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Another Fellowship report —
shows how to estimate & use
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DETERMINATION OF COLUMN FIXITY

AT COLUMN BASES

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INTRODUCTION

One of the first steps in most structural engineering problems is to choose a reasonable model for the structure being analyzed. In modeling a framed structure it is common practice for the engineer to assume that the column bases are either fully fixed or fully pinned. In reality neither of these two assumptions is strictly valid due to the difficulty in obtaining either of these ideal conditions in practice. In frame analysis, regardless of which of these two assumptions is used, the calculated bending moments in the columns of the first tier are generally larger than those which will actually exist in the structure. These larger moments normally lead to the use of heavier columns than are really necessary and therefore result in a more expensive and less competitive structure.

The assumptions made concerning the degree of fixity also have a significant effect on the calculated drift of a structural frame. If the column bases are assumed to be pinned the calculated drift may be so large that stiffer members are required to reduce the drift to acceptable levels. Therefore if partial base fixity is considered, additional savings may be realized because of the lighter members needed to control drift. At the other extreme, the assumption of full base fixity in drift calculations may result in an underestimation of the actual drift.

In the design of steel columns by AISC guidelines and specifications, an attempt is made to account for the difficulty in obtaining either perfectly fixed or perfectly pinned column bases in practice. This difficulty is accounted for by recommending rigidity ratios $[G_B = \Sigma(\text{COLUMN / BEAM})\ \text{RIGIDITIES}]$, for use in effective length alignment charts, of 1.0 rather than zero for fixed bases and of 10.0 rather than infinity for pinned bases.[1] While these assumptions seem reasonable, they are quite subjective and place all base connections into two categories with no provision for additional variation of base fixity.

Galambos[2] showed that by accounting for partial base fixity of a typical pinned base detail the theoretical buckling strength of a rigid frame can be increased significantly. Preliminary results of current research[3] indicate that for a typical pinned base detail a rigidity ratio (G_B) for the column base as low as 1.50 (rather than the recommended value of 10.0) may be justified. Such a drastic reduction in the rigidity ratio will result in significant reduction of the effective length factor and may allow the use of lighter columns. These results tend to concur with results of similar studies[4, 5, 6] which consider the effect of partial restraint provided by simple beam-to-column connections. These studies have also indicated that significant reductions in the column effective length may be justified by considering the partial restraint of a simple beam-to-column connection.

The preceding discussion has made it clear that consideration of the degree of fixity of column bases would result in more realistic and more accurate structural analysis, thereby resulting in a more efficient design. Unfortunately, the determination of the degree of restraint offered by column bases is not treated explicitly in existing literature. There have, however, been some attempts to develop reasonable methods for estimating the rotational characteristics of column bases.[2, 7, 8] The PCI Design Handbook,[9] published by the Prestressed Concrete Institute, presents a method for approximating the degree of fixity of column bases for precast concrete columns which is fairly easy to use.

The primary purpose of this study is to develop a rational method for approximating the degree of fixity of a typical steel column base detail. The method will be similar to the method presented in the PCI Design Handbook and will consider the combined effects of footing rotation, anchor bolt elongation, and base plate bending on the fixity of the column base. In addition to the development of the relationships for the base stiffness, a program for the Hewlett-Packard HP-41 calculator will be presented as an aid in using the proposed method. Using the calculator program, a sensitivity analysis will be conducted to determine the relative significance of the variables involved in the expressions. Tables and charts will be presented, as an additional design aid, to allow the engineer to obtain a reasonable estimate of column base fixity for preliminary input into an analysis. Finally, an example will be provided on the use of the design aids in structural analysis and design.

DEVELOPMENT OF THE METHOD

The base detail studied is shown in Figure 1. The nomenclature used is defined in Appendix A. The degree of fixity of a column base is the ratio of the stiffness of the base to the sum of the base and column stiffnesses. This ratio is analogous to the distribution factors used in moment distribution. The stiffness of the column base is defined as the moment required to rotate the base through a unit angle. Similarly, the absolute stiffness of a member is defined as the moment required to rotate one end of the member through a unit angle when the far end of the member is fully fixed. The inverse of stiffness is defined as flexibility. The approach used in this development is to apply a unit moment (P * e) to the column and determine the expression for the resulting base rotation. This expression is then divided by the applied moment to give an expression for the column base flexibility. Finally, the expression for the flexibility is inverted to obtain an expression for the base stiffness.

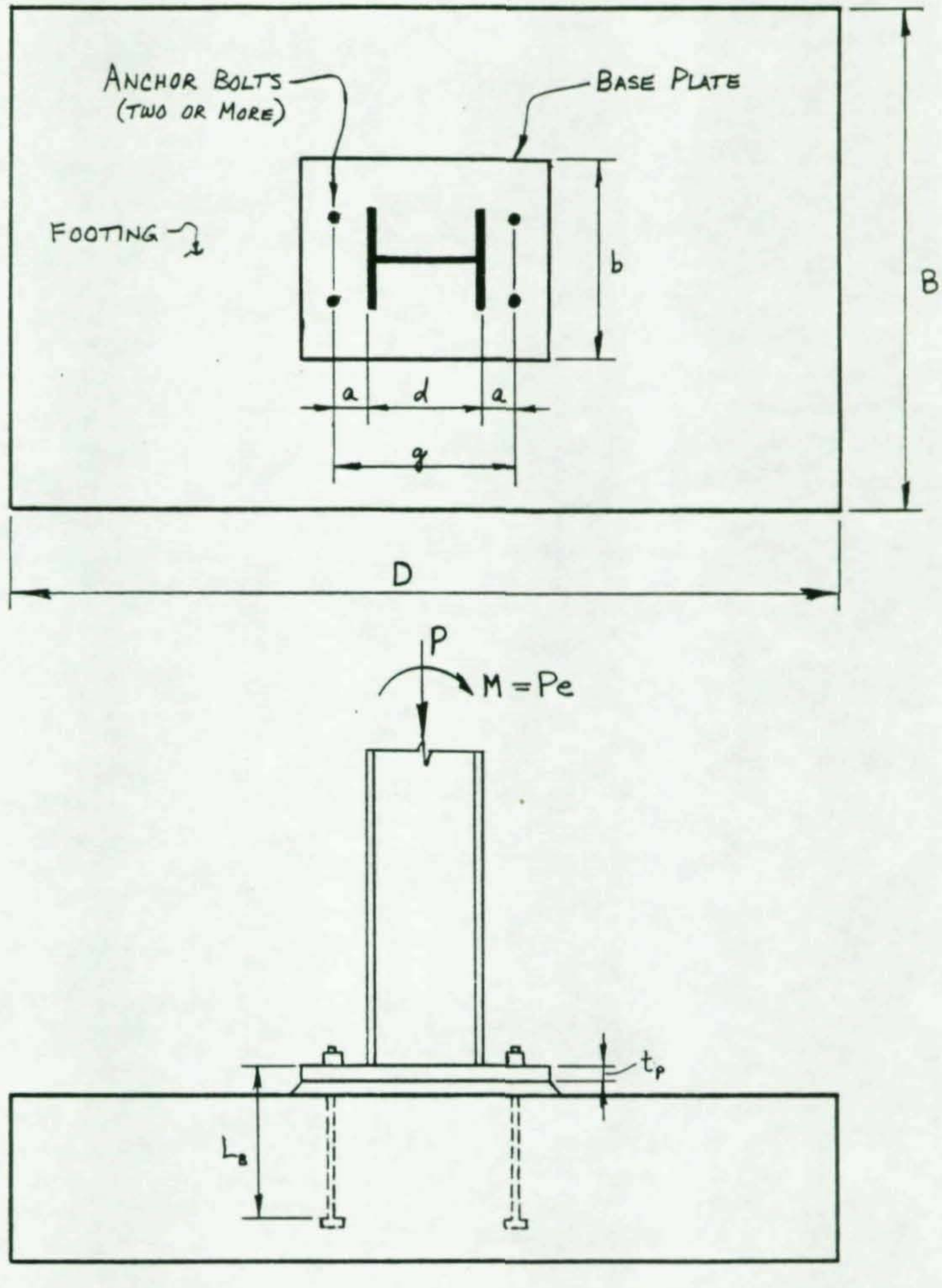


FIGURE 1 - BASE DETAIL STUDIED.

The total rotation of the base is the sum of the rotations due to three factors (Fig. 2). One of the components is the rotation of the column footing due to the elastic behavior of the soil beneath the footing. The development of an expression for the rotation of the footing is relatively simple, but it is based on the modulus of subgrade reaction. The modulus of subgrade reaction (K_s) is the ratio of the pressure exerted on the soil to the settlement of the soil, and is difficult to obtain with any degree of accuracy.[10, 11] The expression for the footing rotation is now developed assuming the footing rotates about its center (Fig. 3):

$$K_s = \frac{q}{\Delta_F}$$

Rearranging;

$$\Delta_F = \frac{q}{K_s}$$

Assuming the rotation is through a small angle;

$$\theta_F = \frac{\Delta_F}{D/2} = \frac{q/K_s}{D/2}$$

Substituting $q = \frac{Mc}{I}$ AND $c = \frac{D}{2}$;

$$\theta_F = \frac{M(D/2)}{K_s I_F (D/2)} = \frac{M}{K_s I_F}$$

Since M = Pe;

$$\theta_F = \frac{Pe}{K_s I_F} \qquad (1)$$

where $I_F = \frac{BD^3}{12}$

Now a fairly simple expression for the footing rotation is available. This equation is valid only if there is no separation between the footing and the soil.

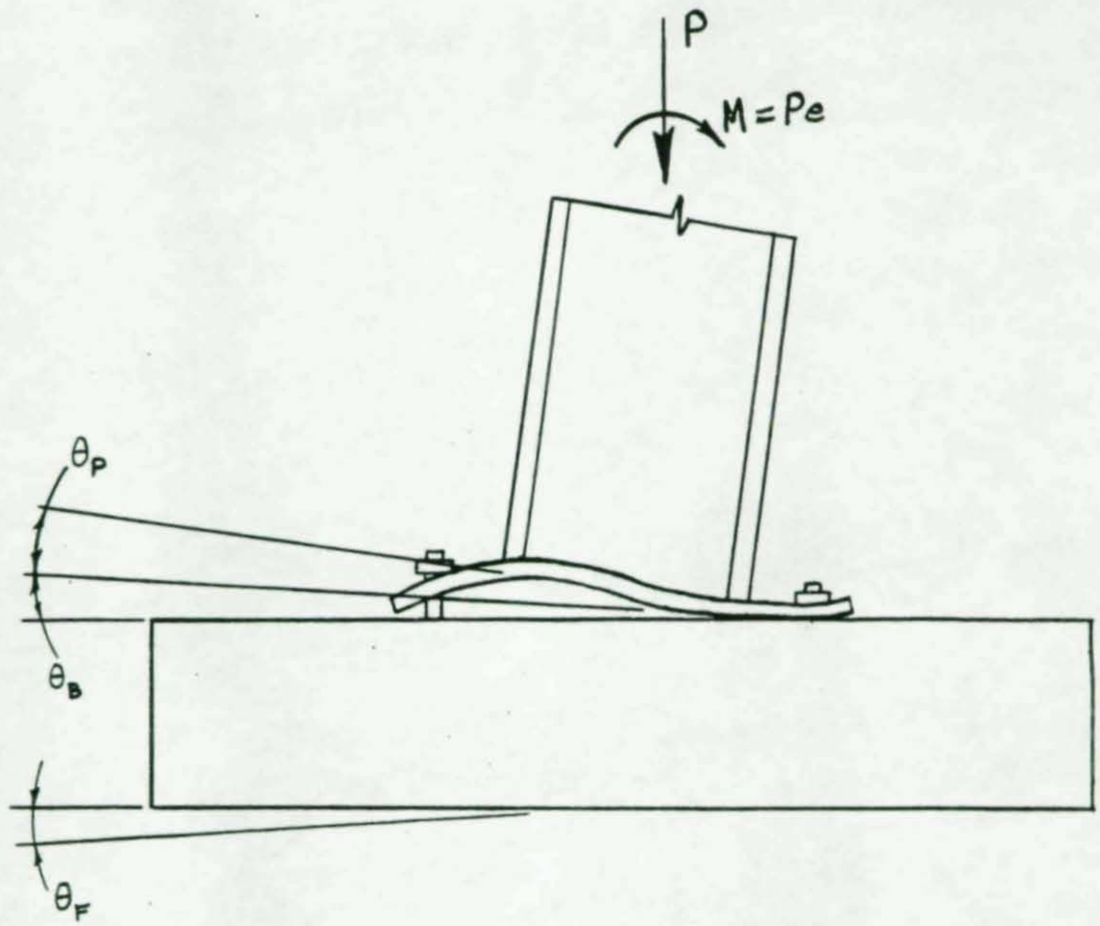
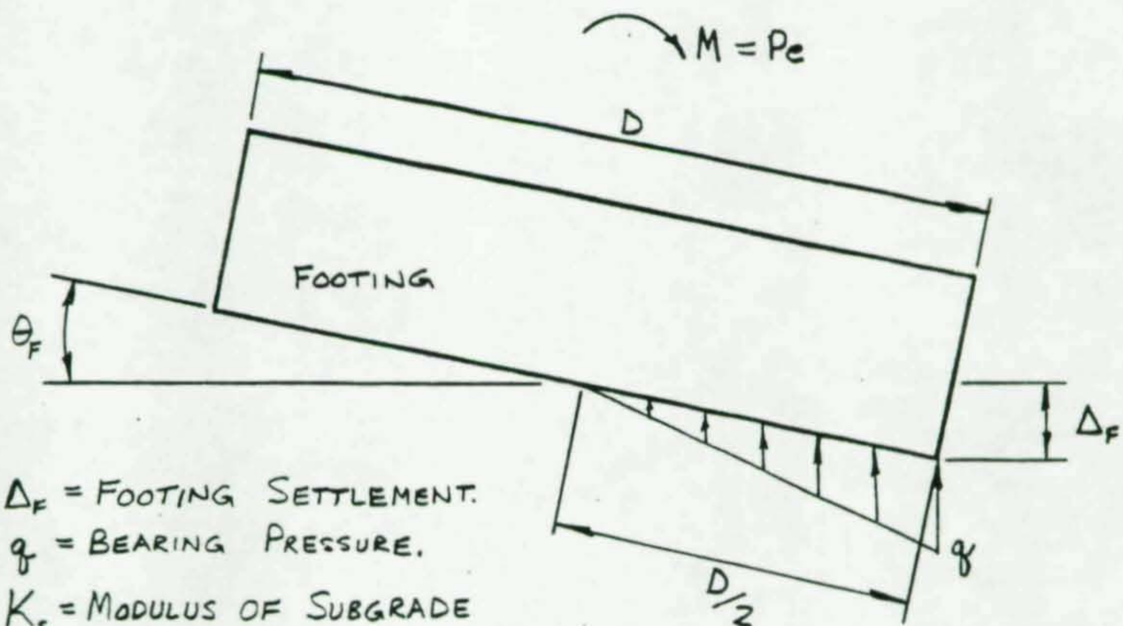


FIGURE 2 - THREE COMPONENTS OF ROTATION.



Δ_F = FOOTING SETTLEMENT.
q = BEARING PRESSURE.
K_s = MODULUS OF SUBGRADE REACTION.

$$K_s = \frac{q}{\Delta_F}$$

θ_F = FOOTING ROTATION.

FIGURE 3 - FOOTING ROTATION.

This is not a serious limitation since the rotation of the footing normally reduces the vertical compressive stress in the soil on one side of the footing rather than actually causing separation on the tension side. The problem is now one of obtaining a reasonable estimate of K_s.

To simplify the use of the modulus of subgrade reaction it is assumed that (a) the value of K_s is independent of the magnitude of pressure and (b) the value of K_s has the same value at every point on the footing surface.[10] Actually the value of the modulus of subgrade reaction is highly dependent on the size, shape, and depth of the footing. For the same soil, the value of K_s decreases with increasing width of the footing and also decreases with increasing length of the footing. The value of K_s increases with increasing depth below original gound surface. Teng[10] and Bowles[11] present expressions to adjust the modulus of subgrade reaction obtained by a one foot square bearing plate test to take account for these footing size and shape effects. Some guidelines for selecting K_s are given in several references.[9, 10, 11, 12] Table 1 gives a range of values for K_s to use as a guide for approximations.[11]

Another component of the base rotation is the rotation due to bending in the base plate. Several assumptions are made in the derivation of an expression for this component of the base rotation. First, bending in

TABLE 1 - RANGE OF VALUES OF MODULUS OF SUBGRADE REACTION K_s.[11]

SOIL	K_s (k/ft^3)
LOOSE SAND	30 - 100
MEDIUM DENSE SAND	60 - 500
DENSE SAND	400 - 800
CLAYEY MEDIUM DENSE SAND	200 - 500
SILTY MEDIUM DENSE SAND.	150 - 300
CLAYEY SOIL:	
$q_u \leq 4$ ksf	75 - 150
$4\text{ ksf} \leq q_u \leq 8$ ksf	150 - 300
$q_u > 8$ ksf	>300

the base plate is assumed to be elastic. Second, it is assumed that the axial forces and the moments due to the stresses in the column web can be neglected without appreciable error. Figure 4 shows that under an applied moment the forces generated in the web are indeed small with respect to the forces generated in the column flanges. The third assumption is that the ratio of the stiffness of the base plate between the column flanges to the stiffness of the base plate outside the column flanges can be represented by the factor lambda (λ);

$$\lambda = \frac{I_{\text{BETWEEN FLANGES}}}{I_{\text{OUTSIDE FLANGES}}}$$

where $1.0 \leq \lambda \leq \infty$

This factor allows any additional stiffness provided by welding the column web to the base plate to be accounted for in the expression for rotation due to base plate bending. A fourth assumption is that, because of moment transfer between the base plate and the column flanges, the moment in the base plate just inside the column flange can be represented by a factor beta (β) times the base plate moment just outside the column flange (where $0 \leq \beta \leq 1.0$).

