# EXPERIMENTAL AND ANALYTICAL FORCE DEFORMATION CURVES FOR BOLTED DOUBLE ANGLE CONNECTIONS

Prepared for

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by

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1984

Nestor
Did Ralph's work result in significant changes in the parameters derived by Reidar?
see Sect. A.7, pgs. 16-18

key words: 1-Connections Flexible

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A Proud Beginning

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18 December 1984

→ GH, RAOD, EB, CP, JE, library (last)

Mr. Nestor Iwankiw
Assistant Director of Engineering
American Institute of Steel Construction, Inc.
400 N. Michigan Avenue
Chicago, ILLINOIS 60611

F.Y.I. - The feuding between Richard & Bjorhovde appears rather serious! Nestor

Dear Nestor:

Enclosed are three (3) copies of my report entitled, "Experimental and Analytical Force Deformation Curves for Bolted Double Angle Connections." This report is to supercede the earlier report by Reidar Bjorhovde entitled, "Strength and Behavior of Connections Elements." In your review of the Bjorhovde report (letter to Bjorhovde dated August 17, 1983) you recommended that Bjorhovde make a shear test as discussed at a Project Task Force meeting in the Spring of 1983 because his shear test was not accurately designed. You further recommended a shear test configuration designed by Mr. William Milek.

Because the results of this research strongly impacts on my gusset plate design studies, I have reviewed the Bjorhovde test specimens and reported results. The following items were either inadequately or incorrectly reported:

1) Certain of the angle tension tests with 1/4" and 5/16" plates were not representative. The specimens failed by tear-out of the top of the plate in the loading fixture (see Figure 4.1a).

2) The compression tests were incorrectly designed with a single bolt. Moreover, the WT section in several of these tests buckled just above the angles. Both of these resulted in incorrect deformation readings.

3) As indicated above, the shear test was incorrectly designed and run.

4) The stiffness and strength parameters K, $K_p$, n, and $R_o$ of the Richard Equation were not synthesized and reported in a useful way.

In the report submitted herein, the following correction and additions to the research have been made.

1) Certain of the 1/4" and 5/16" plate/angle test results have been deleted because of inaccurate deformation readings.

2) Compression test specimens were designed, fabricated, and tested.

3) The shear test specimens, as designed by the AISC Engineering Staff, were fabricated and tested.

4) The parameters for the Richard curve for all the specimens have been synthesized and documented in a way useful for analysis and design.

It is my recommendation that all copies of the Bjorhovde report be returned to him for disposal. I estimate that the Bjorhovde report cost AISC approximately $15,000 in research funding.

wow!

Sincerely yours,

Ralph M. Richard, Ph.D., P.E.

RMR:sst

Enclosures

xc: P. Mather, EES

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## 1. INTRODUCTION

The accurate prediction of connection deformation under loading is one of the most important aspects involved in the analysis of structural connections. The development of design criteria for standard connections is the final objective of the detailed series of finite element analyses of gusset plate framing connections currently under way at the University of Arizona. Force distributions in connections of various configurations under ultimate loading conditions are being compiled by means of the nonlinear finite element program, INELAS [1]. Using a method unique to INELAS, the force-deformation characteristics of the connection elements, obtained from laboratory tests and defined by the Richard equation, are used in the finite element analysis.

One of the most popular framing connections for structural members is the double angle connection. Due to the anisotropic behavior of this connection, the finite element model must simulate the action of tensile, compressive, and shear forces as shown in Figure 1. Test data from the University of Illinois [2] and the work of Crawford and Kulak [3] have provided strength and stiffness properties needed to formulate the Richard curve parameters for a limited number of double angle framing connections. In order to define the force-deformation relationships for a wider range of angle and connecting plate thicknesses and gage lengths than provided by these studies, additional testing of double angles has been conducted here at the University of Arizona [4]. This report recompiles the results of these tests along with the analytical Richard equation parameters.

Additionally, the results of correctly modeled compression and shear tests are reported.

## 2. BACKGROUND AND SCOPE

In all of the tests performed (tension, compression, and shear), there are four variables in connection geometry of primary importance. These are: gage length (g), angle thickness ($t_a$), connecting plate thickness ($t_p$), and the bolt diameter ($d_b$). The results of nineteen tension tests, two compression tests, and one shear test are presented herein.

The large number of tension tests were performed for the following reasons. First, the tension specimen results were expected to be highly dependent upon the gage length, so three gage lengths were investigated. Second, it was shown in the University of Illinois study [2] that similar angle tests, with a small gage length, will respond almost identically in tension and compression at low magnitudes of load. That is, a partial knowledge of the compressive response can be obtained from the tension tests. Finally, the accurate representation of double angles subject to tensile loading is a more complex, and a more critical aspect of connection analysis.

The compression and shear tests reported herein were made to replace those in Reference 4. The previous compression tests used only a single bolt and gave inconsistent results. Moreover, the authors of this report observed that the excessive height of the WT section, used as the connecting plate, combined with the single bolt configuration, gave results which included the effects of a) plate buckling and b) plate rotation. In order to eliminate these effects, the new compression tests consisted of a two bolt connection with a WT section connecting plate that extended above the vertical angle legs by less than 1/2 inch. The design of the shear test specimen was discussed with the engineering staff of the American Institute of Steel Construction. That recommended test configuration is shown in Figures 4 and 5.

The objective of this study was to define the force-deformation characteristics for a wide range of double angle connection geometries. To accomplish this it is crucial to determine what factors of the geometry dominate the response of a given connection. Once the relative effect each component of a connection has on the inelastic response was determined, generalized force-deformation curves or semi-empirical relationships were developed which define the inelastic response as a function of the connection geometry. The ability to predict analytically the force distribution in complex connections is dependent on providing sufficiently accurate force-deformation relationships for the finite elements of the mathematical model.

## 3. TESTING DETAILS AND PROCEDURES

### 3.1 Test Configurations

The configurations of each test specimen are shown in Figures 1 through 6. The dimensions of each component and the diameter of the bolts used for all of the specimens are given in Table 1. The combination of used for all of the specimens are given in Table 1. The combination of angles and plate thicknesses, gage lengths, and bolt diameters were based upon those generally encountered in practice.

The tension specimens consisted of combinations of 1/4, 3/8, and 1/2 inch angle and connecting plate thicknesses, with gage lengths of 1-3/4, 2-1/4, and 3 inches. The vertical legs of all the tension specimens were 5 inches. The compression specimens consisted of 3/8 inch angle, with one specimen using a 1/2 inch connecting plate with 7/8 inch diameter A325X bolts and the other using a 3/8 inch plate with 3/4 inch diameter A325X bolts. The shear test specimen consisted of 3/8 inch angles, a 1/2 inch connecting plate and 7/8 inch diameter A325X bolts.

The connecting plates used in the tension tests were flat plates with two bolt holes through the top to provide a means of applying the tensile load. The compression and shear test connecting plates were WT sections to accommodate compression loading in the testing machine.

The tension and compression tests both used the common three-inch pitch, which was also used in the finite element connection models. The shear test specimen had a two-inch pitch, with a 1-1/2 inch edge distance in order to insure adequate ductility under ultimate loading. Additional details of the tension specimens, as well as complete details of the compression and shear specimens are shown in Figures 1 through 6.

### 3.2 Specimen Assembly

The specimens were assembled in the laboratory and the bolts were pretensioned in the following manner. All bolts were first hand-tightened and the components of the specimen were aligned. Next, the bolts were tightened with an eighteen-inch spud wrench. Bolts in opposing locations were tightened in a two-cycle rotation to insure a relatively uniform clamping of the specimen elements. The tension specimens were bolted to a rigid base plate, 1-1/2 inches thick, which was securely fastened to the base of the testing machine. The connecting plate between the two angles was then fastened to the top loading head. The compression and shear specimens were placed directly on the base of the testing machine and were loaded by a spherical bearing head. The tension specimens were preloaded to five kips and the compression and shear specimens to ten kips. This was done to insure that all bolts were in full bearing contact with the elements of the specimen. While under the action of the respective preloads, all bolts were then tightened in a two-cycle rotation by the one-half-turn-of-the-nut method. A careful sequence of tightening was followed to insure full contact between the plates due to bolt pretension. The preload was then removed and the test specimen was fitted with the appropriate gages.

### 3.3 Testing Procedure

The dial gages used provided accuracy of up to 0.0001 inches. The tension specimens were fitted with two gages as shown in Figure 2, and the readings were averaged. It is noted that the placement of the gages is such that the deformation in the connecting plate, due to the bearing of the top two bolts, did not affect the readings. In an actual connection this connecting plate would be a continuous beam web or gusset plate. The recorded the deformation of the specimen by measuring the change in distance between the top spherical loading head and the support base of the testing machine.

For all of the configurations, three identical specimens were assembled and tested. Eight of the tension test specimens and the shear test specimen provided useful data for only two of the three tested. This was due to loading head and dial gage difficulties in the tension test specimen and a cracked weldment in one of the shear test specimens.

All of the tests were conducted at the same slow rate of loading (about 0.03 inches per minute) on a Tinius Olsen 200 kip universal screw drive testing machine. Each test was performed by two experienced research assistants to insure accurate results.

## 4. FORCE-DEFORMATION RELATIONSHIPS

### 4.1 The Richard Curve

A typical Richard Curve is shown in Figure 7. The four parameters defining the shape of the curve are K, $K_p$, $R_o$, and N. The parameter K is the initial slope of the curve which represents the initial or elastic stiffness of the specimen. The value $K_p$ is the final slope of the curve and represents the final or plastic stiffness. An asymptote to the curve at a slope equal to $K_p$ will intercept the vertical axis at the value $R_o$, called the reference load. Finally, the value N, called the Richard parameter, defines the sharpness of the transition between the two portions of the curve defined by K and $K_p$. A more detailed presentation of the Richard equation is included in Appendix A.

## 4.2 Connection Deformations

One of the questions addressed here is how much of the force-deformation curve must be provided to adequately access the connection behavior. Based on displacement data from the finite element analyses of gusset plate framing connections [5], which was in turn supported by actual full scale testing at the University of Alberta, the authors concluded that it was sufficient to represent, analytically, deformations up to approximately three- to five-tenths of an inch. Although the ultimate strength of the specimen may not have been reached, combined tensile, compressive and shear deformations of this magnitude are adequate to assess the ultimate strength of the entire connection. It is noted that the point of termination of the curve has no effect on the finite element analysis, since the solution is dependent only on that portion of the curve corresponding to the computed displacements at the ultimate load of the connection. However, for the purposes of this study, a correlation between connection geometry and the Richard equation parameters was made by applying a consistent maximum range of deformation.

## 4.3 Curve Fitting Technique

The Richard curve parameters presented in this report were obtained by a least squares curve fitting routine. The program XYPLOT (Williams, 1982) contains the subroutine RCFIT (Gillett and Hormby, 1978), which gives the least squares Richard curve fit and provides the corresponding Richard equation parameters for a given set of data points. The plotting of the curves was done on a CalComp 1051 plotter from commands generated by XYPLOT. The coordinates of the data points to be fitted along with the elastic stiffness. K, are input to program XYPLOT.

## 4.4 Determination of the Elastic Stiffness

In order to achieve a successful curve fit, the elastic stiffness had to be calculated beforehand as a function of the connection geometry. As indicated previously, it was anticipated that the tension and compression specimens should have basically the same elastic stiffness given a small gage length. Also, since there was a large number of tension tests performed, any analytical or semi-empirical equation for the elastic stiffness would have significant experimental confirmation. Accordingly, the initial effort was made to develop a definition of the elastic stiffness for the tension test results.

In the tension specimens there are four significant variables contributing to the elastic stiffness: the angle thickness ($t_a$), the plate thickness ($t_p$), the gage length (g), and the bolt diameter ($d_b$). Attempting to formulate a strictly analytical result for the elastic stiffness is beyond the scope of this study. However, in general, the total elastic stiffness may be given as:

$$\frac{1}{K_T} = \frac{1}{K_a} + \frac{1}{K_{PL}} + \frac{1}{K_b} + \frac{1}{K_g} \quad (1)$$

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where:

$K_a$ = The elastic stiffness of the angles

$K_{PL}$ = The elastic stiffness of the plate

$K_b$ = The elastic stiffness of the bolts

$K_g$ = The elastic stiffness contribution of the outstanding angle legs

The elastic stiffness due to the flexing of the outstanding legs of the angles can be approximated by modeling the horizontal leg of the angle, from the heel to the center line of the bolt hole, as a beam of length L, fixed at both ends, as shown in Figure 8. This yields the following result for the two angles:

$$K_g = 2 \left[\frac{12\ EI}{L^3}\right] \text{ kips/in} \qquad (2)$$

The gage length is L, and a three-inch pitch is assumed. If the elastic modulus E is taken as 30,000 ksi, Eqn. 2 becomes the elastic stiffness, $K_T$, of the tension specimen,

$$K_T = 18 * 10^4 \left[\frac{t_1}{g}\right]^3 \text{ kips/in} \qquad (3)$$

where:

$t_1$ = The thickness of the angle leg in inches

g = The gage length in inches

since $K_{PL}$, and $K_b$ are at least an order-of-magnitude larger than $K_g$ as discussed below. The values of $K_T$ for each specimen, rounded to the nearest 50 kips/inch, were input as the initial stiffnesses for the tension test force-deformation curves. As shown in the curves of Figures 10 through 28, the experimental data supports this result extremely well.

Although Eqn. 3 provides an accurate value of the elastic stiffness, a further argument is presented to support its use. First, the elastic stiffness of a single plate connection ($K_{PL}$) is easily an order-of-magnitude greater than that provided by Eqn. 4 (Richard, Gillett, Kriegh, and Lewis, 1980). Secondly, the elastic stiffness of the A325 bolts in shear or tension ($K_b$) is also much greater than the values of $K_T$ as obtained from Eqn. 3 (Crawford and Kulak, 1971) and (Fisher and Struik, 1974) [3,8]. Finally, it is noted that the additional stiffness contribution of the vertical leg of the angles ($K_a$) is considered to be incorporated into $K_{PL}$. It is, therefore, concluded that due to the nature of Eqn. 1, the two stiffnesses, $K_{PL}$ and $K_b$ would lower the value of $K_T$ by an insignificant amount and may be ignored.

The results obtained above, combined with the observation that the tensile and compressive elastic stiffnesses would be essentially equal, led to the following conclusion: Regardless of the actual gage length, the elastic stiffness of the compression curves can be defined from Eqn. 3 by using a gage length of 1-3/4 inches. The value of $K_C$ was then doubled to account for the six-inch-wide angle with two bolts that was used in the compression tests. This yields the following equation for the elastic stiffness of the compression specimens:

$$K_C = 36 * 10^4 \left(\frac{t_1}{g}\right)^3 \text{ kips/in} \quad (4)$$

Equation 4 was used to calculate the value of $K_C$ for the compression test force-deformation curves shown in Figures 40 and 41. Since the angle thickness ($t_1$) in both of the compression test specimens was 3/8 inch, the value of $K_C$ = 3550 kips/inch, is obtained from Eqn. 4 and used in both curves. It is apparent that the data from the compression tests supports this result.

The elastic stiffness of the shear specimen was determined in a similar manner. That is, the connection was expected to have a similar initial response, regardless of the manner of loading. Accordingly, the value of $K_C$ from Eqn. 4 was multiplied by a factor of 2, to account for the use of four angles in the shear specimens versus two in the compression specimens. This value was then reduced by one-sixth to account for the use of five-inch-wide angles in the shear test compared to six-inch-wide angle in the compression tests. This yielded a value of $K_S$ = 5900 kips/inch for the elastic stiffness of the shear test specimen. This value of $K_S$ was used as the elastic stiffness of the shear test force-deformation curve shown in Fig. 44. It is noted that this value is somewhat higher than the test results indicate. The reason for this was apparent upon observation of the specimen behavior during loading. It was observed that the legs of the angles perpendicular to the connecting plate had undergone a significant amount of rotation in the early stages of loading until a somewhat shifted seating of the bolts was achieved. This rotation led to an increased rate of deformation which resulted in the apparent decrease in the elastic stiffness. Since this rotation would not exist in an actual connection it is asserted that this reduction in initial stiffness would be negligible in a typical connection. Therefore, an elastic stiffness of K = 5900 kips/inch is recommended as the appropriate value for this double angle specimen loaded in shear.

### 4.5 Tension Test Results and Conclusions

Two different modes of deformation occurred in the tension test specimens. The first occurred as a result of the gage length, where the heels of the angles "dished" up as they separated from the base plate, a distortion indicated by the dashed lines in Fig. 8 and seen in the actual test specimens in Fig. 9. The second occurred as a result of bearing stresses around the bolt holes. In the majority of tests, a combination of these two effects contributed to essentially all deformation of the specimen. The tested tension specimens are shown in Fig. 9.

The elastic stiffness for each tension test was calculated from Eqn. 3 and the data for deformations of 0.3 to 0.5 inches was used to generate the curves presented in Figures 10 through 28. A summary of the Richard curve parameters for each of the tension specimen geometries is presented in Table 2.

Certain reasoning used in obtaining the curve parameters follows. For example, the thicker the plate or angle, or the shorter the gage length, the stiffer and stronger the connection should become (i.e., K, $K_p$, and $R_o$ should increase as $t_a$ and $t_p$ increase, or as g decreases). Also, it was reasoned that the Richard parameter, N, should follow some consistent pattern. When these concepts were applied to the data for the tension tests, the Richard equation parameters converged on those values given in Table 2. In effect, the data from nearly fifty individual tension tests provided mutually supporting and consistent results.

A review of the parameters of the tension test curves indicates that the magnitudes of $R_o$, Kp, and N are virtually independent of the connecting plate thickness. It is noted that the tests of 1/2 and 3/8 inch angles with the two smaller gage lengths did yield slightly different results for different plate thickness, and in fact, all of the curves have different values of N. However, these variations in magnitude, when used for analyses in program INELAS, resulted in literally no difference in the force distributions for gusset plate framing connections. It is thereby concluded that the force-deformation curves for double angle framing connections loaded in tension are essentially independent of the connecting plate thickness. The curves obtained by applying this conclusion are presented in Figures 29 through 36. The values of the Richard curve parameters in these figures were obtained by averaging the values in Table 2. The parameters for the eight combinations of gage length and angle thickness, independent of the connecting plate thickness, are listed in Table 3.

In order to apply this result to a still wider range of connection geometries, curves representing the two Richard equation parameters, $K_p$ and $R_o$ were plotted as a function of the gage length for each of the tested angle thicknesses. These curves are shown in Figures 37 and 38. The solid lines represent the experimentally obtained results, while the dotted lines are interpolated (or extrapolated) values for other typical angle sizes.

It is noted that the $R_o$ curves given in Figure 38 show a virtually linear relationship between $R_o$ and g. Accordingly, the following equations are recommended for $R_o$ in kips,

$$R_{o4} = 16.0 - 4.0g \quad (5)$$

$$R_{o5} = 32.0 = 8.4g \quad (6)$$

$$R_{o6} = 52.2 - 14.2g \quad (7)$$

$$R_{o7} = 63.0 - 16.0g \quad (8)$$

$$R_{o8} = 72.1 - 17.5g \quad (9)$$

$$R_{o9} = 81.4 - 19.2g \quad (10)$$

$$R_{o10} = 90.6 - 20.8g \quad (11)$$

The subscripts of $R_o$ indicate the number of sixteenths of an inch of angle thickness used in the double angle connection. These parameter values apply to a three-inch element of the double angle with bolts for the standard three-inch pitch. Note that these values of $R_o$, as well as the values of $K_p$ obtained from Figures 37 and 38, are based upon final deformations of about 1/2 inch. The values obtained provide the tensile response of a typical double angle connection which, when combined with appropriate compressive and shear force-deformation characteristics, define the inelastic response of a double angle connection in a gusset plate or beam to column framing connection.

### 4.6 Compression Test Results and Conclusions

The deformations of the compression specimens were primarily a result of bearing deformations at the bolt holes. It is noted that only in the compression specimen with the 7/8 inch A325X bolts did the bolts show any visible signs of shear-related distress. However, this minimal bolt shear deformation was apparent at a load of approximately 180 kips. Since the magnitude of loading for a deformation of about 0.3 inches was only 160 kips, the effects of bolt deformation are generally negligible. The tested compression specimens are shown in Figure 39.

The elastic stiffness obtained from Eqn. 4 was used in program XYPLOT along with the experimental data to yield the Richard curves of Figures 41 and 42. The Richard equation parameters from the two curves are given in Table 4. These compression test results lead to the following relationship between the Richard equation parameters. First, the elastic stiffness, K, and the Richard parameter, N, are virtually unaffected by the increase in the connecting plate thickness. Second, the values of the final stiffness, $K_p$, and the reference load $R_o$ increase in direct proportion to the increase in connecting plate thickness. That is, when the connecting plate thickness was increased by 25%, the values of $K_p$ and $R_o$ also increased by 25%. It is noted that in both of the compression specimens the connecting plates represented the controlling thickness for bearing failure. Therefore, if the preceding observation is used to predict the inelastic response of a particular connection where the double angle thickness may be the critical factor, care should be taken when proportioning the magnitudes of $K_p$ and $R_o$. For the purposes of defining the force-deformation characteristics supplied to the finite element program INELAS, the following definitions of the Richard equation parameters are recommended for each three-inch element of the double angle connection.

$$K_C = 18 * 10^4 \left(\frac{t_1}{1.75}\right)^3 \text{ kips/in} \quad (12)$$

$$N_C = 1.2 \quad (13)$$

$$R_{oC} = 142 * \left(\frac{t_c}{8}\right) \text{ kips} \quad (14)$$

$$K_{pC} = 138 * \left(\frac{t_c}{8}\right) \text{ kips/in} \quad (15)$$

where:

$t_1$ = The thickness of the angle leg in inches

$t_p$ = The thickness of the connecting plate in inches

$t_c$ = The thickness of the connection in bearing, in number of sixteenths of an inch ($t_p$ or two times $t_1$, whichever is smaller)

## 4.7 Shear Test Results and Conclusions

Most of the distortion in the shear test specimens was in the connecting plate at the location of the bolt holes. One of the tested shear specimens is shown in Figure 42. The Richard curve for the shear test is shown in Figure 43, and the Richard equation parameters are given in Table 5.

As mentioned in Section 4.4, the angles in the shear test underwent rotation in the plane perpendicular to the connecting plate. The effect of this rotation lowered the experimental elastic shear stiffness of the specimen significantly. In a typical gusset connection, this rotation would be essentially zero. Therefore, the lower experimental value of K determined from this two-bolt test would not be an accurate property of the typical connection. Accordingly, the value of $K_S$ given in Section 4.4. is considered appropriate.

In addition to the bearing deformations in the connecting plate, it was also observed that the heels of the four angles parallel to the connecting plate exhibited some minor "dishing" at the bottom of the specimen. Although this secondary deformation introduced added flexibility to the specimen, it is impossible to quantify it from the recorded data. Even though this effect would not exist in the typical, deeper connection, no attempt is made here to account for the additional stiffness due to the depth of the connection [8]. Therefore, the following conclusions regarding $K_p$ and $R_o$ may be considered conservative.

