1273

INVESTIGATION AND ANALYSIS OF WELDED FRAMED BEAM CONNECTIONS

Interim Report prepared for the

American Institute of Steel Construction

Marc C. LeBouton
AISC Fellow

and

Ralph M. Richard Ph.D.
Professor of Civil Engineering
and Engineering Mechanics
and Optical Sciences

The University of Arizona
Tucson, Arizona 85721

January, 1987

The following is an interim report prepared for the American Institute of Steel Construction by Marc C. LeBouton and Ralph M. Richard of the Department of Civil Engineering and Engineering Mechanics, University of Arizona, Tucson, Arizona. The report is the result of research on welded double framing angle connections from September to December, 1986. The research is being funded by an Educational Fellowship from the American Institute of Steel Construction.

TABLE OF CONTENTS

ABSTRACT

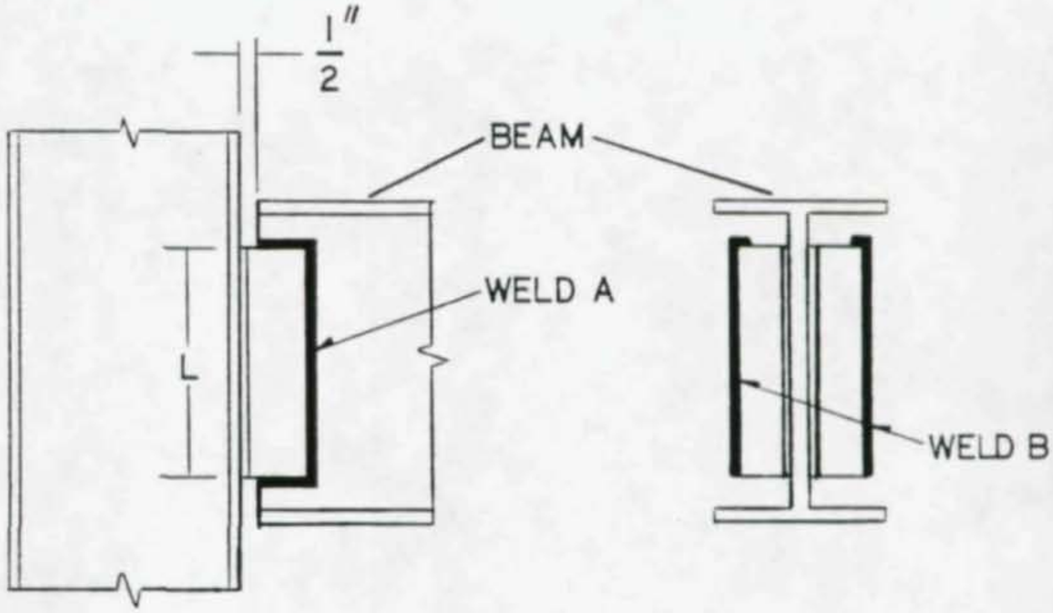
The purpose of the following research is two-fold: first, to establish the moment-rotation characteristics of several common welded double angle framing connections; and second, to review Weld A and Weld B in the Welded Framed Beam Connections Design Table IV on pages 4-36 and 4-37 of the Eighth Edition of the Manual of Steel Construction by the American Institute of Steel Construction.

The results of this research indicate that the weld sizes given for Weld A in the AISC manual are adequate. The weld sizes given for Weld B, however, are undersized. Weld B is under-designed because the connection moment was not properly accounted for in the design of Weld B.

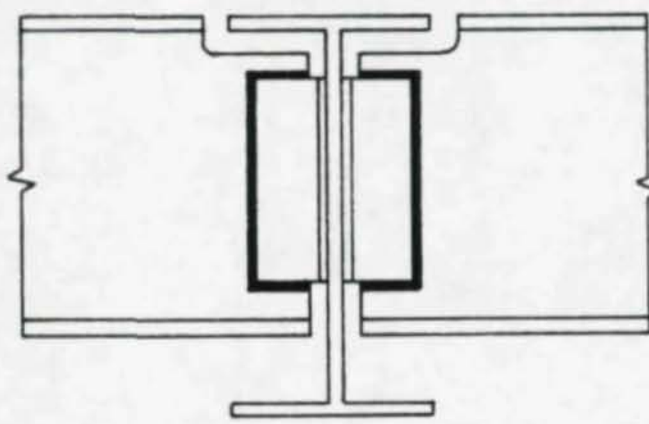
INTRODUCTION

Framed beam connections are used to connect beams to girders and girders to columns. The design of these connections involves the properties of steel angles which are given in the AISC manual. These angle connections may be fastened to the girder web or to the column flange by high strength bolts, fillet welds, or a combination of bolts in one leg of the angle and welds along the other leg. This research involves framed beam connections that are welded to the outstanding legs of the angle (Figure 1).

In order to analyze the structural behavior of double angle connections, the Richard equation (Appendix A) is used to analytically define the load-deformation characteristics of double angle framing connections. A typical Richard curve is defined by four parameters: the elastic stiffness or initial slope of the curve, the plastic stiffness or final slope of the curve, the reference load or the intercept of a line asymptotic to the plastic stiffness with the vertical axis, and the shape parameter, a dimensionless parameter that defines the sharpness of transition between the elastic stiffness and plastic stiffness (Figure 2). This equation, in short, defines the deformation associated with load for a given double angle connection. This deformation may be in the form of a translational displacement associated with a force, or it may be in the form of a rotation associated with an applied moment (Figure 3).

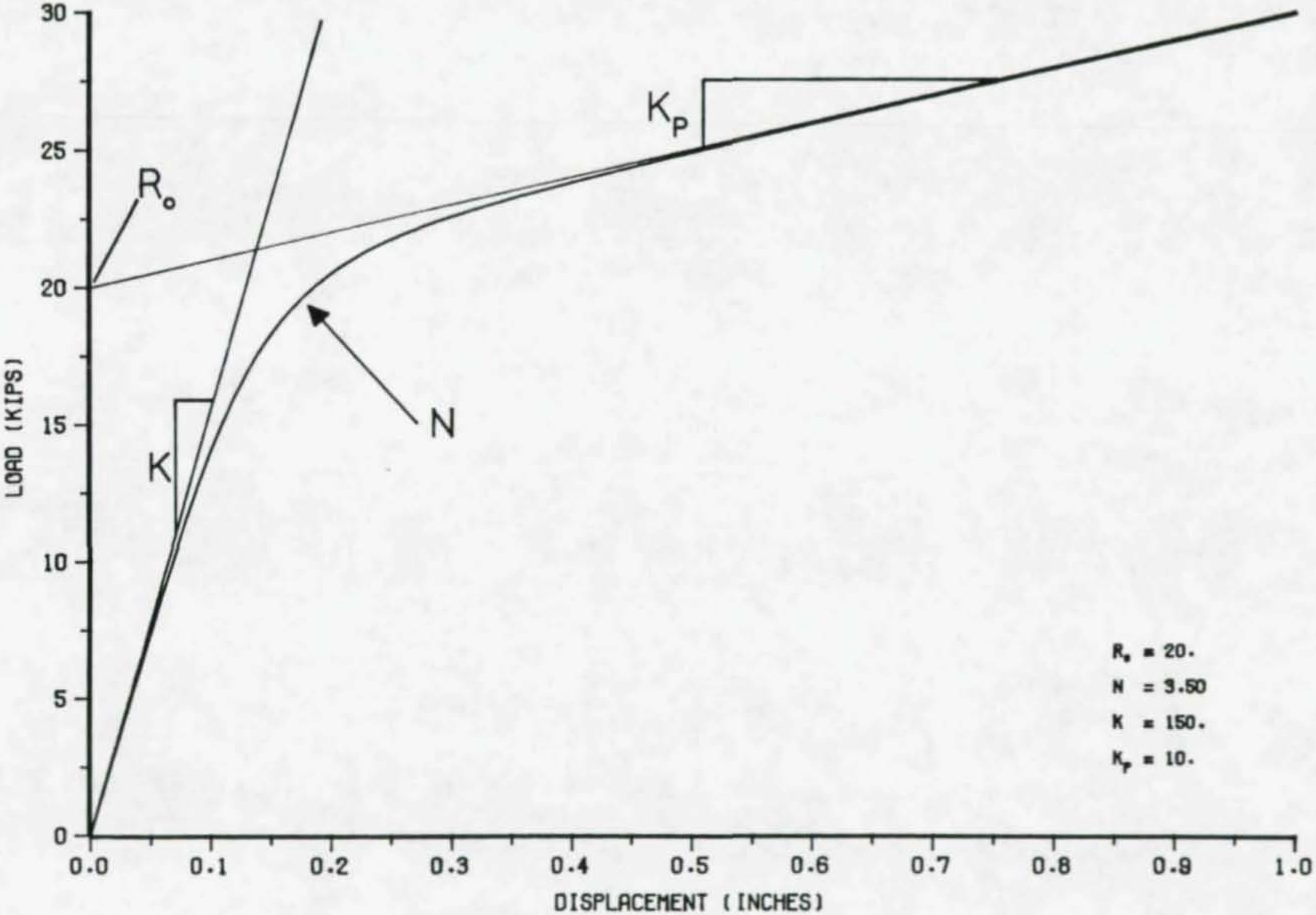


a) Beam web to column flange connection.



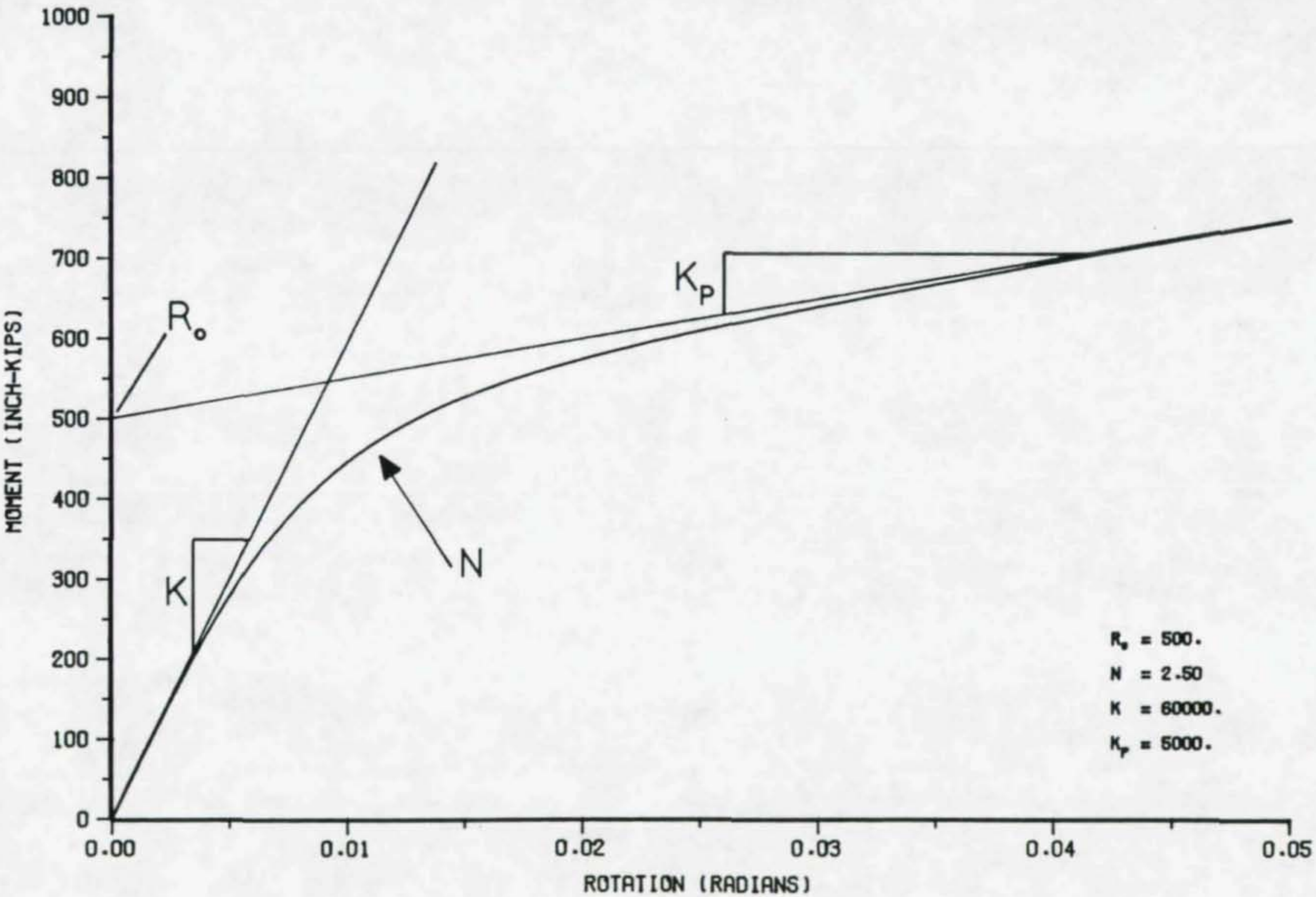
b) Beam web to beam web connection.

Figure 1. Typical Framed Beam Connections.



GENERAL FORCE DEFORMATION CURVE WITH RICHARD PARAMETERS

Figure 2.



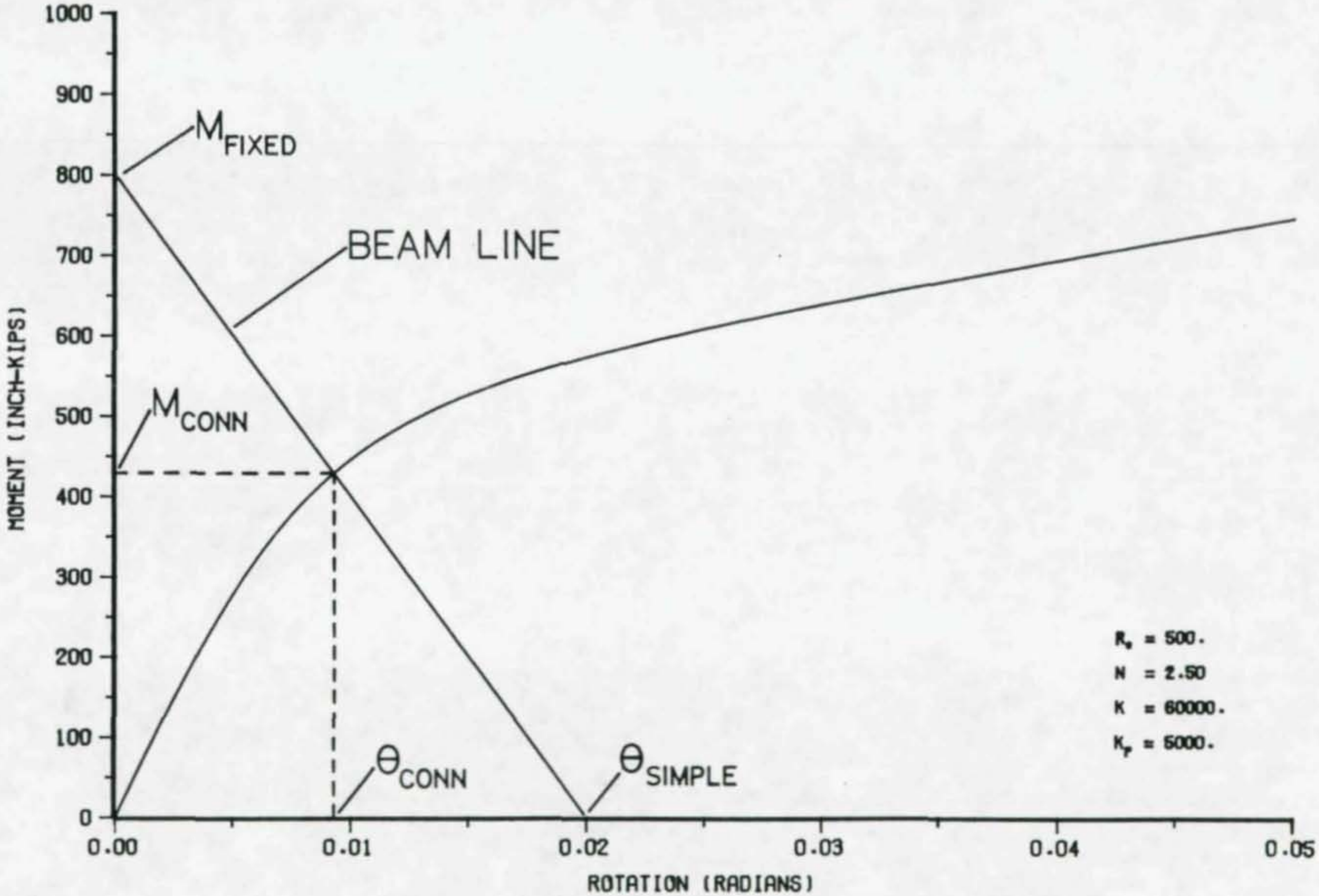
MOMENT ROTATION CURVE WITH RICHARD PARAMETERS FOR A TYPICAL CONNECTION

Figure 3.

When the beam shown in Figure 1 is loaded with a given loading, a displacement and rotation will occur at the ends of the beam. Blewitt and Richard [1] used the Richard equation to describe force-deformation curves for three inch segments of angle clip connections based on experimental testing. Hsia and Richard [2] used the force-deformation characteristics of three inch connection segments to generate moment-rotation curves for various connection geometries. If the moment-rotation curve for a particular connection is known, the moment which occurs at the connection can be determined.

In order to determine the restraining moment which occurs at the connection, the beam line [3] is superimposed on the moment-rotation curve and the intersection of these two curves gives the restraining moment and end rotation that occur at the connection for a beam with a given loading (Figure 4). With the restraining moment known, the welds at the connection can be designed for strength and safety.

This research reviews the strength of Weld A and Weld B in the Welded Framed Beam Connections Design Table IV on pages 4-36 and 4-37 of the Eighth Edition of the <u>Manual of Steel Construction</u> by the American Institute of Steel Construction (AISC). In the AISC design guide, Weld B was designed for only a vertical shear force and a torsional moment in a plane normal to the beam axis which were assumed to act along the interface of the column flange and the angle clip legs. Weld A, however, is designed for this shear force <u>and</u> the torsional moment that this force causes about the centroid of Weld A (see Figure 5).



MOMENT ROTATION CURVE WITH BEAM LINE FOR A TYPICAL CONNECTION

FIXED END MOMENT = 800.0 INCH-KIPS FREE END ROTATION = 0.02 RADIANS

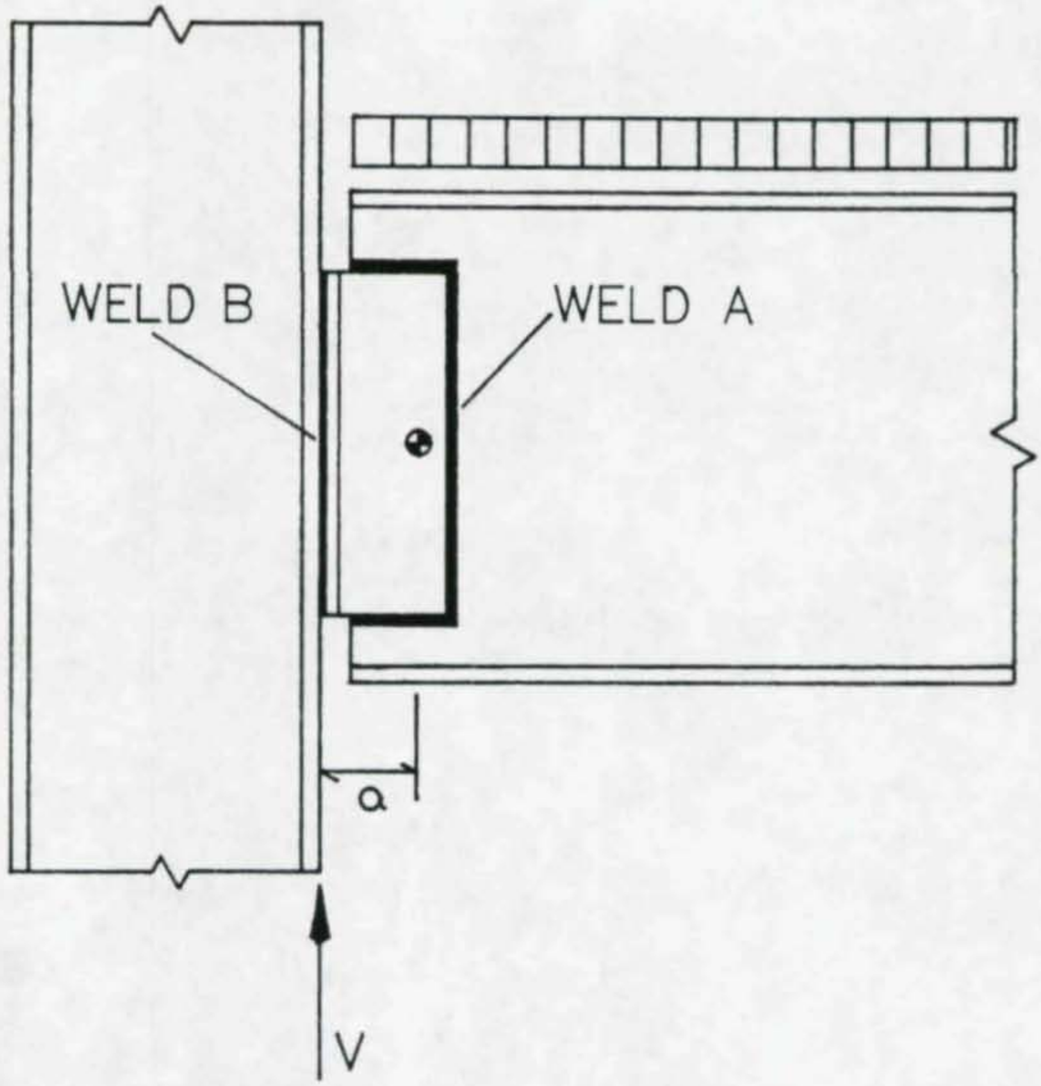
Figure 4.

This design model does not account for the connection moment as shown in Figure 6. The shear force acts at an eccentricity ,e, from the centroid of Weld A, so that <u>both</u> Weld A and Weld B should be designed for both shear and moment acting on the welds (Figure 6).

This is accomplished in the following three steps:

1) Force-deformation curves for double framing angle geometries are generated from physical tests.
2) Moment-rotation curves are then derived from the force-deformation curves for various connections.
3) The beam line for a given loading condition is then superimposed on the moment-rotation curve for the connection under consideration to determine the end moment.

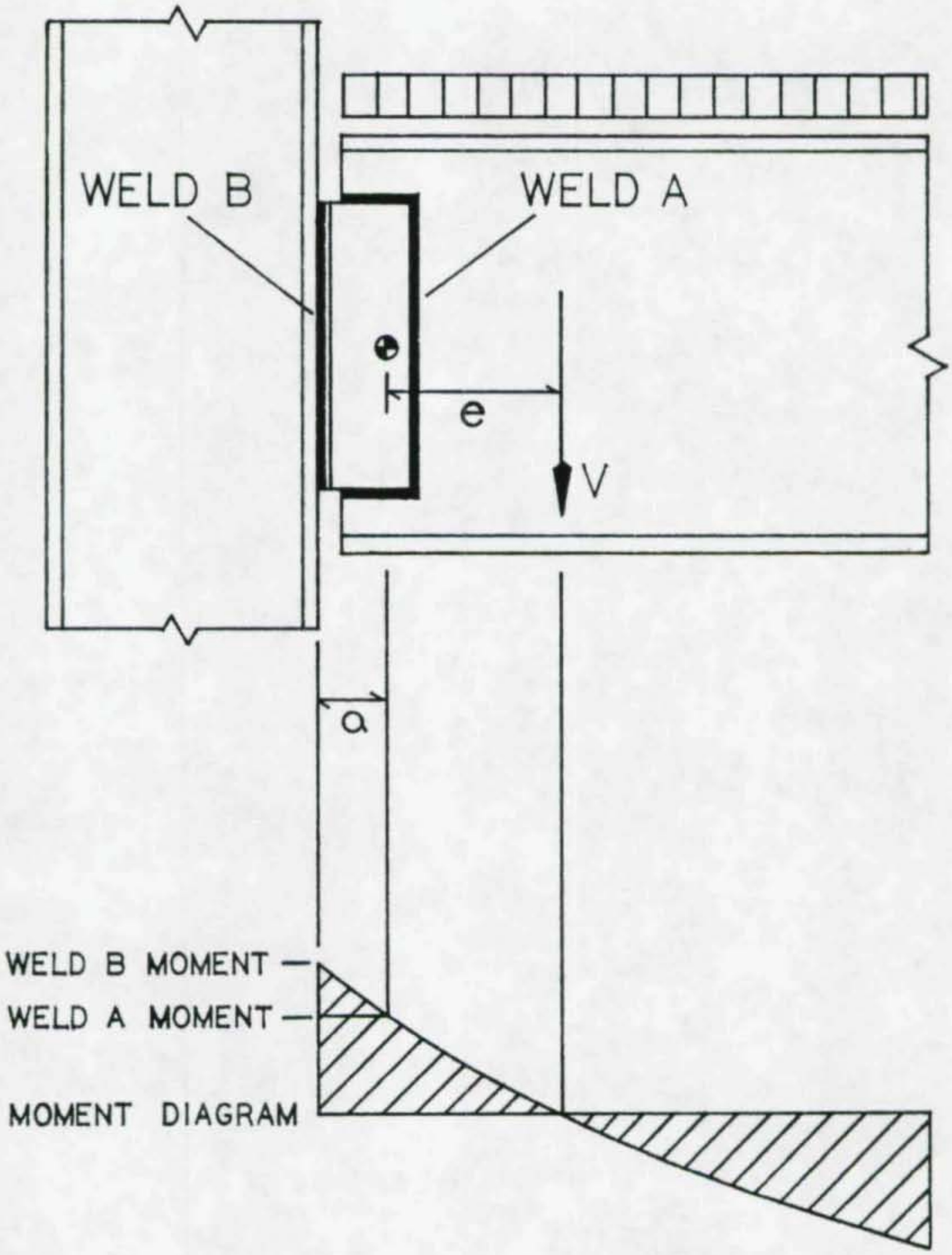
These welds are designed and compared with those given in the AISC manual.



Weld B: Designed for shear force V.

Weld A: Designed for shear force V and torsional Moment M = V x a

Figure 5. AISC design guide philosophy for design of Weld A and Weld B.



Weld A: Designed for shear force V and torsional moment M = V x e

Weld B: Designed for shear force V and bending moment M = V x (a + e)

Figure 6. Research design philosophy for Weld A and Weld B.

FORCE-DEFORMATION CURVES

The Richard equation (Appendix A) is used to analytically describe the force-deflection characteristics of a segment of a welded double framing angle connection. The parameters of this curve are determined from physical testing of segments of welded double framing angle connections. Astaneh and McMullin of the University of Oklahoma performed the physical tests for four different connection angle geometries:

1) L 4 X 3-1/2 X 1/4 X 3 inch segment
2) L 4 X 3-1/2 X 3/8 X 3 inch segment
3) L 4 X 3-1/2 X 1/2 X 3 inch segment
4) L 5 X 3-1/2 X 5/8 X 3 inch segment

For any one segment of connection, a different Richard curve is found for each of the various modes of deformation the connection segment exhibits. There are three modes of deformation to consider: 1) displacement caused by a tensile force; 2) displacement caused by a compressive force; and 3) displacement caused by a shear force. These three modes of deformation in relation to a beam to column connection are shown in Figure 7.

Tension tests

The tension test configuration is shown in Figure 8. In each test, an initial load was applied to the angles to allow the bolts to slip into bearing against the connecting plate and angles. Under static loading conditions (approximately ninety minutes from start to failure) force and displacement readings

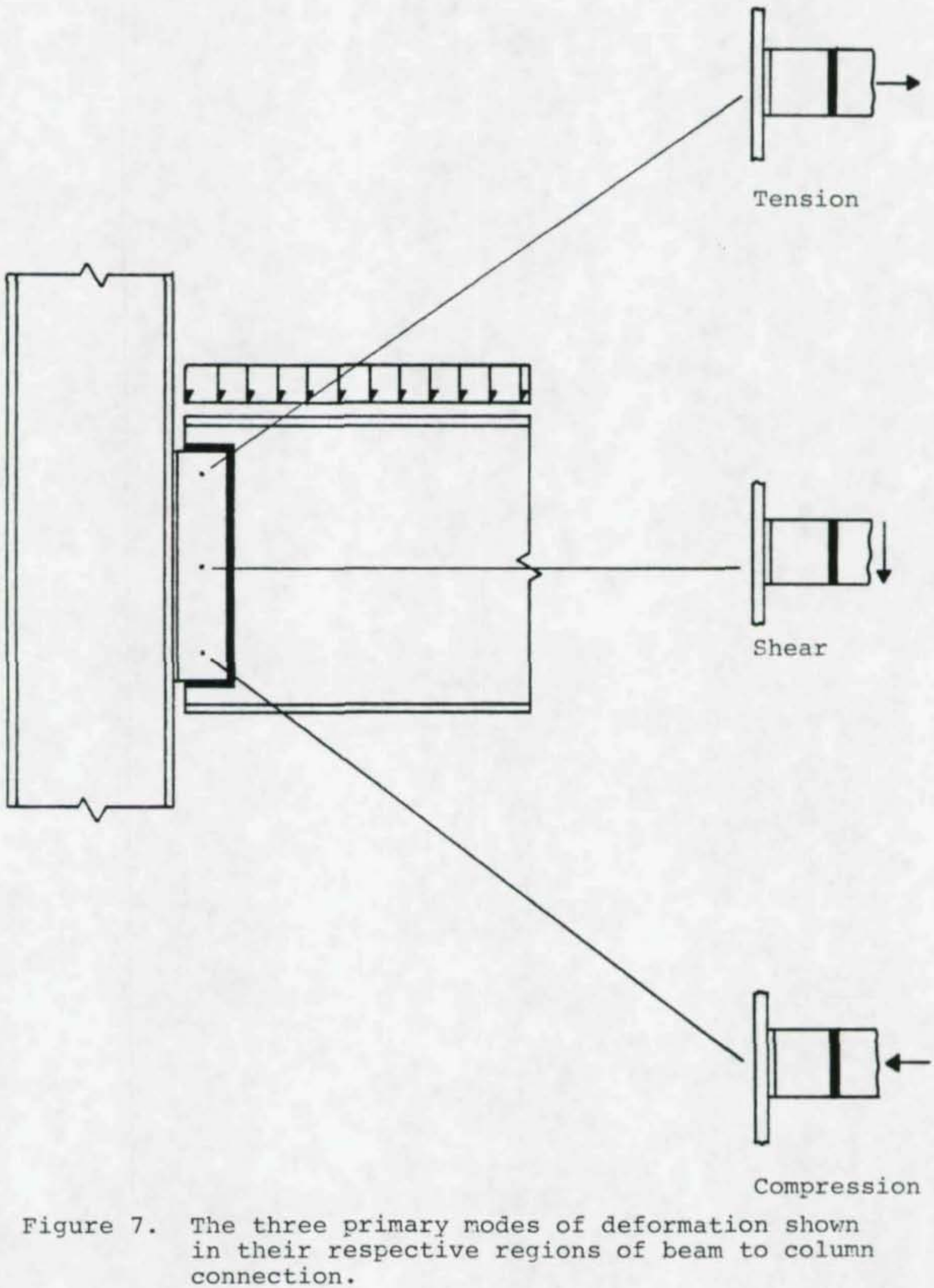


Figure 7. The three primary modes of deformation shown in their respective regions of beam to column connection.

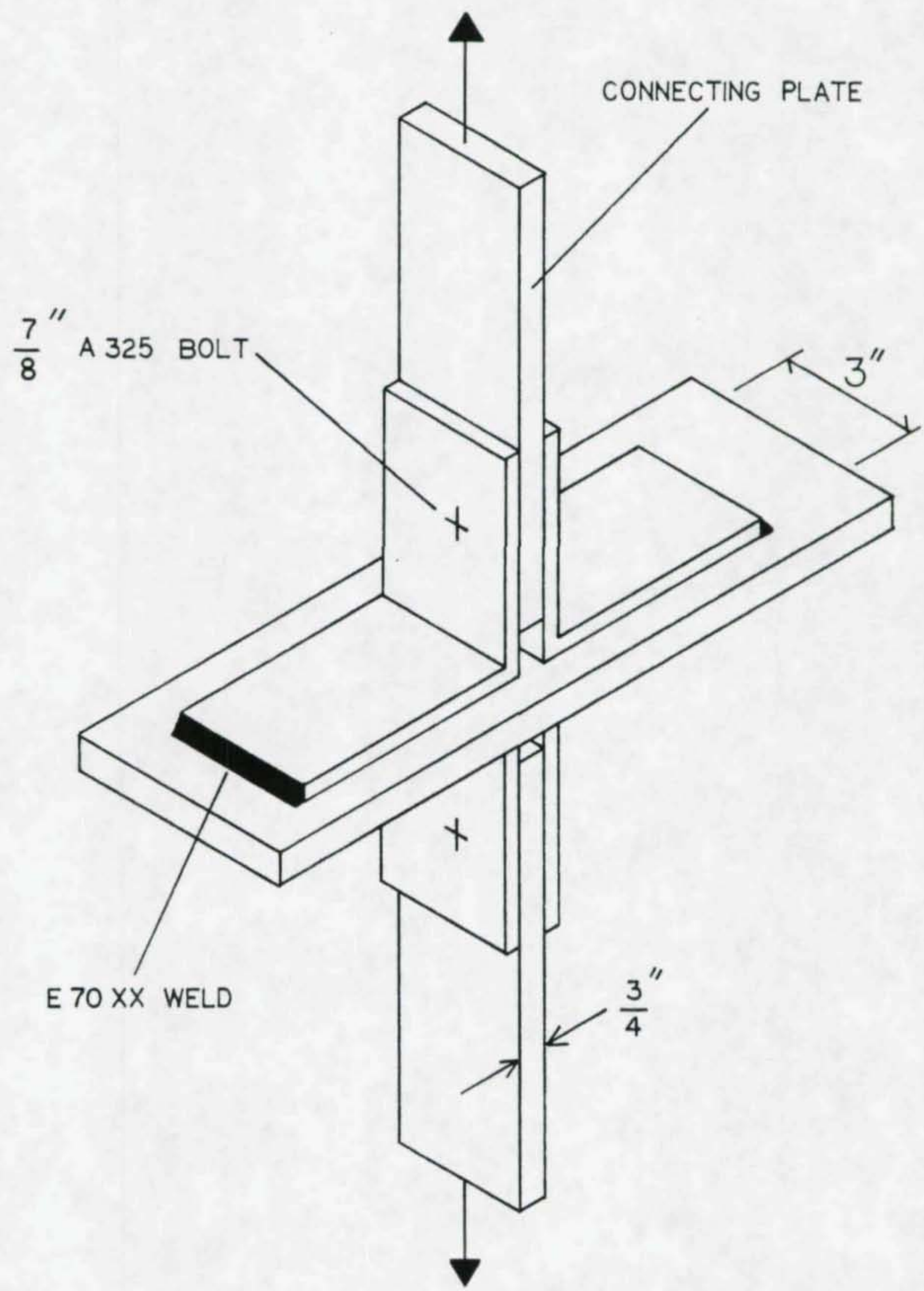


Figure 8. Tension test configuration.

were taken at sufficient intervals until failure occurred. In each test, the failure mode was weld fracture. This force and displacement data is given in Tables 1 through 5.

