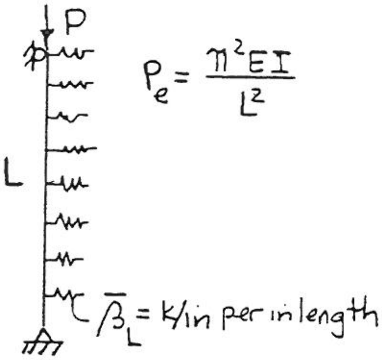
$$I_{REQ'D} = \frac{5.33(120)^3}{48(29000)} = 6.6 \text{ in}^4$$

TRY C5x6.

$F_{br} = 0.01(120) = 1.20\text{k}$ $I_x = 7.5, S_x =$

$f_b = 1.2(120)/4(3.5) = 10.3$ ksi OK

CONTINUOUS COLUMN BRACING



$$P_e = \frac{\pi^2 EI}{L^2}$$

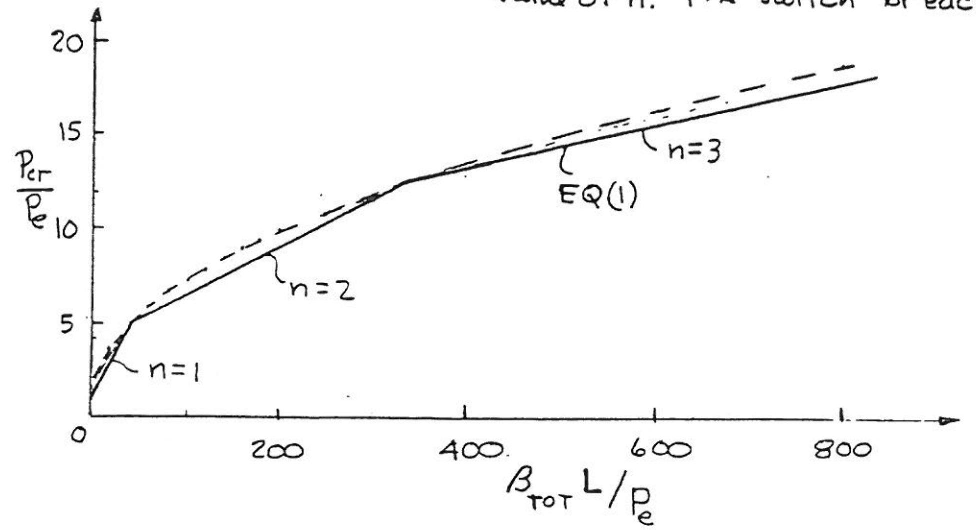
From Timoshenko & Gere

$$P_{cr} = P_e \left(n^2 + \frac{\bar{\beta}_L L^2}{n^2 \pi^2 P_e}\right) \quad (1)$$

where n = number of half sine waves in the buckled shape

To use this solution with a given $\bar{\beta}_L$, you must subsitute n=1, 2, 3 etc in Eq(1) and use the smallest result. Eq(1) has no limit except $P_{cr} \le P_{yield}$. A plot of Eq(1) for values of n up to 3 is given in Fig. 1 where $\beta_{TOT} = \bar{\beta}_L \times L$ = total brace stiffness. The solution is a linear function of $\bar{\beta}_L$ for every value of n. The switch for each n occurs when

$$\frac{\bar{\beta}_L L^2}{\pi^2 P_e} = n^2 (n+1)^2 \quad (2$$



Substitute (2) into (1) gives an approximate solution shown dashed which matches at the switch points as follows

$$\boxed{P_{cr} = 2\sqrt{\bar{\beta}_L E I_y} + P_e} \quad ($$

Eq(3) gives the critical load for any value of $\bar{\beta}_L$ directly so it can be used in design without determining n. Eq(3) should be adjusted for initial-out-of-straightness by using a brace twice as stiff. In LRFD replace P_e in (3) with $P_o = \phi_c (.877) \tau^* P_e$

$$\boxed{p \le \phi_c P_{cr} = P_o + \left(\frac{L}{\pi}\right)\sqrt{2\phi_{br}\bar{\beta}_L P_o} \quad \text{and} \quad F_{br}^{**} = 0.04 P/L_o} \quad (4)$$

$\phi_c = 0.85$, $\phi_b = 0.75$

and where L_o = max. theoretical unbraced length that can supported the factored load P

Eq (4) can also be used for single point braces by defining $\bar{\beta}_L = \frac{\beta_L \times \text{No. of braces}}{L}$

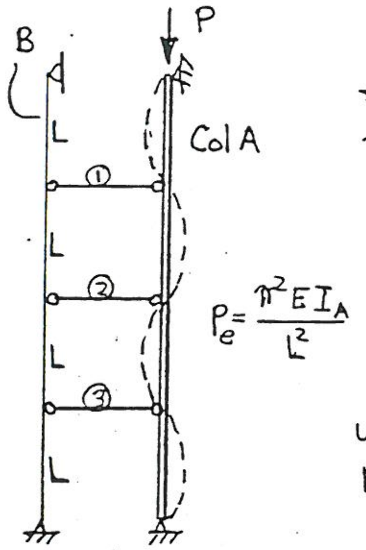
For single point braces, Eq(4) must be limited to $P_{cr} = \pi^2 EI/a^2$ where "a" is the distance between braces. The use of Eq(4) for single point braces is accurate for 2 or more braces. For one brace, use $\bar{\beta}_L = \beta_L / .75L$ or the single brace approach on sheet B.

* In LRFD, $\tau = -7.38 (P/P_y) \log[(P/P_y)/0.85]$; in ASD use the stiffness reduction factor on p 3-8, 9th Ed AISC ASD Manual (sample on p. E

** See State of Art paper for derivation

P = factored column load, P_y = yield load, $F_y A$

LEAN-ON SYSTEMS



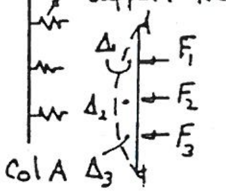
$$P_e = \frac{\pi^2 E I_A}{L^2}$$

FIG 1

Col A is supported by member B. What I_b is req'd for full bracing?

This system is usually modeled as a single point brace system but the spring stiffness is difficult to evaluate as follows:

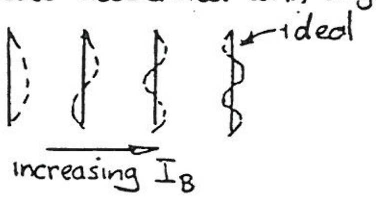
support from B. Are all these springs equal stiffness?



Is $F_1 = F_2 = F_3$?

There are many possibilities what is correct?

Will col A bend through the various modes associated with single point bracing as the I_B is increased?



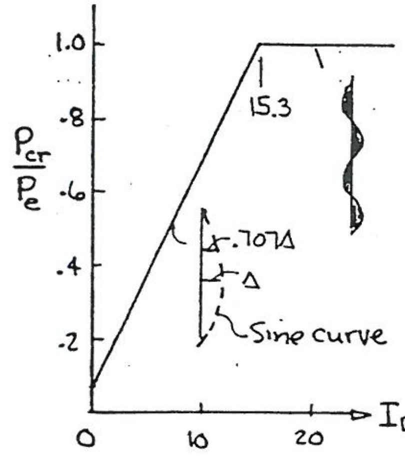


FIG 2

The exact solution to the system shown in Fig 1 indicates that the response is not similar to the single point system. As the I_B is increased, the buckling load increases linearly until the ideal brace situation is reached when buckling occurs between the supports. The response shown in Fig 2 indicates that the buckled shape is always a half sine curve until the full bracing is achieved when $I_B = 15.3 I_A$. There is no switching from one shape to the next higher mode as shown for single point bracing.

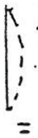
LEAN-ON BRACING IS NOT THE SAME AS SINGLE POINT BRACING

A lean-on system is one in which the "bracing" member must have the same shape as the "buckling" member. Such systems can be solved using the ΣP concept.

EX 1. - Problem above
Elastic behavior

What I_B is necessary to permit $P = \pi^2 E I_A / L^2$

B $\quad \frac{\pi^2 E I_B}{(4L)^2}$



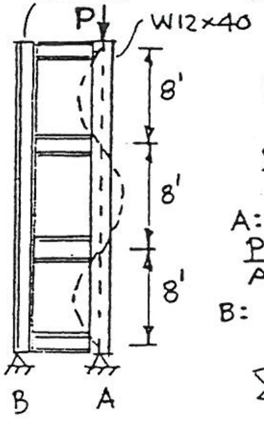
A $\quad \frac{\pi^2 E I_A}{(4L)^2}$

$$\Sigma P_{cr} = P$$

$$\frac{\pi^2 E I_B}{16L^2} + \frac{\pi^2 E I_A}{16L^2} = \frac{\pi^2 E I_A}{L^2}$$

$\therefore I_B = 15 I_A$ - close to 15.3

EX 2. - Crane column - Problem from Lutz-Fisher, AISC Jour, 4th Q, 1985
$\boxed{\text{ASD}}$ - A36 steel



Is the W12×26 sufficient to fully brace the W12×40?

From AISC Manual $P_{allow} = 217^k$ for L = 8'

$\Sigma P_{concept}$: W12×40, $A = 11.8 in^2$, $I_y = 44.1 in^4$, $r_y = 1.93$
W12×26, $A = 7.65$ $I_x = 204 in^4$, $r_x = 5.17$

A: $\frac{P}{A} = \frac{217}{11.8} = 18.4$ ksi, $\tau = .301$; $P_A = \frac{12}{23} \frac{\pi^2 (29000)(44.1)}{(288)^2} (.301) = 24^k$

B: $\tau = 1.0$, $P_B = \frac{12}{23} \frac{\pi^2 (29000)(204)}{(288)^2} = 367^k$

$\Sigma P_{cr} = 367 + 24 = 391k > 217k$ OK

W12×19 would work also

E

τ 9th Edition AISC

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F_y = 36 ksi
F_y = 50 ksi

Table A
Stiffness Reduction Factors f_a/F'_e

f_a	F_y	
	36 ksi	50 ksi
28.0	—	0.097
27.9	—	0.104
27.8	—	0.112
27.7	—	0.120
27.6	—	0.127
27.5	—	0.136
27.4	—	0.144
27.3	—	0.152
27.2	—	0.160
27.1	—	0.168
27.0	—	0.177
26.9	—	0.184
26.8	—	0.193
26.7	—	0.202
26.6	—	0.210
26.5	—	0.218
26.4	—	0.227
26.3	—	0.236
26.2	—	0.245
26.1	—	0.253
26.0	—	0.262
25.9	—	0.271
25.8	—	0.280
25.7	—	0.288
25.6	—	0.297
25.5	—	0.306
25.4	—	0.315
25.3	—	0.324
25.2	—	0.333
25.1	—	0.342
25.0	—	0.350
24.9	—	0.359
24.8	—	0.368
24.7	—	0.377
24.6	—	0.386
24.5	—	0.394
24.4	—	0.403
24.3	—	0.412
24.2	—	0.421
24.1	—	0.430
24.0	—	0.439
23.9	—	0.447
23.8	—	0.456
23.7	—	0.465
23.6	—	0.473
23.5	—	0.482
23.4	—	0.490
23.3	—	0.499
23.2	—	0.507
23.1	—	0.516
23.0	—	0.524
22.9	—	0.533
22.8	—	0.541
22.7	—	0.549
22.6	—	0.557
22.5	—	0.565
22.4	—	0.574
22.3	—	0.582
22.2	—	0.590
22.1	—	0.598
22.0	—	0.606
21.9	—	0.614
21.8	—	0.622
21.7	—	0.630
21.6	—	0.637
21.5	—	0.645
21.4	—	0.653
21.3	—	0.660
21.2	—	0.668
21.1	—	0.675
21.0	—	0.683
20.9	—	0.689
20.8	—	0.697
20.7	—	0.704
20.6	—	0.712
20.5	0.064	0.718
20.4	0.074	0.725
20.3	0.083	0.732
20.2	0.093	0.739
20.1	0.102	0.746
20.0	0.114	0.753
19.9	0.125	0.760
19.8	0.136	0.766
19.7	0.147	0.772
19.6	0.158	0.778
19.5	0.169	0.785
19.4	0.181	0.792
19.3	0.193	0.798
19.2	0.204	0.804
19.1	0.216	0.810
19.0	0.228	0.816
18.9	0.241	0.822
18.8	0.252	0.827
18.7	0.264	0.833
18.6	0.277	0.839
18.5	0.288	0.844
18.4	0.301	0.849
18.3	0.314	0.855
18.2	0.326	0.860
18.1	0.338	0.865
18.0	0.350	0.871
17.9	0.363	0.875
17.8	0.375	0.880
17.7	0.387	0.885
17.6	0.400	0.890
17.5	0.411	0.894
17.4	0.424	0.899
17.3	0.436	0.903
17.2	0.448	0.908
17.1	0.460	0.912
17.0	0.472	0.917
16.9	0.484	0.920
16.8	0.496	0.924
16.7	0.508	0.928
16.6	0.519	0.932
16.5	0.531	0.935
16.4	0.543	0.939
16.3	0.554	0.942
16.2	0.565	0.946
16.1	0.577	0.950
16.0	0.588	0.952
15.9	0.599	0.956
15.8	0.610	0.959
15.7	0.621	0.962
15.6	0.632	0.964
15.5	0.643	0.967
15.4	0.653	0.970
15.3	0.664	0.972
15.2	0.675	0.974
15.1	0.684	0.977
15.0	0.695	0.979
14.9	0.704	0.981
14.8	0.715	0.983
14.7	0.724	0.985
14.6	0.734	0.987
14.5	0.743	0.988
14.4	0.753	0.990
14.3	0.762	0.991
14.2	0.770	0.993
14.1	0.780	0.994
14.0	0.789	0.995
13.9	0.797	0.996
13.8	0.805	0.997
13.7	0.814	0.998
13.6	0.822	0.998
13.5	0.830	0.999
13.4	0.838	0.999
13.3	0.845	1.000
13.2	0.853	—
13.1	0.860	—
13.0	0.868	—
12.9	0.874	—
12.8	0.881	—
12.7	0.888	—
12.6	0.895	—
12.5	0.901	—
12.4	0.907	—
12.3	0.913	—
12.2	0.918	—
12.1	0.924	—
12.0	0.929	—
11.9	0.934	—
11.8	0.939	—
11.7	0.944	—
11.6	0.949	—
11.5	0.953	—
11.4	0.958	—
11.3	0.962	—
11.2	0.966	—
11.1	0.970	—
11.0	0.973	—
10.9	0.976	—
10.8	0.979	—
10.7	0.982	—
10.6	0.984	—
10.5	0.987	—
10.4	0.989	—
10.3	0.991	—
10.2	0.993	—
10.1	0.995	—
10.0	0.996	—
9.9	0.997	—
9.8	0.998	—
9.7	0.999	—
9.6	1.000	—

STIFFNESS OF BRACING SYSTEMS

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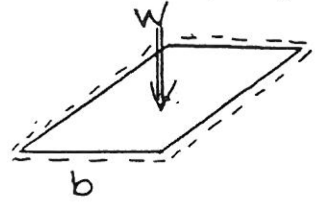
In the evaluation of the brace stiffness, the effect of connection flexibility must be included using the following general expression

$$\boxed{\frac{1}{\beta_{system}} = \frac{1}{\beta_{brace}} + \frac{1}{\beta_{conn}}} \qquad (1)$$

Eq(1) indicates that the system stiffness must always be smaller than the smallest stiffness, either the brace or the connection

Frequently, braces are framed into flat plates, such as webs of W shapes. The deflection of a flat plate subjected to a concentrated load at the center (given in most good strength of materials texts) is,

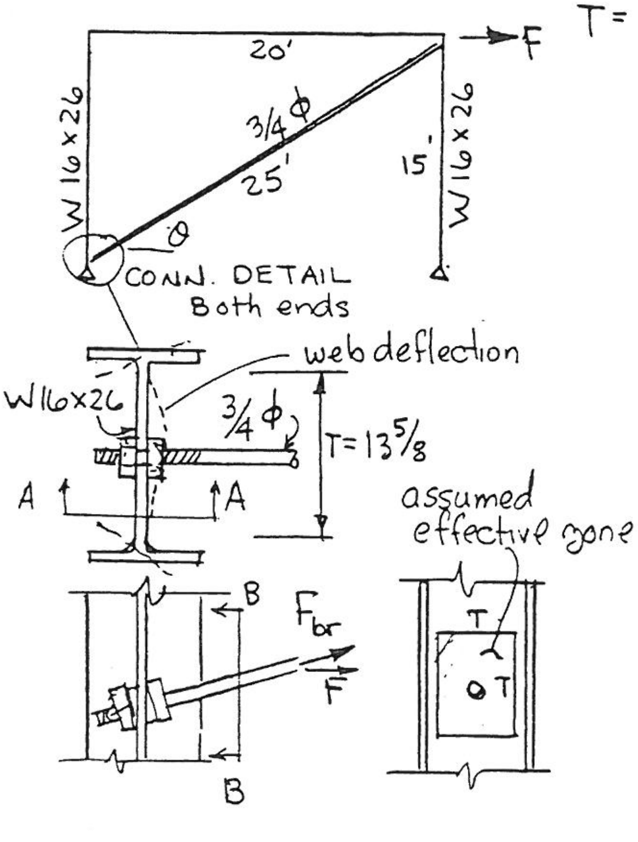


$$\Delta_{\mathbb{C}} = c\,(1-\overset{0.3 \text{ for metals}}{\nu^2})\,W b^2/E t^3 \qquad (2)$$

c:
- 0.138 Simply supported edges
- 0.067 fixed edges

Example:

A 3/4 ϕ diagonal brace is attached to the center of the web of a W16×26. Determine the brace system stiffness.



T = clear distance between fillets

Rod stiffness:

$\frac{3}{4}\phi$, $A = 0.44\ \text{in}^2$

$$\beta_b = \frac{AE}{L_{br}}\cos^2\theta = \frac{0.44(29000)}{25(12)}\left(\frac{20}{25}\right)^2$$

$$= 27\ \text{k/in}$$

Connection:

- assume S.S. edges - Eq(2), $t_w = 0.25$

$$\Delta_{\mathbb{C}} = 0.138(.91)\frac{F\,(13.625)^2}{29000\,(.25)^3}$$

$$\beta_{conn} = \frac{F}{\Delta_{\mathbb{C}}} = 19.4\ \text{k/in} \quad \text{- each end}$$

$$\frac{1}{\beta_{sys}} = \frac{1}{27} + \frac{1}{19.4} + \frac{1}{19.4} \quad \therefore \underline{\underline{\beta_{sys} = 7.1\ \text{k/in}}}$$

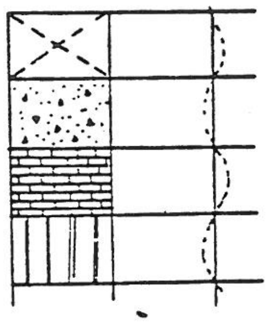
only 25% of rod. stif

BRACING REQUIREMENTS

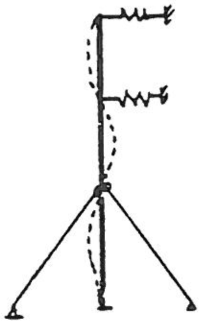
Brace requires STIFFNESS & STRENGTH

- Depends on the type of brace i.e. whether the brace controls the movement at a single particular point or whether it controls the <u>relative</u> movement between two points (or stories)

RELATIVE



SINGLE POINT



THEORY:

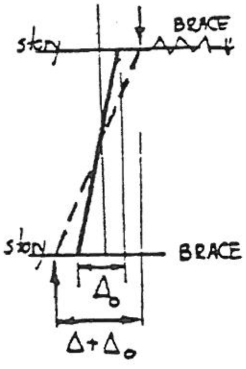
STIFFNESS, $\beta = \frac{P}{L}\left(1+\frac{\Delta_o}{\Delta}\right)$

STRENGTH, $\underset{\llcorner \text{horizontal component}}{F_{br}} = \frac{P}{L}(\Delta+\Delta_o)$

$\beta = A\frac{P}{L}\left(1+\frac{\Delta_o}{\Delta}\right)$ where A varies between 2.0 and 4.0

$F_{br} = 2\frac{P}{L}(\Delta+\Delta_o)$

where P = Sum of column loads in a story to be stabilized by the brace. In the case of a point brace P would be the average load in the compression member above and below the brace point. In the case of a beam or beam-column P would be the compressive force in the member. P is the service load.



L = Story height or distance between braces

Δ_o = small displacement from straight position at the brace point caused by sources other than the gravity loads or compressive forces. For example Δ_o would be a displacement caused by wind or other lateral forces, erection tolerance (initial out-of-plumb), etc.

Δ = additional displacement at the brace point as a result of the compressive forces or gravity loads.

NOTE: TOTAL SMALL DISPLACEMENT AT THE BRACE POINT $= \Delta + \Delta_o$
Δ and Δ_o are measured relative to adjacent brace points.

DESIGN: Assuming $\Delta = \Delta_o$ and $\Delta_o = 0.002L$ and using a safety factor of 2.0 in stiffness requirement (factor of safety for strength is handled directly by allowable stress) the theory requirements above become:

conservatively uses A=4.0

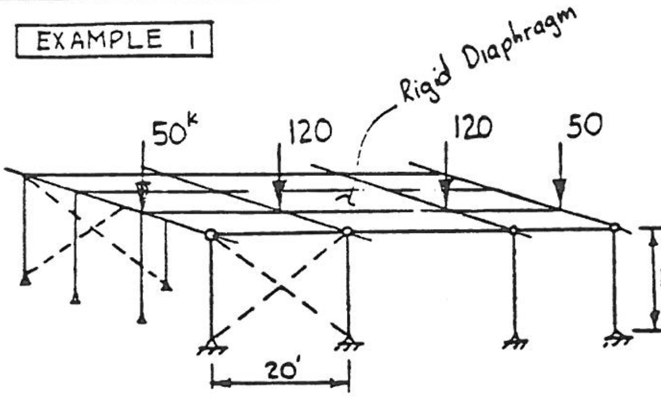
DESIGN FORMULAS

$\boxed{\beta_{REQ'D} = 4P/L \;;\; F_{br} = 0.004P}$ RELATIVE

or

$\boxed{\beta_{REQ'D} = 16P/L \;;\; F_{br} = 0.01P}$ SINGLE POINT

EXAMPLE 1



REQ'D: SIZE OF TENSION BRACES TO STABILIZE THE SYSTEM

$P/\text{brace} = 3(50+120+120+50) = 1020\,k$

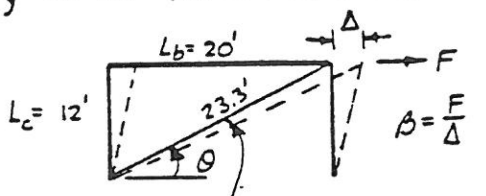
RELATIVE BRACE

$$\beta_{REQ'D} = 4P/L = 4(1020)/12' = 340\,{}^{k}/_{FT}$$

$$F_{br} = 0.004\,(1020) = 4.08\,k$$

These requirements assume the brace is perpendicular to the columns to be braced

Tension Bracing system every third bent
Roof acts as rigid diaphragm to support "unbraced" bents
$F_y = 36$ ksi (See Ex. 4 for a case with gravity plus wind loads)



A_{br} = area of brace
$L_{br} = 23.3'$

Brace Force $= F_{br}/\cos\theta = 4.08/\cos\theta = 4.76^{k}$

$F_t = 0.6F_y = 22$ ksi ; $A_{br\,NET} = \frac{4.76}{22} = 0.216\ in^2$

Assuming rod is threaded – $\frac{5}{8}\phi$ REQ'D

Brace Stiffness

$$\left(\frac{A_{br}E}{L_{br}}\right)\cos^2\theta = \frac{F}{\Delta} = 340\,{}^{k}/_{FT}$$

$$\frac{A_{br}\,(29{,}000)}{23.3}\left(\frac{20}{23.3}\right)^2 = 340\ ;\ A_{b_{gross}} = 0.372\ in^2$$

STIFFNESS GOVERNS $\longrightarrow$ $\frac{3}{4}\phi$ REQ'D

USE $\frac{3}{4}\phi$ $F_y = 36$ ksi

EXAMPLE 2

– same problem as Ex. 1 but use steel shear diaphragms instead of tension rods – assume corrugated sheet

Brace requirements are the same : $\beta = 340\,{}^{k}/_{FT}$, $F_{br} = 4.08\,k$

Try 20ga. ($t = 0.036$ in) – get strength and stiffness of corrugated sheet from American Iron and Steel Institute Booklet "Design of Light Gage Steel Diaphragms)

Allowable strength $= 0.22\,{}^{k}/_{FT}$ $F_{allow} = 0.22 \times 20' = 4.4^{k} > 4.08^{k}$ ok

Stiffness ($t = 0.036$ and $L = 20'$) $= 1820\,{}^{k}/_{FT} > 340$ req'd ok

USE 20ga

EXAMPLE 3

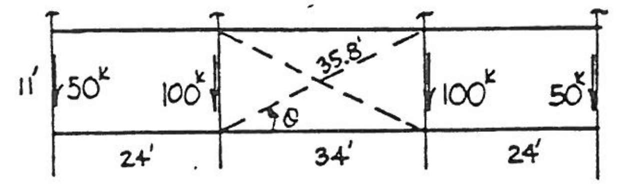
– same problem as Ex. 1 but use brick shear wall
Try 4in. brick wall – From "Recommended Building Code Requirements for Engineered Brick Masonry" min allow shear stress = 40 psi

$F_{allow} = 40[4 \times 20(12)] = 38.4^{k} > 4.08^{k}$ OK

Stiffness – From "The Behavior of One Story Brick Shear Walls" by Benjamin & Williams Proc. ASCE Vol. 84 July 1958 $\beta = 200\,{}^{L_b}/_{L_c} \times$ brick thickness $= 1330\,{}^{k}/_{FT} > 340$ OK

USE 4in brick

EXAMPLE 4 - Multistory Frame



Type 2 Construction - simple framing, $F_y = 36$ ksi.
Bracing every third bent
Wind shear per bent (this level) = 6.5^k

Bracing req'd to stabilize
Wind shear = $3(6.5^k) = 19.5^k$
Col. gravity loads = $3(50+100+100+50) = 900^k$

Notes:
1. Bracing must be adequate to stabilize the structure under gravity load alone and combined wind and gravity load.
2. Axial loads in the columns due to gravity load alone are shown for a typical bent. Axial loads are the same in the "unbraced" bents. Floor diaphragms or bracing are used to transmit the wind shears and PΔ overturning moments to the braced bent.
3. Even though the wind forces alter the distribution of the axial forces in the columns, the sum of the column loads must be equal to the applied gravity loads so the shears due to the PΔ moments are unaffected.
4. Use 33% increase in allowable stresses for the combined load case.

