UNIVERSITY PROGRAMS

Structural Fire Engineering: Learning Module 2a

Column Capacity at a Steady Temperature

Supplemental Material

Overview

Evaluate the effects of temperature on the nominal compression capacity of a column using the equations and information in Appendix 4 and Chapter E of the ANSI/AISC 360-16 *Specification for Structural Steel Buildings*.

Learning Objectives

- Calculate the nominal compression capacity of a column (P_n) at a given temperature using the equations and information in Appendix 4 and Chapter E of the AISC *Specification*.
- Explain the impact that cooler floors above and/or below a heated column will have on the restraint against thermal elongation.
- Calculate the compression capacity of a column (P_n) at a given temperature considering the restraint of cooler floors above and below the heated column using the equations and information in the Commentary of Appendix 4 and in Chapter E of the AISC *Specification*.
- Compare and explain the column capacity at an elevated temperature (both considering the restraint of the cooler floors and not considering the restraint of the cooler floors) with the column capacity at ambient temperature.

Students should already understand:

- How to calculate the nominal compression capacity of a column (P_n) at ambient temperatures using Chapter E of the AISC *Specification*.
- The concept of slenderness ratios and effective length factors for columns with varying boundary conditions (Table C-A-7.1).

Mechanical Properties of Steel at Elevated Temperatures

For steel temperatures less than or equal to 400°F (200°C), the member and connection design strengths is permitted to be determined without consideration of temperature effects on the nominal strengths.

At elevated temperatures, steel deteriorates in strength and stiffness, which needs to be considered in a structural analysis. The temperature-dependent material properties of steel can be calculated through AISC *Specification* Table A-4.2 for the following properties: $F_y(T)$, $F_p(T)$, $F_u(T)$, $E(T)$, and $G(T)$. Table A-4.2.1, shown in Table 1, contains retention factors for these properties. The retention factors are the ratio of the material property at elevated temperature to the value of the material property at ambient temperature, which is assumed to be 68°F (20°C).

Table 1. Retention factors for structural steel per AISC *Specification* Appendix 4

TABLE A-4.2.1
Properties of Steel at Elevated Temperatures

Steel Temperature, °F (°C)	$k_E = E(T)/E = G(T)/G$	$k_p = F_p(T)/F_y$	$k_y = F_y(T)/F_y$	$k_u = F_u(T)/F_y$
68 (20)	1.00	1.00	*	*
200 (93)	1.00	1.00	*	*
400 (200)	0.90	0.80	*	*
600 (320)	0.78	0.58	*	*
750 (400)	0.70	0.42	1.00	1.00
800 (430)	0.67	0.40	0.94	0.94
1000 (540)	0.49	0.29	0.66	0.66
1200 (650)	0.22	0.13	0.35	0.35
1400 (760)	0.11	0.06	0.16	0.16
1600 (870)	0.07	0.04	0.07	0.07
1800 (980)	0.05	0.03	0.04	0.04
2000 (1100)	0.02	0.01	0.02	0.02
2200 (1200)	0.00	0.00	0.00	0.00

*Use ambient properties

Table A-4.2 can be used by first calculating the steel temperature through a heat transfer analysis. For the purposes of this module, the temperatures will be provided to the students as heat transfer analysis is outside of the scope of the learning objectives. The retention factors, (k_E, k_p, k_y, k_u) are used to calculate the elevated temperature material properties by multiplying the retention factors in Table 1 by the ambient material property value (E, F_y, F_u). The values in Table 1 can be linearly interpolated and are consistent with material properties in codes around the world (e.g. Eurocode).

For example, for a steel member with a temperature of 760°C, the *Young's Modulus* value would be calculated as follows:

$k_E = 0.11$	The retention factor for *Young's Modulus* at 760°C
$E = 29{,}000$ ksi	*Young's Modulus* of structural steel at ambient temperature

$$E(760°\text{C}) = k_E E = (0.11)(29{,}000ksi) = 3190ksi$$

This same process is applied to the remaining temperature-dependent values. Figure 1 depicts the retention factors at elevated temperatures.