As explained previously, a Richard curve is defined by the elastic stiffness of the angles K, the plastic stiffness of the angles KP, the reference load R, and the shape parameter N. These values are determined by fitting a least squares curve to the experimental data given in Tables 1-5. The values of KP, R, and N are determined on a trial and error basis using a least squares criterion. The value of K, the elastic stiffness, however, must be determined beforehand using principles of structural mechanics.

To determine the elastic stiffness of the angles, K, the outstanding leg of the angle with the weld is considered as a beam fixed at one end and simply supported at the other (Figure 9). The beam has a modulus of elasticity E, a moment of inertia I, and a length g which is the length of the outstanding leg L minus the dimension k, a detailing dimension given in the AISC manual. The dimension k is subtracted from the overall length because k defines the critical section where the slope of the outstanding leg becomes zero.

Using the moment-area method of structural mechanics along with the stiffness method of structural analysis, the elastic stiffness of the welded double angles is given below (Figure 9).

$$K = [2] \times 3EI/g^3$$

Using $E = 30{,}000$ ksi

$g = L - k$ (inches)

TABLE 1 FORCE-DEFORMATION DATA FOR WELDED DOUBLE ANGLE CONNECTION: 2-L 4 X 3-1/2 X 1/4 X 3 INCH

Load (kips)	Displacement (inches)	
	top angles	bottom angles
0.08	0.000	0.000
0.50	0.016	0.014
1.00	0.046	0.041
1.60	0.085	0.079
2.09	0.119	0.115
2.50	0.147	0.145
3.01	0.207	0.218
3.51	0.320	0.343
4.06	0.443	0.474
6.50	1.125	1.063

TABLE 2 FORCE-DEFORMATION DATA FOR WELDED DOUBLE ANGLE CONNECTION: 2-L 4 X 3-1/2 X 3/8 X 3 INCH

Load (kips)	Displacement (inches)	
	top angles	bottom angles
0.15	0.000	0.000
0.20	0.002	0.001
1.00	0.011	0.022
2.00	0.033	0.037
4.00	0.083	0.083
5.15	0.183	0.184
5.50	0.256	0.236
6.25	0.353	0.294
6.25	0.423	0.335
6.45	0.444	0.355
8.35	0.750	0.625

TABLE 3 FORCE-DEFORMATION DATA FOR WELDED DOUBLE ANGLE CONNECTION: 2-L 4 X 3-1/2 X 3/8 X 3 INCH

Load (kips)	Displacement (inches)	
	top angles	bottom angles
0.26	0.000	0.000
0.50	0.002	0.003
1.04	0.009	0.011
2.06	0.023	0.025
3.01	0.036	0.038
3.99	0.050	0.053
5.13	0.070	0.078
6.07	0.111	0.153
7.15	0.220	0.282
7.52	0.263	0.330
8.08	0.319	0.391
8.56	0.361	0.437
10.50	0.625	0.594

TABLE 4 FORCE-DEFORMATION DATA FOR WELDED DOUBLE ANGLE CONNECTION: 2-L 4 X 3-1/2 X 1/2 X 3 INCH

Load (kips)	Displacement (inches)	
	top angles	bottom angles
0.35	0.000	0.000
1.02	0.002	0.002
1.50	0.004	0.004
2.00	0.006	0.006
3.14	0.011	0.011
4.08	0.017	0.015
5.25	0.025	0.021
6.04	0.029	0.025
7.05	0.036	0.033
8.12	0.046	0.041
9.05	0.055	0.049
10.05	0.067	0.059
11.12	0.085	0.075
12.07	0.118	0.108
13.10	0.167	0.156
13.54	0.194	0.185
14.08	0.231	0.247
14.56	0.269	0.284
15.02	0.314	0.318
15.65	0.369	0.356
16.07	0.413	0.383
16.57	-----	0.407
17.00	-----	0.439
17.50	-----	0.514
17.50	0.500	0.438

TABLE 5 FORCE-DEFORMATION DATA FOR WELDED DOUBLE ANGLE CONNECTION: 2-L 5 X 3-1/2 X 5/8 X 3 INCH

Load (kips)	Displacement (inches)	
	top angles	bottom angles
0.00	0.000	0.000
0.98	0.011	0.008
1.70	0.013	0.012
2.00	0.015	0.013
3.00	0.021	0.018
4.02	0.028	0.024
5.02	0.036	0.032
6.04	0.047	0.039
7.03	0.057	0.045
8.05	0.067	0.053
9.26	0.078	0.063
10.00	0.086	0.069
11.04	0.100	0.084
12.02	0.125	0.109
12.70	0.168	0.132
13.00	0.187	0.171
13.42	0.211	0.190
14.00	0.241	0.220
14.51	0.294	0.247
14.73	0.322	0.263
15.01	0.378	0.273
15.25	0.423	0.290
15.35	0.439	0.303
15.66	0.498	0.379
15.99	0.498	0.379
16.26	0.517	0.402
16.50	0.534	0.428
17.00	0.563	0.468
17.25	0.577	0.490
17.50	0.591	0.509
17.99	0.614	0.544
18.50	0.638	0.580
21.40	0.750	0.750

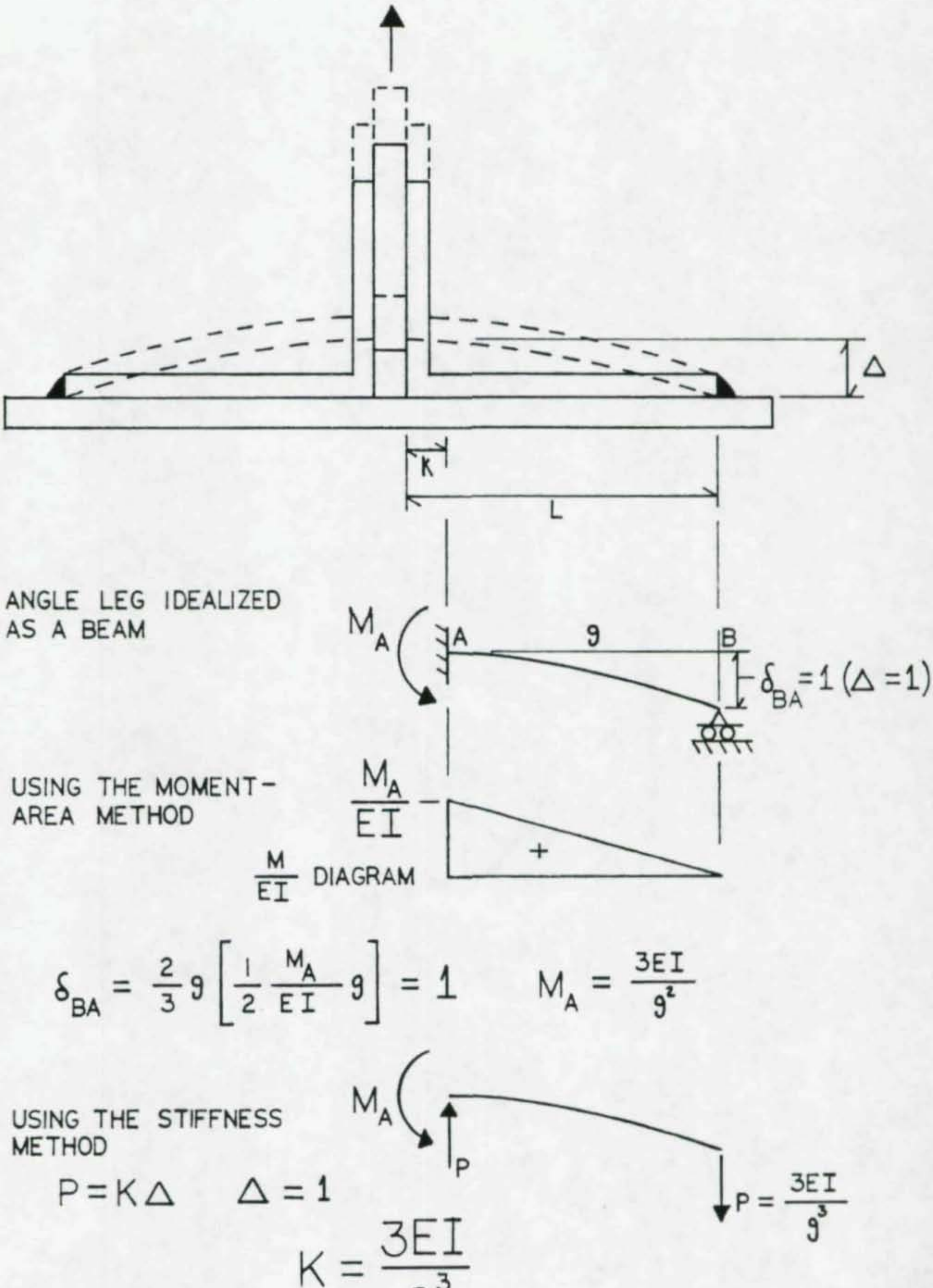


Figure 9. Elastic stiffness of tension specimens.

$I = (1/12) \times (\text{base}) \times (\text{height})^3$

$= (1/12) \times (3 \text{ inch}) \times (t)^3$

$= [t]^3/4$

t = angle leg thickness in inches

the elastic stiffness of two welded double angles (3 inch segments) becomes

$$K = 45{,}000 \times [t/g]^3 \qquad \text{(Kips/inch).}$$

The elastic stiffnesses for each of the four tension test specimens are given in Table 6.

The elastic stiffness along with the data points were input into the computer program XYPLOT (Williams) for each tension test. Program XYPLOT contains a subroutine RCFIT (Gillett and Hormby) which gives the least squares Richard curve fit and supplies the Richard parameters KP, R, and N. Figures 10 through 13 give the force-deformation curves for the four welded double angle specimens in tension.

Compression tests

Physical testing for compression was not necessary since the four Richard parameters have previously been established for a three inch segment of double framing angles loaded in compression. Blewitt and Richard [1] have developed the following empirical formulas for the four Richard parameters for a three inch segment of bolted framing angles loaded in compression.

$$K = 180{,}000 \times [t/1.75] \qquad \text{(Kips/inch)}$$

$$KP = 138 \times [t_c/8] \qquad \text{(Kips/inch)}$$

$$R = 142 \times [t_c/8] \qquad \text{(Kips)}$$

TABLE 6 ELASTIC STIFFNESS, K, FOR TENSION TEST SPECIMENS

Angles	L	k	t	g	K
L-4 x 3-1/2 x 1/4	4	11/16	1/4	3.3125	19
L-4 x 3-1/2 x 3/8	4	13/16	3/8	3.1875	73
L-4 x 3-1/2 x 1/2	4	15/16	1/2	3.0625	196
L-5 x 3-1/2 x 5/8	5	1-1/8	5/8	3.8750	189

L = length of outstanding angle leg in inches

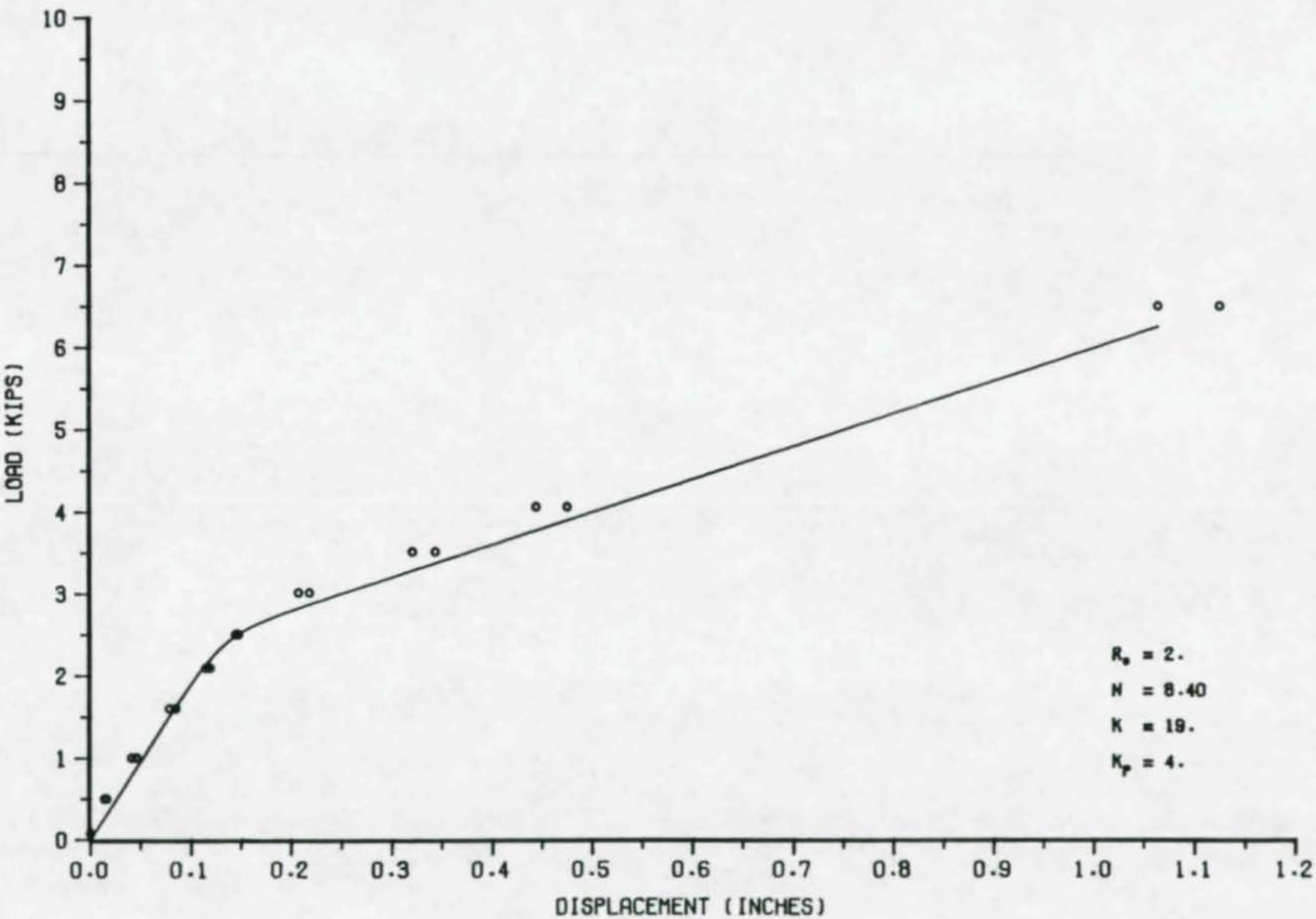
k = AISC dimensioning detail in inches

t = angle thickness in inches

g = L - k

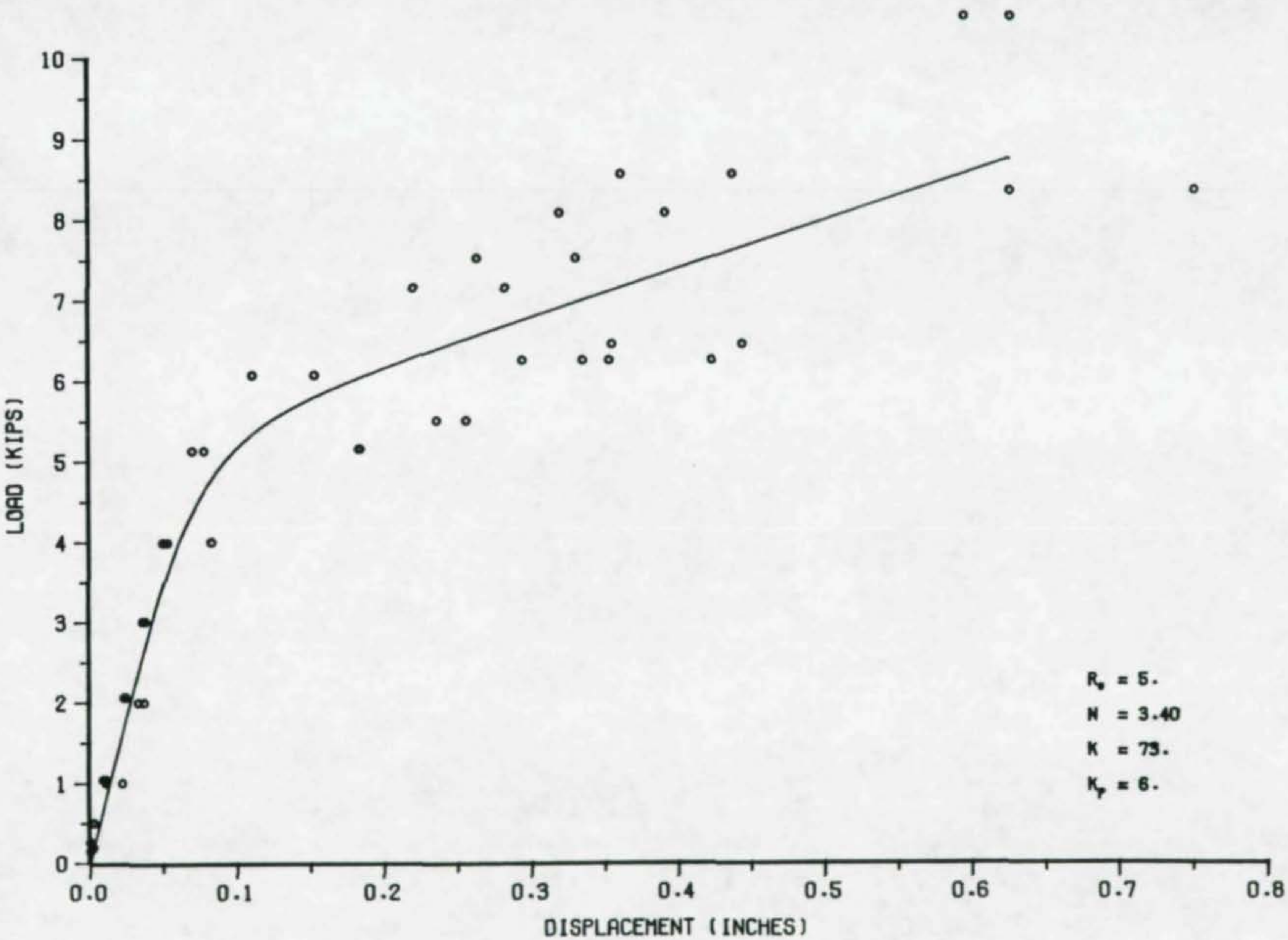
K = elastic stiffness of 3 inch segment of double angles loaded in tension (kips/inch)

$K = 45{,}000 \times [t/g]^3$



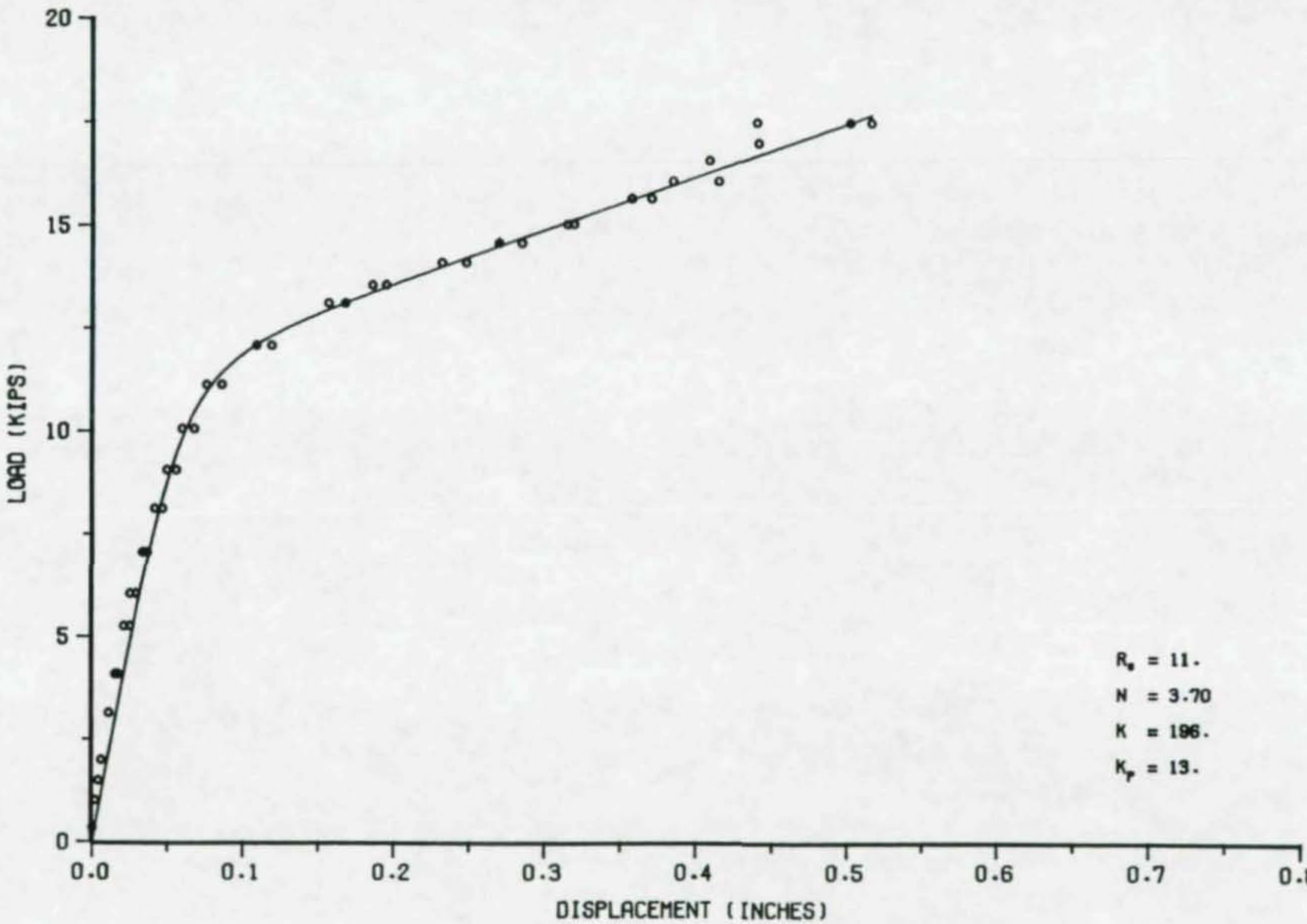
FORCE DEFORMATION CURVE 2-L4 X 3-1/2 X 1/4 X 3

Figure 10.



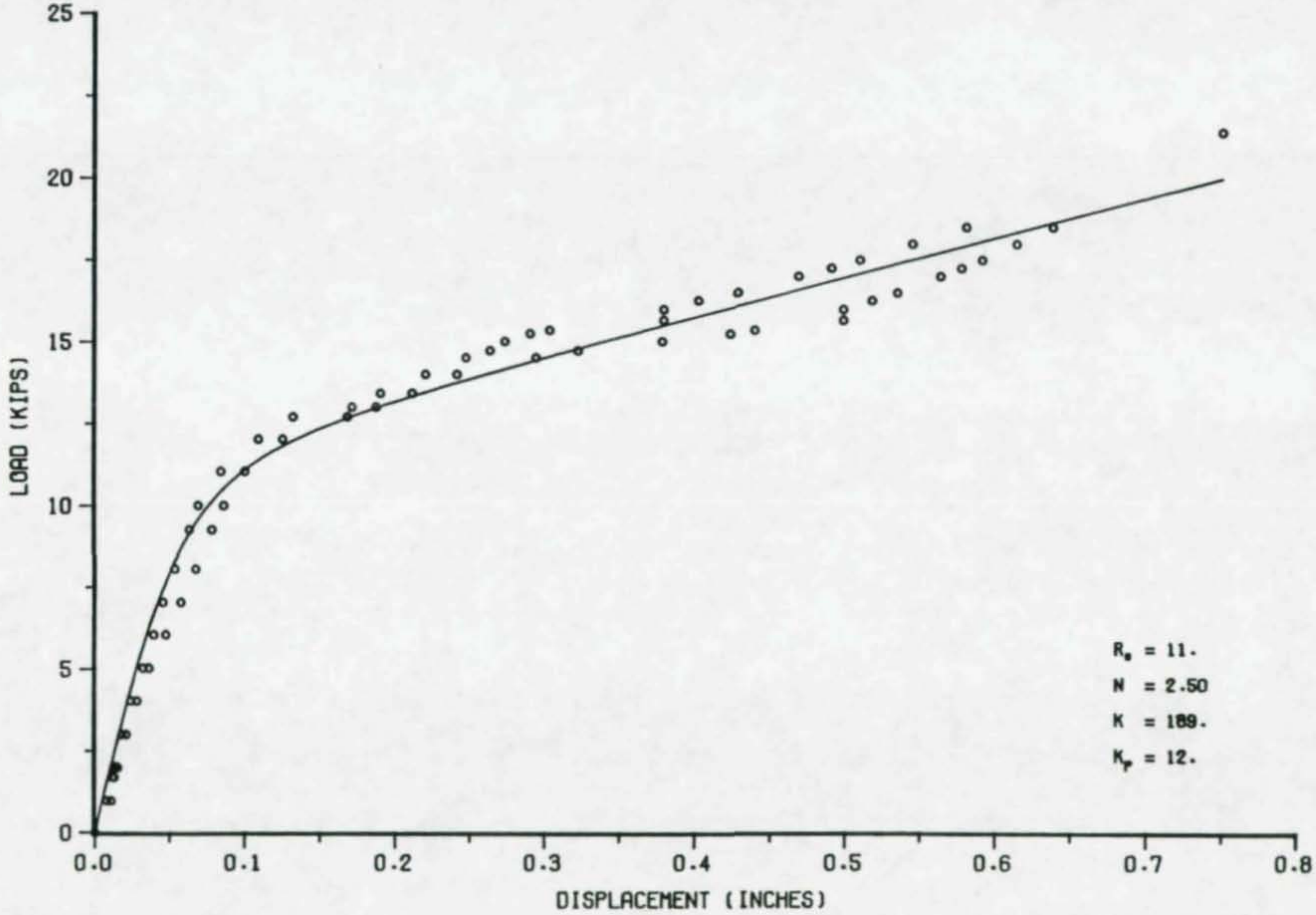
FORCE DEFORMATION CURVE 2-L4 X 3-1/2 X 3/8 X 3

Figure 11.



FORCE DEFORMATION CURVE 2-L4 X 3-1/2 X 1/2 X 3

Figure 12.



FORCE DEFORMATION CURVE 2-L5 X 3-1/2 X 5/8 X 3

Figure 13.

$N = 1.2$

where

t = angle leg thickness in inches

t_p = connecting plate thickness

t_c = critical thickness

= t_p or $2t$ in sixteenths of an inch whichever is smaller.

The fact that the above formulas were developed for compression specimens with bolts in the outstanding angle legs and this research involves welds along the outstanding angle legs is irrelevant. The Richard parameters for compression are only dependent on bearing considerations of the angles, and not on any flexural action of the angles, in which case the support conditions would in fact make a difference in the Richard parameters. The Richard parameters for the four compression specimens are given in Table 7.

Shear tests

Hsia and Richard [2] demonstrated that the deformation caused by shearing forces in a double angle connection are negligible compared to deformations caused by tensile and compressive forces. This agrees with an intuitive understanding of the structural behavior of a double framing angle connection like that shown in Figure 7. Most of the deformation results from the tensile and compressive forces at the connection which are resisting the applied loads, and very little from the shearing forces caused by the loads.

In summary, the Richard parameters for the three inch segments of welded double angle connections loaded in tension

TABLE 7 RICHARD EQUATION PARAMETERS FOR THREE INCH SEGMENTS OF WELDED DOUBLE ANGLES LOADED IN COMPRESSION

Angles	Richard Parameters						
	t	t_p	t_c	K_c	KP_c	R_c	N_c
L-4 x 3-1/2 x 1/4	1/4	3/4	1/2	525	138	142	1.2
L-4 x 3-1/2 x 3/8	3/8	3/4	3/4	1771	207	213	1.2
L-4 x 3-1/2 x 1/2	1/2	3/4	3/4	4198	207	213	1.2
L-5 x 3-1/2 x 5/8	5/8	3/4	3/4	8200	207	213	1.2

t = angle leg thickness in inches

t_p = connecting plate thickness in inches

t_c = critical thickness in <u>sixteenths of an inch</u>
= t_p or $2t$ whichever is smaller

K_c = $180{,}000 \times [t/1.75]^3$ in kips/inch

KP_c = $138 \times [t_c/8]$ in kips/inch

R_c = $142 \times [t_c/8]$ in kips

N_c = 1.2

and loaded in compression are given in Table 8. These values are used in the next section to develop the moment-rotation curves for various connections.

TABLE 8 RICHARD EQUATION PARAMETERS FOR THREE INCH SPECIMENS OF WELDED DOUBLE ANGLES LOADED IN TENSION AND IN COMPRESSION

Angles	Tension Parameters				Compression Parameters			
	K_t	KP_t	R_t	N_t	K_c	KP_c	R_c	N_c
L-4 x 3-1/2 x 1/4	19	4	2	8.4	525	138	142	1.2
L-4 x 3-1/2 x 3/8	73	6	5	3.4	1771	207	213	1.2
L-4 x 3-1/2 x 1/2	196	13	11	3.7	4198	207	213	1.2
L-5 x 3-1/2 x 5/8	189	12	11	2.5	8200	207	213	1.2

K_t, KP_t, K_c, and KP_c are in kips/inch

R_t, and R_c are in kips

MOMENT-ROTATION CURVES

The force-deformation curves defined previously establish the characteristics of a <u>three inch segment</u> of welded double framing angles. Therefore, by "stacking" several three inch segments one on top of the other, a connection of any length can be constructed.

Hsia and Richard [2] used this concept to develop moment-rotation curves for connections of various lengths. They idealized the angle clip connection as a series of three inch rigid bars with a non-linear spring attached to each bar. The force-deformation characteristics of each spring are given by the force-deformation curves established earlier. When the beam is loaded, the ends of the beam will rotate about a point of rotation. Since the connection angles are an integral part of the beam, they must also rotate about this rotation point. This means that some of the non-linear springs used to model the connection will be acted upon by tensile forces and some will be acted upon by compressive forces (shearing deformations are neglected). This concept is illustrated in Figure 14.

For a given end rotation of the connection, the forces that are developed in the non-linear springs must obey the laws of equilibrium. Therefore, by summing moments of forces about the rotation point, the moment that occurs at the connection for a specified rotation of the connection or beam end can be determined. If the end rotation is increased, different forces

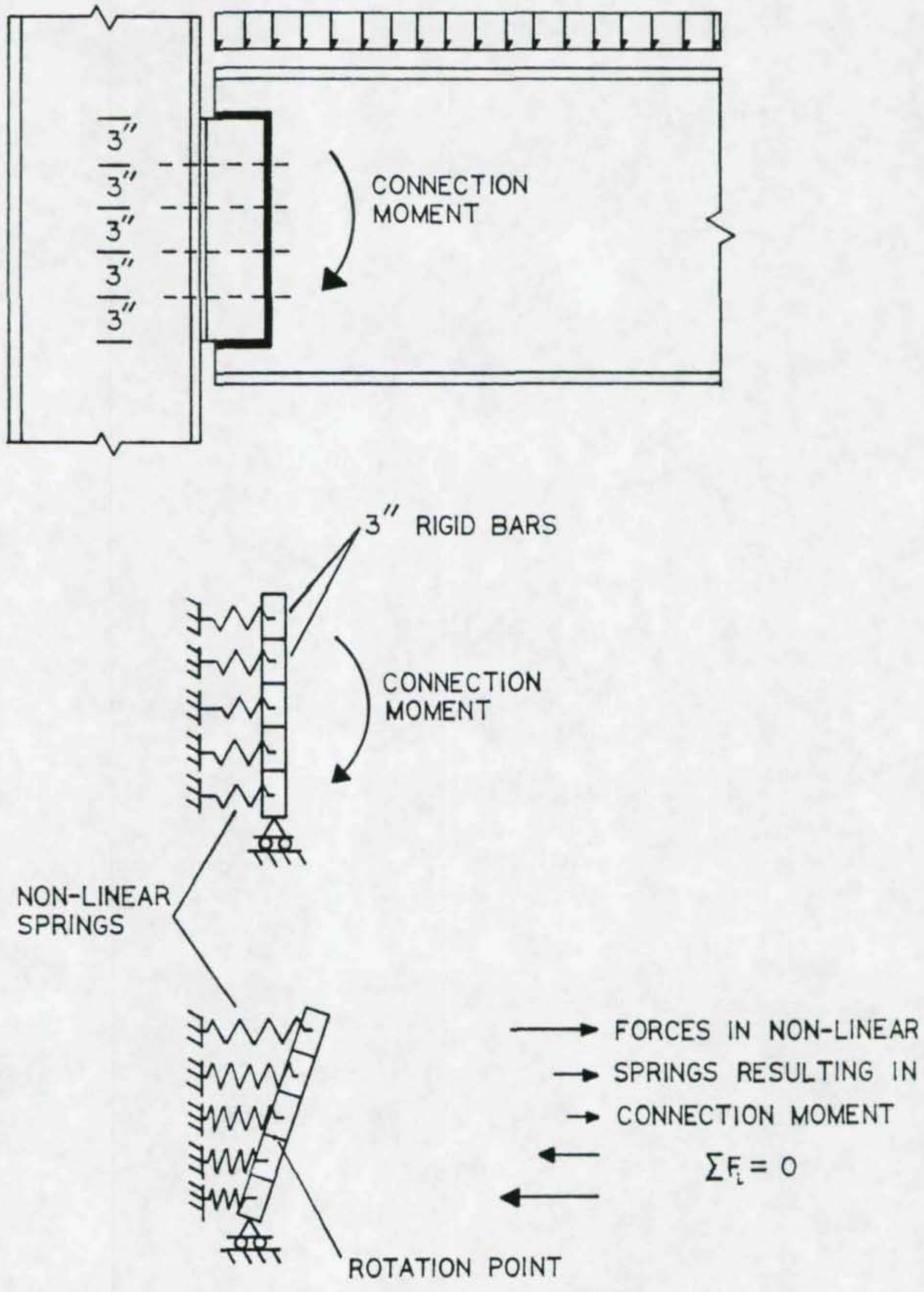


Figure 14. Connection modeled with rigid bars and non-linear springs.

will result in the non-linear springs. Repeating this process, moment-rotation curves are then generated. Using a least squares fit of the Richard equation (Appendix A), an analytical expression for these moment-rotation curves is obtained.