Before an expression for the rotation at the base of the column due to bending of the base plate can be developed, expressions for the forces acting on the base plate must be derived. In the derivation all forces acting on the base plate are considered to be concentrated loads (Fig. 5). The resultant compressive force between

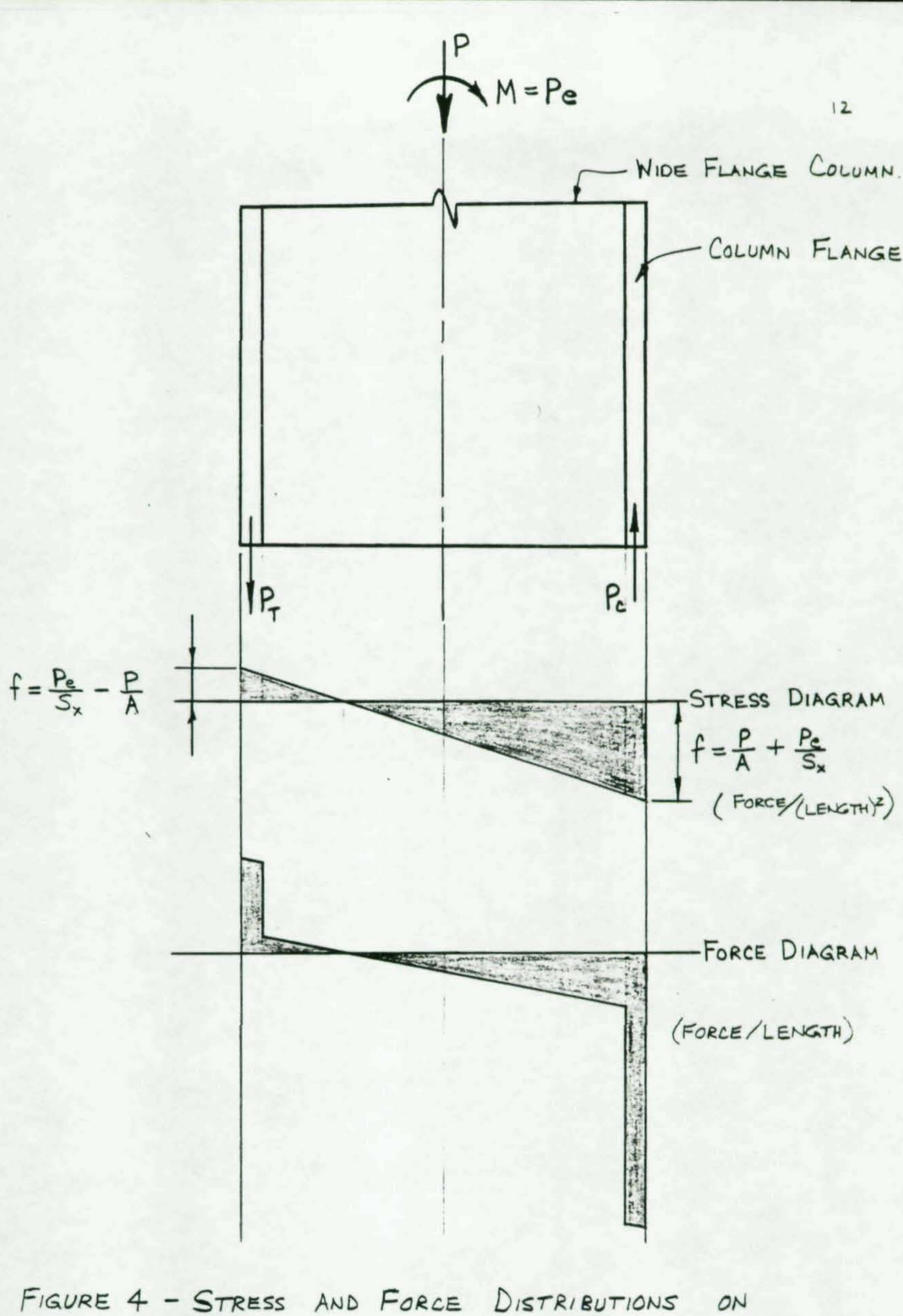
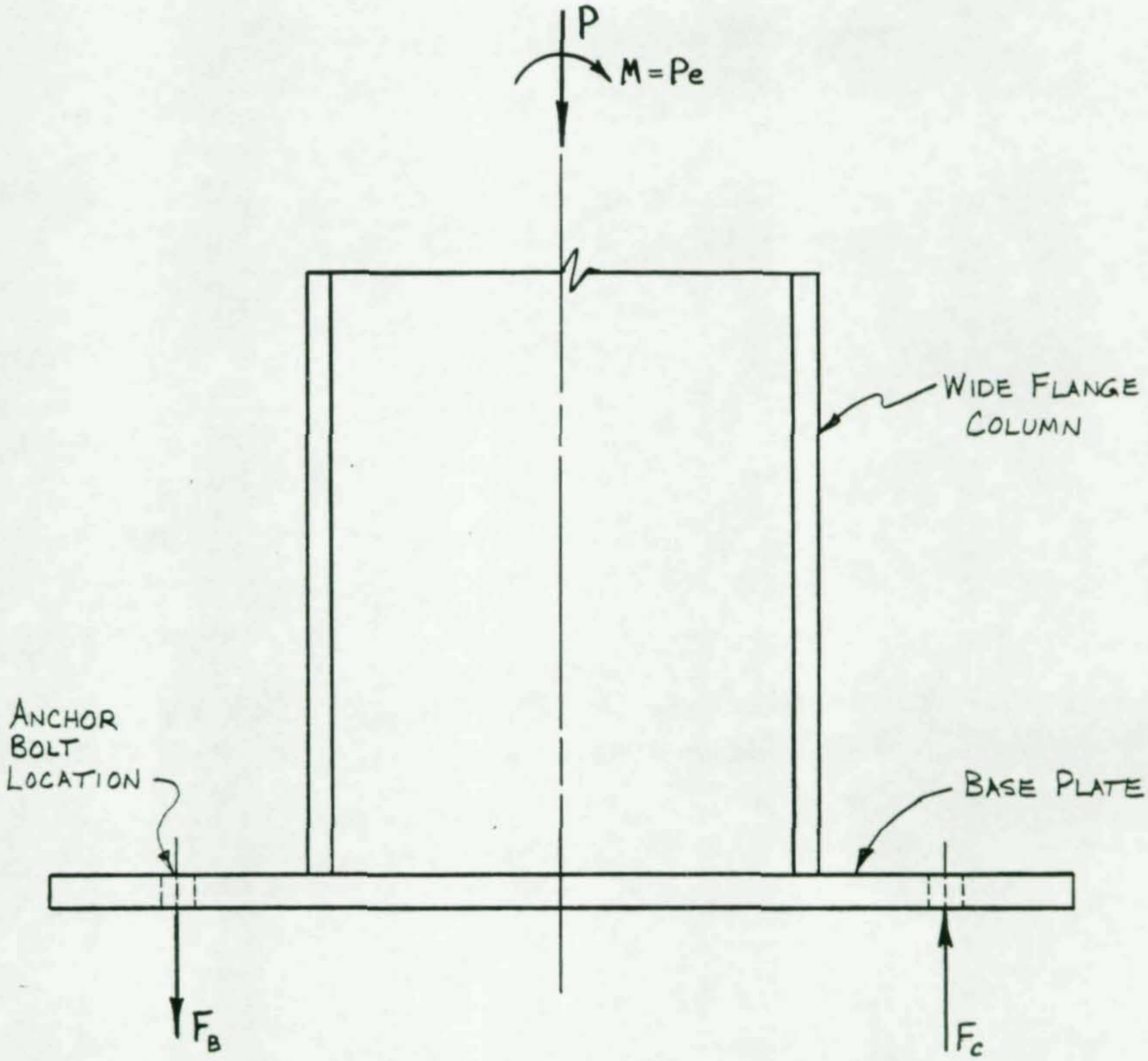


FIGURE 4 - STRESS AND FORCE DISTRIBUTIONS ON COLUMN.



F_B = TENSION FORCE IN ANCHOR BOLTS
F_C = COMPRESSIVE FORCE BETWEEN CONCRETE AND BASE PLATE.

FIGURE 5 – FORCES ACTING ON BASE PLATE.

the concrete and the base plate is assumed to act at the centerline of the anchor bolts. Except for very stiff base plates this position of the resultant is essentially an outer bound for the location of the compressive resultant. These assumptions are generally conservative because they tend to result in an overestimation of base flexibility and, therefore, an underestimation of base stiffness. Referring to Figure 5 and using statics, the tension force (F_B) in the anchor bolts can be expressed as:

$$\Sigma M_{F_c} = 0; \quad -P(a + d/2) + Pe - gF_B = 0$$

Solving this equation for F_B and rearranging:

$$F_B = \frac{-\frac{P}{2}(2a + d) + Pe}{g}$$

$$F_B = \frac{Pe - P/2(g)}{g}$$

$$F_B = P\left[\frac{e}{g} - \frac{1}{2}\right] \qquad (2)$$

Similarly, the resultant compressive force (F_C) may be expressed as:

$$\Sigma M_{F_B} = 0; \quad P(a + d/2) + Pe - gF_c = 0$$

$$\therefore \quad F_c = \frac{Pe + P/2(2a + d)}{g}$$

$$F_c = \frac{Pe + P/2(g)}{g}$$

$$F_c = P\left[\frac{e}{g} + \frac{1}{2}\right] \qquad (3)$$

The remaining forces acting on the base plate are thoseapplied through the column flanges. Referring again to Figure 4, an expression is derived for the force (P_c) applied by the compression flange. It is assumed that the average stress in the flange is equal to the maximum flange stress. This assumption tends to compensate for the force in the web which is neglected. Therefore, from Figure 4:

$$P_c = A_F f_{MAX} \qquad \text{where } A_F = \text{AREA OF THE FLANGE.}$$

FROM; $f_{MAX} = P/A + Pe/S_x$

$$P_c = A_F\left[\frac{P}{A} + \frac{Pe}{S_x}\right]$$

Simplification gives;

$$P_c = \left(\frac{A_F}{A}\right)P\left[\left(\frac{A}{S_x}\right)e + 1\right]$$

Setting $\frac{A_F}{A} = \alpha$ AND $\frac{A}{S_x} = B_x$;

$$P_c = \alpha P\left[B_x e + 1\right] \qquad (4)$$

In equation 4, the coefficient B_x is the same as the B_x that is tabulated in the column tables published in the Manual of Steel Construction.[1] The trends for this coefficient and the newly introducted alpha (α) coefficient have been studied and the results are shown in Table 2.

Using these expressions which have been derived for the forces acting on the base plate, an expression can be developed for the rotation at the base of the column due to base plate bending. The method of virtual work will be used in this derivation. In Figure 6(a) the real forces, as modeled by the assumptions noted previously, are

TABLE 2 - VALUES OF THE COEFFICIENTS ALPHA (α) AND B_x FOR WIDE FLANGE COLUMNS.

NOMINAL COLUMN SIZE	ALPHA (α)			B_x		
	HIGH	LOW	AVG.	HIGH	LOW	AVG.
W14	0.4171	0.3367	0.3886	0.2010	0.1680	0.1840
W12	0.3959	0.3489	0.3824	0.2274	0.2045	0.2155
W10	0.3969	0.3566	0.3846	0.2774	0.2611	0.2660
W8	0.3940	0.3672	0.3855	0.3388	0.3262	0.3313
W6	0.3763	0.3163	0.3825	0.4820	0.4395	0.4608

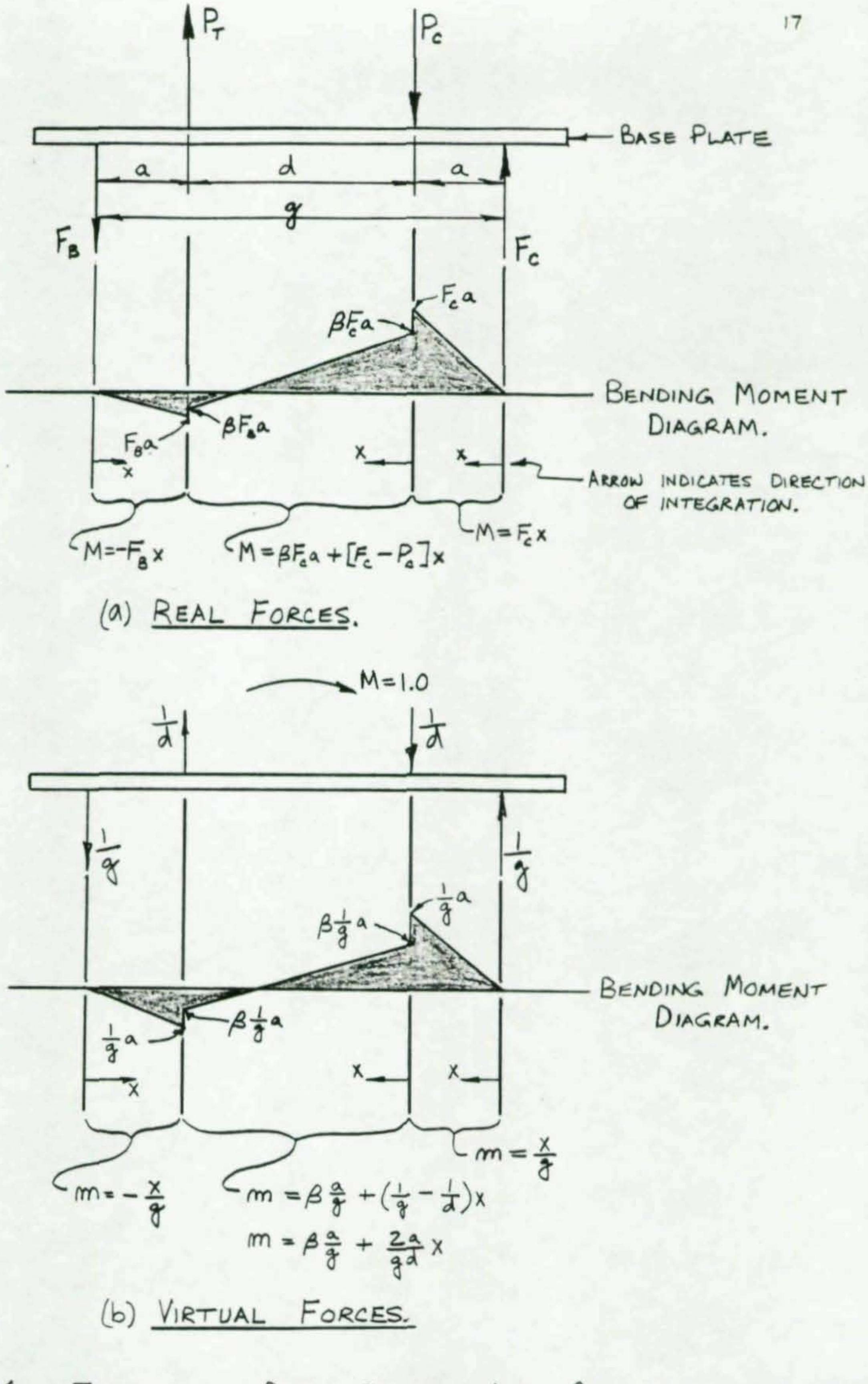


FIGURE 6 - FORCES ON BASE PLATE AND RESULTING BENDING MOMENT DIAGRAMS.

applied to the base plate and expressions are shown for the bending moment in different regions of the plate. Note that the factor beta (β) is applied in the equation for moment. Figure 6(b) shows the application of a unit moment and the resulting virtual forces. Applying the information from Figure 6 to the virtual work equation gives:

$$\theta = \Sigma \int \frac{mM}{EI} dx$$

$$\theta_p = \int_0^a \frac{\frac{x}{g}(F_c x) dx}{E_p I_p} + \int_0^a \frac{-\frac{x}{g}(F_B x) dx}{E_p I_p} + \int_0^d \frac{\left[\beta \frac{a}{g} - \frac{2a}{gd} x\right]\left[\beta F_c x + F_c x - P_c x\right] dx}{\lambda E_p I_p}$$

Note that in the third term of this equation the lambda (λ) factor is applied to the base plate stiffness. Multiplying both sides by E_p I_p and multiplying the polynomials in the third term gives:

$$E_p I_p \theta_p = \int_0^a \frac{F_c}{g} x^2 dx + \int_0^a \frac{F_B}{g} x^2 dx + \frac{1}{\lambda}\int_0^d \left[\frac{\beta^2 a^2 F_c}{g} + \frac{\beta a F_c x}{g} - \frac{\beta a P_c x}{g} - \frac{2\beta a^2 F_c x}{gd} - \frac{2aF_c x^2}{gd} + \frac{2a P_c x^2}{gd}\right] dx$$

Integrating;

$$E_p I_p \theta_p = \frac{F_c a^3}{3g} + \frac{F_B a^3}{3g} + \frac{1}{\lambda}\left[\frac{\beta^2 a^2 F_c d}{g} + \frac{\beta a F_c d^2}{2g} - \frac{\beta a P_c d^2}{2g} - \frac{\beta a^2 F_c d^2}{gd} - \frac{2aF_c d^3}{3gd} + \frac{2a P_c d^3}{3gd}\right]$$

Simplifying;

$$E_p I_p \theta_p = [F_c + F_B]\frac{a^3}{3g} + \frac{ad}{\lambda g}\Big[\beta^2 a F_c + \frac{\beta d}{2} F_c - \frac{\beta d}{2} P_c - \beta a F_c - \frac{2}{3} d F_c + \frac{2}{3} d P_c\Big]$$

From equations 2 and 3;

$$F_c + F_B = P\Big[\frac{e}{g} + \frac{1}{2}\Big] + P\Big[\frac{e}{g} - \frac{1}{2}\Big]$$

$$F_c + F_B = \frac{2Pe}{g}$$

Substituting for $[F_C + F_B]$ and combining terms;

$$E_p I_p \theta_p = \frac{2a^3 P_e}{3g^2} + \frac{ad}{\lambda g}\Big[aF_c(\beta^2 - \beta) - (P_c - F_c)\frac{\beta d}{2} + (P_c - F_c)\frac{2}{3}d\Big]$$

Factoring beta (β), F_C, and d out of the second term;

$$E_p I_p \theta_p = \frac{2a^3 Pe}{3g^2} + \frac{\beta a d^2 F_c}{\lambda g}\Big[\frac{a}{d}(\beta - 1) - \Big(\frac{P_c}{F_c} - 1\Big)\frac{1}{2} + \Big(\frac{P_c}{F_c} - 1\Big)\frac{2}{3\beta}\Big]$$

Simplifying;

$$E_p I_p \theta_p = \frac{2a^3 Pe}{3g^2} + \frac{\beta a d^2 F_c}{\lambda g}\Big[\frac{a}{d}(\beta - 1) + \Big(\frac{P_c}{F_c} - 1\Big)\Big(\frac{2}{3\beta} - \frac{1}{2}\Big)\Big]$$

From equation 3;

$$F_c = P\Big[\frac{e}{g} + \frac{1}{2}\Big]$$

$$F_c = Pe\Big[\frac{1}{g} + \frac{1}{2e}\Big]$$

$$F_c = Pe\Big[\frac{2e + g}{2ge}\Big] \qquad (3a)$$

From equations 3a and 4;

$$\frac{P_c}{F_c} = \frac{\alpha P[B_x e + 1]}{Pe\Big[\frac{2e + g}{2ge}\Big]}$$

$$\frac{P_c}{F_c} = \frac{2g\alpha[B_x e + 1]}{2e + g}$$

Substituting for

$$E_pI_p\theta_p = \frac{2a^3P_e}{3g^2} + \frac{\beta a d^2 P_e}{\lambda g}\left(\frac{2e+g}{2ge}\right)\left[\frac{a}{d}(\beta-1) + \left(\frac{2g\alpha[B_x e+1]}{2e+g} - 1\right)\left(\frac{2}{3\beta} - \frac{1}{2}\right)\right]$$

Simplifying;

$$E_pI_p\theta_p = \frac{2a^3P_e}{3g^2} + \frac{\beta a d^2 P_e}{\lambda g}\left[\frac{a}{d}\left(\frac{1}{g} + \frac{1}{2e}\right)(\beta-1) + \left(\alpha\left[B_x + \frac{1}{e}\right] - \left(\frac{1}{g} + \frac{1}{2e}\right)\right)\left(\frac{2}{3\beta} - \frac{1}{2}\right)\right]$$

$$E_pI_p\theta_p = \frac{2a^3P_e}{3g^2} + \frac{\beta a d^2 P_e}{\lambda g}\left[\frac{a}{d}\left(\frac{1}{g} + \frac{1}{2e}\right)(\beta-1) + \left(\frac{2}{3\beta} - \frac{1}{2}\right)\left(\alpha B_x + \frac{\alpha}{e} - \frac{1}{g} - \frac{1}{2e}\right)\right]$$

$$E_pI_p\theta_p = \frac{2a^3P_e}{3g^2} + \frac{\beta a d^2 P_e}{\lambda g}\left[\frac{a}{d}\left(\frac{1}{g} + \frac{1}{2e}\right)(\beta-1) + \left(\frac{2}{3\beta} - \frac{1}{2}\right)\left(\alpha B_x + \frac{(\alpha-0.5)}{e} - \frac{1}{g}\right)\right]$$

Factoring out P_e and dividing by E_pI_p;

$$\theta_p = \frac{P_e}{E_pI_p}\left[\frac{2a^3}{3g^2} + \frac{\beta a d^2}{\lambda g}\left\{\frac{a}{d}\left(\frac{1}{g} + \frac{1}{2e}\right)(\beta-1) + \left(\frac{2}{3\beta} - \frac{1}{2}\right)\left(\alpha B_x + \frac{(\alpha-0.5)}{e} - \frac{1}{g}\right)\right\}\right] \quad (5)$$

Now an expression for the rotation of the column base due to bending in the base plate is available. While most of the variables in the equation are straightforward and easily obtained for a specific column base detail, the factors beta (β) and lambda (λ) are not readily attainable. If the column web is welded to the base plate it is obvious that lambda will have a value greater than 1.0, but it is not obvious whether lambda should have an order of magnitude of 10.0 or of 100.0. Similarly, the value for beta will probably be only slightly less than 1.0 if the column flange is welded to the base plate on one side of the flange only. If the flange is welded to the base plate on both sides of the flange or is welded with a full penetration groove weld, some transfer of moment will occur between the base plate and the column flange. Either of these weld conditions would require beta to have a value less than one. The selection of reasonable values of the beta and lambda factors will be discussed in another section of this paper.

