COMPANION TO THE
AISC
STEEL CONSTRUCTION MANUAL

Volume 1: Design Examples

For Use in First Semester
Structural Steel Design Classes

Version 15.1

AMERICAN INSTITUTE
OF
STEEL CONSTRUCTION
Notes: Design Examples For Use in First Semester Structural Steel Design Course

The AISC Committee on Manuals prepares design examples to illustrate the application of the provisions in the AISC Specification for Structural Steel Buildings. The complete set of design examples includes 166 example problems totaling 985 pages, and it is a free download that can be found at www.aisc.org/designexamples.

The AISC Partners in Education Committee has condensed the set of Design Examples to include 45 example problems that will be most likely to address topics that are studied in a first semester structural steel design course. These 45 example problems can be found on the following pages.
PREFACE

The primary objective of this Companion is to provide guidance and additional resources of the use of the 2016 AISC Specification for Structural Steel Buildings (ANSI/AISC 360-16) and the 15th Edition AISC Steel Construction Manual.

The Companion consists of design examples in Parts I, II and III. The design examples provide coverage of all applicable limit states, whether or not a particular limit state controls the design of the member or connection. In addition to the examples that demonstrate the use of the AISC Manual tables, design examples are provided for connection designs beyond the scope of the tables in the AISC Manual. These design examples are intended to demonstrate an approach to the design, and are not intended to suggest that the approach presented is the only approach. The committee responsible for the development of these design examples recognizes that designers have alternate approaches that work best for them and their projects. Design approaches that differ from those presented in these examples are considered viable as long as the AISC Specification, sound engineering, and project specific requirements are satisfied.

Part I of these examples is organized to correspond with the organization of the AISC Specification. The Chapter titles match the corresponding chapters in the AISC Specification.

Part II is devoted primarily to connection examples that draw on the tables from the AISC Manual, recommended design procedures, and the breadth of the AISC Specification. The chapters of Part II are labeled II-A, II-B, II-C, etc.

Part III addresses aspects of design that are linked to the performance of a building as a whole. This includes coverage of lateral stability and second-order analysis, illustrated through a four-story braced-frame and moment-frame building.

The Design Examples are arranged with LRFD and ASD designs presented side-by-side, for consistency with the AISC Manual. Design with ASD and LRFD are based on the same nominal strength for each element so that the only differences between the approaches are the set of load combinations from ASCE/SEI 7-16 used for design, and whether the resistance factor for LRFD or the safety factor for ASD is used.

CONVENTIONS

The following conventions are used throughout these examples:


3. The source of equations or tabulated values taken from the AISC Specification or AISC Manual is noted along the right-hand edge of the page.

4. When the design process differs between LRFD and ASD, the designs equations are presented side-by-side. This rarely occurs, except when the resistance factor, \( \phi \), and the safety factor, \( \Omega \), are applied.

5. The results of design equations are presented to three significant figures throughout these calculations.
ACKNOWLEDGMENTS

The AISC Committee on Manuals reviewed and approved V15.1 of the AISC Design Examples:

Mark V. Holland, Chairman  Ronald L. Meng
Gary C. Violette, Vice Chairman  Larry S. Muir
Allen Adams  Thomas M. Murray
Scott Adan  James Neary
Abbas Aminmansour  Davis G. Parsons II, Emeritus
Craig Archacki  John Rolfes
Charles J. Carter  Rafael Sabelli
Harry A. Cole, Emeritus  Thomas J. Schlafly
Brad Davis  Clifford W. Schwinger
Bo Dowswell  William T. Segui, Emeritus
Matt Eatherton  Victor Shneur
Marshall T. Ferrell, Emeritus  William A. Thornton
Patrick J. Fortney  Michael A. West
Timothy P. Fraser  Ronald G. Yeager
Louis F. Geschwindner, Emeritus  Cynthia J. Duncan, Secretary
John L. Harris III  Eric Bolin, Assistant Secretary
Christopher M. Hewitt  Michael Gannon, Assistant Secretary
William P. Jacobs V  Carlo Lini, Assistant Secretary
Benjamin Kaan  Jennifer Traut-Todaro, Assistant Secretary

The committee gratefully acknowledges the contributions made to this document by the AISC Committee on Specifications and the following individuals: W. Scott Goodrich, Heath Mitchell, William N. Scott, Marc L. Sorenson and Sriramulu Vinnakota.
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</tbody>
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Part I
Examples Based on the AISC Specification

This part contains design examples demonstrating select provisions of the AISC Specification for Structural Steel Buildings.
Chapter C
Design for Stability

C1. GENERAL STABILITY REQUIREMENTS

The AISC Specification requires that the designer account for both the stability of the structural system as a whole and the stability of individual elements. Thus, the lateral analysis used to assess stability must include consideration of the combined effect of gravity and lateral loads, as well as member inelasticity, out-of-plumbness, out-of-straightness, and the resulting second-order effects, \( P-\Delta \) and \( P-\delta \). The effects of “leaning columns” must also be considered, as illustrated in the examples in this chapter and in the four-story building design example in Part III of these Design Examples.

\( P-\Delta \) and \( P-\delta \) effects are illustrated in AISC Specification Commentary Figure C-C2.1. Methods for addressing stability, including \( P-\Delta \) and \( P-\delta \) effects, are provided in AISC Specification Section C2 and Appendix 7.

C2. CALCULATION OF REQUIRED STRENGTHS

The calculation of required strengths is illustrated in the examples in this chapter and in the four-story building design example in Part III of these Design Examples.

C3. CALCULATION OF AVAILABLE STRENGTHS

The calculation of available strengths is illustrated in the four-story building design example in Part III of these Design Examples.
EXAMPLE C.1A  DESIGN OF A MOMENT FRAME BY THE DIRECT ANALYSIS METHOD

Given:
Determine the required strengths and effective length factors for the columns in the moment frame shown in Figure C.1A-1 for the maximum gravity load combination, using LRFD and ASD. The uniform load, \( w_D \), includes beam self-weight and an allowance for column self-weight. Use the direct analysis method. All members are ASTM A992 material.

Columns are unbraced between the footings and roof in the \( x \)- and \( y \)-axes and have pinned bases.

Solution:
From AISC Manual Table 1-1, the W12\times65 has \( A = 19.1 \) in.\(^2\)

The beams from grid lines A to B and C to E and the columns at A, D and E are pinned at both ends and do not contribute to the lateral stability of the frame. There are no \( P-\Delta \) effects to consider in these members and they may be designed using \( L_c = L \).

The moment frame between grid lines B and C is the source of lateral stability and therefore will be evaluated using the provisions of Chapter C of the AISC Specification. Although the columns at grid lines A, D and E do not contribute to lateral stability, the forces required to stabilize them must be considered in the moment-frame analysis. The entire frame from grid line A to E could be modeled, but in this case the model is simplified as shown in Figure C.1A-2, in which the stability loads from the three “leaning” columns are combined into a single representative column.

From Chapter 2 of ASCE/SEI 7, the maximum gravity load combinations are:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_u )</td>
<td>( 1.2D + 1.6L )</td>
<td>( D + L )</td>
</tr>
<tr>
<td></td>
<td>( = 1.2(0.400 \text{ kip/ft}) + 1.6(1.20 \text{ kip/ft}) )</td>
<td>( = 0.400 \text{ kip/ft} + 1.20 \text{ kip/ft} )</td>
</tr>
<tr>
<td></td>
<td>( = 2.40 \text{ kip/ft} )</td>
<td>( = 1.60 \text{ kip/ft} )</td>
</tr>
</tbody>
</table>

Per AISC Specification Section C2.1(d), for LRFD, perform a second-order analysis and member strength checks using the LRFD load combinations. For ASD, perform a second-order analysis using 1.6 times the ASD load combinations and divide the analysis results by 1.6 for the ASD member strength checks.
Frame analysis gravity loads

The uniform gravity loads to be considered in a second-order analysis on the beam from B to C are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w'_u = 2.40$ kip/ft</td>
<td>$w'_u = 1.6(1.60$ kip/ft) = $2.56$ kip/ft</td>
</tr>
</tbody>
</table>

Concentrated gravity loads to be considered in a second-order analysis on the columns at B and C contributed by adjacent beams are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P'_u = \frac{w'_u l}{2}$ = $(2.40$ kip/ft$)(30.0$ ft$) = 36.0$ kips</td>
<td>$P'_u = \frac{w'_u l}{2}$ = $(2.56$ kip/ft$)(30.0$ ft$) = 38.4$ kips</td>
</tr>
</tbody>
</table>

Concentrated gravity loads on the representative “leaning” column

The load in this column accounts for all gravity loading that is stabilized by the moment frame, but is not directly applied to it.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P'_{ul} = (60.0$ ft$)(2.40$ kip/ft$) = 144$ kips</td>
<td>$P'_{ul} = (60.0$ ft$)(2.56$ kip/ft$) = 154$ kips</td>
</tr>
</tbody>
</table>

Frame analysis notional loads

Per AISC Specification Section C2.2, frame out-of-plumbness must be accounted for either by explicit modeling of the assumed out-of-plumbness or by the application of notional loads. Use notional loads.