Given in appendix B is a Fortran program called MRCURVE which was adapted from a similar program developed by Hsia and Richard [2]. This program calculates the moment and rotation data points and also gives the four Richard parameters associated with the curve passing through these data points.

The numerical procedure outlined above and used in program MRCURVE was compared by Hsia and Richard [2] to a more advanced non-linear finite element procedure. The two methods gave essentially the same results.

Program MRCURVE was used to develop moment-rotation curves for the following connections.

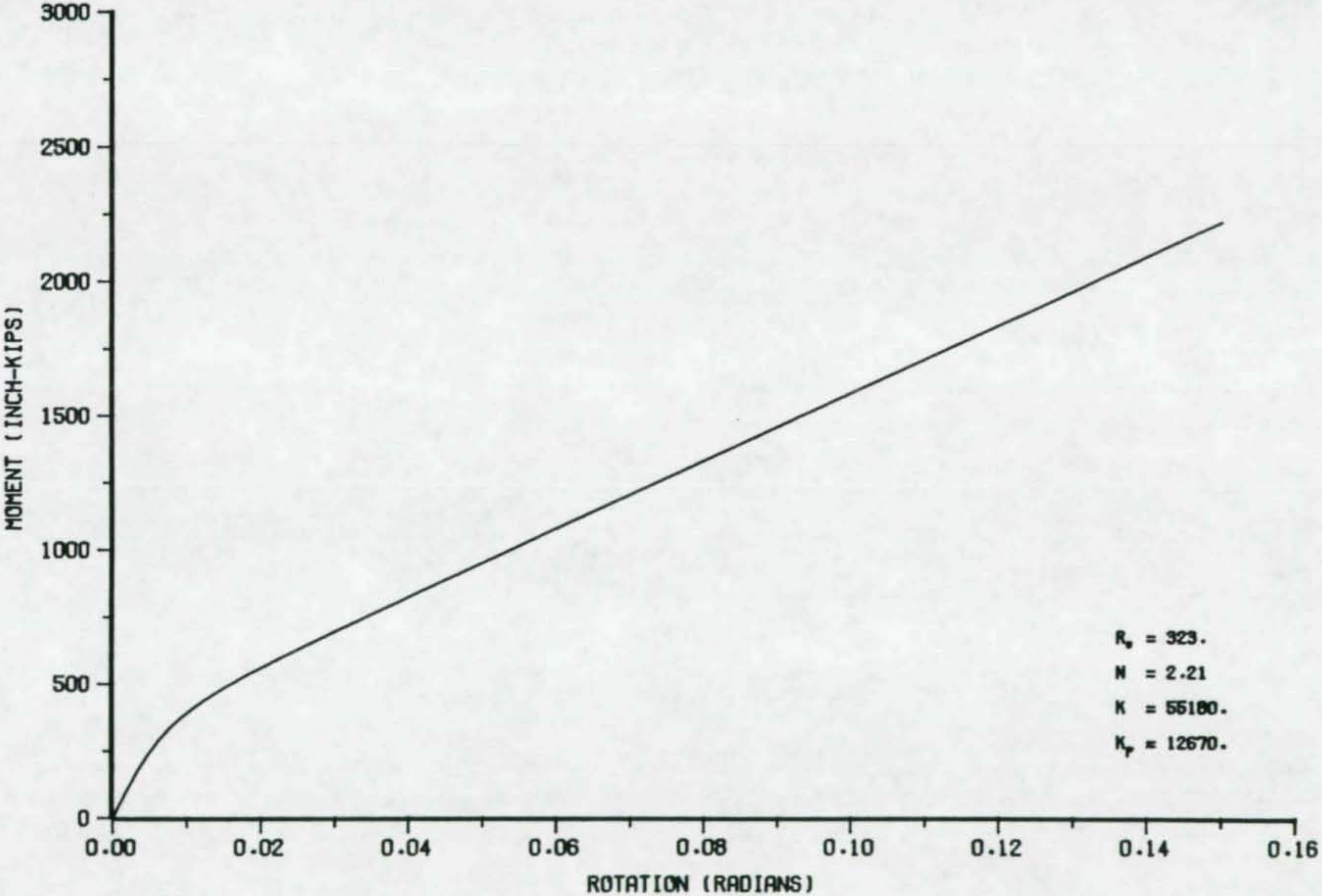
2 - L4 x 3 x 1/4 x 33 inches
2 - L4 x 3 x 1/4 x 30 inches
2 - L4 x 3 x 1/4 x 27 inches (see Figures 15-20)
2 - L4 x 3 x 1/4 x 24 inches
2 - L4 x 3 x 1/4 x 21 inches
2 - L4 x 3 x 1/4 x 18 inches

2 - L4 x 3 x 3/8 x 33 inches
2 - L4 x 3 x 3/8 x 30 inches
2 - L4 x 3 x 3/8 x 27 inches (see Figures 21-26)
2 - L4 x 3 x 3/8 x 24 inches
2 - L4 x 3 x 3/8 x 21 inches
2 - L4 x 3 x 3/8 x 18 inches

2 - L4 x 3 x 1/2 x 33 inches
2 - L4 x 3 x 1/2 x 30 inches
2 - L4 x 3 x 1/2 x 27 inches (see Figures 27-32)
2 - L4 x 3 x 1/2 x 24 inches
2 - L4 x 3 x 1/2 x 21 inches
2 - L4 x 3 x 1/2 x 18 inches

2 - L5 x 3 x 5/8 x 33 inches
2 - L5 x 3 x 5/8 x 30 inches
2 - L5 x 3 x 5/8 x 27 inches (see Figures 33-38)
2 - L5 x 3 x 5/8 x 24 inches
2 - L5 x 3 x 5/8 x 21 inches
2 - L5 x 3 x 5/8 x 18 inches

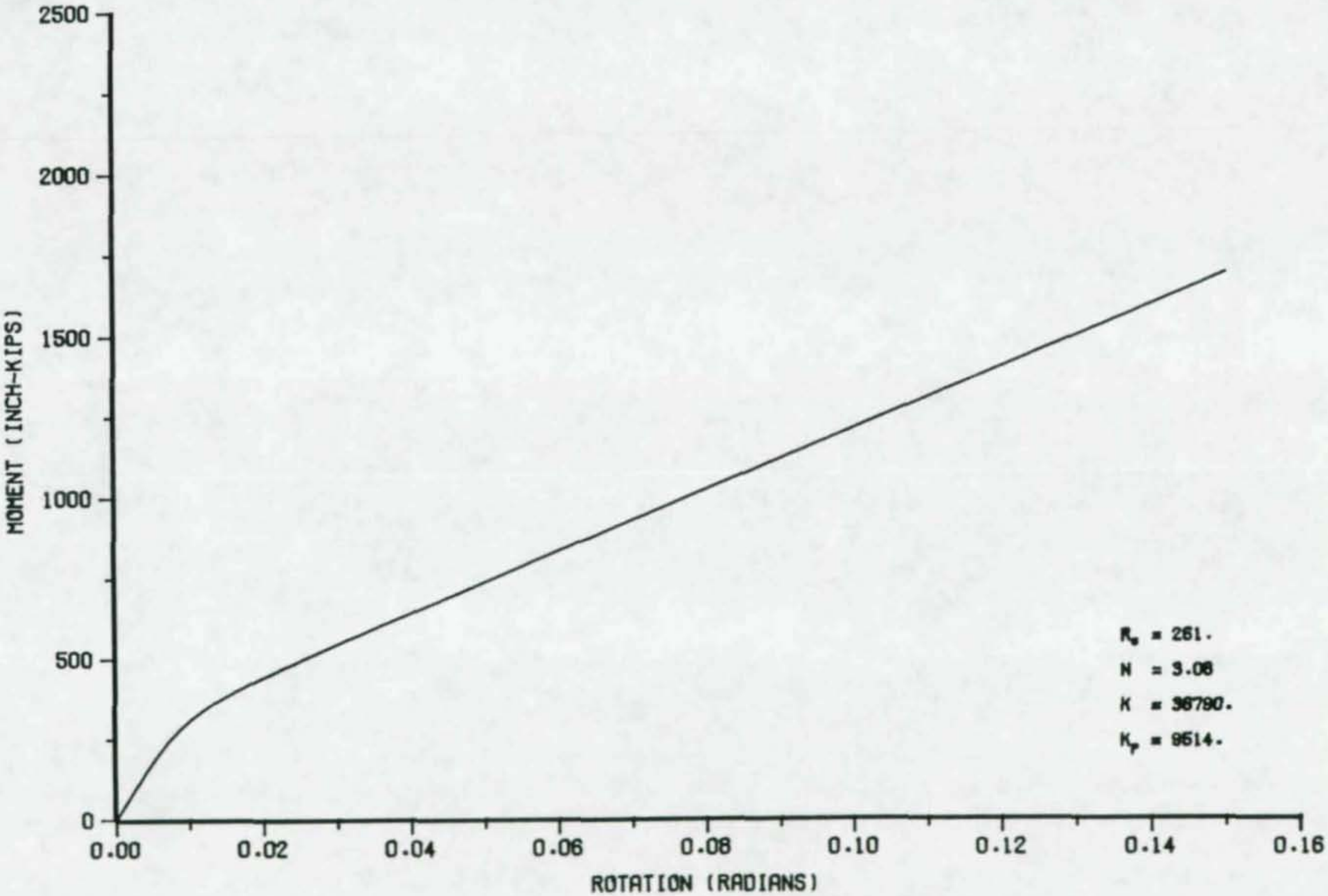
In the next section, beam line theory is used to determine the actual end rotation and end moment that exists at a particular connection.



MOMENT ROTATION CURVE FOR WELDED DOUBLE ANGLE CONNECTION

DOUBLE ANGLES ARE L 4 X 3 X 1/4 X 33 INCHES

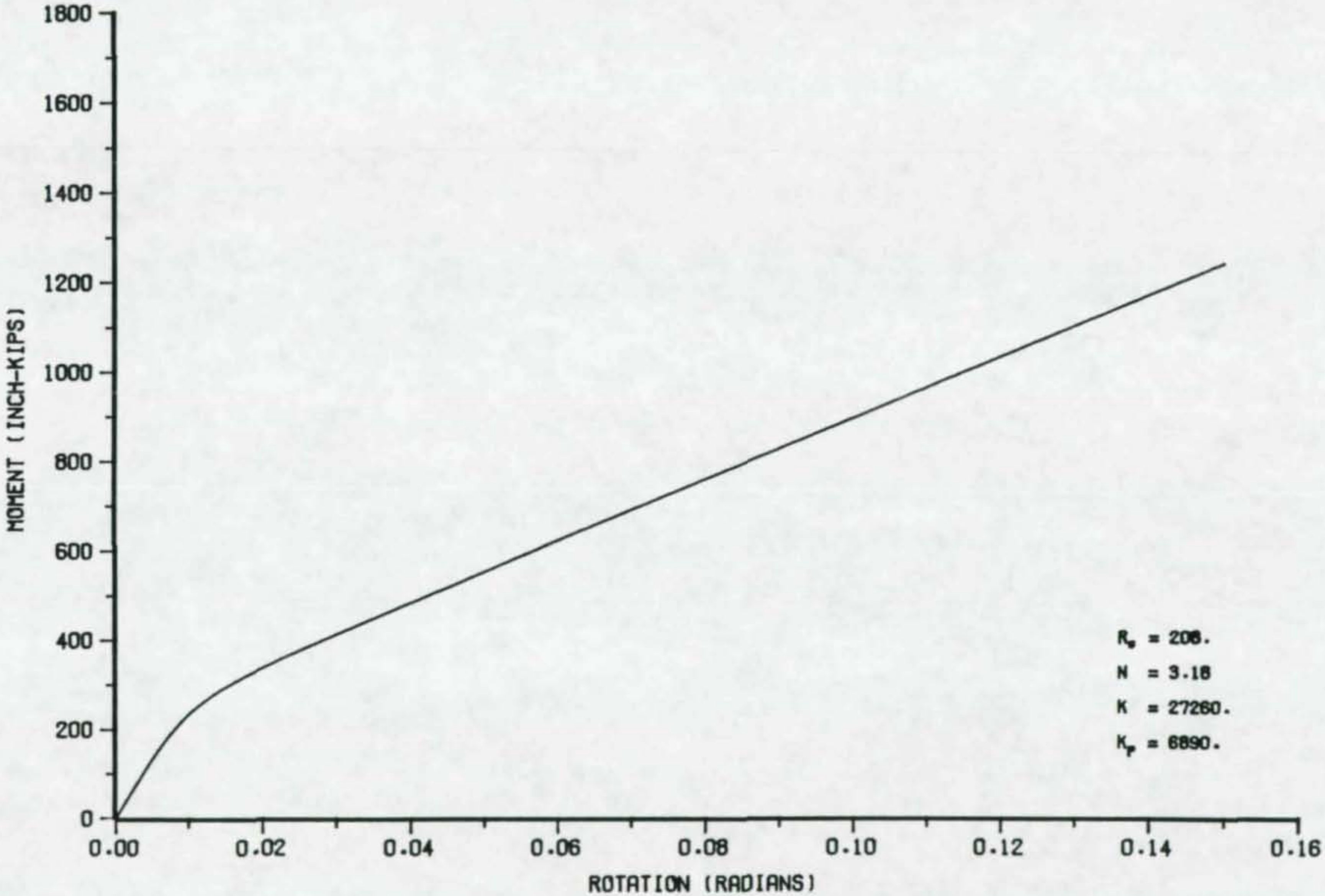
Figure 15.



MOMENT ROTATION CURVE FOR WELDED DOUBLE ANGLE CONNECTION

DOUBLE ANGLES ARE L 4 X 3 X 1/4 X 30 INCHES

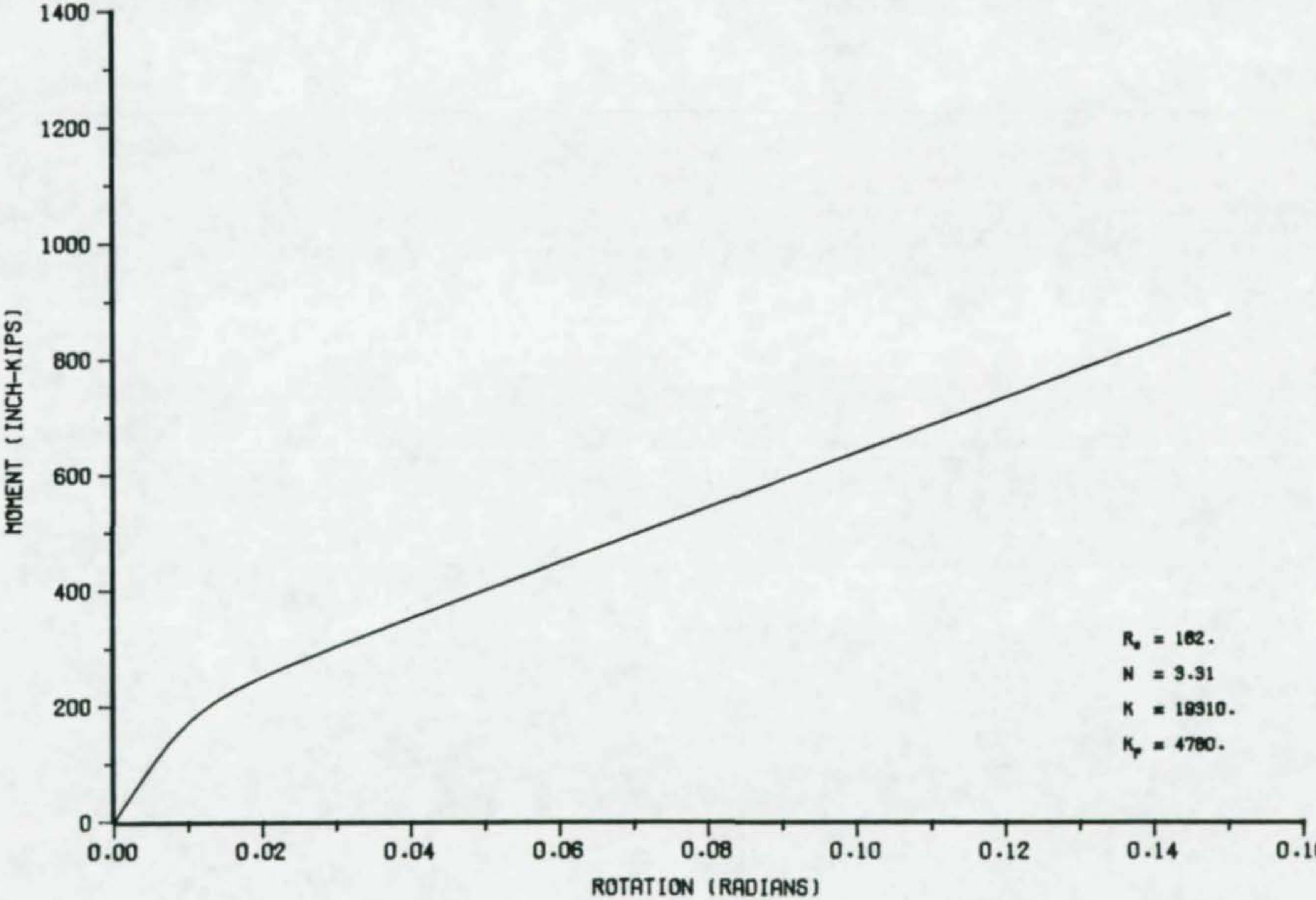
Figure 16.



MOMENT ROTATION CURVE FOR WELDED DOUBLE ANGLE CONNECTION

DOUBLE ANGLES ARE L 4 X 3 X 1/4 X 27 INCHES

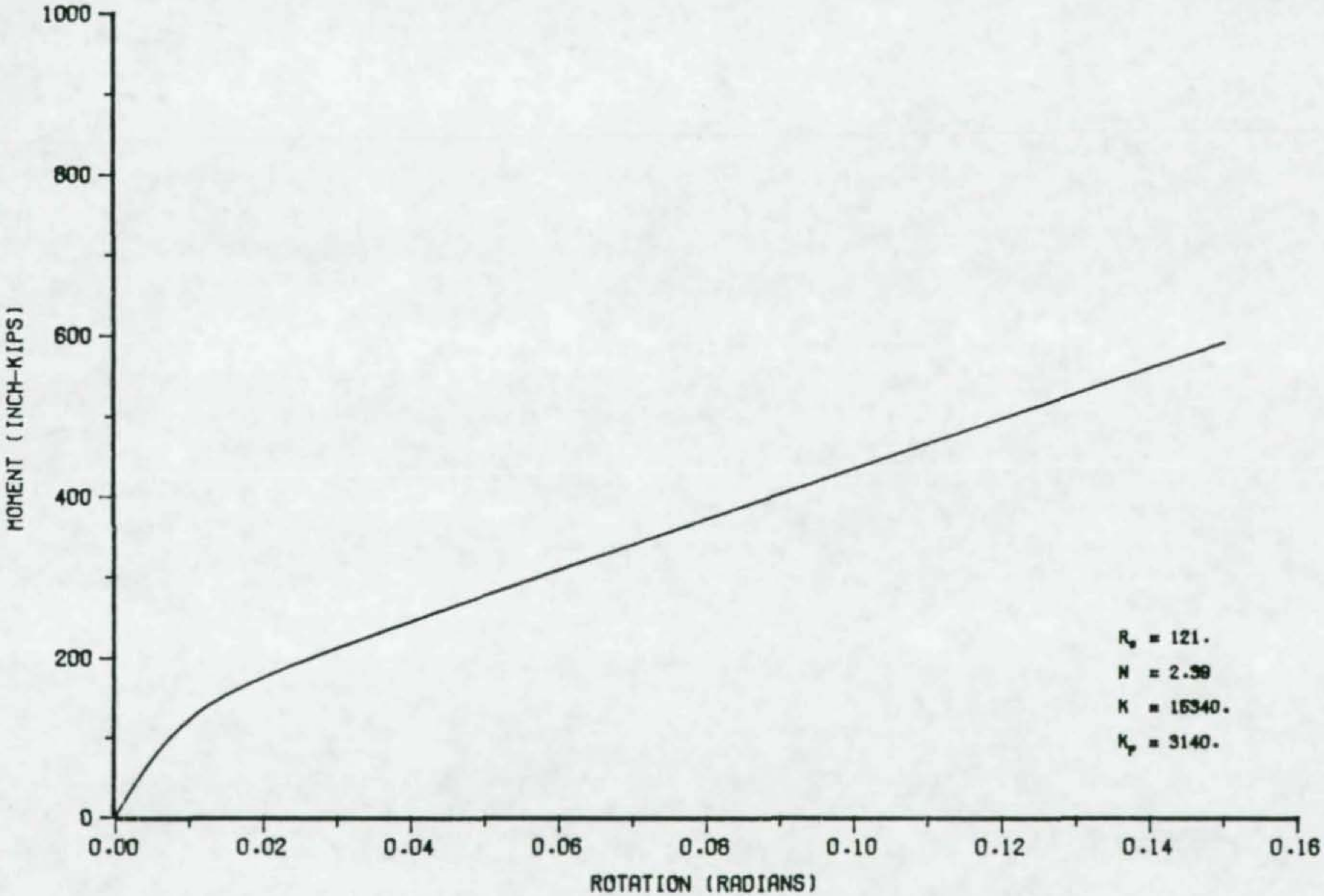
Figure 17.



MOMENT ROTATION CURVE FOR WELDED DOUBLE ANGLE CONNECTION

DOUBLE ANGLES ARE L 4 X 3 X 1/4 X 24 INCHES

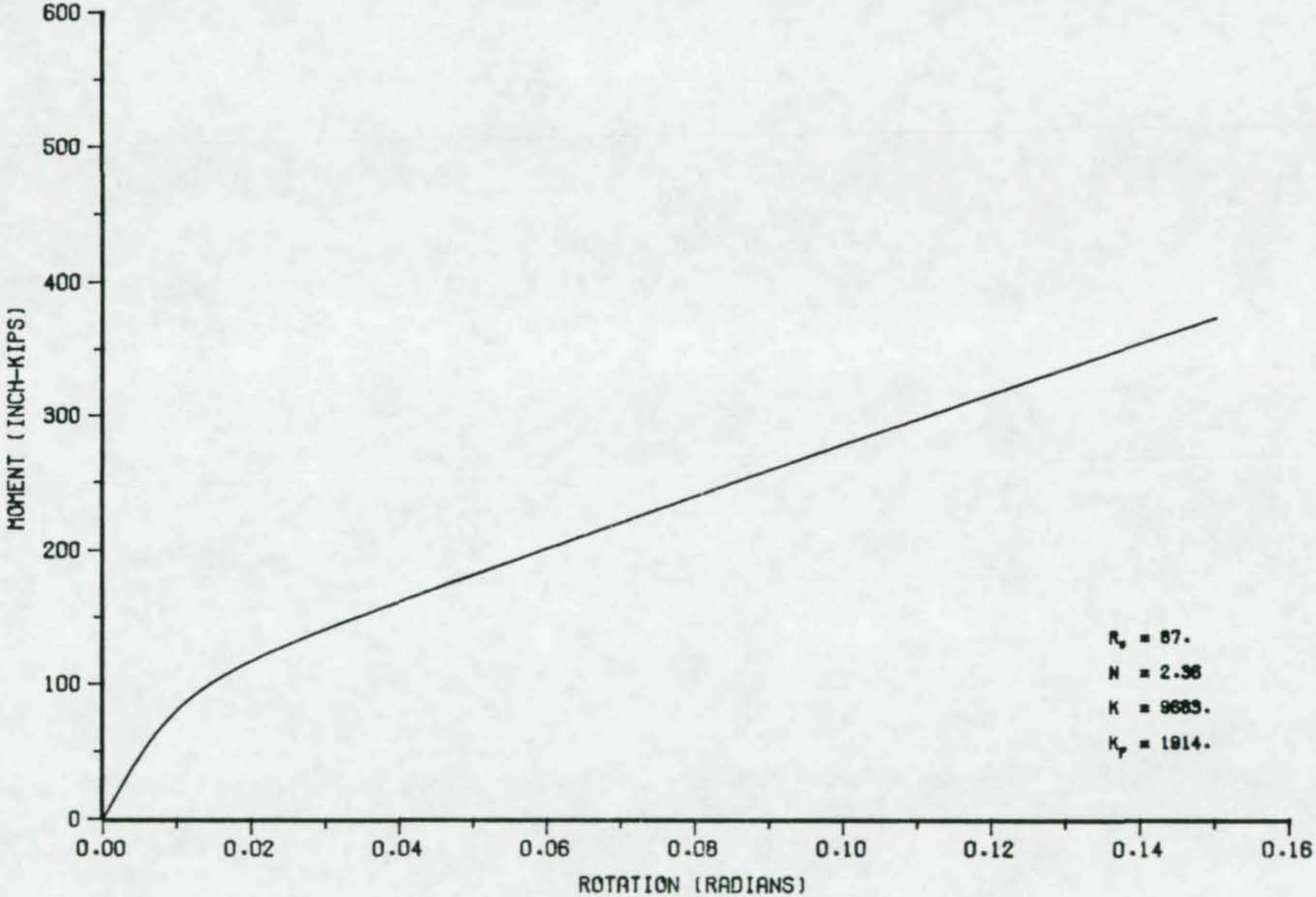
Figure 18.



MOMENT ROTATION CURVE FOR WELDED DOUBLE ANGLE CONNECTION

DOUBLE ANGLES ARE L 4 X 3 X 1/4 X 21 INCHES

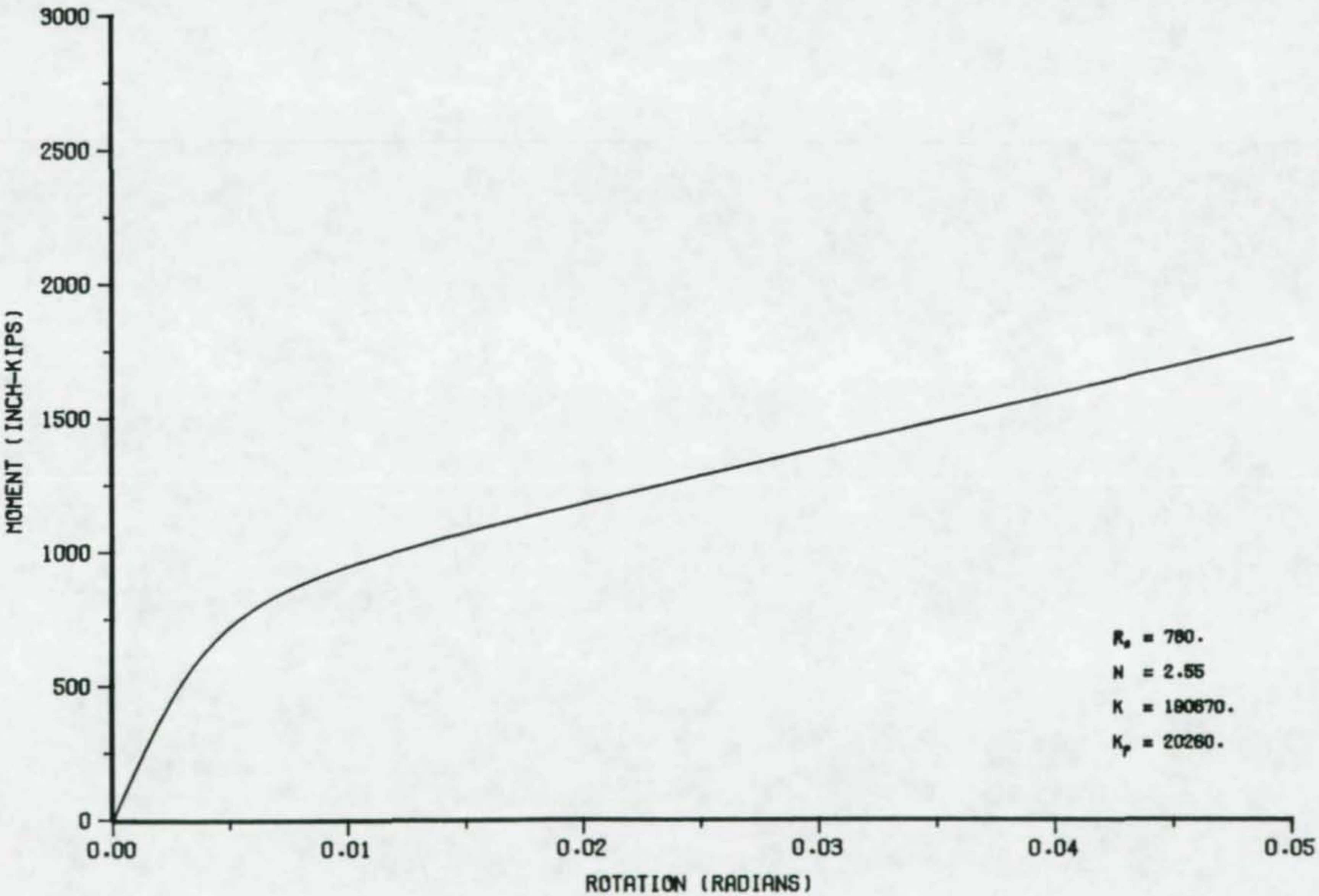
Figure 10



MOMENT ROTATION CURVE FOR WELDED DOUBLE ANGLE CONNECTION

DOUBLE ANGLES ARE L 4 X 3 X 1/4 X 18 INCHES

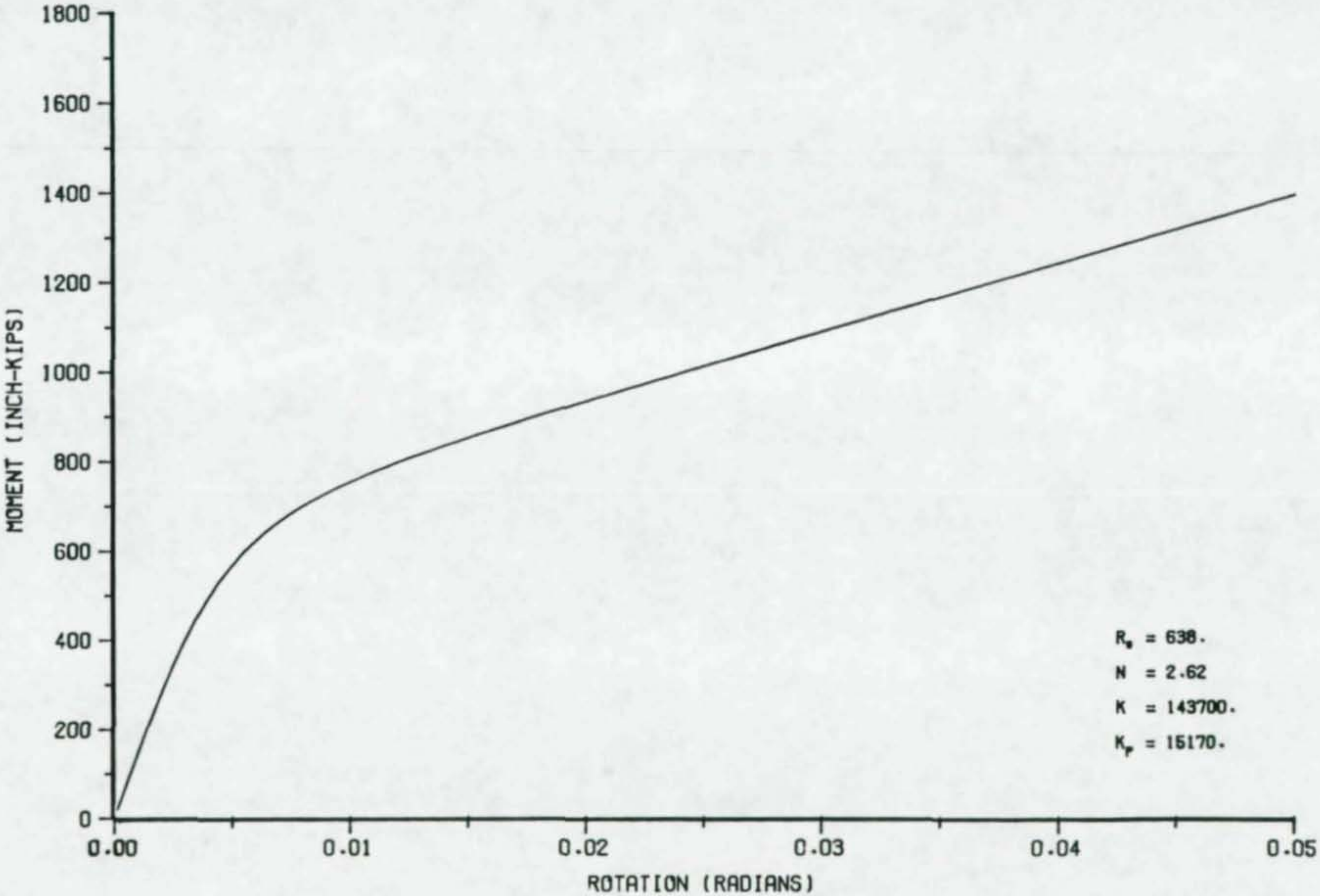
Figure 20.