Based on a comparison of the shear and compression tests with the same angle, connecting plate, and bolt sizes (Figures 40 and 43), it is seen that the experimental values of $K_p$ and $R_o$ are very nearly the same in both tests. However. the shear specimen represents a geometry five-thirds the size of the compression test. Therefore, the values of $K_p$ and $R_o$ defining the inelastic shear characteristics would be three-fifths of the values given for compression. This yields the following recommended definitions of the Richard equation parameters for a three-inch length of a bolted double angle connection loaded in shear.

$$K_S = 18 * 10^4 \left( \frac{t_1}{1.75} \right)^3 \quad \text{kips/in} \tag{16}$$

$$N_S = 0.9 \tag{17}$$

$$R_{oS} = 85 * \left( \frac{t_c}{8} \right) \quad \text{kips} \tag{18}$$

$$K_{pS} = 83 * \left( \frac{t_c}{8} \right) \quad \text{kips/in} \tag{19}$$

The value of K is the same as that given for compression, and the values of $K_p$ and $R_o$ are three-fifths of those recommended for compression. The value of N is the value obtained from the least squares Richard curve fit to the shear test data.

As already noted, these definitions of the inelastic response of the connection in shear act in combination with the tensile and compressive effects to give the inelastic response of the entire connection.

## 5. SUMMARY AND CONCLUSIONS

The results of this study have yielded semi-empirical curves and equations defining the Richard equation parameters for bolted, double angle connections as a function of connection geometry. This study is based on the results of physical testing of double angle connections conducted at the University of Arizona. The primary application of these experimental and analytical Richard curves will be their use in the finite element analysis of gusset plate framing connections.

The following conclusions are considered significant:

1. The inelastic response of a bolted double angle connection loaded in tension is generally independent of the thickness of the connecting plate.

2. The elastic response (i.e. elastic stiffness) of a bolted double angle connection is virtually identical for tension, compression and shear, given a small gage length (less than 1-3/4 inches)

3. The inelastic strength in kips and stiffness in kips per inch ($R_o$ and KP) of a bolted double angle connection loaded in compression is directly proportional to the least thickness of the connection (i.e. the thickness of the connecting plate or two times the thickness of the angles).

4. For a three-inch element of a double angle framing connection loaded in tension, the reference load ($R_o$) in kips as a function of the connection geometry may be summarized as follows:

$$K_T = 18 * 10^4 \left[\frac{t_1}{g}\right]^3 \text{ kips/in} \quad (20)$$

$$R_{o4} = 16.0 - 4.0g \quad (21)$$

$$R_{o5} = 32.0 - 8.4g \quad (22)$$

$$R_{o6} = 52.2 - 14.2g \quad (23)$$

$$R_{o7} = 63.0 - 16.0g \quad (24)$$

$$R_{o8} = 72.1 - 17.5g \quad (25)$$

$$R_{o9} = 81.4 - 19.2g \quad (26)$$

$$R_{o10} = 90.6 - 20.8g \quad (27)$$

where:

$t_1$ = The thickness of the angle leg in inches

$g$ = The gage length in inches

and the subscripts of $R_o$ indicate the number of sixteenths of an inch of angles thickness. The appropriate values of N and $K_p$ must be determined from those presented in Table 3 and Figure 37, respectively.

5. For a three-inch element of a double angle connection loaded in compression, The Richard equation parameters as a function of the connection geometry are:

$$K_C = 18 * 10^4 \left( \frac{t_1}{1.75} \right)^3 \text{ kip/in} \quad (28)$$

$$N_C = 1.2 \quad (29)$$

$$R_{oC} = 142 * \left( \frac{t_c}{8} \right) \text{ kips} \quad (30)$$

$$K_{pC} = 138 * \left( \frac{t_c}{8} \right) \text{ kips} \quad (31)$$

where:

$t_1$ = The thickness of the angle leg

$t_p$ = The thickness of the connecting plate in inches

$t_c$ = The critical thickness of the connection in bearing, in number of sixteenths of an inch (either $t_p$ or two times $t_1$, whichever is less).

6. For a three-inch element of a double angle connection loaded in shear, the Richard equation parameters are:

$$K_S = 18 * 10^4 \left( \frac{t_1}{1.75} \right)^3 \quad \text{kips/in} \qquad (32)$$

$$N_S = 0.9 \qquad (33)$$

$$R_{oS} = 85 * \left( \frac{t_c}{8} \right) \quad \text{kips} \qquad (34)$$

$$K_{pS} = 83 * \left( \frac{t_c}{8} \right) \quad \text{kips/in} \qquad (35)$$

where the variables $t_1$, and $t_c$ are as defined above.

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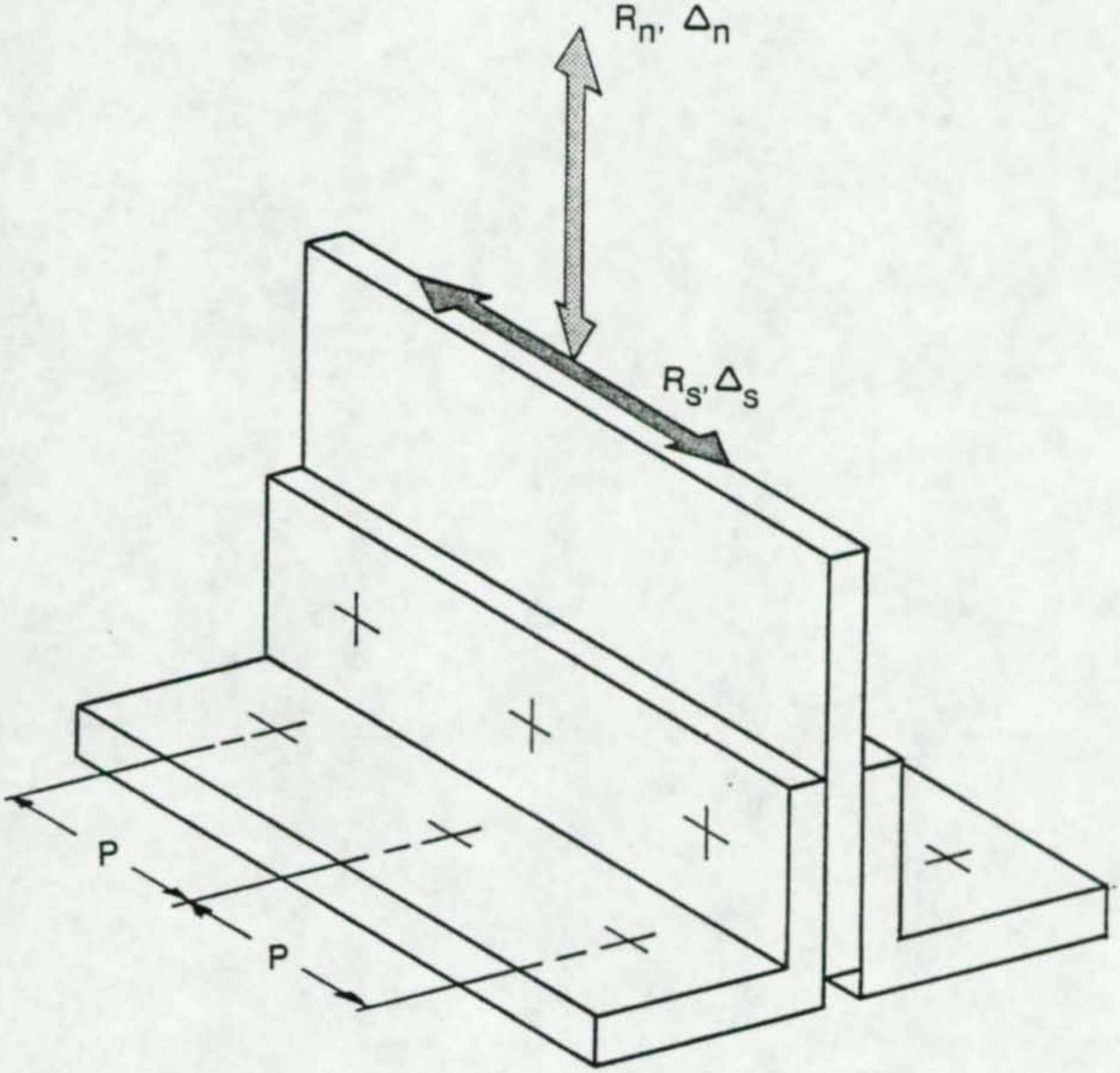


FIGURE 1 - LOADING OF DOUBLE FRAMING ANGLES

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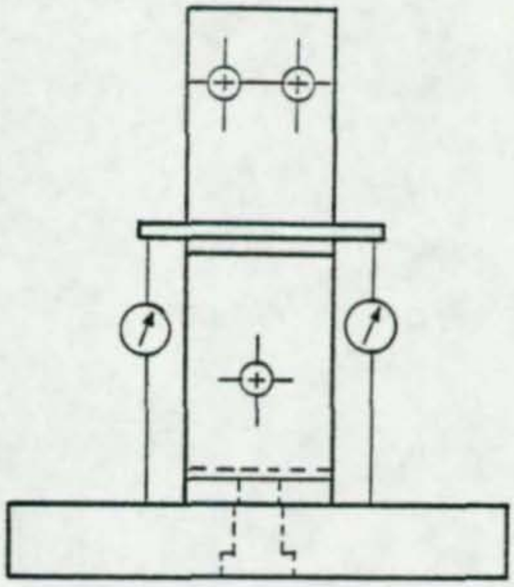
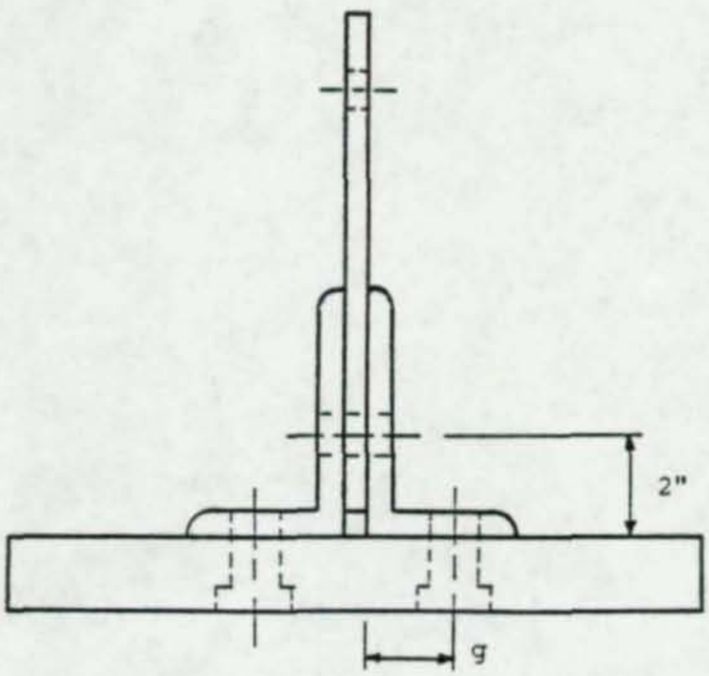


FRONT VIEW




SIDE VIEW

FIGURE 2 - TENSION TEST CONFIGURATIONS

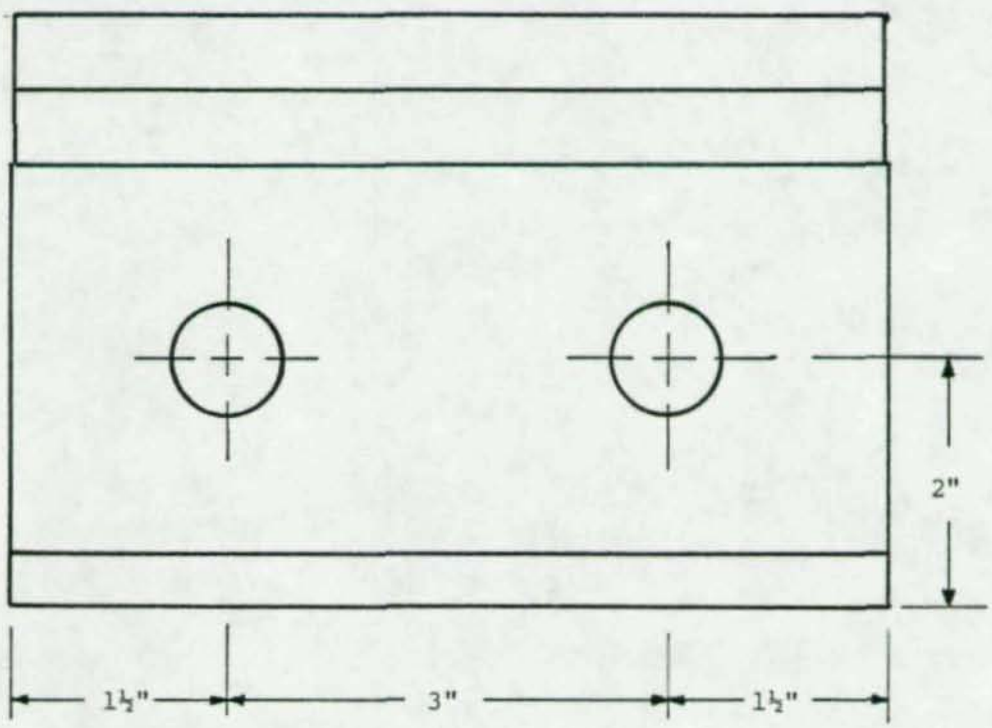



FIGURE 3 - COMPRESSION TEST CONFIGURATION
( FRONT VIEW)

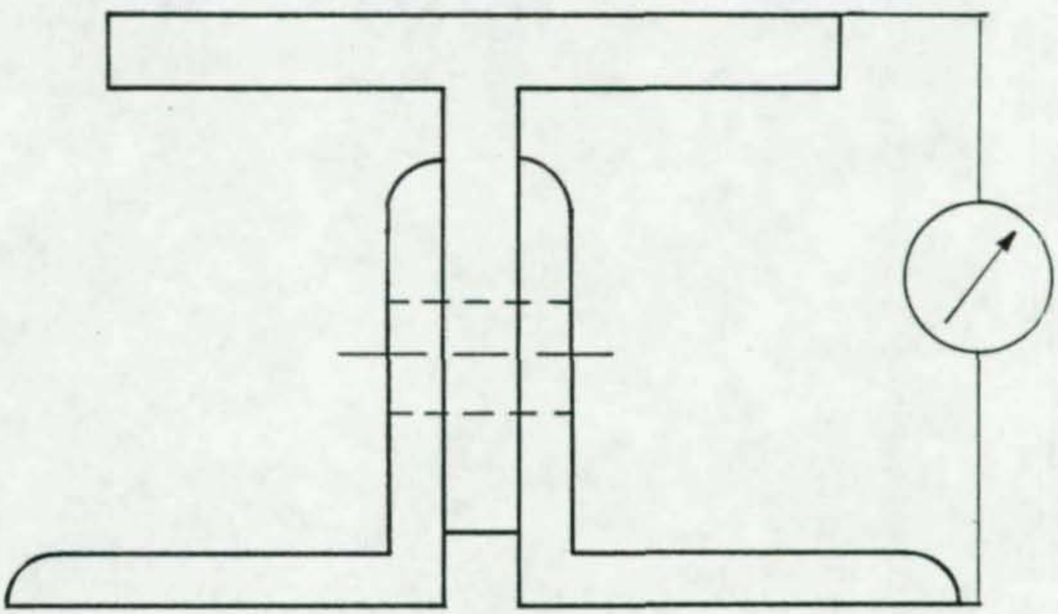


FIGURE 4 - COMPRESSION TEST CONFIGURATION

( SIDE VIEW )

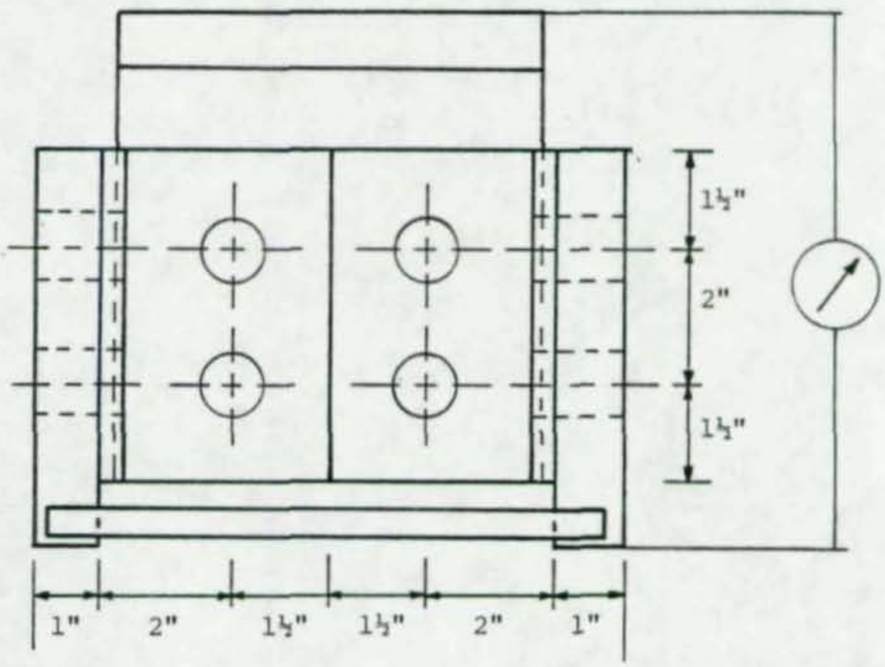



FIGURE 5 - SHEAR TEST CONFIGURATION

( FRONT VIEW )

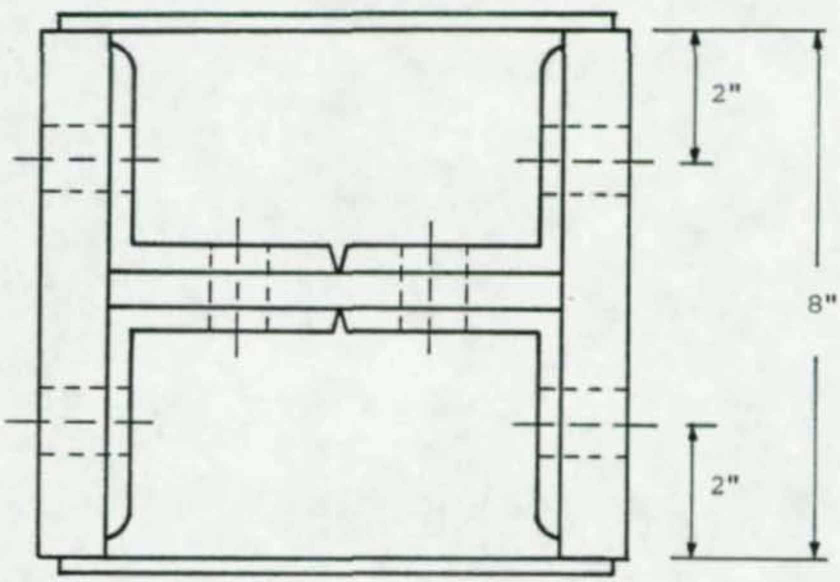



FIGURE 6 - SHEAR TEST CONFIGURATION
( BOTTOM VIEW )