From AISC Specification Equation C2-1, the notional loads are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 1.0$</td>
<td>$\alpha = 1.6$</td>
</tr>
<tr>
<td>$Y_i = (120$ ft$)(2.40$ kip/ft$) = 288$ kips</td>
<td>$Y_i = (120$ ft$)(1.60$ kip/ft$) = 192$ kips</td>
</tr>
<tr>
<td>$N_i = 0.002\alpha Y_i$ = $0.002(1.0)(288$ kips$) = 0.576$ kip</td>
<td>$N_i = 0.002\alpha Y_i$ = $0.002(1.6)(192$ kips$) = 0.614$ kip</td>
</tr>
</tbody>
</table>
Summary of applied frame loads

The applied loads are shown in Figure C.1A-2.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>36.0 kips</td>
<td>38.4 kips</td>
</tr>
<tr>
<td>2.40 kip/ft</td>
<td>2.56 kip/ft</td>
</tr>
<tr>
<td>0.576 kip</td>
<td>0.614 kip</td>
</tr>
<tr>
<td>Leaning column</td>
<td>Leaning column</td>
</tr>
</tbody>
</table>

Fig. C.1A-2. Applied loads on the analysis model.

Per AISC Specification Section C2.3, conduct the analysis using 80% of the nominal stiffnesses to account for the effects of inelasticity. Assume, subject to verification, that $\alpha P_r/P_{ns}$ is not greater than 0.5; therefore, no additional stiffness reduction is required ($\tau_b = 1.0$).

Half of the gravity load is carried by the columns of the moment-resisting frame. Because the gravity load supported by the moment-resisting frame columns exceeds one-third of the total gravity load tributary to the frame, per AISC Specification Section C2.1, the effects of $P-\delta$ and $P-\Delta$ must be considered in the frame analysis. This example uses analysis software that accounts for both $P-\Delta$ and $P-\delta$ effects. (If the software used does not account for $P-\delta$ effects this may be accomplished by subdividing the columns between the footing and beam.)

Figures C.1A-3 and C.1A-4 show results from a first-order and a second-order analysis. (The first-order analysis is shown for reference only.) In each case, the drift is the average of drifts at grid lines B and C.

First-order results

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_{1st} = 0.181$ in.</td>
<td>$\Delta_{1st} = 0.193$ in. (prior to dividing by 1.6)</td>
</tr>
<tr>
<td>113 kip-ft</td>
<td>75.2 kip-ft</td>
</tr>
<tr>
<td>5.64 kips</td>
<td>3.76 kips</td>
</tr>
<tr>
<td>71.6 kips</td>
<td>47.7 kips</td>
</tr>
</tbody>
</table>

Fig. C.1A-3. Results of first-order analysis.
Second-order results

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD (Reactions and moments divided by 1.6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_{2nd} = 0.290$ in.</td>
<td>$\Delta_{2nd} = 0.321$ in. (prior to dividing by 1.6)</td>
</tr>
<tr>
<td>Drift ratio:</td>
<td>Drift ratio:</td>
</tr>
<tr>
<td>$\frac{\Delta_{2nd}}{\Delta_{1st}} = 0.290$ in. $\frac{\Delta_{1st}}{0.181}$ in. $= 1.60$</td>
<td>$\frac{\Delta_{2nd}}{\Delta_{1st}} = 0.321$ in. $\frac{\Delta_{1st}}{0.193}$ in. $= 1.66$</td>
</tr>
</tbody>
</table>

Fig. C.1A-4. Results of second-order analysis.

Check the assumption that $\alpha P_r / P_{ns} \leq 0.5$ on the column on grid line C.

Because a W12×65 column contains no elements that are slender for uniform compression,

$$P_{ns} = F_y A_g$$

$$= (50$ ksi$)(19.1$ in.$^2)$

$$= 955$ kips

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha P_r$ $\frac{F_y A_g}{955}$ kips</td>
<td>$\alpha P_r$ $\frac{F_y A_g}{955}$ kips</td>
</tr>
<tr>
<td>$= 0.0760 \leq 0.5$ o.k.</td>
<td>$= 0.0811 \leq 0.5$ o.k.</td>
</tr>
</tbody>
</table>

The stiffness assumption used in the analysis, $\tau_p = 1.0$, is verified.

Note that the drift ratio, 1.60 (LRFD) or 1.66 (ASD), does not exceed the recommended limit of 2.5 from AISC Specification Commentary Section C1.

The required axial compressive strength in the columns is 72.6 kips (LRFD) or 48.4 kips (ASD). The required bending moment diagram is linear, varying from zero at the bottom to 127 kip-ft (LRFD) or 84.8 kip-ft (ASD) at the top. These required strengths apply to both columns because the notional load must be applied in each direction.
Although the second-order sway multiplier (drift ratio) is fairly large at 1.60 (LRFD) or 1.66 (ASD), the change in bending moment is small because the only sway moments are those produced by the small notional loads. For load combinations with significant gravity and lateral loadings, the increase in bending moments is larger.

Per AISC Specification Section C3, the effective length for flexural buckling of all members is taken as the unbraced length ($K = 1.0$):

\[
\begin{align*}
L_{cx} &= 20.0 \text{ ft} \\
L_{cy} &= 20.0 \text{ ft}
\end{align*}
\]
EXAMPLE C.1B  DESIGN OF A MOMENT FRAME BY THE EFFECTIVE LENGTH METHOD

Given:
Repeat Example C.1A using the effective length method.

Determine the required strengths and effective length factors for the columns in the moment frame shown in Figure C.1B-1 for the maximum gravity load combination, using LRFD and ASD. Use the effective length method.

Columns are unbraced between the footings and roof in the $x$- and $y$-axes and have pinned bases.

Solution:
From AISC Manual Table 1-1, the $W12\times65$ has $I_x = 533$ in.$^4$

The beams from grid lines A to B and C to E and the columns at A, D and E are pinned at both ends and do not contribute to the lateral stability of the frame. There are no $P$-$
\Delta$ effects to consider in these members and they may be designed using $L_e = L$.

The moment frame between grid lines B and C is the source of lateral stability and therefore will be evaluated using the provisions of Chapter C of the AISC Specification. Although the columns at grid lines A, D and E do not contribute to lateral stability, the forces required to stabilize them must be considered in the moment-frame analysis. The entire frame from grid line A to E could be modeled, but in this case the model is simplified as shown in Figure C.1B-2, in which the stability loads from the three “leaning” columns are combined into a single representative column.

Check the limitations for the use of the effective length method given in AISC Specification Appendix 7, Section 7.2.1:

(a) The structure supports gravity loads primarily through nominally vertical columns, walls or frames.
(b) The ratio of maximum second-order drift to the maximum first-order drift (both determined for LRFD load combinations or 1.6 times ASD load combinations, with stiffness not adjusted as specified in AISC Specification Section C2.3) in all stories will be assumed to be no greater than 1.5, subject to verification in the following.
From Chapter 2 of ASCE/SEI 7, the maximum gravity load combinations are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_d = 1.2D + 1.6L )</td>
<td>( w_d = D + L )</td>
</tr>
<tr>
<td>( = 1.2(0.400 \text{ kip/ft}) + 1.6(1.20 \text{ kip/ft}) )</td>
<td>( = 0.400 \text{ kip/ft} + 1.20 \text{ kip/ft} )</td>
</tr>
<tr>
<td>( = 2.40 \text{ kip/ft} )</td>
<td>( = 1.60 \text{ kip/ft} )</td>
</tr>
</tbody>
</table>

Per AISC Specification Appendix 7, Section 7.2.2, the analysis must conform to the requirements of AISC Specification Section C2.1, with the exception of the stiffness reduction required by the provisions of Section C2.1(a).

Per AISC Specification Section C2.1(d), for LRFD perform a second-order analysis and member strength checks using the LRFD load combinations. For ASD, perform a second-order analysis at 1.6 times the ASD load combinations and divide the analysis results by 1.6 for the ASD member strength checks.

**Frame analysis gravity loads**

The uniform gravity loads to be considered in a second-order analysis on the beam from B to C are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_d' = 2.40 \text{ kip/ft} )</td>
<td>( w_d' = 1.6(1.60 \text{ kip/ft}) )</td>
</tr>
<tr>
<td></td>
<td>( = 2.56 \text{ kip/ft} )</td>
</tr>
</tbody>
</table>

Concentrated gravity loads to be considered in a second-order analysis on the columns at B and C contributed by adjacent beams are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_d' = \frac{w_d'f}{2} )</td>
<td>( P_d' = \frac{w_d'f}{2} )</td>
</tr>
<tr>
<td>( = \frac{(2.40 \text{ kip/ft})(30.0 \text{ ft})}{2} )</td>
<td>( = \frac{(2.56 \text{ kip/ft})(30.0 \text{ ft})}{2} )</td>
</tr>
<tr>
<td>( = 36.0 \text{ kips} )</td>
<td>( = 38.4 \text{ kips} )</td>
</tr>
</tbody>
</table>

**Concentrated gravity loads on the representative “leaning” column**

The load in this column accounts for all gravity loads that is stabilized by the moment frame, but not directly applied to it.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_d' = (60.0 \text{ ft})(2.40 \text{ kip/ft}) )</td>
<td>( P_d' = (60.0 \text{ ft})(2.56 \text{ kip/ft}) )</td>
</tr>
<tr>
<td>( = 144 \text{ kips} )</td>
<td>( = 154 \text{ kips} )</td>
</tr>
</tbody>
</table>

**Frame analysis notional loads**

Per AISC Specification Appendix 7, Section 7.2.2, frame out-of-plumbness must be accounted for by the application of notional loads in accordance with AISC Specification Section C2.2b. Note that notional loads need to only be applied to the gravity load combinations per AISC Specification Section C2.2b(d) when the requirement that \( \Delta_{2nd} / \Delta_{1st} \leq 1.7 \) (using stiffness adjusted as specified in Section C2.3) is satisfied. Per the User Note in AISC Specification Appendix 7, Section 7.2.2, Section C2.2b(d) will be satisfied in all cases where the effective length method is applicable, and therefore the notional load need only be applied in gravity-only load cases.