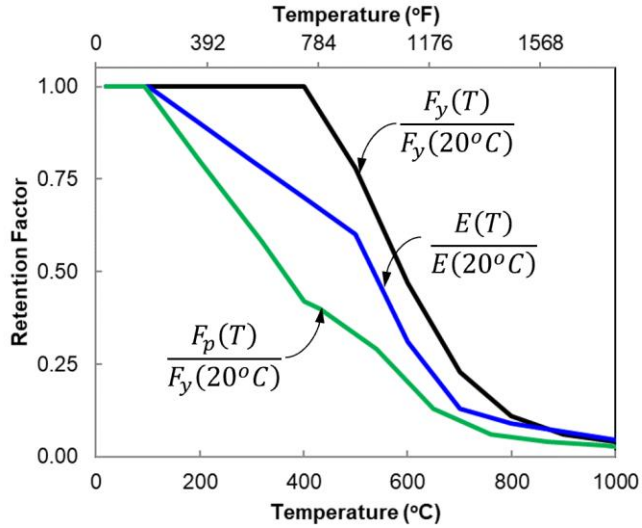


Figure 1: Retention Factors at Elevated Temperatures

Nominal capacity of the column at elevated temperatures per Appendix 4

Appendix 4 Simple Methods of Analysis are intended for single-story fires only. The retention factors (from AISC *Specification* Table A-4.2) will be used to calculate the nominal column buckling capacity (P_n) of a column using Chapter E and Appendix 4 of the AISC *Specification*. The critical buckling stress of the column is calculated per Equation A-4-2 in the AISC *Specification*.

$$F_{cr}(T) = \left[0.42^{\sqrt{\frac{F_y(T)}{F_e(T)}}}\right] * F_y(T)$$ AISC *Specification* Equation A-4-2

Where, $F_{cr}(T)$ is the temperature-dependent critical buckling stress and $F_e(T)$ is the temperature-dependent critical elastic buckling stress (Euler buckling stress) calculated using AISC *Specification* Equation E3-4, shown below:

$$F_e(T) = \frac{\pi^2 * E(T)}{\left(\frac{L_c}{r}\right)^2}$$ AISC *Specification* Equation E3-4

Where, $\frac{L_c}{r}$ is the slenderness ratio of the heated column, and L_c is the effective length of the column and *r* is the radius of gyration of the column section. The content of Appendix 4 and Chapter E assume the slenderness ratio does not change with temperature. However, researchers have concluded that cooler floors above and below a heated column can have restraint on the thermal expansion of the heated column. This restraint may affect the slenderness ratio with temperature, and therefore the Commentary of Appendix 4 addresses this new research.

Discussion Question

Q1. What is the same and what is different with regards to the critical buckling stress and elastic buckling stress from Appendix 4 to Chapter E?

Answer:

Chapter E considers elastic and inelastic buckling of the column (Equations E3-2 and E3-3 respectively). Whereas in Appendix 4, in lieu of considering both elastic and inelastic buckling, Equation A-4-2 is used to calculate the critical buckling stress as a function of temperature. The reasoning for this is because during a fire, there will not be elastic column buckling. Rather during a fire, the buckling will always be due to inelastic behavior.

Restraint of heated column by cooler floors above and below

The cooler floors above and/or below a heated column can act as rotational restraints. These restraints decrease the temperature-dependent slenderness ratio. Typical gravity columns have pinned boundary conditions as shown in Figure 2a. These pinned boundary conditions allow for rotation at the column end as shown by the deformed column shape in Figure 2a. However, when considering a heated column with a cooler floor above and below, the rotation of the column ends is restrained by the cooler floors as shown in Figure 2b. This restraint will change the buckling shape of the column.

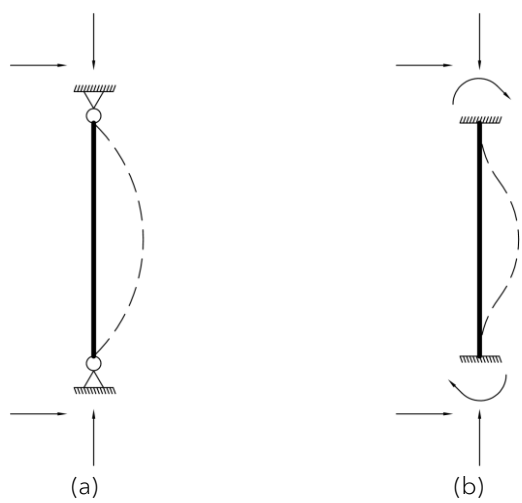


Figure 2: Deformed shape of a column; (a) with pin-pin restraints and (b) with pin-pin restraints and cooler floors above and below the heated column

The dotted line depicts the deformed shape of the column, whereas the solid line is the undeformed shape. When the column has pin-pin restraints, the column ends are not restrained against rotation. When the column is heated, however, and has cooler floors above and below the heated column, the cooler floors prevent rotation at the ends of the column and act as rotational restraints making the column act as if it has fixed- fixed boundary conditions. This causes the deformation to start further away from the ends of the column. As the rest of this learning module will explain, the cooler floors can increase the nominal compression capacity compared to when the changing boundary conditions are not considered.