MOMENT ROTATION CURVE FOR WELDED DOUBLE ANGLE CONNECTION

DOUBLE ANGLES ARE L4 X 3 X 3/8 X 33 INCHES

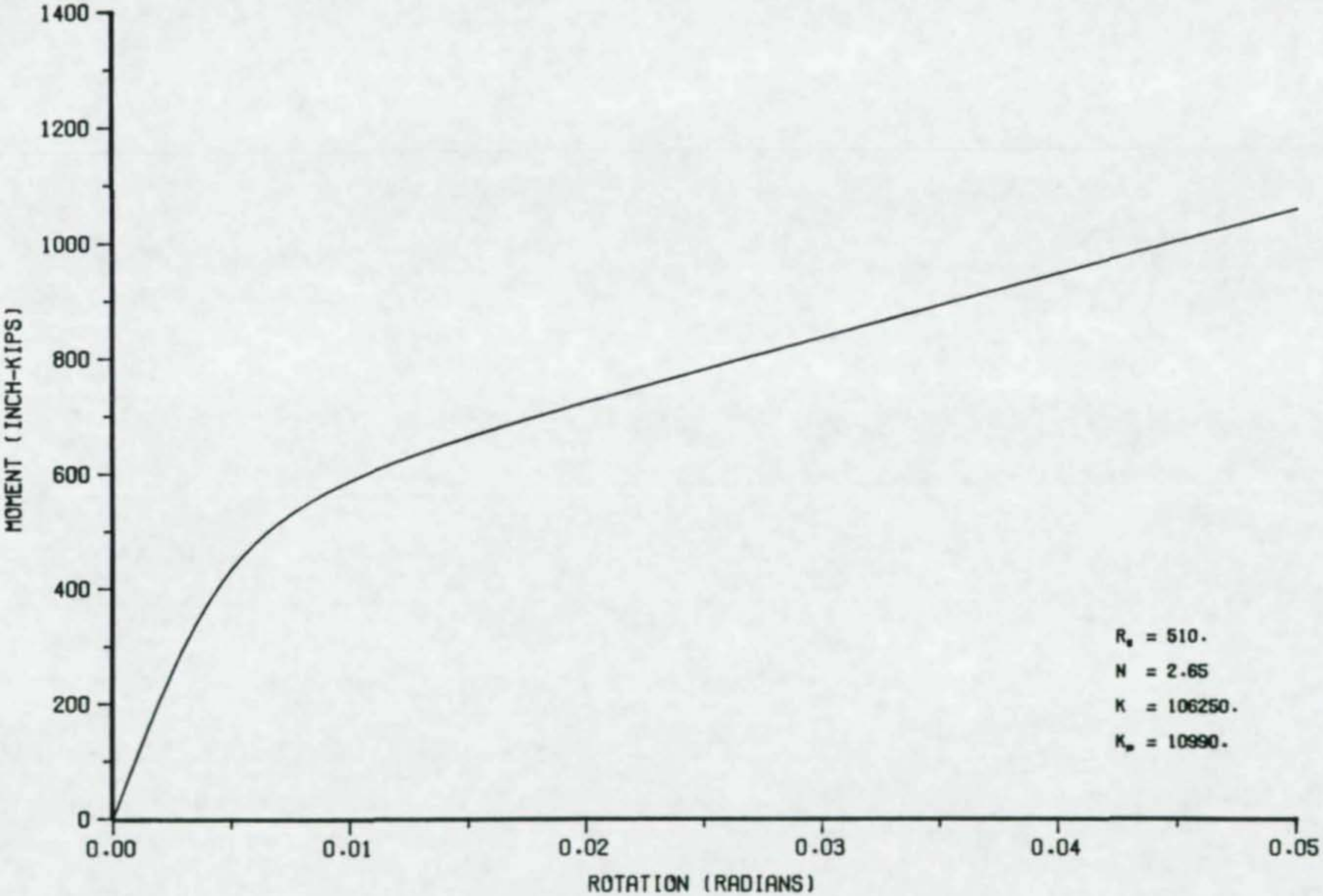
Figure 21.



MOMENT ROTATION CURVE FOR WELDED DOUBLE ANGLE CONNECTION

DOUBLE ANGLES ARE L4 X 3 X 3/8 X 30 INCHES

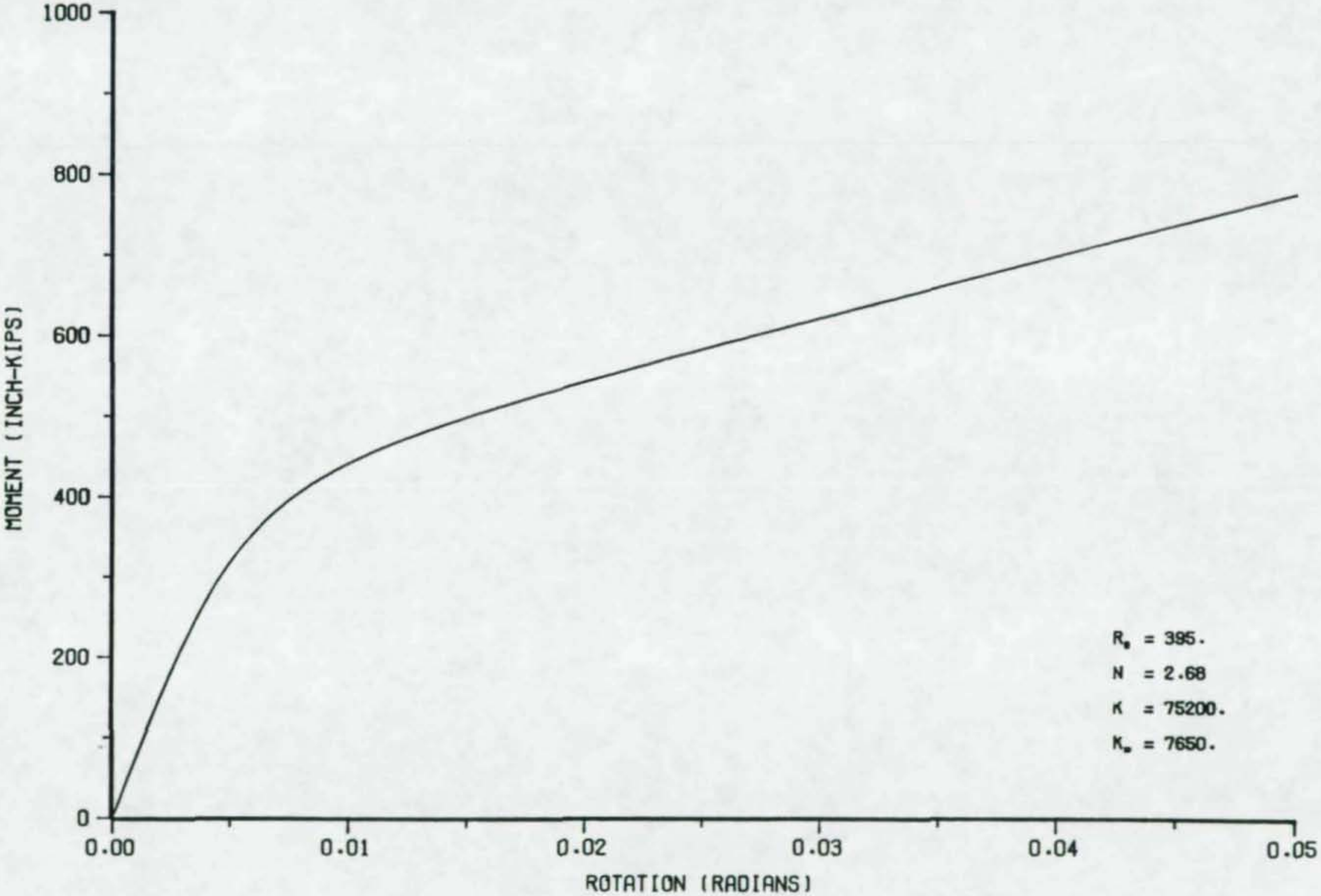
Figure 22.



MOMENT ROTATION CURVE FOR WELDED DOUBLE ANGLE CONNECTION

DOUBLE ANGLES ARE L4 X 3 X 3/8 X 27 INCHES

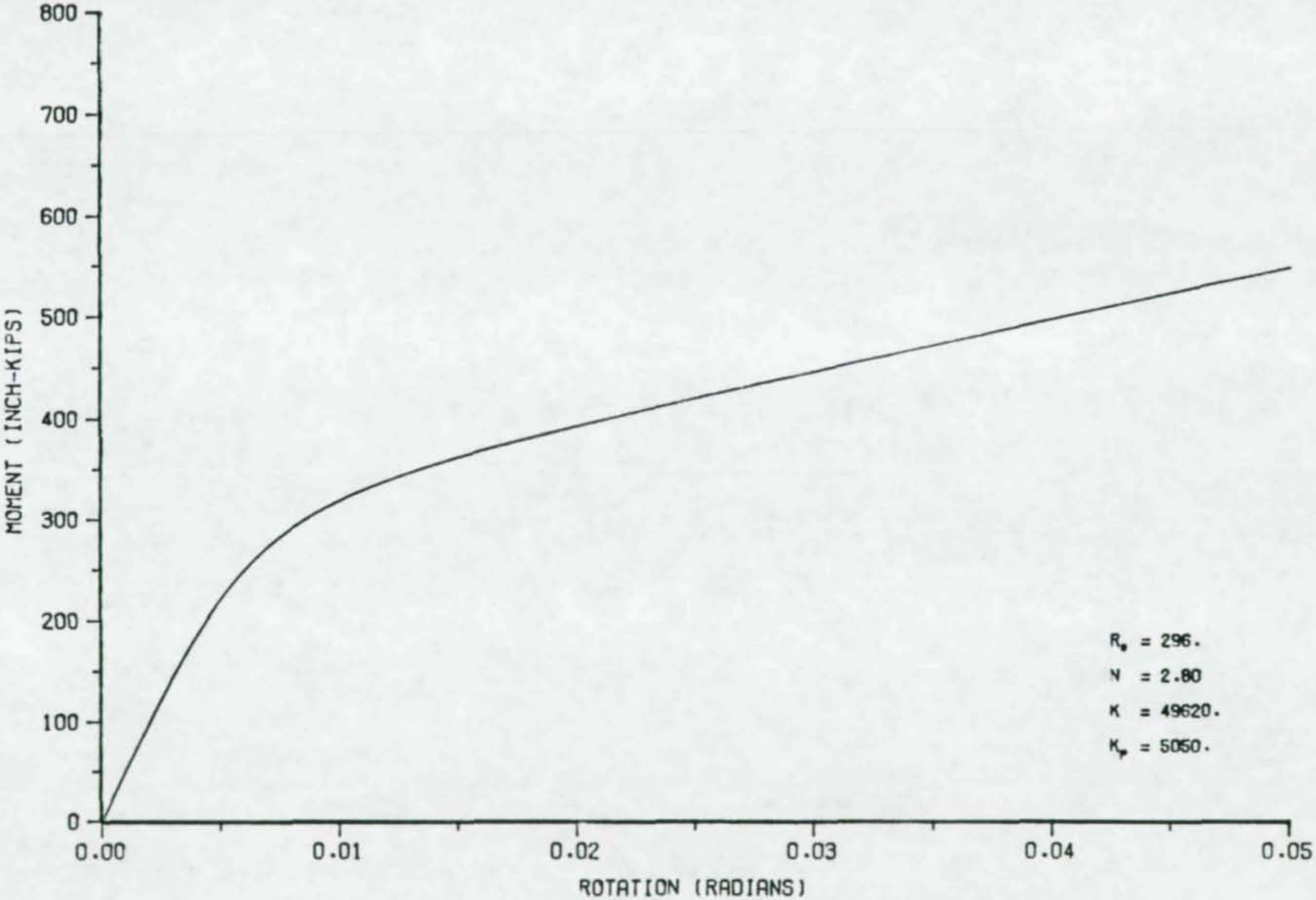
Figure 23.



MOMENT ROTATION CURVE FOR WELDED DOUBLE ANGLE CONNECTION

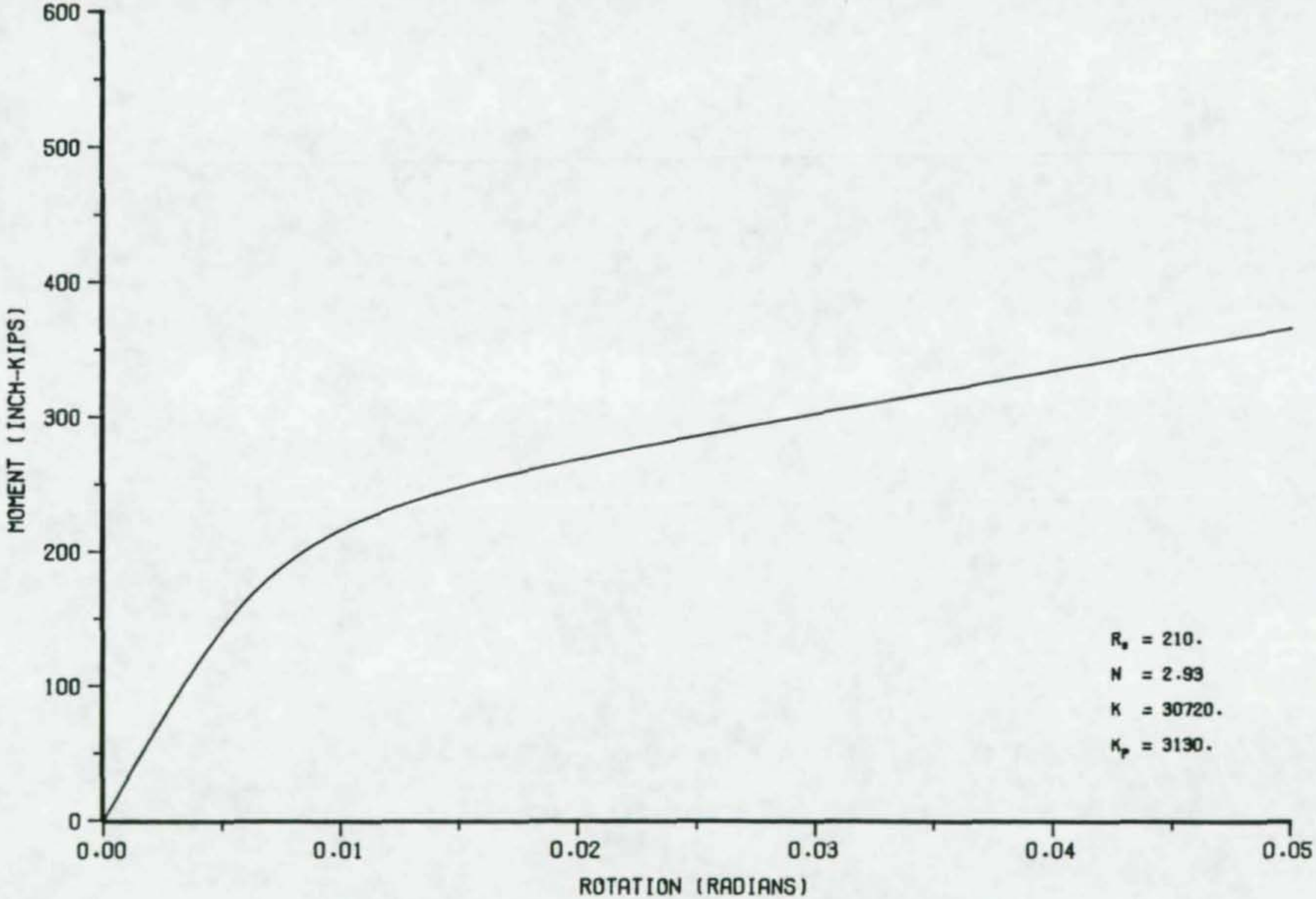
DOUBLE ANGLES ARE L4 X 3 X 3/8 X 24 INCHES

Figure 24.



MOMENT ROTATION CURVE FOR WELDED DOUBLE ANGLE CONNECTION

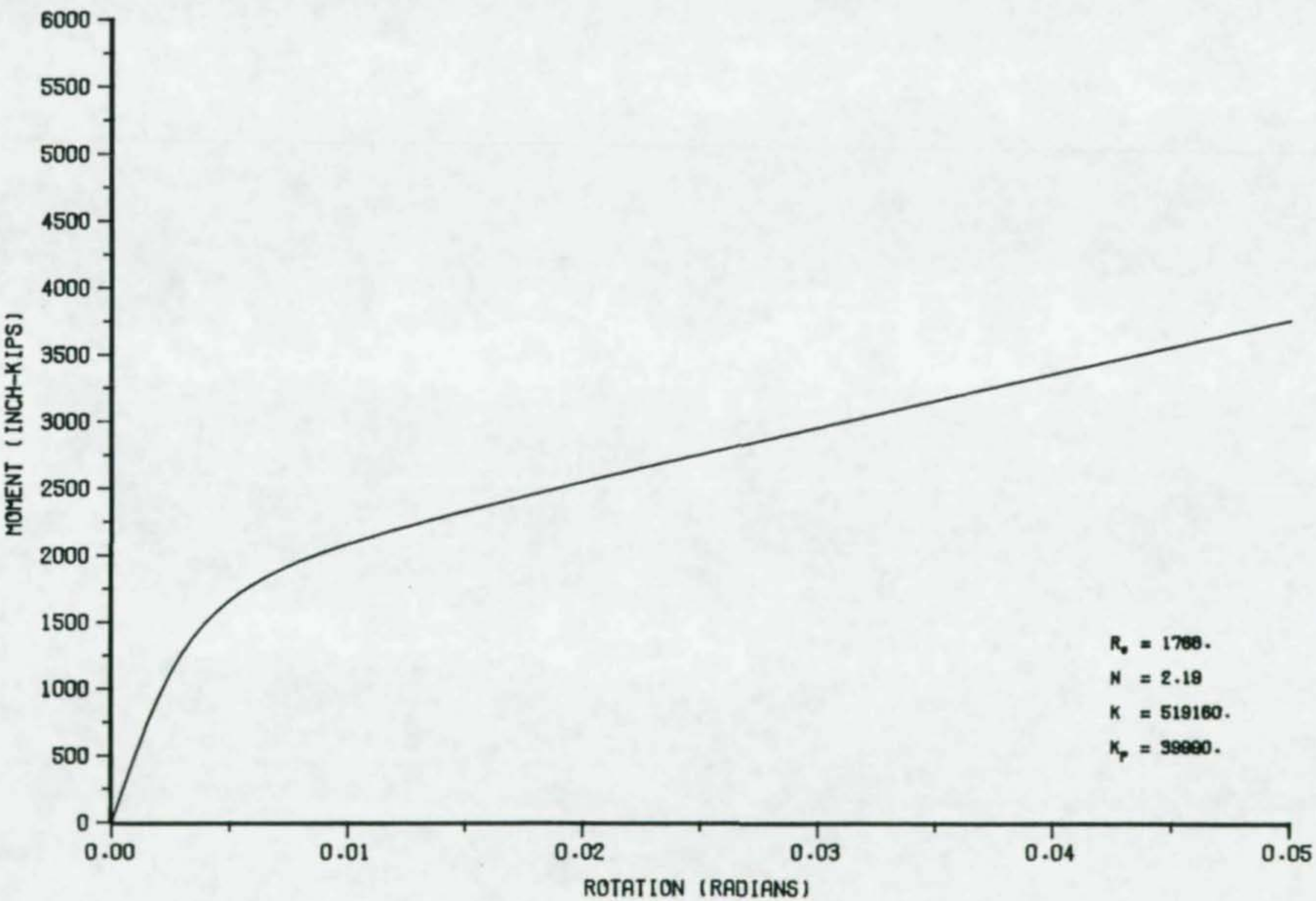
DOUBLE ANGLES ARE L4 X 3 X 3/8 X 21 INCHES



MOMENT ROTATION CURVE FOR WELDED DOUBLE ANGLE CONNECTION

DOUBLE ANGLES ARE L4 X 3 X 3/8 X 18 INCHES

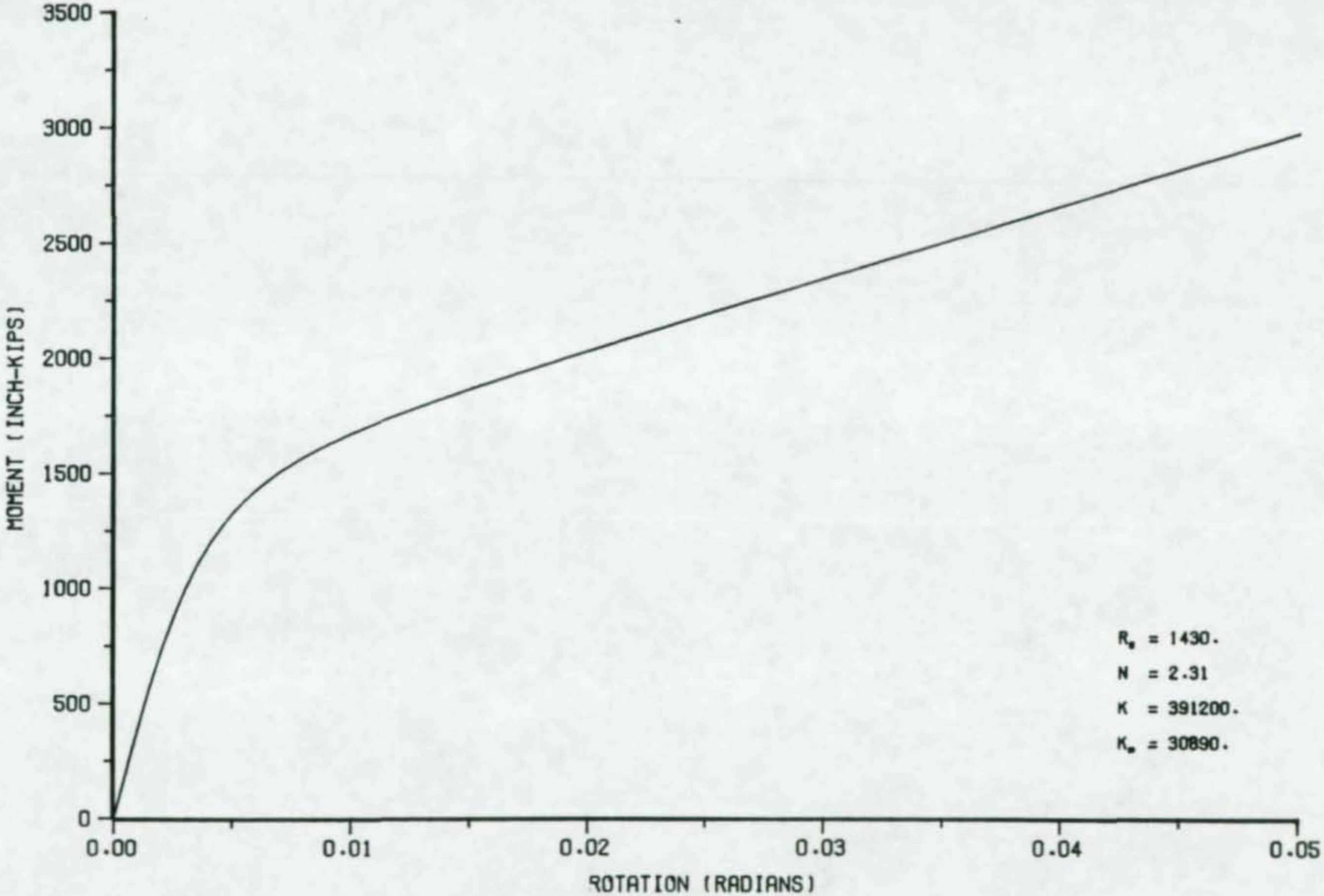
Figure 26.



MOMENT ROTATION CURVE FOR WELDED DOUBLE ANGLE CONNECTION

DOUBLE ANGLES ARE L4 X 3 X 1/2 X 33 INCHES

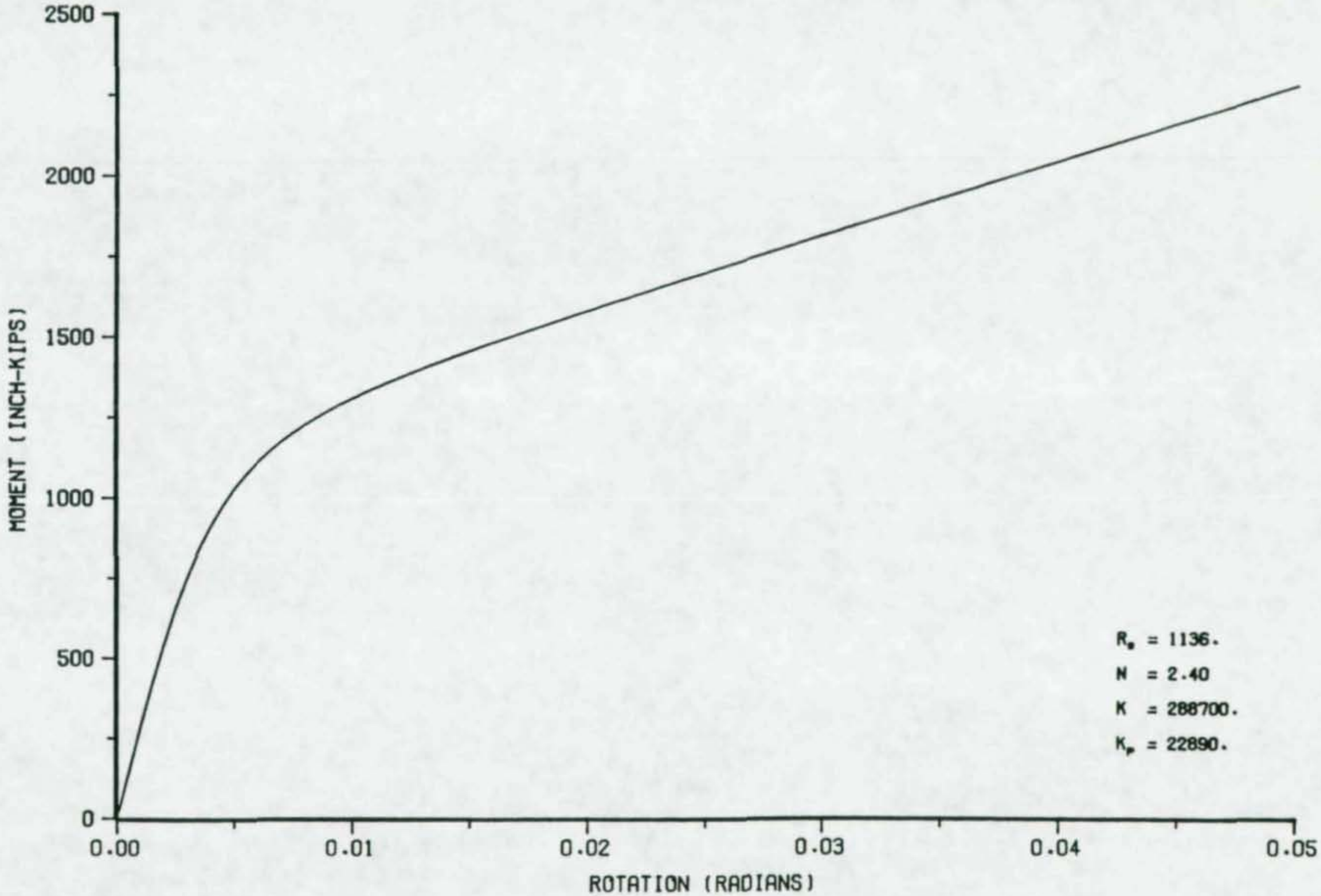
Figure 27.



MOMENT ROTATION CURVE FOR WELDED DOUBLE ANGLE CONNECTION

DOUBLE ANGLES ARE L4 X 3 X 1/2 X 30 INCHES

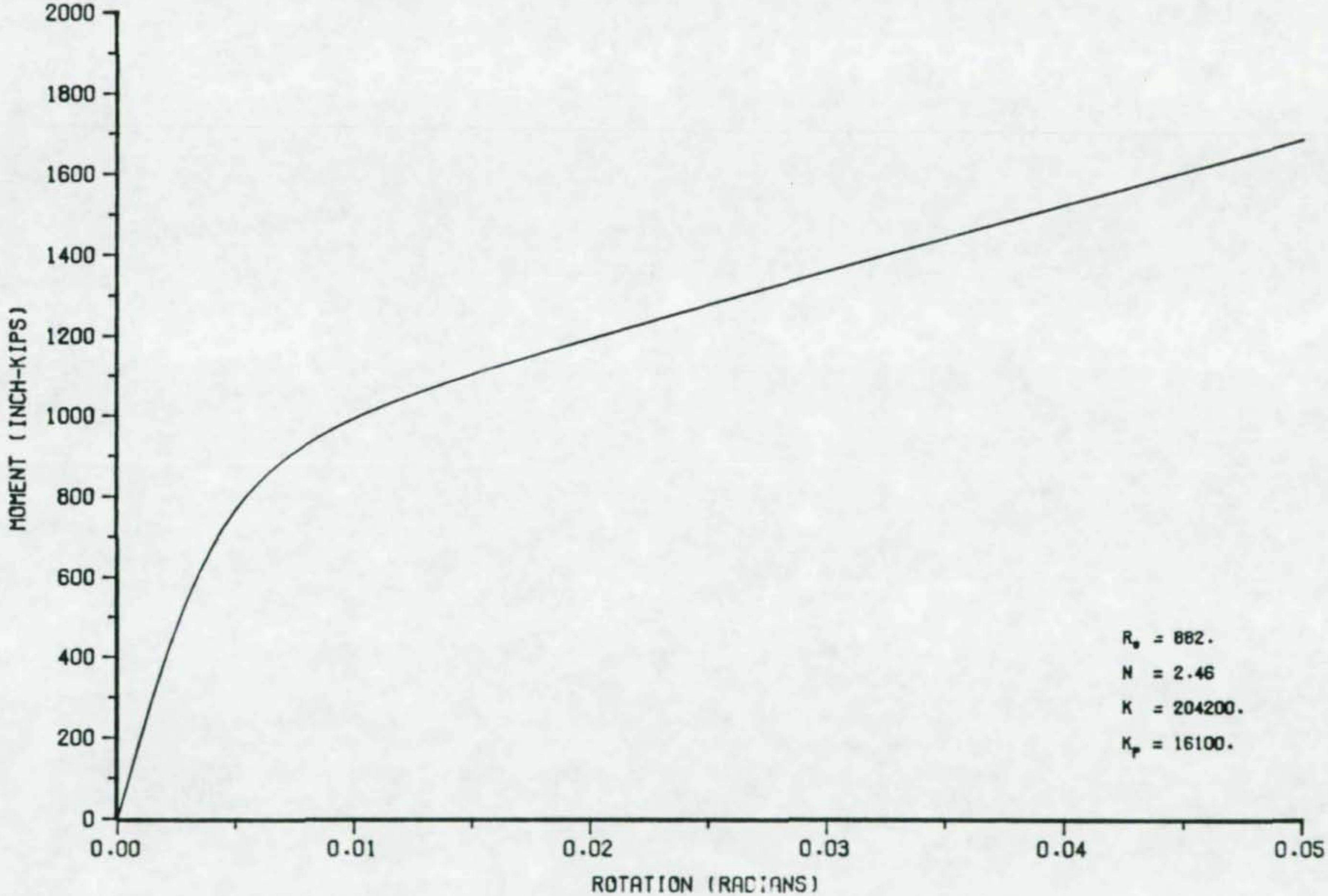
Figure 28.



MOMENT ROTATION CURVE FOR WELDED DOUBLE ANGLE CONNECTION

DOUBLE ANGLES ARE L4 X 3 X 1/2 X 27 INCHES

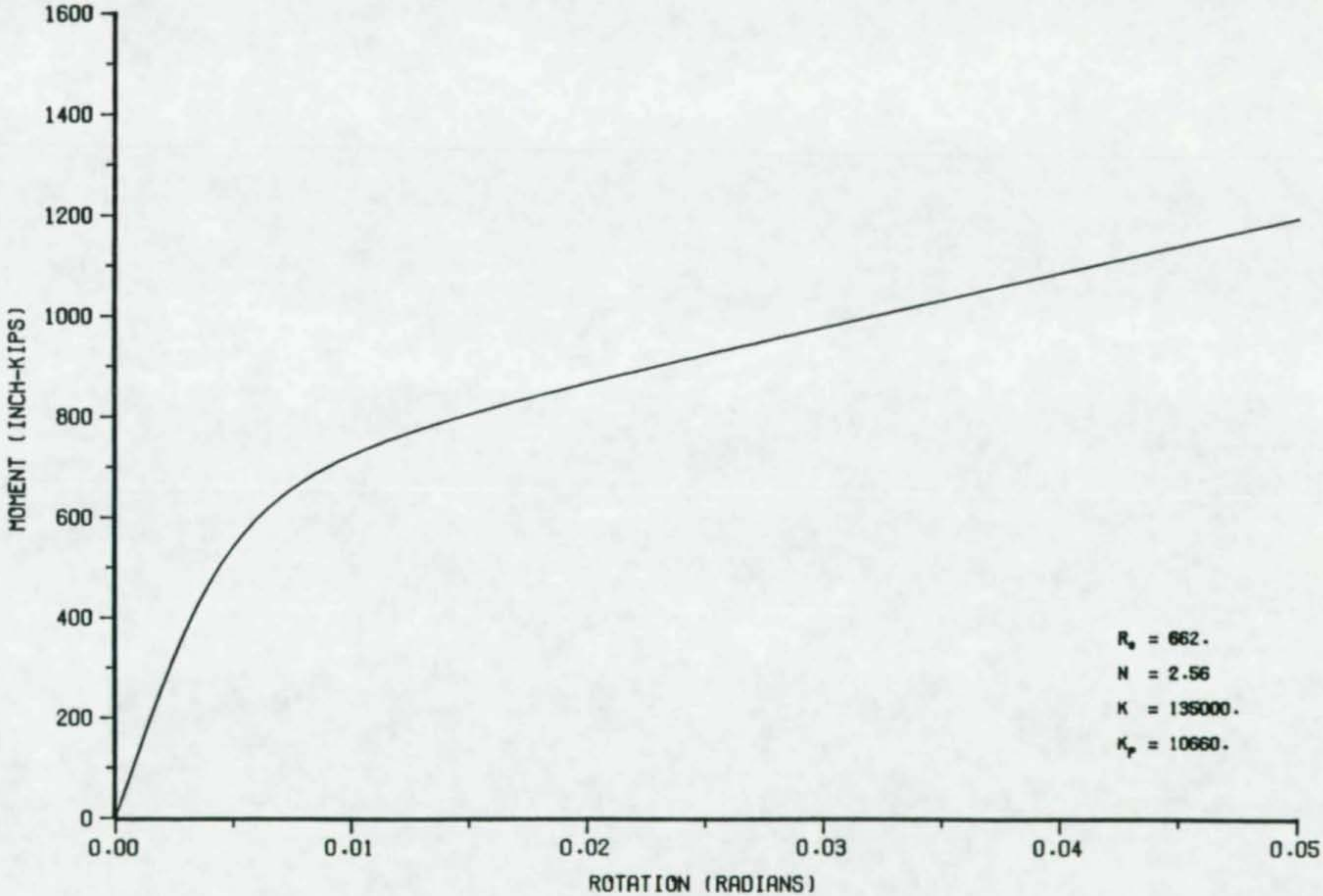
Figure 29.