Brace stability requirements - relative brace

$$F_{br} = 0.004(900^k) = 3.6^k$$
$$\beta_{REQ'D} = 4P/L = 4(900)/11' = 328\ ^k/FT$$

Gravity:

Stiffness: $\frac{A_{br}E}{L_{br}}\cos^2\theta = 328\ ^k/FT$; $A_{br\ gross} = \frac{328\,(35.8)}{29{,}000}\left(\frac{35.8}{34}\right)^2 = 0.451\ in^2$

Strength: $F_{br} = 0.004\,P = 0.004(900) = 3.6^k$ $F_b/\cos\theta = 3.8^k$

$$A_{br\,net} = 3.8/.6F_y = 3.8/22 = 0.173\ in^2$$

Gravity plus Wind:

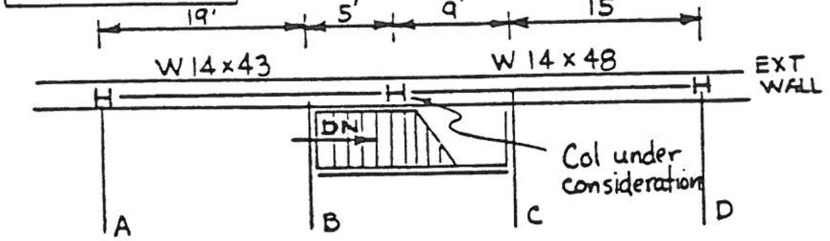
Stiffness: - no change from gravity $A_{br\,gross} = 0.451\ in^2$

Strength: $A_{br} = \frac{19.5}{1.33(22)\cos\theta} + \frac{3.8}{1.33(22)} = 0.830\ in^3$ (Anet) ← controls

$$\boxed{\text{use } 1\tfrac{1}{4}\,\phi \text{ Threaded Rod } (A_{Net} = 0.969\ in^2)}$$

Note that the brace strength requirement for gravity loads is added to the requirement for wind alone. Do not add the larger stiffness requirement.

EXAMPLE 5 - POINT BRACES



12'-0" Story height

Column load above = 175^k
Column load below = 200^k

Exterior column in a multistory frame. The only lateral support of the column under consideration at the floor levels is provided by the weak-axis bending strength and stiffness of the W sections shown. Headroom requirements do not permit a beam to frame directly into the web of the column. Determine if these beams have sufficient strength and stiffness to brace the column at the floor level. It is assumed that the spandrel beams are laterally braced at locations A, B, C and D.

The beams act only to control the column movement at this particular floor level so they are single point braces. Therefore the bracing requirements are:

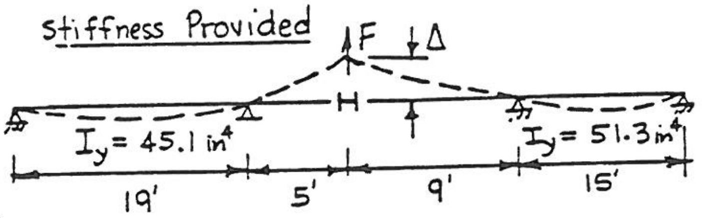
$\beta_{REQ'D} = 16P/L$ and $F_{br} = 0.008P$

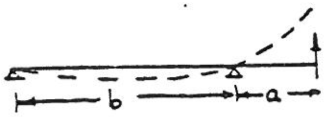
P = avg. column load = $(175+200)/2 = 188^k$; L = 12'

$\beta_{REQ'D} = \frac{16(188)}{12} = 250^{k/'}$

$F_{br} = 0.01(188) = 1.88^k$

Stiffness Provided





$\Delta_1 = \frac{F_1 a^2(a+b)}{3EI}$ (AISC Manual) $\beta = \frac{F_1}{\Delta_1} = \frac{3EI}{a^2(a+b)}$

W 14×43 ; $\beta_1 = 3(29000)45.1/(5)^2(24)144 = 45.4$ k/FT

W 14×48 ; $\beta_2 = 3(29000)51.3/(9)^2(24)144 = \underline{28.7}$ k/FT

$\beta_{TOTAL} = 74.1 < 250$ k/FT N.G.

~~TRY~~ USE

W 14×78 left side $\beta = 208$ k/FT $> \beta_{TOTAL} = 268 > 250$ k/FT OK

W 14×61 right side $\beta = 60$

Check strength - The 1.88^k required force is provided by the beams in proportion to their stiffnesses

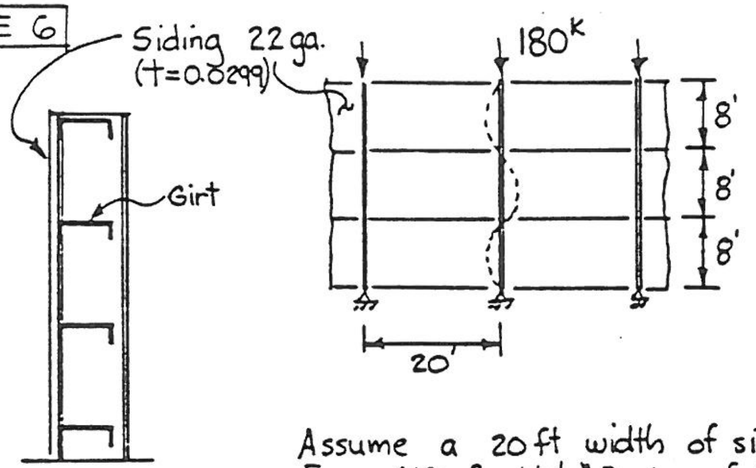
W 14×78 ($S_y = 34.5\ in^3$)

$F_{W14x78} = 1.88\left(\frac{208}{268}\right) = 1.46^k$; $M_y = 1.46^k(5\times12) = 88$ in-k

$f_{by} = M_y/S_y = 88/34.5 = 2.6$ ksi OK when considered with f_{bx} also

$\boxed{\text{USE W 14x78 and W 14x61}}$ - COL CAN NOW BE SAFELY DESIGNED FOR KL = 12'

EXAMPLE 6



Is the 22ga. siding sufficient to support the girts so that the columns are braced at the one-third points? Siding is attached to the girts in a standard manner using screws for fasteners.

Assume a 20 ft width of siding supports each column
From AISI Booklet "Design of Light Gage Steel Diaphragms"
22 ga × 20'

$F_{allow} = 0.180\ ^{K}/_{FT} \times 20' = 3.6^{K}$ using L.F. = 2.7

$\beta = 1260\ ^{K}/_{FT}$

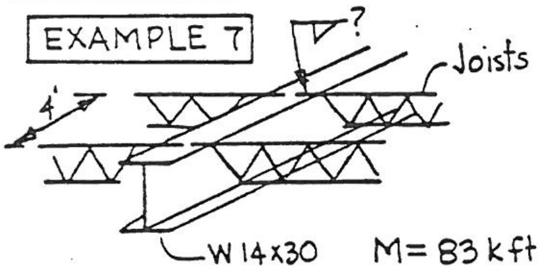
Relative Brace (siding prevents the girts from moving relatively)

$F_{REQ'D} = 0.004(180) = 0.72^{K} < 3.6\,K$ OK

$\beta_{REQ'D} = 4P/L = 4(180)/8 = 90 < 1260\ ^{K}/_{FT}$ OK

22ga. Siding OK

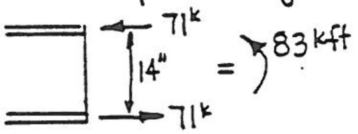
EXAMPLE 7



M = 83 kft

How much weld is required so that the joist will adequately brace the beam

Treat the compression region of the beam as a column.



$P = 71^{K}$

$\beta_{REQ'D} = 71\ ^{K}/_{FT}$

Typical metal floor decks provide approximately 10 times this required stiffness. Metal deck with 2½ in. concrete fill provides 30 times this stiffness. Normal fasteners that connect the deck to the joists can transfer the 0.57^k force.

Point brace (for connector requirements)

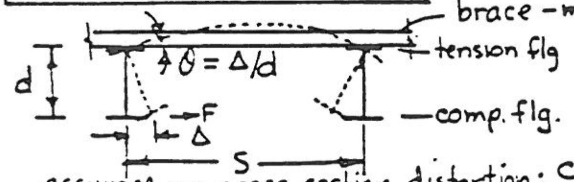
$F_{br} = 0.01\ P = 0.01\ (71) = 0.71^{K}$

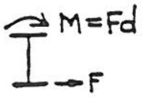
USE $\frac{3}{16}$ 0-$\frac{5}{16}$ (Tack)

Floor system - diaphragm - relative brace prevents the relative movement of adjacent joists

$\beta_{REQ'D} = 4P/L = 4(71)/4' = 71\ ^{K}/_{FT}$

TENSION FLANGE BRACING





$\beta_{REQ'D} = 16P/L$; $F_{br} = 0.008P$

$\beta_{act.} = \frac{F}{\Delta} = \frac{4EI_b}{sd^2}$ - solve for I_b

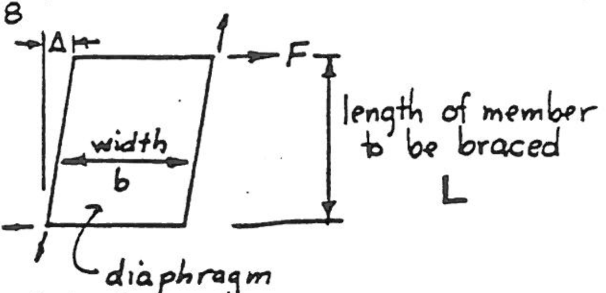
assumes no cross section distortion; check brace for bending moment, $M/2 = Fd/2$

Continuous Bracing by Shear Diaphragms

This development is based on the work of M. A. Larson, Discussion of "Lateral Bracing of Columns and Beams" by G. Winter, Journal of the Structural Division, ASCE, Vol. 84, No ST5, September 1958

Definition: Shear stiffness/ft, G'

$$G' = \frac{F/b}{\Delta/L}$$



shear stiffness ^{k}/radian per foot of width
Get G' for Light Gage Steel Diaphragms from American Iron and Steel Institute booklet "Design of Light Gage Steel Diaphragms".

Theory: $bG' = P\left(1+\frac{\Delta_o}{\Delta}\right) - P_{all}$ and $V_{REQ'D} = P\frac{\pi}{L}(\Delta+\Delta_o) - P_{all}\frac{\pi}{L}\Delta$.

P = compressive load to be carried by the member to which the diaphragm is continuously attached along the length

P_{all} = allowable load on the member in the plane of the diaphragm if no bracing is provided

$V_{REQ'D}$ = required shear strength for the diaphragm

Δ, Δ_o = deformations - see definitions on p. 1

Design: - Same assumptions as on sh. #1

STIFFNESS: $\boxed{bG'_{REQ'D} = 4\left(P - \frac{P_{all}}{2}\right)}$ and STRENGTH $\boxed{V_{all} = 0.013\left(P - \frac{P_{all}}{2}\right)}$

or conservatively $G' = 4P/b$ and $V_{all} = 0.013P$

EXAMPLE 8

Same as Example 6 except that the diaphragm is attached to the columns along the entire 24' length of the column.

Determine if the 22 ga. corrugated siding is sufficient to brace the column carrying 180^k

From the AISI booklet $G' = 2170^k/FT$; $V = 0.487^k/_{FT} \times \overset{\text{width}}{20'} = 9.74^k/\overset{\text{Load Factor}}{2.7}$

Diaphragm Requirements: -Simply $G' = 4(180)/20 = 36^k/FT < 2170^k/FT$ OK

Note that the conservative formulas do not require a knowledge of the column size

$V = 0.013(180) = 2.34^k < \frac{9.74}{2.7} = 3.6$ OK

22 ga. diaphragm is OK even using the conservative formulas

BRACING

TABLE 2.1

LOAD FACTORS FOR DESIGN OF LIGHT GAGE STEEL DIAPHRAGMS

Type of Connections Used	Load Factor*		
	Wind or Earthquake	Gravity Live Load	Gravity Dead Load
Mechanical Fasteners	2.5**	2.7	2.0
Welded Connections	2.4	3.0	2.3

* The load factors given are for diaphragm action only.

** When backed-up fasteners (bolts, rivets, spreading back fasteners or the like) are used as intermediate side lap fasteners, the load factor may be reduced to 2.1 for wind or earthquake only.

The load factors of Table 2.1 are consistent with other pertinent safety provisions of the AISI Specification for the Design of Light Gage Cold-Formed Steel Structural Members.

Diaphragm Strength

$$S_u = .14 + 11.5t$$

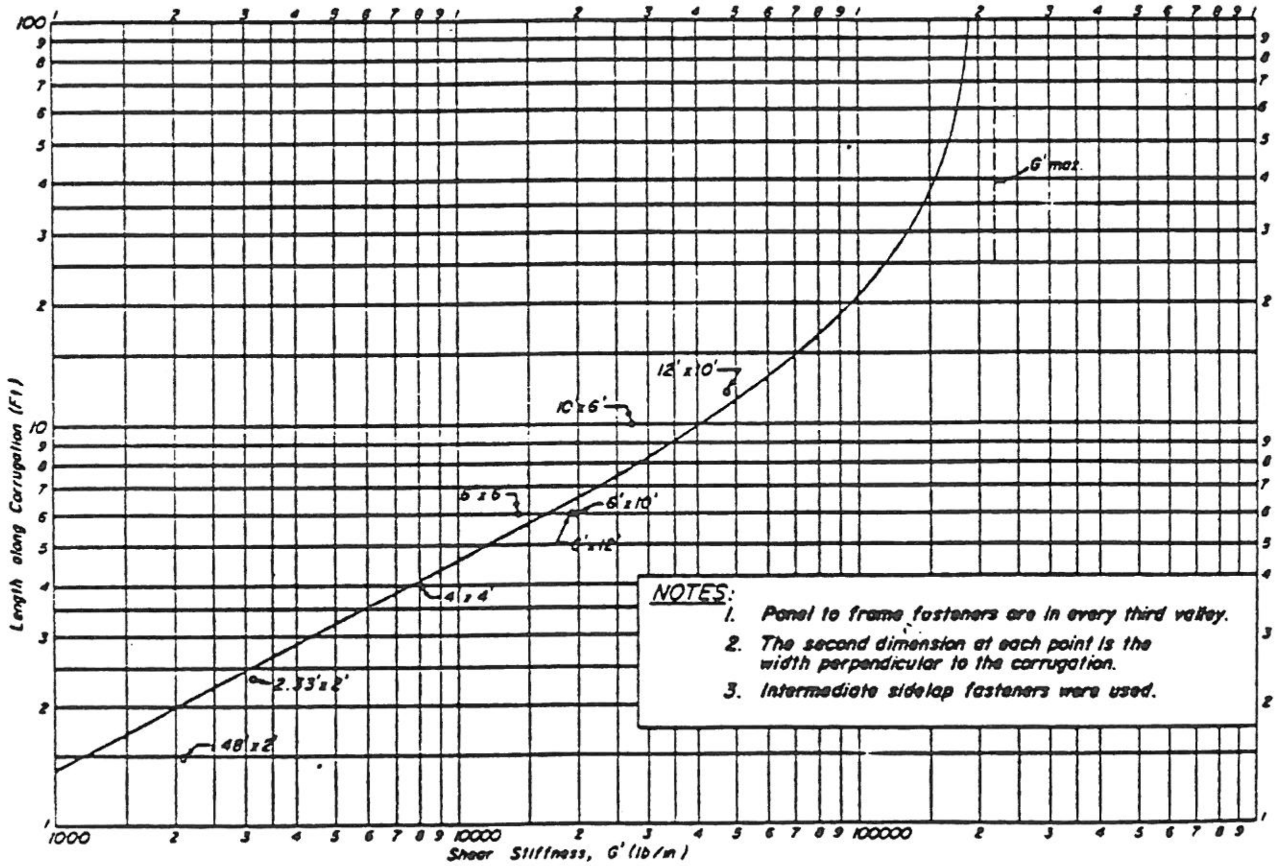
where S_u = kips per foot of diaphragm

t = thickness in inches

$$F_{allow} = \frac{Su}{Load\ Factor}$$

Stiffness: - Use graph below

For other thicknesses, $S = S_{graph} \times \left(\frac{actual\ thickness}{0.0198\ in.} \right)$



NOTES:

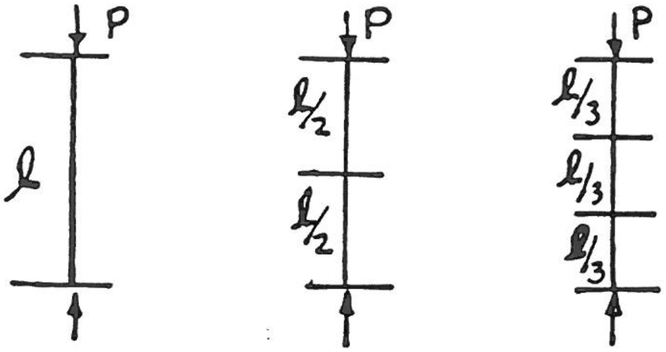
1. Panel to frame fasteners are in every third valley.
2. The second dimension at each point is the width perpendicular to the corrugation.
3. Intermediate sidelap fasteners were used.

Tested Shear Stiffness for 2½" x ½" Standard Corrugated Steel Diaphragms
(Thickness of Panels = 0.0198 in.)

BRACE STIFFNESS REQUIREMENT

$$\beta_{REQ'D} = \frac{4P}{L} \quad \text{or} \quad \beta_{REQ'D} = \frac{16P}{L}$$

relative brace — single point brace

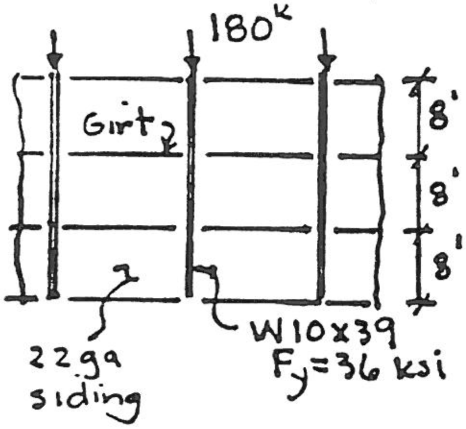


$\beta = \frac{4P}{l}$ $\beta = \frac{8P}{l}$ $\beta = \frac{12P}{l}$ ← more braces require more stiffness ?!

NO.

The derivation assumes that the unbraced length provided is just sufficient to permit a load P on the column. When there are more braces than necessary to permit the column to support the load, it is conservative to use the permissible unbraced length rather than the actual unbraced length in the formulas for $\beta_{REQ'D}$.

EXAMPLE 6a - Redo example 6 with a W10x39 column



P_{allow} = 213 kips (AISC Column Tables) L = 8'

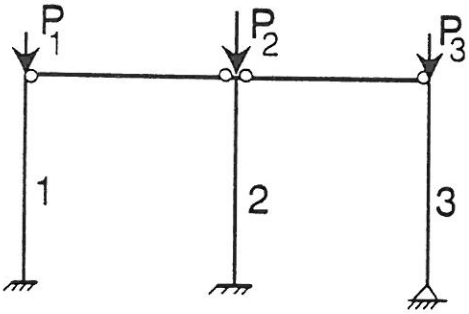
so unbraced length could be greater than 8'. From the column load tables, the unbraced length corresponding to 180 kips is 13'. Therefore

$$\beta_{REQ'D} = \frac{4P}{L} = \frac{4(180)}{13} = 55 \text{ K/FT}$$

(In Ex.6 β = 90 K/FT WHEN L = 8' WAS USED)

ΣP CONCEPT - LEAN ON BRACING

1/8 ©



$\Sigma P_{cr} \rangle \Sigma P$

- service loads in ASD
- factored loads in LRFD

Sway Buckling Capacity

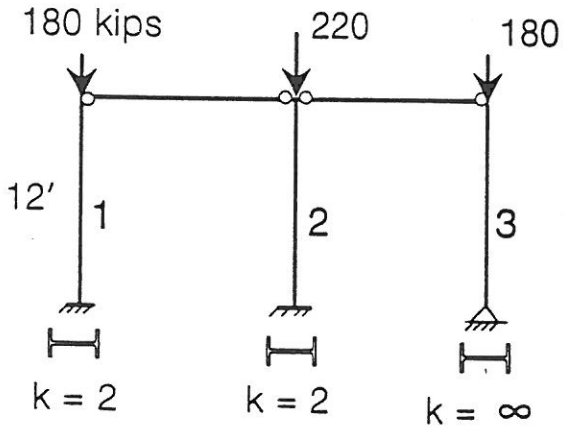
$$P_{cr_1} = A \frac{\pi^2 E I_1 \tau_1}{(k_1 L)^2}$$ based on P_1

$$P_{cr_2} = A \frac{\pi^2 E I_2 \tau_2}{(k_2 L)^2}$$ based on P_2

A:
- 12 / 23 in ASD
- 0.877 ϕ in LRFD

$$P_{cr_3} = 0$$

ASD Example



all columns W10×45

F_y = 36 ksi ASD

$A = 13.3 \text{ in}^2$, $I_x = 248 \text{ in}^4$

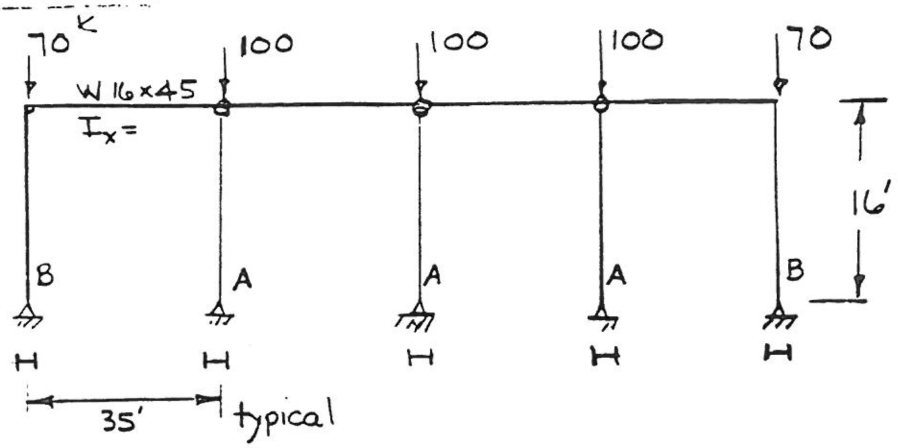
$r_x / r_y = 2.15$

$\frac{180}{13.3}$ =	13.5ksi	16.5
τ =	.830	.531

$$\frac{12}{23} \frac{\pi^2 (29000)(248)(.83)}{(2 \times 144)^2} = 370 + 237 + 0 = 607 \rangle 580 \text{ kips } OK$$

Using col load tables- 2 × 12 / 2.15 = 11.2 ft, ΣP_a = 224 × 2 = 480 *NG*

ΣP CONCEPT



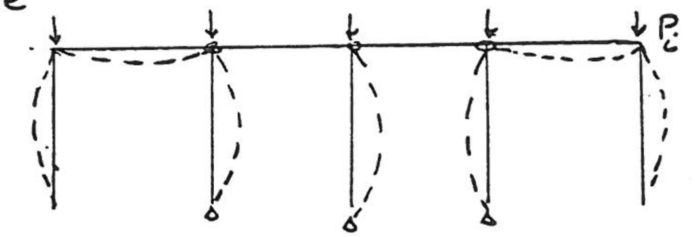
$F_y = 36$ ksi

Columns braced top & bottom out of plane

Design Cols A & B
Factored Loads shown

In an unbraced frame the following requirements must be satisfied

1. Each col must support the load that is on it in the no sway (K=1.0) mode

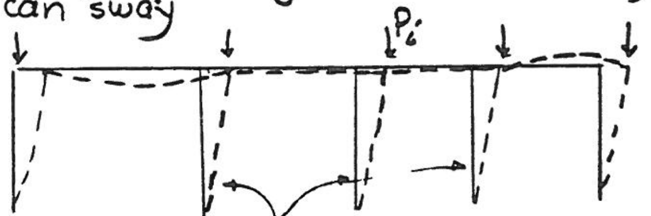


- - - no sway buckling

$$\boxed{P_i \le \phi P_{n_i}\ [K=1.0]}$$

Check both the x-x and y-y axes
This mode of buckling is exactly the same as if the structure had diagonal bracing to prevent sway.

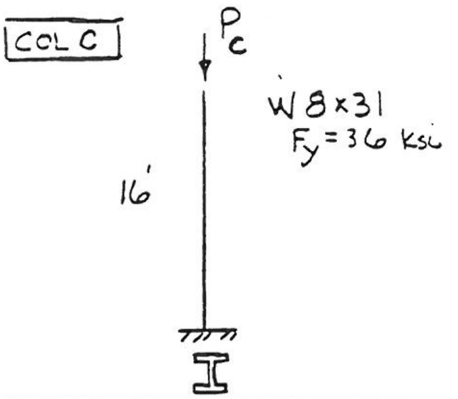
2. Check sway buckling of an entire story in all the direction(s) the frame can sway



These columns do not bend in this particular case (pinned connections at top) because their stiffness is zero in the sway mode $(k=\infty,\ P_{cr}=0)$

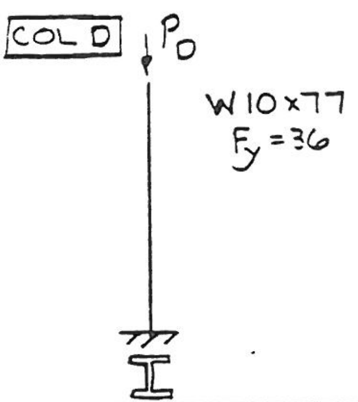
$$\boxed{\Sigma P_i \le \Sigma \phi P_{ni\ [k\ge 1.0]}}$$; ϕP_{ni} is the individual column capacity based on the alignment chart value of k for sway permitted $(k \ge 1.0)$

ILLUSTRATION - HOW ΣP CONCEPT WORKS



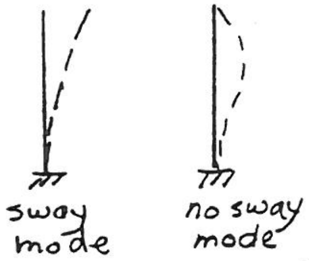
Sway mode: K=2.0
KL_y = 2×16 = 32'
ϕP_n = 54^k (p.2-28)

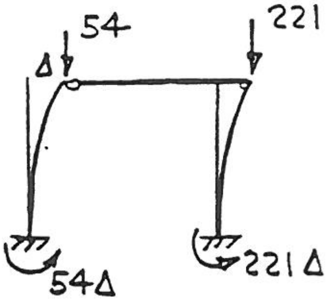
No sway: use K=1.0
KL_y = 1.0×16 = 16'
ϕP_n = 174^k (p2-28)



Sway Mode: K=2.0
KL_y = 32'
ϕP_n = 221^k (p2-26)

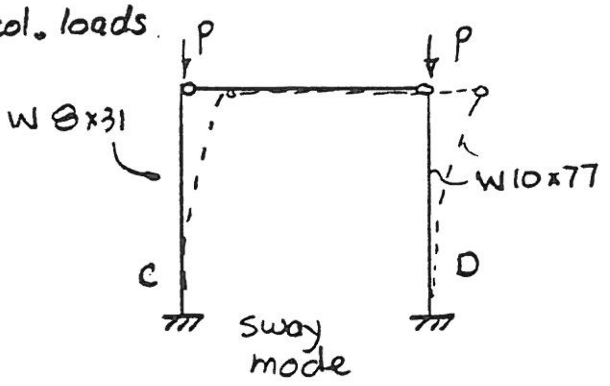
No sway: use K=1.0
KL_y = 16'
ϕP_n = 519^k (p2-26)

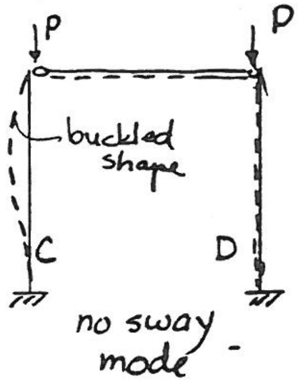




For single columns, or frames with each column loaded up to its own buckling load, the sway mode will control.