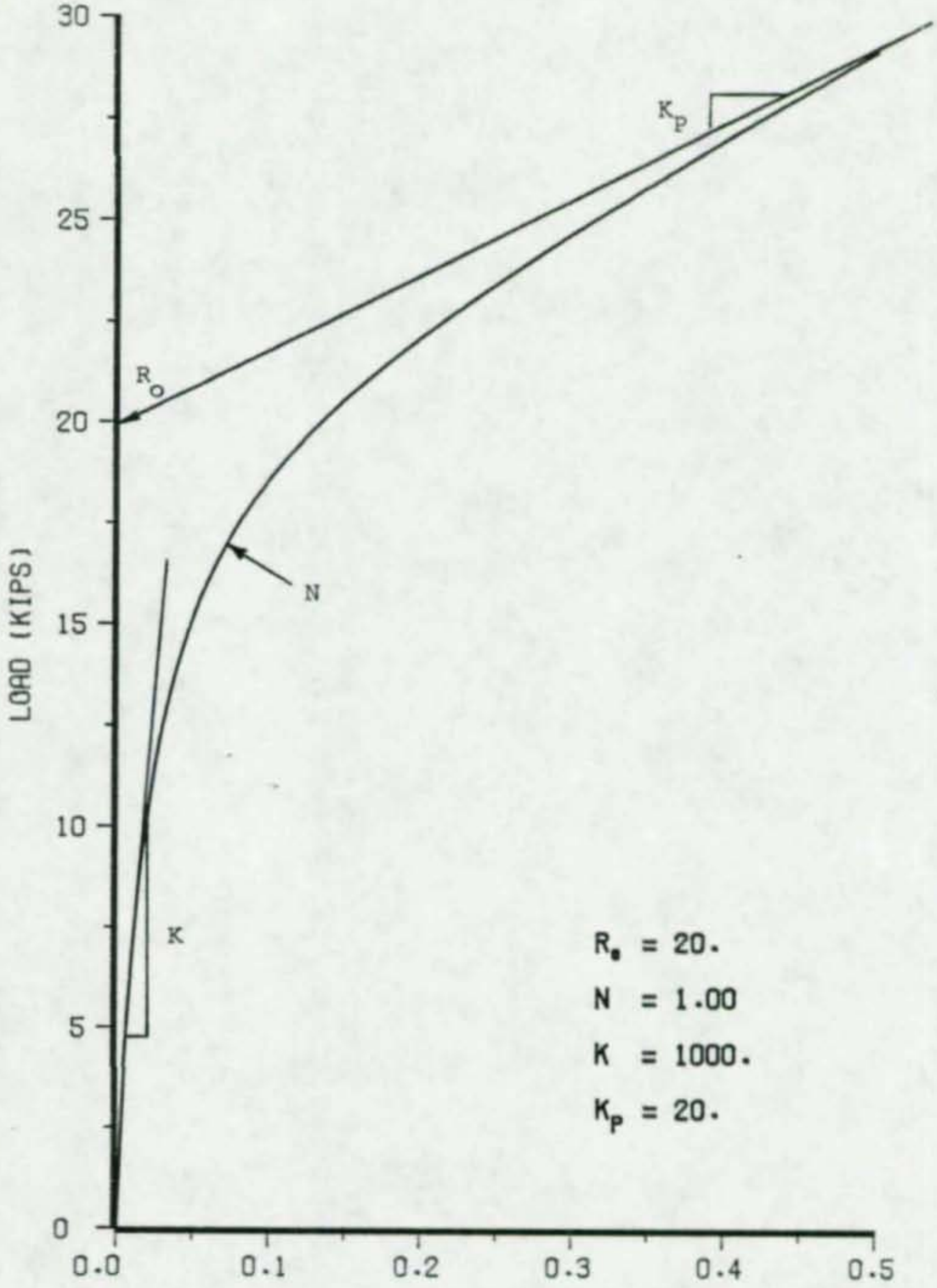



FIGURE 7 - TYPICAL RICHARD CURVE

WITH RICHARD EQUATION PARAMETERS

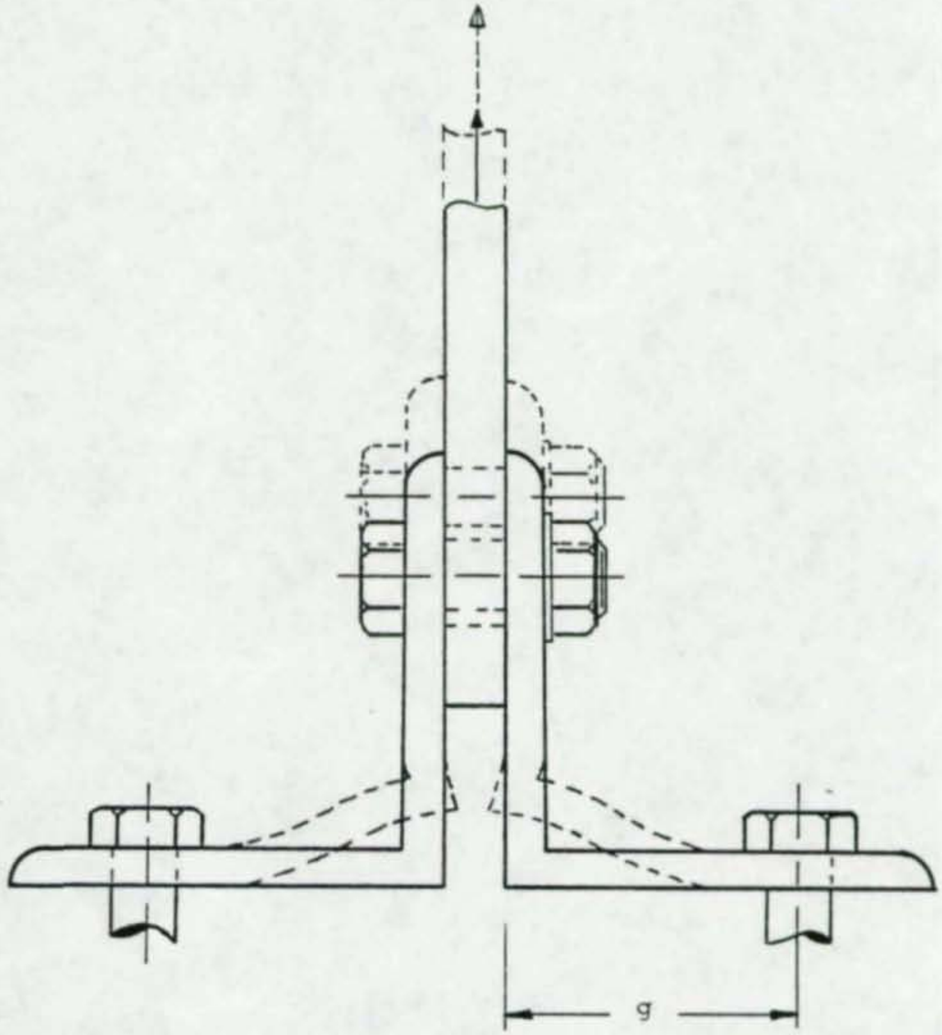



FIGURE 8 - ANGLE PERFORMANCE FOR STIFFNESS

00463

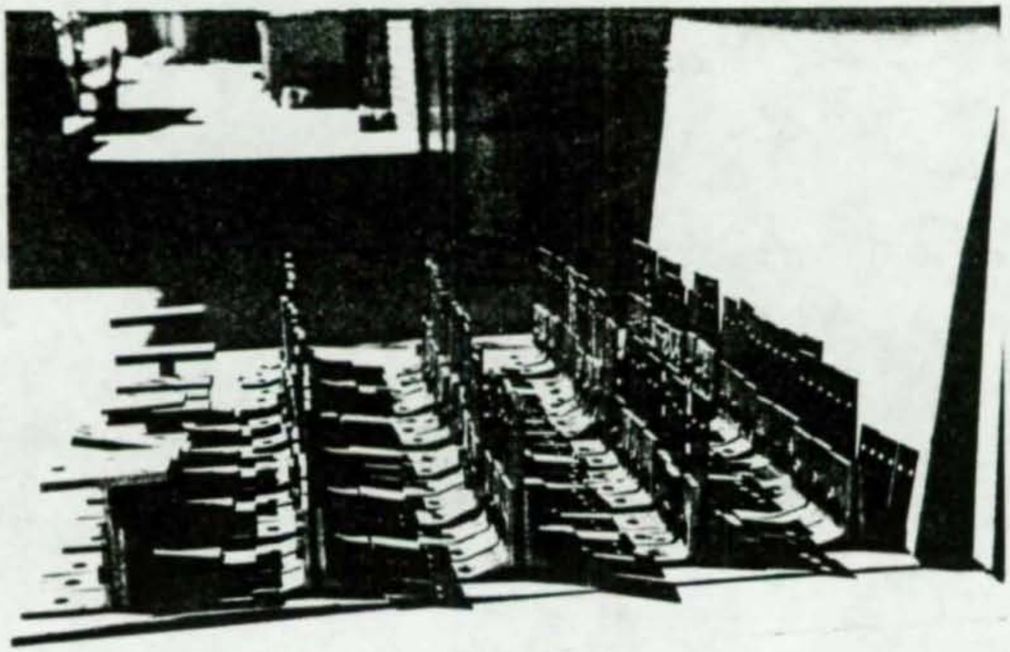


FIGURE 9 - TENSION SPECIMENS AFTER TESTING

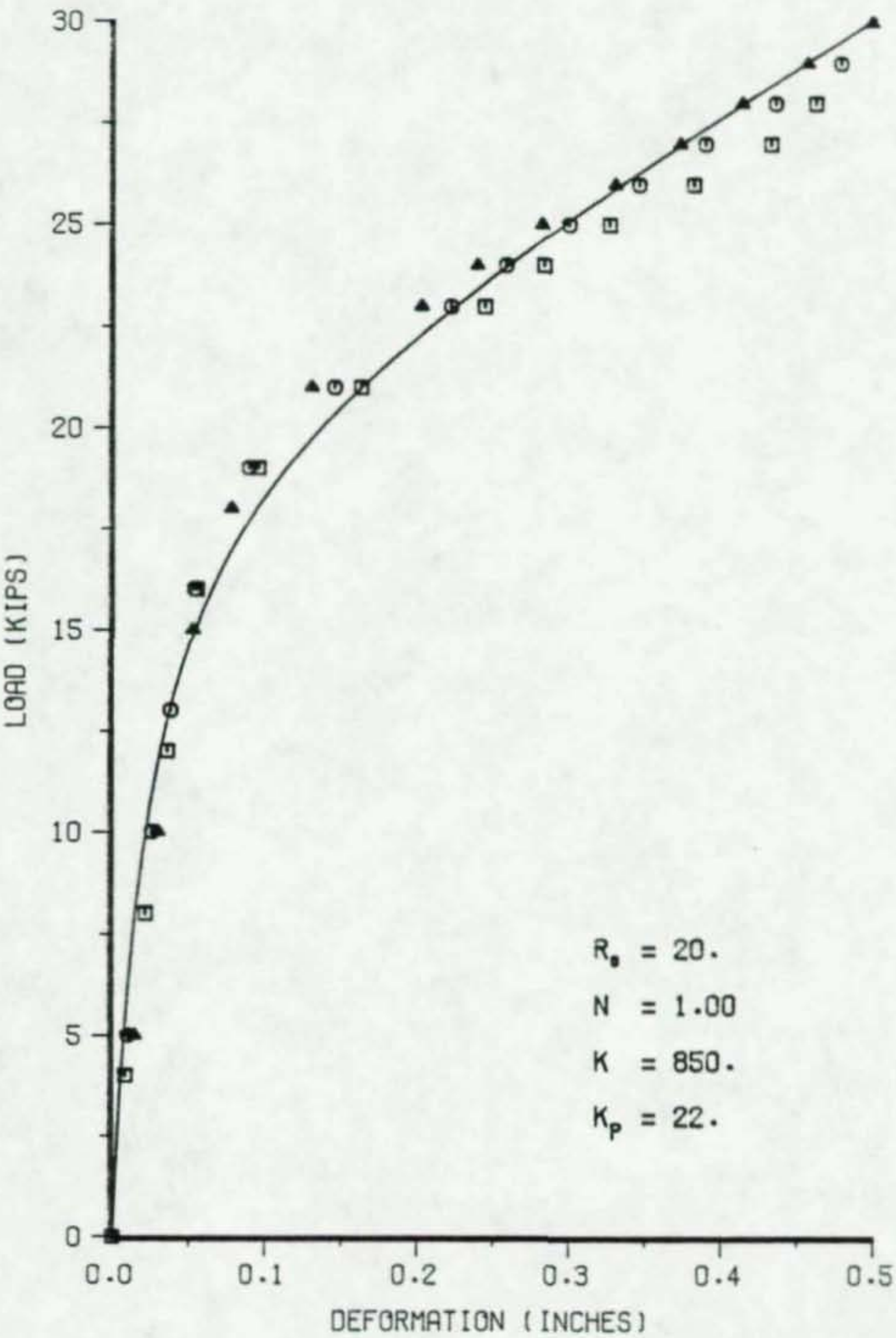



FIGURE 10 - TENSION TESTS: 3 INCH GAGE

1/2 INCH ANGLE AND 1/2 INCH PLATE

00465

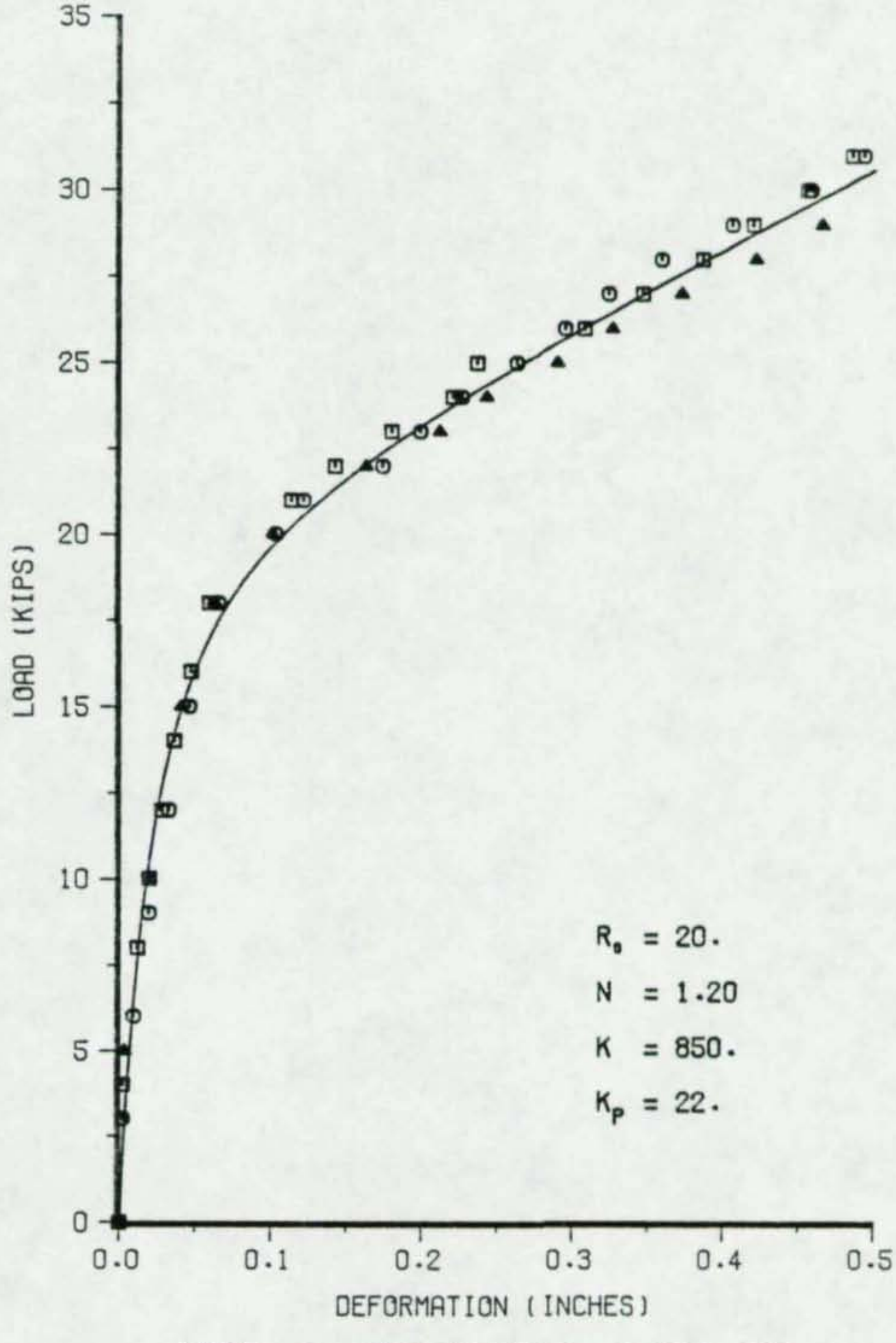



FIGURE 11 - TENSION TESTS: 3 INCH GAGE

1/2 INCH ANGLE AND 3/8 INCH PLATE

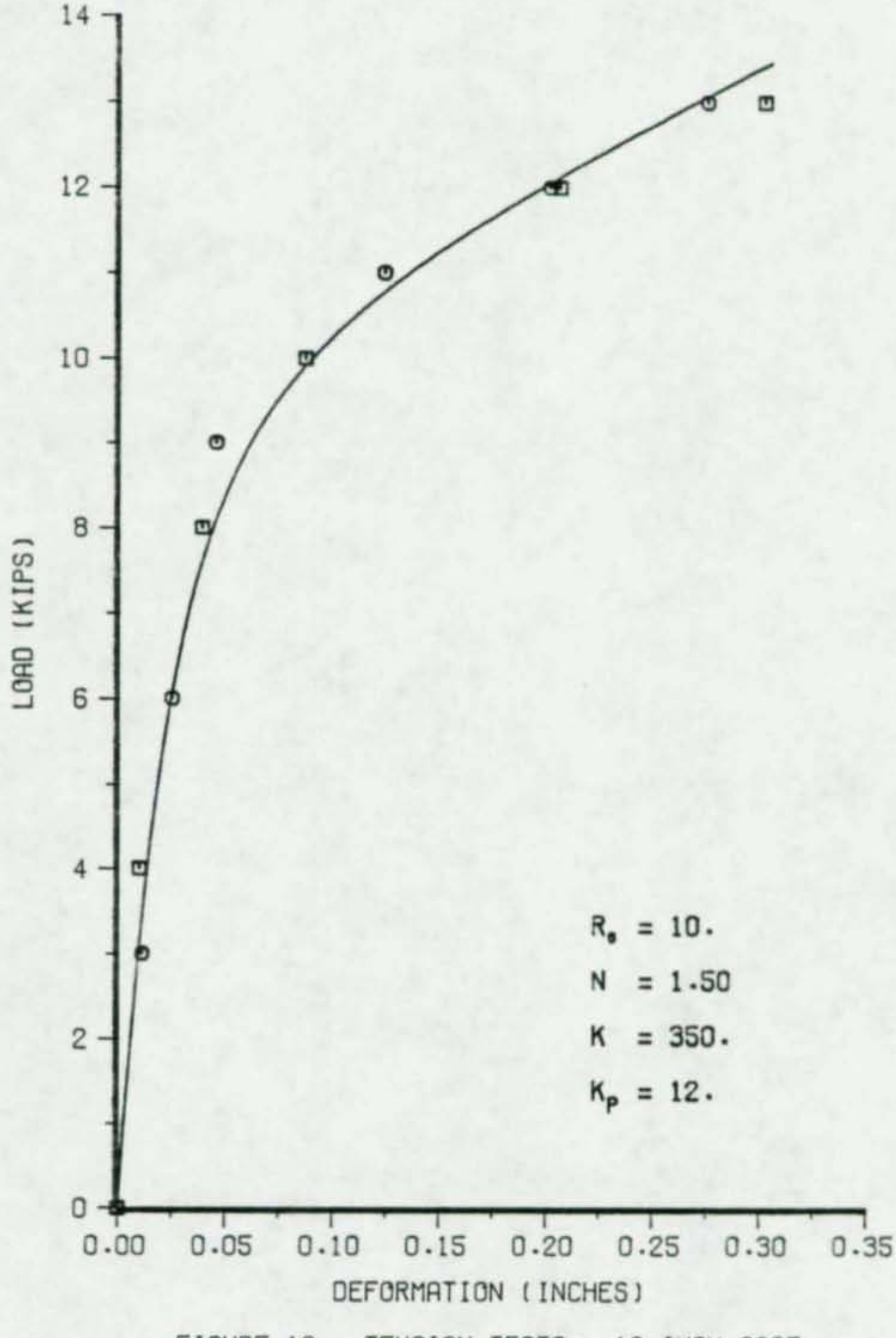



FIGURE 12 - TENSION TESTS: 12 INCH GAGE

3/8 INCH ANGLE AND 1/2 INCH PLATE

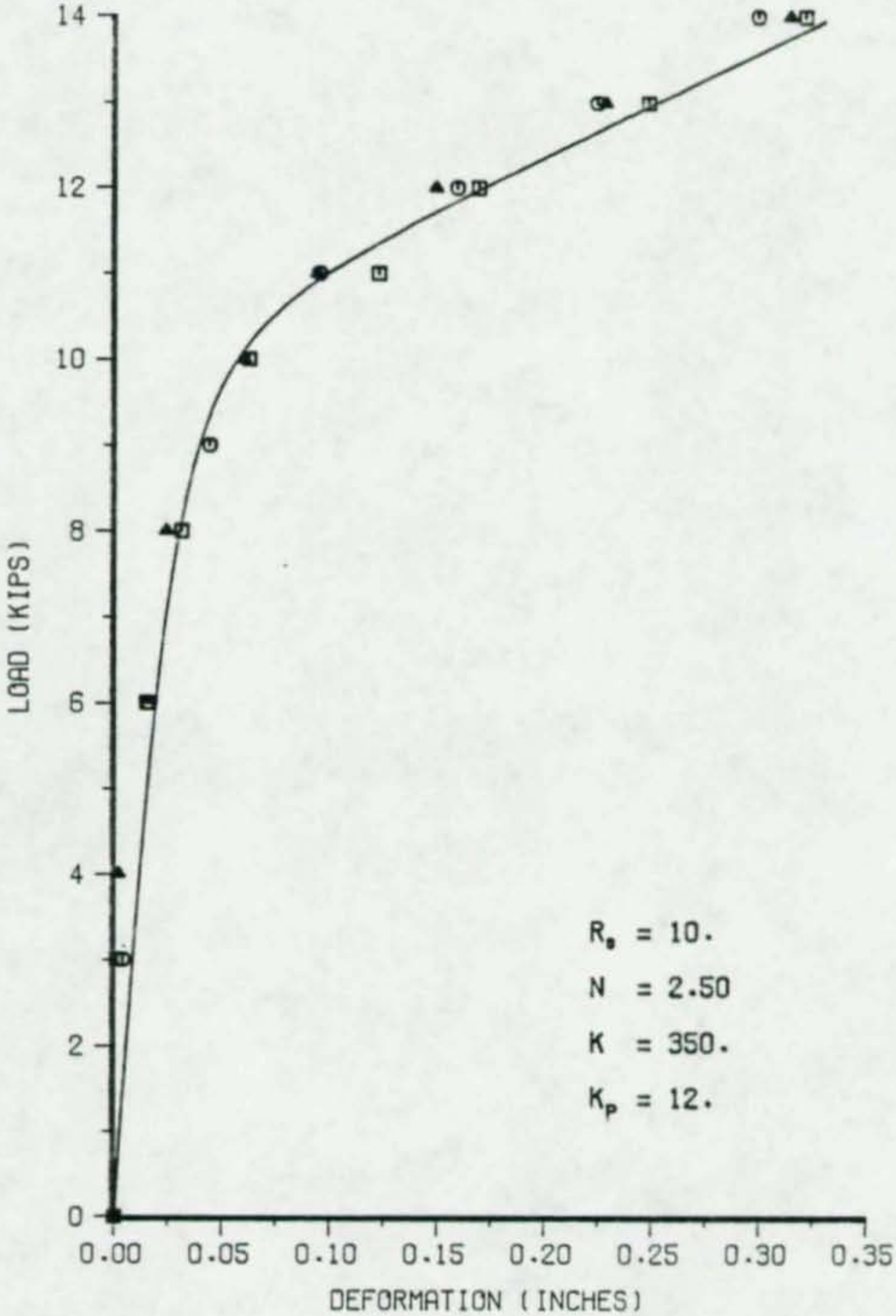



FIGURE 13 - TENSION TESTS: 3 INCH GAGE

3/8 INCH ANGLE AND 3/8 INCH PLATE

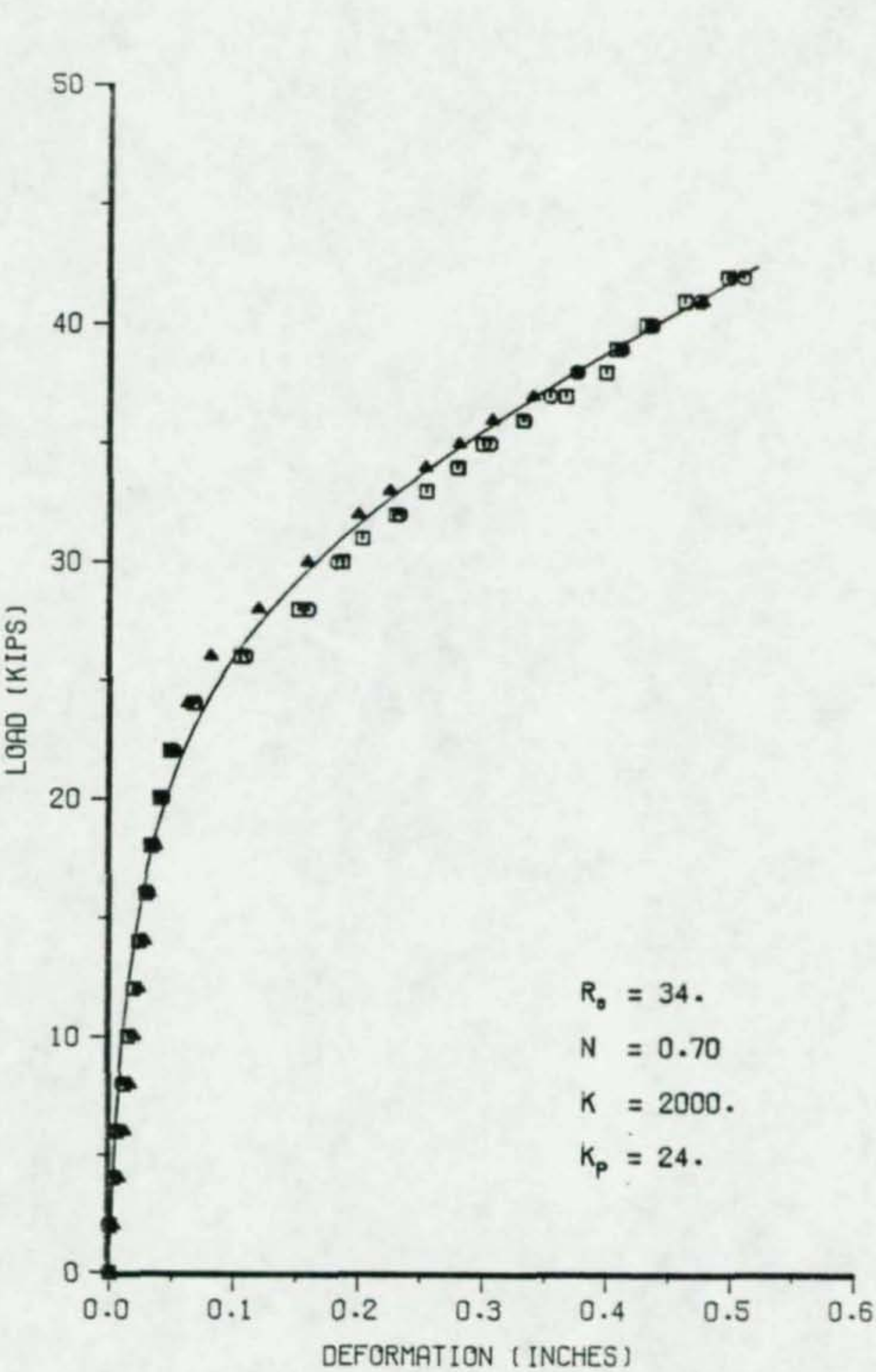



FIGURE 14 - TENSION TESTS: 2-1/4 INCH GAGE

1/2 INCH ANGLE AND 1/2 INCH PLATE

00469

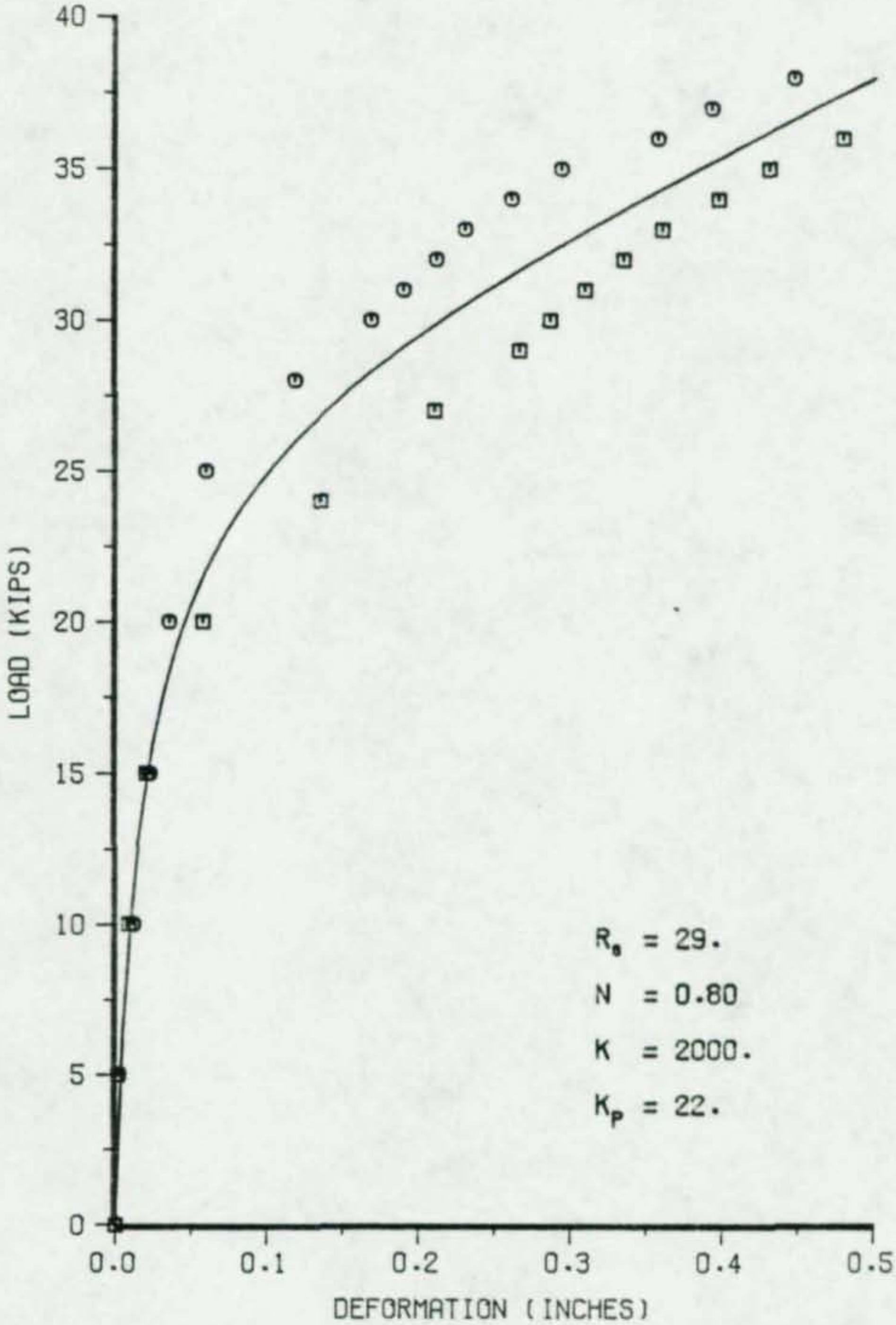



FIGURE 15 - TENSION TESTS: 2-1/4 INCH GAGE

1/2 INCH ANGLE AND 3/8 INCH PLATE

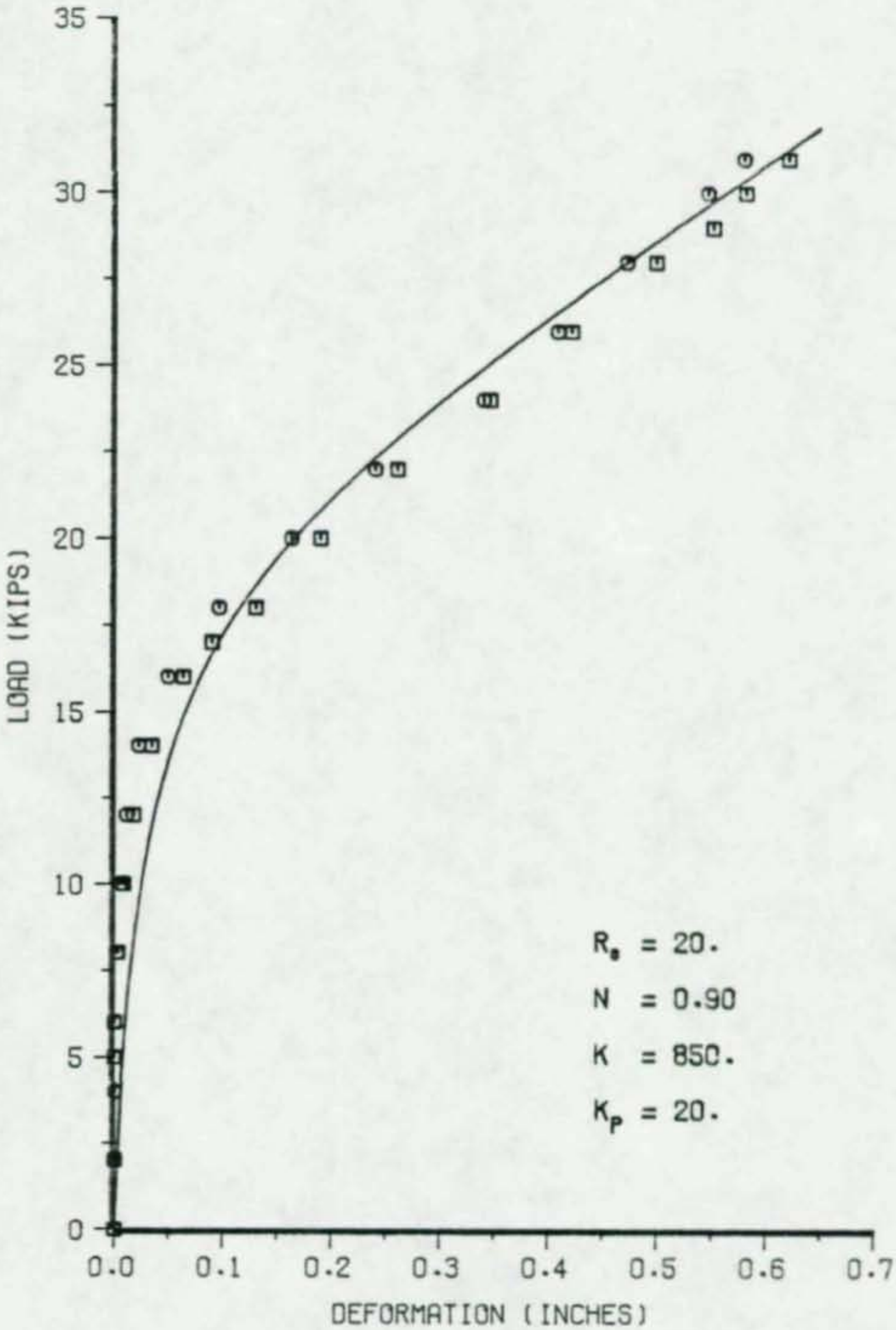



FIGURE 16 - TENSION TESTS: 2-1/4 INCH GAGE

3/8 INCH ANGLE AND 1/2 INCH PLATE

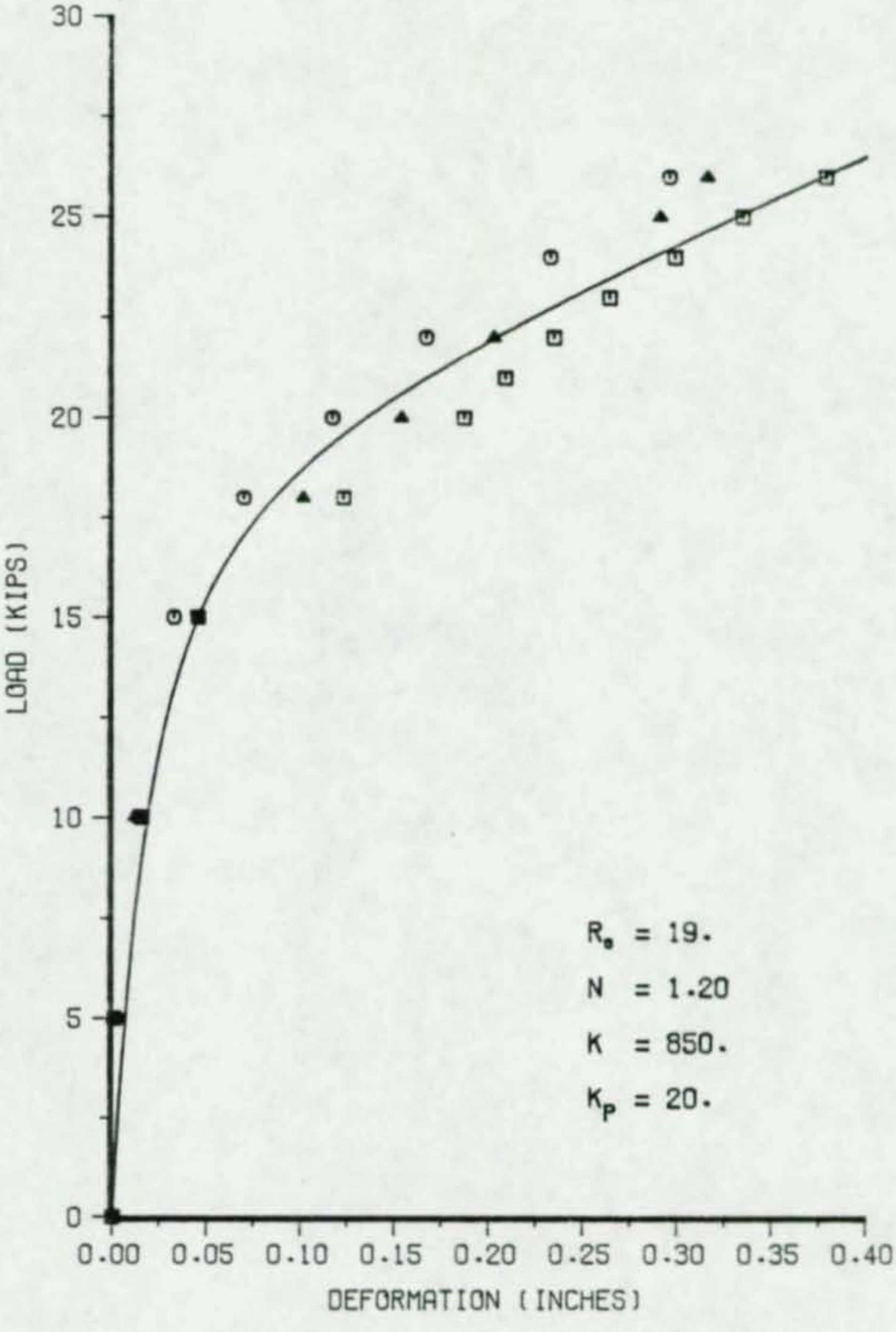



FIGURE 17 - TENSION TESTS: 2-1/4 INCH GAGE

3/8 INCH ANGLE AND 3/8 INCH PLATE

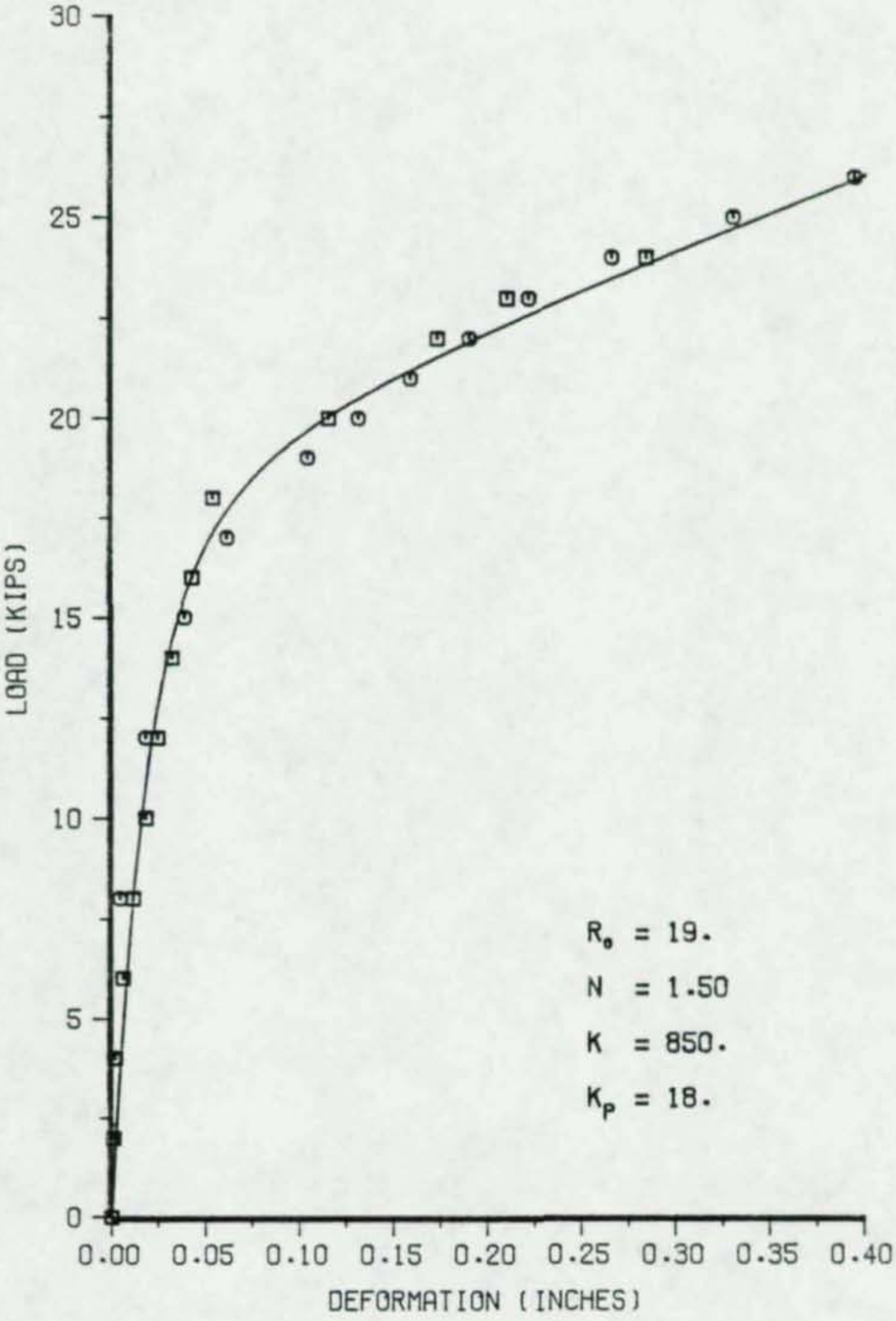



FIGURE 18 - TENSION TESTS: 2-1/4 INCH GAGE

3/8 INCH ANGLE AND 1/4 INCH PLATE

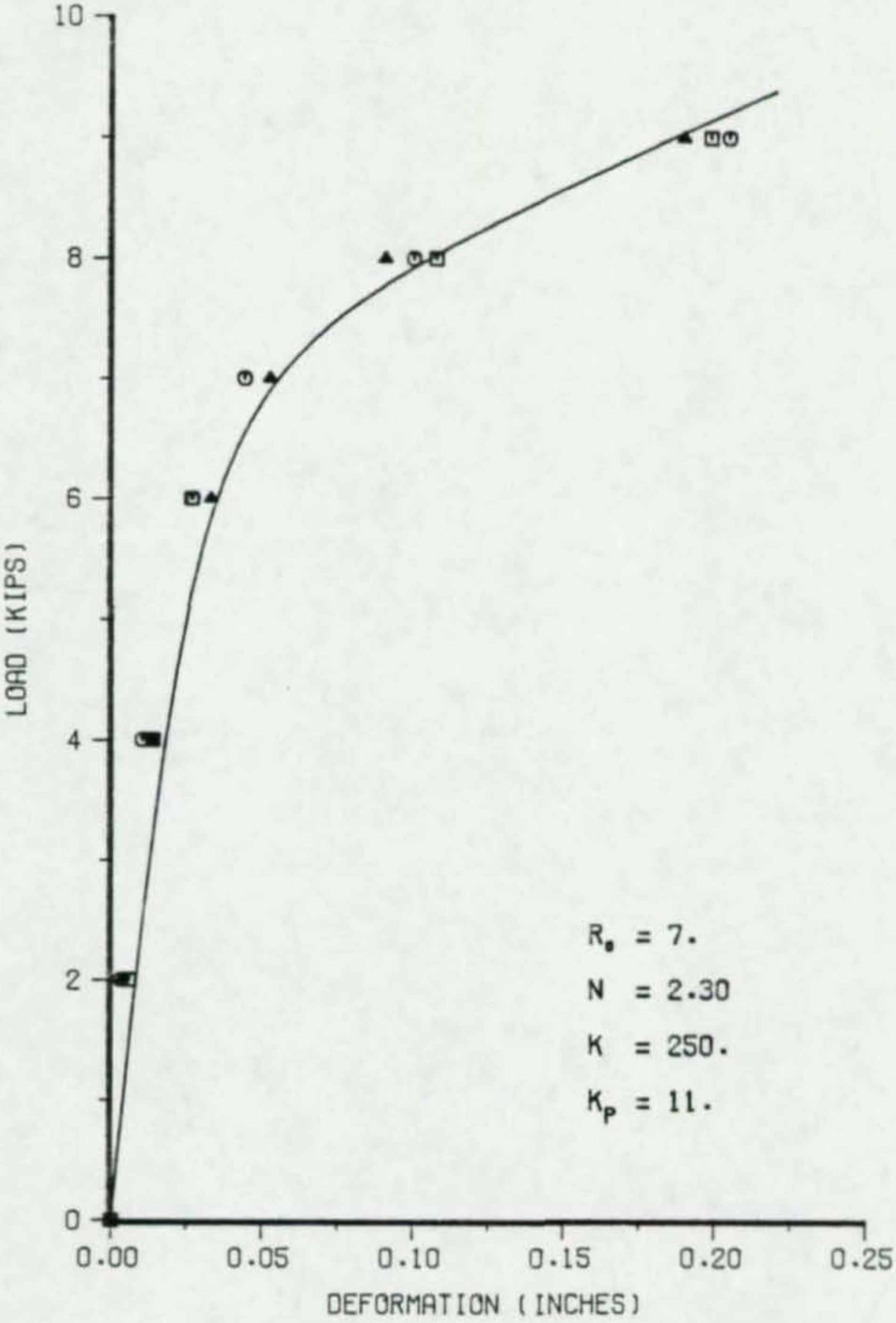