MOMENT ROTATION CURVE FOR WELDED DOUBLE ANGLE CONNECTION

DOUBLE ANGLES ARE L4 X 3 X 1/2 X 24 INCHES

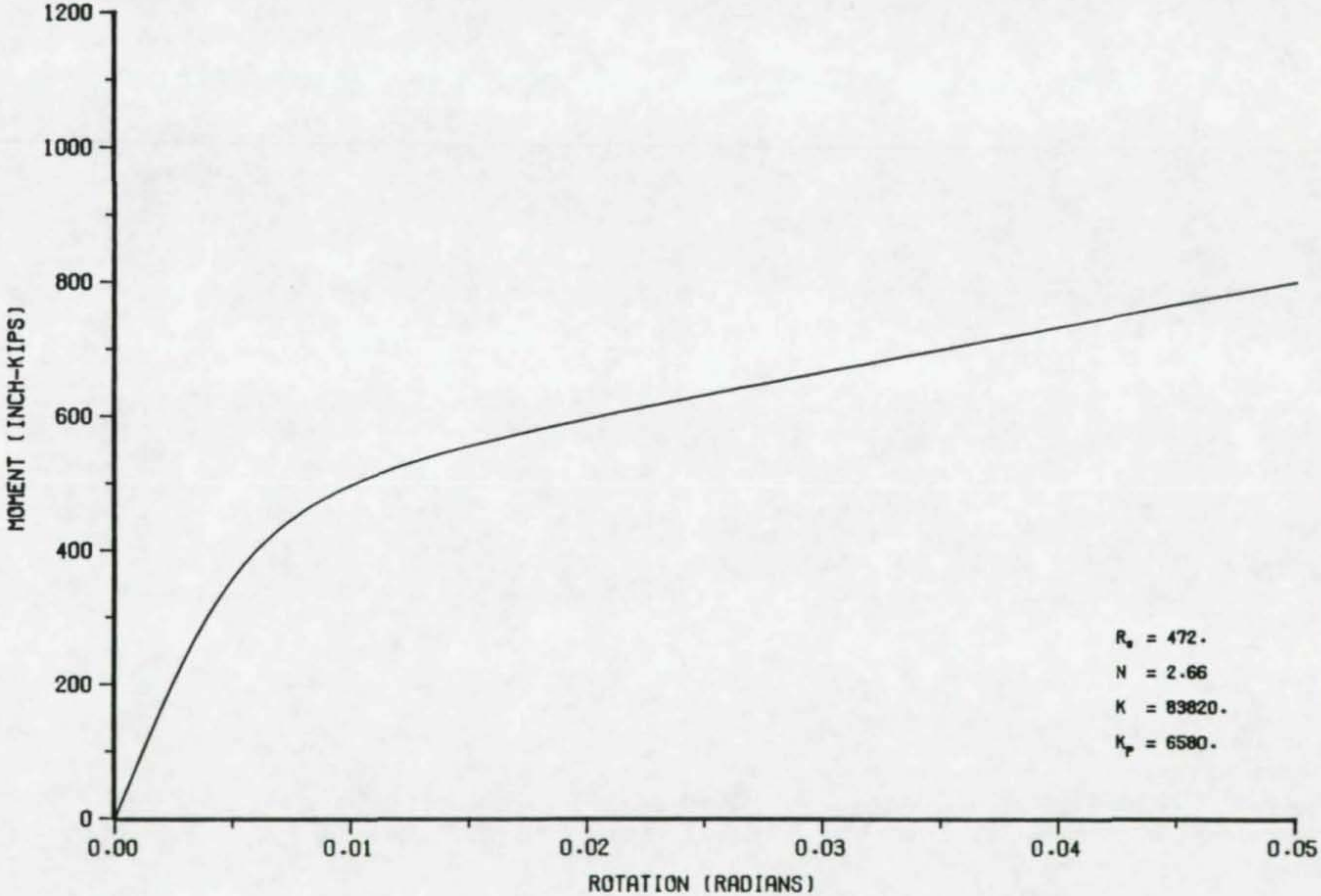
Figure 30.



MOMENT ROTATION CURVE FOR WELDED DOUBLE ANGLE CONNECTION

DOUBLE ANGLES ARE L4 X 3 X 1/2 X 21 INCHES

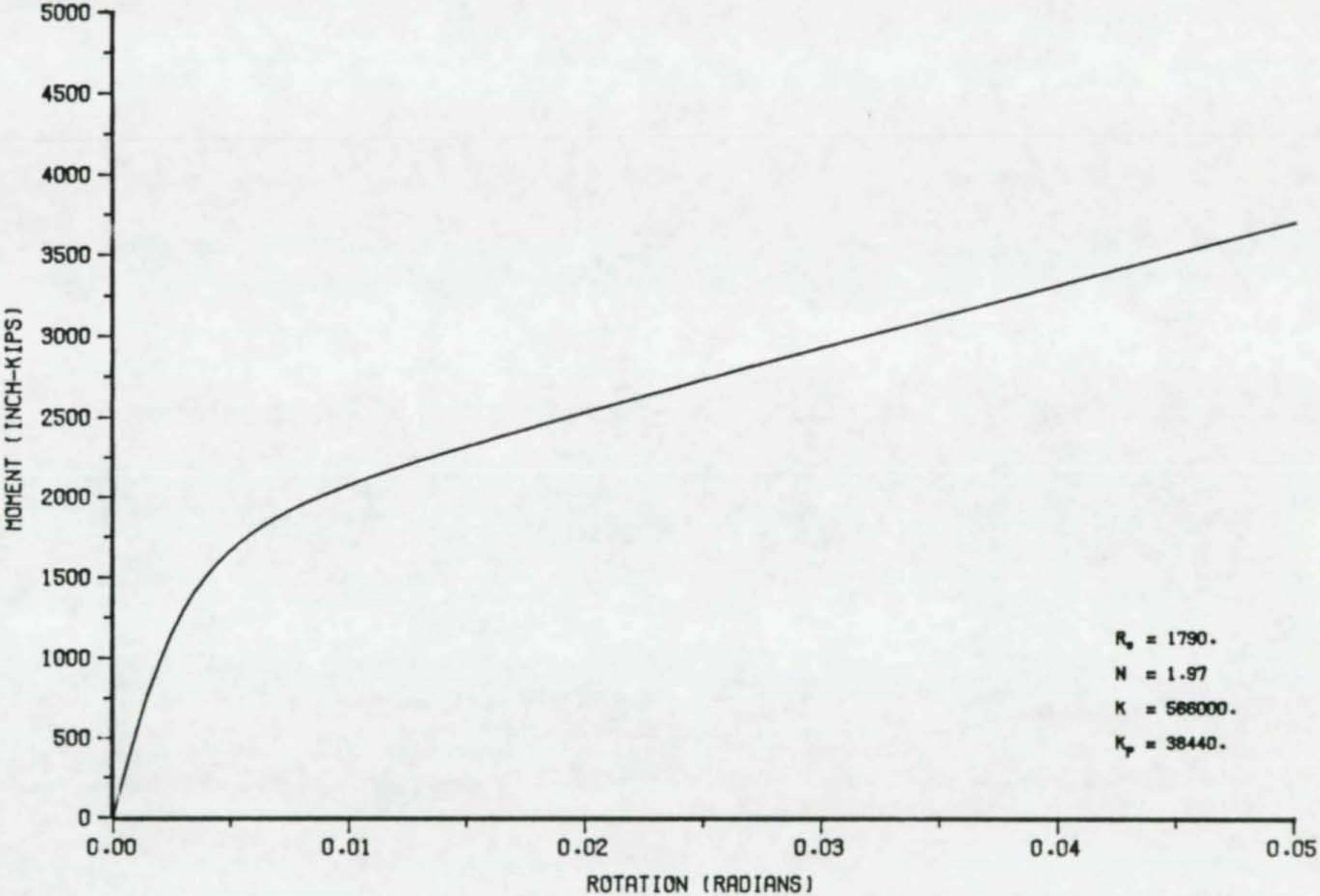
Figure 31.



MOMENT ROTATION CURVE FOR WELDED DOUBLE ANGLE CONNECTION

DOUBLE ANGLES ARE L4 X 3 X 1/2 X 18 INCHES

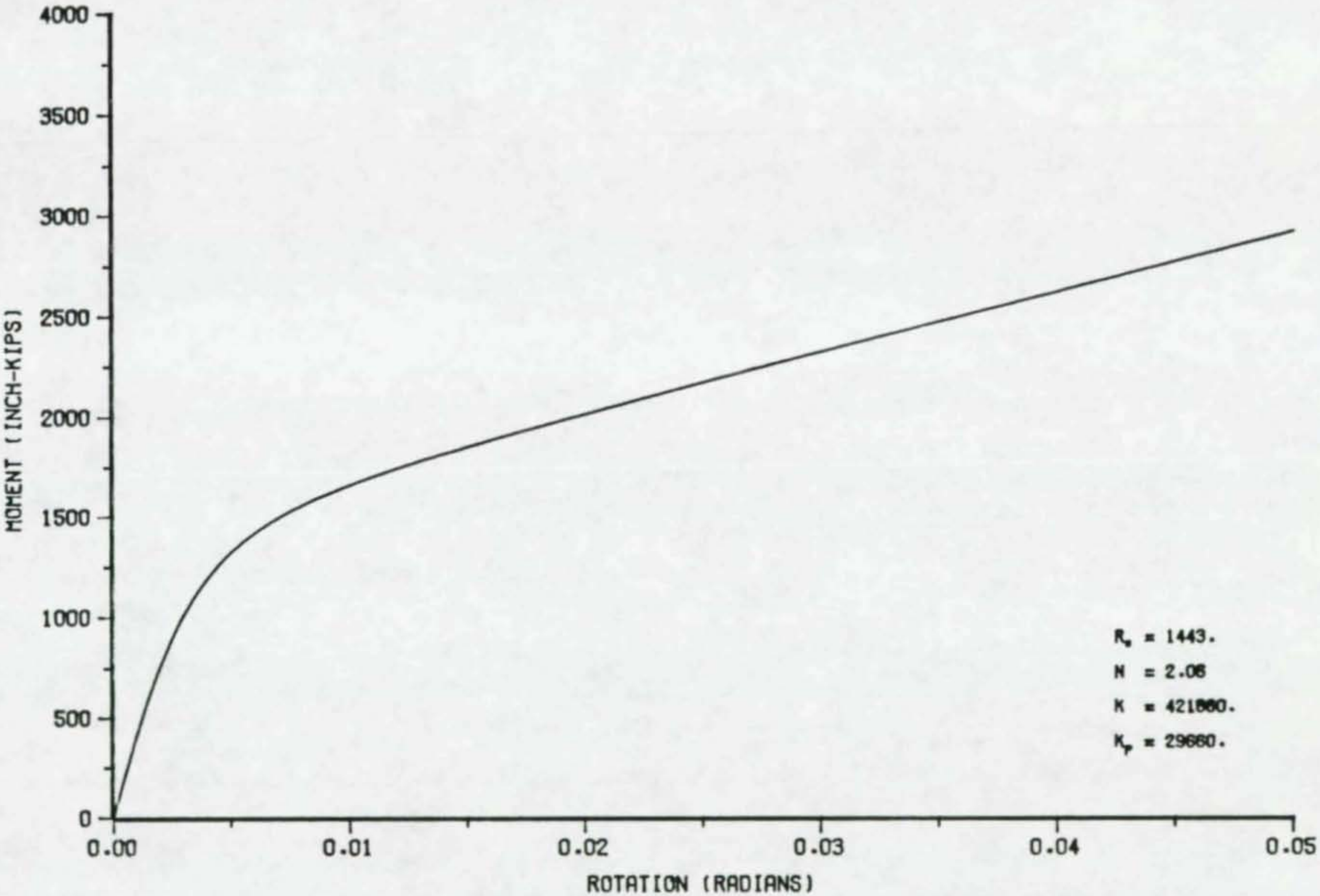
Figure 32.



MOMENT ROTATION CURVE FOR WELDED DOUBLE ANGLE CONNECTION

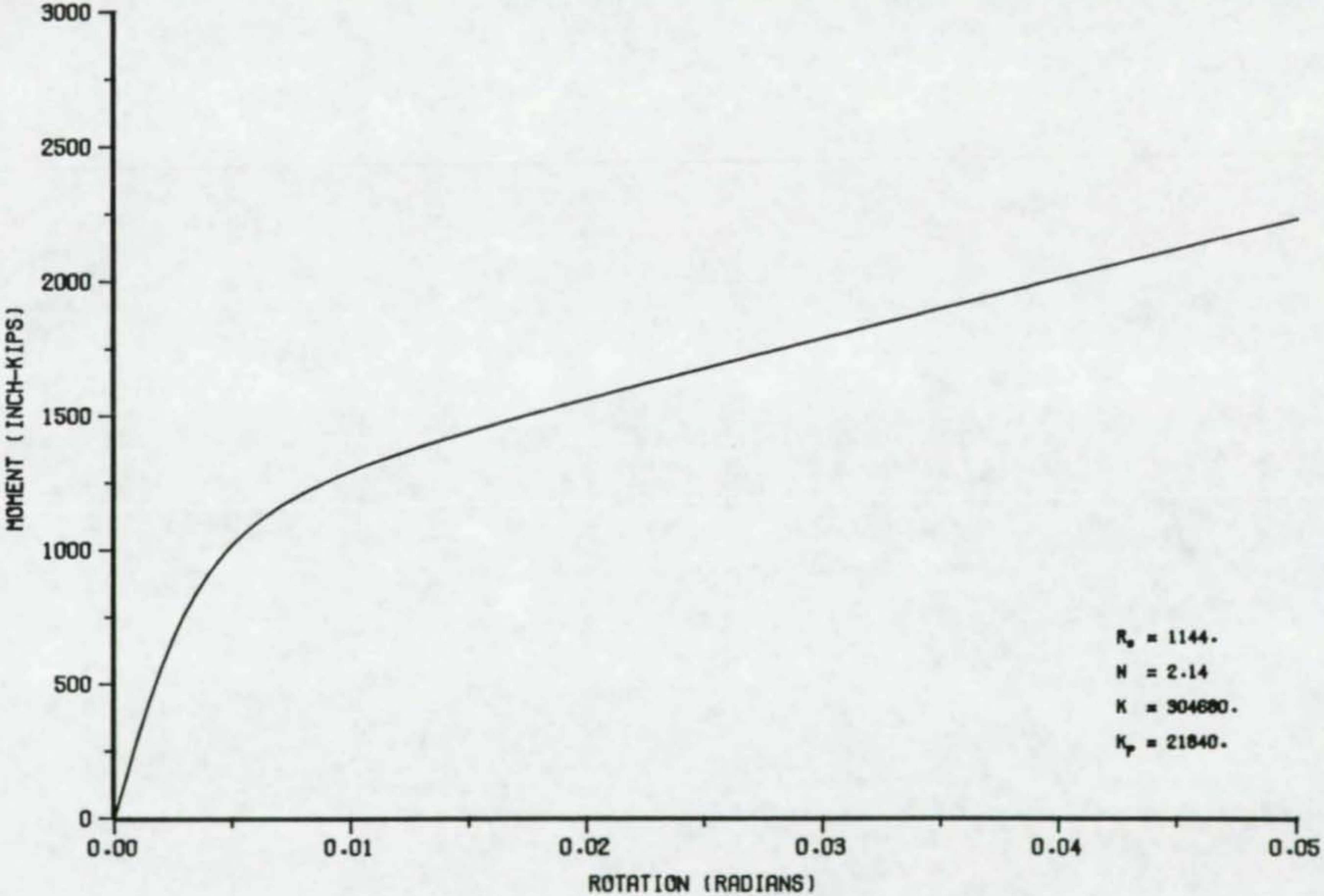
DOUBLE ANGLES ARE L 5 X 3 X 5/8 X 33 INCHES

Figure 33



MOMENT ROTATION CURVE FOR WELDED DOUBLE ANGLE CONNECTION

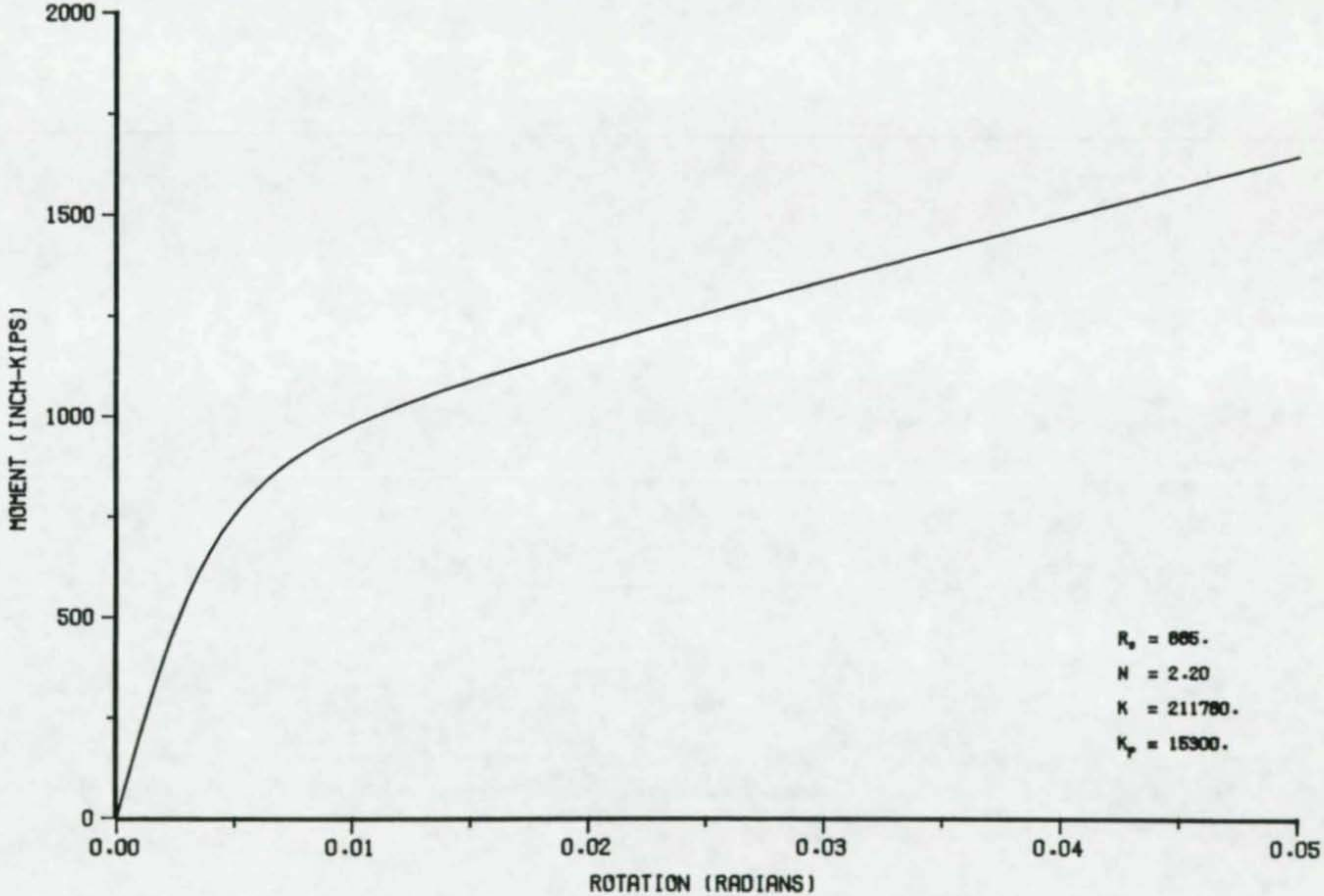
DOUBLE ANGLES ARE L 5 X 3 X 5/8 X 30 INCHES



MOMENT ROTATION CURVE FOR WELDED DOUBLE ANGLE CONNECTION

DOUBLE ANGLES ARE L 5 X 3 X 5/8 X 27 INCHES

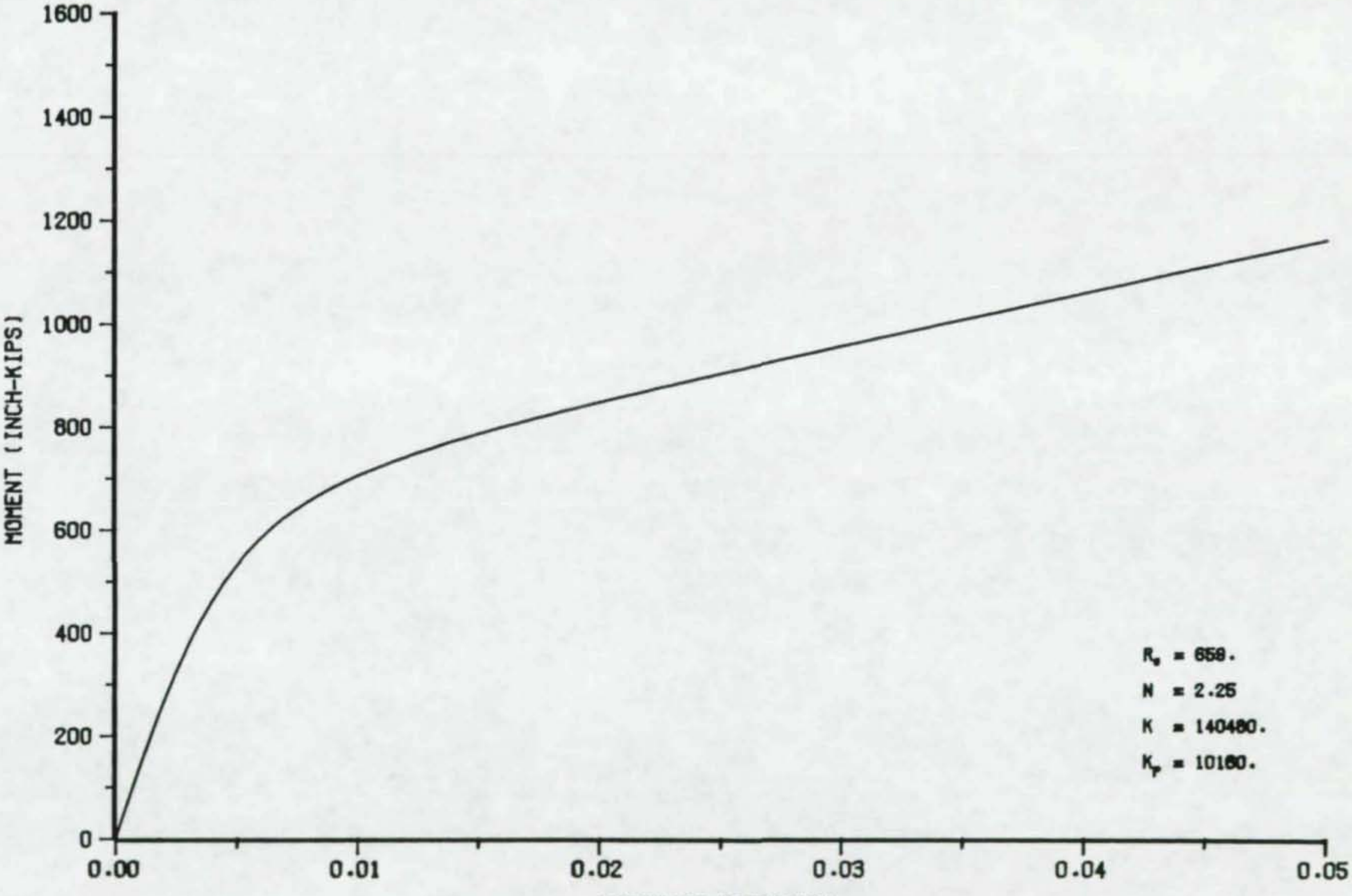
Figure 35.



MOMENT ROTATION CURVE FOR WELDED DOUBLE ANGLE CONNECTION

DOUBLE ANGLES ARE L 5 X 3 X 5/8 X 24 INCHES

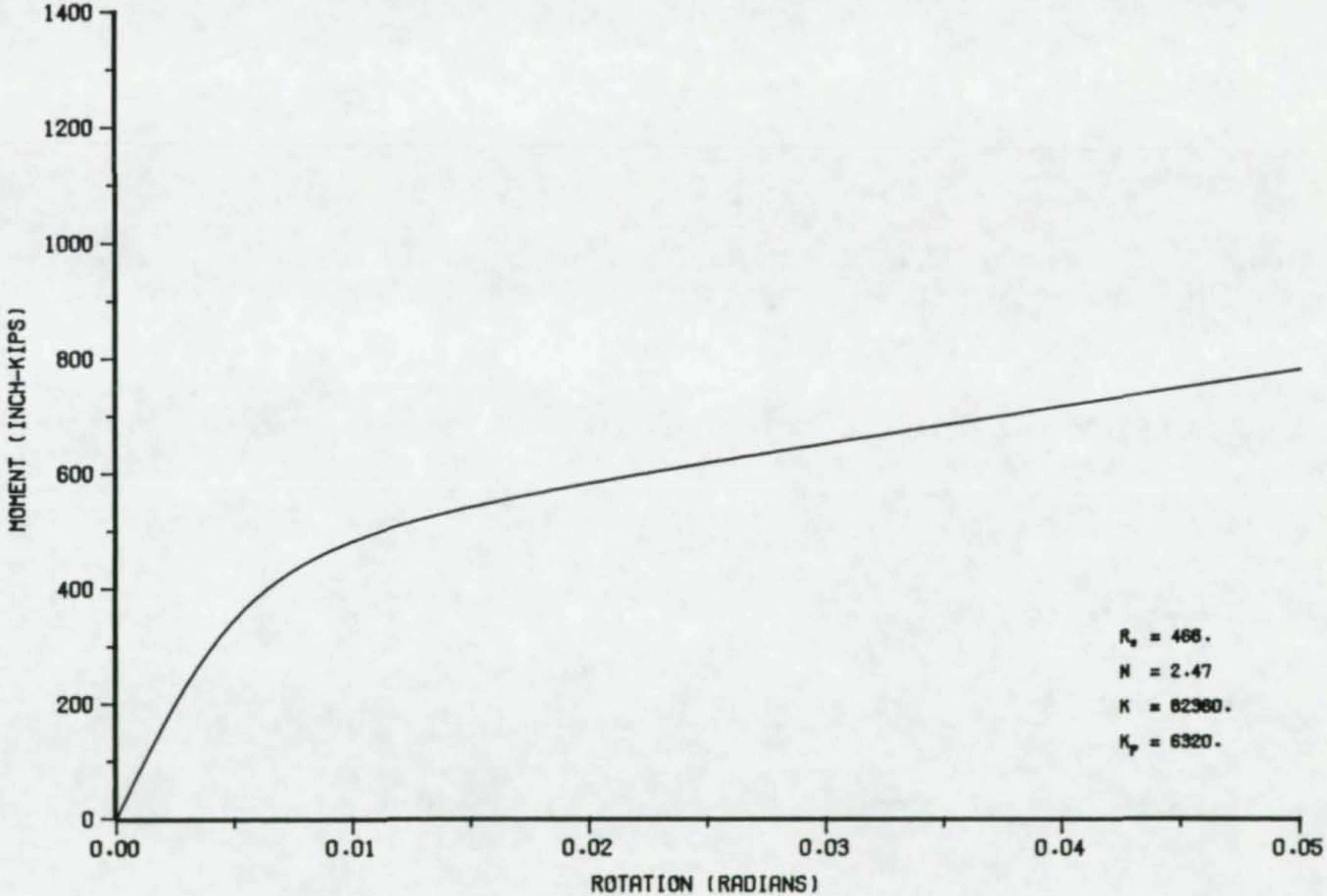
Figure 36.



MOMENT ROTATION CURVE FOR WELDED DOUBLE ANGLE CONNECTION

DOUBLE ANGLES ARE L 5 X 3 X 5/8 X 21 INCHES

Figure 37.



MOMENT ROTATION CURVE FOR WELDED DOUBLE ANGLE CONNECTION

DOUBLE ANGLES ARE L 5 X 3 X 5/8 X 18 INCHES

Figure 38.

BEAM LINE THEORY

The beam line [3] is a linear equation which for a particular loading on a beam gives the relationship between the end rotation and the end restraining moment (Figure 4). The beam line is developed using the slope-deflection equations from structural analysis. For the beam shown in Figure 39 and defining counterclockwise as positive for moments and rotations, the slope-deflection equations are given below.

$$M_A = M_{FA} + 4EI\theta_A/L + 2EI\theta_B/L \qquad (1)$$

$$M_B = M_{FB} + 2EI\theta_A/L + 4EI\theta_B/L \qquad (2)$$

According to our counterclockwise notation, the moment at B is the negative of the moment at A and the rotation at B is the negative of the rotation at A. Subtracting equation (2) from equation (1), with the appropriate substitutions, gives

$$M_A = M_{FA} + 2EI\theta_A/L$$

Note, however, that according to this sign convention, the moment and fixed end moment at A are both positive whereas the rotation at A is negative. Because of the symmetry in loading and geometry of the beam in Figure 39, the following beam line equation is valid for either end of the beam and the subscripts can therefore be eliminated.

$$M = M_{FIXED} - 2EI\theta/L$$

Thus the beam line gives the moment at the beam end for a given end rotation.

AISC-1.2 defines three types of connections.

Type 1 - Rigid-frame connection

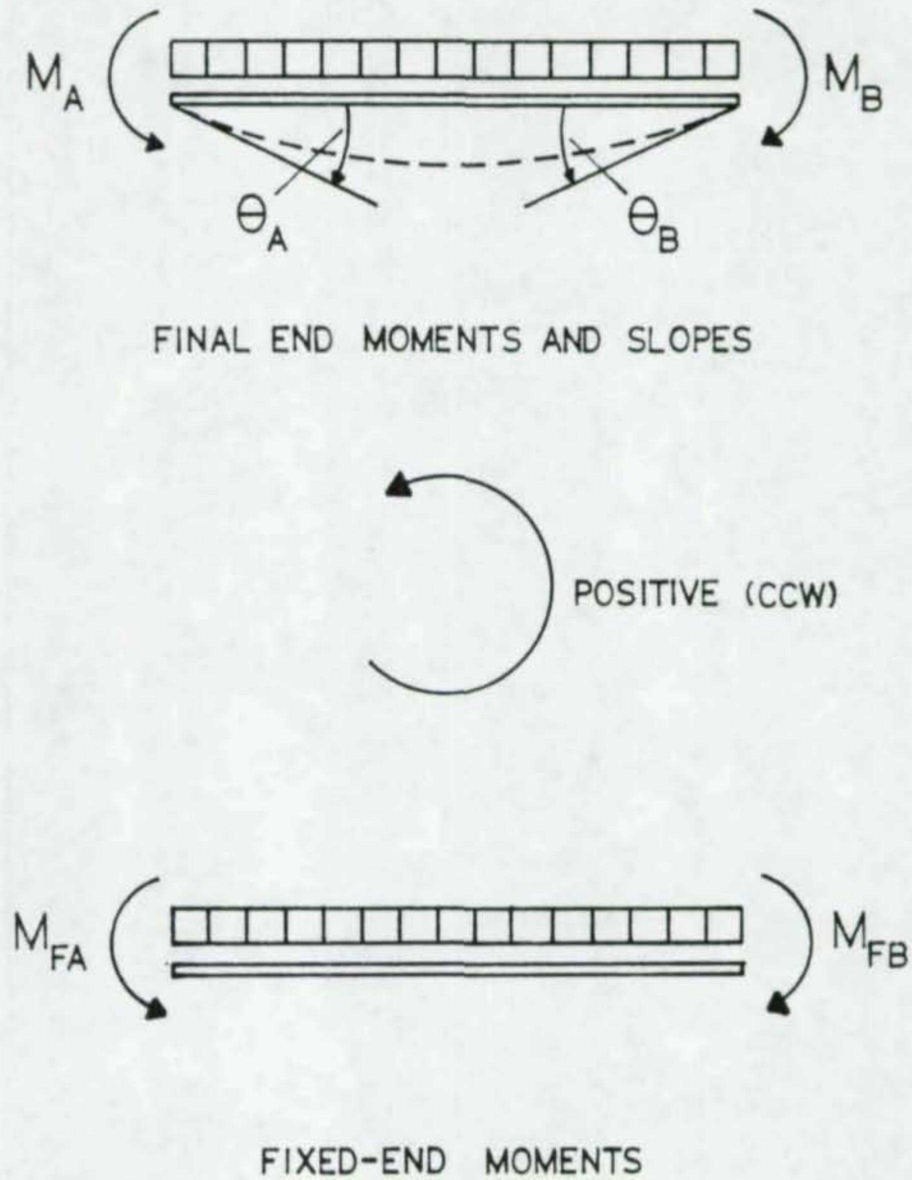


Figure 39. Beam line equation development.

Type 2 - Simple framing connection

Type 3 - Semi-rigid framing connection

These three connection types are shown with the beam line in Figure 40 wherein "fixed" connections are seen to have some small amount of end rotation which results in an end moment that is slightly smaller than the fixed-end moment. Similarly, "simple" connections are not truly simple. These are restrained slightly from rotating and this results in some moment developing at the connection. The moment that actually exists at the connection occurs at the intersection of the beam line and the connection moment-rotation curve. This is true because there can be only <u>one</u> end rotation for a particular loading, or looking at it another way, the end rotation must be compatible with that caused by the loads.

In order to compare Weld A in Table IV of the AISC Manual, the following procedure was used to achieve loading situations that are compatible with those given in Table IV.

1) For a given angle size and angle length and capacity V of weld A from Table IV (AISC pages 4-36 and 4-37), the uniform load on a simply supported beam of length L is

$$w = 2V/L.$$

2) For a simply supported beam of length L and uniform load w, the maximum bending moment occurs at the middle of the beam and is

$$M_{MAX} = wL^2/8$$

3) Using the maximum bending moment and considering beams that have full lateral support of the compression flange

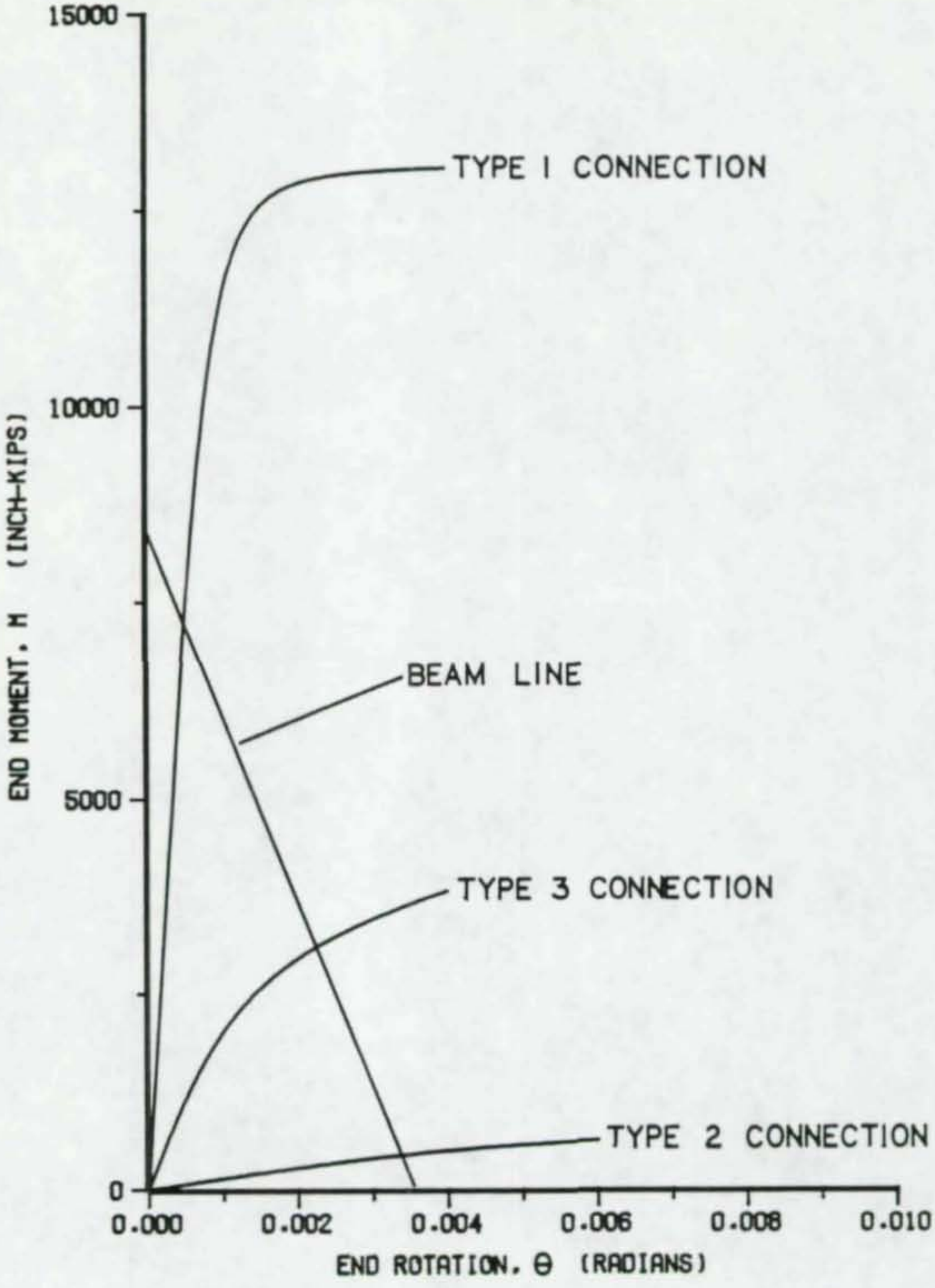


Figure 40. AISC connection types with beam line.