Link the two columns above by a pin - connected beam with equal col. loads





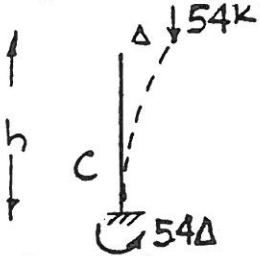
In the no sway mode with equal loads on each column, Col C will buckle first because

$174^k < 519^k$

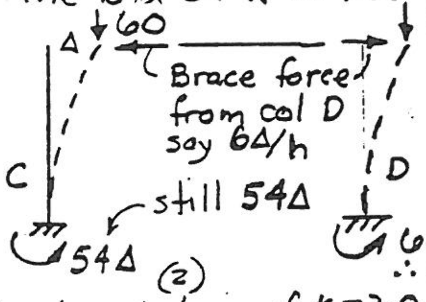
therefore Col D remains straight

Sway mode:

As P is increased from zero to buckling, column C will try to sway when the P reaches 54^k, the individual sway buckling load. At P = say 60^k, Col C wants to sway but Col D braces it. Column D can be a brace as long as the load on it is less than 221^k.

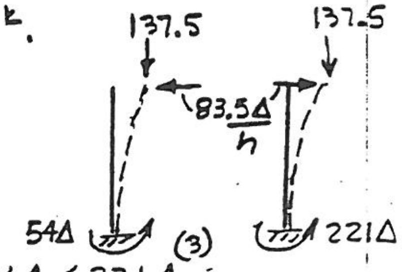


case (1)



(2)

60Δ + 6Δ = 66Δ < 221Δ
∴ sway can't occur in case (2)

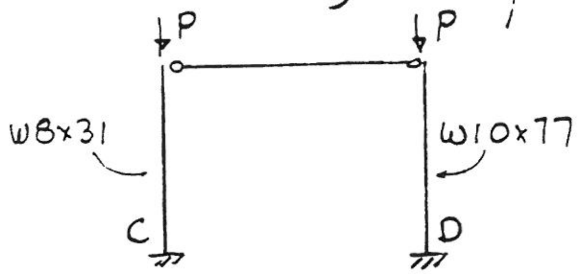


(3)

The alignment chart solution of K=2.0 is based on the assumption that there is no brace, Case (1). With bracing more axial can be added without exceeding a moment of 54Δ at the base (fixed end) as shown

ILLUSTRATION - HOW ΣP CONCEPT WORKS

what is the sway buckling load P



$2P = 54^k + 221^k = 275^k$

$\underline{P_{sway} = 138^k}$

Since $\phi P_{sway} < \phi P_{No\ sway}$ -Sway controls

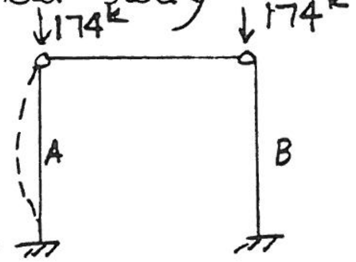
$138^k < 174^k$

What if Col B was a W 10x112 section instead of W 10x77, Then $< \begin{matrix} P_{sway};\ KL = 32',\ \phi P_n = 342^k \\ P_{No\ sway}: KL = 16',\ \phi P_n = 768^k \end{matrix}$

Sway mode: $2P = 54^k + 342 = 396^k$; $\underline{P_{sway} = 198^k}$

Since $P_{sway} > P_{No\ sway}$

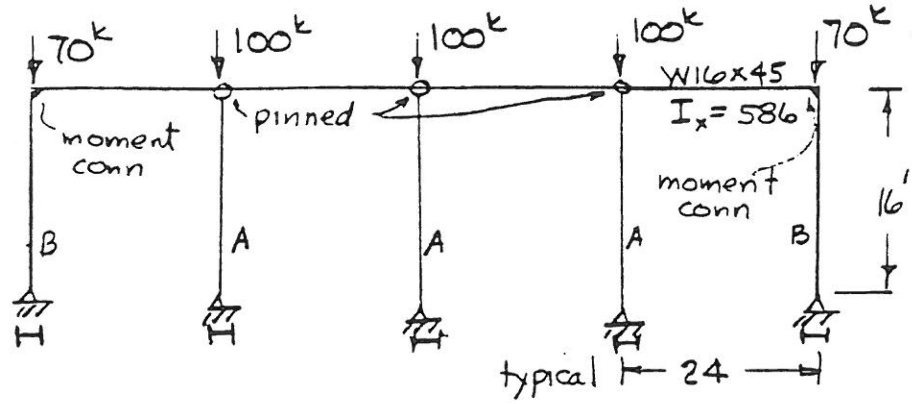
$198^k > 174^k$, the structure will buckle at 174^k without sway



technically, the column will support higher loads because k=0.7 not 1.0 in this case.

EXAMPLE 1

4/4
5/8



← Factored Load

$F_y = 36$ ksi

Columns braced top
& bottom out-of-plane

Design Cols A & B

x-x axis in plane
y-y axis out-of-plane

Col A:

No sway mode - K=1.0 $KL_y = 16'$, $P_{REQ'D} = 100$ kips

USE ~~Try~~ W8×24 $\phi P_n = 102$ kips (p. 2-29) > 100 k OK

Sway mode: $\phi P_{sway} = 0$ since $G_A = G_B = \infty$ and $k = \infty$

∴ use Col. B to brace Col. A

Col B: No sway K=1.0 $KL_y = 16'$, $P_{REQ'D} = 70^k$

Sway K=? - $P_{REQ'D} = \frac{70+3(100)+70}{2} = 220$ kips

two col B

so that $\sum P_i \le \sum \phi P_{n\,sway}$

K_B must be greater than 2.0, ∴ $KL_x > 2\times16 = 32$

W10 has $r_x/r_y \approx 2.15$ so KL_x eq. = 32/2.15 = 14.9 say 15

Try W10×45. [The W10×39 has $\phi P_n = 228$ but since K must be greater than 2.0, the 228 is too close to the req'd 220 kips]

or W12×40, $r_x/r_y = 2.66$, $KL_{eq} = 2\times16/2.66 = 12$ $\phi P_n = 269$ kips

so try W12×40 - lighter

actual load
col. affects τ ?

use ~~Try~~ W12×40 $I_x = 310\text{ in}^4$, $r_x/r_y = 2.66$, $A = 11.8\text{ in}^2$, $\frac{70}{11.8} = 5.93$ ksi

τ = 1.0

COL. B

$G = \frac{1.0(310/16)}{0.5(586/24)} = 1.58$, $G_B = \infty$ ∴ K = 2.5

far end of beam pinned

$KL_{eq} = 2.5\times16/2.66 = 15.0'$ $\phi P_n = 228$ kips

$\Sigma\phi P_n = 2(228) + 3(0) = 456^k$

$\Sigma P = 2(70) + 3(100) = 440^k$ $\Sigma\phi P_n > \Sigma P$ Ok

No sway 1.0 (16) = 16' = KL_y 214 > 70 kips ok

Column Design *

$$\phi P_n = 0.85(.877)\frac{\pi^2 EI}{(KL)^2}\tau = 213000\frac{I}{(KL)^2}\tau$$

where $\tau = 1.0$ for $P/P_y \le 1/3$

$$\tau = -7.38(P/P_y)\log\left(\frac{P/P_y}{0.85}\right)$$

$$P_y = F_y A$$

This is the same as

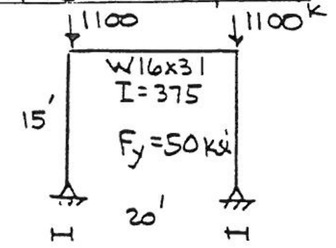
$$\phi P_n = 0.85\,F_{cr}A$$

where $F_{cr} = (0.658^{\lambda_c^2})F_y$ if $\lambda_c < 1.5$

$$F_{cr} = \frac{0.877}{\lambda_c^2}F_y \text{ if } \lambda_c \ge 1.5$$

$$\lambda_c = \frac{KL}{r\pi}\sqrt{F_y/E}$$

AISC Ex. 3-3 p. 3-9



Continuously braced out-of-plane

Try W12×120, $A = 35.3\text{ in}^2$, $I_x = 1070\text{ in}^2$

$P_y = 50(35.3) = 1765$; $P/P_y = 1100/1765 = 0.623 > 1/3$ ∴ τ inelastic

$\tau = -7.38(.623)\log(1.176 \times .623) = 0.621$

$G_T = \dfrac{0.621(1070/15)}{375/20} = 2.36$; $G_B = 10$ ∴ $K = 2.2$ (aligmment cha

$\phi P_n = 213000(1070)(.621)/(2.2\times12\times15)^2 = 903^k < 1100$ N.G.

Try W12×136, $A = 39.9$, $I_x = 1240$

$P_y = 50(39.9) = 1995$; $P/P_y = 1100/1995 = 0.551 > 1/3$

$\tau = -7.38(.551)\log(1.176\times .551) = 0.767$

$G_T = \dfrac{0.767(1240/15)}{375/20} = 3.38$, $G_B = 10$, $K = 2.3$

$\phi P_n = 213000(1240).767/(2.3\times12\times15)^2 = 1182 > 1100$ OK

For the problem above what size beam would be necessary to make the W12×120 satisfactory?

$\tau = 0.621$; $\phi P_n = 1100 = 213000(1070)(.621)/(K\times12\times15)^2$ gives $K = 1.99$

With $G_B = 10$ and $K = 1.99$, $G_{T\,REQ'D} = 1.4 = \dfrac{.621(1070/15)}{I_b/20}$

I_b req'd $= 633\text{ in}^4$ Use W21×44, $I = 843\text{ in}^4$ p. 4-25 AISC Man.

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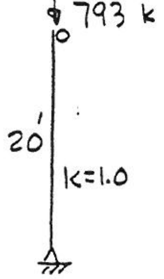
* Compare this method with the standard method



Try W12×120 ($A = 35.3\text{ in}^2$, $I_y = 345\text{ in}^4$, $r_y = 3.13\text{ in}$), $F_y = 36$ ksi

Std. Method: $\lambda_c = \dfrac{20(12)}{\pi(3.13)}\sqrt{\dfrac{36}{29000}} = 0.860 < 1.5$

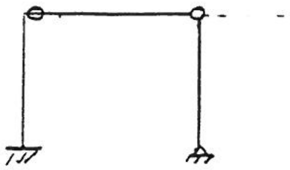
$\phi P_n = 0.85\times(0.658^{(.86)^2})\,36\times35.3 = 793$ kips OK

Equiv. Method: $P/P_y = 793/36\times35.3 = 0.624 > 1/3$ ∴ $\tau < 1.0$

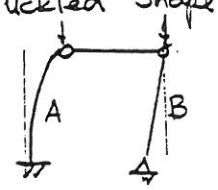
$\tau = -7.38(.624)\log(0.624/.85) = 0.619$

$\phi P_n = 213000(345)(.619)/240^2 = 791$ kips OK

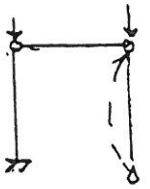
$\approx$ sar



Buckled Shape



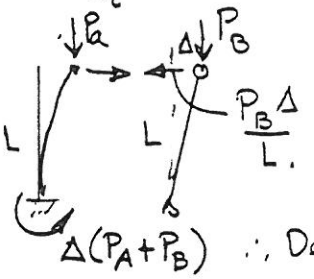
Sway



No Sway

$\Sigma P_{cr_i} > \Sigma P$ $P_{cr_{k=1}} > P_i$

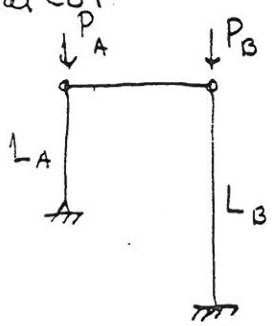
Why does ΣP work

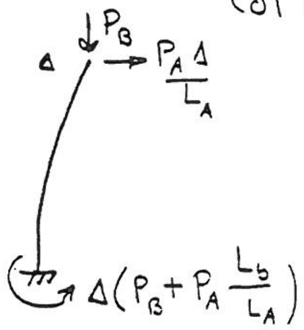


∴ Design Col A to support $P_A + P_B$

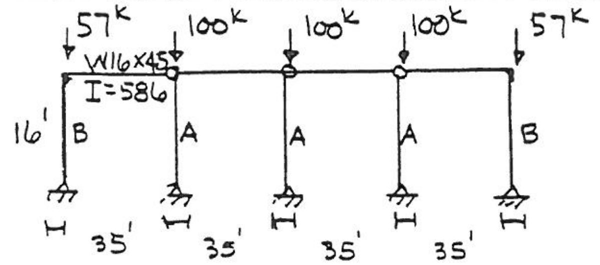
Col B, $P_{cr} = 0$ in sway mode

Unequal Col





Col B must support $P_B + \frac{L_b}{L_A} P_A$



F_y = 36 ksi, Columns braced top and bottom out-of-plane

Design Col A for the no sway case, K=1.0 for P=100k, KL=16'

-Use AISC Col tables use W8×24 $\phi P_n = 102^k$

Col. B. Design Col B to support 57^k out-of-plane with KL=16 and to support 57 + 150 = 207k in-plane with K=?

Try W 12×30, $A = 8.79 \text{ in}^2$, $I_x = 238$, $I_y = 20.3 \text{ in}^4$, $P_y = 8.79(36) = 316\text{k}$

$P/P_y = 57/316 = 0.180 < 1/3 \therefore \tau = 1.0$

Out-of-plane $\phi P_n = 213000(20.3)1.0/(16\times12)^2 = 117\text{k} > 57$ ok

In-plane: $G_T = \dfrac{(1.0)\,238/16}{\frac{3}{6}(586/35)} = 1.78$, $G_B = 10$ $\therefore K = 2.1$

$$\phi P_n = 213000(238)1.0/(2.1\times16\times12)^2 = 312\text{ k}$$

$\Sigma P_{cr} = 2\times312 + 3\times0 = 624$ kips

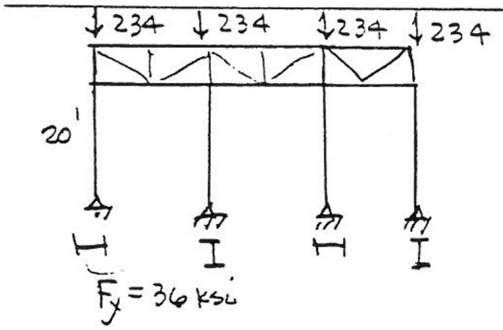
$\Sigma P = 2\times57 + 3(100) = 414 < 624$ ok

Use W 12×30

Note - it is conservative to use the ϕP_{cr} from the column tables with KL in the $\Sigma P_{concept}$ Ex: $\dfrac{KL}{r_x} = \dfrac{2.1\times16\times12}{5.21} = 77.4$ say 77, $\phi F_{cr} = 22.40$

$\phi P_n = 22.40(8.79) = 197^k$

$2\times197 + 3\times0 = 394 < 414$ N.G.



two columns, one with I_x and one with I_y support a sway load of 2 × 234 = 468k with KL. Each column must support a load = 234 with K=1.0 and I_y

Try W 12×53, A = 15.6, $I_x = 425$, $I_y = 95.8$

$P_y = 15.6\times36 = 561.6$

$P/P_y = 234/561.6 = .417 \therefore$ inelastic

$\tau = -7.38(.417)\log(1.176\times.417) = 0.952$

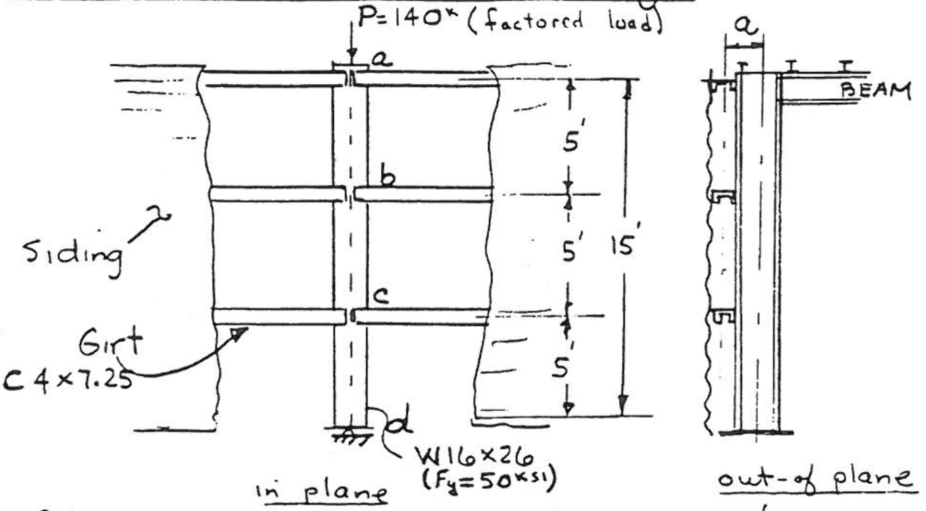
No sway: $\phi P_n = 213000(95.8).952/(1.0\times20\times12)^2 = 337 > 234$ kips OK

Sway $\phi P_{nx} + \phi P_{ny} = 213000(425 + 95.8).952/(1.65\times20\times12)^2$

$\left.\begin{matrix} G_T = 0 \\ G_B = 10 \end{matrix}\right\} K = 1.65$ $= 673 > 2\times234 = 468\text{k}$ OK

W12×50 won't work use W 12×53

Columns Braced on One Flange Example



W 16×26 (F_y = 36 ksi)

I_x = 301 in^4

I_y = 9.59 in^4

r_x = 6.26 in.

r_y = 1.12 in

J = 0.26 in^4

d = 15.69 − .345 = 15.345

A = 7.68 in^2

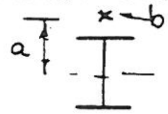
$G = 29000/2.6 = 11,200$ ksi

$P_y = 7.68\text{ in}^2 \times 50^{ksi} = 384^K$

- Column braced in plane by girts attached to one flange by simple connections (no rotational restraint provided by girt, only lateral restraint)
- Out of plane, the column is part of an unbraced frame; assume pinned base and rigid connection to beam gives $K_x = 2.5$
- The column can not twist at points a and d, the top & bottom of the column

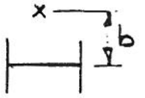
Three types of column buckling must be checked

1. strong axis Euler (flexural) buckling with L = 15' and K_x = 2.5
2. weak axis Euler buckling between girts L = 5' and k_y = 1.0
3. torsional buckling about a restrained axis



$$P_{cr} = \frac{P_{ey}\left(\frac{d^2}{4} + a^2\right) + GJ}{a^2 + r_x^2 + r_y^2}$$

or



$$P_{cr} = \frac{P_{ey}\left(\frac{d^2}{4} + \frac{I_x}{I_y} b^2\right) + GJ}{b^2 + r_x^2 + r_y^2}$$

where $P_{ey} = \tau\pi^2 E I_y / \ell^2$ and ℓ = distance between locations where twist is prevented.

$$\frac{P}{P_y} = \frac{140^k}{384^k} = 0.36 > \frac{1}{3} \Rightarrow \tau = -7.38(0.36)\left[\log\left(\frac{0.36}{0.85}\right)\right] = 0.99$$

1. Strong axis : $P_{cr_x} = 213,000 \frac{\tau I_x}{(K_x L)} = 213,000 \frac{(0.99)301}{(2.5 \times 15 \times 12)^2} = \underline{\underline{313^k}} > 140^k$

2. weak axis : $P_{cr_y} = 213,000 \frac{(0.99)9.59}{(1.0 \times 5 \times 12)^2} = \underline{\underline{562^k}} > 140^k$

3. torsional buckling: $P_{ey} = 0.99 \frac{\pi^2 \times 29\,000(9,59)}{(15 \times 12)^2} = 83.9$

$\ell = 15'$

$a = \frac{15.69}{2} + \frac{4}{2} = 9.84$ to centroid of brace $\quad P_{cr_T} = \overset{\phi}{0.85}\left[\frac{83.9\left(\frac{\overline{15.345}^2}{4} + \overline{9.84}^2\right) + 11,200(0.26)}{\overline{9.84}^2 + \overline{6.26}^2 + \overline{1.12}^2}\right] = 99^k < 140^k$ N G

-or-

or $a = 15.69/2 = 7.845$ to flange of column $\quad P_{cr_T} = 0.85\left[\frac{83.9\left(\frac{\overline{15.345}^2}{4} + \overline{7.845}^2\right) + 11200(.26)}{(7.845)^2 + (6.26)^2 + (1.12)^2}\right] = 108^k < 140^k$ (No Good)

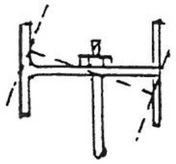
TORSIONAL BRACING

W sections can buckle flexurally (Euler buckling, $P_{ey} = \frac{\pi^2 E I_y}{L_y^2}$) or torsionally

$$P_T = \frac{\frac{\pi^2 E I_y}{L_t^2}\left(\frac{d^2}{4}\right) + GJ}{r_x^2 + r_y^2} \quad \text{where } L_y = \text{distance between lateral braces} \qquad (1)$$

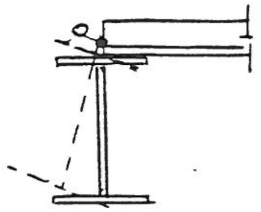
L_t = distance between torsional braces

If a brace prevents lateral movement and twist, then $L_y = L_t$ and Euler type buckling always controls(1). If a brace only prevents lateral movement, then torsional buckling must be considered. Tie rods do not prevent twist.

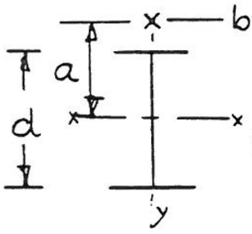


Y-axis movement is prevented but twist can occur
Check Eq.(1). Euler may still control

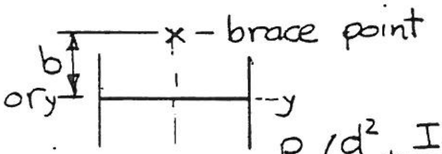
Eq.(1) is valid for twisting about the shear center, but in many practical applications, twisting will occur about some point other than the shear center (centroid for W shapes).



member will twist about point O. In such cases, Eq.(1) is unconservative. From Timoshenko and Gere, "Theory of Elastic Stability" the following two equations can be used to check torsional buckling



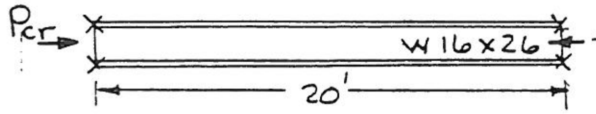
$$P_T = \frac{P_{ey}\left(\frac{d^2}{4} + a^2\right) + GJ}{a^2 + r_x^2 + r_y^2}$$



$$P_T = \frac{P_{ey}\left(\frac{d^2}{4} + \frac{I_x}{I_y} b^2\right) + GJ}{b^2 + r_x^2 + r_y^2} \qquad (2)$$

where $P_{ey} = \pi^2 E I_y / L_t^2$

Some numerical results



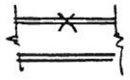
twist and lateral movement prevented at both ends

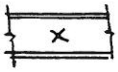
$P_{cr} = \pi^2 E I_y / (240)^2 = \underline{48 \text{ kips}}$

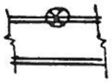
Added brace at midspan

Computer Results

a)  -both flanges laterally supported $L_y = L_t = 120\text{in}$ — $P_{cr} = 191$ kips

b)  only one flange braced laterally — $P_{cr} = 79$ K (82^K EQ.2)

c)  lateral brace at centroid — $P_{cr} = 124$ K (132^K EQ 2)

d)*  lateral movement and twist prevented at one flange only and no stiffener is used  section distorts — $P_{cr} = 169^k$ (191 k Euler)

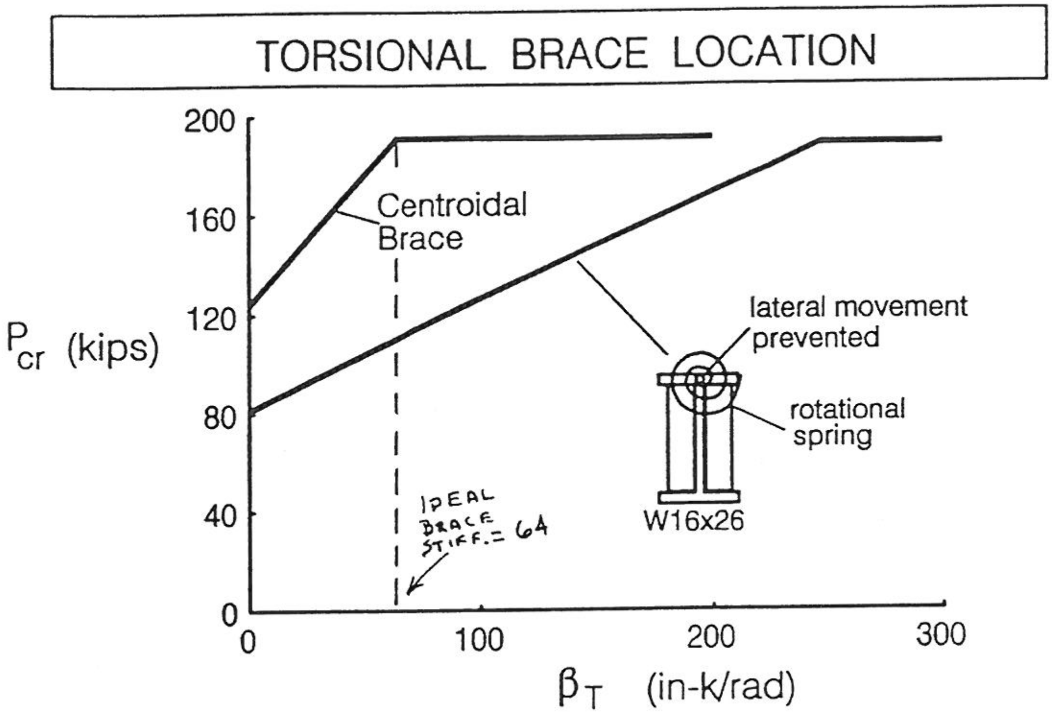
(1) In singly symmetric or unsymmetric sections, lateral torsional buckling must be checked.

* With a stiffener, a torsional brace = 64 in-k/rad is required to prevent twist.

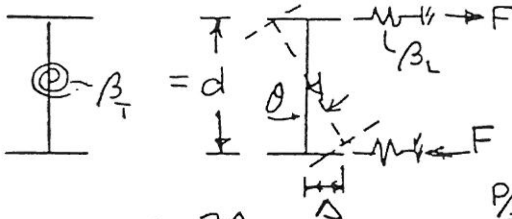
TORSIONAL BRACING

HOW MUCH TORSIONAL BRACING IS REQUIRED @ MIDSPAN TO FORCE BUCKLING BETWEEN THE BRACE POINTS?