These cooler floors above and below are intended to be stories that do not have smoke infiltration or fires thereby keeping these floors near ambient temperatures.

Discussion Question

Q2. How will the thermal restraint and changing slenderness ratio impact the temperature-dependent Euler buckling stress ($F_e(T)$) and temperature-dependent critical buckling stress ($F_{cr}(T)$)?

Answer:

The thermal restraint and decreased slenderness ratio will increase the Euler buckling stress and critical buckling stress, and therefore, increase the nominal buckling capacity of the column at elevated temperatures.

Consideration of temperature-dependent slenderness ratio

The commentary of Appendix 4 provides guidance for engineers to calculate temperature-dependent slenderness ratios that depend on the temperature of the column and if the column has cooler floors above and/or below. AISC *Specification* Equation C-A-4-9 shows how the temperature-dependent slenderness ratio is calculated.

$$\left(\frac{L_c}{r}\right)_T = \left(1 - \frac{T-32}{n(3600)}\right)\left(\frac{L_c}{r}\right) - \frac{35}{n(3600)}(T-32) \geq 0 \qquad \text{AISC } \textit{Specification} \text{ Eqn C-A-4-9}$$

Where T is the temperature of the steel column in Fahrenheit and n considers if there is only a cooler floor (non-heated floor) above or below, or if there are cooler floors above and below the heated column. If both the columns above and below the affected column are cooler, then n is taken as 1. If only one of the columns either above or the below the affected column is cooler, then n is taken as 2.

Once $\left(\frac{L_c}{r}\right)_T$ is calculated, this value is then used in Equation E3-4 of the AISC *Specification* to calculate the Euler buckling stress of the column at elevated temperatures. The rest of the calculations for the nominal compression capacity of the column are the same as previously described.

Resistance Factor (ϕ)

The resistance factor for design, ϕ, is applied the same as with ambient temperatures. The resistance factor can be found in Chapter E of AISC *Specification* when designing the member for compression.

Practice

Steel column example at steady state and uniform temperature

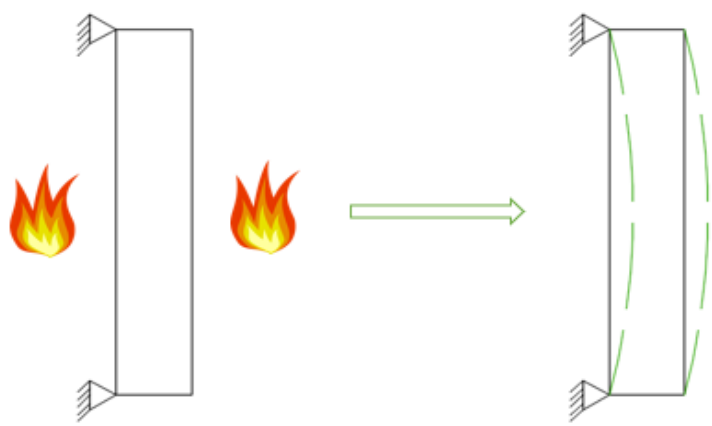
Problem Description

This example problem calculates the column buckling capacity of a W10x54 column at elevated temperature using two different methodologies:

1. Considering the slenderness of the column does not change from ambient temperature, and
2. Considering the slenderness of the column is temperature-dependent, which considers the restraint from the cooler floors above and below the heated column.

This lesson plan then provides prompts for the instructor to lead a discussion on the difference between the two methodologies and the effects that the restraint of the cooler floors have on the column buckling capacity.