FIGURE 19 - TENSION TESTS: 2-1/4 INCH GAGE

1/4 INCH ANGLE AND 1/2 INCH PLATE

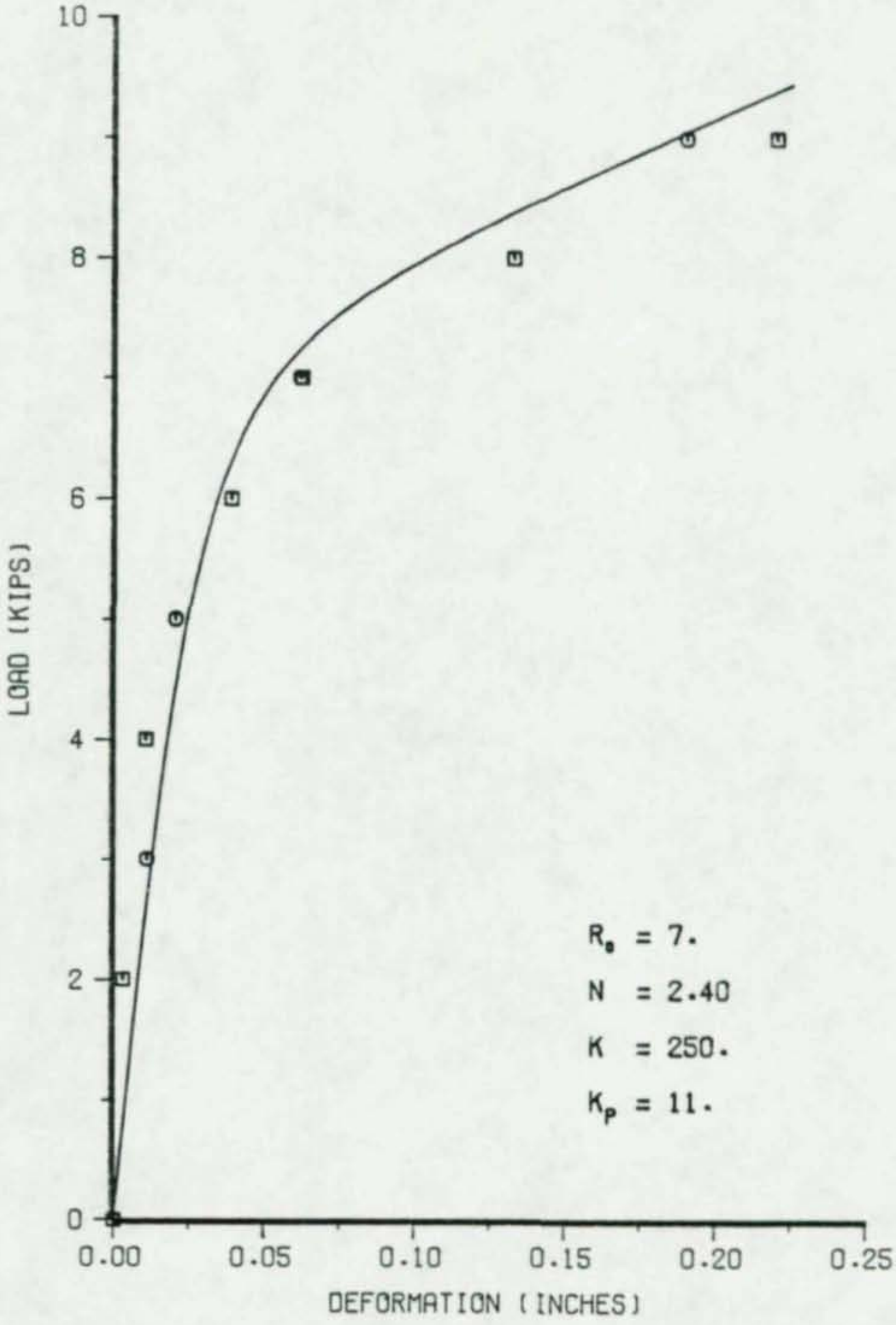



FIGURE 20 - TENSION TESTS: 2-1/4 INCH GAGE

1/4 INCH ANGLE AND 3/8 INCH PLATE

00475

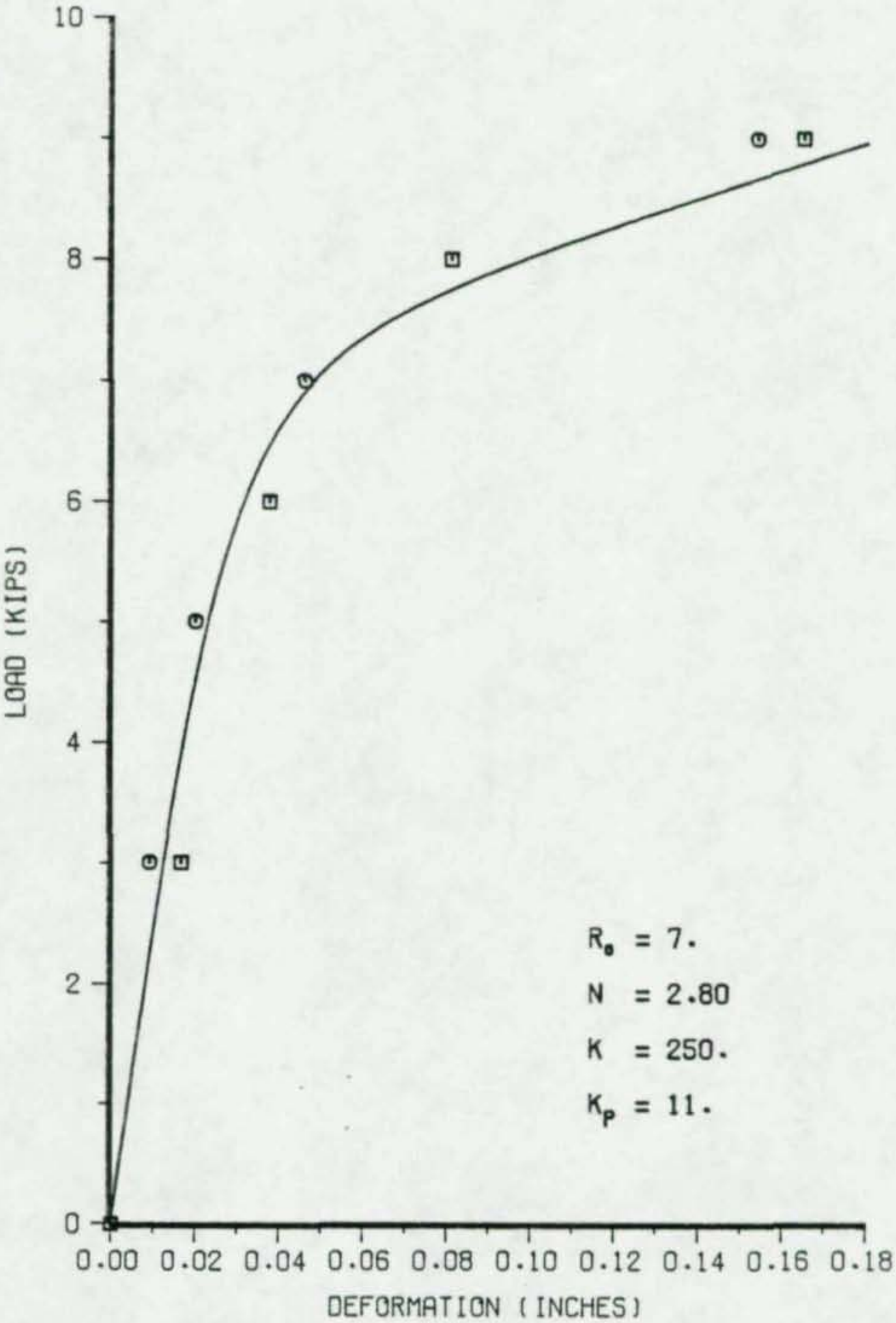



FIGURE 21 - TENSION TESTS: 2-1/4 INCH GAGE
1/4 INCH ANGLE AND 1/4 INCH PLATE

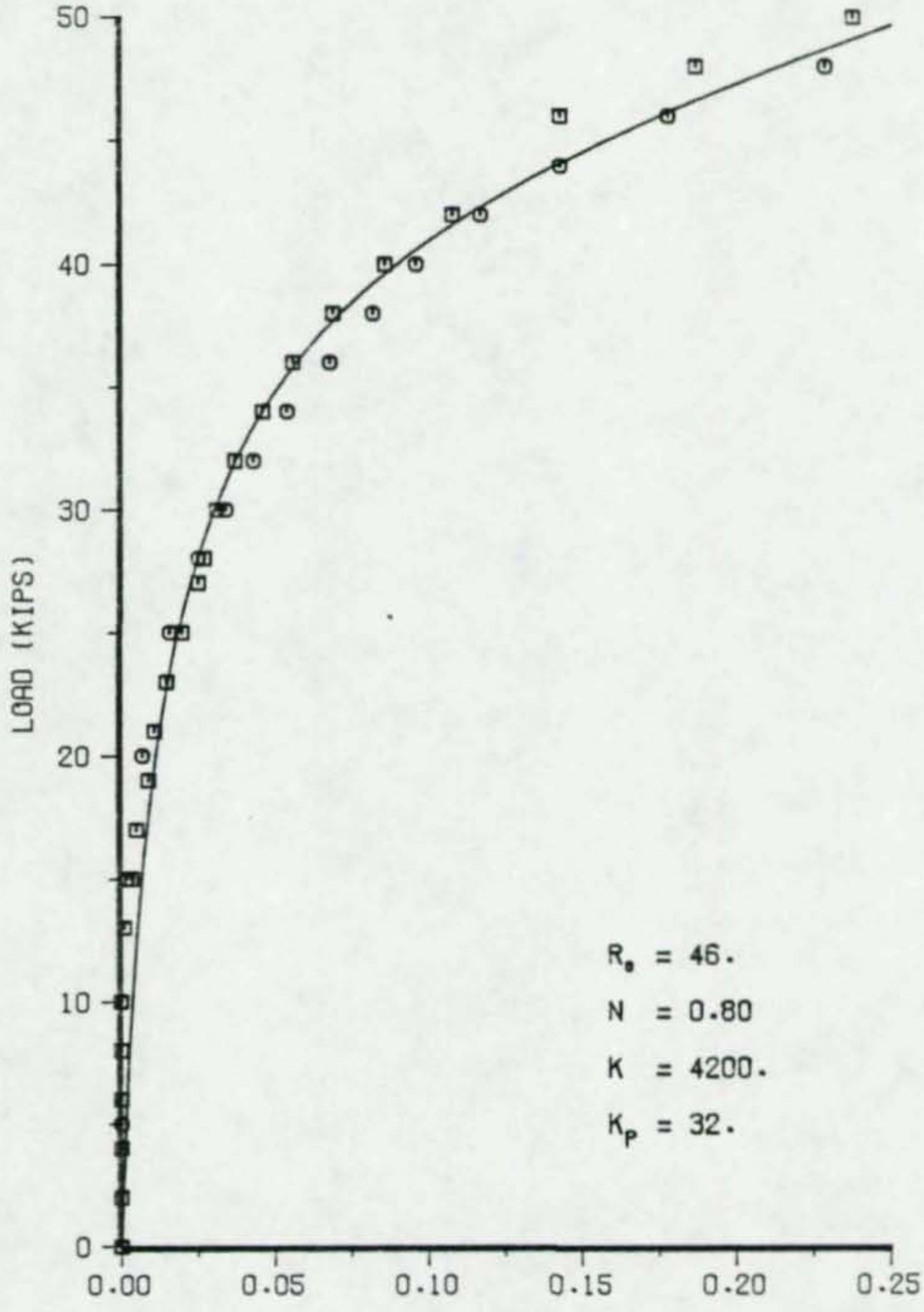



FIGURE 22 - TENSION TESTS: 1-3/4 INCH GAGE

1/2 INCH ANGLE AND 1/2 INCH PLATE

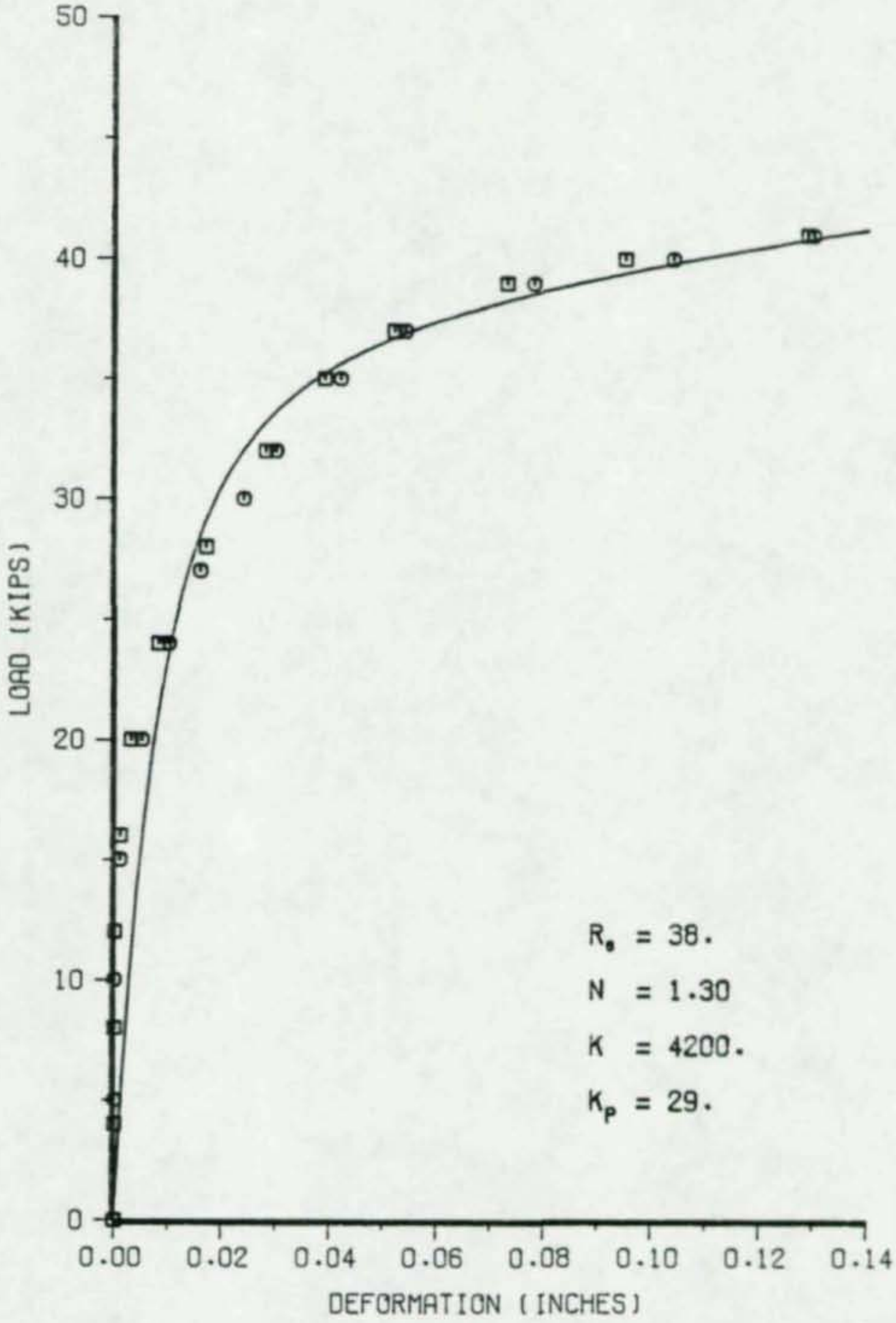



FIGURE 23 - TENSION TESTS: 1-3/4 INCH GAGE

1/2 INCH ANGLE AND 3/8 INCH PLATE

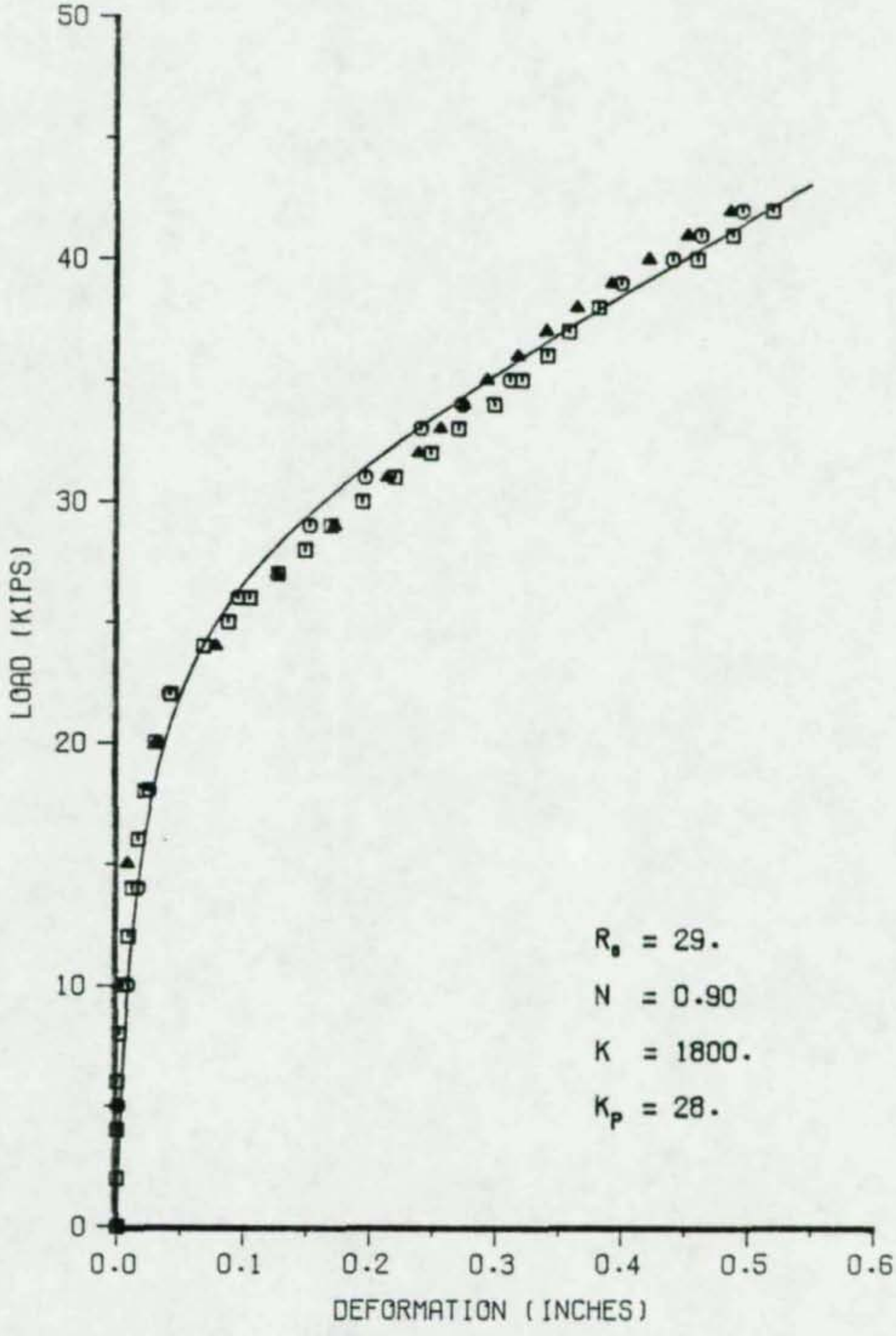



FIGURE 24 - TENSION TESTS: 1-3/4 INCH GAGE

3/8 INCH ANGLE AND 1/2 INCH PLATE

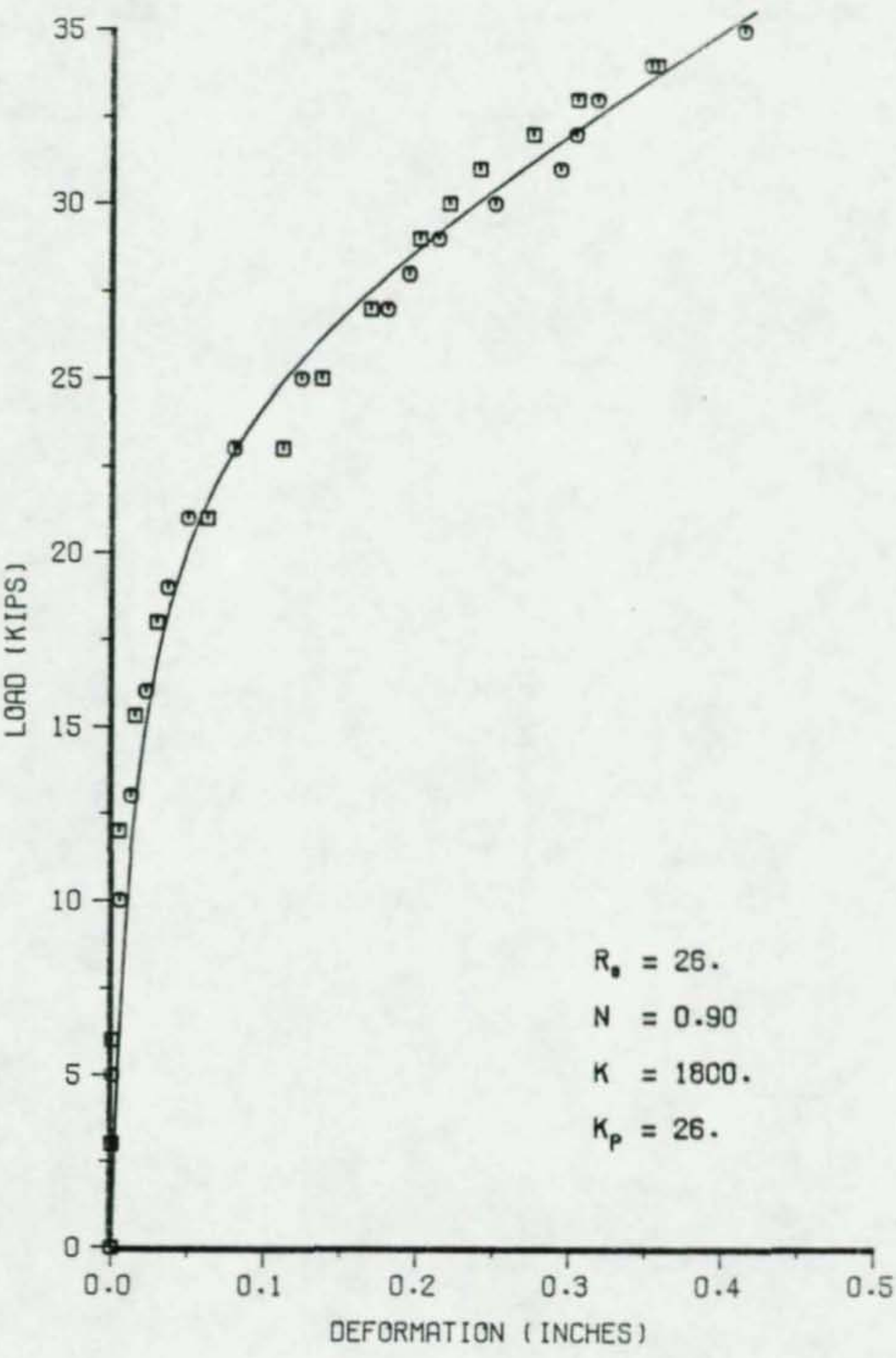



FIGURE 25 - TENSION TESTS: 1-3/4 INCH GAGE

3/8 INCH ANGLE AND 3/8 INCH PLATE

00480

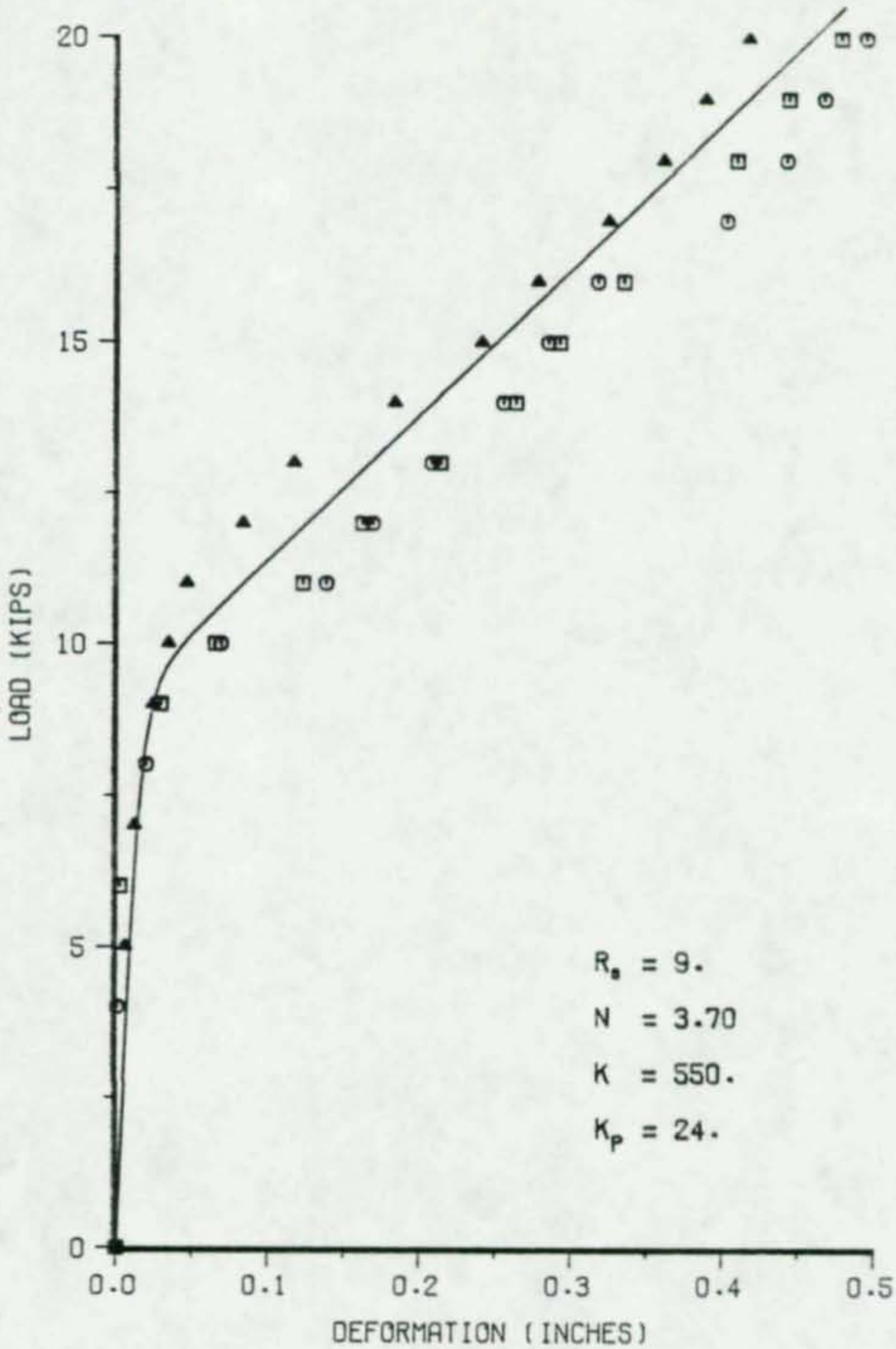



FIGURE 26 - TENSION TESTS: 1-3/4 INCH GAGE

1/4 INCH ANGLE AND 1/2 INCH PLATE

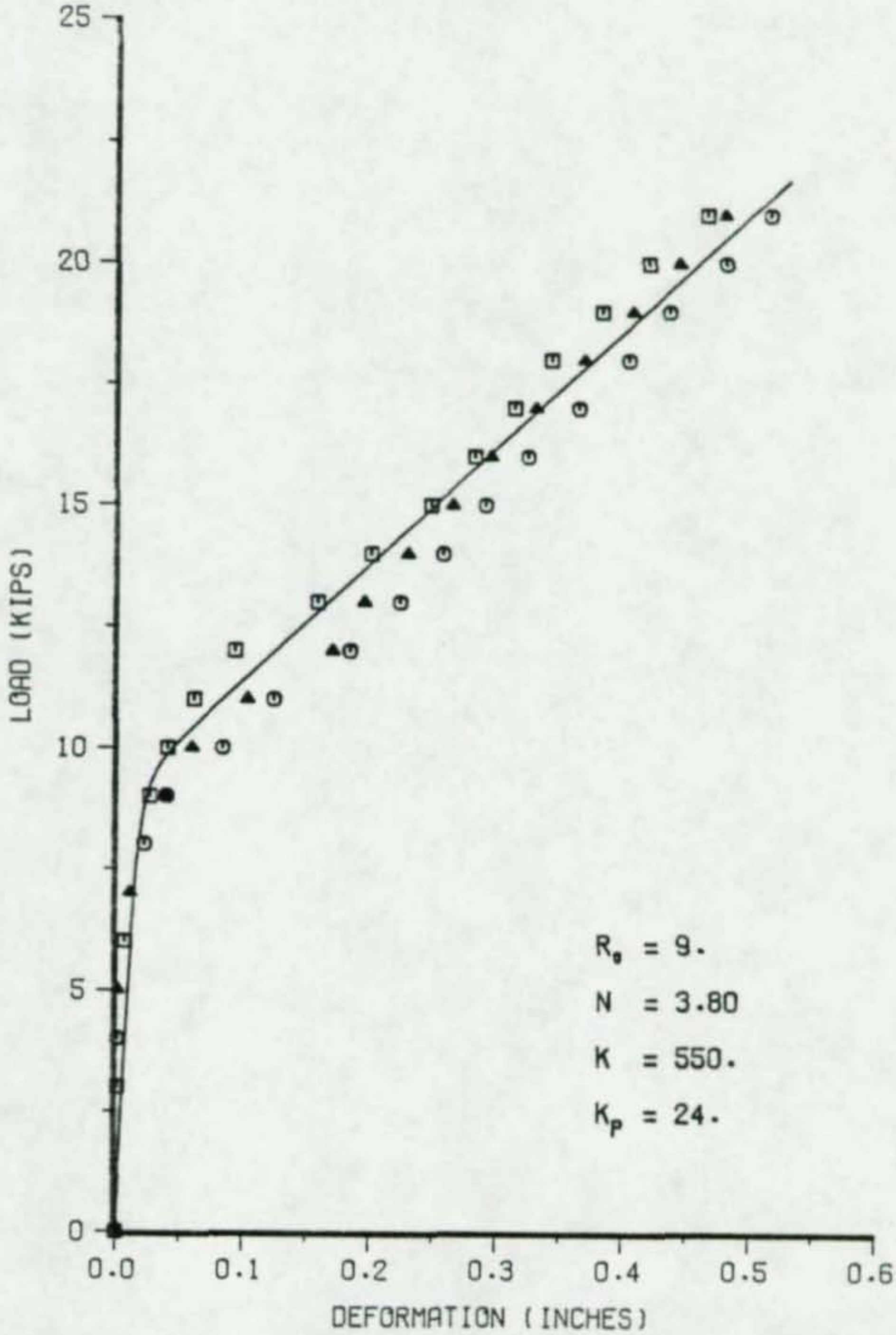



FIGURE 27 - TENSION TESTS: 1-3/4 INCH GAGE

1/4 INCH ANGLE AND 3/8 INCH PLATE

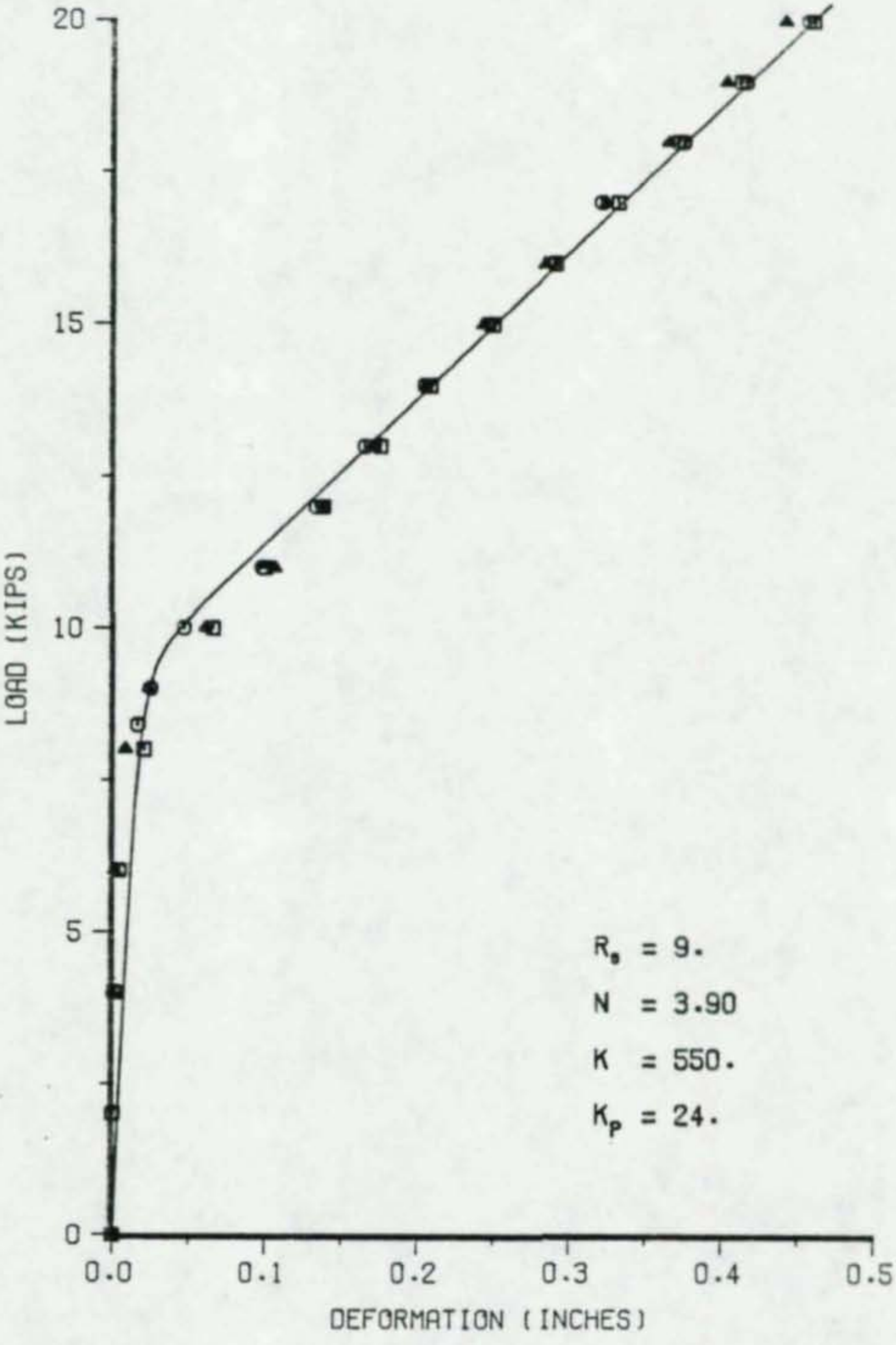



FIGURE 28 - TENSION TESTS: 1-3/4 INCH GAGE

1/4 INCH ANGLE AND 1/4 INCH PLATE

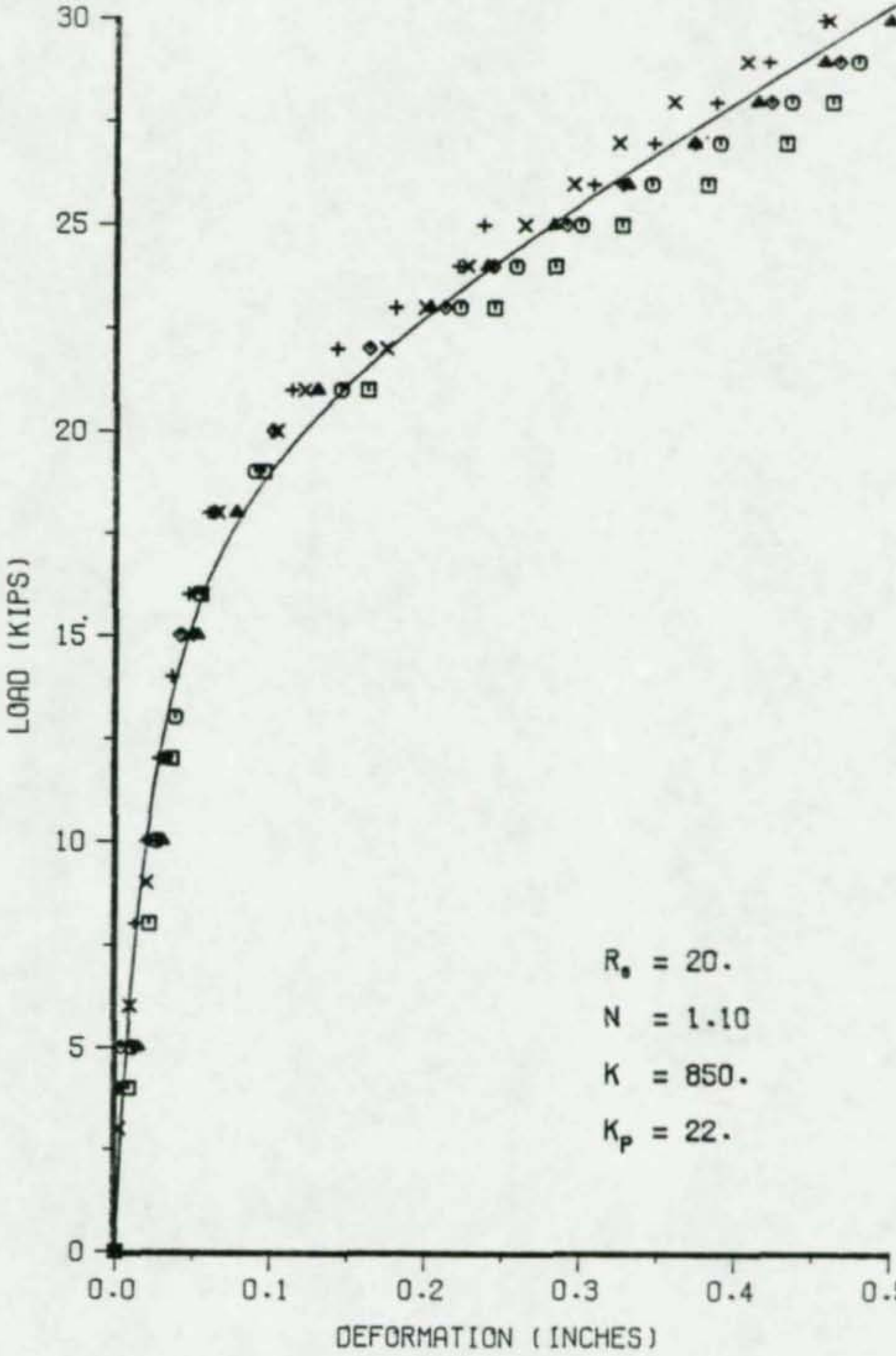



FIGURE 29 - TENSION TESTS: 3 INCH GAGE

1/2 INCH ANGLE

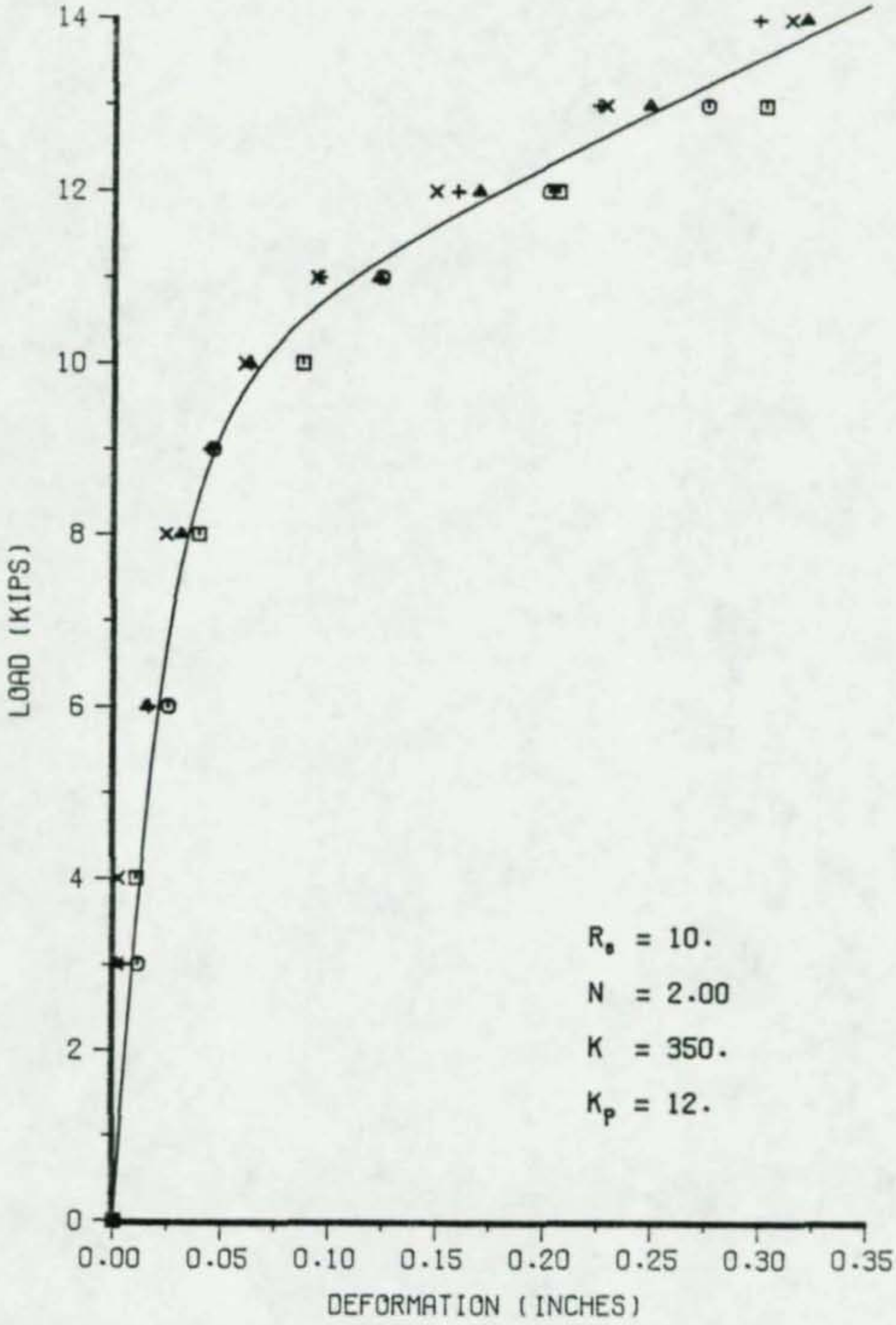



FIGURE 30 - TENSION TESTS: 3 INCH GAGE

3/8 INCH ANGLE

00485