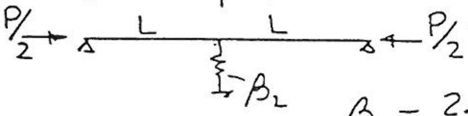
A COMPUTER ANALYSIS YIELDS THE FOLLOWING RESULTS:



How to calculate the ideal torsional brace stiffness



Treat each flange as a column with a lateral brace at midspan. The ideal lateral brace for one brace at midspan is



- one half of col load on each flg.

$$\theta = \frac{2\Delta}{d}$$

$$\beta_L = \frac{2.0\,(P/2)}{L} \quad \text{(see sheet B)}$$

$$M = \beta_L \Delta d = \beta_L\left(\frac{\theta d}{2}\right) d$$

Torsional Brace Stiffness $= \beta_T = M/\theta = \frac{\beta_L d^2}{2} = \frac{Pd^2}{2L}$

$$\boxed{\text{Torsional brace} = \text{Lateral brace of one flange} \times d^2/2}$$

$M_{br} = \theta_{\circ} \beta_T$

assume 1° (.0175 rad)

A load of $P_1 = 124^k$ can be supported with no brace. Therefore the brace must support $191 - 124^k = 67$ K

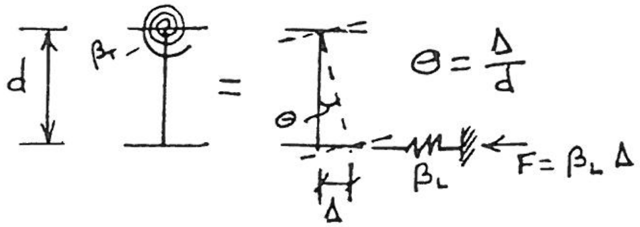
$$\beta_T = \frac{67(15.34)^2}{(2)\,120} = 66 \text{ in-k/rad}$$

very good comparison with above

- In design, double this to account for initial out-of-straightness

TORSIONAL BRACING @ ONE FLANGE:

NO TRANSLATION OF ONE FLANGE.



$$\theta = \frac{\Delta}{d}$$

Treating unbraced flange as a column w/ load P/2:

$$\beta_{L_i} = \frac{2.0\,(P/2)}{L}$$

$$M = \beta_L \Delta d = \beta_L(\theta d)d$$

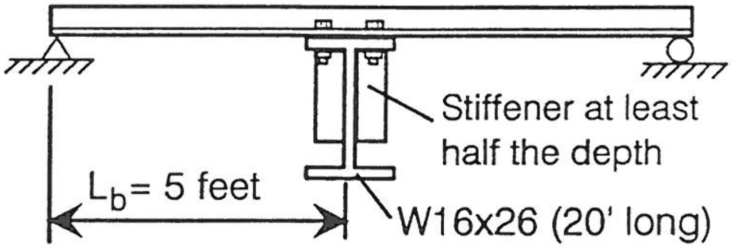
Ideal Torsional Brace stiffness $= \beta_{Ti} = M/\theta = \beta_{Li}\, d^2$

$$\therefore \underline{\underline{\beta_{Ti} = \frac{Pd^2}{L}}}$$

} DOUBLE TO ACCOUNT FOR INITIAL IMPERFECTIONS

$$\boxed{\text{Torsional Brace} = \text{Lateral Brace of one flange} \times d^2}$$

LRFD DESIGN EXAMPLE



Girder is braced on one flange. The capacity of for buckling about restrained axis = 82 k.

Factored Load = 191 kips

$\phi = 0.75$

$P_{brace} = 191 - 82 = 109$ k

$$\beta_T = 2\left(\frac{P_{brace}\ d^2}{\phi L}\right) = 2 \times \frac{109 \times (15.34)^2}{0.75 \times 120} = 570 \text{ in-k/rad}$$

$$\beta_T = 2 \times \frac{3\,E\,I_b}{L_b} = \frac{6\,(29000)\,I_b}{60} = 570 \text{ in-k/rad}$$

$I_b = 0.197 \text{ in}^4$

Try $L\ 2 \times 2 \times \frac{3}{16}$

Check Strength:

$$f_{br} = \frac{M_{br}}{S_x} = \frac{\beta\,\theta}{S_x} = \frac{(570)\times(0.0175 \text{ rad})}{2 \times (0.19 \text{ in}^3)} = 26.2 \text{ ksi} < 32.4 \text{ ksi} = \overset{\boxed{0.9}}{\phi} F_y$$

OK

TORSIONAL BRACING DESIGN RECOMMEDATIONS

Centroid Brace:

$$\beta_{T\,req'd} = 2\beta_{Ti} = \frac{Pd^2}{L}$$

$$\boxed{1^\circ}$$

$$M_{br} = \beta_T\,\theta_o = 0.0175\,\beta_T$$

ASD

$$\beta_T = \frac{2\,Pd^2}{\cancel{2}L}$$

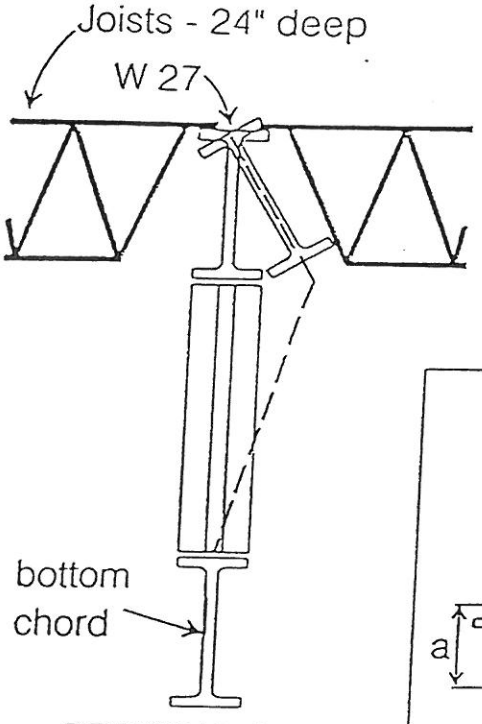
$$M_{br} = 0.0175\,\beta_T$$

LRFD

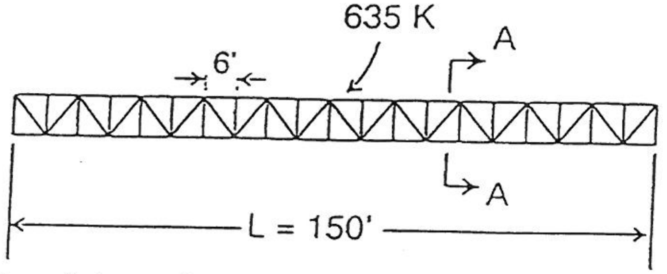
$$\beta_T = \frac{Pd^2}{\phi L}\ ;\ \phi = 0.75$$

$$M_{br} = 0.0175\,\beta_T$$

$$\boxed{\text{Double } \beta_T \text{ for bracing on single flange}}$$



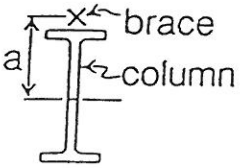
SECTION A-A



L = distance between braces that prevent twist

TORSIONAL BUCKLING

$$P_{cr} = \frac{P_{ey}\left(\frac{d^2}{4} + a^2\right) + GJ}{a^2 + r_x^2 + r_y^2} = 180\text{ k}$$



$$\text{where } P_{ey} = \frac{\pi^2 EI_y}{L^2}$$

Ⓔ

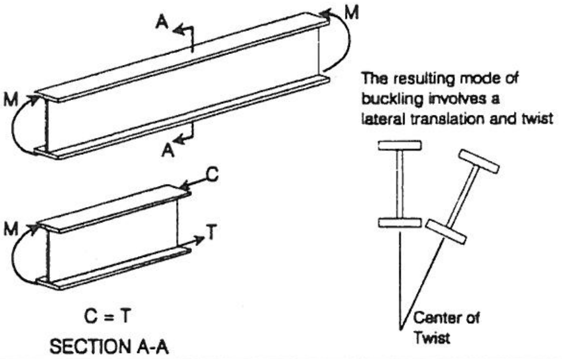
AISC/SSRC Short Course
"BRACING FOR STABILITY"

BEAM BUCKLING

- DOUBLY - SYMMETRIC and SINGLY-SYMMETRIC SECTIONS
  - UNIFORM MOMENT
  - MOMENT GRADIENT
  - LOAD HEIGHT EFFECTS
  - TIPPING EFFECT
- SPECIAL CASES
  - COPED BEAMS
  - TAPERED SECTIONS
  - CONTINUOUS BEAMS BRACED AT THE TOP FLANGE
  - UNBRACED CANTILEVERS
  - INFLECTION POINT AS A BRACE POINT

A

DOUBLY - SYMMETRIC SECTIONS



The resulting mode of buckling involves a lateral translation and twist

B

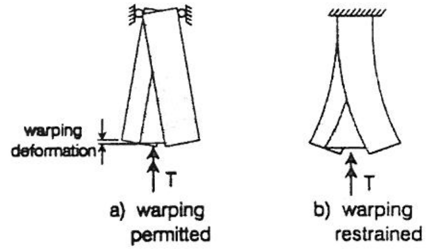
M_{cr} for DOUBLY-SYMMETRIC SECTIONS

$$M_{cr} = \frac{\pi}{L_b}\sqrt{\underbrace{E I_y G J}_{\text{St. Venant Term}} + \underbrace{E^2 I_y C_w \frac{\pi^2}{L_b^2}}_{\text{Warping Term}}}$$

L_b = spacing between points of full bracing
E = modulus of elasticity
I_y = weak-axis moment of inertia
J = torsional constant = $\Sigma (b t^3) / 3$
G = shear modulus
C_w = warping constant = $I_y d^2 / 4$
d = distance between flange centroids

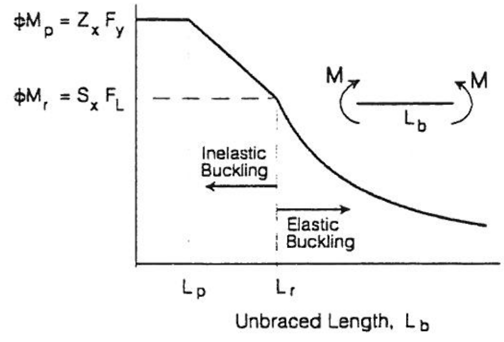
C

WARPING STIFFNESS for WIDE FLANGE SHAPES



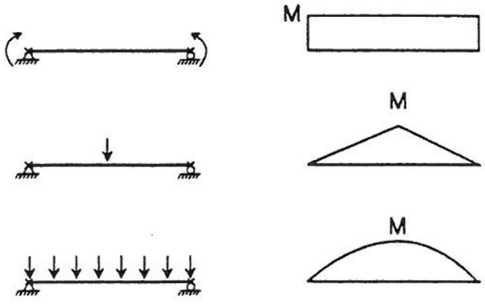
D

AISC LRFD BEAM CURVE



E

VARIABLE MOMENT



F

LOAD HEIGHT EFFECTS on M_{cr} (SSRC - 3rd Ed.)

$$M_{cr} = \frac{C_b \pi^2 E I_y d}{2(KL_b)^2}\left[\sqrt{1 + C_2^2 + \frac{(KL_b)^2 G J}{\pi^2 E C_w}} \pm C_2\right]$$

effective length factor for weak-axis bending

coefficient to account for load height
=0.45 for distributed loads
=0.55 for midspan point loads

A

LOAD HEIGHT EFFECTS on M_{cr} (SSRC - 4th Ed.)

A modified version of C_b is used to account for load height effects (C_b^*):

Top Flange Loading: $C_b^* = C_b / B$

Load at Shear Center: $C_b^* = C_b$

Bottom Flange Loading: $C_b^* = C_b B$

B is defined in the following expressions:

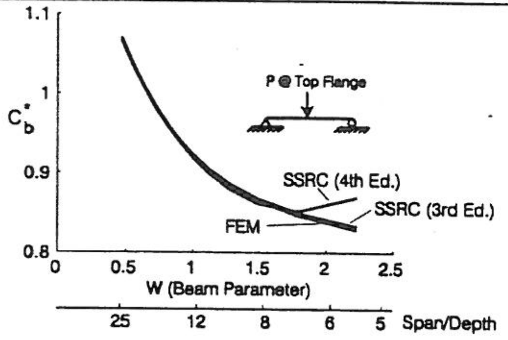
Point Load at Midspan: $B = 1 - 0.180\,W^2 + 0.649\,W$

Uniform Distributed Load: $B = 1 - 0.154\,W^2 + 0.535W$

W, the "beam parameter," is defined as: $W = \frac{\pi}{L_b}\sqrt{\frac{E\,C_w}{G\,J}}$

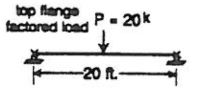
B

C_b^* versus W for DOUBLY - SYMMETRIC SECTIONS



C

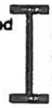
LOAD HEIGHT EXAMPLE (SSRC - 4th Ed.)



$L_b = 20$ ft. > 16.2 ft. = L_r

$W = \frac{\pi}{L_b}\sqrt{\frac{E\,C_w}{G\,J}} = 1.00$

The W21x50 is subjected to a factored midspan load of 20 kips at the top flange. Is the beam stable?



$I_y = 24.9\ in^4$
$J = 1.14\ in^4$
$C_w = 2570\ in^6$

Point Load at Midspan: $B = 1 - 0.160W^2 + 0.649W = 1.47$

Top Flange Loading: $C_b^* = C_b / B = 1.35 / 1.47 = 0.92$

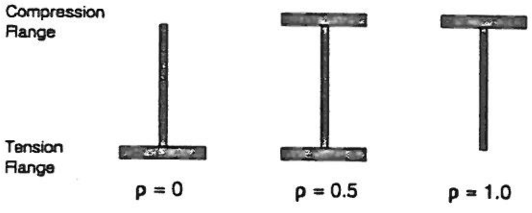
$$M_{cr} = \phi C_b^* \frac{\pi}{L_b}\sqrt{E I_y G J + E^2 I_y C_w \frac{\pi^2}{L_b^2}} = 1470 \text{ k-in} = 122 \text{ k-ft}$$

$$P_{cr} = \frac{4}{20\text{ ft}}\ 122 \text{ k-ft} = \underline{\underline{24.5 \text{ kips} > 20 \text{ kips}\ \ \text{OK}}}$$

D

SINGLY - SYMMETRIC SECTIONS

$$\rho = \frac{I_{yc}}{I_y}$$



E

EXACT SOLUTION for SINGLY - SYMMETRIC SECTIONS

$$M_{cr} = \frac{\pi}{L_b}\sqrt{E I_y G J}\left[B_1 + \sqrt{1 + \frac{\pi^2 a^2}{L_b^2} + B_1^2}\right]$$

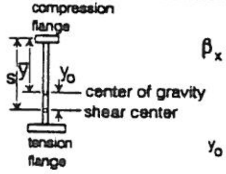
$$B_1 = \frac{\pi}{2}\frac{\beta_x}{L_b}\sqrt{\frac{E I_y}{GJ}}$$

$$a = \sqrt{\frac{EC_w}{GJ}}$$

$$C_w = I_y d^2 \rho (1 - \rho)$$

F

MONO-SYMMETRY PARAMETER for EXACT SOLUTION



$$\beta_x = \frac{1}{I_x}\left(\int_A x^2 y\, dA + \int_A y^3\, dA - 2y_o\right.$$

$y_o = s - \bar{y}$ positive if the shear center is located between the centroid and the tension flange.

$$\beta_x = \frac{1}{I_x}\left((d-\bar{y})\left[\frac{b_t^3 t_t}{12} + b_t t_t (h-\bar{y})^2 + \frac{(h-\bar{y})^3 t_w}{4}\right] - \bar{y}\left[\frac{b_c^3 t_c}{12} + b_c t_c \bar{y}^2 + \frac{\bar{y}^3 t_w}{4}\right]\right) - 2y_o$$

β_x can also be approximated with the following expression:

$$\beta_x \; 0.9d(2\rho - 1)\left[1 - \left(\frac{I_y}{I_x}\right)^2\right]$$

A

APPROXIMATE SOLUTIONS

AISC LRFD Singly-Symmetric Equation:

$$M_{cr} = \frac{57000}{L_b}\sqrt{I_y J}\left(B_1 + \sqrt{1 + B_2 + B_1^2}\right)$$

$$B_1 = 2.25\,(2\rho - 1)\,\frac{d}{L_b}\sqrt{\frac{I_y}{J}} \qquad B_2 = 25\,(1 - \rho)\left(\frac{I_{yc}}{J}\right)\left(\frac{d}{L_b}\right)^2$$

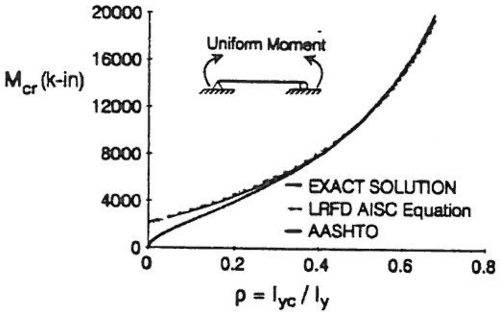
For Tees: $M_{cr} = \frac{57000}{L_b}\sqrt{I_y J}\left(B_1 + \sqrt{1 + B_1^2}\right)$

AASHTO Equation:

$$M_{cr} = \pi E\left(\frac{I_{yc}}{L_b}\right)\sqrt{0.772\left(\frac{J}{I_{yc}}\right) + 9.87\left(\frac{d}{L_b}\right)^2}$$

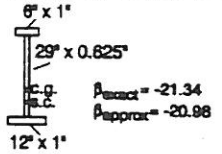
B

VARIATION of M_{cr} with ρ

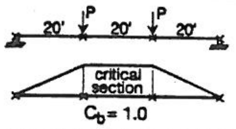


C

EXAMPLE PROBLEM for SINGLY - SYMMETRIC SECTIONS



$\beta_{exact} = -21.34$
$\beta_{approx} = -20.98$



Determine the buckling capacity of the singly-symmetric girder shown.

SOLUTIONS

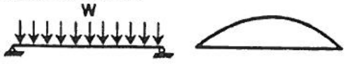
EXACT - M_{cr} = 5788 k-in
DOUBLY-SYMMETRIC EQUATION - M_{cr} = 13563 k-in (-134%)
AISC SING.-SYMMETRIC EQUATION- M_{cr} = 5895 k-in (-1.8%)
AASHTO EQUATION - M_{cr} = 4893 k-in (15.5%)

D

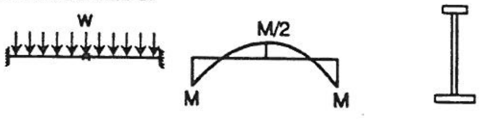
MOMENT GRADIENTS on SINGLY-SYMMETRIC SECTIONS

The C_b factors which have been presented previously are not directly applicable to singly-symmetric girders.

Single Curvature:

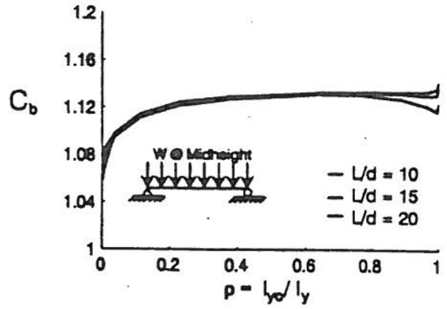


Reverse Curvature:



E

VARIATION of C_b with ρ (finite element)



F

AISC/SSRC Short Course
"BRACING FOR STABILITY

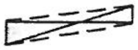
SINGLY-SYMMETRIC SECTIONS with REVERSE CURVATURE

$M_{cr\ top}$ = capacity with top flange critical (use ρ_{top}).
$M_{cr\ bot}$ = capacity with bottom flange critical (use ρ_{bot}).

Design Procedure:

$M_{top} \leq C_b M_{cr\ top}$

$M_{bot} \leq C_b M_{cr\ bot}$

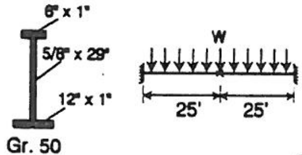


For $0.1 \leq \rho \leq 0.9$:
$C_b = 1.75$
For all other ρ:
$C_b = 1.0$

A

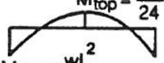
REVERSE CURVATURE DESIGN EXAMPLE

Determine what factored load the beam can safely carry without flexural-torsional buckling.



$\rho_{top} = \frac{I_{y\ top}}{I_y} = \underline{0.11}$, $I_{y\ top} = \underline{18\ in^4}$

$\rho_{bot} = (1 - \rho_{top}) = \underline{0.89}$, $I_{y\ bot} = \underline{144\ in^4}$



$M_{top} = \frac{wL^2}{24}$

$M_{bot} = \frac{wL^2}{12}$

$\therefore \underline{C_b = 1.75}$

B

DESIGN EXAMPLE (continued)

AISC LRFD Singly-Symmetric Equation:

$$M_{cr} = \phi \frac{57000}{L_b} \sqrt{I_y J} \left(B_1 + \sqrt{1 + B_2 + B_1^2} \right)$$

$$B_1 = 2.25\ (2\rho - 1) \frac{d}{L_b} \sqrt{\frac{I_y}{J}} \qquad B_2 = 25\ (1 - \rho) \left(\frac{I_{yc}}{J} \right) \left(\frac{d}{L_b} \right)^2$$

Using $\phi = 0.9$, $L_b = 25'$, $\rho_{top} = 0.11$, and $\rho_{bot} = 0.89$:

$$C_b M_{cr\ top} = \frac{1.75 \times 4210\ k''}{12\ ''/'} = \frac{w(2L_b)^2}{24} \quad ;\ w = 5.9\ k/ft$$

$$C_b M_{cr\ bot} = \frac{1.75 \times 13906\ k''}{12\ ''/'} = \frac{w(2L_b)^2}{12} \quad ;\ w = 9.7\ k/ft$$

<u>Top Flange Controls: w = 5.9 k/ft</u>

C

SLENDERNESS LIMITS for SINGLY - SYMMETRIC SECTION

AISC LRFD APPENDIX F:

$$\lambda_r = \frac{X_1}{F_L}\ 1 + \sqrt{1 + X_2 F_L^2} \qquad X_1 = \frac{\pi}{S_{xc}} \sqrt{\frac{E G J A}{2}} \qquad X_2 = 4 \frac{C_w}{I_y} \left(\frac{S_{xc}}{G J} \right)^2$$

F_L = 50 ksi - 16.5 ksi = 33.5 ksi
$S_{xc} = 291\ in^3$
$J = 8.36\ in^4$
$C_w = 14400\ in^6$
$I_y = 162\ in^4$
$r_{yc} = 1.73\ in$

$X_1 = 2382$ $\qquad X_2 = 0.0035$

$\lambda_r = 127 = \frac{L_r}{r_{yc}}$

$\therefore L_r = 220\ in < 300\ in = L_b$

$\therefore$ <u>ELASTIC BUCKLING</u>

D

LOAD HEIGHT EFFECTS for SINGLY-SYMMETRIC SECTIONS

The method in the 4th edition of the SSRC Guide can be adjusted to work for Singly-Symmetric Sections:

Top Flange Loading: $C_b^* = C_b / B$
Bottom Flange Loading: $C_b^* = C_b B$

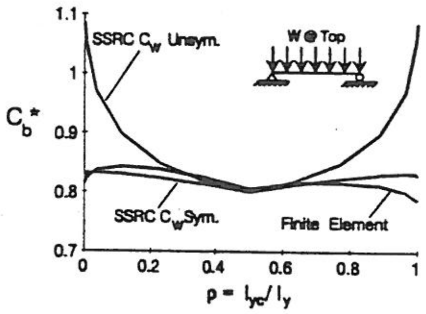
$$W = \frac{\pi}{L_b} \sqrt{\frac{E\ C_w}{G\ J}}$$

Midspan Point Load: $B = 1 - 0.180\ W^2 + 0.649\ W$
Distributed Load: $B = 1 - 0.154\ W^2 + 0.535\ W$

$$C_w = \frac{I_y d^2}{4}$$

E

VARIATION of C_b^* with ρ - Span/depth = 10



F

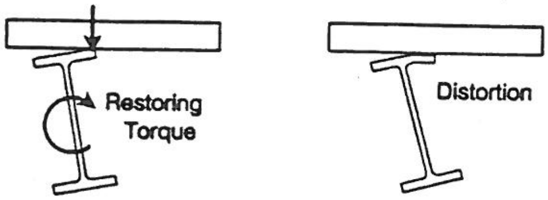
Why do most design specifications ignor the effects of load height on the flexural-torsional buckling capacity?

A) TIPPING EFFECT

B) INTERMEDIATE BRACING

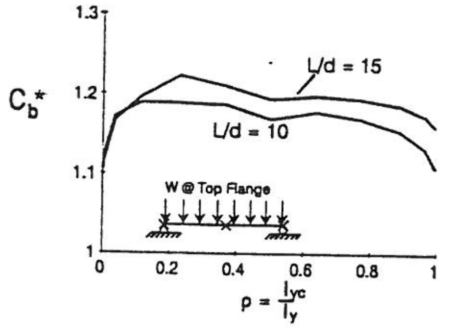
A

TIPPING EFFECT



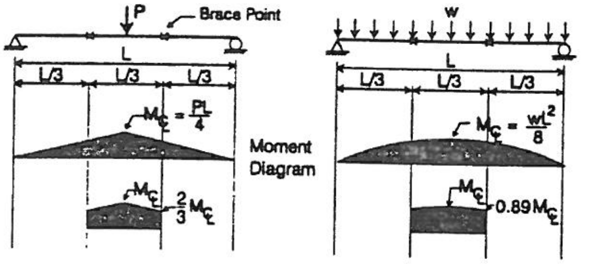
B

VARIATION of C_b^* with ρ – Midspan Brace (finite element)



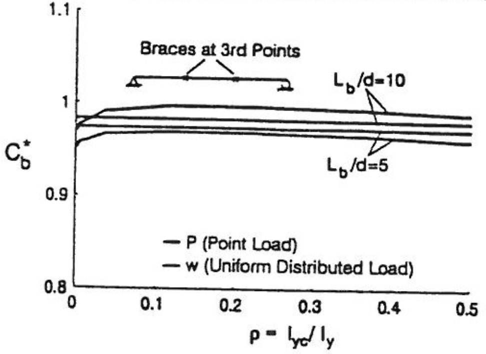
C

LOAD HEIGHT EFFECTS with INTERMEDIATE BRACING



D

VARIATION of C_b^* with ρ – Top Flange Loading



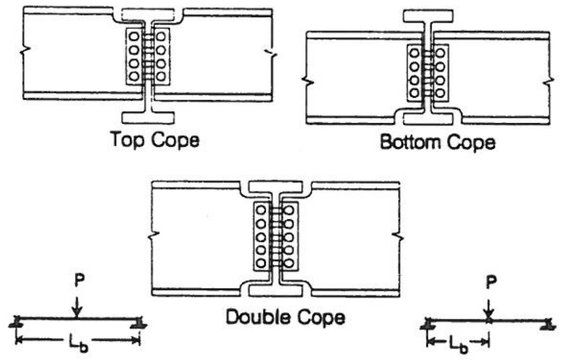
E

WHEN WILL LOAD HEIGHT BE A PROBLEM?