From AISC Specification Equation C2-1, the notional loads are:
Summary of applied frame loads

The applied loads are shown in Figure C.1B-2.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 1.0$</td>
<td>$\alpha = 1.6$</td>
</tr>
<tr>
<td>$Y_i = (120 \text{ ft})(2.40 \text{ kip/ft})$</td>
<td>$Y_i = (120 \text{ ft})(1.60 \text{ kip/ft})$</td>
</tr>
<tr>
<td>= 288 kips</td>
<td>= 192 kips</td>
</tr>
<tr>
<td>$N_i = 0.002 \alpha Y_i$</td>
<td>$N_i = 0.002 \alpha Y_i$</td>
</tr>
<tr>
<td>= 0.002(1.0)(288 kips)</td>
<td>(Spec. Eq. C2-1)</td>
</tr>
<tr>
<td>= 0.576 kip</td>
<td></td>
</tr>
</tbody>
</table>

Per AISC Specification Appendix 7, Section 7.2.2, conduct the analysis using the full nominal stiffnesses.

Half of the gravity load is carried by the columns of the moment-resisting frame. Because the gravity load supported by the moment-resisting frame columns exceeds one-third of the total gravity load tributary to the frame, per AISC Specification Section C2.1(b), the effects of $P-\delta$ on the response of the structure must be considered in the frame analysis. This example uses analysis software that accounts for both $P-\Delta$ and $P-\delta$ effects. When using software that does not account for $P-\delta$ effects, this could be accomplished by subdividing columns between the footing and beam.

Figures C.1B-3 and C.1B-4 show results from a first-order and second-order analysis. In each case, the drift is the average of drifts at grid lines B and C.
First-order results

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Reactions and moments divided by 1.6)</td>
<td></td>
</tr>
<tr>
<td>$\Delta_{1st}$</td>
<td>0.145 in.</td>
<td>0.155 in. (prior to dividing by 1.6)</td>
</tr>
</tbody>
</table>

![Fig. C.1B-3. Results of first-order analysis.](image1)

Second-order results

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_{2nd}$</td>
<td>0.204 in.</td>
<td>0.223 in. (prior to dividing by 1.6)</td>
</tr>
</tbody>
</table>

Drift ratio:

$\frac{\Delta_{2nd}}{\Delta_{1st}} \approx \begin{cases} 1.41 & \text{(LRFD)} \\ 1.44 & \text{(ASD)} \end{cases}$

![Fig. C.1B-4. Results of second-order analysis.](image2)

The assumption that the ratio of the maximum second-order drift to the maximum first-order drift is no greater than 1.5 is verified; therefore, the effective length method is permitted.

Although the second-order sway multiplier is fairly large at approximately 1.41 (LRFD) or 1.44 (ASD), the change in bending moment is small because the only sway moments for this load combination are those produced by the small notional loads. For load combinations with significant gravity and lateral loadings, the increase in bending moments is larger.

Calculate the in-plane effective length factor, $K_x$, using the “story stiffness approach” and Equation C-A-7-5 presented in AISC Specification Commentary Appendix 7, Section 7.2. With $K_x = K_2$: 

\[ K_x = K_2 \]
Calculate the total load in all columns, \( P_{\text{story}} \), as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{\text{story}} = (2.40 \text{ kip/ft})(120 \text{ ft}) )</td>
<td>( P_{\text{story}} = (1.60 \text{ kip/ft})(120 \text{ ft}) )</td>
</tr>
<tr>
<td>= 288 kips</td>
<td>= 192 kips</td>
</tr>
</tbody>
</table>

Calculate the coefficient to account for the influence of \( P-\delta \) on \( P-\Delta \), \( R_M \), as follows, using AISC Specification Commentary Appendix 7, Equation C-A-7-6:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{mf} = 71.5 \text{ kips} + 72.5 \text{ kips} )</td>
<td>( P_{mf} = 47.6 \text{ kips} + 48.4 \text{ kips} )</td>
</tr>
<tr>
<td>= 144 kips</td>
<td>= 96.0 kips</td>
</tr>
<tr>
<td>( R_M = 1 - 0.15 \left( \frac{P_{mf}}{P_{\text{story}}} \right) ) (Spec. Eq. C-A-7-6)</td>
<td>( R_M = 1 - 0.15 \left( \frac{P_{mf}}{P_{\text{story}}} \right) ) (Spec. Eq. C-A-7-6)</td>
</tr>
<tr>
<td>= 1 - 0.15 \left( \frac{144 \text{ kips}}{288 \text{ kips}} \right)</td>
<td>= 1 - 0.15 \left( \frac{96.0 \text{ kips}}{192 \text{ kips}} \right)</td>
</tr>
<tr>
<td>= 0.925</td>
<td>= 0.925</td>
</tr>
</tbody>
</table>

Calculate the Euler buckling strength of one moment frame.

\[
\frac{\pi^2 EI}{L^2} = \frac{\pi^2 (29,000 \text{ ksi})(533 \text{ in.}^4)}{[(20.0 \text{ ft})(12 \text{ in./ft})]^2} = 2,650 \text{ kips}
\]

From AISC Specification Commentary Equation C-A-7-5, for the column at line C:
Note that the column loads are multiplied by 1.6 for ASD in Equation C-A-7-5.

With $K_x = 3.45$ and $K_y = 1.00$, the column available strengths can be verified for the given member sizes for the second-order forces (calculations not shown), using the following effective lengths:

\[ L_{cx} = K_x L_x \]
\[ = 3.45(20.0 \text{ ft}) \]
\[ = 69.0 \text{ ft} \]

\[ L_{cy} = K_y L_y \]
\[ = 1.00(20.0 \text{ ft}) \]
\[ = 20.0 \text{ ft} \]
Chapter D
Design of Members for Tension

D1. SLENDERNESS LIMITATIONS

AISC Specification Section D1 does not establish a slenderness limit for tension members, but recommends limiting $L/r$ to a maximum of 300. This is not an absolute requirement. Rods and hangers are specifically excluded from this recommendation.

D2. TENSILE STRENGTH

Both tensile yielding strength and tensile rupture strength must be considered for the design of tension members. It is not unusual for tensile rupture strength to govern the design of a tension member, particularly for small members with holes or heavier sections with multiple rows of holes.

For preliminary design, tables are provided in Part 5 of the AISC Manual for W-shapes, L-shapes, WT-shapes, rectangular HSS, square HSS, round HSS, Pipe, and 2L-shapes. The calculations in these tables for available tensile rupture strength assume an effective area, $A_e$, of $0.75A_g$. The gross area, $A_g$, is the total cross-sectional area of the member. If the actual effective area is greater than $0.75A_g$, the tabulated values will be conservative and calculations can be performed to obtain higher available strengths. If the actual effective area is less than $0.75A_g$, the tabulated values will be unconservative and calculations are necessary to determine the available strength.

D3. EFFECTIVE NET AREA

In computing net area, $A_n$, AISC Specification Section B4.3b requires that an extra $\frac{3}{32}$ in. be added to the bolt hole diameter. A computation of the effective area for a chain of holes is presented in Example D.9.

Unless all elements of the cross section are connected, $A_e = A_nU$, where $U$ is a reduction factor to account for shear lag. The appropriate values of $U$ can be obtained from AISC Specification Table D3.1.

D4. BUILT-UP MEMBERS

The limitations for connections of built-up members are discussed in Section D4 of the AISC Specification.

D5. PIN-CONNECTED MEMBERS

An example of a pin-connected member is given in Example D.7.

D6. EYEBARS

An example of an eyebar is given in Example D.8. The strength of an eyebar meeting the dimensional requirements of AISC Specification Section D6 is governed by tensile yielding of the body.
EXAMPLE D.1  W-SHAPE TENSION MEMBER

Given:

Select an ASTM A992 W-shape with 8 in. nominal depth to carry a dead load of 30 kips and a live load of 90 kips in tension. The member is 25.0 ft long. Verify the member strength by both LRFD and ASD with the bolted end connection as shown in Figure D.1-1. Verify that the member satisfies the recommended slenderness limit. Assume that connection limit states do not govern.

![Diagram of W-shape with connection details.]

Fig D.1-1. Connection geometry for Example D.1.