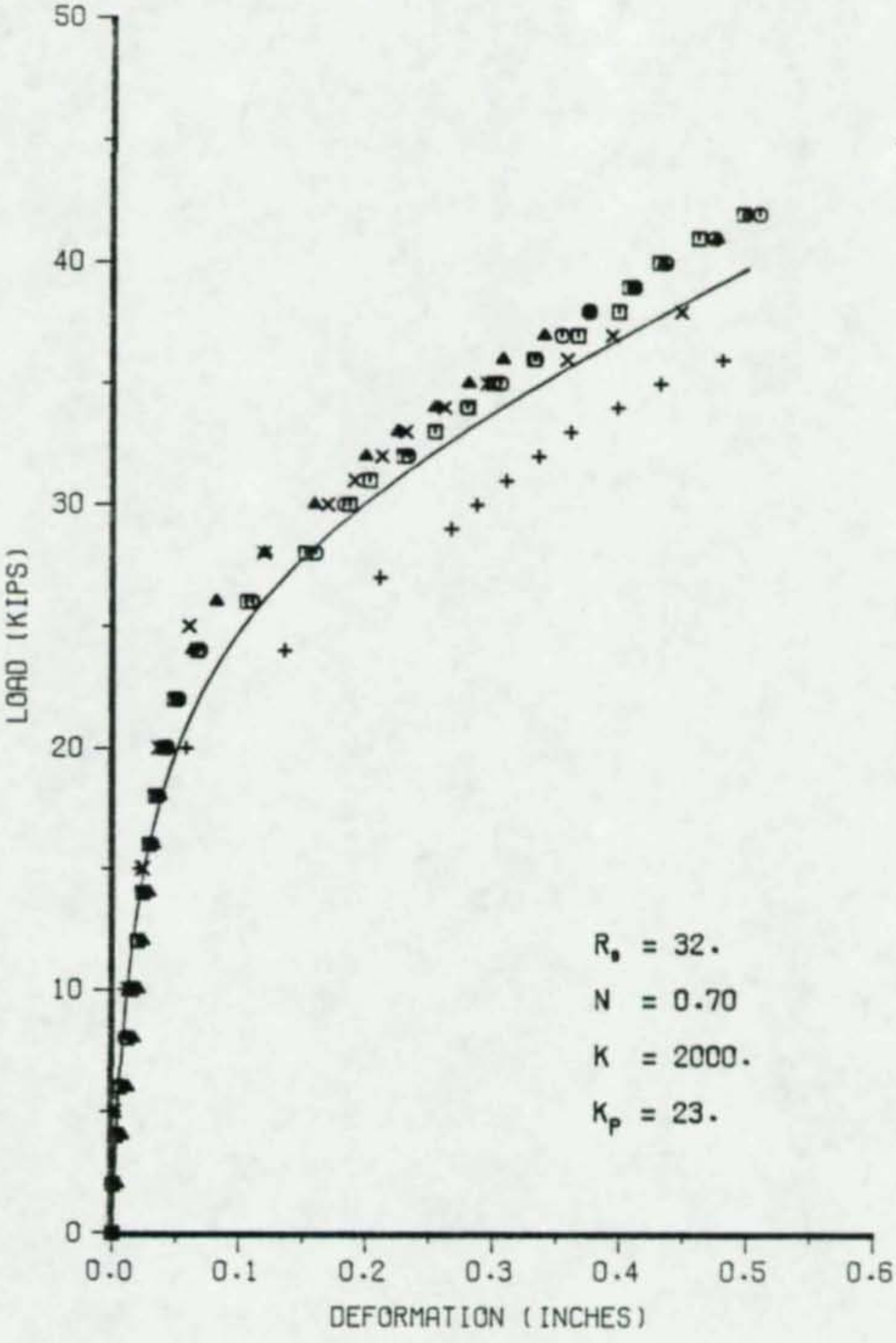



FIGURE 31 - TENSION TESTS: 2-1/4 INCH GAGE
1/2 INCH ANGLE

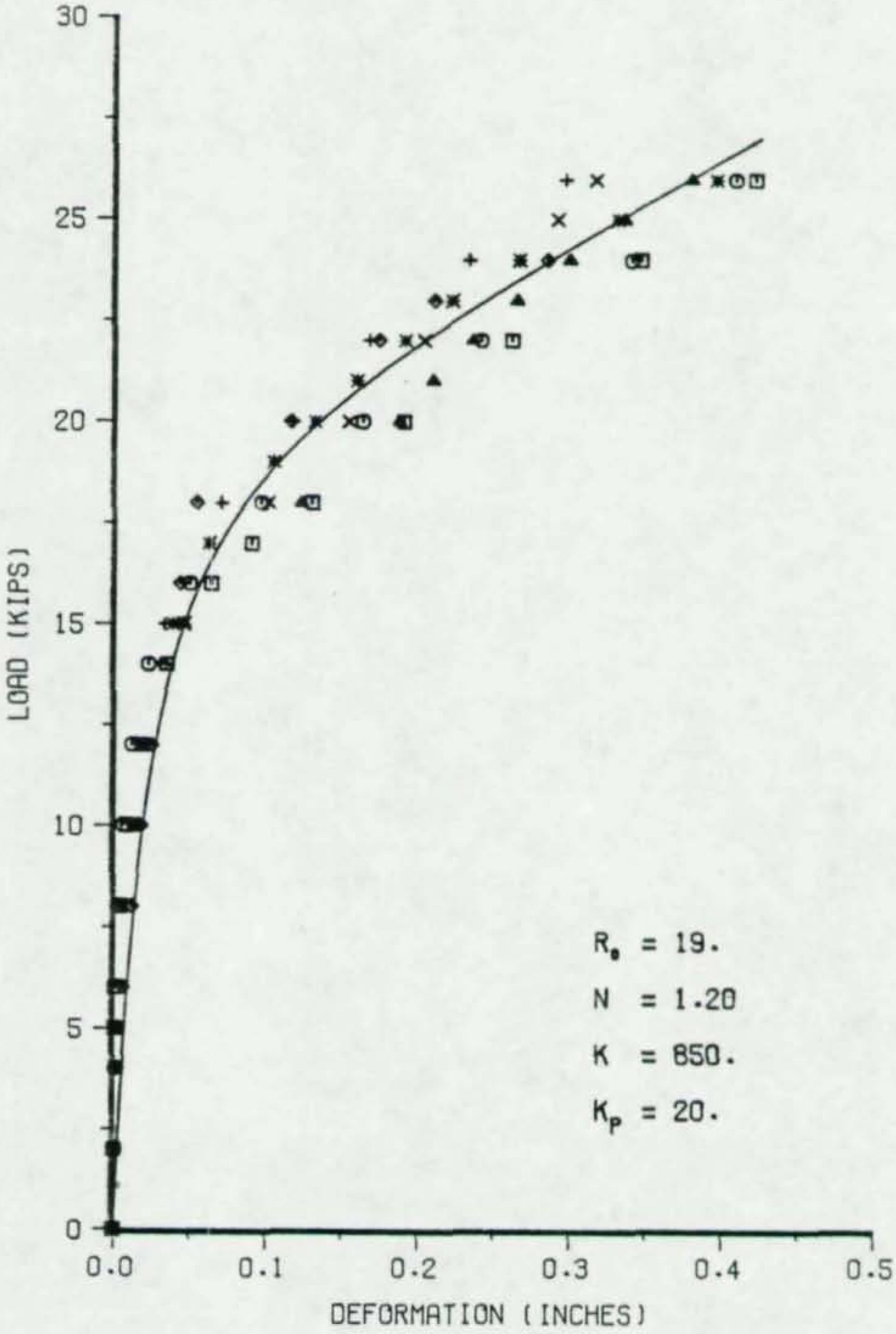



FIGURE 32 - TENSION TESTS: 2-1/4 INCH GAGE

3/8 INCH ANGLE

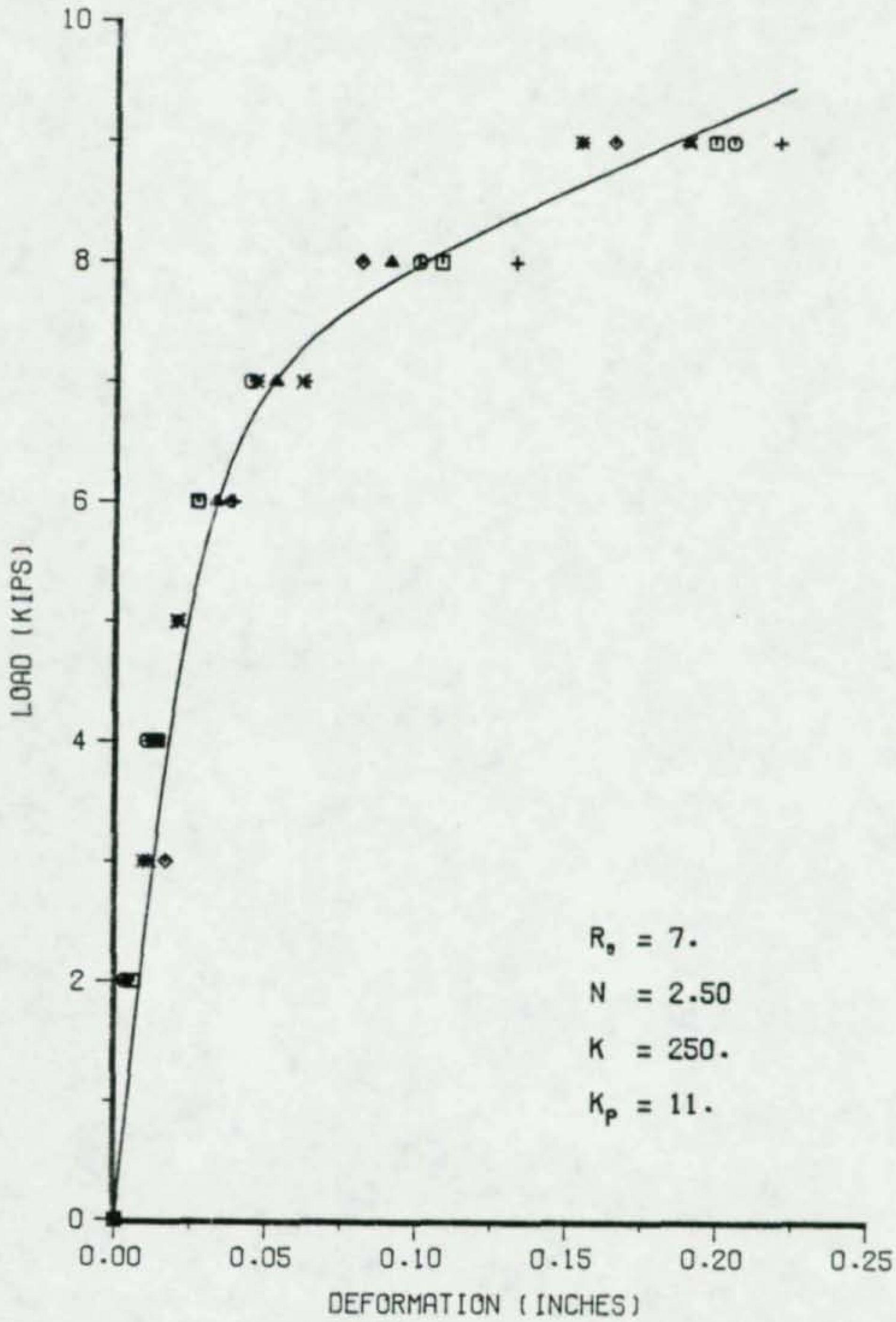



FIGURE 33 - TENSION TESTS: 2-1/4 INCH GAGE

1/4 INCH ANGLE

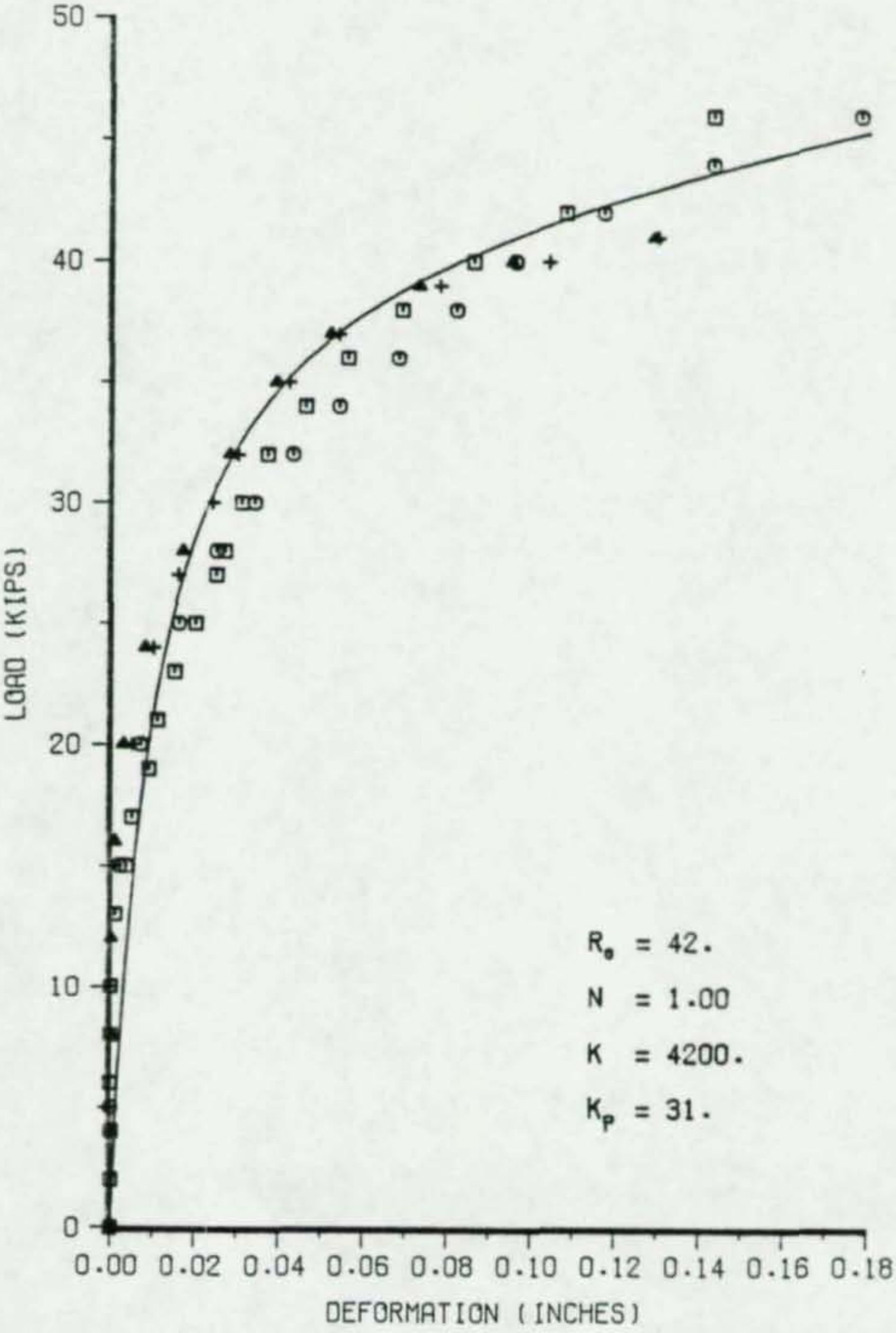



FIGURE 34 - TENSION TESTS: 1-3/4 INCH GAGE

1/2 INCH ANGLE

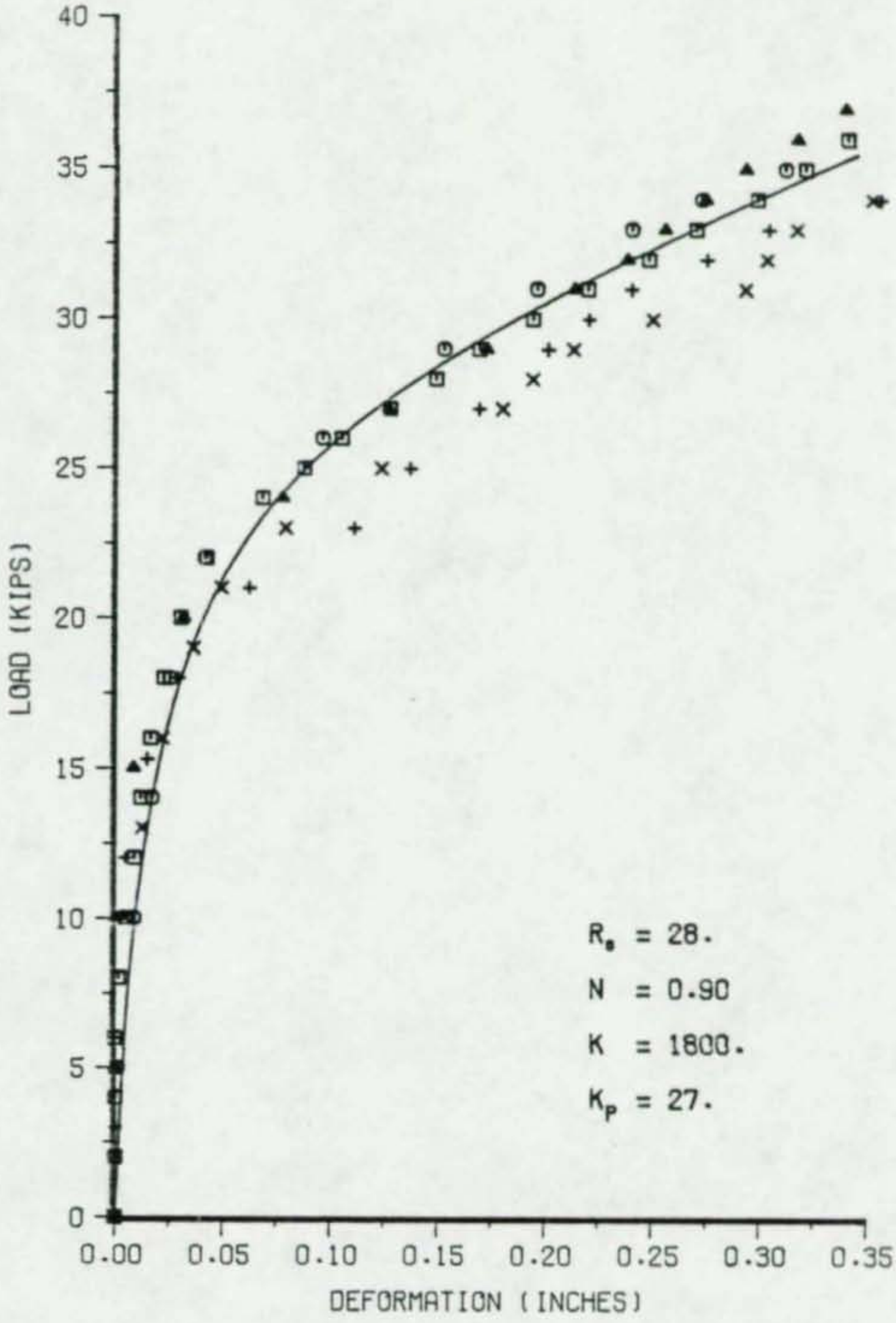



FIGURE 35 - TENSION TESTS: 1-3/4 INCH GAGE

3/8 INCH ANGLE

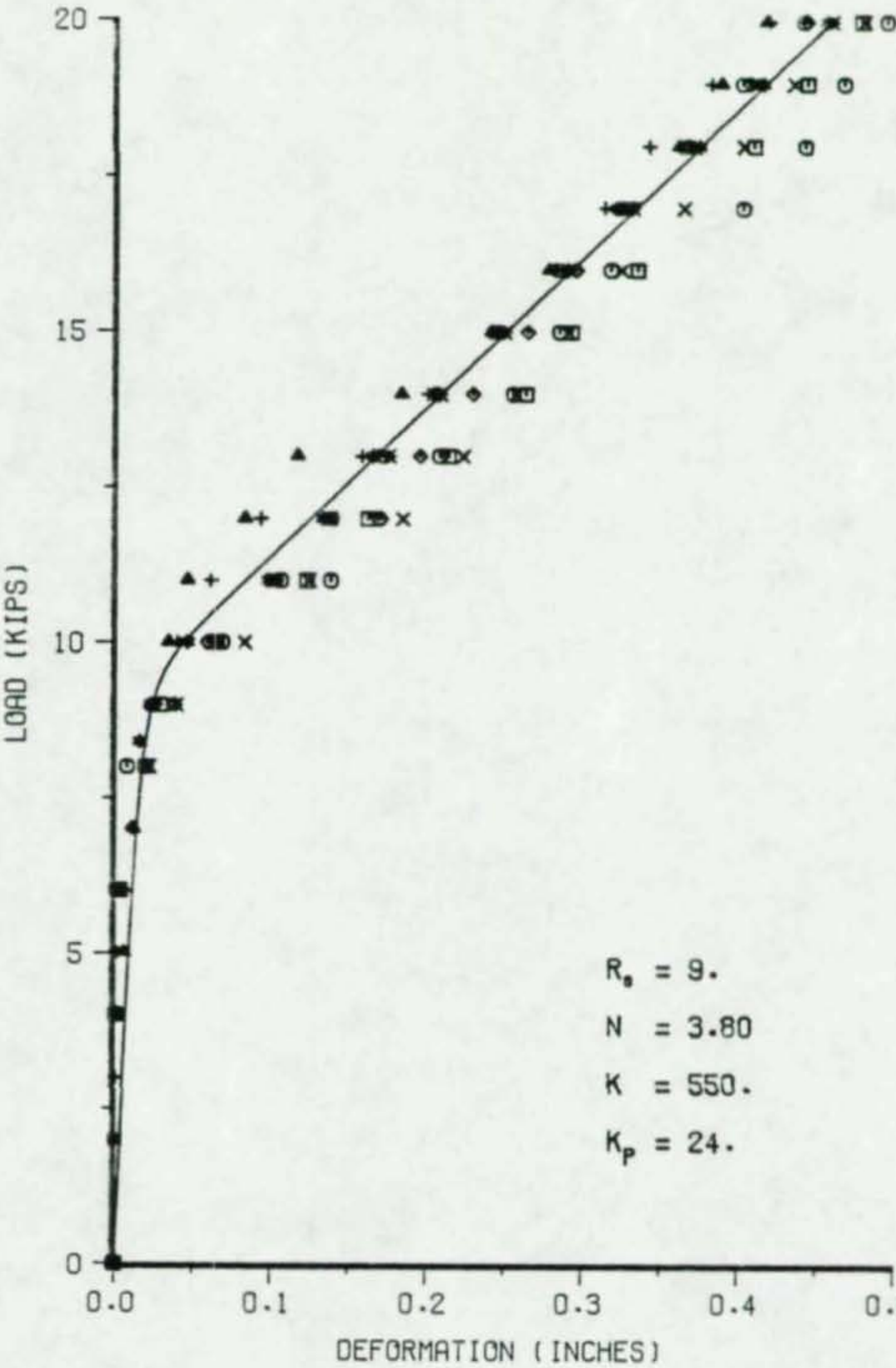



FIGURE 36 - TENSION TESTS: 1-3/4 INCH GAGE

1/4 INCH ANGLE

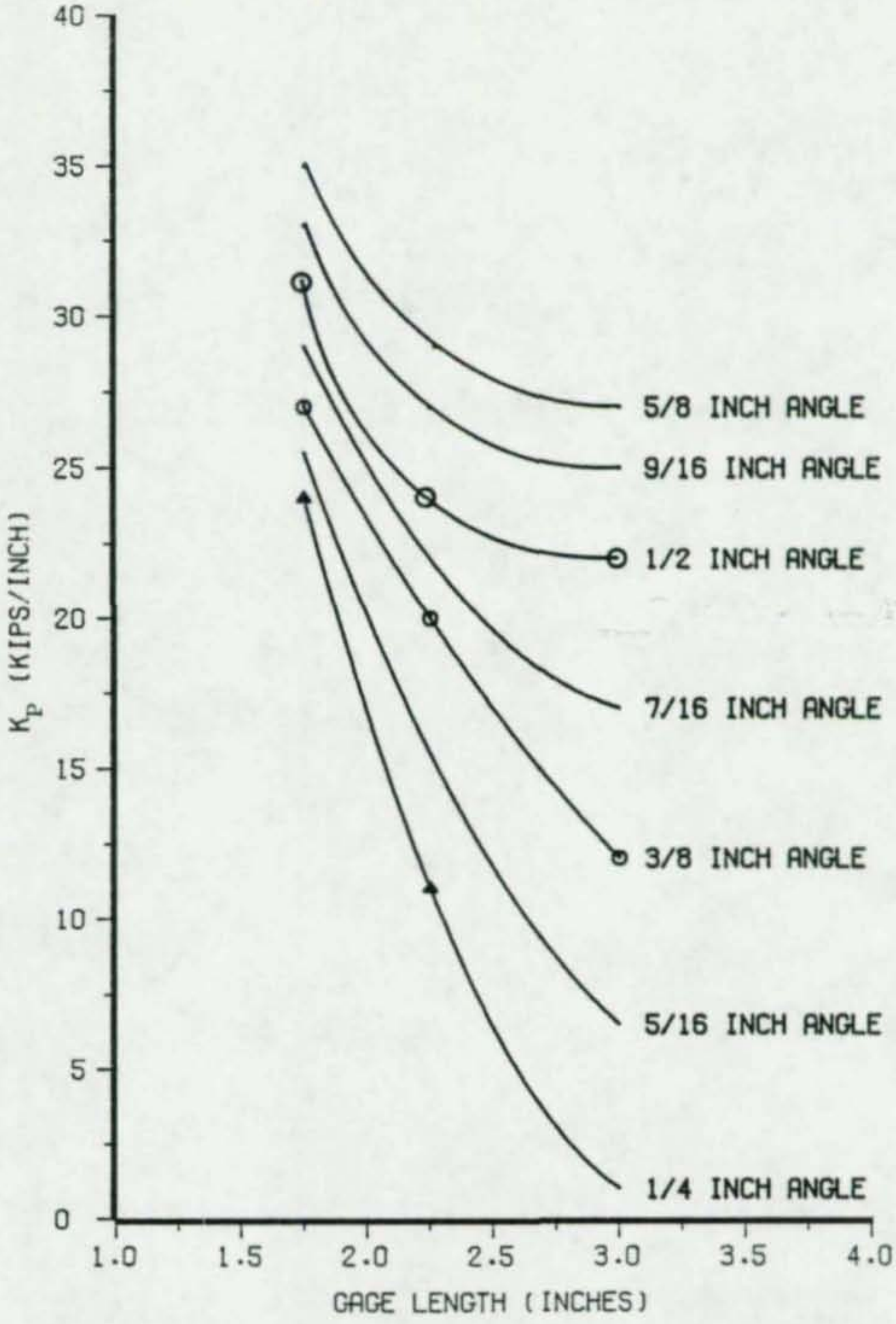



FIGURE 37 - $K_p$ FOR TENSION AS A FUNCTION OF THE GAGE LENGTH FOR VARIOUS ANGLE THICKNESSES

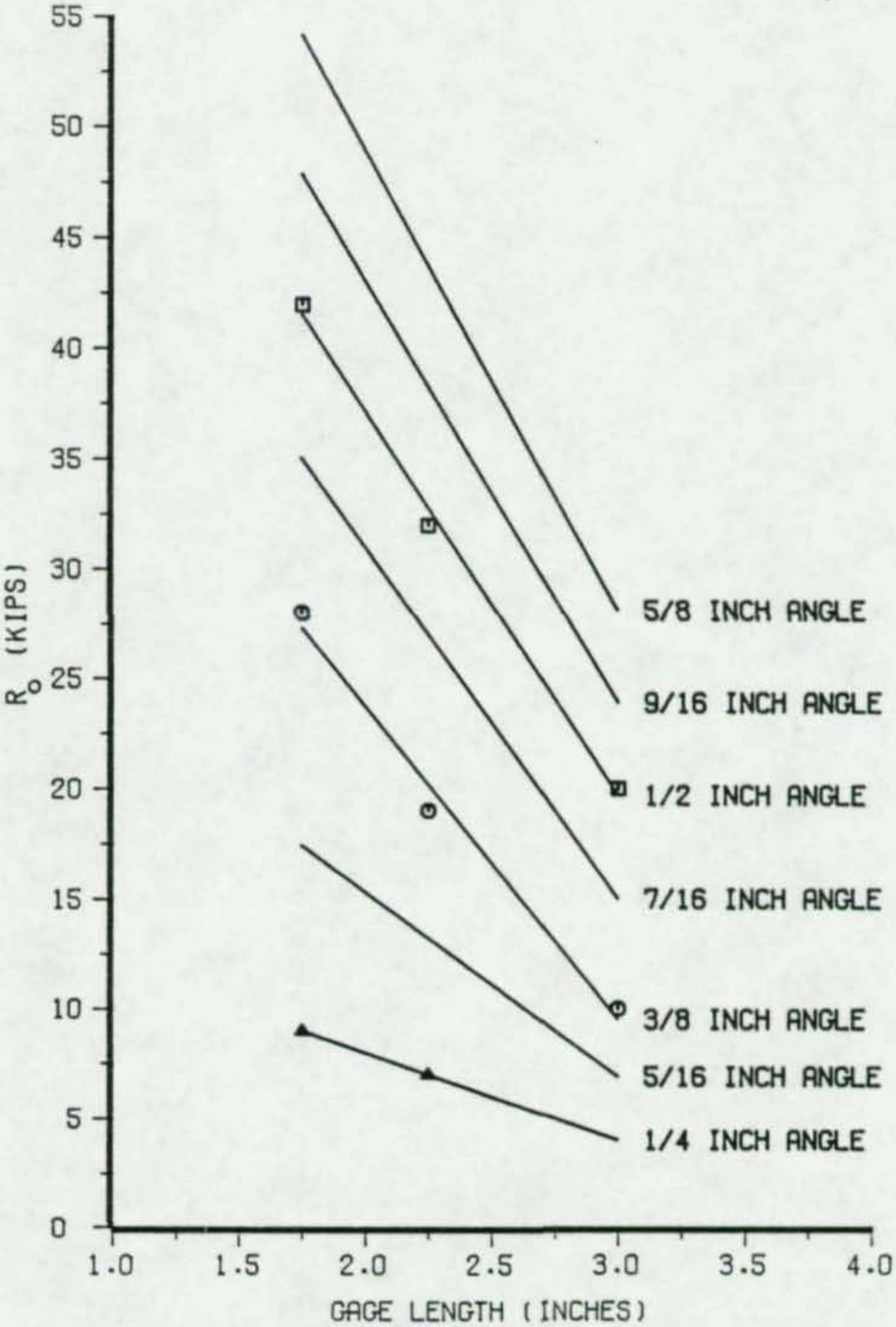



FIGURE 38 - $R_o$ FOR TENSION AS A FUNCTION OF THE GAGE LENGTH FOR VARIOUS ANGLE THICKNESSES

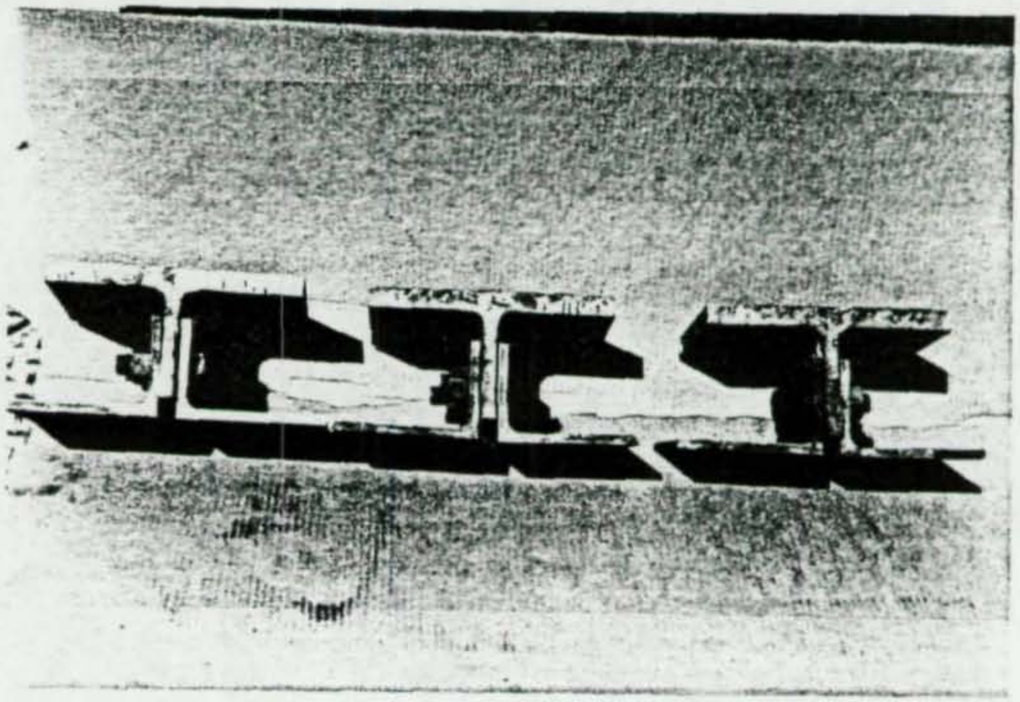


FIGURE 39 - COMPRESSION SPECIMENS
AFTER TESTING

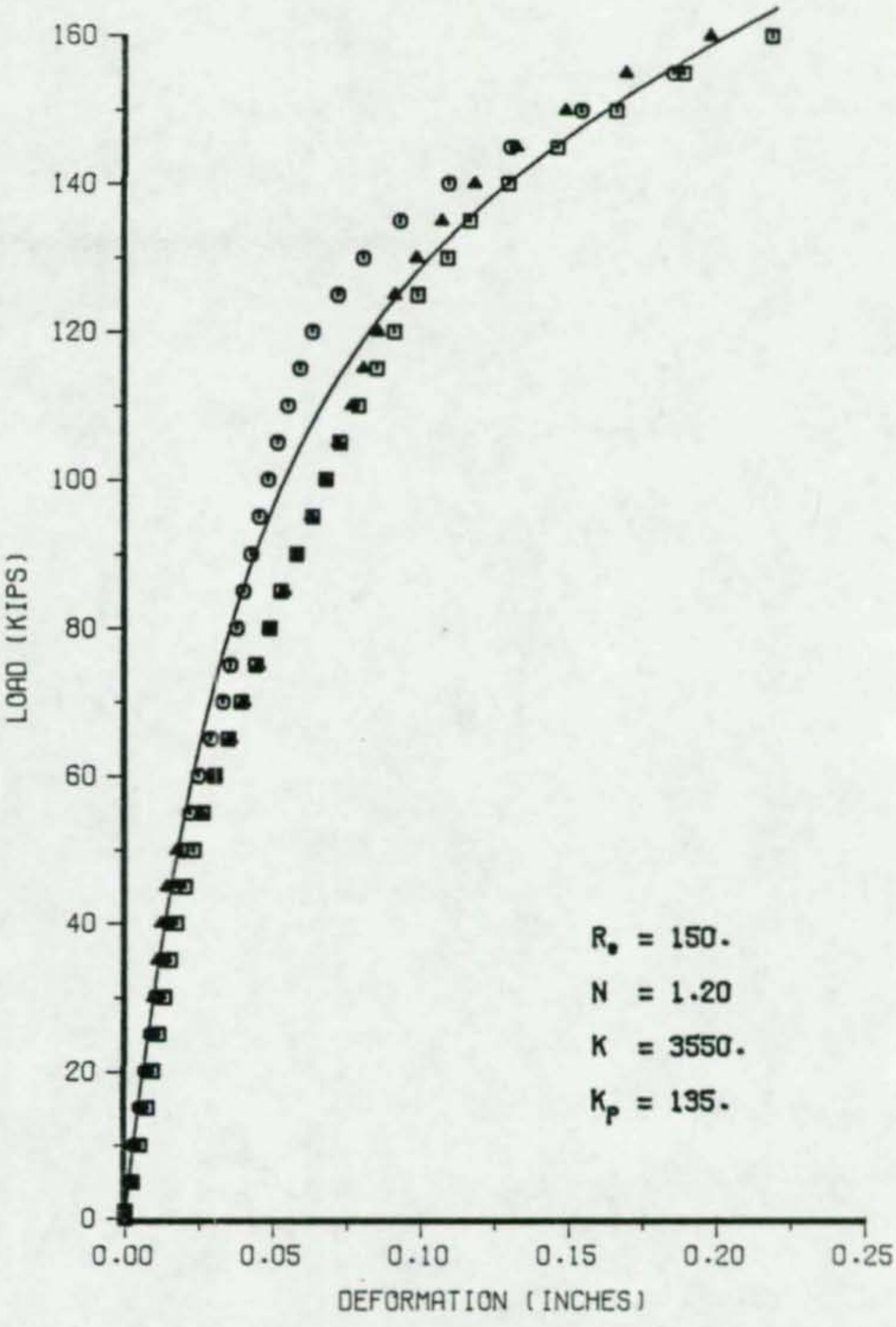



FIGURE 40 - COMPRESSION TEST: 1/2 INCH TEE
3/8 INCH ANGLES AND 7/8 INCH A325 BOLTS

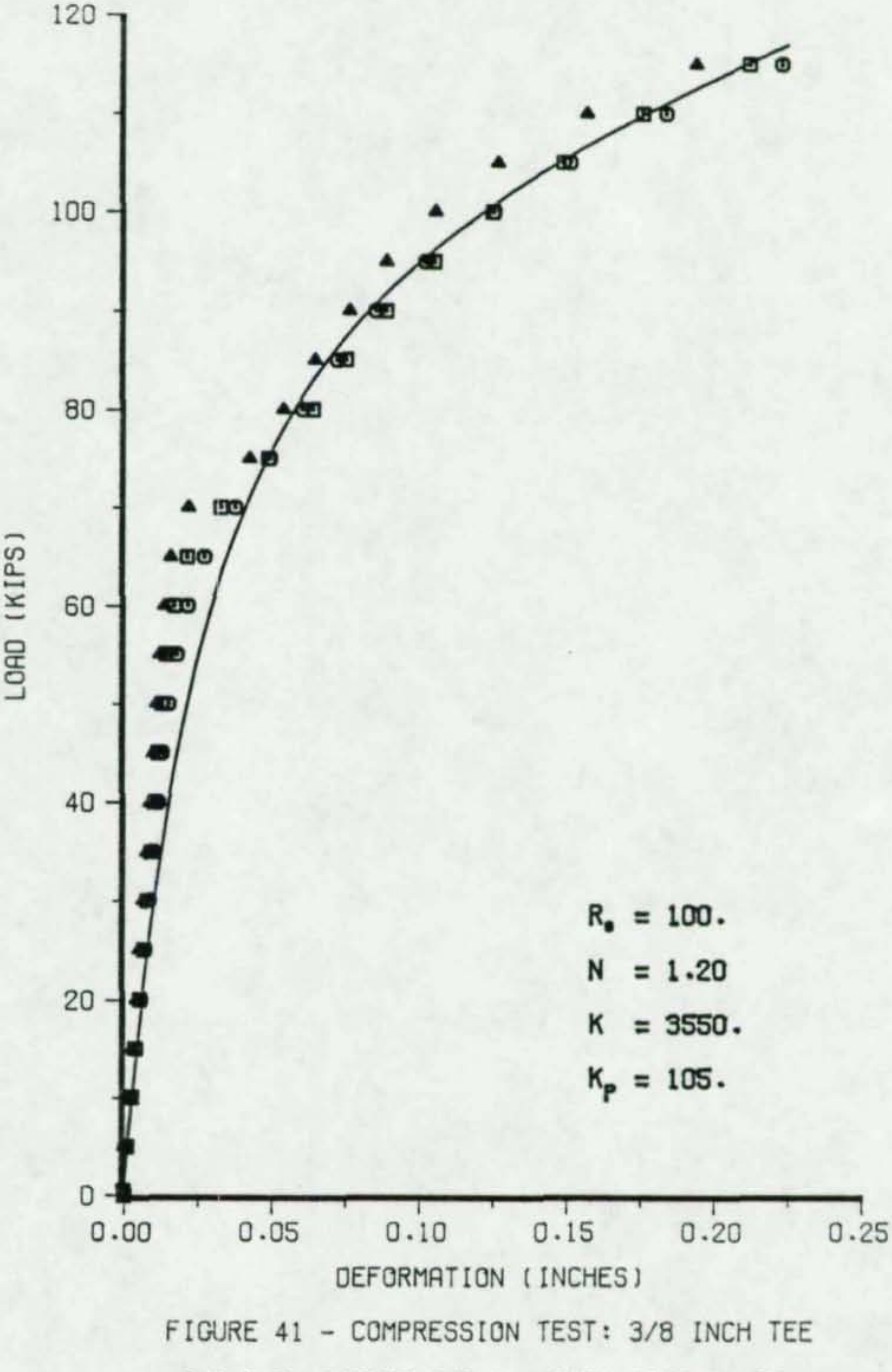



FIGURE 41 - COMPRESSION TEST: 3/8 INCH TEE

3/8 INCH ANGLES AND 3/4 INCH A325 BOLTS

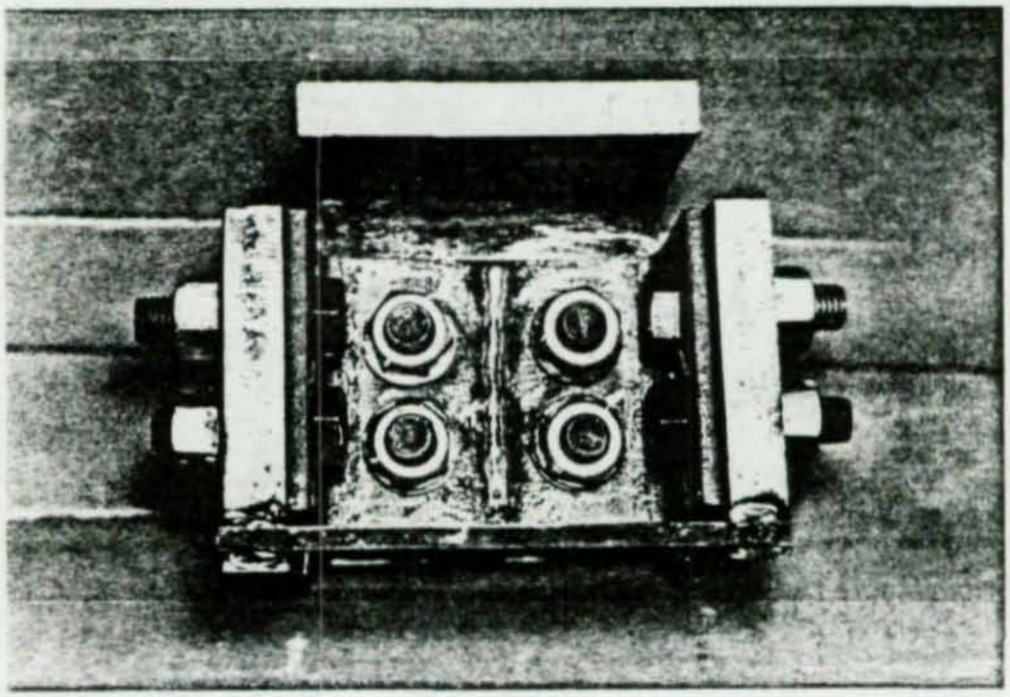


FIGURE 42 - SHEAR SPECIMEN AFTER TESTING

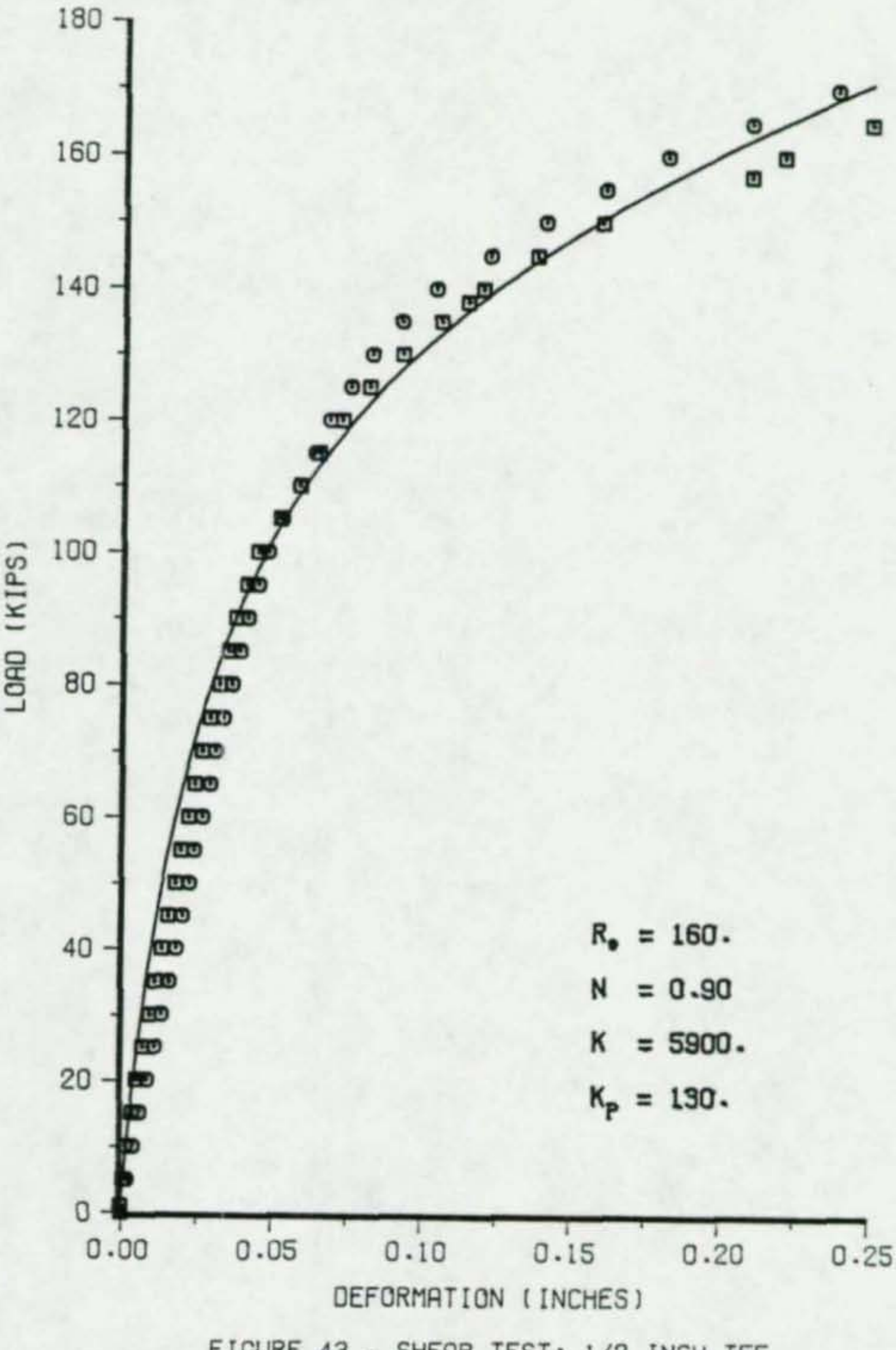



FIGURE 43 - SHEAR TEST: 1/2 INCH TEE