- For most practical situations, load height will not be a problem, however, when in doubt use a C_b^* equal to 1.0.
- Use the provisions outlined for load height in cases with no intermediate bracing, and top flange loading.
- Avoid cases with top flange loading such as the following:

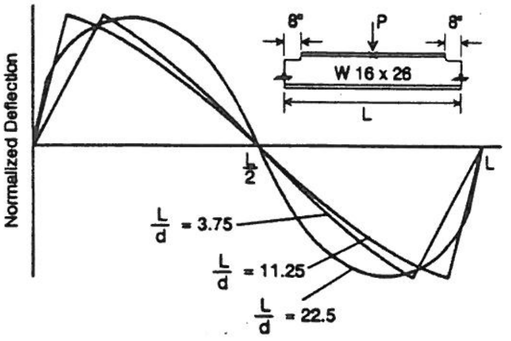
F

BUCKLING OF COPED BEAMS



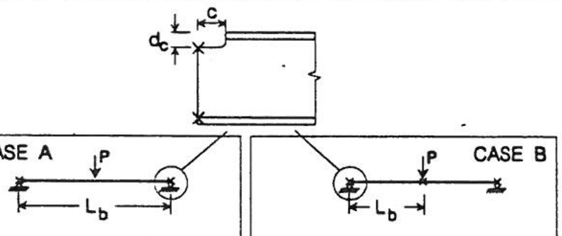
A

BUCKLING OF COPED BEAMS



B

COMPRESSION FLANGE COPED BEAMS



CASE A

$$\frac{1}{M^*_{LTB}} + \frac{1}{\left(\frac{L_b}{2c}\right) C_{dt} M_{TEE}} = \frac{1}{M_{cr}}$$

CASE B

$$\left(\frac{1}{M^*_{LTB}}\right)^2 + \left(\frac{1}{\left(\frac{L_b}{c}\right) C_{dt} M_{TEE}}\right)^2 = \left(\frac{1}{M_{cr}}\right)^2$$

C

COMPRESSION FLANGE COPED BEAMS

CASE A : Copes at both ends

$$\boxed{\frac{1}{M^*_{LTB}} + \frac{1}{\left(\frac{L_b}{2c}\right) C_{dT} M_{TEE}} = \frac{1}{M_{cr}}}$$

For rolled sections w/ $d_c \leq 0.2d$:

$\boxed{L_b = 2c}$

$\boxed{\text{Use AISC Equations for LTB for } M^*_{LTB} \text{ and } M_{TEE} \;;\; C_{dT} = 1.0}$

For rolled sections w/ $d_c \geq 0.2d$, or thin web members ($h/t_w \geq 60$):

$$M^*_{LTB} = \frac{3}{16} \frac{n}{(n+2)} E L_b d \frac{t_w^3}{d_c^2} \qquad n = \frac{16 \, t_f \, b_f^3 \, d_c^2}{(L_b t_w)^3}$$

$$C_{dT} = \left(1 - \frac{3}{2} \frac{d_c}{d}\right)$$

D

COMPRESSION FLANGE COPED BEAMS

CASE B: Cope at one end only

$$\boxed{\left(\frac{1}{M^*_{LTB}}\right)^2 + \left(\frac{1}{\left(\frac{L_b}{c}\right) C_{dT} M_{TEE}}\right)^2 = \left(\frac{1}{M_{cr}}\right)^2}$$

For rolled sections w/ $d_c \leq 0.1d$:

$\boxed{L_b = 2c}$

$\boxed{\text{Use AISC Equations for LTB for } M^*_{LTB} \text{ and } M_{TEE} \;;\; C_{dT} = 1.0}$

For rolled sections w/ $d_c \geq 0.1d$, or thin web members ($h/t_w > 60$):

$$\frac{1}{M^*_{LTB}} = \left(\frac{1}{M_{LTB}}\right)_{AISC} + \frac{1}{M_{dis}} \qquad M_{dis} = \frac{1}{2} E L_b d \frac{t_w^3}{d_c^2}$$

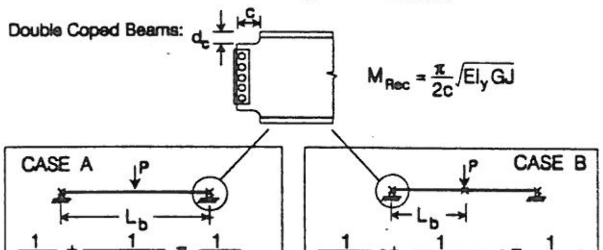
$$C_{dT} = \left(1 - \frac{3}{2} \frac{d_c}{d}\right)$$

E

TENSION FLANGE and DOUBLE COPED BEAMS

Tension Flange Coped Beams: $M_{cr} = 0.9 \, M_{LTB\,AISC}$

Double Coped Beams:



$$M_{Rec} = \frac{\pi}{2c} \sqrt{E I_y G J}$$

CASE A

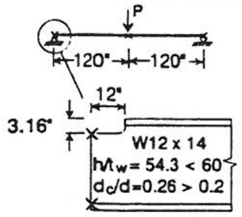
$$\frac{1}{M_{LTB\,AISC}} + \frac{1}{\left(\frac{L_b}{2c}\right) M_{Rec}} = \frac{1}{M_{cr}}$$

CASE B

$$\left(\frac{1}{M_{LTB\,AISC}}\right)^2 + \left(\frac{1}{\left(\frac{L_b}{c}\right) M_{Rec}}\right)^2 = \left(\frac{1}{M_{cr}}\right)^2$$

F

COPED BEAM EXAMPLE (COMPRESSION FLANGE)



Determine the maximum factored load that the coped beam can support without buckling.

Cope at one end only:

$$\frac{1}{\left(M^*_{LTB}\right)^2} + \frac{1}{\left(\left(\frac{L_b}{c}\right) C_{dT} M_{TEE}\right)^2} = \frac{1}{\left(M_{cr}\right)^2}$$

$$\frac{1}{M^*_{LTB}} = \left(\frac{1}{M_{LTB}}\right)_{AISC} + \frac{1}{M_{dis}}$$

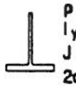
$$M_{LTB} = \overset{1.75}{C_b} \frac{\pi}{\underset{120"}{L_b}} \sqrt{EI_y GJ + E^2 I_y C_w \left(\frac{\pi}{L_b}\right)^2} = 585 \text{ k-in}$$

$$M_{dis} = \frac{1}{2} E L_b d \frac{t_w^3}{d_c^2} = 16603 \text{ k-in}$$

$$M^*_{LTB} = 565 \text{ k-in}$$

A

COPED BEAM EXAMPLE (COMPRESSION FLANGE)



$\rho = 0$
$I_y = 1.17 \text{ in}^4$
$J = 0.038 \text{ in}^4$
$2c = 24 \text{ in}$
$d_T = 8.75 \text{ in}$

AISC LRFD Equation for Tee's:

$$B_1 = 2.25\,(2\rho - 1)\,\frac{d_T}{2c}\sqrt{\frac{I_y}{J}} = -4.55$$

$$M_{TEE} = \frac{57000}{2c}\sqrt{I_y J}\left(B_1 + \sqrt{1 + B_1^2}\right) = 54 \text{ k-in}$$

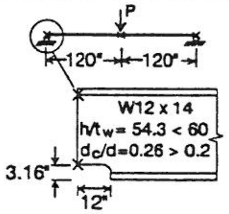
$$C_{dT} = \left(1 - \frac{3}{2}\frac{d_c}{d}\right) = 0.60 \qquad \frac{1}{\left(M^*_{LTB}\right)^2} + \frac{1}{\left(\left(\frac{L_b}{c}\right) C_{dT} M_{TEE}\right)^2} = \frac{1}{\left(M_{cr}\right)^2}$$

$$M_{cr} = 281 \text{ k-in} \Longrightarrow \phi M_{cr} = 0.9 \times 281 \text{ k-in} = 253 \text{ k-in}$$

$$\frac{PL}{4} = \phi M_{cr} \qquad P = \frac{4}{240 \text{ in}}\, 253 \text{ k-in} = \underline{\underline{4.2 \text{ kips}}}$$

B

COPED BEAM EXAMPLE (TENSION FLANGE)



Determine the maximum factored load that the coped beam can support without buckling.

$$M_{LTB} = \overset{1.75}{C_b} \frac{\pi}{\underset{120"}{L_b}} \sqrt{EI_y GJ + E^2 I_y C_w \left(\frac{\pi}{L_b}\right)^2}$$

$$M_{LTB} = 585 \text{ k-in}$$

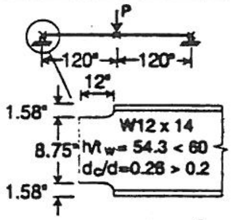
Tension Flange Cope: $M_{cr} = 0.9\, M_{LTB} = 526 \text{ k-in}$

$$\phi M_{cr} = 0.9 \times 526 \text{ k-in} = 474 \text{ k-in}$$

$$\frac{PL}{4} = \phi M_{cr} \qquad P = \frac{4}{240 \text{ in}}\, 474 \text{ k-in} = \underline{\underline{7.9 \text{ kips}}}$$

C

DOUBLE COPED BEAM EXAMPLE



Determine the maximum factored load that the coped beam can support without buckling.

Cope at one end only:

$$\frac{1}{\left(M_{\substack{LTB\\AISC}}\right)^2} + \frac{1}{\left(\left(\frac{L_b}{c}\right) M_{Rec}\right)^2} = \frac{1}{\left(M_{cr}\right)^2}$$

$$M_{Rec} = \frac{\pi}{2c}\sqrt{EI_y GJ} = 27.4 \text{ k-in}$$

$$I_y = \frac{1}{12}(8.75)(0.2)^3 = 0.0058 \text{ in}^4 \qquad J = \frac{1}{3}(8.75)(0.2)^3 = 0.023 \text{ in}^4$$

D

DOUBLE COPED BEAM EXAMPLE (CONTINUED)

$$M_{LTB} = \overset{1.75}{C_b} \frac{\pi}{\underset{120"}{L_b}} \sqrt{EI_y GJ + E^2 I_y C_w \left(\frac{\pi}{L_b}\right)^2} = 585 \text{ k-in}$$

$$M_{cr} = 248 \text{ k-in} \Longrightarrow \phi M_{cr} = 0.9 \times 248 \text{ k-in} = 223 \text{ k-in}$$

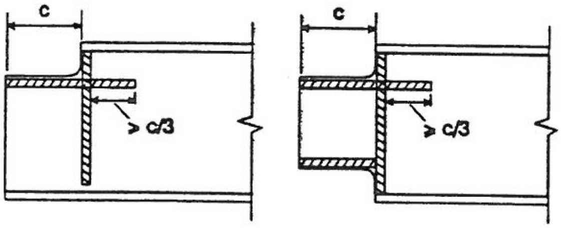
$$P = \frac{4}{240 \text{ in}}\, 223 \text{ k-in} = \underline{\underline{3.7 \text{ kips}}}$$

SUMMARY:

Compression Flange Cope: P = 4.2 kips
Tension Flange Cope: P = 7.9 kips
Double Flange Cope: P = 3.7 kips

E

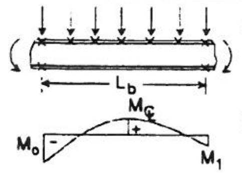
REINFORCING COPED BEAMS



- ▷ No reduction is required if $d_c < 0.2d$
- ▷ If $d_c > 0.2d$ consider effects of cross-section distorsion.

F

C_b - TOP FLANGE BRACED



1. If neither end moment cause compression on the bottom flange there is no buckling.
2. When one or both end moments cause compression on the bottom, use C_b with L_b.

M_0 = end moment that gives the largest comp. stress on the bottom flange

M_1 = the other end moment

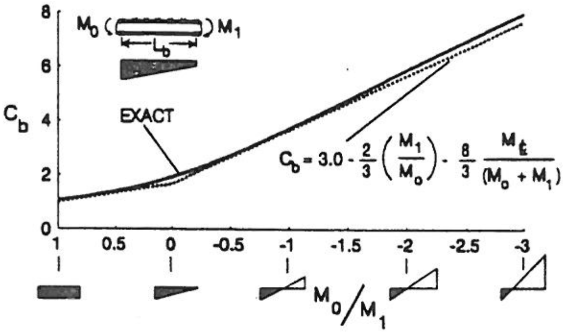
$M_{\text{℄}}$ = moment at midspan

$$\boxed{C_b = 3.0 - \frac{2}{3}\left(\frac{M_1}{M_0}\right) - \frac{8}{3}\,\frac{M_{\text{℄}}}{(M_0 + M_1)^*}}$$

* Take $M_1 = 0$ in this term if M_1 is positive

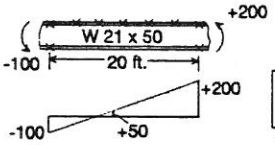
A

C_b - TOP FLANGE BRACED



B

C_b - TOP FLANGE BRACED (EXAMPLE)



The W21x50 has the top flange continuously braced and is subjected to the factored moments shown. Is the beam stable?

$$\boxed{C_b = 3.0 - \frac{2}{3}\left(\frac{M_1}{M_0}\right) - \frac{8}{3}\,\frac{M_{\text{℄}}}{(M_0 + M_1)^*}}$$

* Take $M_1 = 0$ in this term if M_1 is positive

$$C_b = 3.0 - \frac{2}{3}\left(\frac{+200}{-100}\right) - \frac{8}{3}\,\frac{(+50)}{(-100 + 0)} = 5.67$$

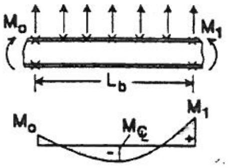
$$M_{cr} = \underset{\boxed{0.9}}{\phi}\,\underset{\boxed{5.67}}{C_b}\,\frac{\pi}{L_b}\sqrt{E I_y G J + E^2 I_y C_w \frac{\pi^2}{L_b^2}} = 9062 \text{ k-in} = \underline{\underline{755 \text{ k-ft} > 100 \text{ k-ft}}}$$

$\boxed{240 \text{ in.}}$

$$M_y = \phi S_x F_y = 0.9(94.5)\ 36/12 = \underline{\underline{255 \text{ k-ft} > 200 \text{ k-ft OK}}}$$

C

C_b - TOP FLANGE BRACED (UPLIFT)



If the applied loading does not cause compression on the bottom flange there is no buckling.

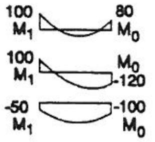
M_0 = end moment that produces the smallest tensile stress or the largest comp. stress in the bottom flange.

THREE CASES

CASE A - Both end moments are positive or zero:

CASE B - M_0 is negative, M_1 is positive or zero:

CASE C - Both end moments are negative:



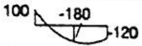
D

C_b - TOP FLANGE BRACED (UPLIFT)

CASE A - Both end moments are positive or zero:

$$\boxed{C_b = 2.0 - \frac{(M_0 + 0.6\,M_1)}{M_{\text{℄}}}} \quad C_b = 2.0 - \frac{(80 + 0.6\text{x}100)}{-150} = 2.93\ ;\ \underline{\underline{C_b M_{cr} > 150}}$$

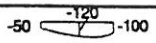
CASE B - M_0 is negative, M_1 is positive or zero:



$$\boxed{C_b = \frac{2M_1 - 2M_{\text{℄}} + 0.165\,M_0}{0.5M_1 - M_{\text{℄}}}} \quad C_b = \frac{2(100) - 2(-180) + 0.165(-120)}{0.5(100) - (-180)} = 2.35$$

$\underline{\underline{C_b M_{cr} > 180}}$

CASE C - Both end moments are negative:

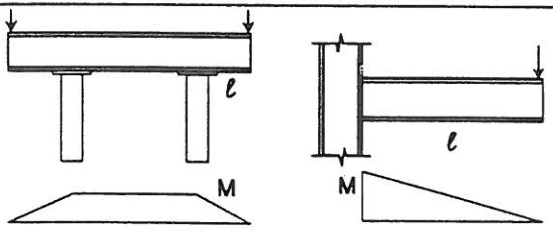


$$\boxed{C_b = 2.0 - \frac{(M_0 + M_1)}{M_{\text{℄}}}\left[0.165 + \frac{1}{3}\frac{M_1}{M_0}\right]} \quad C_b = 2.0 - \frac{(-100-50)}{-120}\left[0.165 + \frac{1}{3}\frac{(-50)}{(-100)}\right]$$

$C_b = 1.50$; $\underline{\underline{C_b M_{cr} > 120}}$

E

BUCKLING of UNBRACED CANTILEVERS



AISC = Unbraced Length = ℓ; $C_b = 1.0$

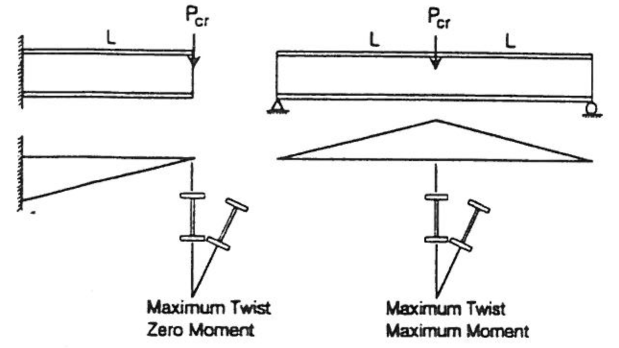
Lower Bound C_b values:

$C_b = 1.28$ $C_b = 2.04$

F

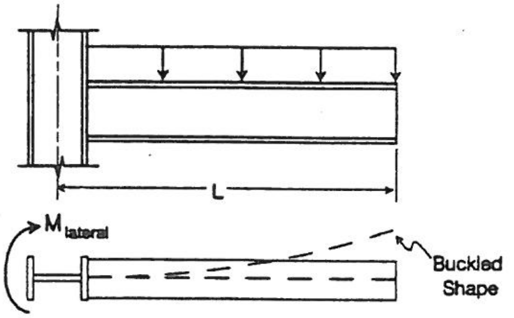
AISC/SSRC Short Course
"BRACING FOR STABILITY

DO NOT USE "2L" FOR UNBRACED CANTILEVERS



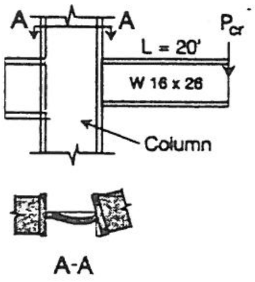
A

CANTILEVER BEAMS



B

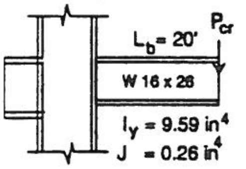
EFFECT OF FIXED END CONDITION



Column Size	P_{cr}
0	2.15 min
W 8 x 24	2.20
W 14 x 211	3.08
fixed	3.45 max

C

APPROXIMATION for UNBRACED CANTILEVERS



Neglect the warping term in the calculation of the buckling capacity.

$$M_{cr} = \phi C_b \frac{\pi}{L_b}\sqrt{EI_yGJ}$$

$$M_{cr} = \overset{\boxed{0.9}}{\phi}\,\overset{\boxed{1.28}}{C_b}\,\frac{\pi}{\overset{\boxed{240"}}{L_b}}\sqrt{EI_yGJ} = 428 \text{ k-in}$$

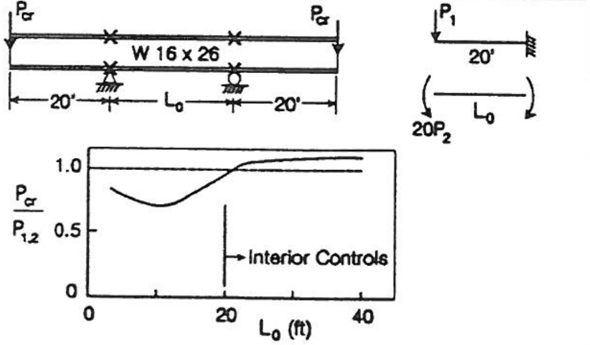
$P_{cr} = (428 \text{ k-in})/240 \text{ in} = \underline{1.78 \text{ k}}$

NOTE: IF WARPING TERM IS NOT NEGLECTED P_{cr} = 2.50 k.

Column Size	P_{cr}
0	2.15 min
W 8 x 24	2.20
W 14 x 211	3.08
fixed	3.45 max

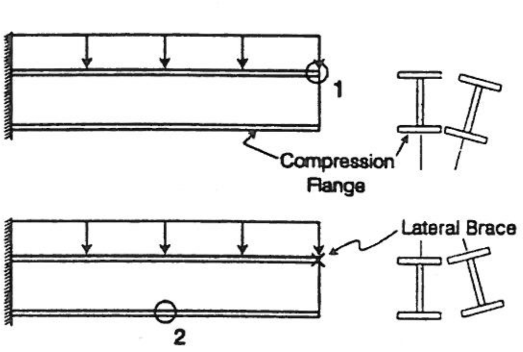
D

OVERHANGING BEAMS



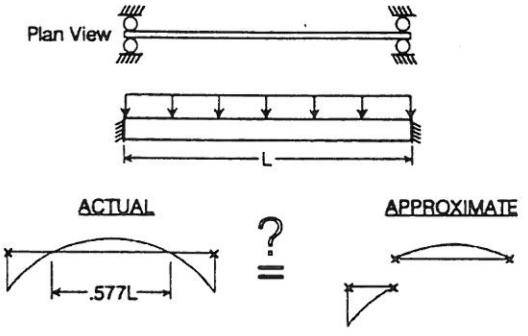
E

BRACING of CANTILEVER BEAMS



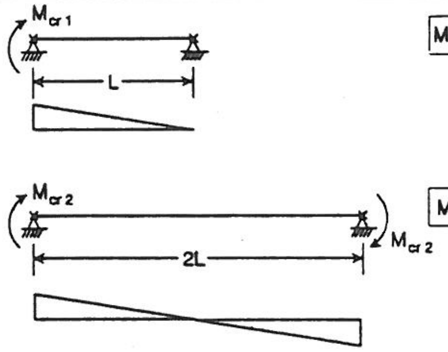
F

IS THE INFLECTION POINT A BRACE POINT?



A

THE INFLECTION POINT IS NOT A BRACED POINT

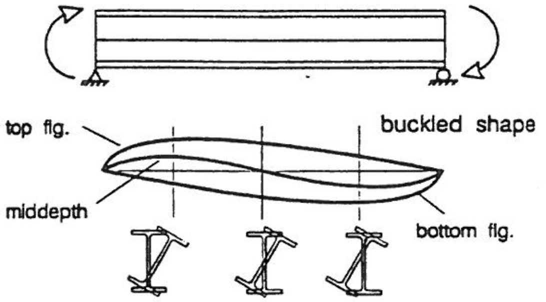


$\boxed{M_{cr\,1} = M}$

$\boxed{M_{cr\,2} = 0.73M}$

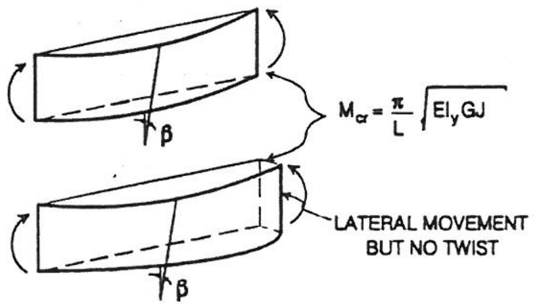
B

AN INFLECTION POINT IS NOT A BRACE POINT



C

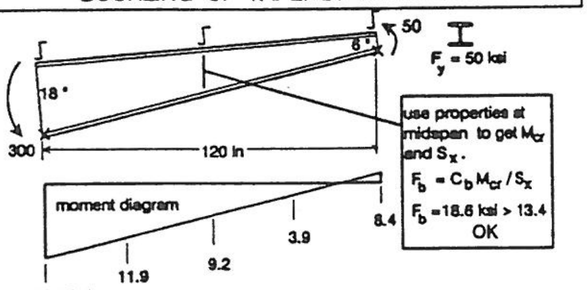
PURE TRANSLATION HAS NO EFFECT ON THE BUCKLING CAPACITY



$M_{cr} = \frac{\pi}{L}\sqrt{EI_yGJ}$

D

BUCKLING OF TAPERED MEMBERS



use properties at midspan to get M_{cr} and S_x.

$F_b = C_b M_{cr} / S_x$

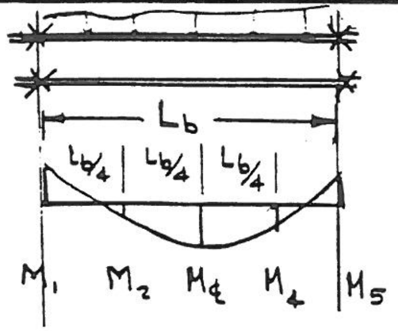
$F_b = 18.6$ ksi > 13.4
OK

new AISC formula $\boxed{C_b = \frac{12.5\ (13.4)}{2.5\ (13.4) + 3\ (11.9) + 4\ (9.2) + 3\ (3.9)} = 1.42}$

E

F

Case I - Braces at the ends of the unbraced length

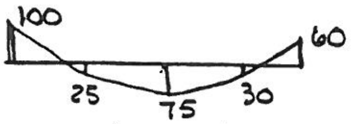


$$\boxed{C_b = \frac{12.5\,M_{max}}{2.5M_{max} + 3M_2 + 4M_℄ + 3M_4}}^{*} \qquad (1)$$

Use absolute value of the moments
M_{max} is the largest absolute value of $M_1, M_2\, M_℄\, M_4$ or M_5

C_b is always used with the maximum moment within the unbraced length, even if M_{max} is at the end

Ex.



$$C_b = \frac{12.5(100)}{2.5(100)+3(25)+4(75)+3(30)} = 1.748$$

Comparison with existing solutions:

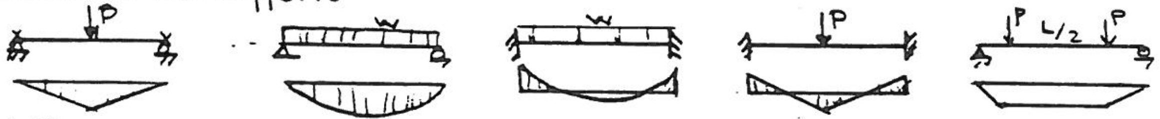
Straight line M-diag

$$M_s = \frac{12{:}5M_L}{2.5M_L + 3\left(\frac{M_s}{4}+\frac{3}{4}M_L\right) + 4\left(\frac{M_s}{2}+\frac{M_L}{2}\right) + 3\left(\frac{M_L}{4}+\frac{3}{4}M_s\right)}$$

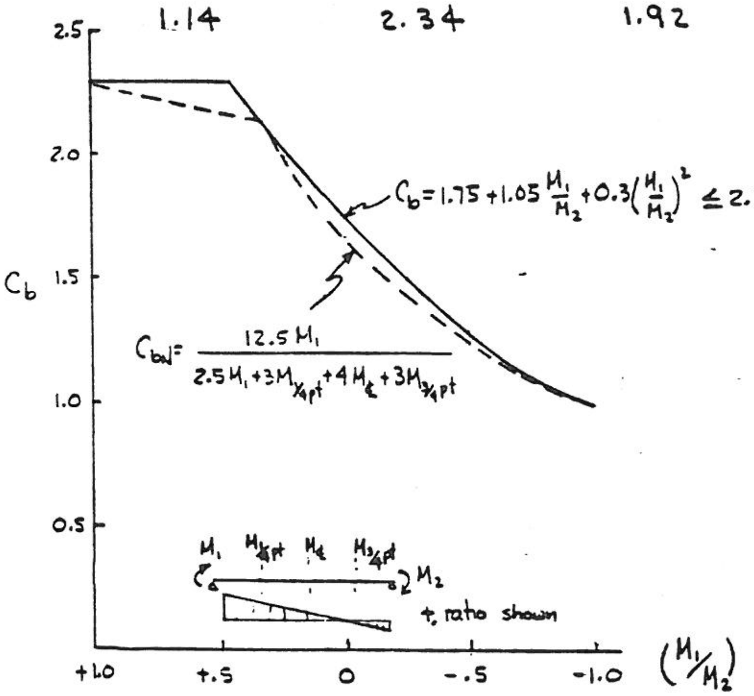
$$= \frac{12.5M_L}{7.5M_L + 5M_s} = \frac{1}{0.6 + 4\frac{M_s}{M_L}}$$

See plot for comparison with AISC $C_b = 1.75 + 1.05\frac{M_s}{M_L} + .3\left(\frac{M_s}{M_L}\right)^2 \leq 2.3$

Braces at end supports



Published**	1.35	1.13	2.60	1.70	1.04
Eq(1)	1.32	1.14	2.34	1.92	1.00



C_b for a straight line moment diagram - prismatic beam

C_{bd} is conservative but difference is < 6%

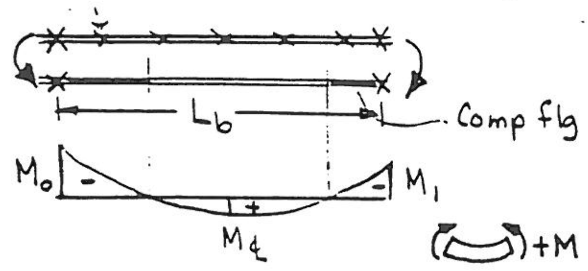
---

* This formula was adopted from a formula in the book by Kirby & Nethercot "Design for Struct. Stab."