(so that the allowable bending stress may be taken as 0.66 times the yield stress of the beam), a beam may be chosen from the Allowable Stress Design Selection Table given in the AISC Manual.

4) The two parameters that define the beam line may now be calculated.

$$M_{FIXED} = wL^2/12$$

$$\theta_{SIMPLE} = wL^3/24EI$$

In the above procedure, E = 29,000 ksi and L = 20 feet for all beams. Table 9 gives the values associated with steps 1 through 4 above for all connection geometries considered.

The beam line for the particular loading can now be superimposed on the moment-rotation curve for the particular connection to determine the end rotation and resisting moment at the connection. Figures 41 through 56 show the beam line with the moment-rotation curve for each of the connection geometries and loadings considered in Table 9.

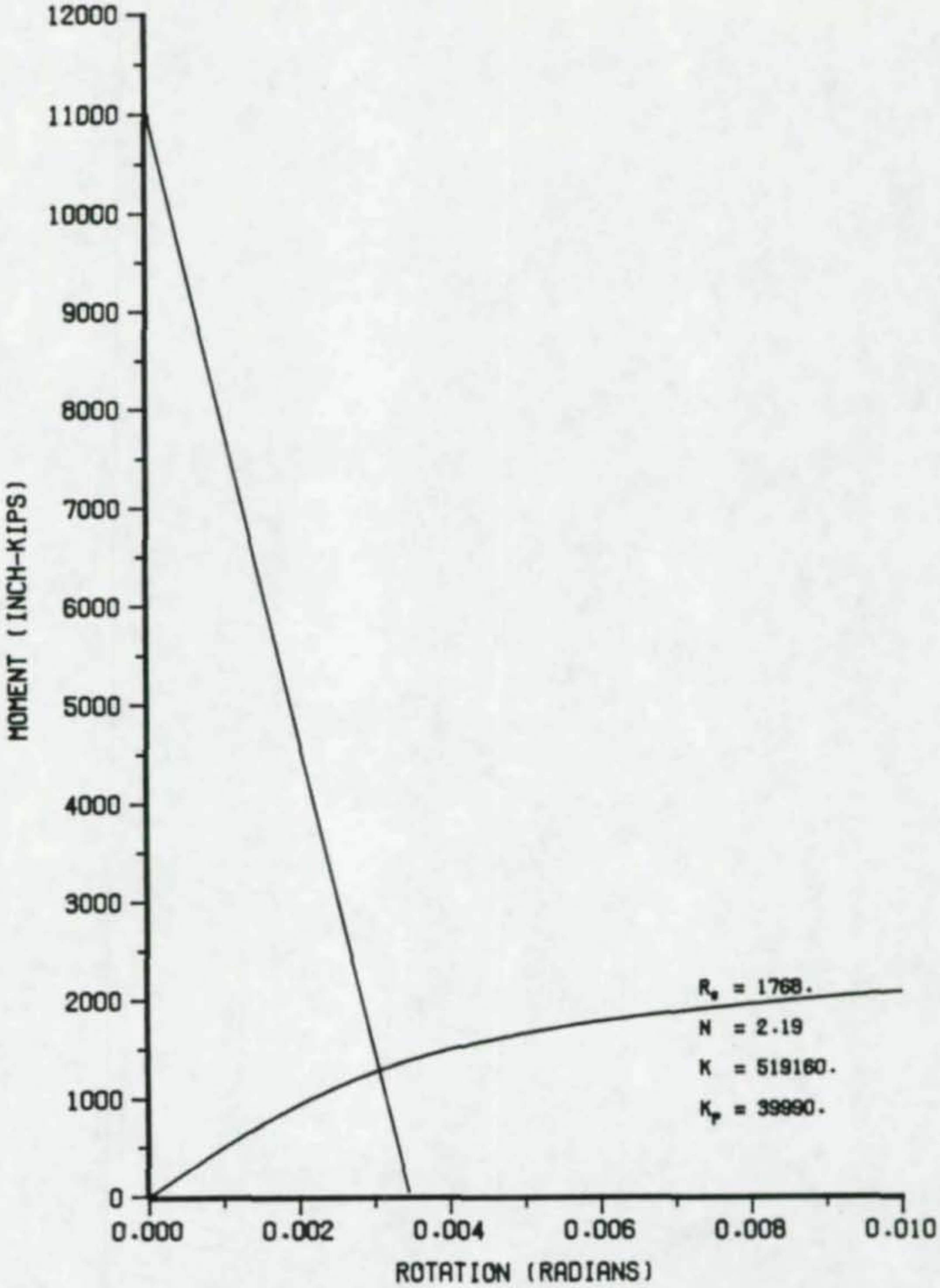
The resisting moment and end rotation, which are represented graphically by the intersection of the beam line and the moment-rotation curve, can be determined numerically using a Newton-Raphson root finding procedure. The Fortran program NRMRSOL (Newton-Raphson Moment Rotation SOLution) given in appendix C uses a Newton-Raphson technique to determine the intersection point of the beam line with the moment-rotation curve. Using this technique, the end rotation and resisting moment for each connection considered are given in Table 10.

With the resisting moment at the connection known, Weld A and Weld B can be designed to resist not only the shear but also the resisting moment that is developed at the connection.

TABLE 9 UNIFORM LOAD, BEAM SELECTION, AND BEAM LINE PARAMETERS FOR WELD "A" COMPARISON

Angle Size & Length	V	w	M_{MAX}	Beam	M_{FIXED}	θ_{SMPL}
L-4 x 3 x 1/2 x 32	277	2.30	1380	W36 x 210	11,040	.00346
L-4 x 3 x 3/8 x 32	221	1.84	1104	W36 x 170	8,832	.00348
L-4 x 3 x 1/2 x 30	262	2.18	1308	W36 x 194	10,464	.00358
L-4 x 3 x 3/8 x 30	210	1.75	1050	W36 x 160	8,400	.00356
L-4 x 3 x 1/2 x 28	248	2.07	1242	W36 x 182	9,936	.00364
L-4 x 3 x 3/8 x 28	198	1.65	990	W36 x 150	7,920	.00363
L-4 x 3 x 1/2 x 26	234	1.95	1170	W36 x 182	9,360	.00343
L-4 x 3 x 3/8 x 26	187	1.56	936	W33 x 152	7,488	.00380
L-4 x 3 x 1/2 x 24	218	1.82	1092	W36 x 170	8,736	.00344
L-4 x 3 x 3/8 x 24	174	1.45	870	W36 x 135	6,960	.00369
L-4 x 3 x 1/2 x 22	204	1.70	1020	W36 x 160	8,160	.00346
L-4 x 3 x 3/8 x 22	163	1.36	816	W36 x 135	6,528	.00346
L-4 x 3 x 1/2 x 20	188	1.57	942	W33 x 152	7,536	.00382
L-4 x 3 x 3/8 x 20	151	1.26	756	W33 x 130	6,048	.00373
L-4 x 3 x 1/2 x 18	172	1.43	858	W36 x 135	6,864	.00364
L-4 x 3 x 3/8 x 18	138	1.15	690	W33 x 118	5,520	.00387

V is chosen from AISC Table IV for weld "A" comparison
w = uniform load in kips/inch = 2V/L (L = 20 feet for all beams)
M_{MAX} = maximum bending moment in beam = $wL^2/8$ (kip-ft)
The beams are selected from AISC Beam Selection Tables
M_{FIXED} = fixed end moment = $wL^2/12$ (kip-inch)
θ_{SMPL} = simple (pinned) end rotation = $wL^3/24EI$ (radians)

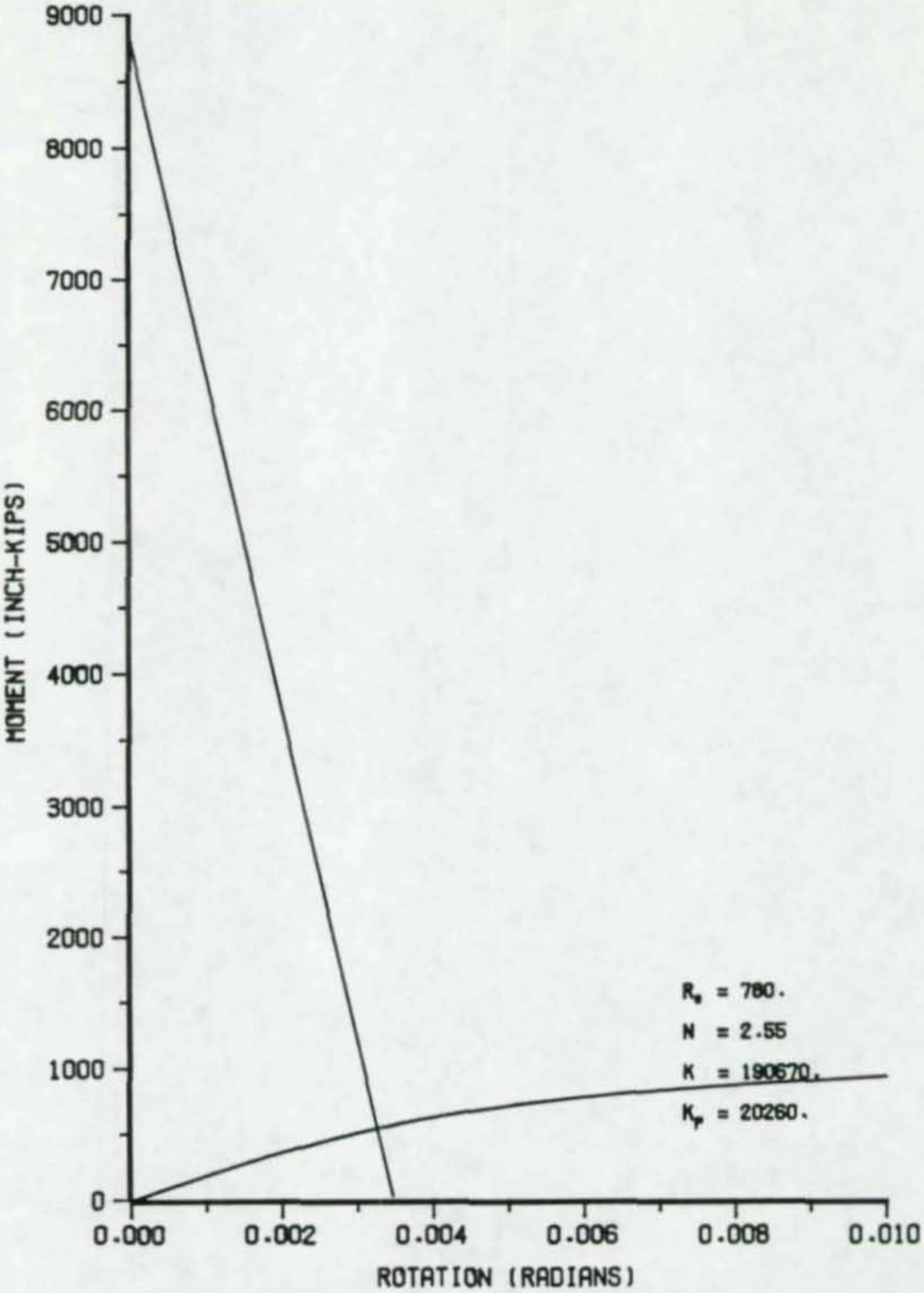


MOMENT ROTATION CURVE WITH BEAM LINE FOR WELDED CONNECTION

DOUBLE ANGLES ARE L4 X 3 X 1/2 X 32 INCHES

BEAM IS A W 36 X 210 WITH UNIFORM LOAD OF 2.3 KIPS/INCH

Figure 41.

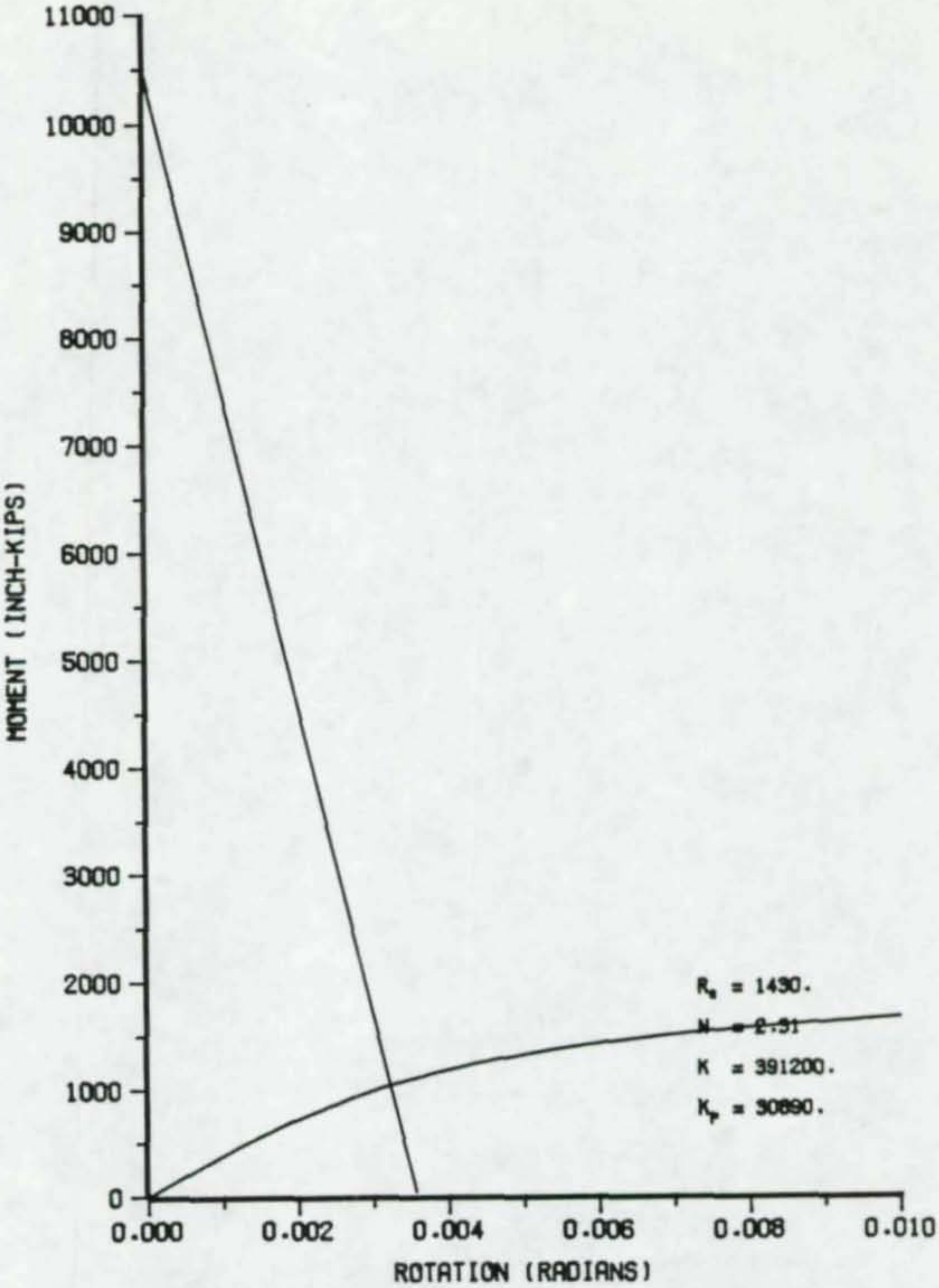


MOMENT ROTATION CURVE WITH BEAM LINE FOR WELDED CONNECTION

DOUBLE ANGLES ARE L4 X 3 X 3/8 X 32 INCHES

BEAM IS A W 36 X 170 WITH UNIFORM LOAD OF 1.84 KIPS/INCH

Figure 42.

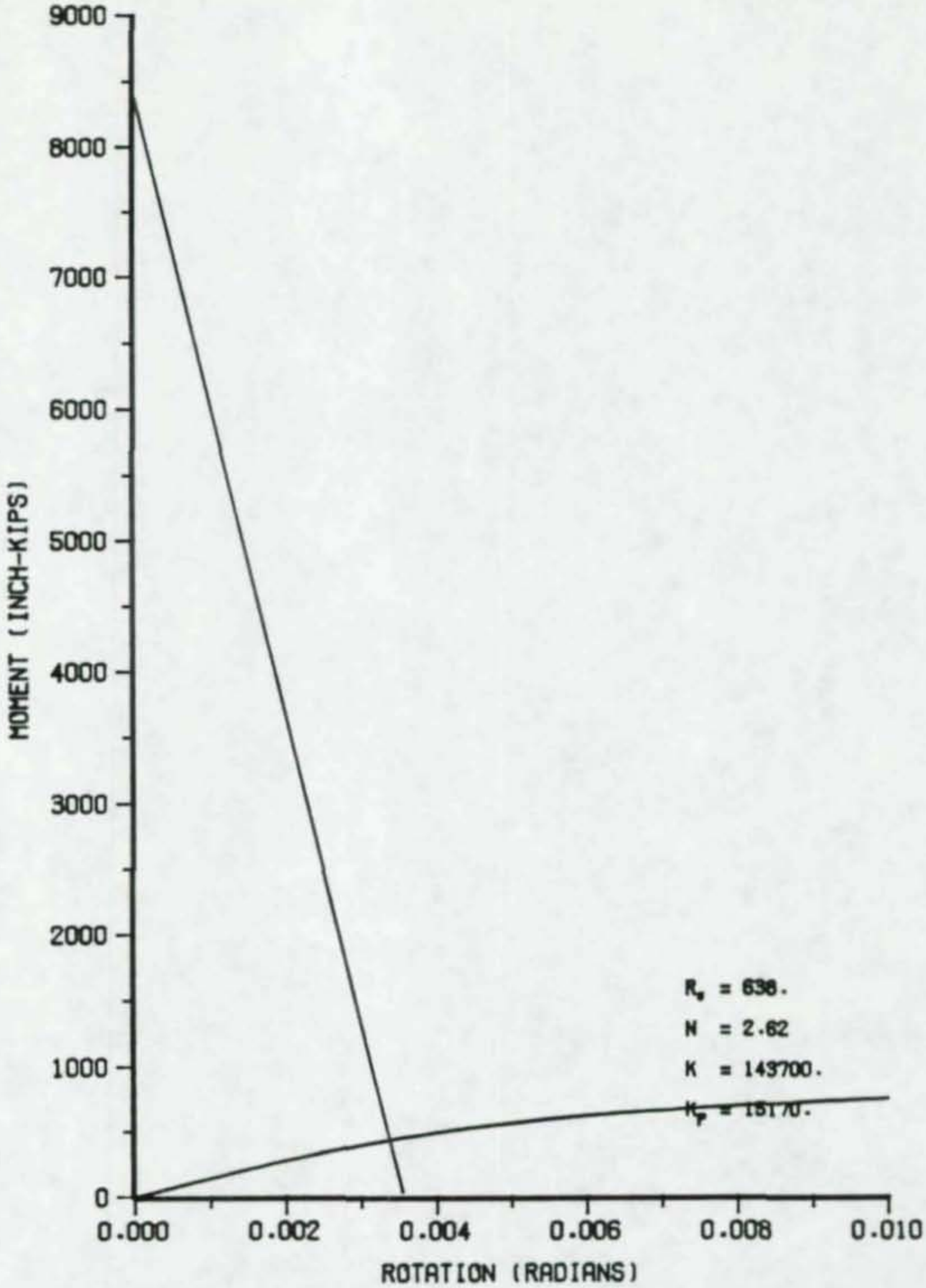


MOMENT ROTATION CURVE WITH BEAM LINE FOR WELDED CONNECTION

DOUBLE ANGLES ARE L4 X 3 X 1/2 X 30 INCHES

BEAM IS A W 36 X 194 WITH UNIFORM LOAD OF 2.18 KIPS/INCH

Figure 43.

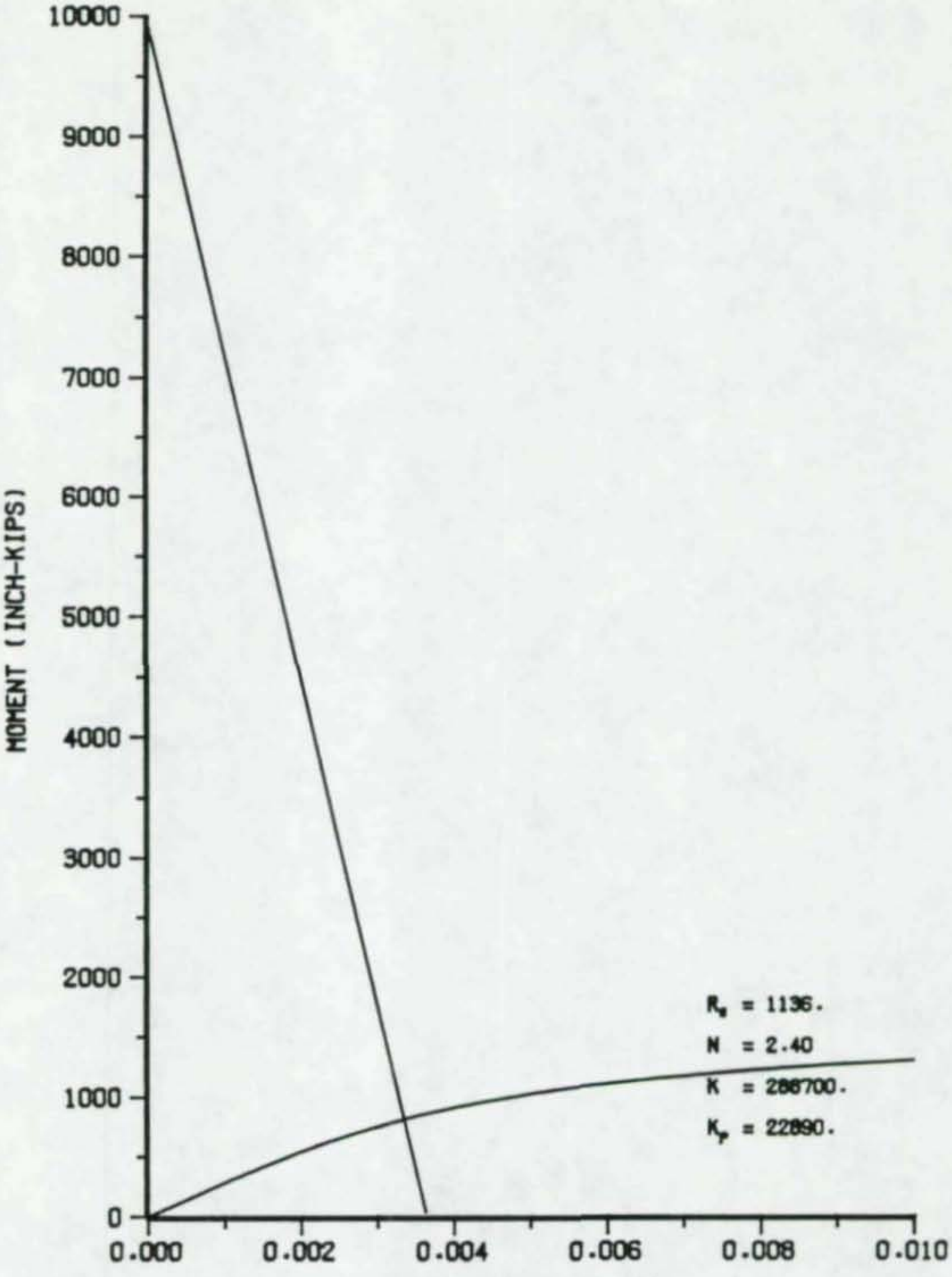


MOMENT ROTATION CURVE WITH BEAM LINE FOR WELDED CONNECTION

DOUBLE ANGLES ARE L4 X 3 X 3/8 X 30 INCHES

BEAM IS A W 36 X 160 WITH UNIFORM LOAD OF 1.75 KIPS/INCH

Figure 44.

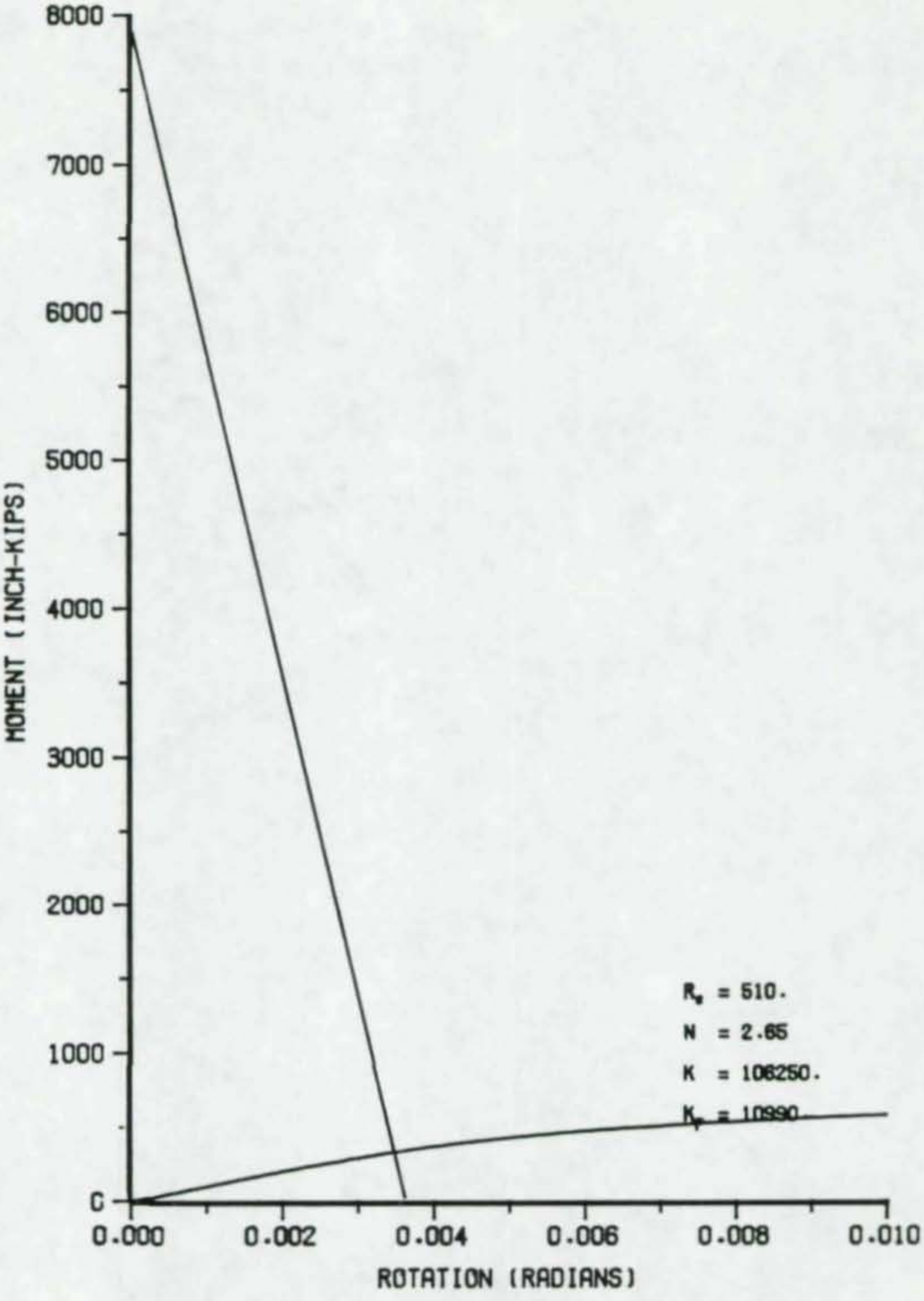


MOMENT ROTATION CURVE WITH BEAM LINE FOR WELDED CONNECTION

DOUBLE ANGLES ARE L4 X 3 X 1/2 X 28 INCHES

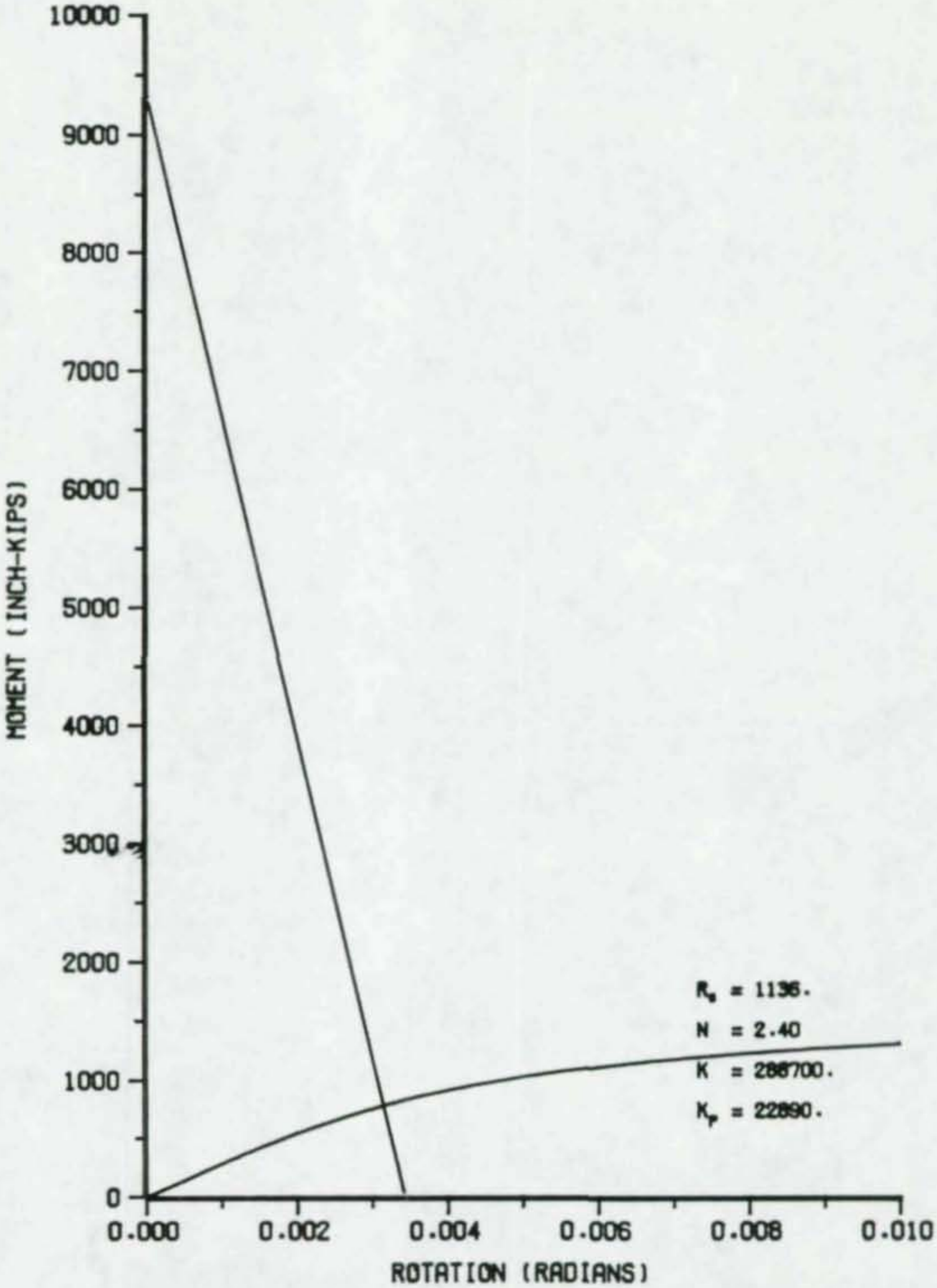
BEAM IS A W 36 X 182 WITH UNIFORM LOAD OF 2.07 KIPS/INCH

Figure 45.



MOMENT ROTATION CURVE WITH BEAM LINE FOR WELDED CONNECTION
DOUBLE ANGLES ARE L4 X 3 X 3/8 X 28 INCHES
BEAM IS A W 36 X 150 WITH UNIFORM LOAD OF 1.65 KIPS/INCH

Figure 46.