Solution:

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_u$</td>
<td>$1.2 \times 30 \text{ kips} + 1.6 \times 90 \text{ kips}$</td>
<td>$P_u = 30 \text{ kips} + 90 \text{ kips}$</td>
</tr>
<tr>
<td></td>
<td>$= 180 \text{ kips}$</td>
<td>$= 120 \text{ kips}$</td>
</tr>
</tbody>
</table>

From AISC Manual Table 5-1, try a W8×21.

From AISC Manual Table 2-4, the material properties are as follows:

- ASTM A992
  - $F_y = 50 \text{ ksi}$
  - $F_u = 65 \text{ ksi}$

From AISC Manual Table 1-1, the geometric properties are as follows:

- W8×21
  - $A_g = 6.16 \text{ in.}^2$
  - $b_f = 5.27 \text{ in.}$
  - $t_f = 0.400 \text{ in.}$
  - $d = 8.28 \text{ in.}$
  - $r_f = 1.26 \text{ in.}$

The WT-shape corresponding to a W8×21 is a WT4×10.5. From AISC Manual Table 1-8, the geometric properties are as follows:

- WT4×10.5
  - $\bar{y} = 0.831 \text{ in.}$
Tensile Yielding

From AISC Manual Table 5-1, the available tensile yielding strength of a W8×21 is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_tP_n$</td>
<td>277 kips &gt; 180 kips</td>
<td>o.k.</td>
</tr>
<tr>
<td>$\frac{P_n}{\Omega_t}$</td>
<td>184 kips &gt; 120 kips</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Tensile Rupture

Verify the table assumption that $A_e/A_g \geq 0.75$ for this connection.

From the description of the element in AISC Specification Table D3.1, Case 7, calculate the shear lag factor, $U$, as the larger of the values from AISC Specification Section D3, Table D3.1 Case 2 and Case 7.

From AISC Specification Section D3, for open cross sections, $U$ need not be less than the ratio of the gross area of the connected element(s) to the member gross area.

$$U = \frac{2b_f l_f}{A_g}$$

$$= \frac{2(5.27 \text{ in.})(0.400 \text{ in.})}{6.16 \text{ in.}^2}$$

$$= 0.684$$

Case 2: Determine $U$ based on two WT-shapes per AISC Specification Commentary Figure C-D3.1, with $\bar{x} = \bar{y} = 0.831 \text{ in.}$ and where $l$ is the length of connection.

$$U = 1 - \frac{\bar{x}}{l}$$

$$= 1 - \frac{0.831 \text{ in.}}{9.00 \text{ in.}}$$

$$= 0.908$$

Case 7:

$$b_f = 5.27 \text{ in.}$$

$$\frac{2}{3}d = \frac{2}{3}(8.28 \text{ in.})$$

$$= 5.52 \text{ in.}$$

Because the flange is connected with three or more fasteners per line in the direction of loading and $b_f < \frac{2}{3}d$:

$$U = 0.85$$

Therefore, use the larger $U = 0.908$.

Calculate $A_e$ using AISC Specification Section B4.3b.
Calculate $A_e$ using AISC Specification Section D3.

$$A_e = A_n U$$

$$= (4.76 \text{ in.}^2)(0.908)$$

$$= 4.32 \text{ in.}^2$$

$$\frac{A_e}{A_g} = \frac{4.32 \text{ in.}^2}{6.16 \text{ in.}^2}$$

$$> 0.701 < 0.75$$

Because $A_e/A_g < 0.75$, the tensile rupture strength from AISC Manual Table 5-1 is not valid. The available tensile rupture strength is determined using AISC Specification Section D2 as follows:

$$P_n = F_n A_e$$

$$= (65 \text{ ksi})(4.32 \text{ in.}^2)$$

$$= 281 \text{ kips}$$

From AISC Specification Section D2, the available tensile rupture strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_t = 0.75$</td>
<td></td>
<td>$\omega_t = 2.00$</td>
</tr>
<tr>
<td>$\phi_t P_n = 0.75(281 \text{ kips})$</td>
<td>$211 \text{ kips} &gt; 180 \text{ kips}$ o.k.</td>
<td>$P_n = 281 \text{ kips}$ $\omega_t = 2.00$ $141 \text{ kips} &gt; 120 \text{ kips}$ o.k.</td>
</tr>
</tbody>
</table>

Note that the W8×21 available tensile strength is governed by the tensile rupture limit state at the end connection versus the tensile yielding limit state.

See Chapter J for illustrations of connection limit state checks.

**Check Recommended Slenderness Limit**

$$\frac{L}{r} = \frac{(25.0 \text{ ft})(12 \text{ in./ft})}{1.26 \text{ in.}}$$

$$= 238 < 300 \text{ from AISC Specification Section D1} \text{ o.k.}$$
EXAMPLE D.2 SINGLE-ANGLE TENSION MEMBER

Given:

Verify the tensile strength of an ASTM A36 L4×4×½ with one line of four ¾-in.-diameter bolts in standard holes, as shown in Figure D.2-1. The member carries a dead load of 20 kips and a live load of 60 kips in tension. Additionally, calculate at what length this tension member would cease to satisfy the recommended slenderness limit. Assume that connection limit states do not govern.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A36

\( F_y = 36 \text{ ksi} \)

\( F_u = 58 \text{ ksi} \)

From AISC Manual Table 1-7, the geometric properties are as follows:

L4×4×½

\( A_g = 3.75 \text{ in.}^2 \)

\( r_x = 0.776 \text{ in.} \)

\( \bar{x} = 1.18 \text{ in.} \)

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

\[
\begin{array}{c|c|c}
\text{LRFD} & \text{ASD} \\
\hline
P_g &= 1.2(20 \text{ kips}) + 1.6(60 \text{ kips}) \\
&= 120 \text{ kips} \\
\hline
P_a &= 20 \text{ kips} + 60 \text{ kips} \\
&= 80.0 \text{ kips} \\
\end{array}
\]

**Tensile Yielding**

\[
P_a = F_y A_g \\
= (36 \text{ ksi})(3.75 \text{ in.}^2) \\
= 135 \text{ kips}
\]

From AISC Specification Section D2, the available tensile yielding strength is:
\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
\phi_t = 0.90 & \Omega_t = 1.67 \\
\phi_t P_n = 0.90(135 \text{ kips}) & \frac{P_n}{\Omega_t} = \frac{135 \text{ kips}}{1.67} \\
= 122 \text{ kips} > 120 \text{ kips} & = 80.8 \text{ kips} > 80.0 \text{ kips} \textbf{ o.k.} \\
\hline
\end{array}
\]

**Tensile Rupture**

From the description of the element in AISC Specification Table D3.1 Case 8, calculate the shear lag factor, \( U \), as the larger of the values from AISC Specification Section D3, Table D3.1 Case 2 and Case 8.

From AISC Specification Section D3, for open cross sections, \( U \) need not be less than the ratio of the gross area of the connected element(s) to the member gross area. Half of the member is connected, therefore, the minimum value of \( U \) is:

\[
U = 0.500
\]

Case 2, where \( l \) is the length of connection and \( y = \bar{y} \):

\[
U = 1 - \frac{\bar{y}}{l} = 1 - \frac{1.18 \text{ in.}}{9.00 \text{ in.}} = 0.869
\]

Case 8, with four or more fasteners per line in the direction of loading:

\[
U = 0.80
\]

Therefore, use the larger \( U = 0.869 \).

Calculate \( A_n \) using AISC Specification Section B4.3b.

\[
A_n = A_k - (d_k + \frac{1}{16} \text{ in.})t = 3.75 \text{ in.} - (\frac{3}{16} \text{ in.} + \frac{1}{16} \text{ in.})(\frac{1}{2} \text{ in.}) = 3.31 \text{ in.}^2
\]

Calculate \( A_e \) using AISC Specification Section D3.

\[
A_e = A_n U = (3.31 \text{ in.}^2)(0.869) = 2.88 \text{ in.}^2
\]

\[
P_e = F_u A_e = (58 \text{ ksi})(2.88 \text{ in.}^2) = 167 \text{ kips}
\]

From AISC Specification Section D2, the available tensile rupture strength is:
\[ \phi_t = 0.75 \]

\[ \phi_t P_n = 0.75(167 \text{ kips}) = 125 \text{ kips} > 120 \text{ kips} \quad \text{o.k.} \]

\[ \Omega_r = 2.00 \]

\[ \frac{P_n}{\Omega_r} = \frac{167 \text{ kips}}{2.00} = 83.5 \text{ kips} > 80.0 \text{ kips} \quad \text{o.k.} \]

The L4\(\times\)4\(\times\)\(\frac{3}{4}\) available tensile strength is governed by the tensile yielding limit state.

\[ \phi_t P_n = 122 \text{ kips} > 120 \text{ kips} \quad \text{o.k.} \]

\[ \frac{P_n}{\Omega_r} = \frac{80.8 \text{ kips}}{80.0 \text{ kips}} \quad \text{o.k.} \]

*Recommended \(L_{\text{max}}\)*

Using AISC *Specification* Section D1:

\[ L_{\text{max}} = 300r_z \]

\[ = 300 \left( \frac{0.776 \text{ in.}}{12 \text{ in./ft}} \right) \]

\[ = 19.4 \text{ ft} \]

Note: The \(L/r\) limit is a recommendation, not a requirement.