Given



W10x54 column	Column size
T = 600°C	Temperature of the column
H=L_c=12′0″	Effective length of the column
Pin-Pin	Boundary conditions
F_y = 50 ksi	Ambient temperature yield stress of the steel
E=29,000 ksi	Ambient temperature *Young's Modulus* of the steel
Cooler floors both above and below	

Find

Nominal column buckling capacity (P_n)

a) Using AISC Specification Equation A-4-2 and the ambient temperature slenderness ratio
b) Considering temperature-dependent slenderness ratios using AISC *Specification* Equation C-A-4-9

Solution Part (a):

The column buckling capacity (P_n) is calculated using Chapter E and Appendix 4. Equation A-4-2 is for the critical buckling stress of the column as a function of temperature:

$$F_{cr}(T) = \left[0.42^{\sqrt{\frac{F_y(T)}{F_e(T)}}}\right] * F_y(T)$$

Where, $F_y(T)$ is the temperature-dependent *Yield Stress* calculated using the retention factors (k_y) in Table A-4.2.1:

$$F_y(T) = k_y * F_y$$

From Table A-4.2.1, the following reduction factors are obtained:

$$T = 650°C,\ k_y = 0.35$$
$$T = 540°C,\ k_y = 0.66$$

TABLE A-4.2.1
Properties of Steel at Elevated Temperatures

Steel Temperature, °F (°C)	$k_E = E(T)/E = G(T)/G$	$k_p = F_p(T)/F_y$	$k_y = F_y(T)/F_y$	$k_u = F_u(T)/F_y$
68 (20)	1.00	1.00	*	*
200 (93)	1.00	1.00	*	*
400 (200)	0.90	0.80	*	*
600 (320)	0.78	0.58	*	*
750 (400)	0.70	0.42	1.00	1.00
800 (430)	0.67	0.40	0.94	0.94
1000 (540)	0.49	0.29	0.66	0.66
1200 (650)	0.22	0.13	0.35	0.35
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1800 (980)	0.05	0.03	0.04	0.04
2000 (1100)	0.02	0.01	0.02	0.02
2200 (1200)	0.00	0.00	0.00	0.00

*Use ambient properties

Through interpolation, the retention factor for *Yield Stress* at 600 °C:

$$T = 600°C,\ k_y = 0.49$$

This retention factor can be used to calculate the *Yield Stress* of the steel at 600 °C:

$$F_y(T) = 0.49 * 50ksi = 24.5ksi$$

The Euler buckling stress at elevated temperature is calculated per Chapter E with the temperature-dependent *Young's Modulus* (Equation E3-4)

$$F_e(T) = \frac{\pi^2 * E(T)}{(\frac{L_c}{r})^2}$$

Similar to the *Yield Stress*, the *Young's Modulus* is calculated using the retention factors in Table A-4.2.1. Through interpolation of the values in the table, we can calculate the retention factor for the *Young's Modulus*.

$$E(T) = k_E * E$$

Through interpolation, the retention factor for *Yield Stress* at 600 °C:

$$T = 650°C,\ k_E = 0.22$$
$$T = 540°C,\ k_E = 0.49$$

$$T = 600°C,\ k_E = 0.34$$

$$E(T) = 0.34 * 29{,}000ksi = 9939ksi$$

Discussion Question

How does the retention factor for the *Yield Stress* compare with the retention factor for *Young's Modulus*? How might that play out in the overall calculation of the design buckling capacity of the column?

<u>Answer</u>

The radius of gyration per AISC 360 for a W10x54 in Table 1-1

$$r_y = 2.56"$$

The temperature-dependent Euler buckling stress can be calculated with Equation E-4.

$$F_e(T) = \frac{\pi^2 * E(T)}{(\frac{L_c}{r})^2}$$

$$F_e(T) = \frac{\pi^2 * 9939ksi}{(\frac{144in}{2.56in})^2} = 31ksi$$

The calculated *Yield Stress* at 600°C and the Euler Buckling Stress at 600°C are used as inputs into the critical buckling stress equation.

$$F_{cr}(T) = \left[0.42^{\sqrt{\frac{F_y(T)}{F_e(T)}}}\right] * F_y(T)$$

$$F_{cr}(T) = \left[0.42^{\sqrt{\frac{24.5ksi}{31ksi}}}\right] * 24.5ksi$$

$$F_{cr}(T) = 11.3ksi$$

The nominal compressive strength, P_n, is found by multiplying the gross area of the steel cross-section by the critical buckling stress, found in Equation E4-1 of the AISC *Specification*. From AISC 360, Table 1-1, the gross area of a W10x54 is $15.8in^2$.

$$P_n = A_g * F_{cr}(T)$$

$$P_n = 15.8in^2 * 11.3ksi = 179kips$$

Solution Part (b)

The column buckling capacity is now computed considering the slenderness ratio as a function of temperature.