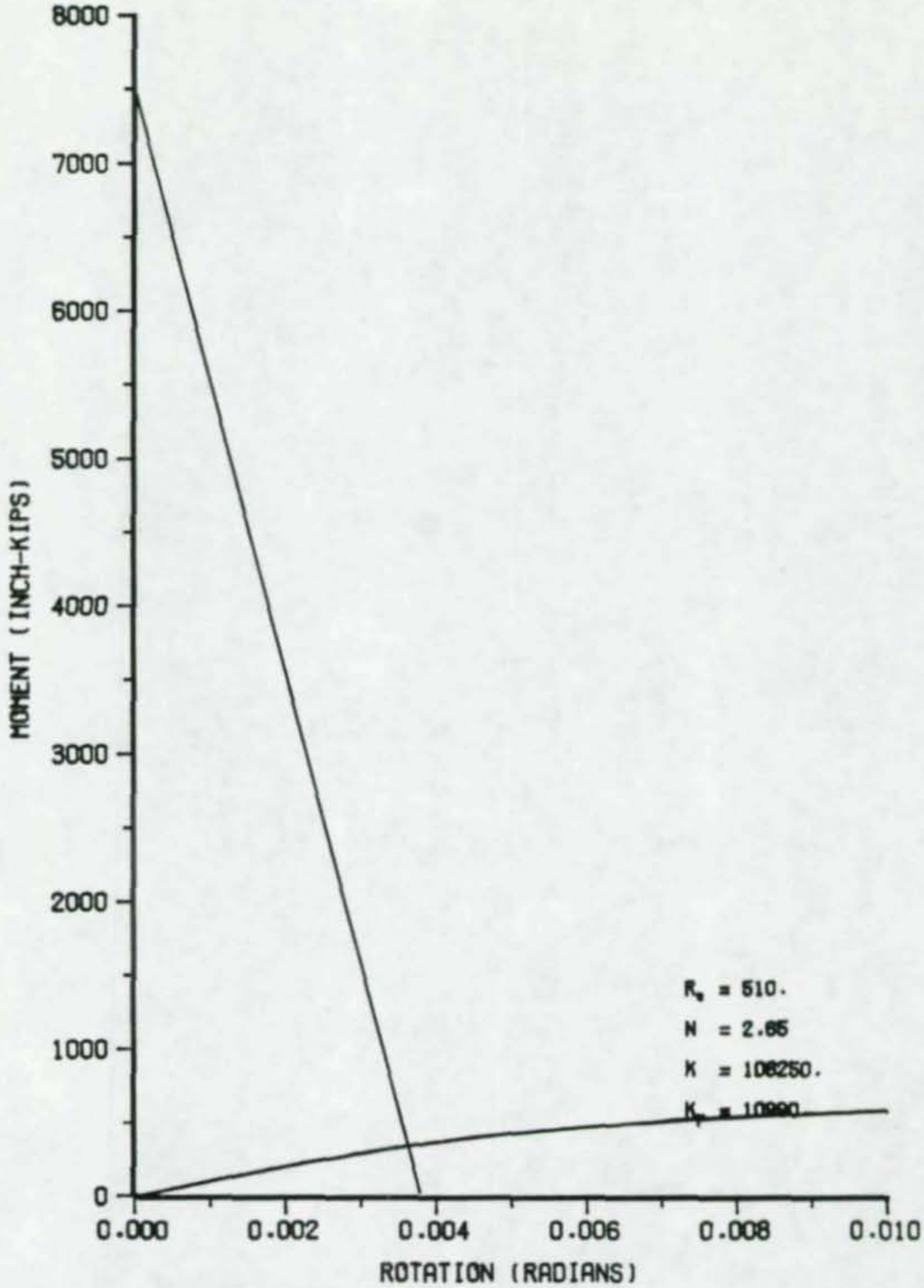


MOMENT ROTATION CURVE WITH BEAM LINE FOR WELDED CONNECTION

DOUBLE ANGLES ARE L4 X 3 X 1/2 X 26 INCHES

BEAM IS A W 36 X 182 WITH UNIFORM LOAD OF 1.95 KIPS/INCH

Figure 47.

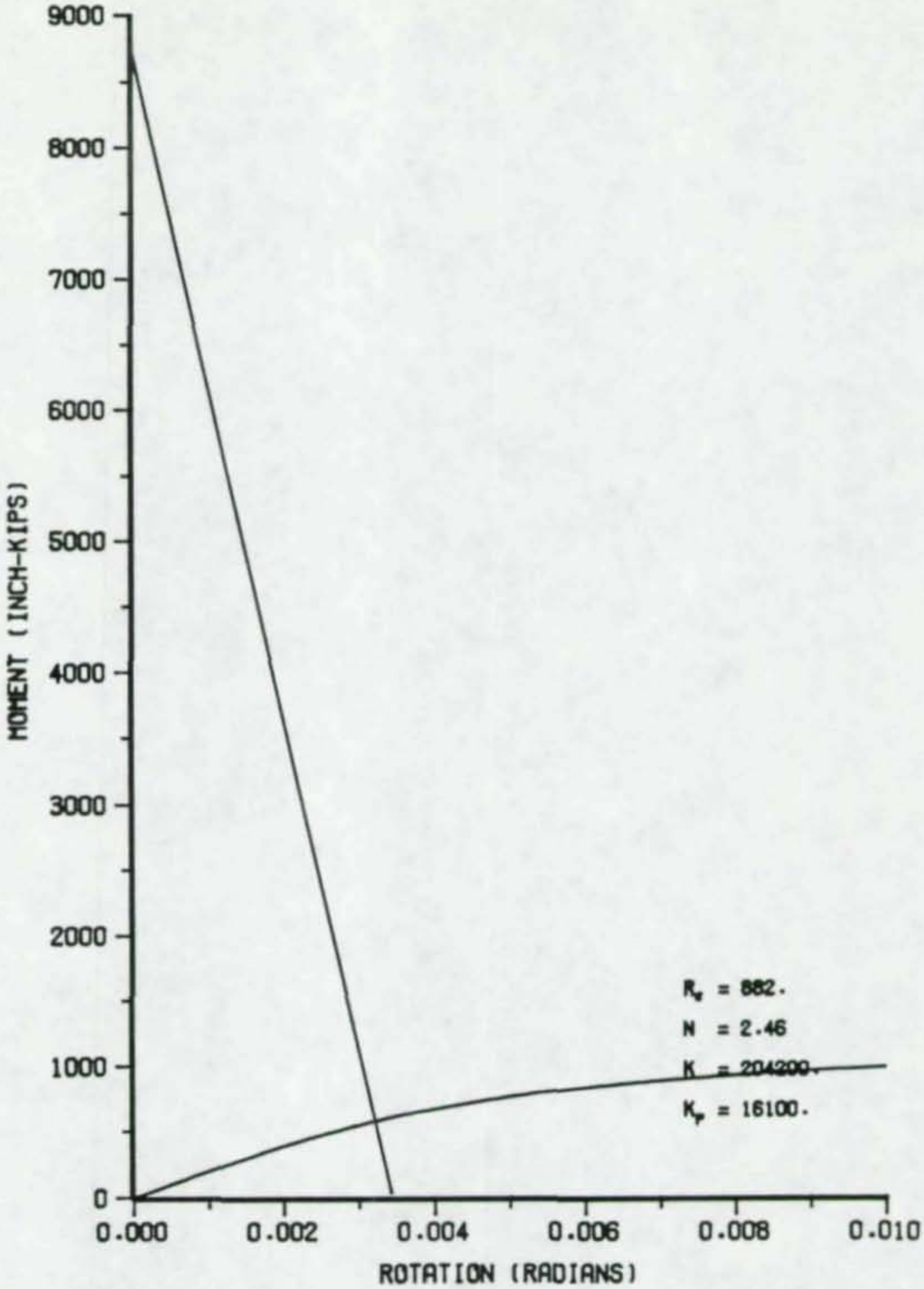


MOMENT ROTATION CURVE WITH BEAM LINE FOR WELDED CONNECTION

DOUBLE ANGLES ARE L4 X 3 X 3/8 X 26 INCHES

BEAM IS A W 33 X 152 WITH UNIFORM LOAD OF 1.56 KIPS/INCH

Figure 48.

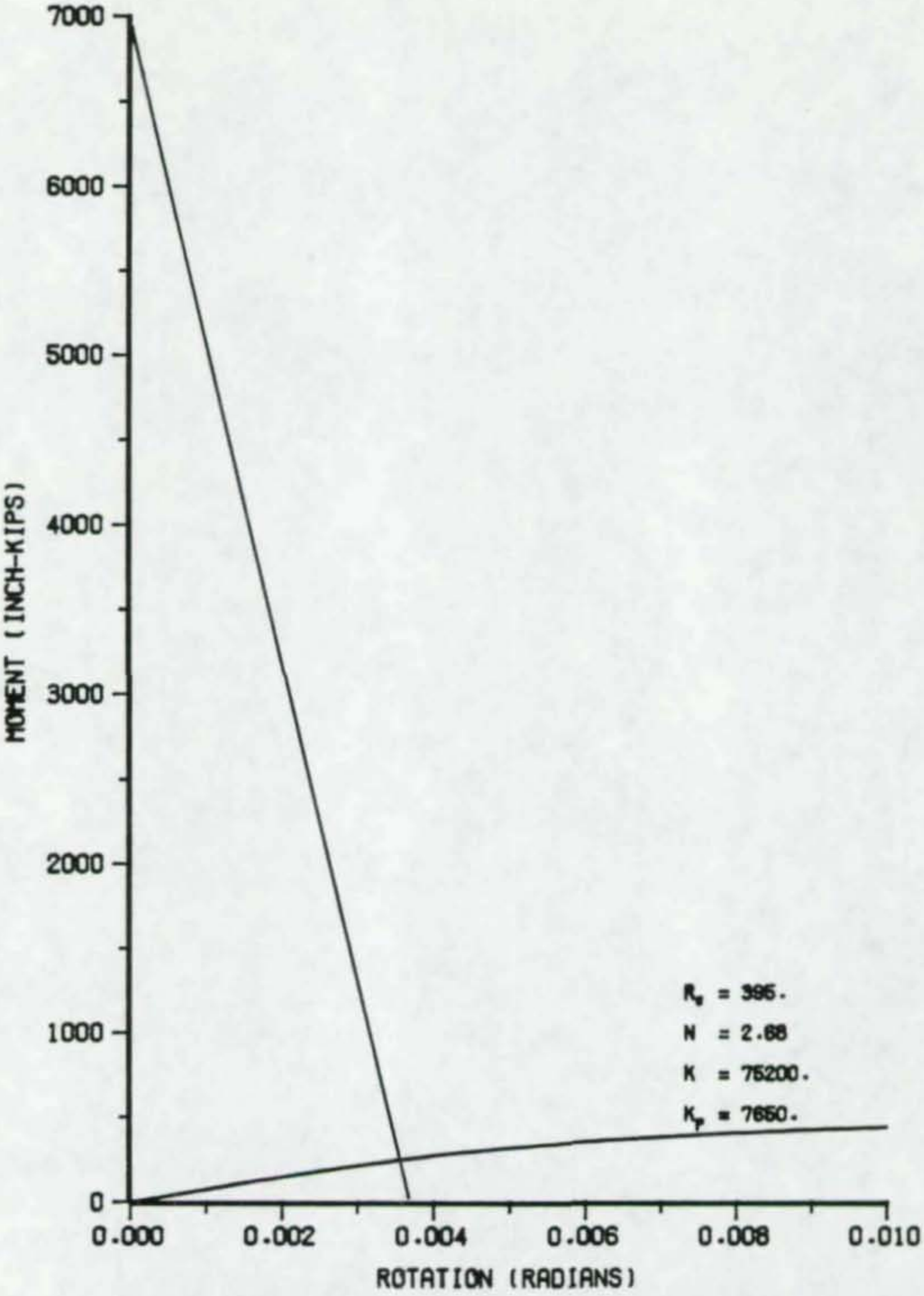


MOMENT ROTATION CURVE WITH BEAM LINE FOR WELDED CONNECTION

DOUBLE ANGLES ARE L4 X 3 X 1/2 X 24 INCHES

BEAM IS A W 36 X 170 WITH UNIFORM LOAD OF 1.82 KIPS/INCH

Figure 49.

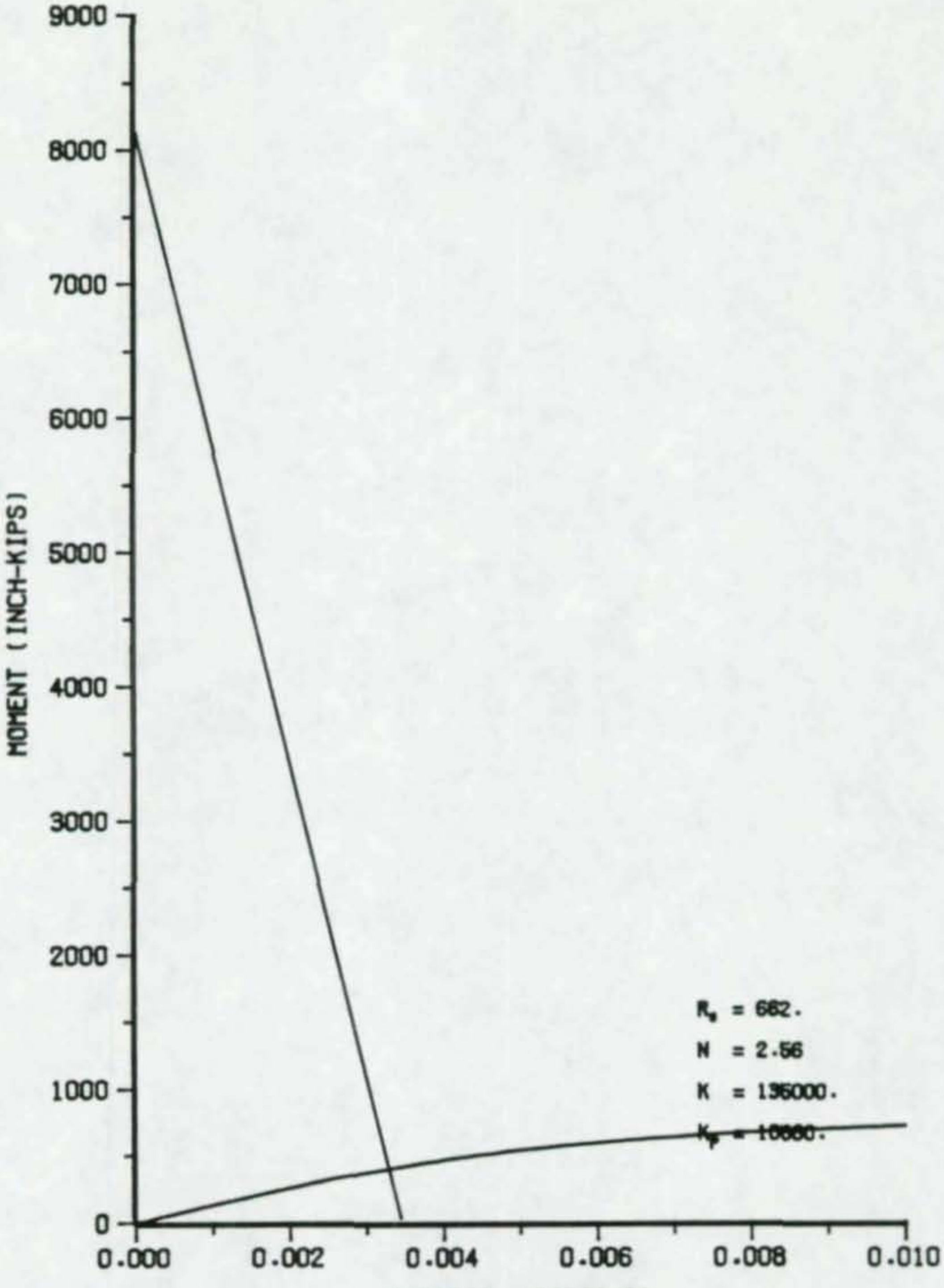


MOMENT ROTATION CURVE WITH BEAM LINE FOR WELDED CONNECTION

DOUBLE ANGLES ARE L4 X 3 X 3/8 X 24 INCHES

BEAM IS A W 36 X 135 WITH UNIFORM LOAD OF 1.45 KIPS/INCH

Figure 50.

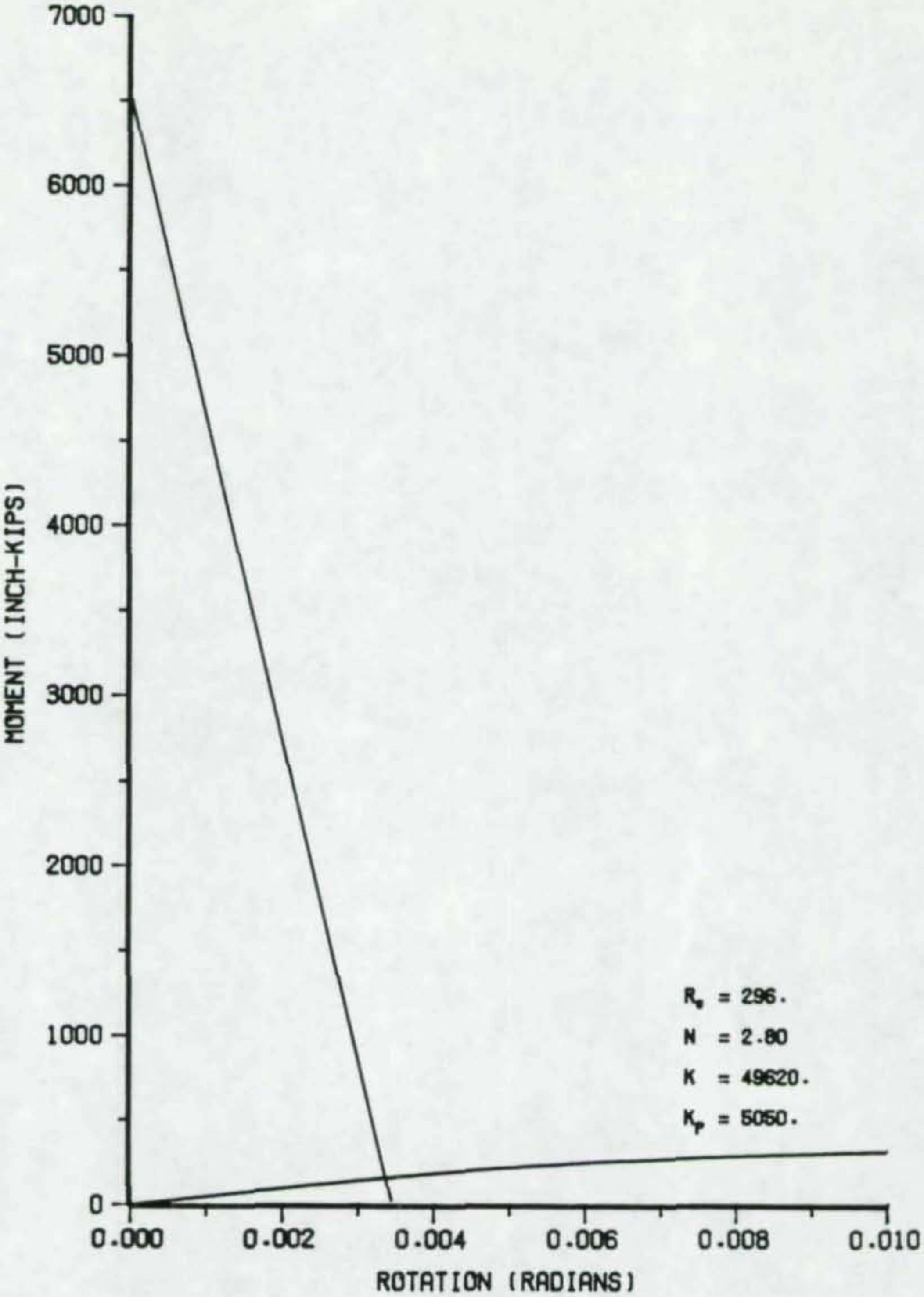


MOMENT ROTATION CURVE WITH BEAM LINE FOR WELDED CONNECTION

DOUBLE ANGLES ARE L4 X 3 X 1/2 X 22 INCHES

BEAM IS A W 36 X 160 WITH UNIFORM LOAD OF 1.70 KIPS/INCH

Figure 51.

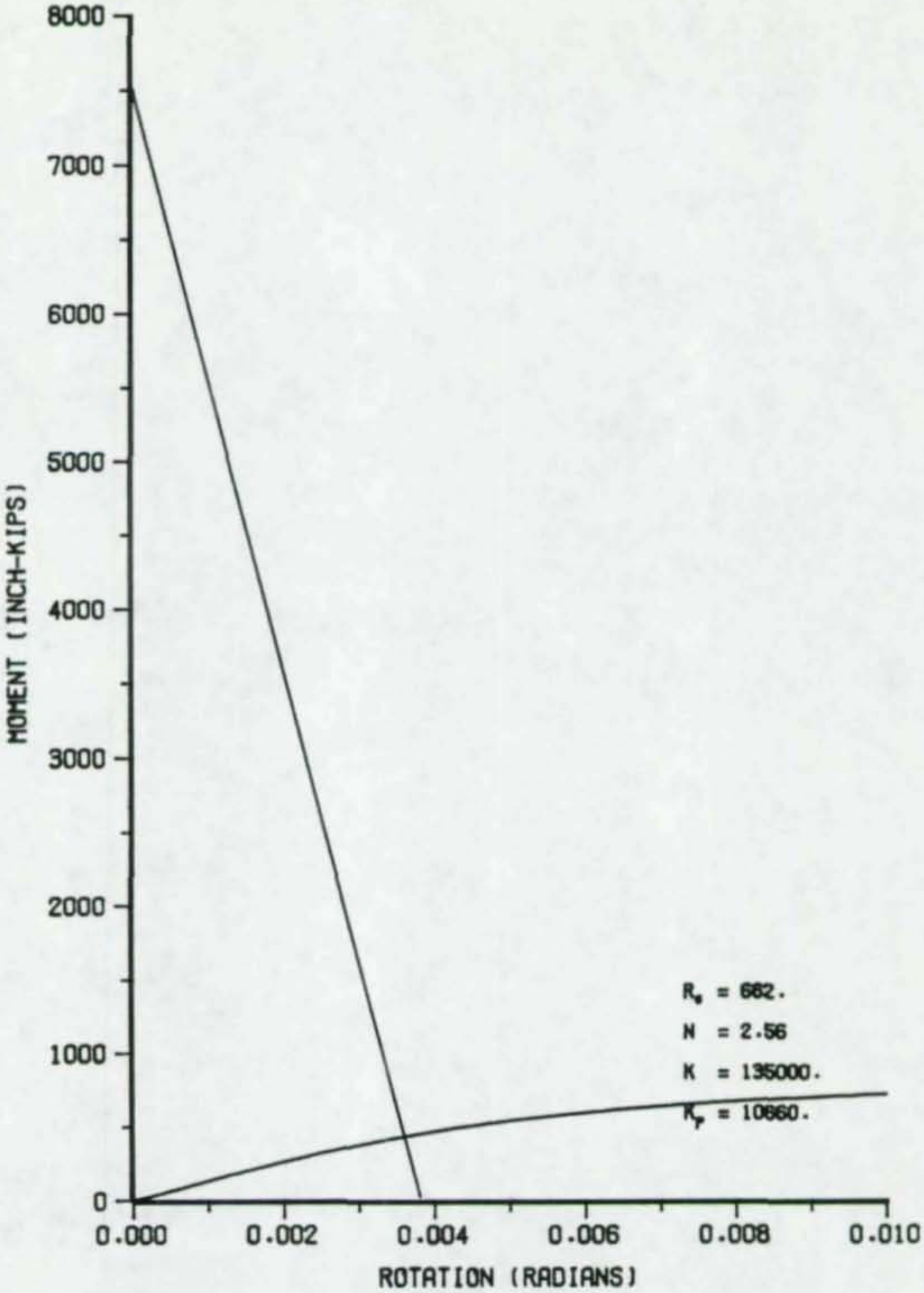


MOMENT ROTATION CURVE WITH BEAM LINE FOR WELDED CONNECTION

DOUBLE ANGLES ARE L4 X 3 X 3/8 X 22 INCHES

BEAM IS A W 36 X 135 WITH UNIFORM LOAD OF 1.36 KIPS/INCH

Figure 52.

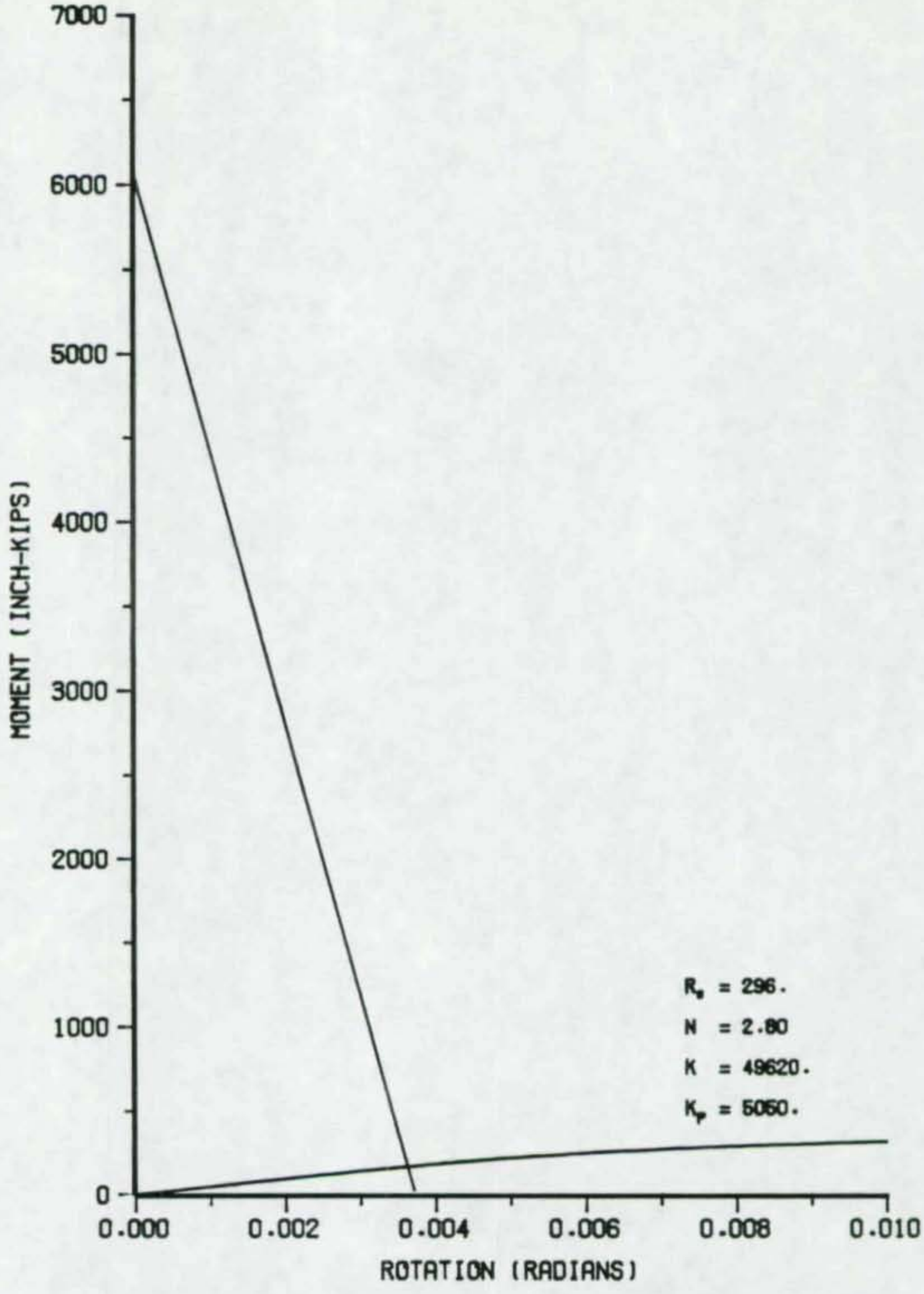


MOMENT ROTATION CURVE WITH BEAM LINE FOR WELDED CONNECTION

DOUBLE ANGLES ARE L4 X 3 X 1/2 X 20 INCHES

BEAM IS A W 33 X 152 WITH UNIFORM LOAD OF 1.57 KIPS/INCH

Figure 53.

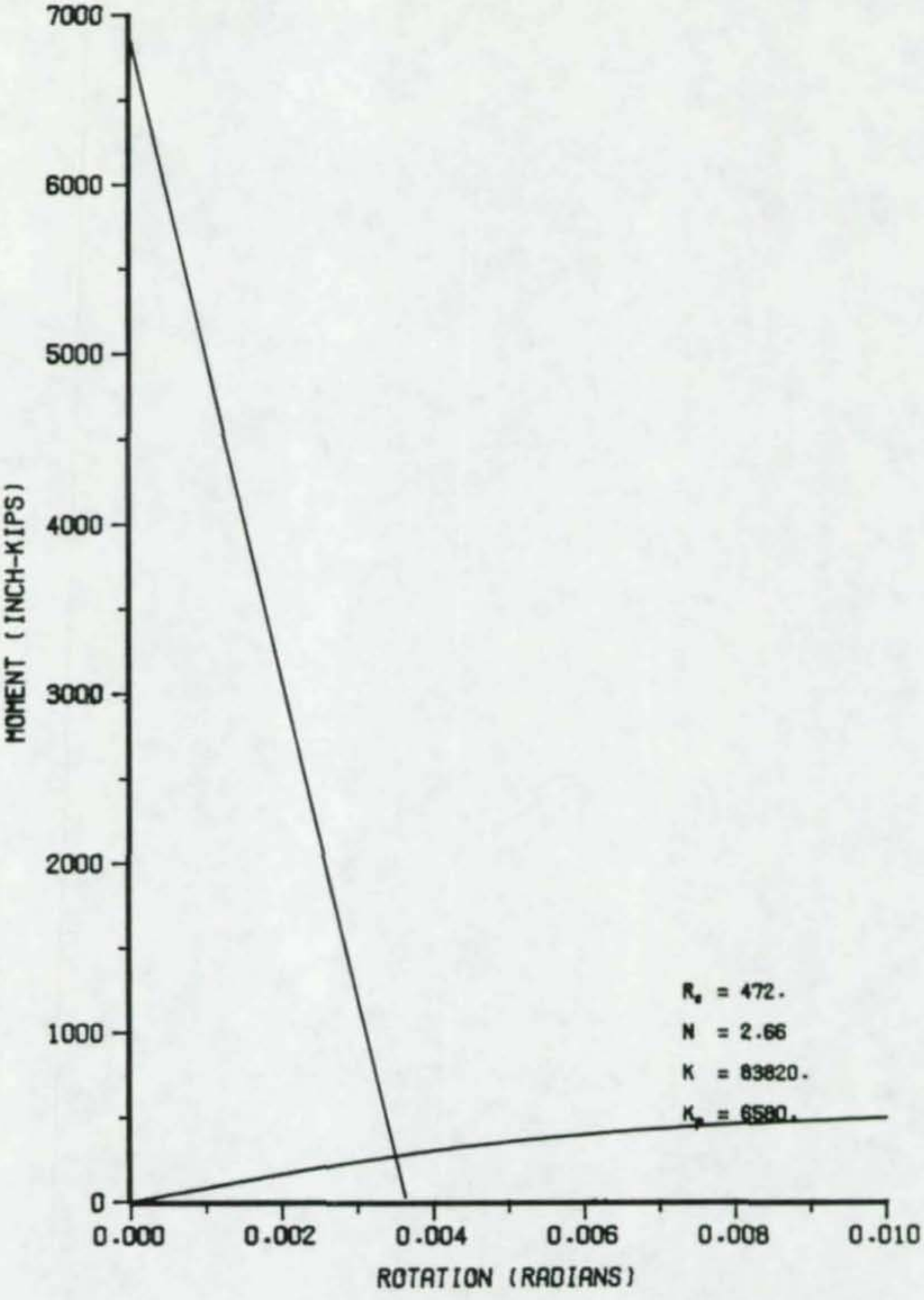


MOMENT ROTATION CURVE WITH BEAM LINE FOR WELDED CONNECTION

DOUBLE ANGLES ARE L4 X 3 X 3/8 X 20 INCHES

BEAM IS A W 33 X 130 WITH UNIFORM LOAD OF 1.26 KIPS/INCH

Figure 54.

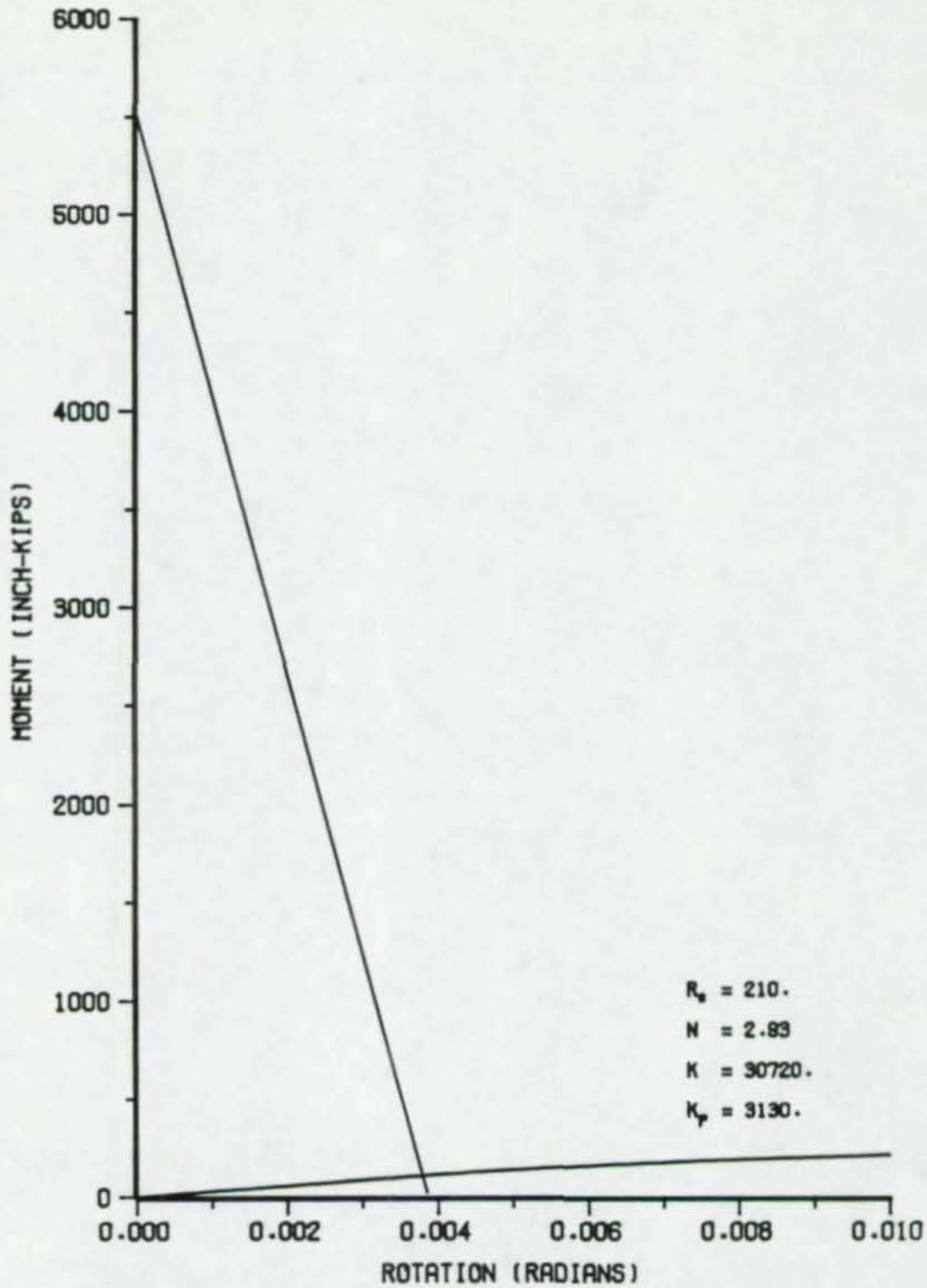


MOMENT ROTATION CURVE WITH BEAM LINE FOR WELDED CONNECTION

DOUBLE ANGLES ARE L4 X 3 X 1/2 X 18 INCHES

BEAM IS A W 36 X 135 WITH UNIFORM LOAD OF 1.43 KIPS/INCH

Figure 55.