If a value of 1.0 for beta (β) is used the rotation can be simplified somewhat. Substituting a value of 1.0 for beta leads to the following form for Equation 5:

$$\theta_p = \frac{P_e}{E_p I_p}\left[\frac{2a^3}{3g^2} + \frac{ad^2}{6\lambda g}\left(\alpha B_x + \frac{(\alpha - 0.5)}{e} - \frac{1}{g}\right)\right] \qquad (5a)$$

The third component of the column base rotation is the rotation due to the elongation of the tension anchor bolts. In the derivation of an expression for this rotation, two simplifying assumptions are made. First, it is assumed that the plate rotates about the anchor boltson the compression side. This assumption is consistent with the assumed location of the resultant compressive force. Second, the derivation assumes that the anchor bolts do not yield and do follow a linear stress-strain relationship. From Figure 7 the rotation due to anchor bolt elongation can be derived as follows:

$$\Delta_B = \frac{F_B L_B}{A_B E_B}$$

where A_B = SUM OF AREA OF ANCHOR BOLTS ON TENSION SIDE.

Using Equation 2;

$$F_B = P\left[\frac{e}{g} - \frac{1}{2}\right] \qquad (2)$$

and substituting for F_B ;

$$\Delta_B = \left[\frac{e}{g} - \frac{1}{2}\right] \frac{P L_B}{A_B E_B}$$

assuming the angle of rotation is small;

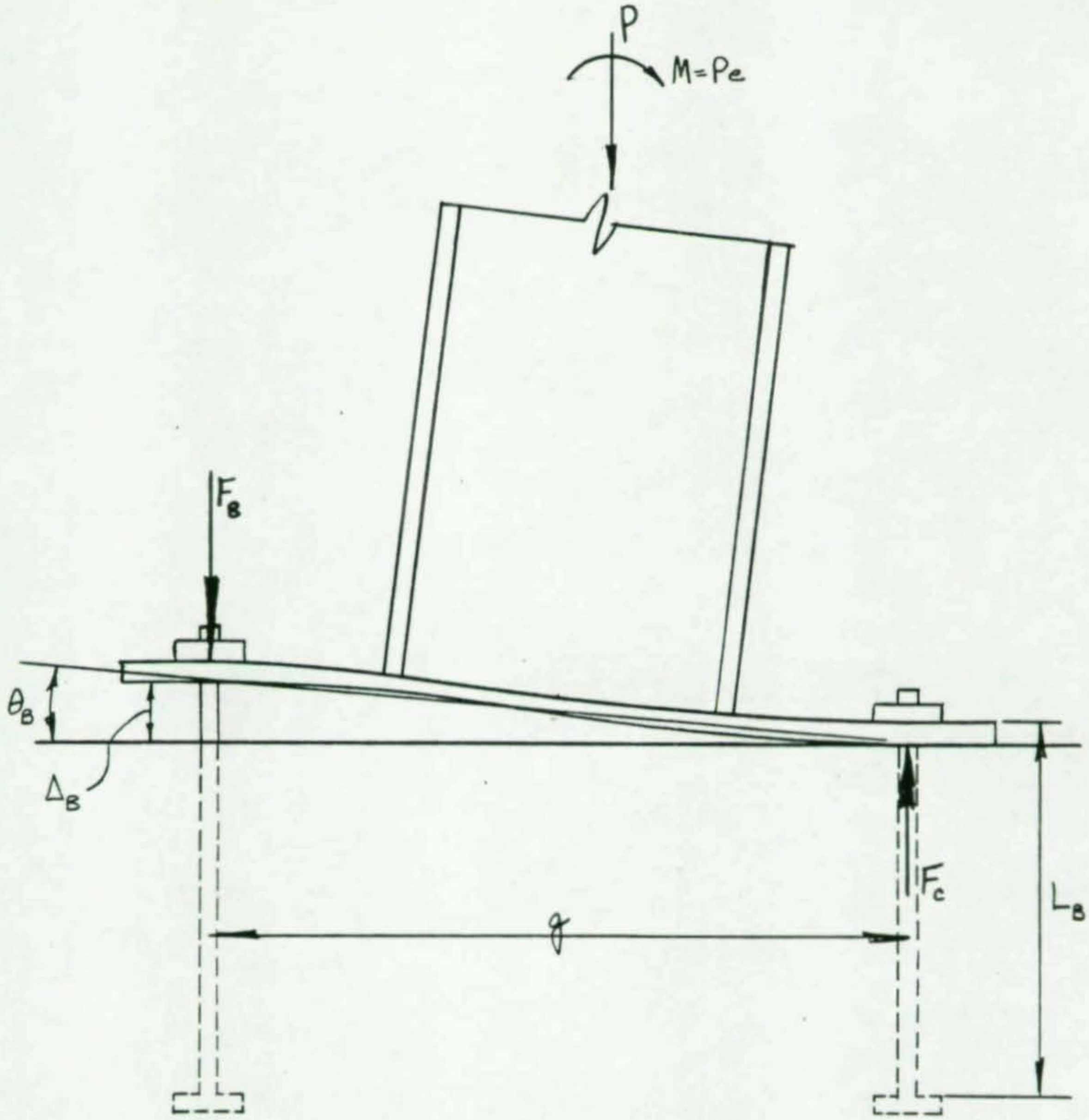
$$\theta_B = \Delta_B / g$$

$$\therefore \ \theta_B = \frac{\left[\frac{e}{g} - \frac{1}{2}\right]}{g} \frac{P L_B}{A_B E_B}$$

Simplifying this equation gives;

$$\theta_B = \frac{[2e - g]}{2g^2} \frac{P L_B}{A_B E_B} \geq 0 \qquad (6)$$

Now a fairly simple expression is available for determining the rotation at the base of a column due to anchor bolt elongation. Observing the first term of the equation, it is apparent that for eccentricities (e) less than $g/2$ the result would be a negative rotation. Therefore, for $e < g/2$ this expression is invalid.



A_B = TOTAL AREA OF TENSION ANCHOR BOLTS.

FIGURE 7 - ROTATION DUE TO ANCHOR BOLT ELONGATION.

There are two factors which this expression for θ_B neglects. The effects of pretensioning the anchor bolts and the effects of crushing of the concrete under anchor bolt head are both neglected. Pretensioning the anchor bolts tends to decrease the rotation and increase the stiffness of the column base. Conversely, crushing of the concrete under the bolt head causes a reduction in stiffness. Pretensioning the anchor bolts tends to counteract the flexibility increase due to the crushing of the concrete because some of the crushing is complete before any loads are applied to the column. Neglecting the effects of pretensioning the anchor bolts is not usually critical because it results in underestimating the stiffness.

The expressions, for the three rotational components of the column base, which have been derived can be summed to find the total rotation due to an applied moment (Pe):

$$\theta_{TOTAL} = \theta_F + \theta_B + \theta_P$$

$$\therefore \theta_{TOTAL} = \frac{Pe}{K_s I_F} + \frac{(2e-g)}{2g^2}\frac{PL_B}{A_B E_B} + \frac{Pe}{E_P I_P}\left\{\frac{2a^3}{3g^2} + \frac{\beta a d^2}{\lambda g}\left[\frac{a}{d}\left(\frac{1}{g}+\frac{1}{2e}\right)(\beta-1) + \left(\frac{2}{3\beta}-\frac{1}{2}\right)\left(\alpha B_x + \frac{(\alpha-0.5)}{e} - \frac{1}{g}\right)\right]\right\} \quad (7)$$

Since the flexibility (γ) is the rotation due to an applied moment, divide through by Pe;

$$\gamma = \frac{\theta_{TOTAL}}{Pe}$$

$$\gamma = \frac{1}{K_s I_F} + \frac{(2e-g)L_B}{2g^2 e A_B E_B} + \frac{1}{E_P I_P}\left\{\frac{2a^3}{3g^2} + \frac{\beta a d^2}{\lambda g}\left[\frac{a}{d}\left(\frac{1}{g} + \frac{1}{2e}\right)(\beta - 1) + \left(\frac{2}{3\beta} - \frac{1}{2}\right)\left(\alpha B_x + \frac{(\alpha - 0.5)}{e} - \frac{1}{g}\right)\right]\right\} \quad (8)$$

Substituting a Beta (β) of 1.0;

$$\gamma = \frac{1}{K_s I_F} + \frac{(2e-g)L_B}{2g^2 e A_B E_B} + \frac{1}{E_P I_P}\left\{\frac{2a^3}{3g^2} + \frac{a d^2}{6\lambda g}\left(\alpha B_x + \frac{(\alpha - 0.5)}{e} - \frac{1}{g}\right)\right\} \quad (8a)$$

Since the stiffness (K) is the inverse of the flexibility;

$$K = \frac{1}{\gamma}$$

$$\therefore K = \frac{1}{\frac{1}{K_s I_F} + \frac{(2e-g)L_B}{2g^2 e A_B E_B} + \frac{1}{E_P I_P}\left\{\frac{2a^3}{3g^2} + \frac{\beta a d^2}{\lambda g}\left[\frac{a}{d}\left(\frac{1}{g} + \frac{1}{2e}\right)(\beta - 1) + \left(\frac{2}{3\beta} - \frac{1}{2}\right)\left(\alpha B_x + \frac{(\alpha - 0.5)}{e} - \frac{1}{g}\right)\right]\right\}} \quad (9)$$

Substituting Beta (β) equals 1.0;

$$K = \frac{1}{\frac{1}{K_s I_F} + \frac{(2e-g)L_B}{2g^2 e A_B L_B} + \frac{1}{E_P I_P}\left\{\frac{2a^3}{3g^2} + \frac{a d^2}{6\lambda g}\left[\alpha B_x + \frac{(\alpha - 0.5)}{e} - \frac{1}{g}\right]\right\}} \quad (9a)$$

Using Equations 7, 8, and 9, the engineer can obtain an estimate of the flexibility and stiffness of a column base of the type studied.

A few notes on the use of these equations are called for. First, since Equation 6 is invalid for eccentricities (e) less than $g/2$, the equations given for flexibility and for stiffness (Equations 8 and 9) are not valid for values of e less than $g/2$. Second, the equation derived for the stiffness at the base of the column gives an approximation of the absolute stiffness. Therefore, when comparing the base stiffness with the column stiffness, the absolute stiffness ($4EI/L$) of the column should be used, or alternately, the base stiffness should be divided by $4E$ (where E is the modulus of elasticity of the base materials) and compared with the relative stiffness of the column (I/L).

CALCULATOR PROGRAM

A program has been developed for the Hewlett-Packard HP-41 calculator which solves the expressions previously developed to determine the stiffness of column bases. A listing of the program and sample output are provided in Appendix B. The program prompts the user to input the following variables:

B = Footing width

D = Footing Length

K_s = Modulus of subgrade reaction

L_B = Anchor bolt length

A_B = Summation of area of tension anchor bolts

E_B = Young's Modulus for anchor bolts

d = Depth of column section

B_x = Ratio of column area to column section modulus

α = Ratio of A_F/A for column (See Table 2)

a = Distance from column face to centerline of anchor bolts

E_p = Young's Modulus for base plate

b = Base plate width

t_p = Base plate thickness

β = Ratio of base plate moment just inside flange to base plate moment just outside flange

λ = Ratio of base plate stiffness between column flanges to base plate stiffness outside column flanges

P = Axial load

e = Load eccentricity

Once the variables have been set, the program calculates and prints each of the three components of the rotation, the three components of the flexibility, the total flexibility, and the total stiffness of the column base. The user is then given the opportunity to change the column base variables and the applied loading (P and e).

This program works very well and saves considerable time in computing the column base stiffness. It is very useful in the design process because it allows one to vary the inputs and compare resulting stiffnesses fairly quickly.

BEHAVIOR

It is very important in structural engineering, as well as other fields, for the engineer to have a good understanding of the principles involved and the behavior which the equations exhibit before using them. The purpose of this section is to observe the behavior of the derived expressions as small changes are made in the variables involved. The behavior is studied by beginning with a set base configuration and varying values of one variable at a time. The beginning base configuration selected is:

$L_B = 24$ in.	$b = 30$ in.	$D = 10$ ft.
$A_B = 6$ in.^{2}	$t_p = 2$ in.	$B = 6$ ft.
$a = 4$ in.	$\beta = 1.0$	$K_s = 250$ lb/in^3
	$\lambda = 20.0$	

First, the relative values of the three components of the column base flexibility are compared. The flexibilities due to each of the three rotation components are determined for a W14 x 43 column with the base configuration given. These flexibilities are determined for eccentricities of load varying from zero to 120 inches. The three components of the flexibility are plotted in Figure 8 together with the total flexibility of the column base connection. For this particular case the footing rotation component accounts for the largest percentage of the total base flexibility. Note that for values of eccentricity of less than about 12 inches the equations behave much differently than they do at larger eccentricities. This graph amplifies the importance of the limitation of the equation to values of eccentricity (e) greater than $\gamma/2$.

Figure 9 is a comparison of the full stiffness of the selected base for a heavy 14 inch column (W14 x 426) and a light 14 inch column (W14 x 43). Superimposed on the same graph is a plot of the stiffness for a representative 14 inch column. For this representative column alpha (α) was taken as 0.385, B_x was taken as 0.185, and the column depth (d) was taken as 14 inches. The graph indicates that using these values of α, B_x, and d results in a lower bound estimate for the column base stiffness. Figure 10 shows similar results for 12 inch wide flange columns; where $\alpha = 0.385$, $B_x = 0.215$, and d = 12 inches for the representative 12 inch column. It

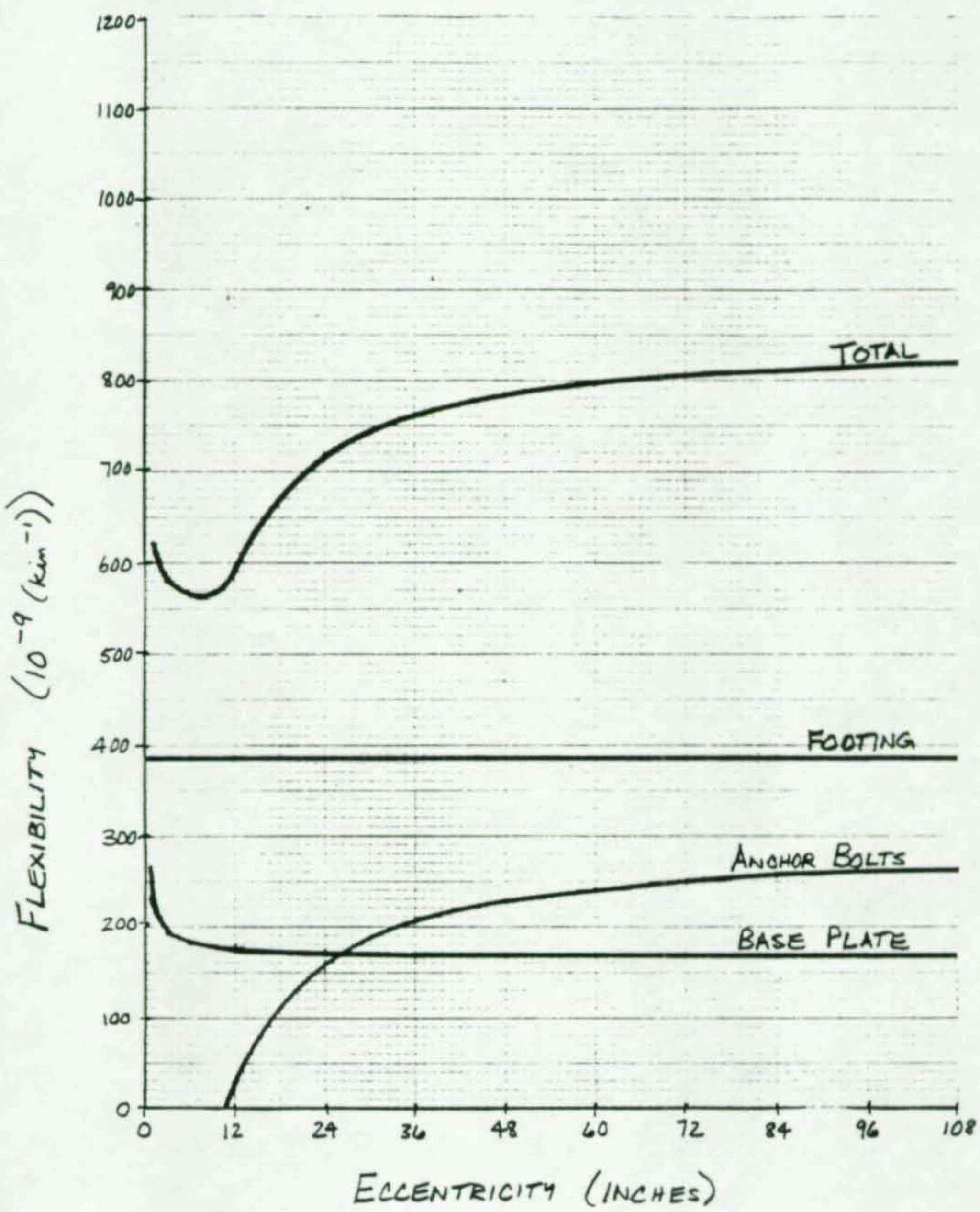


FIGURE 8 - COMPARISON OF THREE COMPONENTS OF THE BASE FLEXIBILITY.