3/8 INCH ANGLES AND 7/8 INCH A325 BOLTS

Table 1. Geometry of the Tension Tests (Dimensions in Inches).

| Test | Gage Length | Angle Thickness | Plate Thickness | Bolt Diameter |
|---|---|---|---|---|
| T1 | 3 | 1/2 | 1/2 | 3/4 |
| T2 | 3 | 1/2 | 3/8 | 3/4 |
| T3 | 3 | 3/8 | 1/2 | 3/4 |
| T4 | 3 | 3/8 | 3/8 | 3/4 |
| T5 | 2-1/4 | 1/2 | 1/2 | 3/4 |
| T6 | 2-1/4 | 1/2 | 3/8 | 3/4 |
| T7 | 2-1/4 | 3/8 | 1/2 | 3/4 |
| T8 | 2-1/4 | 3/8 | 3/8 | 3/4 |
| T9 | 2-1/4 | 3/8 | 1/4 | 3/4 |
| T10 | 2-1/4 | 1/4 | 1/2 | 3/4 |
| T11 | 2-1/4 | 1/4 | 3/8 | 3/4 |
| T12 | 2-1/4 | 1/4 | 1/4 | 3/4 |
| T13 | 1-3/4 | 1/2 | 1/2 | 3/4 |
| T14 | 1-3/4 | 1/2 | 3/8 | 3/4 |
| T15 | 1-3/4 | 3/8 | 1/2 | 3/4 |
| T16 | 1-3/4 | 3/8 | 3/8 | 3/4 |
| T17 | 1-3/4 | 1/4 | 1/2 | 3/4 |
| T18 | 1-3/4 | 1/4 | 3/8 | 3/4 |
| T19 | 1-3/4 | 1/4 | 1/4 | 3/4 |

Table 1a. Geometry of Compression and Shear Tests (Dimensions in Inches).

| Test | Angle Thickness | Plate Thickness | Bolt Diameter |
|---|---|---|---|
| C1 | 3/8 | 1/2 | 7/8 |
| C2 | 3/8 | 3/8 | 3/4 |
| S1 | 3/8 | 1/2 | 7/8 |

Table 2. Tension Test Richard Equation Parameters (Units in Kips and Inches).

| Test | Elastic Stiffness $K$ | Plastic Stiffness $K_p$ | Reference Load $R_o$ | Richard Parameter $N$ |
|---|---|---|---|---|
| T1 | 850 | 22 | 20 | 1.0 |
| T2 | 850 | 22 | 20 | 1.2 |
| T3 | 350 | 12 | 10 | 1.5 |
| T4 | 350 | 12 | 10 | 2.5 |
| T5 | 2000 | 24 | 34 | 0.7 |
| T6 | 2000 | 22 | 29 | 0.8 |