** See "Guide to Design of Metal Compression Members" (Johnston) or Salmon & Johnson "Steel Structures"

CASE 2 - Beams with top flange braced continuously - downward load

(It is assumed that loading is also applied at the top flange)



1. If neither end moment produces a compressive stress on the bottom flange, there is no buckling

2. When one or both end moments produce bottom flange compression

$$\boxed{C_b = 3.0 - \frac{2}{3}\left(\frac{M_1}{M_0}\right) - \frac{8}{3}\,\frac{M_{\text{℄}}}{(M_0+M_1)^*}} \qquad (2)$$

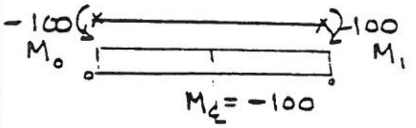
* take $M_1 = 0$ in this term if M_1 is positive. This means that the (M_0+M_1) term must always be between $1.0M_0$ and $2.0M_0$

· M_0, M_1 and $M_{\text{℄}}$

- M_1 — the other end moment
- M_0 — end moment that produces the largest comp. stress on the bottom flg.

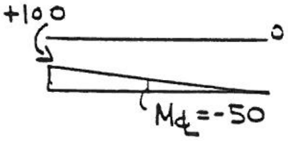
C_b is used with M_0 to check LTB.
Use M_{max} to check yielding

Examples



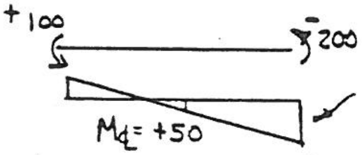
$$C_b = 3.0 - \frac{2}{3}\left(\frac{-100}{-100}\right) - \frac{8}{3}\left(\frac{-100}{-100-100}\right)$$

$= 1.00$



$$C_b = 3.0 - \frac{8}{3}\left(\frac{-50}{+100}\right)$$

$= 1.67$

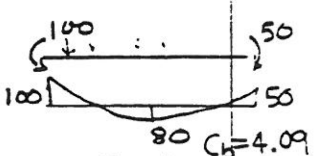


$$C_b = 3.0 - \frac{2}{3}\left(\frac{-200}{+100}\right) - \frac{8}{3}\left(\frac{+50}{100+0}\right)$$

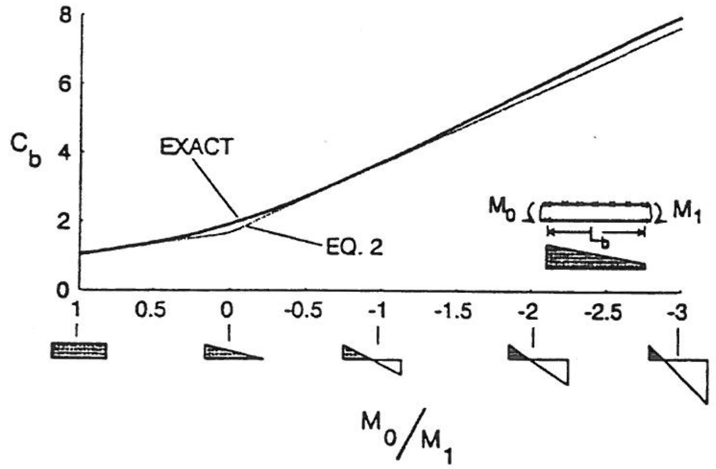
$= 5.67$

since M_1 is negative

$100 \le 5.67 \times$ Basic buckling formula
Use 200 to check yielding



$C_b = 4.09$



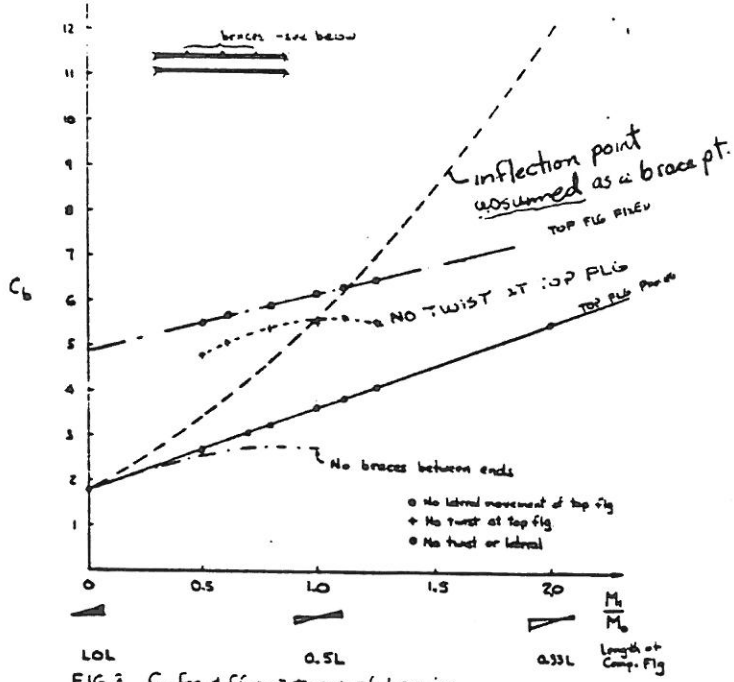
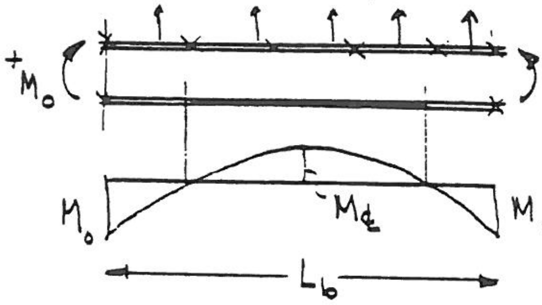


FIG. 3 C_b for different types of bracing

Case 3 Uplift or Suction Load - top flange braced continuously

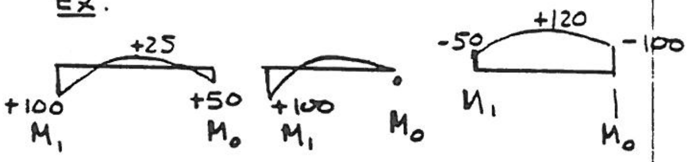
(Load also applied at top flange)



All moments shown are positive

M_o = end moment that produces the smallest tensile stress or largest compressive stress on the bottom flg., i.e., the smallest algebraic end moment

Ex.



Solution

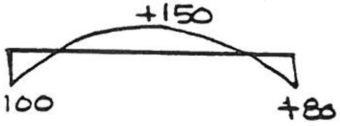
1. If M_o, M_1 or M_{max} do not produce a compressive stress on the bottom flange, there is no buckling. Check only yielding.

2. When both end moments are positive or zero as shown above

$$\boxed{C_b = 2.0 + \frac{(M_o + 0.6 M_1)}{M_{℄}}} \qquad (3)$$

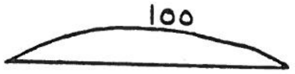
use with largest moment that produces compression on the bottom flange

Examples



$$C_b = 2.0 + \frac{(80 + 0.6 \times 100)}{150} = 2.93$$

use with 150

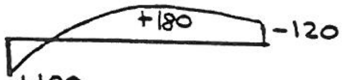


$C_b = 2.0$ since $M_o = M_1 = 0$

3. When M_o is negative and M_1 is positive or zero

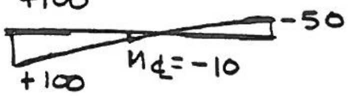
$$\boxed{C_b = \frac{2 M_{℄} + 2 M_1 + 0.165 M_o}{M_{℄} + 0.5 M_1}} \qquad (4)$$

Examples



$$C_b = \frac{2(180) + 2(100) + .165(-120)}{180 + .5(100)} = 2.35$$

with 180



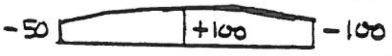
$$C_b = \frac{2(-10) + 2(100) + .165(-50)}{-10 + .5(100)} = 4.29$$

with 50

4. When both M_o and M_1 are negative

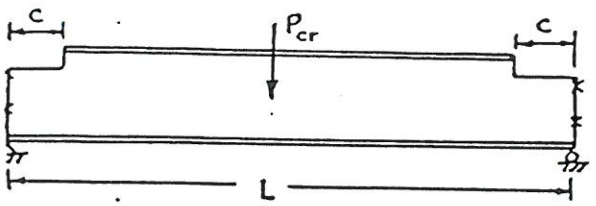
$$C_b = 2 + \frac{(M_o + M_1)}{M_{℄}}\left(0.165 + \frac{1}{3}\left(\frac{M_1}{M_o}\right)\right) \qquad (5)$$

Example



$$C_b = 2 + \frac{(-100-50)}{100}\left(.165 + \frac{1}{3}\left(\frac{-50}{-100}\right)\right) = 1.50$$

LATERALLY UNSUPPORTED COPED BEAMS



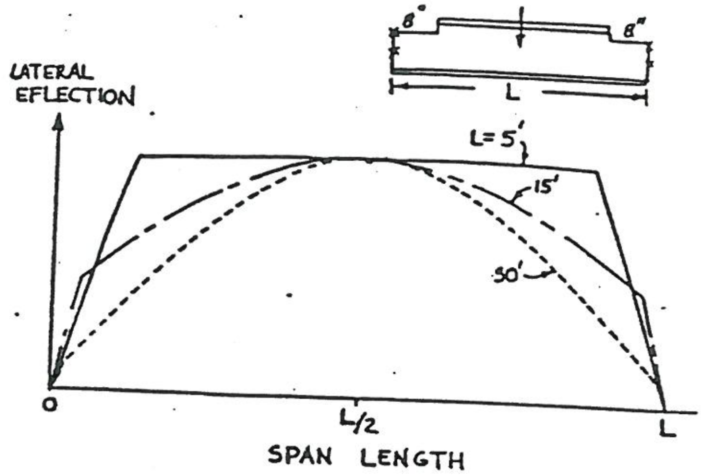
Design Recommendation:

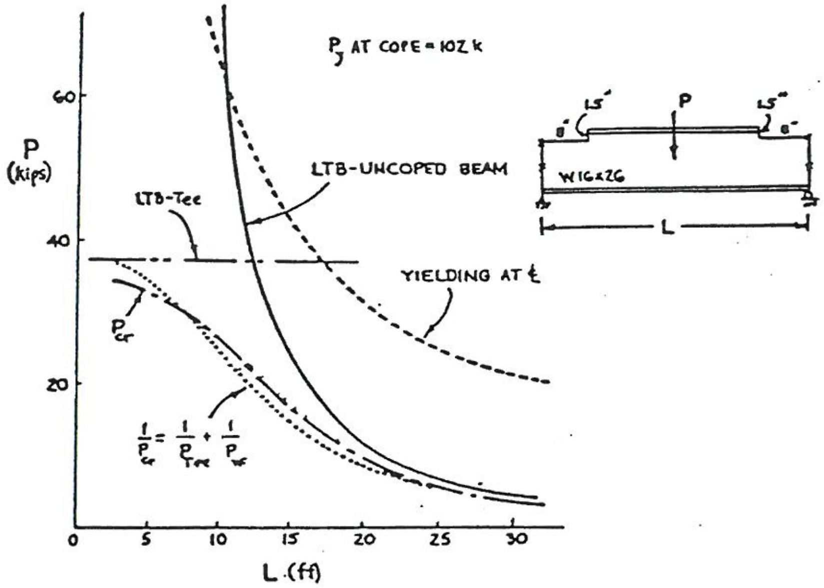
$$\boxed{\frac{1}{P_{cr}} = \frac{1}{P_{WF}} + \frac{1}{P_{Tee}}}$$

LTB OF UNCOPED BEAM WITH SPAN = L

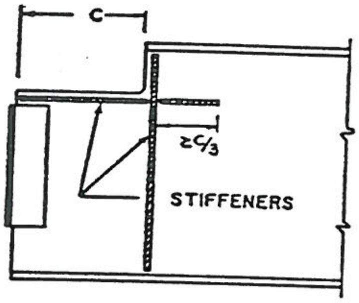
LTB OF Tee BEAM WITH SPAN 2C

BUCKLED SHAPES





COPE REINFORCEMENT FOR LATERAL BUCKLING



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Technical Papers

shown in Fig. 2(a). For example, at $P = 0.95P_{cr}$ and $\Delta_o = L_b / 500$, the brace force is 7.6% of P_e which is off the scale of the graph. Theoretically the brace force will be infinity when the buckling load is reached if the ideal brace stiffness is used. Thus, a brace system will not be satisfactory if the theoretical ideal stiffness is provided because the brace forces get too large. If the brace stiffness is overdesigned, as represented by the $\beta_L = 2\beta_i$ and $3\beta_i$ curves in Fig. 2(b), then the brace forces will be more reasonable. For a brace stiffness twice the ideal value and a $\Delta_o = L_b / 500$, the brace force is only 0.8%P_e at $P = P_e$, not infinity as in the ideal brace stiffness case. For a brace stiffness ten times the ideal value, the brace force will reduce even further to 0.44%. The brace force cannot be less than 0.4%P corresponding to $\Delta = 0$ (an infinitely stiff brace) for $\Delta_o = L_b / 500$. For design $F_{br} = 1\%P$ is recommended based on a brace stiffness of twice the ideal value and an initial out-of-straightness of $L_b / 500$ because the Winter model gives slightly unconservative results for the midspan brace problem (Plaut, 1994).

Published bracing requirements for beams usually only consider the effect of brace stiffness because perfectly straight beams are considered. Such solutions should not be used directly in design. Similarly, design rules based on strength considerations only, such as a 2% rule, can result in inadequate bracing systems. Both strength and stiffness of the brace system must be checked.

Beam Bracing Systems

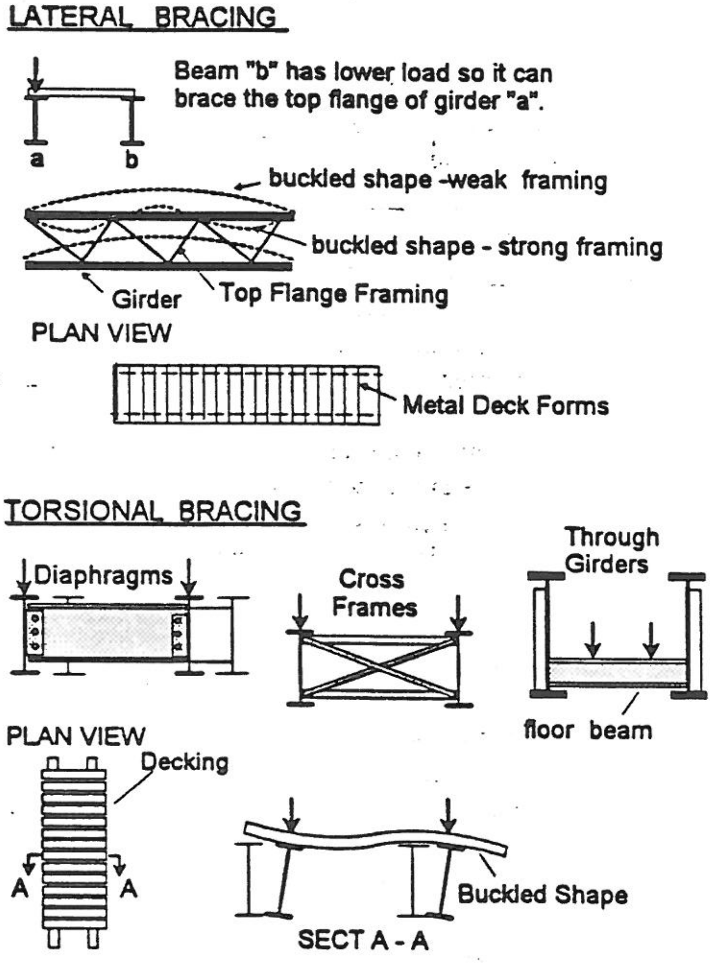


Fig. 3 Types of Beam Bracing

Beam bracing is a much more complicated topic compared to column bracing. This is due mainly to the fact that most column buckling involves primarily bending whereas beam buckling involves both flexure and torsion. An effective beam brace resists twist of the cross section. In general bracing may be divided into two main categories, lateral and torsional bracing as illustrated in Fig. 3. Lateral bracing restrains lateral displacement as its name implies. The effectiveness of a lateral brace is related to the degree that twist of the cross section is restrained. For a simply supported beam subjected to uniform moment, the center of twist is located at a point outside the tension flange; the top flange moves laterally much more than the bottom flange. Therefore, a lateral brace restricts twist best when it is located at the top flange. Lateral bracing attached at the bottom flange of a simply supported beam is almost totally ineffective. A torsional brace can be differentiated from a lateral brace in that twist of the cross section is restrained directly, as in the case of twin beams with a cross frame or diaphragm between the members. The cross frame location, while able to displace laterally, is still considered a brace point because twist is prevented. Some systems such as concrete slabs can act both as lateral and torsional braces. Bracing that controls both lateral movement and twist is more effective than lateral or torsional braces acting alone (Tong and Chen, 1988; Yura, 1992). However, since bracing requirements are so minimal, it is more practical to develop separate design recommendations for these two types of systems.

Lateral bracing can be divided into four categories: relative, discrete, continuous and lean-on. A relative brace system controls the relative lateral movement between two points along the span of the a half sine curve. Even though there is lateral movement at the brace point, the load increase can be more than three times the unbraced case. The ideal brace stiffness required to force the beam to buckle between lateral supports is 1.6 k/in. in this example. Any brace stiffness greater than this value does not increase the beam buckling capacity and the buckled shape is a full sine curve. When the brace is attached at the top flange, there is no cross section distortion. No stiffener is required at the brace point.

A lateral brace placed at the centroid of the cross section requires an ideal stiffness of 11.4 k/in. if a 4 x 1/4 stiffener is attached at midspan and 53.7 k/in. (off scale) if no stiffener is used. Substantially more bracing is required for the no stiffener case because of web distortion at the brace point. The centroid bracing system is less efficient than the top flange brace because the centroid brace force causes the center of twist to move <u>above</u> the bottom flange and closer to the brace point which is undesirable for lateral bracing.

For the case of a beam with a concentrated centroid load at midspan, shown in Fig. 7, the moment varies along the length. The ideal centroid brace (110 k/in.) is 44 times larger than the ideal top flange brace (2.5 k/in.). For both brace locations cross section distortion had a minor effect (<3%). The maximum beam moment at midspan when the beam buckles between the braces is 1.80 times greater than the uniform moment case which is close to the C_b factor = 1.75 given in specifications (AISC, AASHTO). This higher buckling moment is the main reason why the ideal top flange brace requirement is 1.56 times greater (2.49 vs. 1.6 k/in.) than the uniform moment case.

Figure 8 shows the effects of load and brace position on the buckling strength of laterally braced beams. If the load is at the top flange, the effectiveness of a top flange brace is greatly reduced. For example, for a brace stiffness of 2.5 k/in., the beam would buckle between the ends and the midspan brace at a centroid load close to 50 kips. If the load is at the top flange, the beam will buckle at a load of 28 kips. For top flange loading, the ideal top flange brace would have to be increased to 6.2 k/in. to force buckling between the braces. The load position effect must be considered in the brace design requirements. This effect is even more important if the lateral brace is attached at the centroid. The results shown in Fig. 8 indicate that a centroid brace is almost totally ineffective for top flange loading. This is not due to

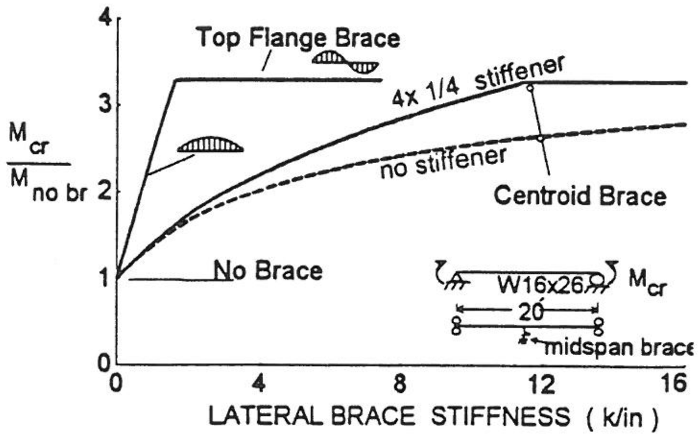


Fig. 6 Effect of Lateral Brace Location

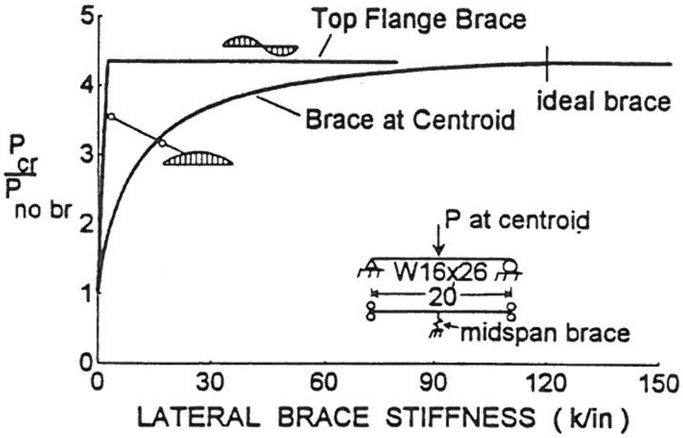


Fig. 7 Midspan Load at Centroid

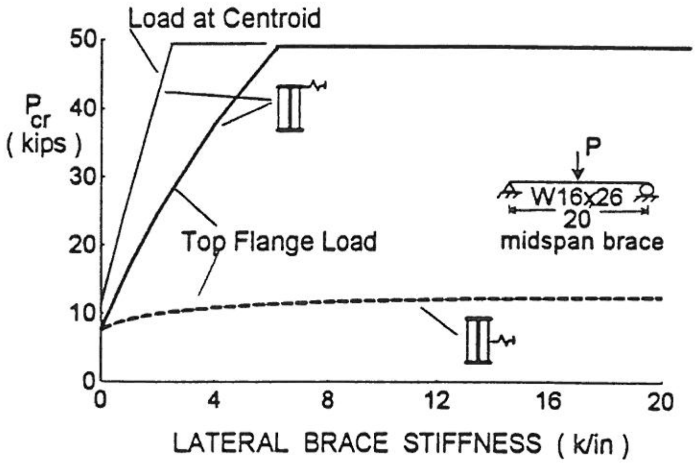
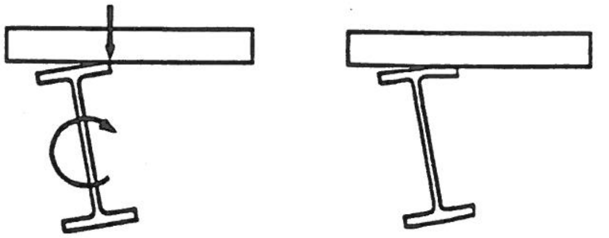


Fig. 8 Effect of Load and Brace Position

cross section distortion since a stiffener was used at the brace point. The top flange loading causes the center of twist at buckling to shift to a position close to mid-depth for most practical unbraced lengths, as shown in Fig. 5. Since there is virtually no lateral displacement near the centroid for top flange loading, a lateral brace at the centroid will not brace the beam. Because of cross-section distortion and top flange loading effects, lateral braces at the centroid are not recommended. Lateral braces must be placed near the top flange of simply supported and overhanging spans. Design recommendations will be developed only for the top flange lateral bracing situation. Torsional bracing near the centroid or even the bottom flange can be effective as discussed later.



Restoring Torque (a)

Cross Section Distortion (b)

Fig. 9 Tipping Effect

The load position effect discussed above assumes that the load remains vertical during buckling and passes through the plane of the web. In the laboratory, a top flange loading condition is achieved by loading through a knife edge at the middle of the flange. In structures the load is applied to the beams through secondary members or the slab itself. Loading through the deck can provide a beneficial "tipping" effect illustrated in Fig. 9. As the beam tries to buckle, the contact point shifts from mid-flange to the flange tip resulting in a restoring torque which increases the buckling capacity. Unfortunately, cross- section distortion severely limits the beneficial effects of tipping. Linder (1982, in German) has developed a solution for the tipping effect which considers the flange-web distortion. The test data (Linder,1982; Raju, 1992)indicates that a cross member merely resting (not positively attached) on the top flange can significantly increase the lateral buckling capacity. The tipping solution is sensitive to the initial shape of the cross section and location of the load point on the flange. Because of these difficulties, it is recommended that the tipping effect not be considered in design.