The slenderness ratio, Equation C-A-4-9, considers the possible increased strength as a result of the restraint provided by the cooler floors above and below. *T* is the temperature in Fahrenheit, *n* is equal to 1 for columns with cooler columns both above and below, and *n* is equal to 2 for columns with cooler columns either above or below only.

$$\left(\frac{L_c}{r}\right)_T = \left(1 - \frac{T-32}{n(3600)}\right)\left(\frac{L_c}{r}\right) - \frac{35}{n(3600)}(T-32) \geq 0$$

$$T = 600°C\left(\frac{9}{5}\right) + 32 = 1{,}112°\text{F}$$

$$\frac{L_c}{r} = \frac{144"}{2.56"} = 56.25$$

$$n = 1$$

$$\left(\frac{L_c}{r}\right)_T = \left(1 - \frac{1{,}112°\text{F} - 32}{(1)(3600)}\right)(56.25) - \frac{35}{(1)(3600)}(1{,}112°\text{F} - 32)$$

$$\left(\frac{L_c}{r}\right)_T = 28.9$$

Discussion Question

How does the slenderness ratio at 600°C compare to what we calculated at ambient temperature (in Part a)? How much did this change? What would the slenderness ratio be if there was only a cooler column above <u>or</u> below the heated column (n = 2)?

<u>Answer:</u>

The slenderness ratio at 600°C is half of the slenderness ratio at ambient temperature.

$$\frac{\left(\frac{L_c}{r}\right)_{600}}{\left(\frac{L_c}{r}\right)} = 0.5$$

The slenderness ratio when there is a cooler column above or below the heated column (n = 2) is 42.56, which is 74% of the original slenderness ratio (about halfway between the slenderness ratio at ambient temperature and 600°C).

The Euler buckling stress is calculated per Chapter E with the temperature-dependent *Young's Modulus* (Equation E3-4) and now the temperature dependent slenderness ratio:

$$F_e(T) = \frac{\pi^2 * E(T)}{\left[\left(\frac{L_c}{r}\right)_T\right]^2}$$

$$F_e(T) = \frac{\pi^2 * 9939ksi}{(28.9)^2} = 117.4ksi$$

The calculated *Yield Stress* at 600°C and the Euler Buckling Stress at 600°C (with the temperature-dependent slenderness ratio) are used as inputs into the critical buckling stress equation.

$$F_{cr}(T) = \left[0.42^{\sqrt{\frac{F_y(T)}{F_e(T)}}}\right] * F_y(T)$$

$$F_{cr}(T) = \left[0.42^{\sqrt{\frac{24.5ksi}{117.4ksi}}}\right] * 24.5ksi$$

$$F_{cr}(T) = 16.5ksi$$

The nominal compressive strength, P_n, is found multiplying the gross area of the steel section by the critical buckling stress. From AISC 360 Table 1-1, the gross area of a W10x54 is 15.8 in^2.

$$P_n = A_g * F_{cr}(T) = 16.5ksi * 15.8in^2 = 261kips$$

Discussion Questions

How do the two answers compare in part (a) and (b)? How do these values compare to the compressive strength at ambient temperature? How do these comparisons relate to the retention factor for *Yield Stress* and for *Young's Modulus*?

$P_{n(a)}$	Part (a)	179 kips
$P_{n(b)}$	Part (b)	261 kips
$P_{n_ambient}$	20°C	564 kips

Answer:

- Calculation of buckling capacity using A-4-2 was 179 kips. This is 31% lower than the buckling capacity calculated using C-A-4-9. Or using C-A-4-9 and considering the restraint of the cooler floors above and below results in a 45.5% increase in buckling capacity of the column.
- The buckling capacity of the column using A-4-2 is 32% of the ambient capacity of the column.
- The buckling capacity of the column using C-A-4-9 is 46% of the ambient capacity of the column.
- The retention factor for *Young's Modulus* at 600°C is 0.34
- The retention factor for the *Yield Stress* at 600°C is 0.49

Additional Resources

AISC (2016), *Specification for Structural Steel Buildings* (Appendix 4), Standard ANSI/AISC 360-16, American Institute of Steel Construction, Inc., Chicago. IL. *(Available for free download at* aisc.org/Specification-for-Structural-Steel-Buildings-ANSIAISC-360-16-1*)*

Agarwal, A. and Varma, A.H. (2011). "Design of Steel Columns at Elevated Temperatures Due to Fire: Effects of Rotational Restraints," *Engineering Journal*, American Institute of Steel Construction, Vol. 48, pp. 297-314. (*Available for free download for AISC Educator and Student Members at* aisc.org/Design-of-Steel-Columns-at-Elevated-Temperatures-Due-to-Fire-Effects-of-Rotational)