MOMENT ROTATION CURVE WITH BEAM LINE FOR WELDED CONNECTION

DOUBLE ANGLES ARE L4 X 3 X 3/8 X 18 INCHES

BEAM IS A W 33 X 118 WITH UNIFORM LOAD OF 1.15 KIPS/INCH

Figure 56.

TABLE 10 RESISTING MOMENT AND ROTATION AT END CONNECTION FOR VARIOUS WELDED CONNECTION GEOMETRIES

Connection Geometry	Resisting Moment (inch-kips)	End Rotation (radians)
L-4 x 3 x 1/2 x 32	1284	.003058
L-4 x 3 x 3/8 x 32	551	.003264
L-4 x 3 x 1/2 x 30	1042	.003222
L-4 x 3 x 3/8 x 30	437	.003379
L-4 x 3 x 1/2 x 28	815	.003340
L-4 x 3 x 3/8 x 28	336	.003471
L-4 x 3 x 1/2 x 26	782	.003141
L-4 x 3 x 3/8 x 26	347	.003621
L-4 x 3 x 1/2 x 24	580	.003214
L-4 x 3 x 3/8 x 24	248	.003561
L-4 x 3 x 1/2 x 22	405	.003291
L-4 x 3 x 3/8 x 22	160	.003378
L-4 x 3 x 1/2 x 20	435	.003601
L-4 x 3 x 3/8 x 20	170	.003625
L-4 x 3 x 1/2 x 18	273	.003496
L-4 x 3 x 3/8 x 18	112	.003793

WELD DESIGN AND COMPARISON

WELD "A" DESIGN

To design the weld that connects the angle clips to the beam web (Weld A), both the shear and the connection moment must be considered. Instead of considering the shear force acting at the centroid of the weld together with an applied moment, the vertical shear may be transferred a distance $e = M/V$ from the weld centroid (see Figure 57). This gives a statically equivalent loading and the weld may be designed by considering eccentric shear (shear and moment).

The allowable stresses for shear on the effective area of all welds is equal to 0.30 times the electrode tensile strength. The electrode tensile strength for the welds involved in this research and for the welds in Table IV of the AISC Manual is equal to 70 ksi. The effective area of fillet welds is equal to the product of the effective throat dimension times the length of the weld [4]. The effective throat dimension of a fillet weld is equal to 0.707 times the weld size, "a" [4]. The allowable shear stress per unit length of weld is therefore equal to the following.

$$f_{allowable} = (0.30)(70\text{ ksi})(0.707)(a/16)$$

$$= 0.928a \quad \text{kips/inch}$$

Here, a is the weld size in sixteenths of an inch.

The actual stresses that occur on Weld A of Figure 57 are caused by shear and by moment. The stress per unit length of weld caused by the direct shear force V is the following.

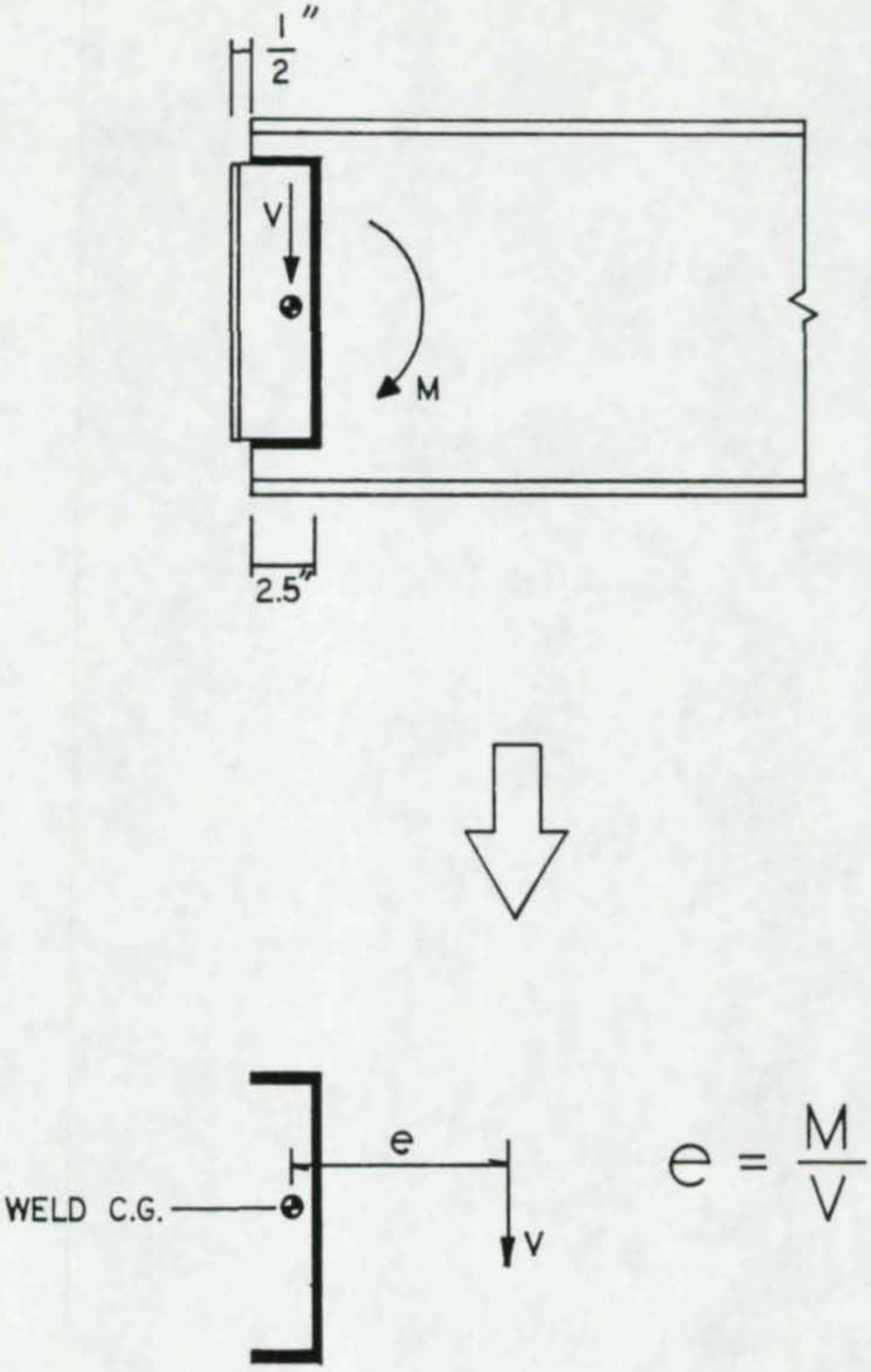


Figure 57. Shear force transfer for Weld A design.

$$f' = V/(\text{length of weld}) \qquad \text{kips/inch}$$

The stress per unit length of weld caused by the torsional moment is the following.

$$f'' = Tr/J \qquad \text{kips/inch}$$

In the above equation, T is the torsional moment and is equal to the connection moment M. The polar moment of inertia is J, and r is the radial distance from the weld centroid to the point of stress computation.

In order to derive useful expressions for the polar moment of intertia and the location of the centroid of the weld, the welds are treated as lines ignoring the thickness (weld size "a") of the welds. For the geometry of Weld A shown in Figure 58, the location of the centroid is given by the following expression.

$$X_{c.g.} = b^2/(2b + d)$$

The polar moment of inertia of the weld geometry shown in Figure 58 is given by the following expression.

$$J = I_p = I_x + I_y$$

$$= I_{xx} + (Area)(d_y)^2 + I_{yy} + (Area)(d_x)^2$$

$$J = \{(8b^3 + 6bd^2 + d^3)/12\} - b^4/(2b + d)$$

<u>Weld "A" Design Example</u>

<u>Given</u>: W36 x 210 beam with a uniform load of 2.3 kips/inch
L = 20 feet, T = 32-1/8 inch, t(web) = .83 inch
Double angles are 2-L4 x 3 x 1/2 x 32 inches

<u>Solution</u>:

$$\bar{x} = \frac{b^2}{2b + d}$$

$$J = \frac{8b^3 + 6bd^2 + d^3}{12} - \frac{b^4}{2b + d}$$

Figure 58. Center of gravity and polar moment of inertia for weld geometry shown.

$V = wL/2 = (2.3 \text{ kips/inch})(240 \text{ inch})/2 = 276 \text{ kips}$

$M = 1284$ inch-kips (see Table 10)

$e = M/V = 1284/276 = 4.65$ inch

$J = \{8(2.5)^3 + 6(2.5)(32)^2 + (32)^3\}/12$

$\quad - (2.5)^4/(2(2.5) + 32)$

$J = 4020 \text{ inch}^3$

$X_{c.a.} = (2.5)^2/(2(2.5) + 32) = 0.17$ inch

$f_x' = 0$

$f_y' = V/(2b + d) = 276/(5 + 32) = 7.5$ kips/inch

$f'' = Tr/J \quad T = M = 1284$ inch-kips

$f_x'' = (1284)(32/2)/4020 = 5.11$ kips/inch

$f_y'' = (1284)(2.5 - 0.17)/4020 = 0.74$ kips/inch

$f = [(f_x' + f_x'')^2 + (f_y' + f_y'')^2]^{1/2}$

$f = [(0 + 5.11)^2 + (7.5 + 0.74)^2]^{1/2}$

$f = 9.7$ kips/inch

This actual stress must be less than or equal to the allowable stress.

$9.7 = 0.928a$ or $a = 10.5$ sixteenths

The stress is resisted by two welds, one on either side of the beam web. Therefore:

$a = (10.5/2)/16 = 5.25/16$ inch

or <u>Weld size a = 5/16 inch</u>

<u>Check minimum web thickness</u>

Shear stress on base metal shall not exceed 0.40 times yield stress of base metal:

$(0.928)(10) = (0.40)(36 \text{ ksi})t$

$t_{web\ min} = 0.64$ inch

Table 11 compares the sizes of Weld A for different connection geometries obtained by using the procedures outlined in this research with those given in the AISC Manual in Table IV.

WELD "B" DESIGN

To design the weld that connects the angle clips to the column flange (Weld B), consider the effects of shear and moment. Thus Weld B is designed for shear and bending by considering the shear force V acting at a distance (a + e) from Weld B (see Figure 6). This requires designing Weld B for a bending moment $M = V \times (a + e)$.

For the purpose of comparison, instead of designing the size of Weld B and comparing this to the AISC Manual, the capacity of Weld B for the given weld size is determined and compared to the capacities given in Table IV of the AISC Manual (This procedure is used because the beams and loadings have already been selected for Weld A to compare with AISC Table IV).

The actual stresses that occur on Weld B of Figure 6 are caused by shear and by bending moment. The stress per unit length of weld caused by the direct shear force V is the same as for Weld A.

$$f' = V/(\text{length of weld}) \qquad \text{Kips/inch}$$

The stress per unit length of weld caused by the bending moment is determined from the flexure formula.

$$f'' = Mc/I \qquad \text{Kips/inch}$$

In the above equation, M is the bending moment which may be calculated from

$$M = V \times (a + e)$$

where V is the capacity of Weld B, "a" is the distance from Weld B to the centroid of Weld A, and "e" is the eccentricity defined as the distance from the centroid of Weld A to the point where the shear force V acts. The moment of inertia of the weld geometry is I, and "c" is the distance (perpendicular to the axis of bending) to the point of stress computation.

Again, the welds are treated as lines ignoring the thickness (weld size "a") of the welds. The necessary equations for the stresses per unit length of weld are given in Figure 59.

Weld "B" Design Example

Given: W36 x 210 beam, length = 20 feet, T = 32-1/8 inch, t(web) = .83 inch, Weld B size = 3/8 inch.

Solution:

$$f_y' = V/2L = V/2(32) = 0.0156\ V \quad \text{kips/inch}$$

$$f_x'' = 3V(a + e)/L^2$$

$$e = 4.65 \text{ inch} \quad \text{(Table 11)}$$

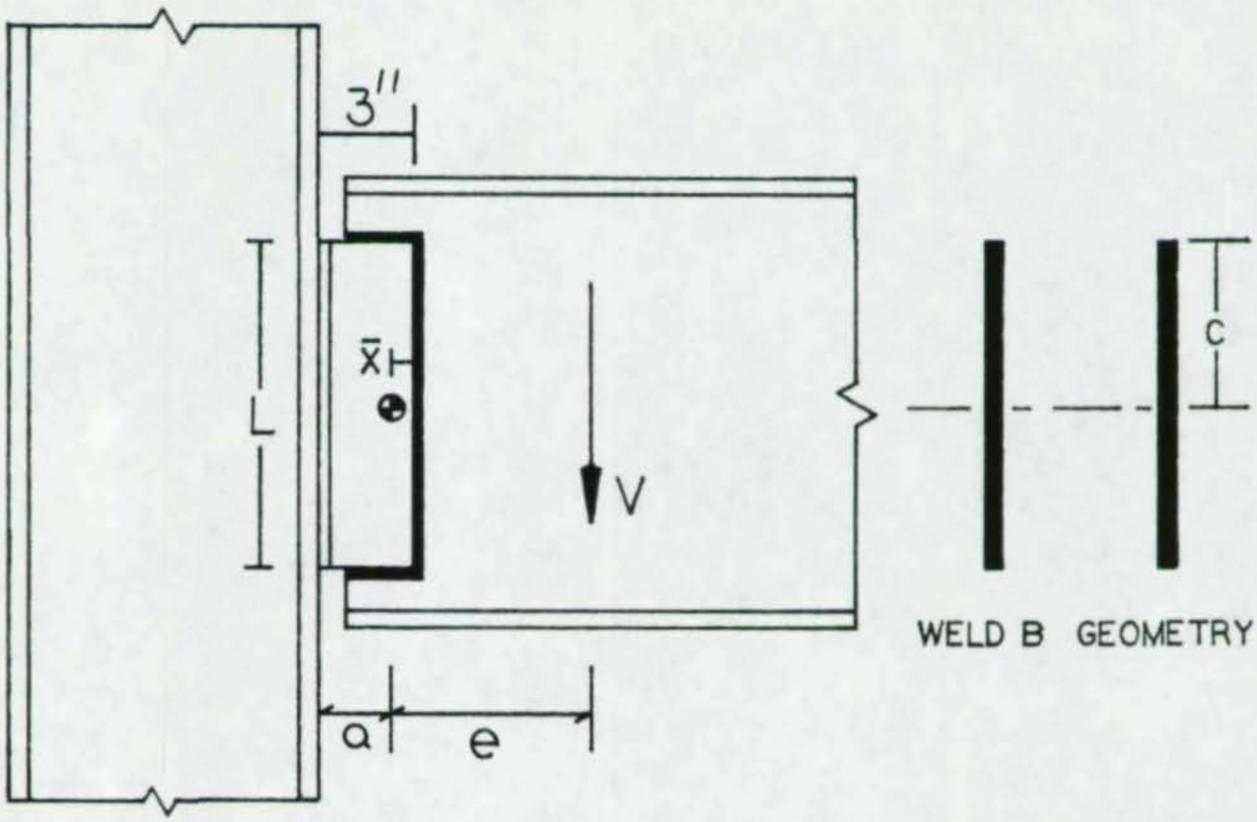
$$a = 3 - 0.17 = 2.83 \text{ inch}$$

$$L = 32 \text{ inch}$$

$$f_x'' = 3V(2.83 + 4.65)/32^2 = 0.0219\ V \quad \text{kips/inch}$$

$$f = [(f_y')^2 + (f_x'')^2]^{1/2}$$

$$f = [(0.0156\ V)^2 + (0.0219\ V)^2]^{1/2}$$



$$f' = \frac{V}{2L}$$

$$f'' = \frac{Mc}{I} \quad \text{where}$$

$$I = 2 \times (1/12)(1)(L)^3 = \frac{L^3}{6}$$

$$c = L/2$$

$$M = V \times (a + e)$$

$a = 3 - \bar{x}$ (see Figure 58)

$e =$ (see Table 11)

$$f'' = \frac{3V(a + e)}{L^2}$$

Figure 59. Equations for computing stresses on Weld B.

$f = 0.0269\ V$ kips/inch

This stress must be less than or equal to the allowable stress.

$$0.0269\ V = .928(6)$$

$$\underline{V = 207 \text{ kips}}$$

In Table 12, the capacities of Weld B given in the AISC Table IV are compared with the capacities of Weld B determined using the method outlined in this research.

TABLE 11 WELD "A" COMPARISON

Shear (kip)	Moment (in-kip)	e (inch)	Connection Geometry	Weld "A" (beam line)	Weld "A" (AISC)
276	1284	4.65	4 x 3 x 1/2 x 32	5/16	5/16
221	551	2.49	4 x 3 x 3/8 x 32	1/4	1/4
262	1042	3.98	4 x 3 x 1/2 x 30	5/16	5/16
210	437	2.08	4 x 3 x 3/8 x 30	1/4	1/4
248	815	3.29	4 x 3 x 1/2 x 28	5/16	5/16
198	336	1.70	4 x 3 x 3/8 x 28	1/4	1/4
234	782	3.34	4 x 3 x 1/2 x 26	5/16	5/16
187	347	1.86	4 x 3 x 3/8 x 26	1/4	1/4
218	580	2.66	4 x 3 x 1/2 x 24	5/16	5/16
174	248	1.43	4 x 3 x 3/8 x 24	1/4	1/4
204	405	1.99	4 x 3 x 1/2 x 22	5/16	5/16
163	160	0.98	4 x 3 x 3/8 x 22	1/4	1/4
188	435	2.31	4 x 3 x 1/2 x 20	5/16	5/16
151	170	1.13	4 x 3 x 3/8 x 20	1/4	1/4
172	273	1.59	4 x 3 x 1/2 x 18	5/16	5/16
138	112	0.81	4 x 3 x 3/8 x 18	1/4	1/4

TABLE 12 WELD "B" COMPARISON

Connection Geometry	Moment (in-kip)	e (inch)	a (inch)	Capacity (kips) Weld B (Beam line)	Weld B (AISC)
4 x 3 x 1/2 x 32	1284	4.65	2.83	207	326
4 x 3 x 3/8 x 32	551	2.49	2.83	210	271
4 x 3 x 1/2 x 30	1042	3.98	2.82	198	302
4 x 3 x 3/8 x 30	437	2.08	2.82	199	251
4 x 3 x 1/2 x 28	815	3.29	2.81	189	278
4 x 3 x 3/8 x 28	336	1.70	2.81	187	231
4 x 3 x 1/2 x 26	782	3.34	2.80	167	254
4 x 3 x 3/8 x 26	347	1.86	2.80	164	211
4 x 3 x 1/2 x 24	580	2.66	2.78	158	230
4 x 3 x 3/8 x 24	248	1.43	2.78	153	191
4 x 3 x 1/2 x 22	405	1.99	2.77	150	206
4 x 3 x 3/8 x 22	160	0.98	2.77	143	171
4 x 3 x 1/2 x 20	435	2.31	2.75	123	181
4 x 3 x 3/8 x 20	170	1.13	2.75	121	152
4 x 3 x 1/2 x 18	273	1.59	2.73	114	157
4 x 3 x 3/8 x 18	112	0.81	2.73	108	131

CONCLUSIONS

All of the connections that were tested and reviewed show no difference in the size of weld A required. This is because the method used to design Weld A in the AISC Manual accounted for both direct shear and a moment, and the moment arm for the calculation of the torsional moment was approximately the same as the moment arm or eccentricity, e, determined in this research. Thus, the two methods gave the same weld size since both had about the same torsional moment.

The capacities of Weld B, however, varied significantly. This is because the method used to calculate the weld sizes for Weld B in the AISC Manual assumed that only a shearing force was transmitted at the interface of the column flange and the outstanding angle legs, which is not correct. The shear force is transmitted through the centroid of Weld A, and since there is a restraining moment being developed in the connection, this shear force and moment is statically equivalent to a shear force acting at an eccentric distance, e, away from the Weld A centroid. Thus, Weld B is not acted upon by only the shear force, but also a bending moment. As a result, the capacities of Weld B given in the AISC Manual are too large for the weld sizes stated. Or, in other words, the sizes of welds given in the AISC Table IV (Weld B) are undersized and should be larger in order to support the loads given in the table.

In summary, the design philosophy for Weld A in the AISC Manual, as well as for Weld B, is incorrect. The sizes of welds for Weld A, however, are satisfactory because the torsional moments used in these designs were approximately equal to those determined in this research.

Only those angles for which moment-rotation curves could be generated from adequate test data were compared. Consequently, only angles with an outstanding leg length of four inches and an angle thickness of either 1/2 or 3/8 inch could be compared. Angles with an outstanding leg length of four inches but with an angle thickness of 5/16 inch, and all angles with an outstanding leg length of three inches could not be compared because of a lack of adequate test data.

If more test data were available, generalized curves for the three Richard equation parameters, R, KP, and N, could be constructed as was done by Blewitt and Richard [1]. These curves plot either the value of R, the reference load, or the value of KP, the plastic stiffness, or the value of N, the shape parameter, for various angle lengths and for various angle thicknesses. The elastic stiffness, K, can always be determined if the angle length and thickness are known, and with these four Richard parameters, the moment-rotation curves can be constructed.

In further research, different loading situations on different beams will be examined, and the welds compared to those given in the AISC Manual.

Appendix A: The Richard Equation

The Richard Equation, published by Richard and Abbott in 1975, is the equation used to describe the non-linear behavior of welded connections presented in this research. This relationship, shown in Figure 2, relates the strength to the stiffness of a structural system, in this case, welded double framing angles. The Richard Equation is given below along with an explanation of the parameters.

$$M = \frac{(K - KP) \times \theta}{\left[1 + \left| \frac{(K - KP) \times \theta}{R_o} \right|^N \right]^{1/N}} + (KP \times \theta)$$

M = Load (moment or force)

θ = Deformation (rotation or displacement)

K = Elastic stiffness or initial slope of the curve

KP = Plastic stiffness or final slope of the curve

N = Shape parameter or the sharpness in transition in slope from K to KP

R_o = Reference load or the intersection of a line asymptotic to the curve at a slope equal to KP with the load axis

Appendix B: MRCURVE

Program MRCURVE is a Fortran computer program that gives the moment and rotation data points and the four Richard equation parameters for the connection under consideration.

The program reads from the input data file, FDINPUT.DAT, and writes to the output data file, OUTPUT.DAT. The input file consists of three lines:

Line 1 = N,DL

Line 2 = TK,TKP,TRO,TN

Line 3 = CK,CKP CRO,CN

where N = number of three inch segments that the connection can be divided into. Suppose the connection is 24 inches long, then N = 24/3 = 8. Suppose the connection is 22 inches long, however, then N must be 7 to give a length of 21 inches, and the half inch lost on either end of the connection is negligible.

DL = maximum rotation of connection to be considered. Choose a rotation that is consistent with the type of connection considered. For welded connections, let DL = 0.05 to DL = 0.2 radians.

TK = elastic stiffness of three inch segment of connection loaded in tension

TKP = plastic stiffness of three inch segment of connection loaded in tension

TRO = reference load for three inch segment of connection loaded in tension

TN = shape parameter for three inch segment of connection loaded in tension

CK = elastic stiffness (compression)

CKP = plastic stiffness (compression)

CRO = reference load (compression)

CN = shape parameter (compression)

After these three lines, which represent one connection, another three lines of input data representing another connection may be input, and so on for all connections being considered. After all connection data, the user must include a final line to stop the program.

Final line = 0,0 (two zeros)

As for the program itself, the first ten lines are simply dimensioning arrays, opening input and output files, and reading input data from the input file. Do loop 100 together with Do loop 200 determine the point of rotation and the forces associated with each three inch segment of connection by invoking equilibrium of forces. The resisting moment for that particular rotation angle is then calculated by summing moments of forces about the bottom of the connection. This gives one moment-rotation data point. This process is repeated ten times (Do loop 100) to give eleven moment-rotation data points.

The program then computes the four Richard equation parameters that are associated with a least squares curve passing through the moment-rotation data points. The elastic stiffness is computed by calculating the slope of the line passing through the origin and the first moment-rotation point. The plastic stiffness is computed by calculating the slope of the line passing through the last two moment-rotation data points. The reference load is computed by calculating the intercept of the line asymptotic to the curve and with a slope equal to the plastic stiffness with the load axis. The shape parameter is computed by starting with a value of 0.01 for the shape parameter and incrementing this value by 0.01 until the sum of the least square errors between the data points obtained earlier and the data points obtained using the incremented value of the shape parameter is a minimum.

SAMPLE INPUT FOR PROGRAM MRCURVE

The following input data is for a 30 inch long welded double framing angle connection with angles that are 2L-4 x 3 x 3/8 inch.

```
10,0.05
73,6,5,3.4
1771,207,213,1.2
0,0
```

SAMPLE OUTPUT FOR PROGRAM MRCURVE

The following output is for the above input data.

```
MOMENT =       7.02       THETA =      .0000
MOMENT =      14.03       THETA =      .0001
MOMENT =      28.99       THETA =      .0002
MOMENT =      57.98       THETA =      .0004
MOMENT =     115.84       THETA =      .0008
MOMENT =     229.00       THETA =      .0016
MOMENT =     421.53       THETA =      .0031
MOMENT =     631.39       THETA =      .0063
MOMENT =     804.98       THETA =      .0125
MOMENT =    1016.79       THETA =      .0250
MOMENT =    1396.06       THETA =      .0500

ELASTIC MODULUS K =  143704.38

PLASTIC MODULUS KP =   15171.11

REFERENCE MOMENT MO =      637.51

SHAPE PARAMETER N =     2.6200
```

PROGRAM MRCURVE

```
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      REAL MOMENT
      DIMENSION ARM(50),DELTA(50),R(50),MOMENT(50),BM1(50)
      OPEN(UNIT=5,FILE='FDINPUT.DAT',STATUS='OLD')
      OPEN(UNIT=6,FILE='OUTPUT.DAT',STATUS='NEW')
501   READ(5,*) N,DL
      IF(N.EQ.0) GO TO 999
      H=3.*N-1.5
      READ(5,*) TK,TKP,TRO,TN
      READ(5,*) CK,CKP,CRO,CN
      DO 100 I=1,11
      THETA = DL/(2.**(11-I))
      X1 = H
      X2 = 0.0
900   X = (X1+X2)/2.
      DO 200 J=1,N
      ARM(J) = (J-1)*3. + 1.5
      DELTA(J) = (ARM(J)-X)*THETA
      IF(DELTA(J).GE.0.0) GO TO 250
      IF(DELTA(J).LT.0.0) GO TO 260
250   T1 = (TK-TKP)*DELTA(J)
      T2 = (ABS(T1/TRO))**TN
      T3 = (1.+T2)**(-1./TN)
      T4 = TKP*DELTA(J)
      R(J) = (T1*T3) + T4
      GO TO 200
260   T1 = (CK-CKP)*DELTA(J)
      T2 = (ABS(T1/CRO))**CN
      T3 = (1.+T2)**(-1./CN)
      T4 = CKP*DELTA(J)
      R(J) = (T1*T3) + T4
200   CONTINUE
      SUM = 0.0
      DO 300 K=1,N
300   SUM = SUM + R(K)
      IF(ABS(SUM).LT.0.1) GO TO 400
      IF(SUM.GT.0.0) X2=X
      IF(SUM.LT.0.0) X1=X
      IF(SUM.EQ.0.0) GO TO 400
      GO TO 900
400   MOMENT(I)=0.0
      DO 500 K=1,N
500   MOMENT(I) = MOMENT(I) + R(K)*ARM(K)
      WRITE(6,7) MOMENT(I),THETA
7     FORMAT(1X,'MOMENT = ',1F10.2,5X,'THETA = ',1F10.4)
100   CONTINUE
      TK = MOMENT(1)/((0.5)**10*DL)
      TKP = (MOMENT(11)-MOMENT(10))/(0.5*DL)
      TRO = MOMENT(11)-2.0*(MOMENT(11)-MOMENT(10))
      WRITE(6,1) TK
1     FORMAT(//1X,'ELASTIC MODULUS K = ',1F10.2)
```

```
      WRITE(6,2) TKP
2     FORMAT(//1X,'PLASTIC MODULUS KP = ',1F10.2)
      WRITE(6,3) TRO
3     FORMAT(//1X,'REFERENCE MOMENT MO = ',1F10.2)
      TN1 = 0.0
      CHECK2 = 1.0E25
101   TN1 = TN1 + 0.01
      IF(TN1.GT.100.0) GO TO 999
      CHECK1 = 0.0
      DO 201 I=1,11
      THETA = ((0.5)**(11-I))*DL
      T1 = (TK-TKP)*THETA
      T2 = (ABS(T1/TRO))**TN1
      T3 = (1.+T2)**(-1./TN1)
      T4 = TKP*THETA
      BM1(I) = (T1*T3) + T4
201   CHECK1 = CHECK1 + ((MOMENT(I)-BM1(I))**2)*THETA
      IF(CHECK1.GT.CHECK2) GO TO 301
      CHECK2 = CHECK1
      TN2 = TN1
      GO TO 101
301   WRITE(6,6) TN2
      GO TO 501
6     FORMAT(//1X,'SHAPE PARAMETER N = ',1F10.4)
999   STOP
      END
```

Appendix C: NRMRSOL

Program NRMRSOL is a fortran computer program that uses a Newton-Raphson root-finding technique to determine the intersection point (moment,rotation) of the beam line and the moment-rotation curve.

The program is very easy to use. The computer will prompt the user via the screen three times. The first prompt will ask for the uniform load in kips/inch on the beam. The second prompt will ask for the length of the beam in inches and the moment of inertia of the beam (strong axis bending). Finally, the third prompt will ask for the four Richard equation parameters defining the moment-rotation curve for the connection under consideration. The input format is free, so decimal points are not necessary after whole numbers, but commas must separate entries. The output of the program appears on the screen and consists of one line. This output line gives the end rotation in radians and the end moment in inch-kips, the two coordinates corresponding to the intersection of the beam line and the moment-rotation curve.

To understand how the computer program works, the theory behind the Newton-Raphson method must first be explained. Consider a point $\underline{x}$ which is not a root of the function f(x) but is "reasonably close" to a root. The function f(x) can be expanded in a Taylor's series expansion about $\underline{x}$:

$$f(x) = f(\underline{x}) + f'(\underline{x})(x - \underline{x}) + f''(\underline{x})(x - \underline{x})^2/2! + \ldots +$$

Taking only the first two terms in the expansion:

$$f(x) = f(\underline{x}) + (x - \underline{x})f'(\underline{x})$$

Setting f(x) = 0 and solving for x gives:

$$x = \underline{x} - f(\underline{x})/f'(\underline{x})$$

The function is a function of the rotation, theta, and to obtain this function the expression for the end moment using the moment-rotation curve must be set equal to the expression for the end moment using the beam line equation. Doing this gives:

$$M_{conn} = M_{beam\ line} \quad \text{or} \quad M_{conn} - M_{beam\ line} = 0 = f(\theta)$$

$$f(\theta) = \frac{K_1\theta}{\left[1 + \left|\frac{K_1\theta}{M_o}\right|^N\right]^{1/N}} + K_p\theta - M_{fixed} + \frac{2EI}{L}\theta = 0$$

Using the Newton-Raphson method:

$$\theta = \theta_o - f(\theta_o)/f'(\theta_o) \qquad \text{or}$$

$$\theta - \theta_o = \delta = -f(\theta_o)/f'(\theta_o)$$

To obtain the derivatives, the Quotient Rule and the Power Rule of differentiation must be used, and in doing so, the following expression for the root of the function of theta becomes:

$$\theta = \theta_o - \frac{\dfrac{K_1\theta_o}{\left[1 + \left|\dfrac{K_1\theta_o}{M_o}\right|^N\right]^{1/N}} + K_p\theta_o - M_{fixed} + \dfrac{2EI}{L}\theta_o}{K_p + \dfrac{2EI}{L} + \dfrac{K_1}{\left[1 + \left|\dfrac{K_1\theta_o}{M_o}\right|^N\right]^{\frac{N+1}{N}}}}$$

The above equation is programmed in NRMRSOL as the subroutine FUNCTN. The program converges on the solution very rapidly and stops when the absolute value of delta (δ) is less than a predetermined epsilon or error (= 0.000001). No error exits have been included in the program in case the method diverges or does not find a root in a reasonable number of iterations. The error exits were not necessary because the function is well defined near the root.

PROGRAM NRMRSOL

```
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      WRITE (*,1)
1     FORMAT(1X,'ENTER UNIFORM LOAD IN KIPS/INCH')
      READ (*,*) W
      WRITE (*,2)
2     FORMAT(1X,'ENTER BEAM LENGTH (INCHES), MOMENT OF INERTIA')
      READ (*,*) XL,XI
      WRITE (*,3)
3     FORMAT(1X,'ENTER RICHARD PARAMETERS K,KP,R,N')
      READ (*,*) TK,TKP,TR,TN
      EPS = 0.000001
      E = 29000.
      THETA0 = 0.
      CALL FUNCTN (DELTA,TK,TKP,TR,TN,THETA0,W,E,XI,XL)
      THETA = THETA0 + DELTA
100   CALL FUNCTN (DELTA,TK,TKP,TR,TN,THETA,W,E,XI,XL)
      THETA = THETA + DELTA
      IF(ABS(DELTA).LT.EPS) GO TO 200
      GO TO 100
200   XMOM = (W*(XL**2)/12.)-(2.*E*XI*THETA/XL)
      WRITE (*,4) THETA,XMOM
4     FORMAT (1X,'END ROTATION =',1F10.8,'RADIANS',5X,'END MOMENT
     $ =',1F10.0,'INCH-KIPS')
      STOP
      END
C
      SUBROUTINE FUNCTN (DL,TK,TKP,TR,TN,ROT,W,E,XI,XL)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      T1 = TK-TKP
      T2 = (ABS(T1*ROT/TR))**TN
      T3 = (1 + T2)**(1./TN)
      T4 = (T1*ROT)/T3
      FEM = W*(XL**2)/12.
      XNUM = T4 + (TKP*ROT) - FEM + (2.*E*XI*ROT/XL)
      XDEN = TKP + (2.*E*XI/XL) + (T1/((1 + T2)**((TN + 1.)/TN)))
      DL = -1.*XNUM/XDEN
      RETURN
      END
```

REFERENCES

1. Blewitt, Russell J., and Richard, Ralph M. "Experimental and Analytical Force Deformation Curves for Bolted Double Angle Connections." University of Arizona, Department of Civil Engineering and Engineering Mechanics, 1984.

2. Hsia, Wei-Kung, and Richard, Ralph M. "Double Angle Connection Moments." M.S. Thesis, University of Arizona, Department of Civil Engineering and Engineering Mechanics, 1986.

3. Salmon, Charles G., and Johnson, John E. *Steel Structures: Design and Behavior.* Second Edition, New York: Harper & Row, Publishers, 1980, pg. 731-733.

4. Salmon, Charles G., and Johnson, John E. *Steel Structures: Design and Behavior.* Second Edition, New York: Harper & Row, Publishers, 1980, pg. 209.