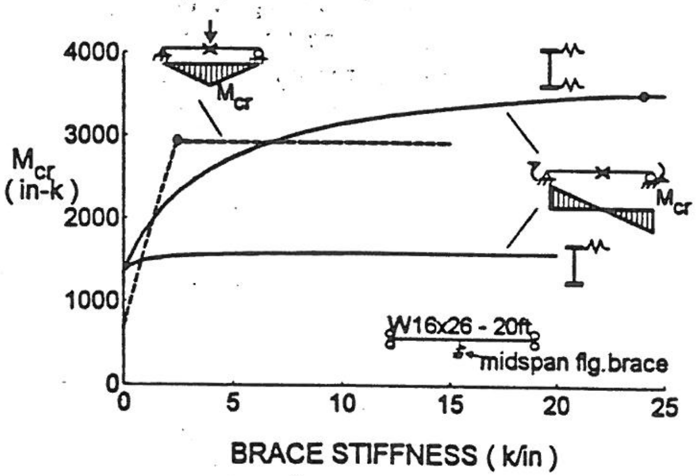


Fig. 10 Beams with Inflection Points

When a beam is bent in double curvature the compression flange switches from the top flange to the bottom flange at the inflection point. Beams with compression in both the top and bottom flanges along the span have more severe bracing requirements than beams with compression on just one side as illustrated by the comparison of the cases given in Fig. 10. The solid lines are BASP solutions for a 20 ft long W16x26 beam subjected to equal but opposite end moments and with lateral bracing at the midspan inflection point. For no bracing the buckling moment is 1350 in-k. A brace attached to one flange is ineffective for reverse curvature because twist at midspan is not prevented. If lateral bracing is attached to both flanges, the buckling moment increases nonlinearly as the brace stiffness increases to 24 k/in, the ideal value shown by the black dot. Greater brace stiffness has no effect because buckling occurs between the brace points. The ideal brace stiffness for a beam with a concentrated midspan load is 2.6 k/in at M_{cr} = 2920 in-k as shown by the dashed lines. For the two load cases the moment diagrams between brace points are similar, maximum moment at one end and zero moment at the other end. In design a C_b = 1.75 is used for these cases which corresponds to an expected maximum moment of 2810 in-k. The double curvature case reached a maximum moment 25% higher because of warping restraint at midspan provided by the adjacent tension flange. In the concentrated load case no such restraint is available since the compression flanges of both unbraced segments are adjacent to each other. On the other hand, the brace stiffness at each flange must be 9.2 times the ideal value of the concentrated load case to achieve the 25 % increase. Since warping restraint is usually ignored in design M_{cr} = 2810 in-k is the maximum

Table 1. Brace Coefficient

Number of Braces	Brace Coef.
1	2
2	3
3	3.41
4	3.63
Many	4.0

For elastic beams under uniform moment the Winter ideal lateral brace stiffness required to force buckling between the braces is $\beta_i = \# P_f / L_b$ where $P_f = \pi^2 EI_{yc} / L^2_b$, I_{yc} is the out-of-plane moment of inertia of the compression flange which is $I_y/2$ for doubly symmetric cross sections, and # is a coefficient depending on the number of braces n within the span, as given in Table 1(Winter, 1960) or approximated by $\# = 4 - (2/n)$. The C_b factor given in design specifications for nonuniform moment diagrams can be used to estimate the increased brace requirements for other loading cases. For example, for a simply supported beam with a load and brace at midspan shown in Fig. 7, the full bracing stiffness required is 1.56 times greater than the uniform moment case. The $C_b = 1.75$ for this loading case provides a conservative estimate of the increase. An additional modifying factor $C_d = 1 + (M_S / M_L)^2$ is required when there are inflection points along the span (double curvature), where M_S and M_L are the maximum moments causing compression in the top and bottom flanges as shown in Fig. 13. The moment ratio must be equal to or less than one, so C_d varies between 1 and 2. In double curvature cases lateral braces must be attached to both flanges. Top flange loading increases the brace requirements even when bracing is provided at the load point. The magnitude of the increase is affected by the number of braces along the span as given by the modifying factor $C_L = 1 + (1.2/n)$. For one brace $C_L = 2.2$; for many braces top flange loading has no effect on brace requirements, i.e. $C_d = 1.0$.

In summary, a modified Winter's ideal bracing stiffness can defined as follows,

$$\beta_i^* = \frac{\# C_b P_f}{L_b} C_L C_d \qquad (1)$$

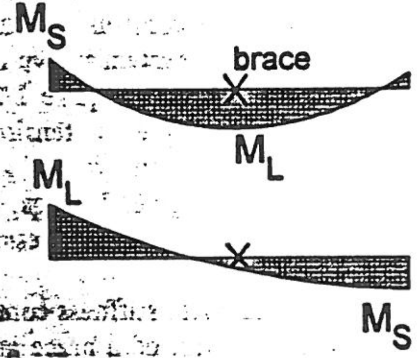


Fig. 13 Double Curvature

For the W12x14 beams laterally braced at midspan shown in Fig. 12, $L_b = 144$ in., $\# = 2$, $C_b = 1.75$, $C_L = 1 + 1.2/1 = 2.2$, and $P_f = \pi^2 (29000) (2.32/2)/(144)^2 = 16.01$ kips, $\beta_i^* = 0.856$ k/in. which is shown by the * in Fig. 12. Equation (1) compares very favorably with the test results and with the theoretical BASP results. For design the ideal stiffness given by Eq. (1) must be doubled for beams with initial out-of-straightness so brace forces can be maintained at reasonable levels as discussed earlier. The brace force requirement for beams follows directly from the column $F_{br} = 0.01P$ for discrete braces given earlier. The column load P is replaced with the equivalent compressive beam flange force, either $(C_b P_f)$ or M_f / h, where M_f is the maximum beam moment and h is the distance between flange centroids. The M_f / h estimate of the flange force is applicable for both the elastic and inelastic regions. For relative bracing the force requirement is one half the discrete value. The lateral brace design recommendations which follow are based on an initial out-of-straightness of adjacent brace points of $L_b/500$. The combined

LATERAL BRACING DESIGN REQUIREMENTS

Stiffness: $$\beta_L^* = 2 \# (C_b P_f) C_L C_d / L_b \quad \text{or} \quad 2 \# (M_f / h) C_L C_D / L_b \qquad (2)^*$$

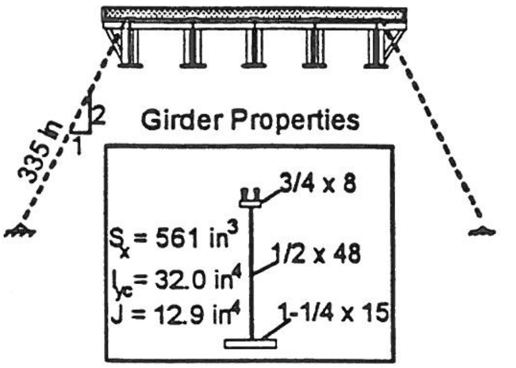
where
$\#$ = 4 - (2/n) or the coefficient in Table 1 for discrete bracing; = 1.0 for relative bracing
$C_b P_f$ = $C_b \pi^2 E I_{yc} / L_b^2$; or = (M_f / h) where M_f is the maximum beam moment
C_L = $1 + (1.2/n)$ for top flange loading; = 1.0 for other loading
C_d = $1 + (M_S / M_L)^2$ for double curvature; = 1.0 for single curvature
n = number of braces

Strength: Discrete bracing: $F_{br} = 0.01 \, C_L C_d M_f / h$ (3)
Relative bracing: $F_{br} = 0.004 \, C_L C_d M_f / h$ (4)

* Conservative simplified Eq(2) $\beta_L^* = 10 (M_f / h) C_d / L_b$ for any n

torsional brace attached to the compression flange, then the buckling strength will increase until buckling occurs between the braces at 3.3 times the no-brace case. The ideal or full bracing requires a stiffness of 1580 in-k/radian for a 4 x 1/4 stiffener and 3700 in-k/radian for a 2.67 x 1/4 stiffener. Tong and Chen (1988) developed a closed form solution for ideal torsional brace stiffness neglecting cross-section distortion that is given by the solid dot at 1450 in-k/radian in Fig. 14. The difference between the Tong solution and the BASP results is due to web distortion . Their solution would require a 6 x 3/8 stiffener to reach the maximum buckling load. If the Tong ideal stiffness (1450 in-k/radian) is used with a 2.67 x 1/4 stiffener, the buckling load is reduced by 14%; no stiffener gives a 51% reduction.

LATERAL BRACING - DESIGN EXAMPLE 1



$S_x = 561 \text{ in}^3$
$I_{yc} = 32.0 \text{ in}^4$
$J = 12.9 \text{ in}^4$

Span = 80 ft. ; 10 in. concrete slab

5 girders @ 8 ft spacing, A36 steel

Design a lateral bracing system to stabilize the girders during the deck pour. Use the external tension system shown. The form supports transmit some load to the bottom flange so assume centroid loading.

Use Load Factor Design for the construction condition - L.F. = 1.3

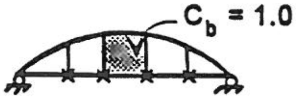
Loads: Steel girder: $A = 48.75 \text{ in.}^2$, wt = 165 lb/ft

Conc. slab: $8' \times \frac{10}{12} \times 150 \text{ lb/ft}^3 = \underline{1000 \text{ lb/ft}}$

$w = 1165 \text{ lb/ft} = 1.165 \text{ k/ft}$

$$M = \frac{1}{8} w L^2 \times \text{L.F.} = \frac{1}{8}(1.165)(80)^2 1.3 = 1211 \text{ k-ft}$$

$$M_y = 36\ (561)/12 = 1682 \text{ k-ft} > 1211 \text{ k-ft}$$



Try 4 lateral braces @ 16-ft spacing

Check lateral buckling - center 16-ft is most critical (AASHTO 10-102c)

$$M = 91 \times 10^6\ (1.0) \frac{32.0}{16 \times 12} \sqrt{0.772 \frac{12.9}{32.0} + 9.87 \left(\frac{50}{16 \times 12}\right)^2}$$

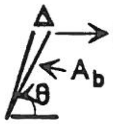
$$= 15020000 \text{ lb-in} = 1251 \text{ k-ft} > 1211 \text{ k-ft} \qquad \underline{\text{4 braces required}}$$

<u>Brace Design:</u> Use the full bracing formula - discrete system -
See Eq 2 & 3

$$P_f = \frac{\pi^2\ (29000)\ (32.0)}{(16 \times 12)^2} = 248 \text{ kips}; \quad \# = 4 - \frac{2}{4} = 3.5\ ; \quad C_b = 1.0\ ; \quad C_L = 1.0$$

$$\beta_L^* = 2\ \frac{3.5\ (248)(1.0)(1.0)}{16 \times 12} = 9.04 \text{ k/in. for ea. girder} = 45.2 \text{ k/in. for 5 girders} = F/\Delta$$

$$\text{Brace stiffness} = \cos^2\theta \left(\frac{AE}{L}\right)_b = \frac{A_b\ (29000)}{(\sqrt{5}\ )^2\ 335} = 45.2 \text{ k/in.}$$



$$\boxed{A_b = 2.61 \text{ in}^2} \longleftarrow \text{CONTROLS}$$

Brace Strength:
(A36 steel)

$$F_{br} = 0.01\ \underbrace{(5)}_{\text{five girders}}\ (1211 \times 12\ /\ 49.0) = 14.83 \text{ k}$$

$$A_b F_y = 14.83\ /\ \cos\theta\ ; \qquad A_b = \frac{14.83 \sqrt{5}}{36} = \underline{0.92 \text{ in}^2}$$

for the unbraced beams (zero brace stiffness). The ideal brace stiffness for top flange loading is 18% greater than for centroid loading. This behavior is different from that shown in Fig. 8 for lateral bracing where the top flange loading ideal brace is 2.5 times that for centroid loading.

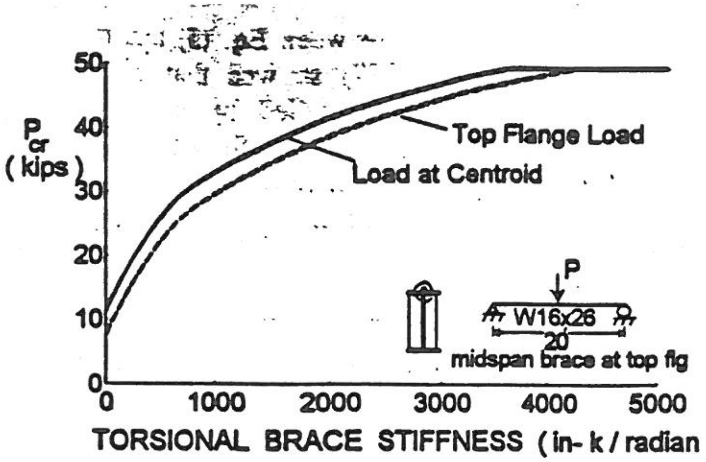


Fig. 17 Effect of Load Position

Figure 18 summarizes the behavior of a 40-ft span with three equal torsional braces spaced 10-ft apart. The beam was stiffened at each brace point to control the distortion. The response is non-linear and follows the pattern discussed earlier for a single brace. For brace stiffness less than 1400 in-k/radian, the stringer buckled into a single wave. Only in the stiffness range of 1400-1600 in-k/radian did multi-wave buckled shapes appear. The ideal brace stiffness at each location was slightly greater than 1600 in-k/radian. This behavior is very different from the multiple lateral bracing case for the same beam shown in Fig. 11. For multiple lateral bracing the beam buckled into two waves when the moment reached 600 in-k and then into three waves at M_{cr} = 1280 in-k. For torsional bracing, the single wave controlled up to M_{cr} = 1520 in-k. Since the maximum moment of 1600 corresponds to buckling between the braces, it can be assumed, for design purposes, that torsionally braced beams buckle in a single wave until the brace stiffness is sufficient to force buckling between the braces. The figure also shows that a single torsional brace at midspan of a 20-ft span (unbraced length = 10 ft) requires about the same ideal brace stiffness as three braces spaced at 10 ft. In the lateral brace case the three brace system requires 1.7 times the ideal stiffness of the single brace system, as shown in Fig. 11.

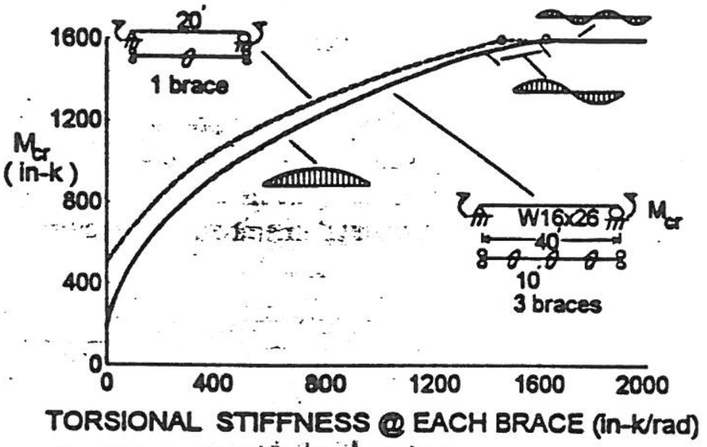


Fig. 18 Multiple Torsional Braces

Tests have been conducted on torsionally braced beams with various stiffener details which are presented elsewhere (Yura, 1992). The tests show good agreement with the Basp solutions.

Buckling Strength of Torsionally Braced Beams. Taylor and Ojalvo (1973) give the following exact equation for the critical moment of a doubly symmetric beam under uniform moment with continuous torsional bracing

$$M_{cr} = \sqrt{M_o^2 + \overline{\beta}_b E I_y} \qquad (5)$$

where M_o is the buckling capacity of the unbraced beam and $\overline{\beta}_b$ = attached torsional brace stiffness (k-in/rad per in. length). Equation (5), which assumes no cross section distortion, is shown by the dot-dash line in Fig. 19. The solid lines are BASP results for a W16x26 section with no stiffeners and spans of 10 ft, 20 ft, and 30 ft under uniform moment with braces attached to the compression flange. Cross-section distortion

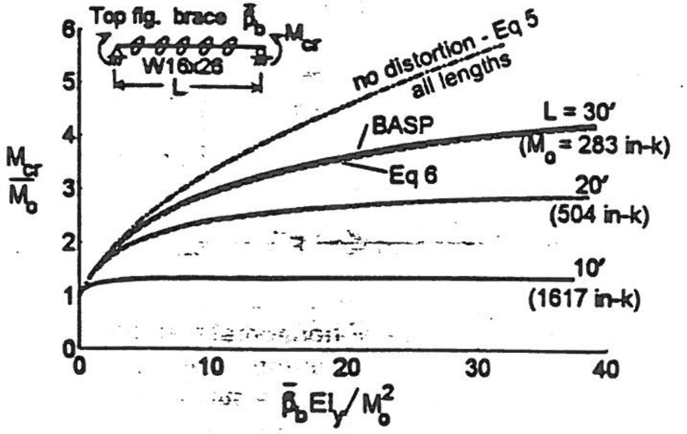


Fig. 19 Approximate Buckling Formula

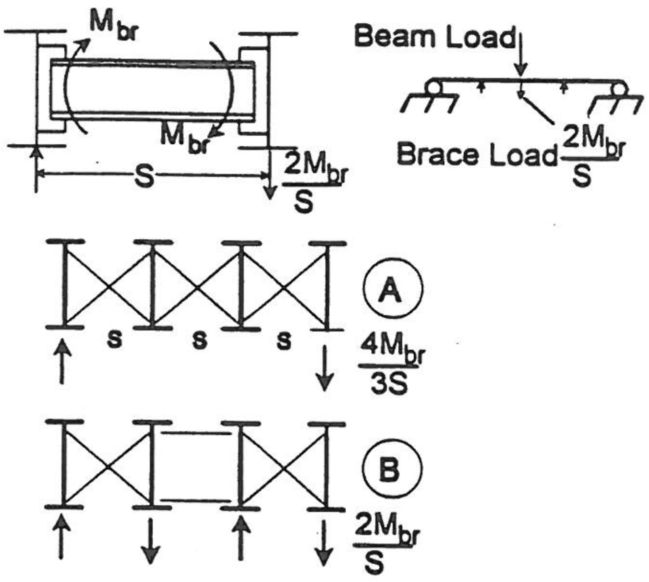


Fig. 22 Beam Load from Braces

In crossframes and diaphragms the brace moments M_{br} are reacted by vertical forces on the main girders as shown in Fig. 22. These forces increase some main girder moments and decreases others. The effect is greater for the twin girder system **B** compared to the interconnected system A. The vertical couple causes a differential displacement in adjacent girders which reduces the torsional stiffness of the cross frame system. For a brace only at midspan in a twin girder system the contribution of the inplane girder flexibility to the brace system stiffness is

$$\beta_g = \frac{12\, S^2\, E I_x}{L^3} \qquad (7)$$

where I_x is the strong axis moment of inertia of one girder and L is the span length. As the number of girders increase, the effect of girder stiffness will be less significant. In multi-girder systems, the factor 12 in Eq. 7 can be conservatively changed to 24 $(n_g - 1)^2/n_g$ where n_g is the number of girders. For example, in a six-girder system, the factor becomes 100 or more than eight times the twin girder value of 12. Helwig (1993) has shown that for twin girders the strong axis stiffness factor β_g is significant and Eq. (7) can be used even when there is more than one brace along the span.

Cross-section distortion can be approximated by considering the flexibility of the web, including full depth stiffeners if any, as follows:

$$\beta_{sec} = 3.3\, \frac{E}{h} \left[\frac{(\, N + 1.5h\,)\, t_w^3}{12} + \frac{t_s\, b_s^3}{12} \right] \qquad (8)$$

where t_w = thickness of web, h = depth of web, t_s = thickness of stiffener, b_s = width of stiffener, and N = contact length of the torsional brace as shown in Fig. 23. For continuous bracing use an effective net width of 1 in. instead of (N + 1.5h) in β_{sec} and $\overline{\beta}_b$ in place of β_b to get $\overline{\beta}_T$. The dashed lines in Fig. 19 based on Eqs. (5) and (6) show good agreement with the BASP theoretical solutions . For the 10 ft and 20 ft spans, BASP and Eq. (6) are almost identical. Other cases with discrete braces and different size stiffeners also show good agreement.

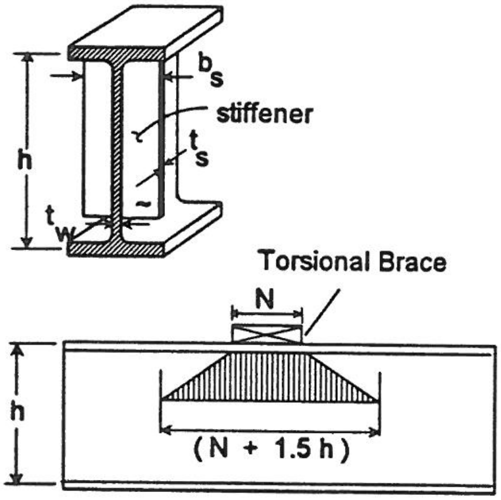


Fig. 23 Effective Web Width

In general, stiffeners or connection details such as clip angles, can be used to control distortion. For decks and through girders, the stiffener must be attached to the flange that is braced. Diaphragms are usually W shapes or channel sections connected to the web of the stringer or girders through clip angles, shear tabs or stiffeners. When full depth stiffeners or connection details are used to control distortion,the stiffener size to give the desired stiffness can be determined from Eq. (8). For partial depth stiffening illustrated in Fig. 24, the stiffness of the various sections of the web can be evaluated separately, then combined as follows:

$$\beta_i = \frac{3.3E}{h_i} \left(\frac{h}{h_i} \right)^2 \left(\frac{(N + 1.5h_i)\, t_w^3}{12} + \frac{t_s\, b_s^3}{12} \right) \qquad (9)$$

where $h_i = h_c, h_s$. or h_t and

$$\frac{1}{\beta_{sec}} = \frac{1}{\beta_c} + \frac{1}{\beta_s} + \frac{1}{\beta_t} \qquad (10)$$

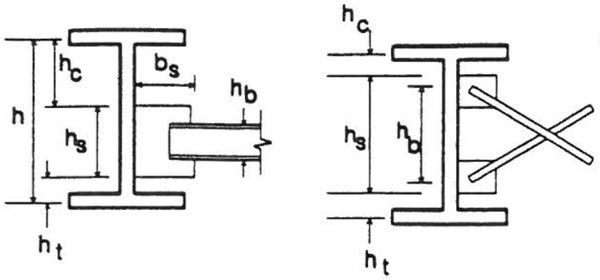


Fig. 24 Partially Stiffened Webs

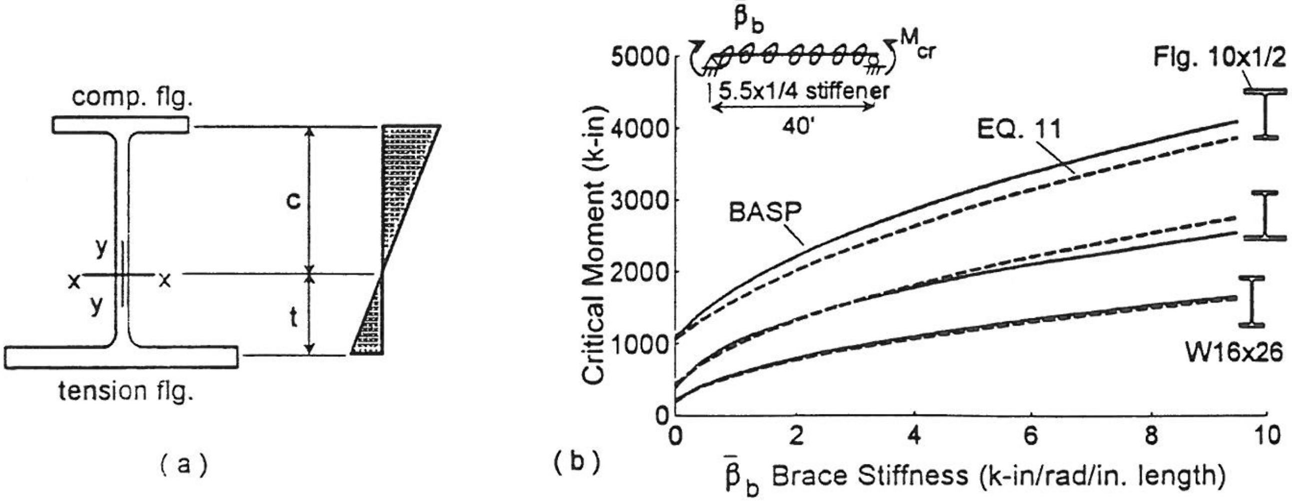
The portion of the web within h_b can be considered infinitely stiff. For rolled sections, if the diaphragm connection extends over at least one-half the beam depth, then cross-section distortion will not be significant because the webs are fairly stocky compared to built-up sections. The depth of the diaphragm, h_s, can be less than one-half the girder depth as long as it provides the necessary stiffness to reach the required moment. Cross frames without web stiffeners should have a depth h_s of at least 3/4 of the beam depth to minimize distortion. The location of a diaphragm or cross frame on the cross section is not very important; it does not have to be located close to the compression flange. The stiffeners or connection angles do not have to be welded to the flanges when diaphragms are used. For cross frames, β_s, should be taken as infinity; only h_t and h_c will affect distortion. If stiffeners are required for flange connected torsional braces on rolled beams, they should extend at least 3/4 depth to be fully effective.

Equation (5) was developed for doubly-symmetric sections. The torsional bracing effect for singly-symmetric sections can be approximated by replacing I_y in Eqs. (5) with I_{eff} defined as follows:

$$I_{eff} = I_{yc} + \frac{t}{c} I_{yt} \qquad (11)$$

where I_{yc} and I_{yt} are the lateral moment of inertia of the compression flange and tension flange respectively, and c and t are the distances from the neutral bending axis to the centroid of the compression and tension flanges respectively, as shown in Fig. 25(a). For a doubly symmetric section c = t and Eq. (11) reduces to I_y. A comparison between BASP solutions and Eqs. (5) and (11) for three different girders with torsional braces is shown in Fig. 25(b). The curves for a W16x26 show very good agreement. In the other two cases, one of the flanges of the W16x26 section was increased to 10x1/2. In one case the small flange is in tension and in the other case, the compression flange is the smallest. In all cases Eq. (11) is in good agreement with the theoretical buckling load given by BASP.

Equation (5) shows that the buckling load increases without limit as the continuous torsional brace stiffness increases. When enough bracing is provided, yielding will control the beam strength so M_{cr} can not exceed M_y, the yield or plastic strength of the section. It was found that Eq. (5) for continuous bracing could be adapted for discrete torsional braces by summing the stiffness of each brace along the span and dividing by the beam length to get an equivalent continuous brace stiffness. In this case M_{cr}



(a) (b)

Fig. 25 Singly Symmetric Girders

will be limited to M_s, the moment corresponding to buckling between the brace points. By adjusting Eq. (5) for top flange loading and other loading conditions, the following general formula can be used for the buckling strength of torsionally braced beams :

$$M_{cr} = \sqrt{C_{bu}^2 M_o^2 + \frac{C_{bb}^2 \overline{\beta}_T E I_{eff}}{C_T}} \leq M_y \; or \; M_s \tag{12}$$

where C_{bu} and C_{bb} are the two limiting C_b factors corresponding to an unbraced beam (very weak braces) and an effectively braced beam (buckling between the braces); C_T is a top flange loading modification factor; $C_T = 1.2$ for top flange loading and $C_T = 1.0$ for centroid loading; and $\overline{\beta}_T$ is the equivalent effective continuous torsional brace (in-k/radian/in. length) from Eq.(6). The following two cases illustrate the accuracy of Eq. (12).