| T7 | 850 | 20 | 20 | 0.9 |
| T8 | 850 | 20 | 19 | 1.2 |
| T9 | 850 | 18 | 19 | 1.5 |
| T10 | 250 | 11 | 7 | 2.3 |
| T11 | 250 | 11 | 7 | 2.4 |
| T12 | 250 | 11 | 7 | 2.8 |
| T13 | 4200 | 32 | 46 | 0.8 |
| T14 | 4200 | 29 | 38 | 1.3 |
| T15 | 1800 | 28 | 29 | 0.9 |
| T16 | 1800 | 26 | 26 | 0.9 |
| T17 | 550 | 24 | 9 | 3.7 |
| T18 | 550 | 24 | 9 | 3.8 |
| T19 | 550 | 24 | 9 | 3.9 |

Table 3. Recommended Richard Equation Parameters for Tension (Units in Kips and Inches).

| Gage Length | Angle Thickness | $K$ | $K_p$ | $R_o$ | $N$ |
|---|---|---|---|---|---|
| 3 | 1/2 | 850 | 22 | 20 | 1.1 |
| 3 | 3/8 | 350 | 12 | 10 | 2.0 |
| 2-1/4 | 1/2 | 2000 | 23 | 32 | 0.7 |
| 2-1/4 | 3/8 | 850 | 20 | 19 | 1.2 |
| 2-1/4 | 1/4 | 250 | 11 | 7 | 2.5 |
| 1-3/4 | 1/2 | 4200 | 31 | 42 | 1.0 |
| 1-3/4 | 3/8 | 1800 | 27 | 28 | 0.9 |
| 1-3/4 | 1/4 | 550 | 24 | 9 | 3.8 |

Table 4. Compression Test Richard Equation Parameters (Units in Kips and Inches).

| Test | Elastic Stiffness $K$ | Plastic Stiffness $K_p$ | Reference Load $R_o$ | Richard Parameter $N$ |
|---|---|---|---|---|
| C1 | 3550 | 135 | 150 | 1.2 |
| C2 | 3550 | 105 | 100 | 1.2 |

Table 5. Shear Test Richard Equation Parameters (Units in Kips and Inches).

| Test | Elastic Stiffness $K$ | Plastic Stiffness $K_p$ | Reference Load $R_o$ | Richard Parameter $N$ |
|---|---|---|---|---|
| S1 | 5900 | 130 | 100 | 0.9 |