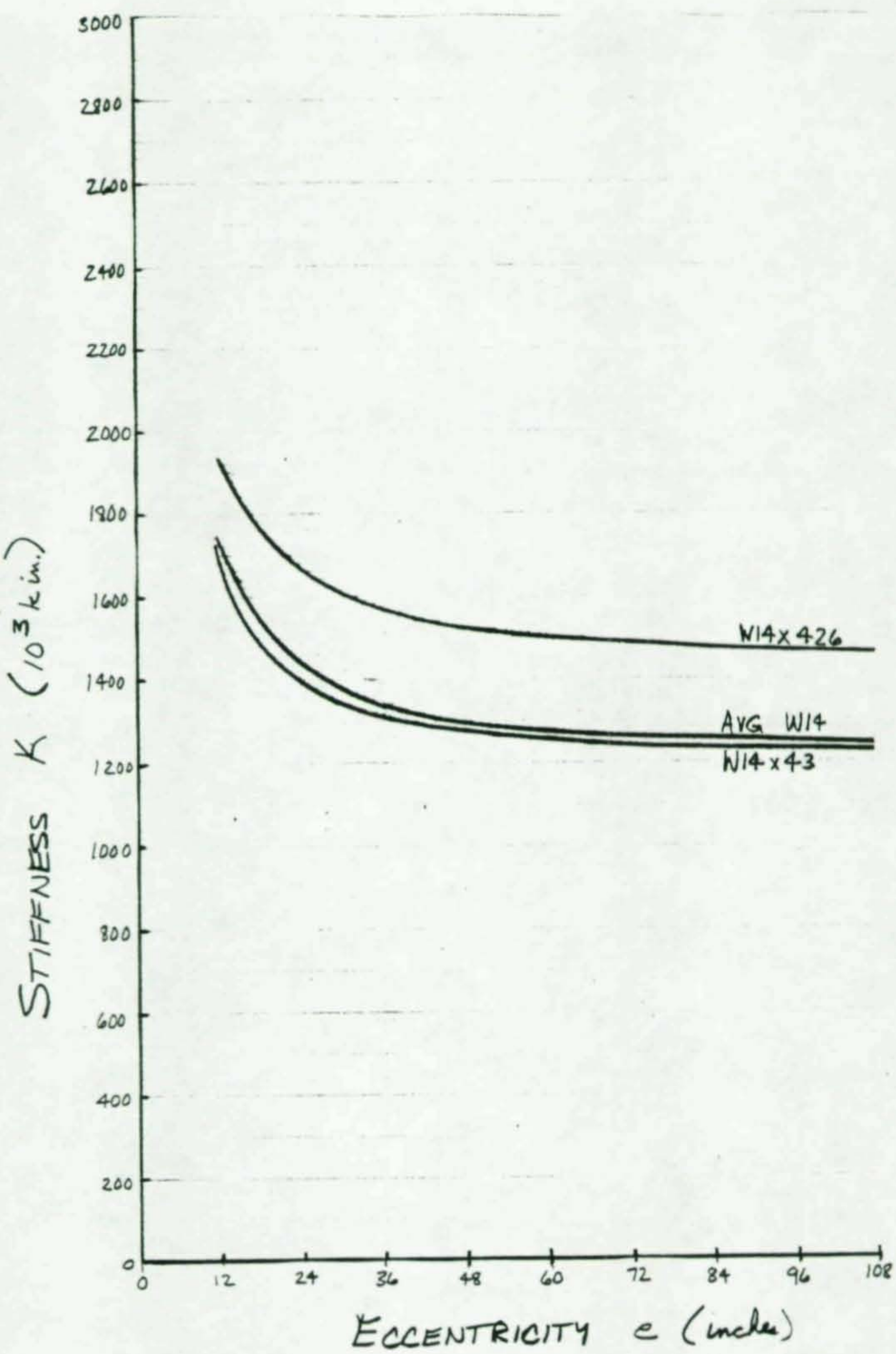


FIGURE 9 - COMPARISON OF STIFFNESS FOR W14 COLUMNS.

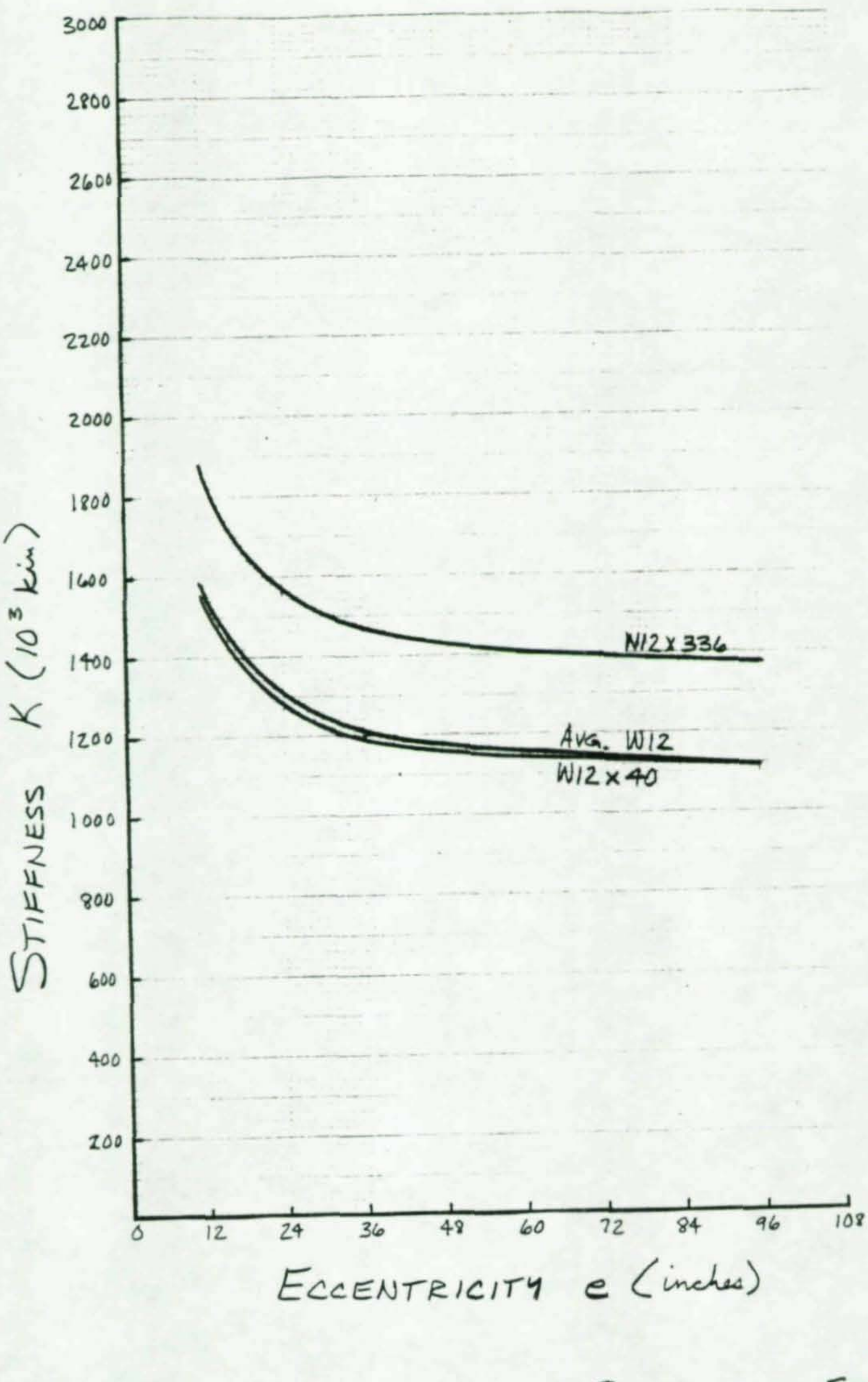


FIGURE 10 - COMPARISON OF STIFFNESS FOR W12 COLUMNS.

should be noted that the percent change in the base stiffness with column weight is quite small in comparison with the percent change in column weight.

The final observations of behavior are made by varying the values of K_s, D, B, L_B, A_B, d, a, b, t_p, , and one at a time. The reference curve is for a representative 14 inch column with the base configuration given earlier. The variation of base stiffness with small changes in these variables is illustrated in Figures 11 through 21.

In reviewing the graphs of these variations several general characteristics are noted. First, for all of the variables except Beta (β) small variations from the reference value cause variance in the stiffness (K) of at least 10 percent. The variation of stiffness is particularly large (20 to 50%) for changes in K_s, in D, and in values of t_p less than 3.0 inches. Small changes in B, L_B, A_B, d, a, b, and result in variations of stiffness of roughly 10 to 20 percent. This would indicate that, if the engineer had a fairly good estimation for these variables and took care in estimating K_s, D, and t_p, an estimate of the base stiffness could be determined within about 10 to 20 percent.

The distressing point is that the modulus of subgrade reaction (K_s) is probably the most variable and the most difficult to determine factor, in addition to being one of the most critical. As a possible approach to counter this problem one could choose upper and lower

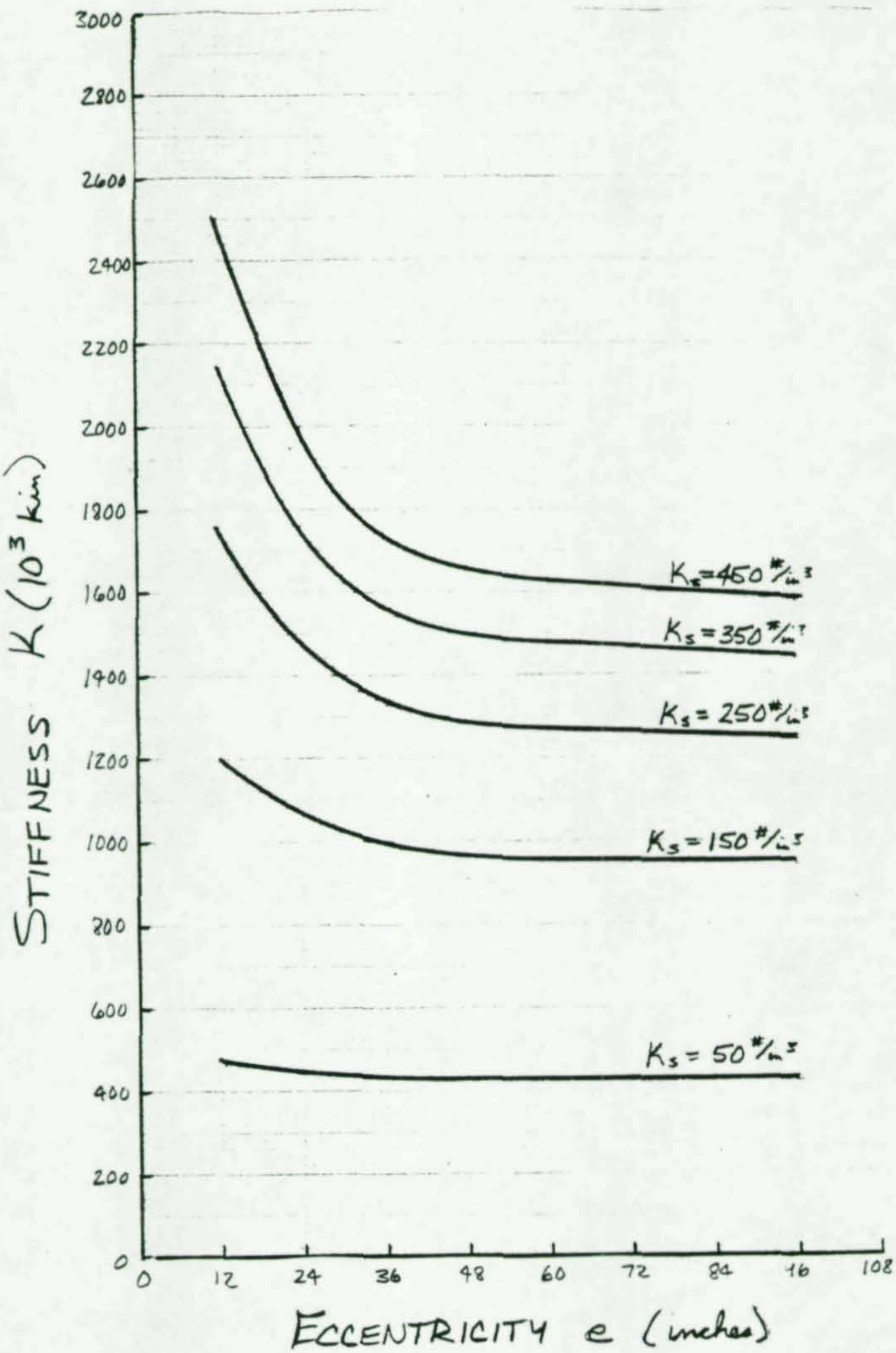


FIGURE 11 - VARIATION OF K WITH CHANGES IN K_s.

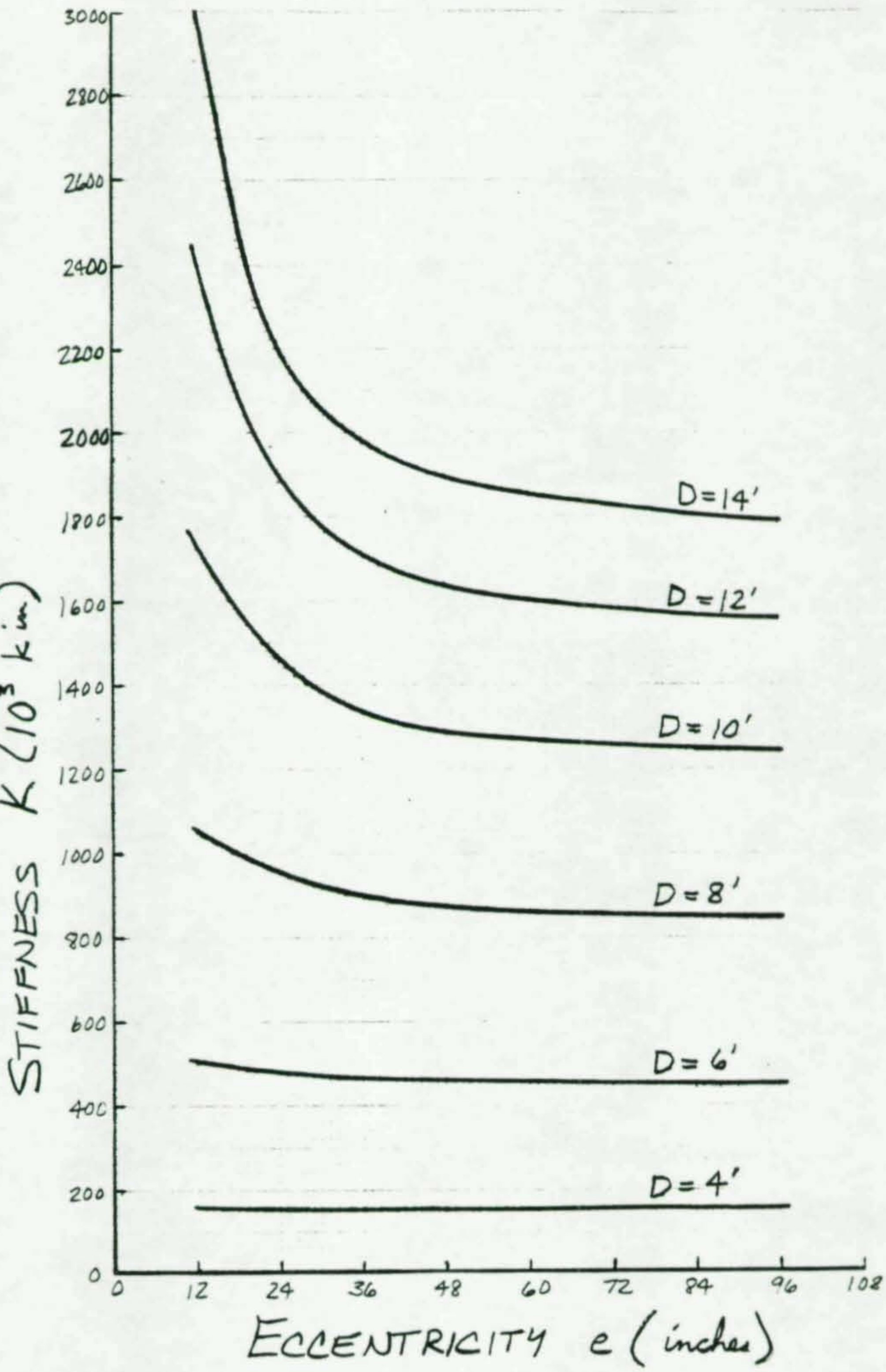


FIGURE 12 - VARIATION OF K WITH CHANGES IN D.

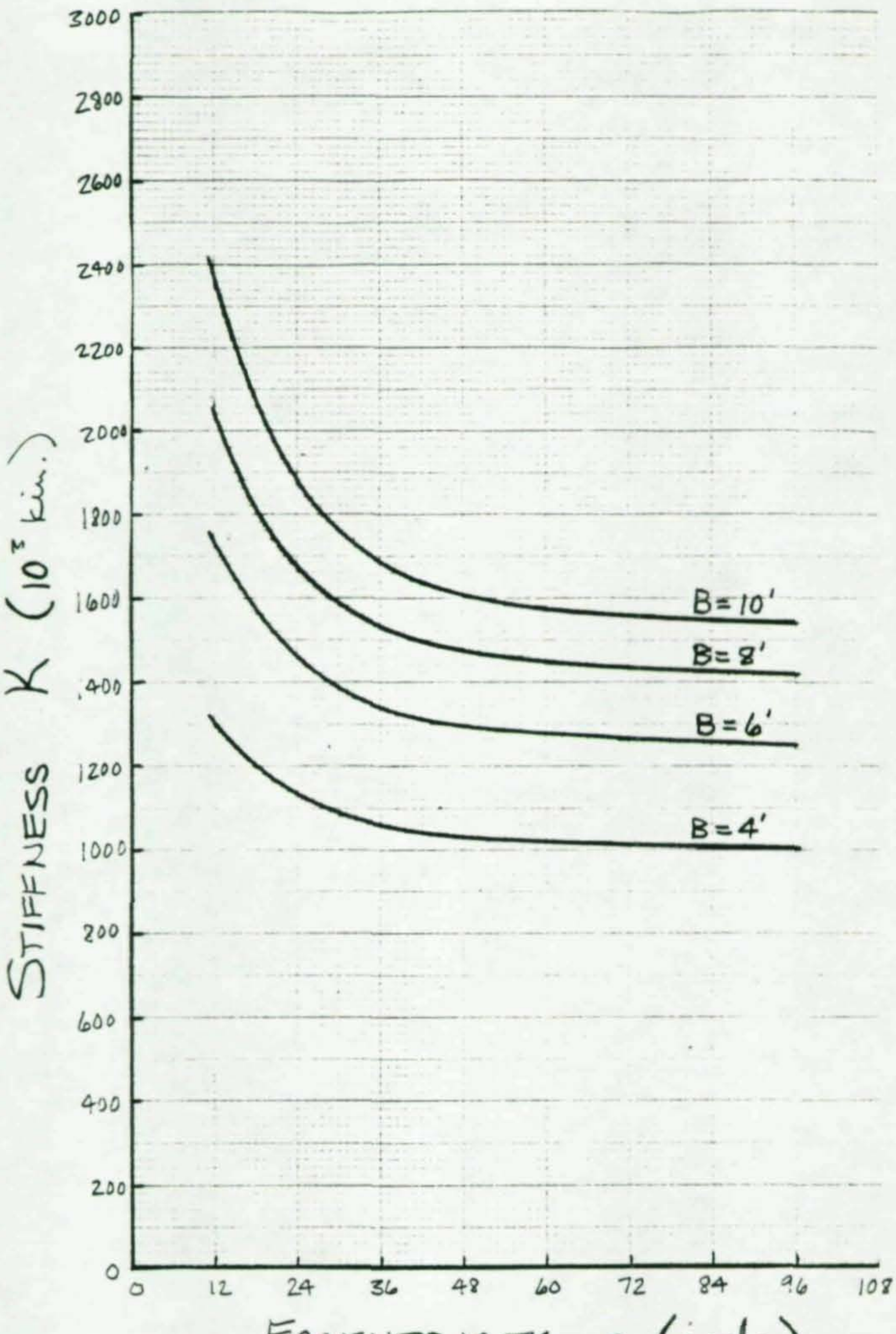


FIGURE 13 - VARIATION OF K WITH CHANGES IN B.