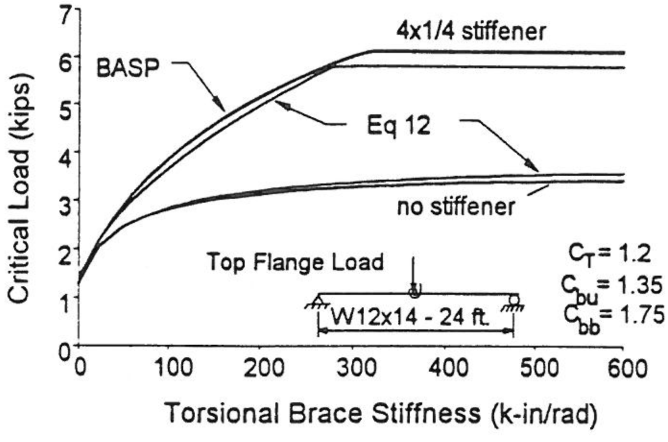


Fig. 26 Effect of Stiffener

For the case of a single torsional brace at midspan shown in Fig. 26, C_{bu}= 1.35 for a concentrated load at the midspan of an unbraced beam (Galambos, 1988). Usually designers conservatively use C_b= 1.0 for this case. For the beam assumed braced at midspan, C_{bb}= 1.75 for a straight line moment diagram with zero moment at one end of the unbraced length. These two values of C_b are used with any value of brace torsional stiffness in Eq. (12). For accuracy at small values of brace stiffness the unbraced buckling capacity $C_{bu}M_o$ should also consider top flange loading effects. Equation (12) shows excellent agreement with the BASP theory. With no stiffener, β_{sec} from Eq. (8) is 114 in-k/radian, so the effective brace stiffness β_T from Eq. (6) cannot be greater than 114 regardless of the brace stiffness magnitude at midspan. Equations (6), (8) and (12) predict the buckling very accurately for all values of attached bracing, even at very low values of bracing stiffness. A 4 x 1/4 stiffener increased β_{sec} from 114 to 11000 in-k/radian. This makes the effective brace stiffness very close to the applied stiffness, β_b. With a 4 x 1/4 stiffener, the effective stiffness is 138 in-k/radian if the attached brace stiffness is 140 in-k/radian. The bracing equations can be used to determine the required stiffener size to reduce the effect of distortion to some tolerance level, say 5%.

Figure 27 shows the correlation between the approximate buckling strength, Eq. (12) and the exact BASP solution for the case of a concentrated midspan load at the centroid with three equally spaced braces along the span. Stiffeners at the three brace points prevent cross-section distortion so $\overline{\beta}_T = 3\beta_b/288$ in.. Two horizontal cutoffs for Eq. (12) corresponding to the theoretical moment at buckling between the braces are shown. The K = 1.0 limit assumes that the critical unbraced length, which is adjacent to the midspan load, is not restrained by the more lightly loaded end spans. To account for the effect of the end span restraint, an effective length factor K = 0.88 was calculated using the procedure given in the SSRC Guide (Galambos, 1988). Figure 27 shows that it is impractical to rely on side span end restraint in determining the buckling load between braces. An infinitely stiff brace is required to reach a moment corresponding to K = 0.88. If a K factor of 1 is used in the buckling strength formula, the

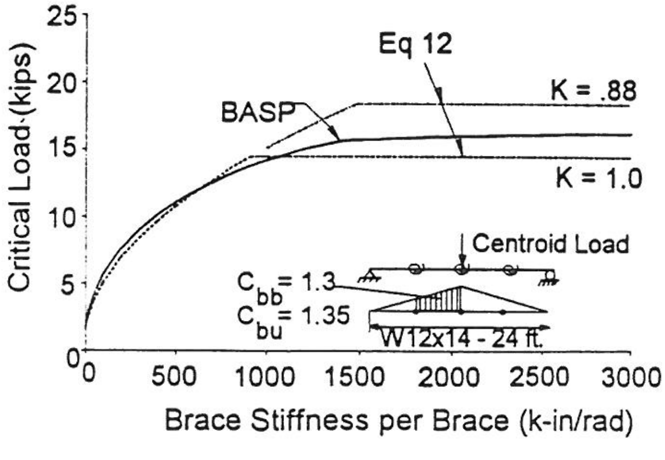


Fig. 27 Multiple Discrete Braces

The torsional brace stiffness requirement, Eq. (14), must be adjusted for the different design specifications as discussed earlier for the lateral brace requirements:

AISC-LRFD:	$\beta_T \geq \beta_T^* / \phi$	where $\phi = 0.75$ is suggested
AISC-ASD:	$\beta_T \geq 2\, \beta_T^*$	where 2 is a safety factor
AASHTO-LFD:	$\beta_T \geq \beta_T^*$	no change

Torsional Brace Design Examples. In Example 3 a diaphragm torsional bracing system is designed by the AASHTO-LFD specification to stabilize the five steel girders during construction as described in Examples 1 and 2 for lateral bracing. The strength criterion, Eq. 15, is initially assumed to control the size of the diaphragm. A C10×15.3 is sufficient to brace the girders. Both yielding and buckling of the diaphragm are checked. The stiffness of the C10×15.3 section, 195,500 in-k/radian, is much greater than required but the connection to the web of the girder and the in-plane girder flexibility also affect the stiffness. In this example, the in-plane girder stiffness is very large and its affect on the brace system stiffness is only 2%. In most practical designs, except for twin girders, this effect can be ignored. If a full depth connection stiffener is used, a 3/8 × 3-1/2 in. section is required. The weld design between the channel and the stiffener, which is not shown, must transmit the bracing moment of 293 in-k.

The 40-in. deep cross frame design in Example 4 required a brace force of 7.13 kips from Eq. (15). The factored girder moment of 1211 k-ft. gives an approximate compression force in the girder of 1211 × 12/49 = 296 kips. Thus, the brace force is 2.5% of the equivalent girder force in this case. The framing details provide sufficient stiffness. The 3-in. unstiffened web at the top and bottom flanges was small enough to keep β_{sec} well above the required value. For illustration purposes, a 30-in. deep cross frame attached near the compression flange is also considered. In this case, the cross frame itself provides a large stiffness, but the 14-in. unstiffened web is too flexible. Cross-section distortion reduces the system stiffness to 16,900 in.-k/radian, which is less than the required value. If this same cross frame was placed at the girder midheight, the two 7-in. unstiffened web zones top and bottom would be stiff enough to satisfy the brace requirements. For a fixed depth of cross frame, attachment at the mid-depth provides more effective brace stiffness than attachment close to either flange

Closing Remarks and Limitations

Two general structural systems are available for bracing beams, lateral systems and torsional systems. Torsional bracing is less sensitive than lateral bracing to conditions such as top flange loading, brace location, and number of braces, but more affected by cross-section distortion. The bracing recommendations can be used in the inelastic buckling range up to M_p if the M_f form of the lateral brace stiffness equation is used (Ales, 1993).

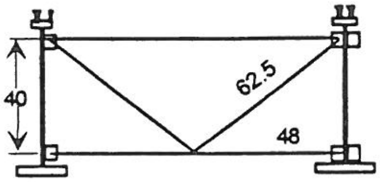
The recommendations do not address the bracing requirements for moment redistribution or ductility in seismic design. The bracing formulations will be accurate for design situations in which the buckling strength does not rely on effective lengths less than one. Lateral restraint provided by lightly loaded side spans should, in general, not be considered because the brace requirements would be much larger than the recommendations herein. Also, laboratory observations in the author's experience (usually unplanned failures of test setups) show that brace forces can be very large when local flange or web buckling occurs prior to lateral instability. After local buckling the cross section is unsymmetric and vertical loads develop very significant out of plane load components. The bracing recommendations do not address such situations.

References

Akay, H.U., Johnson, C.P., and Will, K.M., 1977, "Lateral and Local Buckling of Beams and Frames," *Journal of the Structural Division*, ASCE, ST9, September, pp. 1821-1832.

Ales, J. M. and Yura, J. A., 1993, "Bracing Design for Inelastic Structures", Structural Stability Research Council Conference-Is Your Structure Suitably Braced?, April 6-7, Milwaukee, WI..

TORSIONAL BRACING - DESIGN EXAMPLE 4



Same as Example 3, but use cross frames. Make all member sizes the same. A K-frame system will be considered using double angle members welded to connection gusset plates. Member lengths are shown in inches. Use four crossframes. See Examples 1 and 3 for section properties. Use A36 steel.

Assume brace strength criterion controls - Eq. (15)

$$F_{br}\,(40) = \frac{0.04\,(80 \times 12)\,(1211 \text{ X } 12)^2}{4\,(29000)\,239\,(1.0)^2} = 293 \text{ in-k} \quad ; \quad \underline{F_{br} = 7.31 \text{ kips}}$$

From Fig. 21 : Max force = diagonal force = $\dfrac{2F_{br}\,L_c}{S} = \dfrac{2(7.31)\,62.5}{96} = 9.52$ kips - comp

The AASHTO Load Factor method does not give a strength formula for compression members so the formulation in Allowable Stress Design will be used. Convert to ASD by dividing the member force by the 1.3 load factor to get an equivalent service load force.

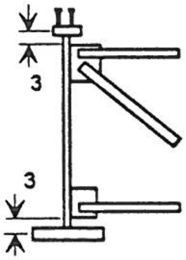
Diagonal Force (ASD) $= 9.52/1.3 = 7.3$ kips

<u>Try 2L - 2 1/2 x 2 1/2 x 1/4</u> $\qquad r_x = .769$ in. , $A = 2.38$ in.^{2}

$\ell / r = 62.5 / .769 = 81.2$; $F_a = 16980 - .53\,(81.2)^2 = 13490$ psi $= 13.5$ ksi

$P_{allow} = 13.49\,(2.38) = 32.1$ kips > 7.3 kips *OK*

Check brace stiffness:



Eq. (14) ; $\beta_{T\ req'd} = 17550$ in-k/radian - see Example 3

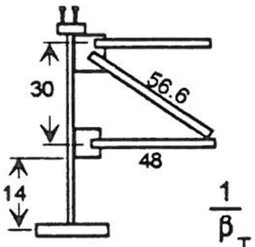
Fig. 21 : $\beta_b = \dfrac{2(29000)\,(96)^2\,(40)^2\,(2.38)}{8\,(62.5)^3 + (96)^3} = 717000$ in-k/radian

Girder : $\beta_g = 406000$ in-k/radian - see Example 3

$$\beta_c = \beta_t = \frac{3.3\,(29000)}{3.0}\left(\frac{49}{3.0}\right)^2\left(\frac{1.5\,(3.0)\,(.5)^3}{12}\right) = 399000 \text{ in-k/rad}$$

Eq. (16) : $\dfrac{1}{\beta_T} = \dfrac{1}{717000} + \dfrac{1}{406000} + \dfrac{2}{399000}$; $\beta_T = \underline{113000 > 17550}$ in-k/rad *OK*

Evaluate the cross frame shown below



$$\beta_b = \frac{2(29000)\,(96)^2(30)^2(2.38)}{8\,(56.6)^3 + (96)^3} = 490000 \text{ in-k/radian}$$

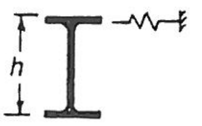
$$\beta_t = \frac{3.3\,(29000)}{14.0}\left(\frac{49}{14.0}\right)^2\left(\frac{1.5\,(14.0)\,(.5)^3}{12}\right) = 18300 \text{ in-k/rad}$$

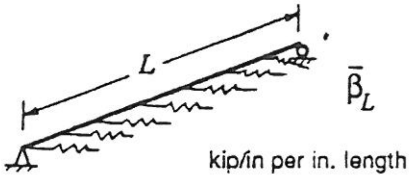
$\dfrac{1}{\beta_T} = \dfrac{1}{490000} + \dfrac{1}{406000} + \dfrac{1}{18300}$; $\beta_T = \underline{16900 < 17550}$ in-k/rad *NG*

Choo, K.M., 1987, , "Buckling Program BASP for Use on a Microcomputer", Thesis presented to The University of Texas at Austin, May

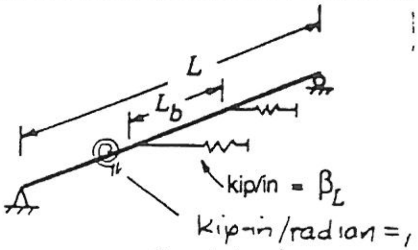
Galambos, T.V., Ed., 1988, Structural Stability Research Council, *Guide to Stability Design Criteria for Metal Structures*, 4th Edition, New York: John Wiley & Sons, Inc.

BEAM BRACING DESIGN RECOMMENDATIONS - J.A. YURA UNIV OF TEXAS

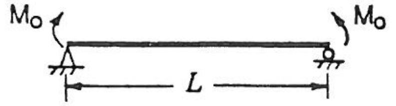




continuous bracing



discrete bracing



convert to continuous $\bar{\beta}_L = \dfrac{\beta_L \times \text{\# of braces}}{L}$

use .75L for one brace

LATERAL BRACING

There are two approaches. A continuous bracing equation is given which gives the critical buckling load for any value of bracing stiffness. Discrete or single-point systems can be converted to continuous as shown above, Eq(1). A modification to Winters column bracing (full bracing) is also given. This gives the required stiffness to force the buckling between braces. The continuous approach must be used if combined with torsional braces. Reductions for out-of-straightness are included.

Continuous Bracing

$$M_{cr} = \sqrt{\left[(C_{bu}M_o)^2 + (.5\, C_{bb} P_y h)^2 A\right](1+A)} \quad \leq M_y \text{ or } M_s$$

where $P_y = \dfrac{\pi^2 E I_y}{L^2}$; $A = L\sqrt{\dfrac{.33\,\bar{\beta}_L}{C_L P_y}}$; $C_L = 1 + \dfrac{1.2}{n}$ for top flange loading

$= 1.0$ for centroid loading

M_o is the buckling strength of the unbraced beam including any top flange loading effects. $= \dfrac{\pi E}{L}\sqrt{\dfrac{I_y J}{2.6} + \dfrac{\pi^2 I_y^2 h^2}{4L^2}}$ or Eq F1-13 in AISC LRFD

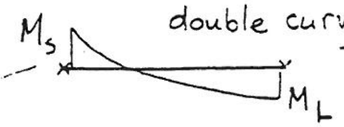
C_{bu} is the C_b factor assuming the beam is unbraced

C_{bb} is the C_b factor assumming the beam braces are fully effective

M_s is the buckling load between the braces

n = number of single-point braces

Modified Winter's Full Lateral Bracing Requirements



$$\beta_{L\ \text{REQ'D}} = \frac{2\#(M/h)C_L C_d}{\phi L_b}$$

or $C_{bb}\dfrac{\pi^2 E I_{yc}}{L_b^2}$

where $\# = 4 - (2/n)$ for discrete bracing

$\# = 1$ for relative bracing

$C_d = 1 + (M_s/M_L)^2$ double curvature

where M = maximum factored moment, $\phi = 0.75$. In ASD use 2× service M and

M_s = maximum moment on one side

$F_{br} = 0.01\ (M/h)C_L C_d$ discrete brace

$= 0.004\,(M/h)C_L C_d$ relative brace

↳ factored moment in LRFD service moment in ASD

TORSIONAL BRACING DESIGN REQUIREMENTS

Stiffness:

$$M_{cr} = \sqrt{\underbrace{C_{bu}^2 M_o^2}_{\text{capacity of unbraced beam}} + \frac{C_{bb}^2 E I_{eff} \overline{\beta}_T}{2 C_T}} \le M_y \text{ or } M_s \qquad (13)$$

M_s: buckling between braces

M_y: yield moment

$C_T = 1.2$ for top flange loading, $C_T = 1.0$ for other

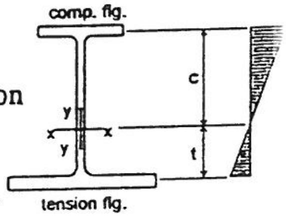
Instead of solving Eq. 13 for the required $\overline{\beta}_T$, the following expression can be used

$$\beta_T = \overline{\beta}_T L / n = 2.4 L M_f^2 / (n E I_{eff} C_{bb}^2) \qquad (14)$$

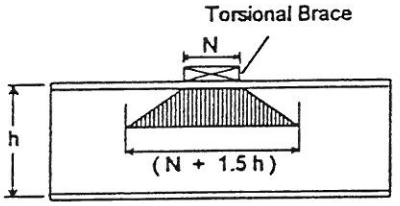
Strength:

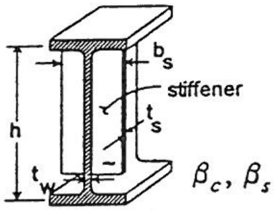
$$M_{br} = F_{br} h_b = 0.04 L M_f^2 / (n E I_{eff} C_{bb}^2) \qquad (15)$$

where M_f = maximum beam moment
$I_{eff} = I_{yc} + (t/c) I_{yt}$; $= I_y$ for doubly symmetric sections
C_{bb} = moment diagram modification factor for the full bracing condition
L = span length
n = number of braces along the span



The available effective stiffness of the brace system β_T is calculated as follows:





$$\frac{1}{\beta_T} = \frac{1}{\beta_c} + \frac{1}{\beta_s} + \frac{1}{\beta_t} + \frac{1}{\beta_b} + \frac{1}{\beta_g} \qquad (16)$$

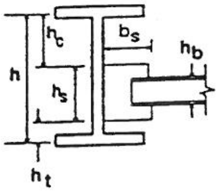
$$\beta_c, \beta_s, \beta_t = \frac{3.3E}{h_i}\left(\frac{h}{h_i}\right)^2 \left(\frac{(N+1.5h_i)t_w^3}{12} + \frac{t_s b_s^3}{12}\right) \qquad (17)$$

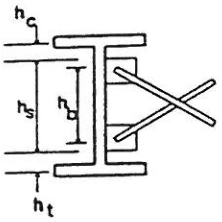
where $h_i = h_c$, h_s, or h_t ; N = bearing length

β_b = stiffness of attached brace (see below)

$$\beta_g = 24(n_g - 1)^2 S^2 EI_x / (L^3 n_g) \qquad (18)$$

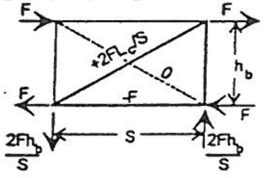
where n_g is the number of interconnected girders

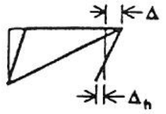




CROSS FRAMES

TENSION SYSTEM





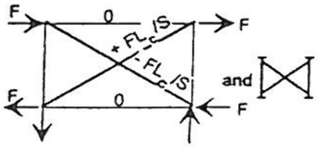
$\theta = \frac{\Delta + \Delta_h}{h_b}$; $M = F h_b$

$\beta_b = M / \theta$

$$\boxed{\beta_b = \frac{E S^2 h_b^2}{\frac{2L_c^3}{A_c} + \frac{S^3}{A_h}}}$$

Tension System - horizontals are required

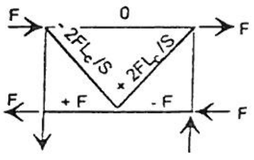
COMPRESSION SYSTEM



$$\boxed{\beta_b = \frac{A_c E S^2 h_b^2}{L_c^3}}$$

Tension - Compression System horizontals not required

K BRACE

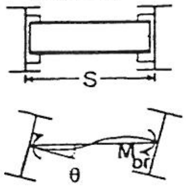


$$\boxed{\beta_b = \frac{2 E S^2 h_b^2}{\frac{8L_c^3}{A_c} + \frac{S^3}{A_h}}}$$

K Brace System - diagonals designed for tension and compression

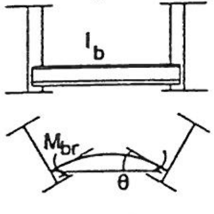
A_h = area of horizontal members
A_c = area of diagonal members
E = modulus of elasticity
L_c = length of diagonal members
S = spacing of girders
h_b = height of the cross frame

Diaphragms or Decks



$$\beta_b = \frac{6 E I_b}{S}$$

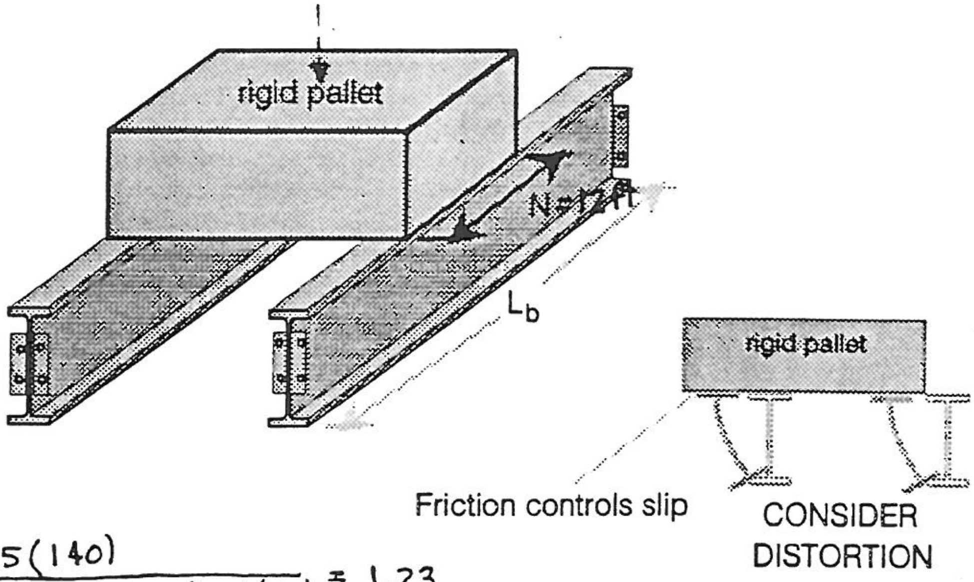
Through Girders



$$\beta_b = \frac{2 E I_b}{S}$$

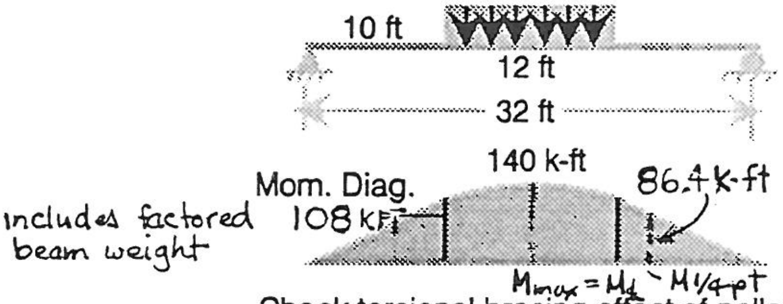
I1
3/

TORSIONAL BRACING - DESIGN EXAMPLE



$C_{bu} = \frac{12.5(140)}{2.5(140) + 6(86.4) + 4(140)} = 1.23$

$C_{bb} = 1.67$



AISC-LRFD Spec-2nd Ed.

A36 Steel - W21X44 $J = 0.77\ in^4$

Factored loads shown $C_w = 2110\ in^6$

$\phi M_p = 258$ k-ft $I_y = 20.7\ in^4$

$t_w = 0.350$ in $h = 20.21$ in

$C_{bu}M_o = 878$ in-k $= 73.2$ k-ft *

Check torsional bracing effect of pallet

$N = 12 \times 12 = 144$ in

Eq. 17 $\beta_{sec} = \frac{3.3\ (29000)}{20.21}\left(\frac{(0.35)^3}{12}(144 + 1.5 \times 20.21)\right) = 2949$ in-k/ rad ; $\beta_b(\text{pallet}) = $

$\frac{1}{\beta_T} = \frac{1}{2949} + \frac{1}{\infty}$

$\therefore \beta_T = 29$

Eq. 13 $M_{cr} = \sqrt{(878)^2 + (1.67)^2\, 29000\ (20.7)\ 2949 / 2\ (1.2)\ (32 \times 12)} = 206$ k-ft

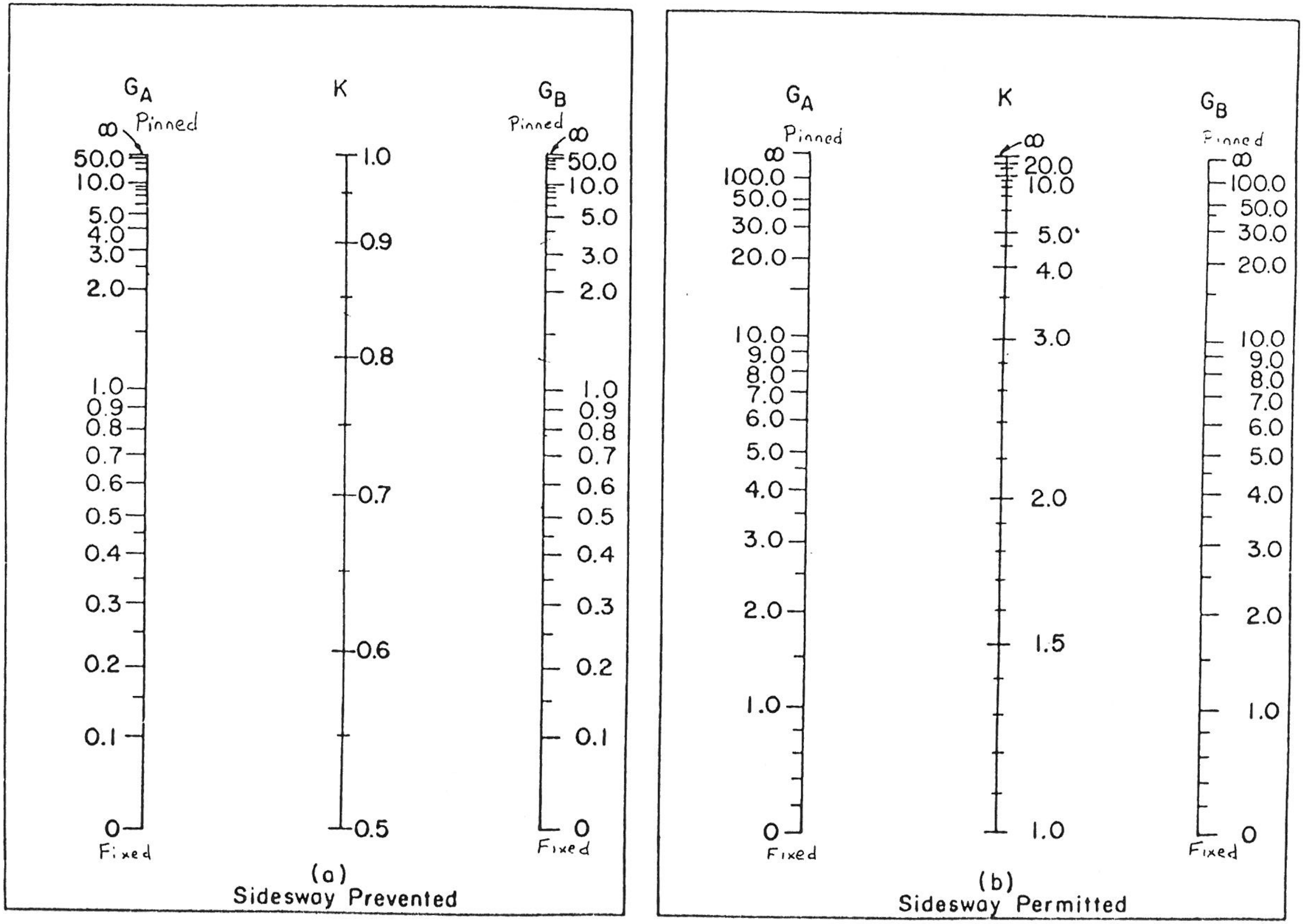
$\underset{\phi}{0.9} \times 206 = 186 > 140$ k-ft *OK*

* If pallet provides no bracing, $L_b = 32'$, $C_b = 1.23$ - Use AISC-LRFD Eq F1-13

$M_n = C_b M_o = 1.23 \frac{\pi}{32(12)} 29000 \sqrt{\frac{20.7(.77)}{2.6} + \frac{\pi^2}{(32 \times 12)^2} 20.7(2110)} = 878$ in-k
$= 73.2$ k-ft

$$G = \frac{\sum I/L \text{ COL}}{\sum I/L \text{ BEAM}}$$

FIGURE 1—Effective Length Factor In Column Design



(a) Sidesway Prevented

(b) Sidesway Permitted