See Chapter J for illustrations of connection limit state checks.
EXAMPLE D.3  WT-SHAPE TENSION MEMBER

Given:

An ASTM A992 WT6×20 member has a length of 30 ft and carries a dead load of 40 kips and a live load of 120 kips in tension. As shown in Figure D3-1, the end connection is fillet welded on each side for 16 in. Verify the member tensile strength by both LRFD and ASD. Assume that the gusset plate and the weld are satisfactory.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992

\[ F_y = 50 \text{ ksi} \]

\[ F_u = 65 \text{ ksi} \]

From AISC *Manual* Table 1-8, the geometric properties are as follows:

WT6×20

\[ A_g = 5.84 \text{ in.}^2 \]

\[ b_y = 8.01 \text{ in.} \]

\[ t_y = 0.515 \text{ in.} \]

\[ r_y = 1.57 \text{ in.} \]

\[ y = 1.09 \text{ in.} \]

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

\[ P_u = 1.2(40 \text{ kips}) + 1.6(120 \text{ kips}) = 240 \text{ kips} \]

\[ P_u = 40 \text{ kips} + 120 \text{ kips} = 160 \text{ kips} \]

*Tensile Yielding*

Check tensile yielding limit state using AISC *Manual* Table 5-3.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \phi \cdot P_u = 263 \text{ kips} &gt; 240 \text{ kips} \text{ o.k.} ]</td>
<td>[ \frac{P_u}{\Omega_t} = 175 \text{ kips} &gt; 160 \text{ kips} \text{ o.k.} ]</td>
</tr>
</tbody>
</table>
Tensile Rupture

Check tensile rupture limit state using AISC Manual Table 5-3.

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_t P_n = 214 \text{ kips} &lt; 240 \text{ kips}$</td>
<td>n.g.</td>
<td>$P_n = 142 \text{ kips} &lt; 160 \text{ kips}$</td>
</tr>
</tbody>
</table>

The tabulated available rupture strengths don’t work and may be conservative for this case; therefore, calculate the exact solution.

Calculate $U$ as the larger of the values from AISC Specification Section D3 and Table D3.1 Case 4.

From AISC Specification Section D3, for open cross sections, $U$ need not be less than the ratio of the gross area of the connected element(s) to the member gross area.

$$U = \frac{b t f}{A_g}$$
$$= \frac{(8.01 \text{ in.})(0.515 \text{ in.})}{5.84 \text{ in.}^2}$$
$$= 0.706$$

Case 4, where $l$ is the length of the connection and $\bar{x} = \bar{y}$:

$$U = \frac{3l^2}{3l^2 + w^2} \left( 1 - \frac{1}{l} \right)$$
$$= \frac{\left[ \frac{3(16.0 \text{ in.})^2}{3(16.0 \text{ in.})^2 + (8.01 \text{ in.})^2} \right]}{1 - \frac{1.09 \text{ in.}}{16.0 \text{ in.}}}$$
$$= 0.860$$

Therefore, use $U = 0.860$.

Calculate $A_e$ using AISC Specification Section B4.3. Because there are no reductions due to bolt holes or notches:

$$A_e = A_g$$
$$= 5.84 \text{ in.}^2$$

Calculate $A_e$ using AISC Specification Section D3.

$$A_e = A_g U \quad \text{(Spec. Eq. D3-1)}$$
$$= (5.84 \text{ in.}^2)(0.860)$$
$$= 5.02 \text{ in.}^2$$

Calculate $P_e$.

$$P_e = F_{ce} A_e \quad \text{(Spec. Eq. D2-2)}$$
$$= (65 \text{ ksi})(5.02 \text{ in.}^2)$$
$$= 326 \text{ kips}$$
From AISC Specification Section D2, the available tensile rupture strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_r = 0.75 )</td>
<td>( \Omega_r = 2.00 )</td>
<td>( \phi_r P_n = 0.75(326 \text{ kips}) )</td>
</tr>
<tr>
<td>( \phi_r P_n = 0.75(326 \text{ kips}) )</td>
<td>( \Omega_r = 2.00 )</td>
<td>( P_n = 326 \text{ kips} )</td>
</tr>
<tr>
<td>( 245 \text{ kips} &gt; 240 \text{ kips} )</td>
<td>( \frac{P_n}{\Omega_r} = \frac{326 \text{ kips}}{2.00} )</td>
<td>( 163 \text{ kips} &gt; 160 \text{ kips} )</td>
</tr>
</tbody>
</table>

Alternately, the available tensile rupture strengths can be determined by modifying the tabulated values. The available tensile rupture strengths published in the tension member selection tables are based on the assumption that \( A_e = 0.75A_g \). The actual available strengths can be determined by adjusting the values from AISC Manual Table 5-3 as follows:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_r P_n = (214 \text{ kips}) \left( \frac{A_e}{0.75A_g} \right) )</td>
<td>( \frac{P_n}{\Omega_r} = (142 \text{ kips}) \left( \frac{A_e}{0.75A_g} \right) )</td>
<td>( \frac{P_n}{\Omega_r} = (142 \text{ kips}) \left( \frac{A_e}{0.75A_g} \right) )</td>
</tr>
<tr>
<td>( = (214 \text{ kips}) \left[ \frac{5.02 \text{ in.}^2}{0.75(5.84 \text{ in.}^2)} \right] )</td>
<td>( = (142 \text{ kips}) \left[ \frac{5.02 \text{ in.}^2}{0.75(5.84 \text{ in.}^2)} \right] )</td>
<td>( = (142 \text{ kips}) \left[ \frac{5.02 \text{ in.}^2}{0.75(5.84 \text{ in.}^2)} \right] )</td>
</tr>
<tr>
<td>( = 245 \text{ kips} &gt; 240 \text{ kips} )</td>
<td>( = 163 \text{ kips} &gt; 160 \text{ kips} )</td>
<td>( = 163 \text{ kips} &gt; 160 \text{ kips} )</td>
</tr>
</tbody>
</table>

**Recommended Slenderness Limit**

\[
\frac{L}{r_x} = \frac{(30.0 \text{ ft})(12 \text{ in./ft})}{1.57 \text{ in.}}
\]

\[
= 229 < 300 \text{ from AISC Specification Section D1} \text{ o.k.}
\]

Note: The \( L/r_x \) limit is a recommendation, not a requirement.

See Chapter J for illustrations of connection limit state checks.
EXAMPLE D.6 DOUBLE-ANGLE TENSION MEMBER

Given:

An ASTM A36 2L4×4×½ (¾-in. separation) has one line of eight ¾-in.-diameter bolts in standard holes and is 25 ft in length as shown in Figure D.6-1. The double angle is carrying a dead load of 40 kips and a live load of 120 kips in tension. Verify the member tensile strength. Assume that the gusset plate and bolts are satisfactory.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A36

\[ F_y = 36 \text{ ksi} \]
\[ F_u = 58 \text{ ksi} \]

From AISC Manual Tables 1-7 and 1-15, the geometric properties are as follows:

\[ L4\times4\times\frac{1}{2} \]
\[ x = 1.18 \text{ in.} \]

\[ 2L4\times4\times\frac{1}{2} (s = \frac{3}{8} \text{ in.}) \]
\[ A_g = 7.50 \text{ in.}^2 \]
\[ r_y = 1.83 \text{ in.} \]
\[ r_x = 1.21 \text{ in.} \]

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

\[ P_u = 1.2(40 \text{ kips}) + 1.6(120 \text{ kips}) = 240 \text{ kips} \]

\[ P_u = 40 \text{ kips} + 120 \text{ kips} = 160 \text{ kips} \]

Tensile Yielding

Check tensile yielding limit state using AISC Manual Table 5-8.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \phi\alpha P_u = 243 \text{ kips} &gt; 240 \text{ kips} ] o.k.</td>
<td>[ \frac{P_u}{\alpha_2} = 162 \text{ kips} &gt; 160 \text{ kips} ] o.k.</td>
</tr>
</tbody>
</table>

Fig. D.6-1. Connection geometry for Example D.6.
Tensile Rupture

Determine the available tensile rupture strength using AISC Specification Section D2. Calculate $U$ as the larger of the values from AISC Specification Section D3, Table D3.1 Case 2 and Case 8.

From AISC Specification Section D3, for open cross sections, $U$ need not be less than the ratio of the gross area of the connected element(s) to the member gross area. Half of the member is connected, therefore, the minimum $U$ value is:

$$U = 0.500$$

From Case 2, where $l$ is the length of connection:

$$U = 1 - \frac{x}{l}$$

$$= 1 - \frac{1.18 \text{ in.}}{21.0 \text{ in.}}$$

$$= 0.944$$

From Case 8, with four or more fasteners per line in the direction of loading:

$$U = 0.80$$

Therefore, use $U = 0.944$.