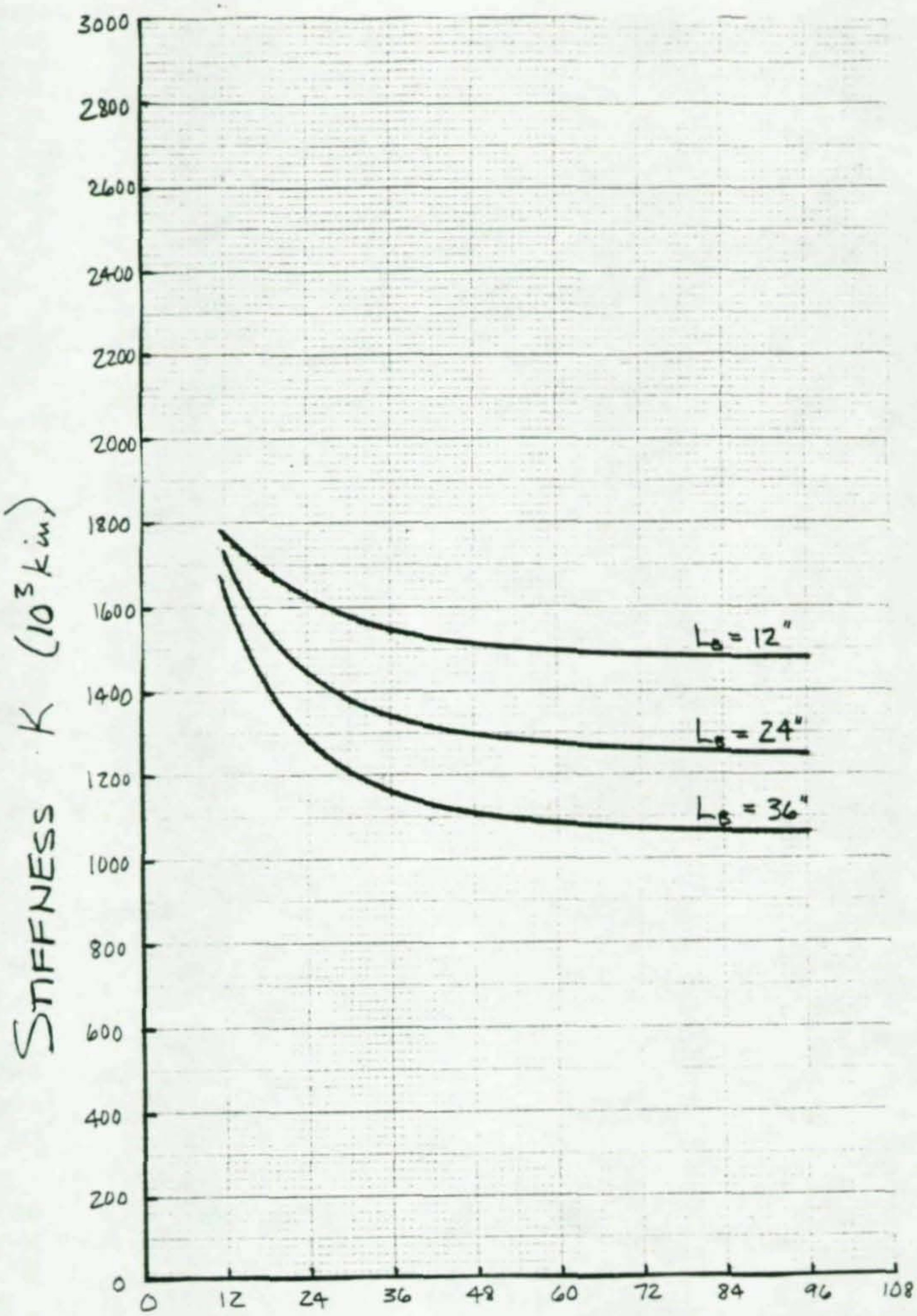


FIGURE 14 - VARIATION OF K WITH CHANGES IN L_B.

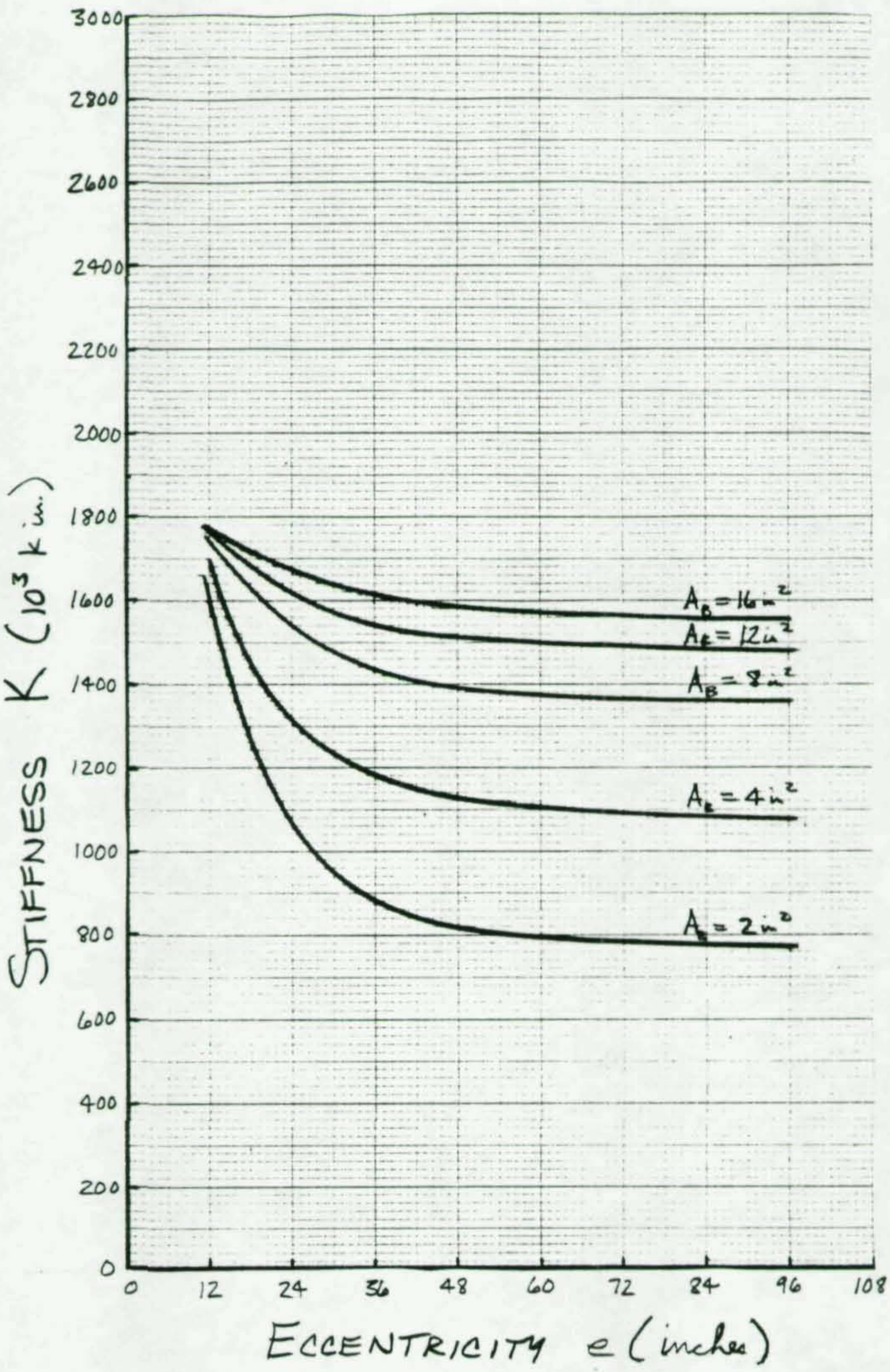


FIGURE 15 - VARIATION OF K WITH CHANGES IN A_B.

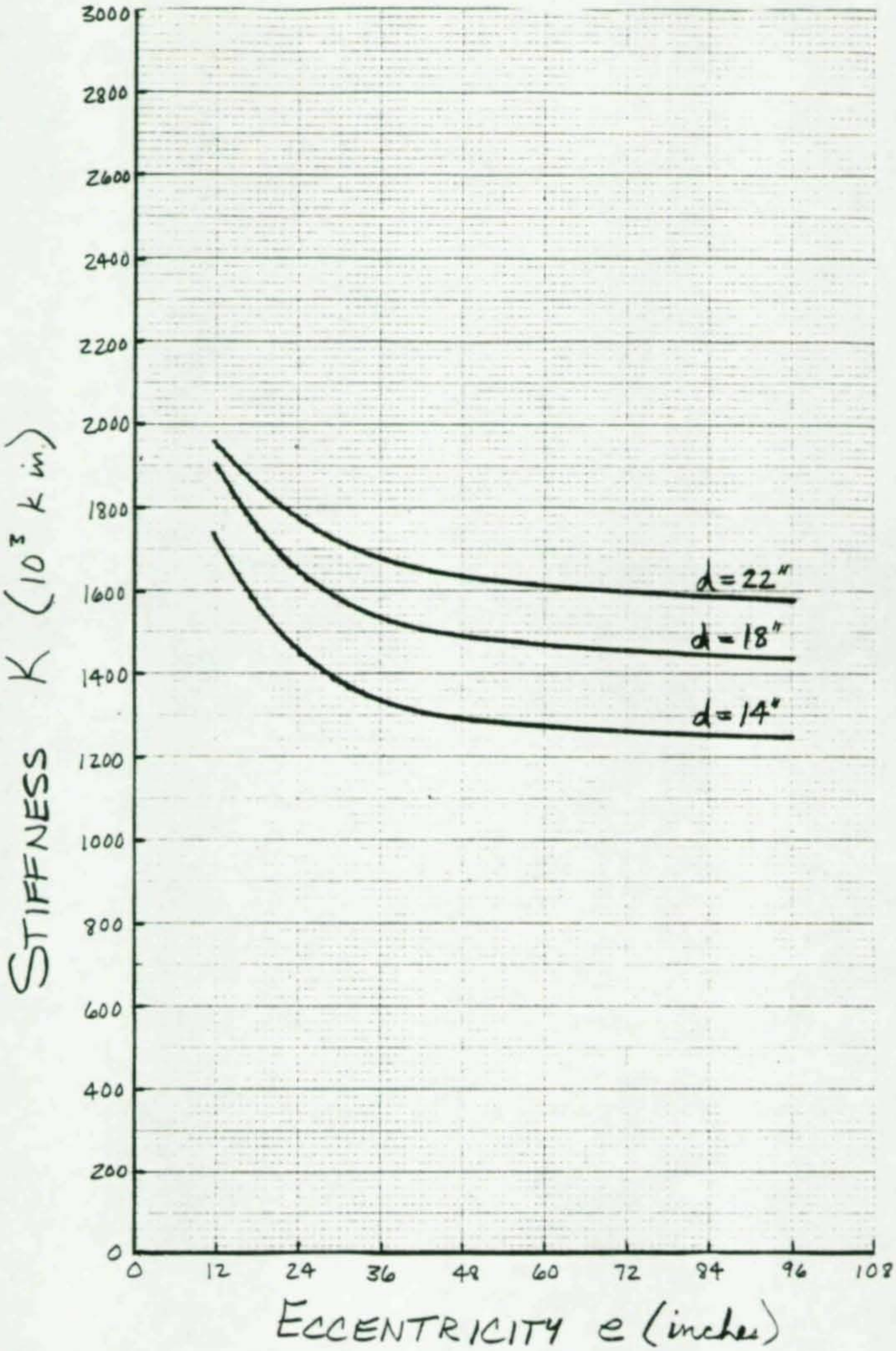


FIGURE 16 - VARIATION IN K WITH CHANGES IN d.

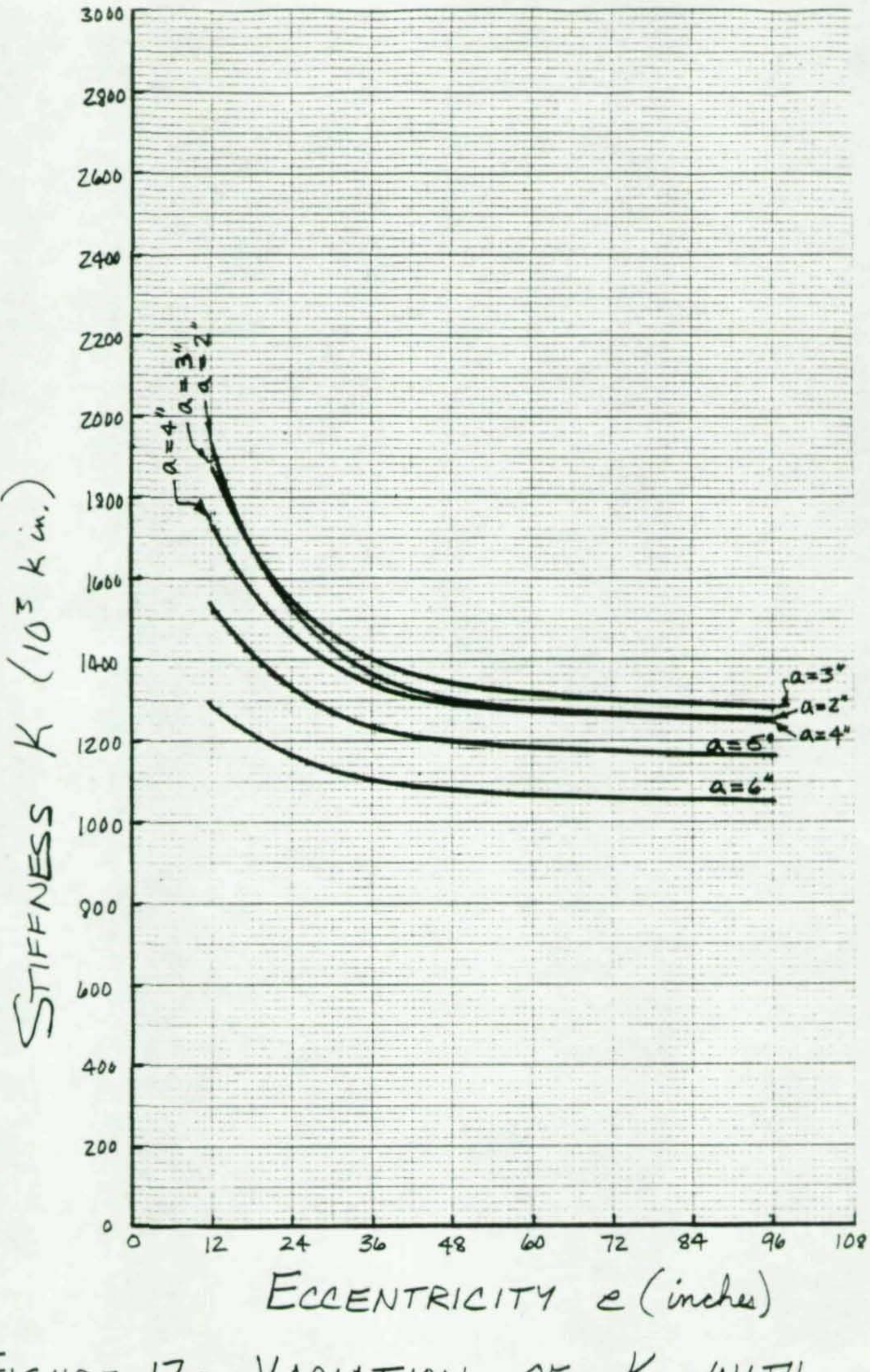


FIGURE 17 - VARIATION OF K WITH CHANGES IN a.

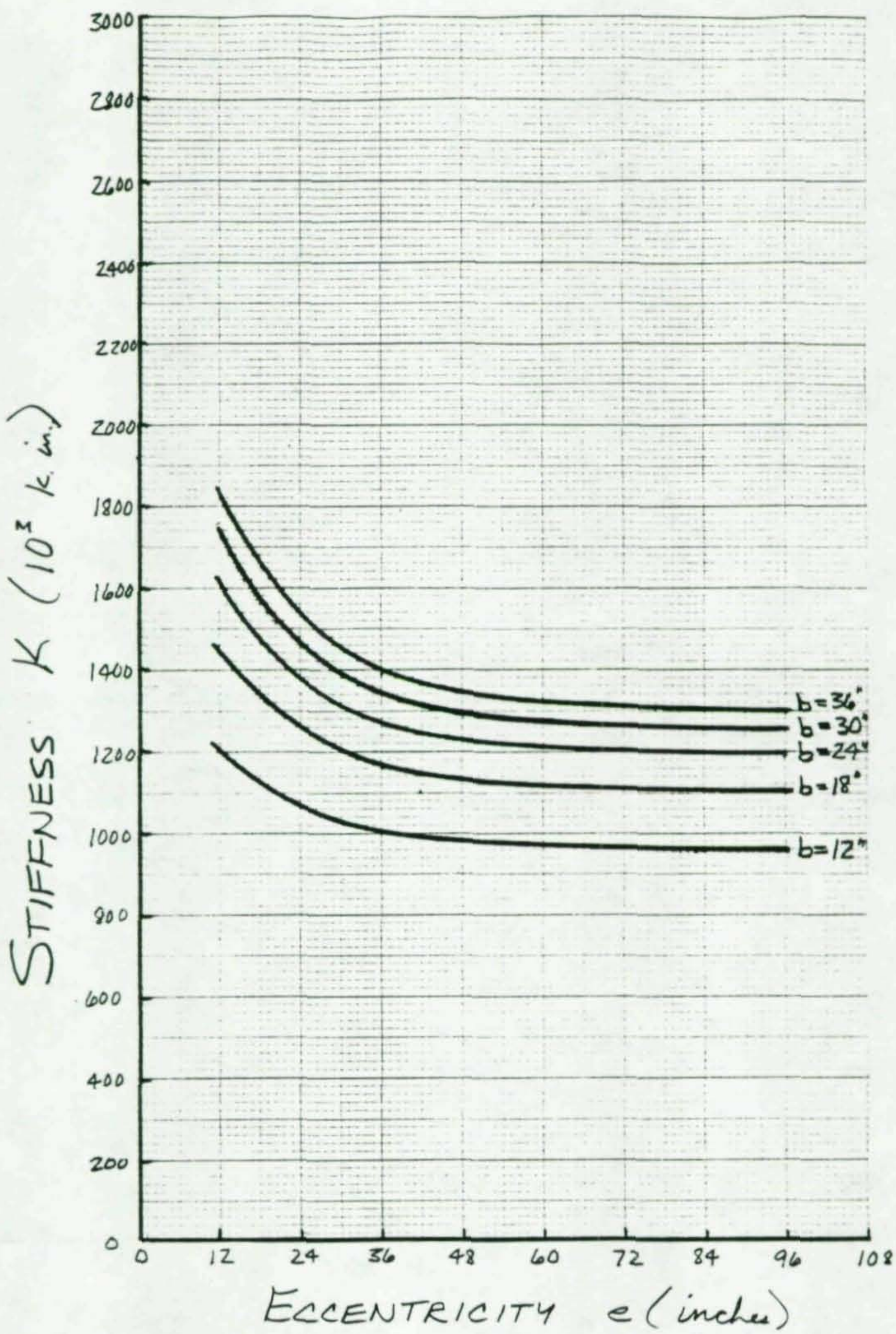


FIGURE 18 – VARIATION OF K WITH CHANGES IN b.