## APPENDIX A

## THE RICHARD EQUATION

The Richard Equation, developed by Richard and Abbott, 1975 is the elastic-plastic, stress-strain formula defining the force-deformation curves presented in this report. The Richard Equation is a continuous analytical expression that describes the relationship between the strength and stiffness of the system in question as follows:

$$R = \frac{K_1 \times \Delta}{\left[1 + \left|\frac{K_1 \times \Delta}{R_o}\right|^N\right]^{1/N}} + K_p \times \Delta$$

where:

$R$ = Load

$\Delta$ = Deformation

$R_o$ = Intersection of a line asymptotic to the curve at a slope equal to $K_p$

$N$ = The sharpness of the transition in slope from K to $K_p$

$K_p$ = The plastic stiffness, or final slope of the curve

$K_1$ = $(K - K_p)$, where K is the elastic stiffness, or the initial slope of the curve

## APPENDIX B

## REFERENCES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3. Crawford, S. F. and Kulak, G. L., "Eccentrically Loaded Bolted Connections," Journal of the Structural Division, ASCE, March 1971.

4. Irish, D. J., "Strength of Connection Elements," M.S. Thesis, Department of Civil Engineering and Engineering Mechanics, University of Arizona, Tucson, Arizona, 1983.

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8. Hamm. K. R. and Richard, R. M., "The Analysis and Behavior fo Deep Bolted Angle Connections," M. S. Thesis, Department of Civil Engineering and Engineering Mechanics, University of Arizona, Tucson, Arizona, 1984.