Calculate $A_n$ using AISC Specification Section B4.3.

$$A_n = A_g - 2(d_h + \frac{1}{16} \text{ in.})t$$

$$= 7.50 \text{ in.}^2 - 2(\frac{1}{32} \text{ in.} + \frac{1}{16} \text{ in.})(\frac{1}{2} \text{ in.})$$

$$= 6.63 \text{ in.}^2$$

Calculate $A_e$ using AISC Specification Section D3.

$$A_e = A_n U$$

$$= (6.63 \text{ in.}^2)(0.944)$$

$$= 6.26 \text{ in.}^2$$

Calculate $P_{ne}$

$$P_n = F_n A_e$$

$$= (58 \text{ ksi})(6.26 \text{ in.}^2)$$

$$= 363 \text{ kips}$$

From AISC Specification Section D2, the available tensile rupture strength is:
Note that AISC Manual Table 5-8 could also be conservatively used since $A_e \geq 0.75A_g$.

The double-angle available tensile strength is governed by the tensile yielding limit state.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega_r = 2.00$</td>
</tr>
<tr>
<td>$\phi_rP_n = 0.75 \times 363 \text{ kips}$</td>
<td>$P_n = 363 \text{ kips}$</td>
</tr>
<tr>
<td>$= 272 \text{ kips}$</td>
<td>$\frac{P_n}{\Omega_r} = 2.00$</td>
</tr>
<tr>
<td></td>
<td>$= 182 \text{ kips}$</td>
</tr>
</tbody>
</table>

Recommended Slenderness Limit

$$L = \frac{(25.0 \text{ ft})(12 \text{ in./ft})}{1.21 \text{ in.}} = 248 < 300 \text{ from AISC Specification Section D1} \quad \text{o.k.}$$

Note: From AISC Specification Section D4, the longitudinal spacing of connectors between components of built-up members should preferably limit the slenderness ratio in any component between the connectors to a maximum of 300.

See Chapter J for illustrations of connection limit state checks.
EXAMPLE D.9  PLATE WITH STAGGERED BOLTS

Given:

Compute $A_e$ and $A_r$ for a 14-in.-wide and ½-in.-thick plate subject to tensile loading with staggered holes as shown in Figure D.9-1.

Solution:

Calculate the net hole diameter using AISC Specification Section B4.3b.

$$d_{net} = d_h + \frac{1}{80} \text{ in.}$$

$$= \frac{1}{80} \text{ in.} + \frac{1}{80} \text{ in.}$$

$$= 0.875 \text{ in.}$$

Compute the net width for all possible paths across the plate. Because of symmetry, many of the net widths are identical and need not be calculated.

$$w = 14.0 \text{ in.} - \sum d_{net} + \sum \frac{s^2}{4g}$$

from AISC Specification Section B4.3b.

Line A-B-E-F:

$$w = 14.0 \text{ in.} - 2(0.875 \text{ in.})$$

$$= 12.3 \text{ in.}$$

Line A-B-C-D-E-F:

$$w = 14.0 \text{ in.} - 4(0.875 \text{ in.}) + \frac{(2.50 \text{ in.})^2}{4(3.00 \text{ in.})} + \frac{(2.50 \text{ in.})^2}{4(3.00 \text{ in.})}$$

$$= 11.5 \text{ in.}$$

Line A-B-C-D-G:
\[ w = 14.0\text{in.} - 3(0.875\text{in.}) + \frac{(2.50\text{in.})^2}{4(3.00\text{in.})} \]
\[ = 11.9\text{ in.} \]

Line A-B-D-E-F:
\[ w = 14.0\text{ in.} - 3(0.875\text{ in.}) + \frac{(2.50\text{ in.})^2}{4(7.00\text{ in.})} + \frac{(2.50\text{ in.})^2}{4(3.00\text{ in.})} \]
\[ = 12.1\text{ in.} \]

Line A-B-C-D-E-F controls the width, \( w \), therefore:
\[ A_n = wt \]
\[ = (11.5\text{ in.})(\frac{1}{2}\text{ in.}) \]
\[ = 5.75\text{ in.}^2 \]

Calculate \( U \).

From AISC Specification Table D3.1 Case 1, because tension load is transmitted to all elements by the fasteners,
\[ U = 1.0 \]
\[ A_r = A_n U \quad \text{(Spec. Eq. D3-1)} \]
\[ = (5.75\text{ in.}^2)(1.0) \]
\[ = 5.75\text{ in.}^2 \]
Chapter E
Design of Members for Compression

This chapter covers the design of compression members, the most common of which are columns. The AISC Manual includes design tables for the following compression member types in their most commonly available grades:

- W-shapes and HP-shapes
- Rectangular, square and round HSS
- Pipes
- WT-shapes
- Double angles
- Single angles

LRFD and ASD information is presented side-by-side for quick selection, design or verification. All of the tables account for the reduced strength of sections with slender elements.

The design and selection method for both LRFD and ASD is similar to that of previous editions of the AISC Specification, and will provide similar designs. In this AISC Specification, LRFD and ASD will provide identical designs when the live load is approximately three times the dead load.

The design of built-up shapes with slender elements can be tedious and time consuming, and it is recommended that standard rolled shapes be used whenever possible.

E1. GENERAL PROVISIONS

The design compressive strength, $\phi_c P_n$, and the allowable compressive strength, $P_n/\Omega_c$, are determined as follows:

$$P_n = \text{nominal compressive strength is the lowest value obtained based on the applicable limit states of flexural buckling, torsional buckling, and flexural-torsional buckling, kips}$$

$$\phi_c = 0.90 \text{ (LRFD)} \quad \Omega_c = 1.67 \text{ (ASD)}$$

Because the critical stress, $F_{cr}$, is used extensively in calculations for compression members, it has been tabulated in AISC Manual Table 4-14 for all of the common steel yield strengths.

E2. EFFECTIVE LENGTH

In the AISC Specification, there is no limit on slenderness, $L_c/r$. Per the User Note in AISC Specification Section E2, it is recommended that $L_c/r$ not exceed 200, as a practical limit based on professional judgment and construction economics.

Although there is no restriction on the unbraced length of columns, the tables of the AISC Manual are stopped at common or practical lengths for ordinary usage. For example, a double L3×3×¼, with a ½-in. separation has an $r_y$ of 1.38 in. At a $L_c/r$ of 200, this strut would be 23 ft long. This is thought to be a reasonable limit based on fabrication and handling requirements.

Throughout the AISC Manual, shapes that contain slender elements for compression when supplied in their most common material grade are footnoted with the letter “c.” For example, see a W14×22.
E3. FLEXURAL BUCKLING OF MEMBERS WITHOUT SLENDER ELEMENTS

Nonslender-element compression members, including nonslender built-up I-shaped columns and nonslender HSS columns, are governed by these provisions. The general design curve for critical stress versus $L_c/r$ is shown in Figure E-1.

The term $L_c$ is used throughout this chapter to describe the length between points that are braced against lateral and/or rotational displacement.

E4. TORSIONAL AND FLEXURAL-TORSIONAL BUCKLING OF SINGLE ANGLES AND MEMBERS WITHOUT SLENDER ELEMENTS

This section is most commonly applicable to double angles and WT sections, which are singly symmetric shapes subject to torsional and flexural-torsional buckling. The available strengths in axial compression of these shapes are tabulated in AISC *Manual* Part 4 and examples on the use of these tables have been included in this chapter for the shapes.

E5. SINGLE-ANGLE COMPRESSION MEMBERS

The available strength of single-angle compression members is tabulated in AISC *Manual* Part 4.

E6. BUILT-UP MEMBERS

The available strengths in axial compression for built-up double angles with intermediate connectors are tabulated in AISC *Manual* Part 4. There are no tables for other built-up shapes in the AISC *Manual*, due to the number of possible geometries.

E7. MEMBERS WITH SLENDER ELEMENTS

The design of these members is similar to members without slender elements except that a reduced effective area is used in lieu of the gross cross-sectional area.

The tables of AISC *Manual* Part 4 incorporate the appropriate reductions in available strength to account for slender elements.

Design examples have been included in this Chapter for built-up I-shaped members with slender webs and slender flanges. Examples have also been included for a double angle, WT and an HSS with slender elements.
Table E-1
Limiting Values of $L_c/r$ and $F_e$

<table>
<thead>
<tr>
<th>$F_y$, ksi</th>
<th>Limiting $L_c/r$</th>
<th>$F_e$, ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>134</td>
<td>15.9</td>
</tr>
<tr>
<td>50</td>
<td>113</td>
<td>22.4</td>
</tr>
<tr>
<td>65</td>
<td>99.5</td>
<td>28.9</td>
</tr>
<tr>
<td>70</td>
<td>95.9</td>
<td>31.1</td>
</tr>
</tbody>
</table>
EXAMPLE E.1A  W-SHAPE COLUMN DESIGN WITH PINNED ENDS

Given:

Select a W-shape column to carry the loading as shown in Figure E.1A. The column is pinned top and bottom in both axes. Limit the column size to a nominal 14-in. shape. A column is selected for both ASTM A992 and ASTM A913 Grade 65 material.

![Fig. E.1A. Column loading and bracing.](image)

Solution:

Note that ASTM A913 Grade 70 might also be used in this design. The requirement for higher preheat when welding and the need to use 90-ksi filler metals for complete-joint-penetration (CJP) welds to other 70-ksi pieces offset the advantage of the lighter column and should be considered in the selection of which grade to use.