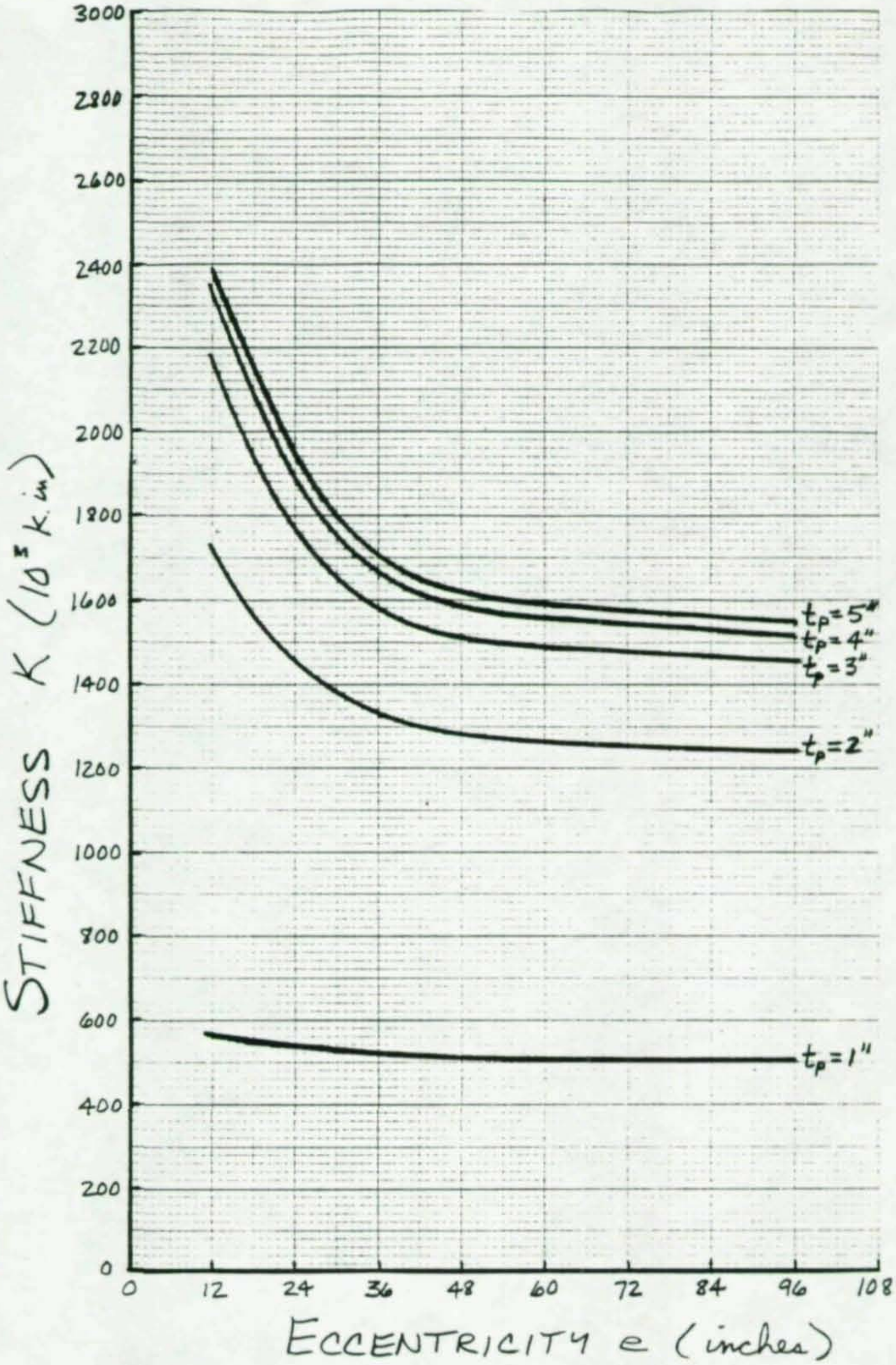


FIGURE 19 - VARIATION OF K WITH CHANGES IN t_p.

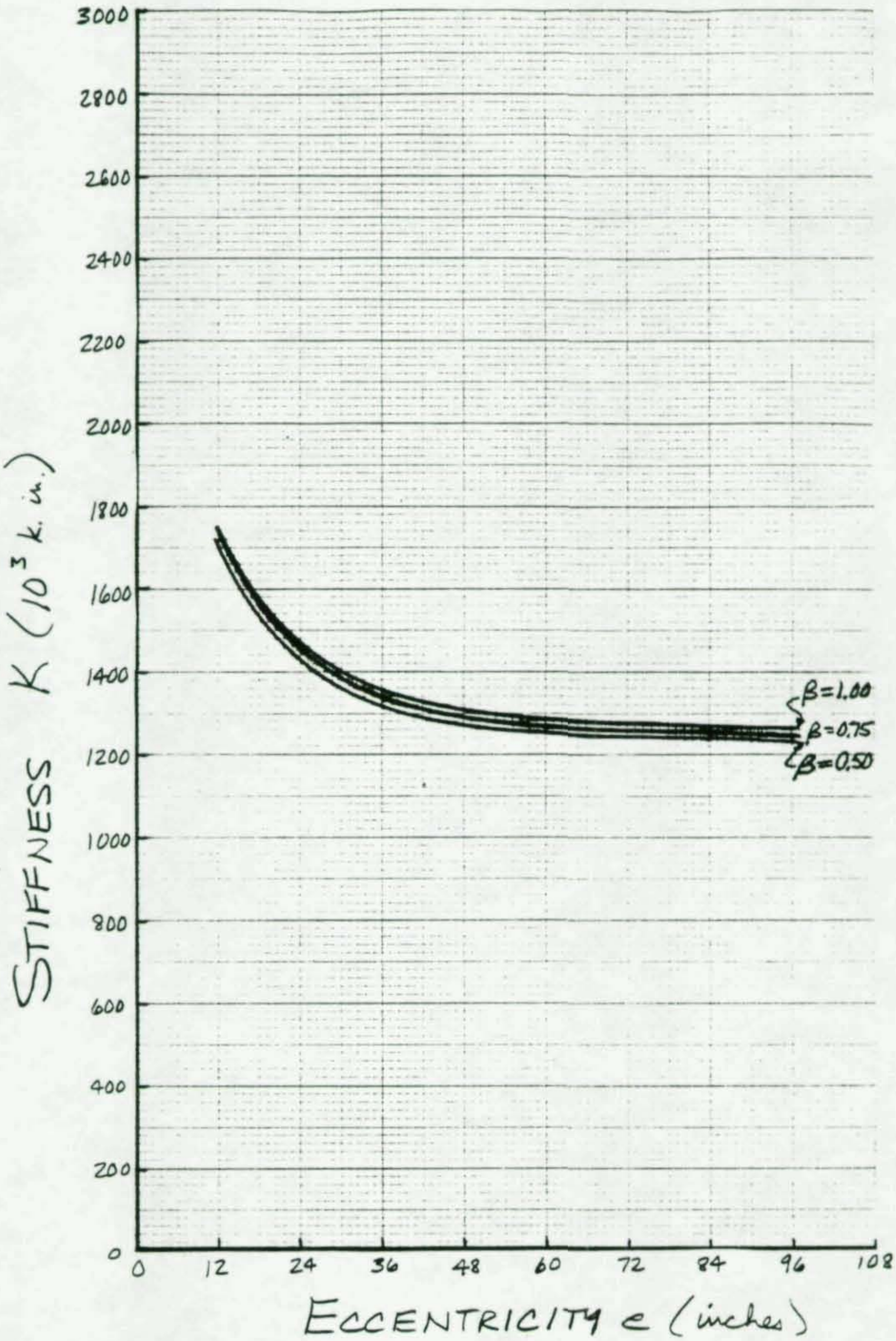


FIGURE 20 - VARIATION OF K WITH CHANGES IN β.

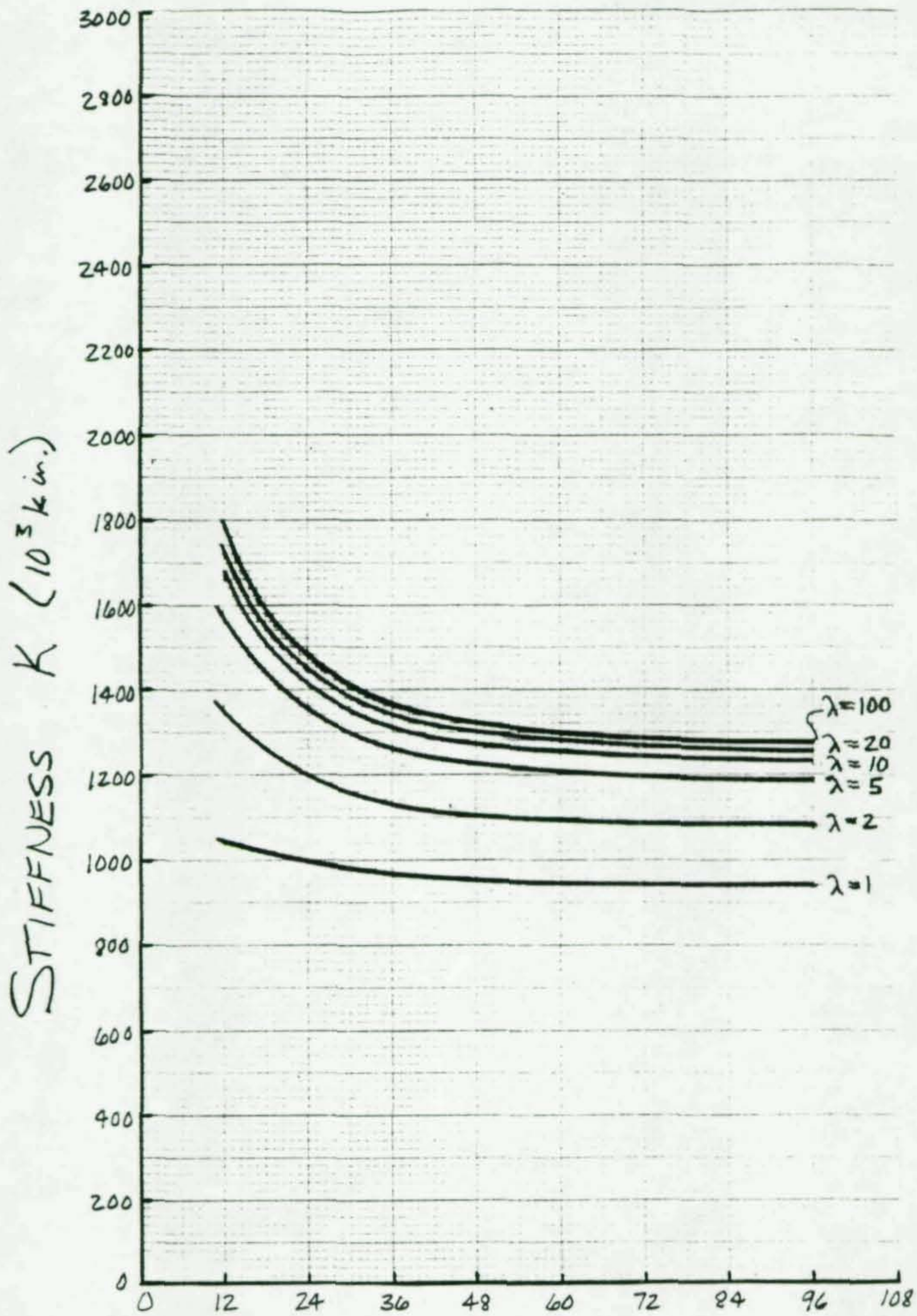


FIGURE 21 - VARIATION OF K WITH CHANGES IN λ.

bounds for the subgrade modulus and calculate coorresponding upper and lower bounds of stiffness. The other variables are not quite as critical since they can be altered in the design if need be.

Focusing attention on coefficients introduced in this study; β and λ . First, it was found that if β could be taken as unity the expression for base plate bending could be simplified significantly. Based on the graph in Figure 20, it appears that assuming Beta (β) equals unity would be very much acceptable for approximating base stiffness. In comparison with the variations of the stiffness with changes in other variables, the variation of stiffness with changes in Beta is negligible. Nevertheless, if more accurate estimates of the base stiffness are desired, the possibility of using Beta less than one should be explored.

Finally, some conclusions can be drawn concerning Lambda (λ) from the plot of Figure 21. There are basically two situations to be considered in choosing values of this coefficient. First, if the column web is not welded to the base plate it appears that Lambda (λ) should be taken conservatively as 1.0. Second, if the web is welded to the base plate some value greater than 1.0 should be chosen. The question of how much the column web stiffens the base plate is one without an obvious answer. Ideally, a finite element analysis, backed by experimental tests, could be used to get an approximate range for Lambda (λ) in this case. Looking at Figure 21 carefully reveals that the difference in stiffness between $\lambda = 10.0$ and $\lambda = 100.0$ is very small. Based on this graphical representation it appears that assuming Lambda (λ) equals either 5.0 or 10.0 would give a reasonable lower bound value for the stiffness.

RECOMMENDATIONS

Application of this expression in structural engineering practice should be done only after obtaining a good understanding of the behavior of the equation and of the assumptions involved in the derivation. If the engineer understands the behavior and the assumptions made, he can easily choose values for the variables which will give conservative estimates of the stiffness, without being overly conservative.

For the use of the equations to obtain approximate values of stiffness and flexibility, several recommendations are made. First, it is recommended that the average values given in Table 3 be used for representative nominal column sizes. Second, it is recommended that Beta (β) be taken as unity. For cases where the column web is not welded to the base plate, Lambda (λ) can be taken conservatively as unity. Finally, if the column web is welded to the base plate, a value for Lambda of 10.0 is probably a slightly conservative assumption.

TABLE 3 - RECOMMENDED VALUES FOR d, B_x, AND α.

COLUMN SIZE	d	B_x	α
W14	14 in.	0.184	0.385
W12	12 in.	0.216	0.385
W10	10 in.	0.266	0.385
W8	8 in.	0.331	0.385
W6	6 in.	0.461	0.385

Tables are provided in Appendix C which allow the engineer to very quickly obtain a rough estimate for the column base flexibility and stiffness. The three tables give values of flexibility for each of the three components of rotation. Table C1 tabulates flexibility due to the footing for different sizes of square footings and different values for the modulus of subgrade reaction. Table C2 tabulates the flexibility caused by anchor bolt elongation for various values of L_B and A_B, and for different nominal column sizes. Finally, Table C3 tabulates values of flexibility due to base plate bending for different nominal column sizes as the base plate thickness varies. These tables are provided as a quick way to obtain a first estimate of column base stiffness. A value for each of the three components is selected. These three values are then summed to determine the total base flexibility. Finally, the inverse of the total flexibility is taken to give the total base stiffness.

The recommended procedure for using these relationships and design aids in structural design is as follows:

1. Use portal method or some other approximate method to obtain preliminary estimates of member forces.
2. Assume trial member sizes and column base configuration.
3. Use the tables in Appendix C to approximate the flexibility due to each of the three rotational components (γ_F, γ_B, & γ_P).

4. Determine the approximate base stiffness (K) by:

$$K = \frac{1}{\gamma_F + \gamma_B + \gamma_P}$$

5. Using this base stiffness to either:

   (a) Determine a distribution factor for the base of the column, for use in moment distribution;

$$D.F. = \frac{K_{COL.}}{K_{BASE} + K_{COL.}} \quad \text{where } K_{COL.} = \frac{4EI_{COL}}{L_{COL}}$$

   or (b) Choose an equivalent member, to attach to the column base just above a hinged support (see Figure 22), which has the same stiffness that the assumed column base has;

$$K_{BASE} = \frac{4EI_{EM}}{L_{EM}}$$

$$\therefore \frac{I_{EM}}{L_{EM}} = \frac{K_{BASE}}{4E} \quad \text{where } I_{EM} = I \text{ FOR EQUIV. MEMBER}, \; L_{EM} = \text{LENGTH OF EQUIV. MEMBER.}$$

6. Analyze the structure by the method selected to obtain design member forces.

7. Calculate the rigidity ratio:

$$G_B = \frac{K_{COL}}{K_{BASE}} \quad \text{where } K_{COL} = \frac{4EI_{COL}}{L_{COL}}$$

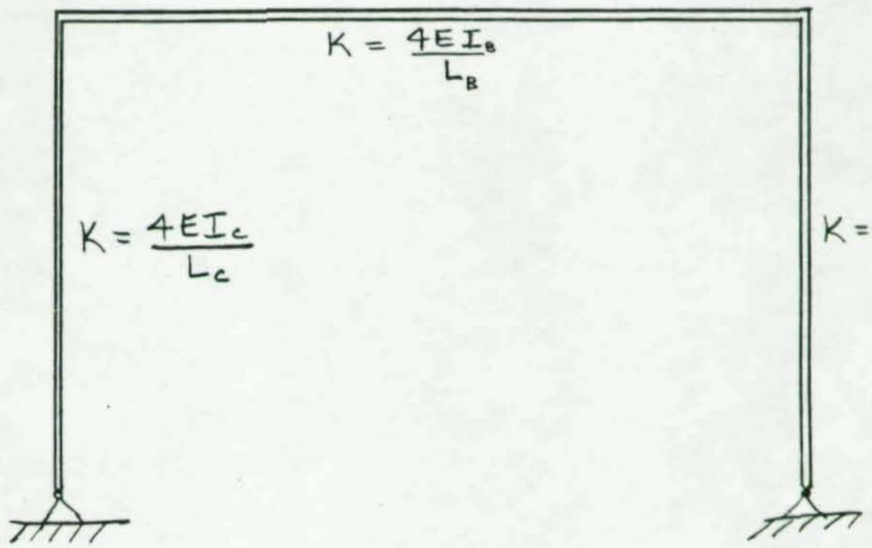
8. Determine effective length of the column using the effective length factor alignment charts.

9. Revise column size and base detail.

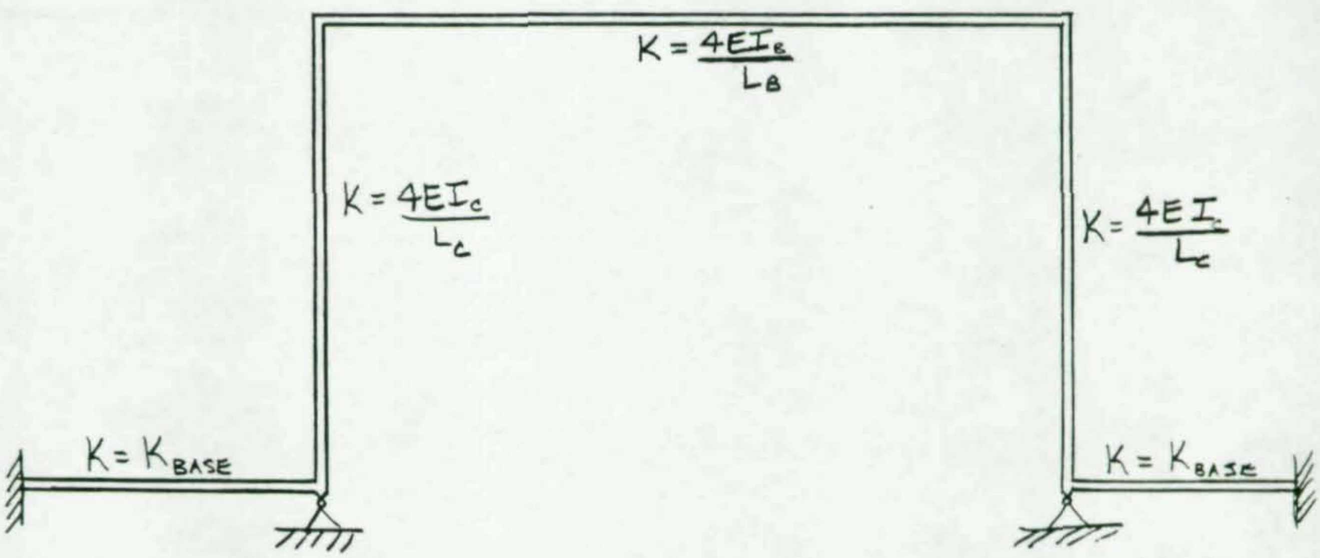
10. Revise approximation of column base stiffness using Equation 9 or calculator program.

11. Repeat Steps 5 through 10 if necessary.

This recommended procedure is illustrated in Appendix D.



(a) SIMPLE FRAME



(b) FRAME W/ EQUIVALENT MEMBERS

FIGURE 22 — APPLICATION OF BASE STIFFNESS WITH EQUIVALENT MEMBERS.

SUMMARY AND CONCLUSIONS

An expression has been developed which allows an engineer to approximate the stiffness of column bases. The approximation considers the combined effects of footing rotation, elongation of anchor bolts, and bending in the base plate. Several simplifying assumptions have been made:

1. Base plate, anchor bolts, and soil behave elastically.
2. Forces in the column web are neglected.
3. Forces on the base plate are modeled as concentrated forces.
4. Resultant compressive force in the concrete acts at the bolt line.

After developing the expression a sensitivity analysis was performed to investigate the relative significance of the variables involved. The stiffness was found to be a very strong function of the modulus of subgrade reaction, the footing length, and the base plate thickness. Variations in the values of the other functions caused only a moderate variation in the base stiffness with the single exception of Beta (β), which caused very little variation.

Based on the assumptions made in the development and the results of the sensitivity analysis, recommendations were made for the practical application of these expressions. Examples of the recommended applications are given in Appendix D. A program for the Hewlett-Packard HP-41 calculator and approximate flexibility charts are presented as design aids for utilizing the proposed method.

This method provides a rational approach for estimating the column base stiffness for a specific base detail. Research is needed, however, to verify the validity of this method. The engineer should recognize that the calculated base stiffness using this method is accurate to only three significant digits at best because of the assumptions and approximations made. If used intelligently the method presented will be of significant practical value to the practicing engineer.

REFERENCES

1. American Institute of Steel Construction, Manual of Steel Construction, 8th Ed., Chicago, Ill., 1980.

2. Galambos, T. V., "Influence of Partial Base Fixity on Frame Stability," Transactions, American Socity of Civil Engineers, Vol. 126, Paper No. 3256, 1961, pp 929-951.

3. Beaulieu, D., Samson, G., and Picard, A., "A Study of the Stabilizing Action of a Simple Column Base Connection," 3rd International Colloquium Proceedings, Stability of Metal Structures, Structural Stability Research Council, Bethlehem, PA, 1983, pp 21-35.

4. Jones, S. W., Kirby, P. A., and Nethercot, D. A., "Columns with Semirigid Joints," Journal of the Structural Division, ASCE, Vol. 108, No. ST2, February, 1982, pp 361-372.

5. Sugimoto, H., and Chen, W. F., "Small End Restraint Effects on Strength of H-Columns," Journal of the Structural Division, ASCE, Vol. 108, No. ST3, March, 1982, pp 661-681.

6. Bjorhovde, R., "Effect of End Restraint on Column Strength - Practical Applications," AISC Engineering Journal, Vol. 21, No. 1, 1984, Chicago, Ill., pp 1-13.

7. Salmon, C. G., Schenker, L., and Johnston, B. G., "Moment-Rotation Characteristics of Column Anchorages," Transactions, American Society of Civil Engineers, Vol. 122, Paper No. 2852, 1957, pp 132-154.

8. LaFraugh, R. W., and Magura, D. D., "Connections in Precast Concrete Structures - Column Base Plates," Journal of the Prestressed Concrete Institute, Vol. 2., No. 6, December 1966, pp 18-39.

9. Prestressed Concrete Institute, PCI Design Handbook, 2nd Ed., Chicago, Ill., 1978, pp. 4-21 thru 4-25.

10. Teng, W. C., Foundation Design, Prentice-Hall, Inc., Englewood Cliffs, NJ, 1962, pp 184-189.

11. Bowles, J. E., Foundation Analysis and Design, 3rd Ed., McGraw-Hill, Inc., New York, NY, 1982, pp 320-325.

12. Department of the Navy, Naval Facilities Engineering Command, Design Manual - Soil Mechanics, Foundations, and Earth Structures, NAVFAC DM-7, March, 1971, p 7-11-10 and p. 7-11-18.

APPENDIX A – NOMENCLATURE.

P = AXIAL LOAD ON COLUMN.

e = ECCENTRICITY OF LOAD ON COLUMN.

K_s = MODULUS OF SUBGRADE REACTION.

q = PRESSURE EXERTED BY FOOTING ON THE SOIL.

Δ_F = SETTLEMENT OF SOIL DUE TO PRESSURE q.

θ_F = ROTATION OF FOOTING.

D = LENGTH OF FOOTING (PERPENDICULAR TO AXIS OF ROTATION).

B = WIDTH OF FOOTING.

I_F = MOMENT OF INERTIA OF FOOTING (PLAN VIEW) = $\frac{BD^3}{12}$.

λ = RATIO OF BASE PLATE I BETWEEN FLANGES TO OUTSIDE FLANGES.

β = PERCENTAGE OF MOMENT IN BASE PLATE OUTSIDE OF COLUMN FLANGES THAT CARRIES OVER TO BASE PLATE BETWEEN FLANGES.

F_B = FORCE IN TENSION ANCHOR BOLTS.

F_c = COMPRESSIVE FORCE BETWEEN CONCRETE AND BASE PLATE.

d = COLUMN DEPTH.

a = DISTANCE FROM FACE OF COLUMN TO CENTERLINE OF ANCHOR BOLTS.

$g = d + 2a$

f_{MAX} = MAXIMUM STRESS IN COLUMN FLANGE.

A_F = AREA OF COLUMN FLANGE.

P_C = FORCE IN COMPRESSIVE COLUMN FLANGE.

P_T = FORCE IN TENSILE COLUMN FLANGE.

A = AREA OF COLUMN CROSS SECTION.

S_x = SECTION MODULUS OF COLUMN ABOUT ITS X-AXIS.

$\alpha = A_F/A$

$B_x = A/S_x$

b = WIDTH OF BASE PLATE.

t_p = THICKNESS OF BASE PLATE.

E_p = MODULUS OF ELASTICITY OF BASE PLATE.

I_p = MOMENT OF INERTIA OF BASE PLATE IN PLANE OF BENDING.

θ_p = ROTATION DUE TO BASE PLATE BENDING.

Δ_B = CHANGE IN LENGTH OF TENSION ANCHOR BOLTS.

L_B = LENGTH OF TENSION ANCHOR BOLTS FROM BOLT HEAD TO TOP OF BASE PLATE.

A_B = SUM OF AREA OF ALL TENSION ANCHOR BOLTS.

E_B = MODULUS OF ELASTICITY OF ANCHOR BOLTS.

θ_B = THE ROTATION DUE TO EXTENSION OF ANCHOR BOLTS.

γ = BASE FLEXIBILITY.

K = BASE STIFFNESS.

Appendix B - HP41 Calculator Program.

Listing:

```
PRP "BASFLEX"

01♦LBL "BASFLEX"
02 CF 01
03 CLRG
04 "PRINTER ON"
05 AVIEW
06 PSE
07 "NORMAL MODE?"
08 AVIEW
09 PSE
10 ADV
11 "INPUT DATA"
12 AVIEW
13 ADV
14 "UNITS:"
15 AVIEW
16 " KIPS, INCHES"
17 AVIEW
18 ADV

19♦LBL 01
20 "FTG WIDTH?"
21 PROMPT
22 STO 01
23 "FTG LENGTH?"
24 PROMPT
25 STO 02
26 FS? 01
27 GTO 11

28♦LBL 02
29 "SUB MODULUS?"
30 PROMPT
31 STO 03
32 FS? 01
33 GTO 12

34♦LBL 03
35 "AB LENGTH?"
36 PROMPT
37 STO 04
38 "AB AREA?"
39 PROMPT
40 STO 05
41 "AB MOD ELAST?"
42 PROMPT
43 STO 06
44 FS? 01
45 GTO 13

46♦LBL 04
47 "COL DEPTH?"
48 PROMPT
49 STO 07
50 "COL BX?"
51 PROMPT
52 STO 12
53 "ALPHA?"
54 PROMPT
55 STO 21
56 FS? 01
57 GTO 14

58♦LBL 05
59 "AB OFFSET?"
60 PROMPT
61 STO 08
62 FS? 01
63 GTO 15

64♦LBL 06
65 "BP MOD ELAST?"
66 PROMPT
67 STO 09
68 "BP WIDTH?"
69 PROMPT
70 STO 10
71 "BP THICKNESS?"
72 PROMPT
73 STO 11
74 FS? 01
75 GTO 16

76♦LBL 07
77 "BETA?"
78 PROMPT
79 STO 22
80 "LAMBDA?"
81 PROMPT
82 STO 23
83 SF 01
```

```
 84+LBL 08
 85 "AXIAL LOAD?"
 86 PROMPT
 87 STO 14
 88 "ECCENTRICITY?"
 89 PROMPT
 90 STO 15
 91 RCL 07
 92 RCL 08
 93 2
 94 *
 95 +
 96 STO 17
 97 ADV
 98 ADV

 99+LBL A
100 "THETA FTG="
101 AVIEW
102 RCL 02
103 3
104 Y↑X
105 RCL 01
106 *
107 12
108 /
109 RCL 03
110 *
111 1/X
112 RCL 14
113 *
114 RCL 15
115 *
116 STO 16
117 VIEW 16
118 PSE
119 "FTGFLEX="
120 AVIEW
121 RCL 16
122 RCL 14
123 /
124 RCL 15
125 /
126 STO 27
127 VIEW 27
128 PSE
129 " "

130+LBL B
131 "THETA AB="
132 AVIEW
133 2
134 ENTER↑
135 RCL 15
136 *
137 RCL 17
138 -
139 RCL 17
140 X↑2
141 2
142 *
143 /
144 RCL 04
145 *
146 RCL 05
147 /
148 RCL 06
149 /
150 RCL 14
151 *
152 X<=0?
153 0
154 STO 18
155 VIEW 18
156 PSE
157 "ABFLEX="
158 AVIEW
159 RCL 18
160 RCL 14
161 /
162 RCL 15
163 /
164 STO 26
165 VIEW 26
166 PSE
167 " "

168+LBL C
169 "THETA BP="
170 AVIEW
171 RCL 21
172 RCL 12
173 *
174 RCL 21
175 .5
176 -
177 RCL 15
178 /
179 +
180 RCL 17
181 1/X
182 -
183 RCL 22
184 1/X
185 2
186 *
187 3
188 /
189 .5
190 -
191 *
192 STO 26
193 RCL 22
194 1
195 -
196 RCL 17
197 1/X
198 RCL 15
199 2
200 *
201 1/X
202 +
203 *
204 RCL 08
205 *
206 RCL 07
207 /
208 RCL 26
209 +
210 RCL 07
211 X↑2
212 *
213 RCL 08
214 *
215 RCL 22
216 *
217 RCL 23
218 /
219 RCL 17
220 /
221 RCL 08
222 ENTER↑
223 3
224 Y↑X
225 RCL 17
226 X↑2
227 /
228 2
229 *
230 3
231 /
232 +
233 RCL 14
234 *
235 RCL 15
236 *
```

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237 RCL 09
238 /
239 RCL 10
240 /
241 RCL 11
242 ENTER↑
243 3
244 Y↑X
245 /
246 12
247 *
248 STO 19
249 VIEW 19
250 PSE
251 "BPFLEX="
252 AVIEW
253 RCL 19
254 RCL 14
255 /
256 RCL 15
257 /
258 STO 28
259 VIEW 28
260 PSE
261 ADV
262 ADV
263 " "

264♦LBL D
265 "THETA TTL="
266 AVIEW
267 RCL 16
268 RCL 18
269 +
270 RCL 19
271 +
272 STO 20
273 VIEW 20
274 PSE
275 ADV
276 ADV
277 " "

278♦LBL E
279 "FLEX TTL="
280 AVIEW
281 RCL 20
282 RCL 14
283 /
284 RCL 15
285 /
286 STO 24
287 VIEW 24
288 PSE
289 ADV
290 ADV
291 " "

292♦LBL F
293 "STIFF TTL="
294 AVIEW
295 RCL 24
296 1/X
297 STO 25
298 VIEW 25
299 PSE
300 ADV
301 ADV
302 " "

303♦LBL G
304 "CHG BASE-Y/N?"
305 AVIEW
306 PSE
307 XEQ "Y/N"
308 X=Y?
309 GTO 10
310 "CHG LOADS-Y/N?"
311 AVIEW
312 PSE
313 XEQ "Y/N"
314 X=Y?
315 GTO 08
316 RTN

317♦LBL 10
318 "CHG FTG-Y/N?"
319 AVIEW
320 PSE
321 XEQ "Y/N"
322 X=Y?
323 GTO 01

324♦LBL 11
325 "CHG SUBMOD-Y/N?"
326 AVIEW
327 PSE
328 XEQ "Y/N"
329 X=Y?
330 GTO 02

331♦LBL 12
332 "CHG AB-Y/N?"
333 AVIEW
334 PSE
335 XEQ "Y/N"
336 X=Y?
337 GTO 03

338♦LBL 13
339 "CHG COL-Y/N?"
340 AVIEW
341 PSE
342 XEQ "Y/N"
343 X=Y?
344 GTO 04

345♦LBL 14
346 "CHG AB LOC-Y/N?"
347 AVIEW
348 PSE
349 XEQ "Y/N"
350 X=Y?
351 GTO 05

352♦LBL 15
353 "CHG BP-Y/N?"
354 AVIEW
355 PSE
356 XEQ "Y/N"
357 X=Y?
358 GTO 06

359♦LBL 16
360 "CHG COEF-Y/N?"
361 AVIEW
362 PSE
363 XEQ "Y/N"
364 X=Y?
365 GTO 07
366 GTO 08

367♦LBL "Y/N"
368 "Y"
369 ASTO Y
370 CLA
371 AON
372 PROMPT
373 ASTO X
374 AOFF
375 RTN
376 END
```

OUTPUT :

```
        XEQ "BASFLEX"
PRINTER ON
NORMAL MODE?

INPUT DATA

UNITS:
 KIPS, INCHES

FTG WIDTH?
     72.00000000    RUN
FTG LENGTH?
     72.00000000    RUN
SUB MODULUS?
      .200000000    RUN
AB LENGTH?
     24.00000000    RUN
AB AREA?
     4.000000000    RUN
AB MOD ELAST?
     29000.00000    RUN
COL DEPTH?
     14.00000000    RUN
COL BX?
      .184000000    RUN
ALPHA?
      .385000000    RUN
AB OFFSET?
     4.000000000    RUN
BP MOD ELAST?
     29000.00000    RUN
BP WIDTH?
     24.00000000    RUN
BP THICKNESS?
     1.500000000    RUN
BETA?
     1.000000000    RUN
LAMBDA?
     10.00000000    RUN
```

```
AXIAL LOAD?
     200.0000000    RUN
ECCENTRICITY?
     12.00000000    RUN

THETA FTG=
            0.005358366
FTGFLEX=
            0.000002233
THETA AB=
            0.000085494
ABFLEX=
            0.000000036
THETA BP=
            0.001335462
BPFLEX=
            0.000000556


THETA TTL=
            0.006779324


FLEX TTL=
            0.000002825


STIFF TTL=
            354017.6001


CHG BASE-Y/N?

N                   RUN
CHG LOADS-Y/N?

Y                   RUN
```

```
AXIAL LOAD?
     200.0000000    RUN
ECCENTRICITY?
     36.00000000    RUN

THETA FTG=
            0.016075103
FTGFLEX=
            0.000002233
THETA AB=
            0.002689655
ABFLEX=
            0.000000374
THETA BP=
            0.002088345
BPFLEX=
            0.000000290


THETA TTL=
            0.020853103


FLEX TTL=
            0.000002896


STIFF TTL=
            345272.3562


CHG BASE-Y/N?

Y                   RUN
CHG FTG-Y/N?

Y                   RUN
FTG WIDTH?
     72.00000000    RUN
FTG LENGTH?
     96.00000000    RUN
```

```
CHG SUBMOD-Y/N?

N                    RUN
CHG AB-Y/N?

N                    RUN
CHG COL-Y/N?

N                    RUN
CHG AB LOC-Y/N?

N                    RUN
CHG BP-Y/N?

N                    RUN
CHG COEF-Y/N?

N                    RUN
AXIAL LOAD?
      200.0000000    RUN
ECCENTRICITY?
      12.00000000    RUN


THETA FTG=
               0.002260561
FTGFLEX=
               0.000000942
THETA AB=
               0.000206897
ABFLEX=
               0.000000086
THETA BP=
               0.000734497
BPFLEX=
               0.000000306


THETA TTL=
               0.003201955


FLEX TTL=
               0.000001334


STIFF TTL=
               749542.0388


CHG BASE-Y/N?