From AISC Manual Table 2-4, the material properties are as follows:

- ASTM A992
  \[ F_y = 50 \text{ ksi} \]

- ASTM A913 Grade 65
  \[ F_y = 65 \text{ ksi} \]

From ASCE/SEI 7, Chapter 2, the required compressive strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ P_u = 1.2(140 \text{ kips}) + 1.6(420 \text{ kips}) ] = 840 kips</td>
<td>[ P_u = 140 \text{ kips} + 420 \text{ kips} ] = 560 kips</td>
</tr>
</tbody>
</table>

Column Selection—ASTM A992

From AISC Specification Commentary Table C-A-7.1, for a pinned-pinned condition, \( K_x = K_y = 1.0 \). The effective length is:
\[ L_c = K_x L_x = K_y L_y = 1.0 \times 30 \text{ ft} = 30.0 \text{ ft} \]

Because the unbraced length is the same in both the \(x\)-\(x\) and \(y\)-\(y\) directions and \(r_x\) exceeds \(r_y\) for all W-shapes, \(y\)-\(y\) axis buckling will govern.

Enter AISC Manual Table 4-1a with an effective length, \(L_c\), of 30 ft, and proceed across the table until reaching the least weight shape with an available strength that equals or exceeds the required strength. Select a W14×132.

From AISC Manual Table 4-1a, the available strength for a \(y\)-\(y\) axis effective length of 30 ft is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi P_n = 893 \text{ kips} &gt; 840 \text{ kips})</td>
<td>(P_n = 594 \text{ kips} &gt; 560 \text{ kips})</td>
</tr>
</tbody>
</table>

Column Selection—ASTM A913 Grade 65

Enter AISC Manual Table 4-1b with an effective length, \(L_c\), of 30 ft, and proceed across the table until reaching the least weight shape with an available strength that equals or exceeds the required strength. Select a W14×120.

From AISC Manual Table 4-1b, the available strength for a \(y\)-\(y\) axis effective length of 30 ft is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi P_n = 856 \text{ kips} &gt; 840 \text{ kips})</td>
<td>(P_n = 569 \text{ kips} &gt; 560 \text{ kips})</td>
</tr>
</tbody>
</table>
EXAMPLE E.1B  W-SHAPE COLUMN DESIGN WITH INTERMEDIATE BRACING

Given:

Verify a W14×90 is adequate to carry the loading as shown in Figure E.1B. The column is pinned top and bottom in both axes and braced at the midpoint about the y-y axis and torsionally. The column is verified for both ASTM A992 and ASTM A913 Grade 65 material.

![Figure E.1B. Column loading and bracing.](image)

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

- **ASTM A992**  
  $F_y = 50$ ksi

- **ASTM A913 Grade 65**  
  $F_y = 65$ ksi

From ASCE/SEI 7, Chapter 2, the required compressive strength is:

- **LRFD**  
  $P_{u} = 1.2(140 \text{ kips}) + 1.6(420 \text{ kips}) = 840 \text{ kips}$

- **ASD**  
  $P_{u} = 140 \text{ kips} + 420 \text{ kips} = 560 \text{ kips}$

From AISC Specification Commentary Table C-A-7.1, for a pinned-pinned condition, $K_x = K_y = 1.0$. The effective length about the y-y axis is:

$$L_{cy} = K_y L_y = 1.0(15 \text{ ft}) = 15.0 \text{ ft}$$

The values tabulated in AISC Manual Tables 4-1a, 4-1b and 4-1c are provided for buckling in the y-y direction. To determine the buckling strength in the x-x axis, an equivalent effective length for the y-y axis is determined using the $r_x/r_y$ ratio provided at the the bottom of these tables. For a W14×90, $r_x/r_y = 1.66$, and the equivalent y-y axis effective length for x-x axis buckling is computed as: 
\[ L_{cx} = K_x L_x \]
\[ = 1.0 \times 30 \text{ ft} \]
\[ = 30.0 \text{ ft} \]

\[ L_{cy\, eq} = \frac{L_{cx}}{r_x / r_y} \]
\[ = \frac{30.0 \text{ ft}}{1.66} \]
\[ = 18.1 \text{ ft} \]

Because 18.1 ft > 15.0 ft, the available compressive strength is governed by the \(x\)-\(x\) axis flexural buckling limit state.

**Available Compressive Strength—ASTM A992**

The available strength of a \(W14\times90\) is determined using AISC *Manual* Table 4-1a, conservatively using an unbraced length of \(L_c = 19.0 \text{ ft}\).

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_c P_n = 903 \text{ kips} &gt; 840 \text{ kips} \quad \text{o.k.})</td>
<td>(\frac{P_n}{\Omega_c} = 601 \text{ kips} &gt; 560 \text{ kips} \quad \text{o.k.})</td>
</tr>
</tbody>
</table>

**Available Compressive Strength—ASTM 913 Grade 65**

The available strength of a \(W14\times90\) is determined using AISC *Manual* Table 4-1b, conservatively using an unbraced length of \(L_c = 19.0 \text{ ft}\).

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_c P_n = 1,080 \text{ kips} &gt; 840 \text{ kips} \quad \text{o.k.})</td>
<td>(\frac{P_n}{\Omega_c} = 719 \text{ kips} &gt; 560 \text{ kips} \quad \text{o.k.})</td>
</tr>
</tbody>
</table>

The available strengths of the columns described in Examples E.1A and E.1B are easily selected directly from the AISC *Manual* Tables. The available strengths can also be determined as shown in the following Examples E.1C and E.1D.
EXAMPLE E.1C  W-SHAPE AVAILABLE STRENGTH CALCULATION

Given:

Calculate the available strength of the column sizes selected in Example E.1A with unbraced lengths of 30 ft in both axes. The material properties and loads are as given in Example E.1A.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

- ASTM A992
  - $F_y = 50$ ksi
- ASTM A913 Grade 65
  - $F_y = 65$ ksi

From AISC Manual Table 1-1, the geometric properties are as follows:

- W14×120
  - $A_g = 35.3$ in.$^2$
  - $r_x = 6.24$ in.
  - $r_y = 3.74$ in.

- W14×132
  - $A_g = 38.8$ in.$^2$
  - $r_x = 6.28$ in.
  - $r_y = 3.76$ in.

Column Compressive Strength—ASTM A992

Slenderness Check

From AISC Specification Commentary Table C-A-7.1, for a pinned-pinned condition, $K_x = K_y = 1.0$. The effective length about the $y$-$y$ axis is:

$$L_{cy} = K_x L_y = 1.0(30 \text{ ft}) = 30.0 \text{ ft}$$

Because the unbraced length for the W14×132 column is the same for both axes, the $y$-$y$ axis will govern.

$$\frac{L_{cy}}{r_y} = \frac{(30.0 \text{ ft})(12 \text{ in./ft})}{3.76 \text{ in.}} = 95.7$$

Critical Stress

For $F_y = 50$ ksi, the available critical stresses, $\phi F_{cr}$ and $F_{cr}/\phi$, for $L_{c}/r = 95.7$ are interpolated from AISC Manual Table 4-14 as follows. The available critical stress can also be determined as shown in Example E.1D.
From AISC Specification Equation E3-1, the available compressive strength of the W14×132 column is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_c F_{cr} = 23.0 \text{ ksi})</td>
<td>(F_{cr} = 15.4 \text{ ksi})</td>
</tr>
</tbody>
</table>

Column Compressive Strength—ASTM A913 Grade 65

Slenderness Check

From AISC Specification Commentary Table C-A-7.1, for a pinned-pinned condition, \(K_x = K_y = 1.0\). The effective length about the y-y axis is:

\[
L_{cy} = K_x L_y \\
= 1.0 (30 \text{ ft}) \\
= 30.0 \text{ ft}
\]

Because the unbraced length for the W14×120 column is the same for both axes, the y-y axis will govern.

\[
\frac{L_{cy}}{r_y} = \frac{(30.0 \text{ ft}) (12 \text{ in./ft})}{3.74 \text{ in.}} \\
= 96.3
\]

Critical Stress

For \(F_y = 65 \text{ ksi}\), the available critical stresses, \(\phi_c F_{cr}\) and \(F_{cr}/\Omega_c\), for \(L_{cy}/r_y = 96.3\) are interpolated from AISC Manual Table 4-14 as follows. The available critical stress can also be determined as shown in Example E.1D.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_c F_{cr} = 24.3 \text{ ksi})</td>
<td>(F_{cr} = 16.1 \text{ ksi})</td>
</tr>
</tbody>
</table>

From AISC Specification Equation E3-1, the available compressive strength of the W14×120 column is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_c P_u = (\phi_c F_{cr}) A_g)</td>
<td>(P_u = (F_{cr}/\Omega_c) A_g)</td>
</tr>
</tbody>
</table>

\[
= (24.3 \text{ ksi})(35.3 \text{ in.}^2) \\
= 858 \text{ kips} > 840 \text{ kips} \text{ o.k.}
\]

Note that the calculated values are approximately equal to the tabulated values.
EXAMPLE E.1D  W-SHAPE AVAILABLE STRENGTH CALCULATION

Given:

Calculate the available strength of a $W14 \times 90$ with a $x-x$ axis unbraced length of 30 ft and $y-y$ axis and torsional unbraced lengths of 15 ft. The material properties and loads are as given in Example E.1A.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi

ASTM A913 Grade 65
$F_y = 65$ ksi

From AISC Manual Table 1-1, the geometric properties are as follows:

$W14 \times 90$
$A_g = 26.5$ in.$^2$
$r_x = 6.14$ in.
$r_y = 3.70$ in.
$b_f = 10.2$
$t_w = 25.9$

Slenderness Check

From AISC Specification Commentary Table C-A-7.1, for a pinned-pinned condition, $K_x = K_y = 1.0$.