N                    RUN
CHG LOADS-Y/N?
N                    RUN
```

APPENDIX C – TABLES FOR APPROXIMATING BASE FLEXIBILITY AND STIFFNESS.

TABLE C1 – FLEXIBILITY DUE TO FOOTING ROTATION

FOOTING FLEXIBILITY (10^{-6} kin^{-1})

FOOTING	K_s (lb/in^3)							
DIMENSION	100	150	200	250	300	350	400	450
2'x2'	361.690	241.127	180.845	144.676	120.563	103.340	90.422	80.376
3'x3'	71.445	47.630	35.722	28.578	23.815	20.413	17.861	15.877
4'x4'	22.606	15.070	11.303	9.042	7.535	6.459	5.651	5.023
5'x5'	9.259	6.173	4.630	3.704	3.086	2.646	2.315	2.058
6'x6'	4.465	2.977	2.233	1.786	1.488	1.276	1.116	0.992
7'x7'	2.410	1.607	1.205	0.964	0.803	0.689	0.603	0.536
8'x8'	1.413	0.942	0.706	0.565	0.471	0.404	0.353	0.314
9'x9'	0.882	0.588	0.441	0.353	0.294	0.252	0.221	0.196
10'x10'	0.579	0.386	0.289	0.231	0.193	0.165	0.145	0.129
11'x11'	0.395	0.264	0.198	0.158	0.132	0.113	0.099	0.088
12'x12'	0.279	0.186	0.140	0.112	0.093	0.080	0.070	0.062
13'x13'	0.203	0.135	0.101	0.081	0.068	0.058	0.051	0.045
14'x14'	0.151	0.100	0.075	0.060	0.050	0.043	0.038	0.033

Table C2 – Flexibility Due to Anchor Bolt Elongation

Anchor Bolt Flexibility (10^{-6} kin^{-1})

Anchor Bolt Length	Column Size	Eccentricity	$A_B = 2 in^2$	$A_B = 4 in^2$	$A_B = 6 in^2$	$A_B = 8 in^2$	$A_B = 10 in^2$
$L_B = 18''$	W14	e = 12"	0.053	0.027	0.018	0.013	0.011
		e = 24"	0.347	0.174	0.116	0.087	0.069
		e = 36"	0.445	0.223	0.148	0.111	0.089
		e = 48"	0.494	0.247	0.165	0.124	0.099
	W12	e = 12"	0.129	0.065	0.043	0.032	0.026
		e = 24"	0.453	0.226	0.151	0.113	0.091
		e = 36"	0.560	0.280	0.187	0.140	0.112
		e = 48"	0.614	0.307	0.205	0.154	0.123
	W10	e = 12"	0.239	0.120	0.080	0.060	0.048
		e = 24"	0.599	0.299	0.200	0.150	0.120
		e = 36"	0.718	0.359	0.239	0.180	0.144
		e = 48"	0.778	0.389	0.259	0.195	0.156
$L_B = 24''$	W14	e = 12"	0.071	0.036	0.024	0.018	0.014
		e = 24"	0.463	0.232	0.154	0.116	0.093
		e = 36"	0.594	0.297	0.198	0.148	0.119
		e = 48"	0.659	0.330	0.220	0.165	0.132
	W12	e = 12"	0.172	0.086	0.057	0.043	0.034
		e = 24"	0.603	0.302	0.201	0.151	0.121
		e = 36"	0.747	0.374	0.249	0.187	0.149
		e = 48"	0.819	0.409	0.273	0.205	0.164
	W10	e = 12"	0.319	0.160	0.106	0.080	0.064
		e = 24"	0.798	0.399	0.266	0.200	0.160
		e = 36"	0.958	0.479	0.319	0.239	0.192
		e = 48"	1.038	0.519	0.346	0.259	0.208
$L_B = 30''$	W14	e = 12"	0.089	0.045	0.030	0.022	0.018
		e = 24"	0.579	0.289	0.193	0.145	0.116
		e = 36"	0.742	0.371	0.247	0.186	0.148
		e = 48"	0.824	0.412	0.275	0.206	0.165
	W12	e = 12"	0.216	0.108	0.072	0.054	0.043
		e = 24"	0.754	0.377	0.251	0.189	0.151
		e = 36"	0.934	0.467	0.311	0.233	0.187
		e = 48"	1.024	0.512	0.341	0.256	0.205
	W10	e = 12"	0.399	0.200	0.133	0.100	0.080
		e = 24"	0.998	0.499	0.333	0.249	0.200
		e = 36"	1.197	0.599	0.399	0.299	0.239
		e = 48"	1.297	0.649	0.432	0.324	0.259

Note: $E_B = 29000$ ksi

TABLE C3 - FLEXIBILITY DUE TO BASE PLATE BENDING.

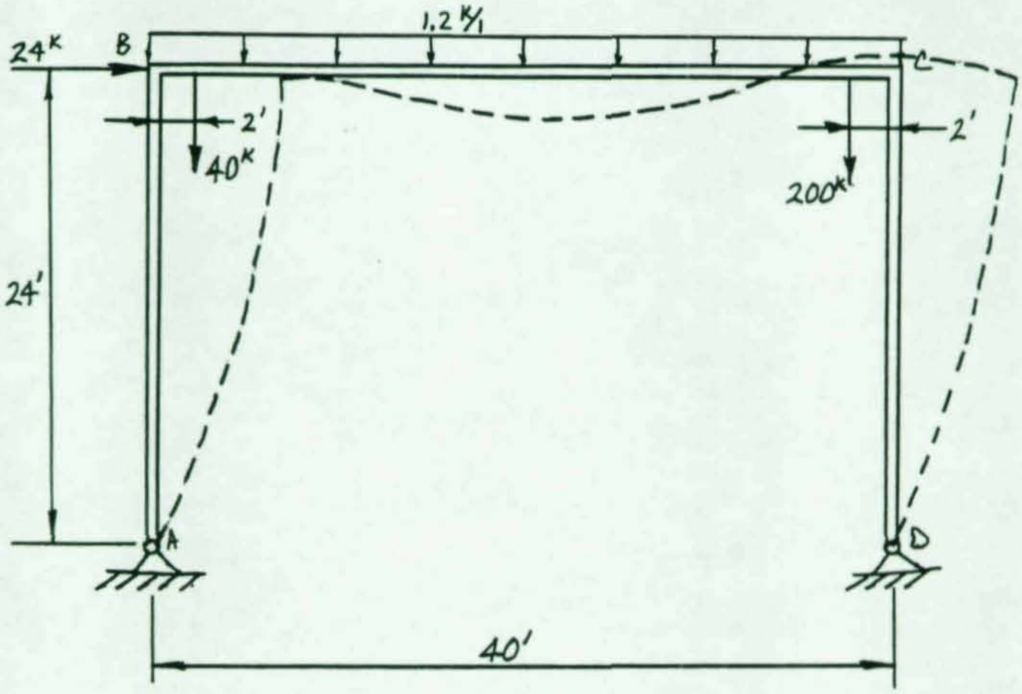
COLUMN SIZE	ECCENTRICITY	BASE PLATE FLEXIBILITY (10^{-6} kin^{-1}) $t_p = 1.0''$	$t_p = 1.25''$	$t_p = 1.50''$	$t_p = 1.75''$	$t_p = 2.0''$	$t_p = 2.5''$	$t_p = 3.0''$
W14	e=12"	1.859	0.952	0.551	0.347	0.232	0.119	0.069
	e=24"	1.957	1.002	0.580	0.365	0.245	0.125	0.072
	e=36"	1.990	1.019	0.590	0.371	0.249	0.127	0.074
	e=48"	2.007	1.027	0.595	0.374	0.251	0.128	0.074
W12	e=12"	2.223	1.138	0.659	0.415	0.278	0.142	0.082
	e=24"	2.302	1.179	0.682	0.430	0.288	0.147	0.085
	e=36"	2.329	1.192	0.690	0.435	0.291	0.149	0.086
	e=48"	2.342	1.199	0.694	0.437	0.293	0.150	0.087
W10	e=12"	2.742	1.404	0.812	0.512	0.343	0.175	0.102
	e=24"	2.803	1.435	0.830	0.523	0.350	0.179	0.104
	e=36"	2.823	1.445	0.836	0.527	0.353	0.181	0.105
	e=48"	2.833	1.451	0.840	0.529	0.354	0.181	0.105

NOTE: THIS TABLE ASSUMES THE FOLLOWING VALUES;

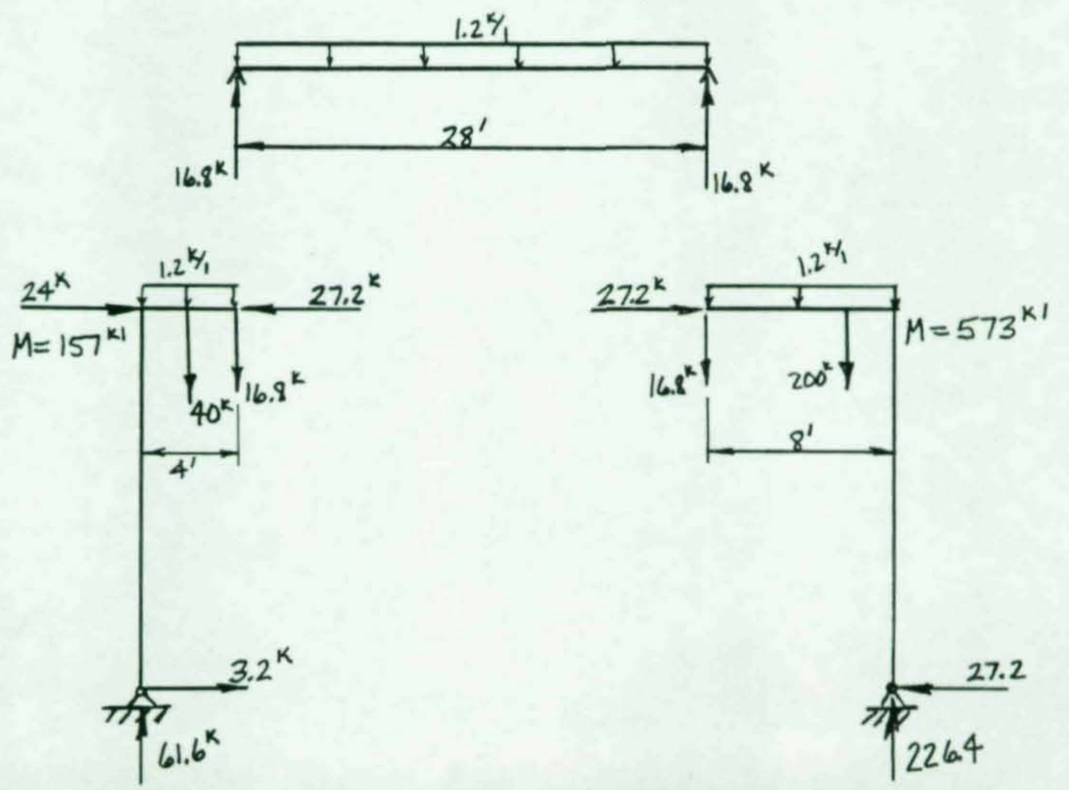
$a = 4''$
$b = 24''$
$E_p = 29000$ ksi
$\beta = 1.0$
$\lambda = 5.0$

APPENDIX D - EXAMPLE

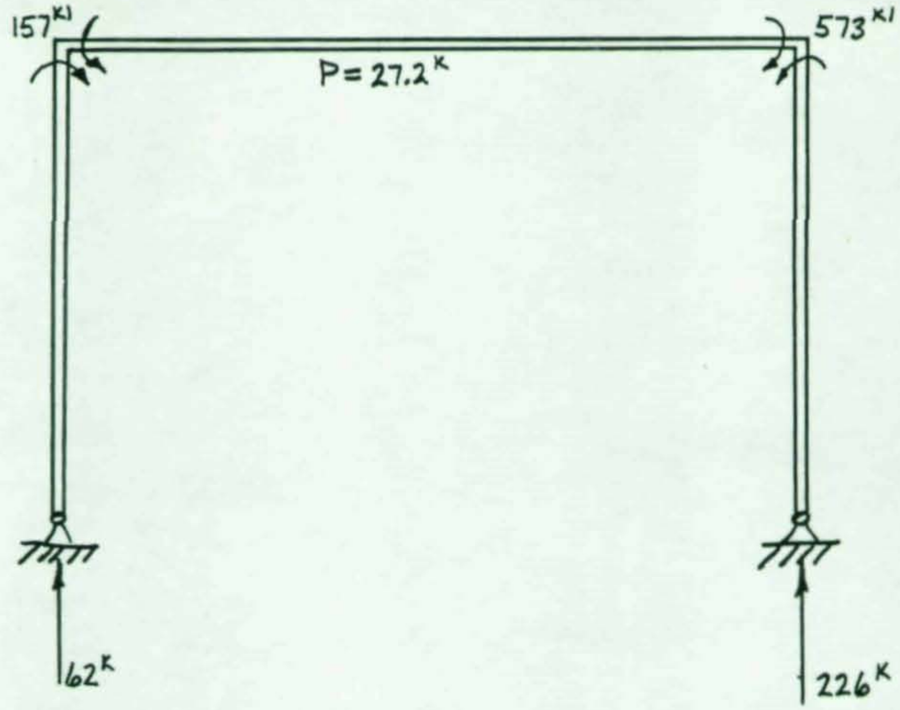
THE FRAME SHOWN REPRESENTS A BAY OF AN INDUSTRIAL BUILDING WITH A CRANE.



PRELIMINARY ANALYSIS - ASSUME POINTS OF CONTRAFLEXURE OCCUR AT POINTS 4ft FROM POINT B AND 8ft FROM C.



RESULTS IN THE FOLLOWING FORCES AND MOMENTS.



CHOOSE TRIAL MEMBER SIZES:

<u>COLUMNS</u> $P = 226^K$ $M = 573^{K'}$ ASSUME W14 $\therefore m = 1.7$

$P_{eff} = P_o + m M_x = 226 + 1.7(573)$

$P_{eff} = 1200^K$

ASSUME $K = 2.0$ $\therefore Kl = 48'$

FROM COLUMN TABLES TRY W14X455 ($I = 7190 \text{ in}^4$

<u>BEAMS</u> ASSUME $F_B = 22$ ksi (Adequate bracing)

$S_{REQ'D} = \frac{573 \times 12}{22} = 313 \text{ in}^3$

TRY W30×116 ($I = 4930 \text{ in}^4$)

COLUMN BASES - ARBITRARILY CHOOSE:

$D = 8'$	$L_B = 24''$	$d = 14 in$	$a = 4''$	$\beta = 1$
$B = 8'$	$A_B = 4 in^2$	$B_x = 0.185$	$E_p = 29000 ksi$	$\lambda = 5$
$K_s = 200 \#/in^3$	$E_B = 29000 ksi$	$\alpha = 0.385$	$b = 24''$	$e = 24''$
			$t_p = 1.5''$	

FROM TABLE C1 - $\gamma_F = 0.707 \times 10^{-6} (kin)^{-1}$

FROM TABLE C2 - $\gamma_B = 0.232 \times 10^{-6} (kin)^{-1}$

FROM TABLE C3 - $\gamma_P = 0.580 \times 10^{-6} (kin)^{-1}$

$$\gamma_{TOTAL} = 1.519 \times 10^{-6} (kin)^{-1}$$

$$\therefore K_{BASE} = 658{,}328 \text{ kin}$$

DETERMINE DISTRIBUTION FACTOR FOR COLUMN BASE:

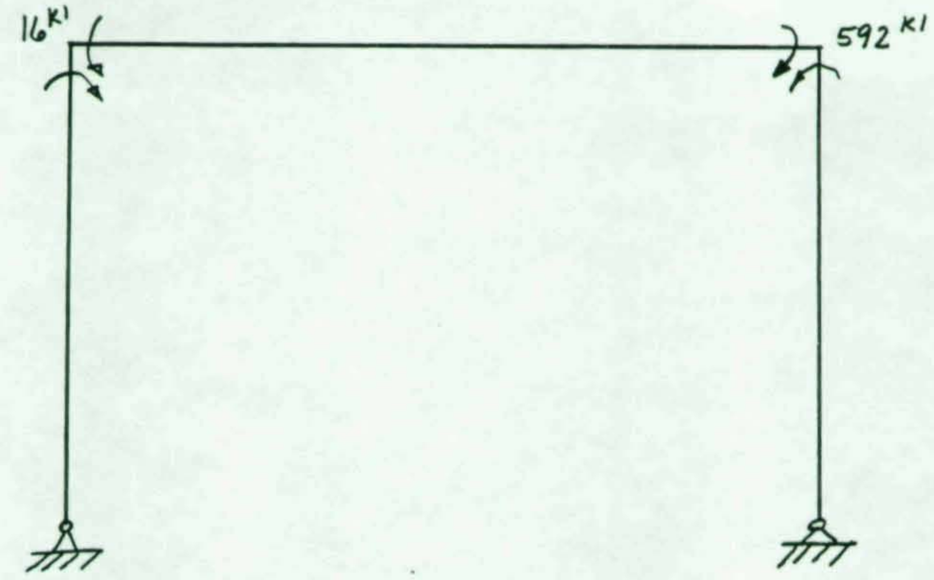
$$K_{COL} = \frac{4(29000)(7190)}{24(12)} = 2{,}895{,}972 \text{ kin}$$

$$\therefore D.F. = \frac{K_{COL}}{K_{BASE} + K_{COL}}$$

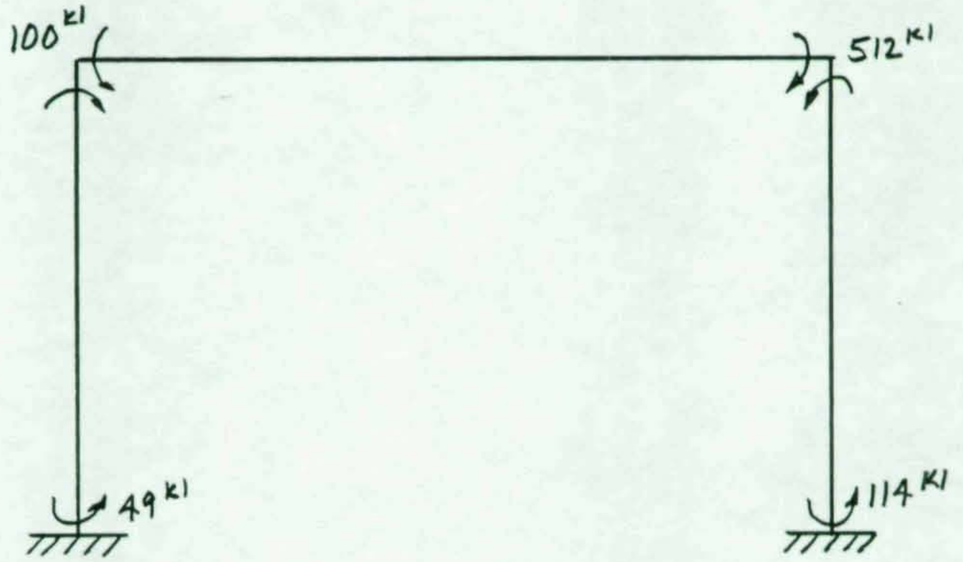
$$D.F. = \frac{2{,}895{,}972 \text{ kin}}{658{,}328 \text{ kin} + 2{,}895{,}972 \text{ kin}}$$

$$D.F. = 0.815$$

ANALYSIS BY MOMENT DISTRIBUTION GIVES THE RESULTS SHOWN ON THE FOLLOWING SHEET FOR THE PINNED BASE AND THE PARTIALLY RESTRAINED BASE.



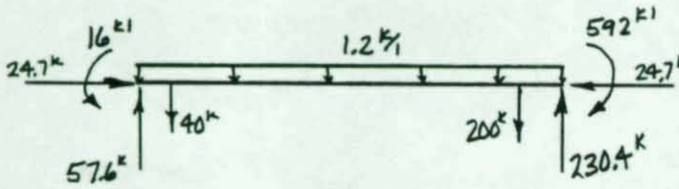
PINNED BASES



PARTIALLY FIXED BASES

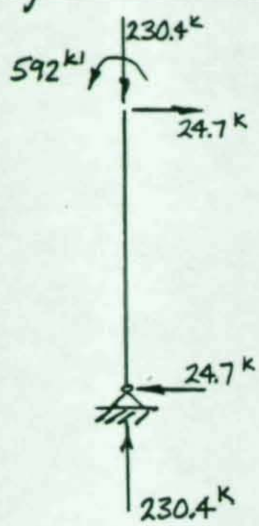
ITERATE TO FINAL DESIGN:

PINNED BASE



$$S_{Req'd} = \frac{592 \times 12}{22} = 323 \text{ in}^3$$

Try W30X116 for beam



$P_{eff} = 230.4 + 1.7(592)$

$P_{eff} = 1237^k$

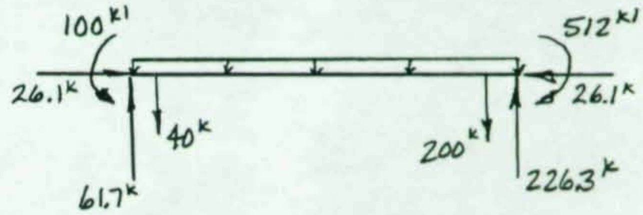
for $K = 2.0$

Try W14X500 for column

REVISE K

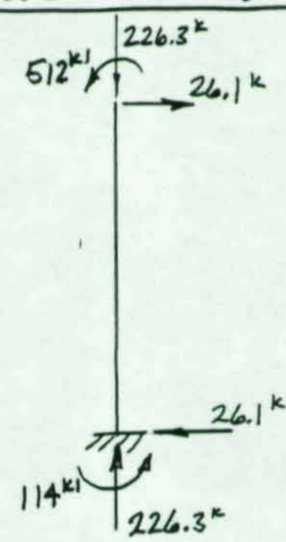
$G_B = 10$

PARTIALLY RESTRAINED BASE



$$S_{Req'd} = \frac{512 \times 12}{22} = 279 \text{ in}^3$$

Try W30X108 for beam



$P_{eff} = 226.3 + 1.7(512)$

$P_{eff} = 1097^k$

for $K = 2.0$

Try W14X455 for column

REVISE K

$$G_B = \frac{2{,}895{,}972}{658{,}328} = 4.40$$

$G_A = \frac{7190}{4930} = 1.46$

FROM EFFECTIVE LENGTH NOMOGRAGH W/ SIDESWAY

$K = 2.0$

$\therefore$ Try W14x500 for column

$G_A = \frac{7190}{4930} = 1.46$

FROM EFFECTIVE LENGTH NOMOGRAPH W/ SIDESWAY

$K = 1.75$

$\therefore Kl = 42'$

$\therefore$ Try W14x370 for column

$\therefore$ BY TAKING ACCOUNT OF PARTIAL RESTRAINT AT THE COLUMN BASES, A SAVINGS HAS ALREADY BEEN REALIZED OF:

8 lbs/ft OF BEAM

&

130 lbs/ft OF COLUMN

THIS IS A SAVINGS OF 6560 lbs per BENT.

NOTE: BEFORE THIS DESIGN CAN BE ACCEPTED THE ADEQUACY OF THE MEMBERS SHOULD VERIFIED USING THE AISC INTERACTION EQUATIONS.

01168

FROM: Nestor

→ TO: ~~GH, RD, RS, CP, DE~~ DATE: 3-25-85

report library (last)

report — Cannon

key words —

1. base plates, columns
2. connections, flexible

Another Fellowship report — shows how to estimate & use column base fixity.

1106

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