$L_{cx} = K_x L_x$
$= 1.0(30 \text{ ft})$
$= 30.0 \text{ ft}$

$L_{cx} = \frac{(30.0 \text{ ft})(12 \text{ in./ft})}{6.14 \text{ in.}}$
$= 58.6 \text{ governs}$

$L_{cy} = K_y L_y$
$= 1.0(15 \text{ ft})$
$= 15.0 \text{ ft}$

$L_{cy} = \frac{(15.0 \text{ ft})(12 \text{ in./ft})}{3.70 \text{ in.}}$
$= 48.6$

Column Compressive Strength—ASTM A992

Width-to-Thickness Ratio
The width-to-thickness ratio of the flanges of the \( W14 \times 90 \) is:

\[
\frac{b_f}{2t_f} = 10.2
\]

From AISC *Specification* Table B4.1a, Case 1, the limiting width-to-thickness ratio of the flanges is:

\[
0.56 \sqrt{\frac{E}{F_y}} = 0.56 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}}
\]

\[
= 13.5 > 10.2; \text{ therefore, the flanges are nonslender}
\]

The width-to-thickness ratio of the web of the \( W14 \times 90 \) is:

\[
\frac{h}{t_w} = 25.9
\]

From AISC *Specification* Table B4.1a, Case 5, the limiting width-to-thickness ratio of the web is:

\[
1.49 \sqrt{\frac{E}{F_y}} = 1.49 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}}
\]

\[
= 35.9 > 25.9; \text{ therefore, the web is nonslender}
\]

Because the web and flanges are nonslender, the limit state of local buckling does not apply.

**Critical Stresses**

The available critical stresses may be interpolated from AISC *Manual* Table 4-14 or calculated directly as follows.

Calculate the elastic critical buckling stress, \( F_{cr} \), according to AISC *Specification* Section E3. As noted in AISC *Specification* Commentary Section E4, torsional buckling of symmetric shapes is a failure mode usually not considered in the design of hot-rolled columns. This failure mode generally does not govern unless the section is manufactured from relatively thin plates or a torsional unbraced length significantly larger than the \( y \)-\( y \) axis flexural unbraced length is present.

\[
F_e = \frac{\pi^2 E}{(L_e/r)^2}
\]

\[
= \frac{\pi^2 \left(29,000 \text{ ksi}\right)}{(58.6)^2}
\]

\[
= 83.3 \text{ ksi}
\]

Calculate the flexural buckling stress, \( F_{cr} \).

\[
4.71 \sqrt{\frac{E}{F_y}} = 4.71 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}}
\]

\[
= 113
\]
Because $\frac{L_c}{r} = 58.6 < 113$,

$$F_{cr} = \left(0.658\frac{F_y}{F_c}\right)F_y$$

$$= \left(0.658\frac{50}{83.3}\right)(50)$$

$$= 38.9 \text{ ksi}$$

Nominal Compressive Strength

$$P_n = F_{cr}A_g$$

$$= (38.9 \text{ ksi})(26.5 \text{ in.}^2)$$

$$= 1,030 \text{ kips}$$

From AISC Specification Section E1, the available compressive strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_c = 0.90$</td>
<td>$\Omega_c = 1.67$</td>
</tr>
<tr>
<td>$\phi_cP_n = 0.90(1,030 \text{ kips})$</td>
<td>$P_n = \frac{1,030 \text{ kips}}{1.67}$</td>
</tr>
<tr>
<td>$= 927 \text{ kips} &lt; 840 \text{ kips}$</td>
<td>$= 617 \text{ kips} &lt; 560 \text{ kips}$ o.k.</td>
</tr>
</tbody>
</table>

Column Compressive Strength—ASTM A913 Grade 65

Width-to-Thickness Ratio

The width-to-thickness ratio of the flanges of the W14×90 is:

$$\frac{b_f}{2t_f} = 10.2$$

From AISC Specification Table B4.1a, Case 1, the limiting width-to-thickness ratio of the flanges is:

$$0.56\sqrt{\frac{E}{F_y}} = 0.56\sqrt{\frac{29,000}{65}}$$

$$= 11.8 > 10.2; \text{ therefore, the flanges are nonslender}$$

The width-to-thickness ratio of the web of the W14×90 is:

$$\frac{h}{t_w} = 25.9$$

From AISC Specification Table B4.1a, Case 5, the limiting width-to-thickness ratio of the web is:
Because the web and flanges are nonslender, the limit state of local buckling does not apply.

Critical Stress

\[ F_c = 83.3 \text{ ksi (calculated previously)} \]

Calculate the flexural buckling stress, \( F_{cr} \).

\[ 4.71 \sqrt{\frac{E}{F_y}} = 4.71 \sqrt{\frac{29,000 \text{ ksi}}{65 \text{ ksi}}} = 99.5 \]

Because \( \frac{L_e}{r} = 58.6 < 99.5 \),

\[ F_{cr} = \left( 0.658 \frac{F_y}{F_c} \right) F_y = \left( 0.658 \frac{83.3 \text{ ksi}}{65 \text{ ksi}} \right) (65 \text{ ksi}) = 46.9 \text{ ksi} \]

Nominal Compressive Strength

\[ P_n = F_{cr} A_g \]

\[ = (46.9 \text{ ksi}) (26.5 \text{ in.}^2) = 1,240 \text{ kips} \]

From AISC Specification Section E1, the available compressive strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_c )</td>
<td>0.90</td>
<td>( \Omega_c ) = 1.67</td>
</tr>
<tr>
<td>( \phi_c \cdot P_n ) = 0.90(1,240 kips)</td>
<td>1,120 kips &gt; 840 kips o.k.</td>
<td>[ \frac{P_n}{\Omega_c} = \frac{1,240 \text{ kips}}{1.67} = 743 \text{ kips} &gt; 560 \text{ kips} \text{ o.k.} ]</td>
</tr>
</tbody>
</table>
EXAMPLE E.4A W-SHAPE COMPRESSION MEMBER (MOMENT FRAME)

This example is primarily intended to illustrate the use of the alignment chart for sidesway uninhibited columns in conjunction with the effective length method.

**Given:**

The member sizes shown for the moment frame illustrated here (sidesway uninhibited in the plane of the frame) have been determined to be adequate for lateral loads. The material for both the column and the girders is ASTM A992. The loads shown at each level are the accumulated dead loads and live loads at that story. The column is fixed at the base about the $x$-$x$ axis of the column.

Determine if the column is adequate to support the gravity loads shown. Assume the column is continuously supported in the transverse direction (the $y$-$y$ axis of the column).

**Solution:**

From AISC Manual Table 2-4, the material properties are as follows:

- ASTM A992
  - $F_y = 50$ ksi
  - $F_u = 65$ ksi

From AISC Manual Table 1-1, the geometric properties are as follows:

- W18×50
  - $I_x = 800 \text{ in.}^4$

- W24×55
  - $I_x = 1,350 \text{ in.}^4$

- W14×82
  - $A_g = 24.0 \text{ in.}^2$
  - $I_x = 881 \text{ in.}^4$

**Column B-C**

From ASCE/SEI 7, Chapter 2, the required compressive strength for the column between the roof and floor is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_d = 1.2(41.5 \text{ kips}) + 1.6(125 \text{ kips})$</td>
<td>$P_d = 41.5 \text{ kips} + 125 \text{ kips}$</td>
</tr>
<tr>
<td>$= 250 \text{ kips}$</td>
<td>$= 167 \text{ kips}$</td>
</tr>
</tbody>
</table>
Effective Length Factor

Using the effective length method, the effective length factor is determined using AISC Specification Commentary Appendix 7, Section 7.2. As discussed there, column inelasticity should be addressed by incorporating the stiffness reduction parameter, \( \tau_b \). Determine \( G_{\text{top}} \) and \( G_{\text{bottom}} \) accounting for column inelasticity by replacing \( E_{\text{col}} L_{\text{col}} \) with \( \tau_b (E_{\text{col}} L_{\text{col}}) \). Calculate the stiffness reduction parameter, \( \tau_b \), for the column B-C using AISC Manual Table 4-13.

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_a = 250 ) kips</td>
<td>( P_a = 167 ) kips</td>
<td></td>
</tr>
<tr>
<td>( A_g = 24.0 ) in.(^2)</td>
<td>( A_g = 24.0 ) in.(^2)</td>
<td></td>
</tr>
<tr>
<td>( = 10.4 ) ksi</td>
<td>( = 6.96 ) ksi</td>
<td></td>
</tr>
<tr>
<td>( \tau_b = 1.00 )</td>
<td>( \tau_b = 1.00 )</td>
<td></td>
</tr>
</tbody>
</table>

Therefore, no reduction in stiffness for inelastic buckling will be required.

Determine \( G_{\text{top}} \) and \( G_{\text{bottom}} \).

\[
G_{\text{top}} = \tau_b \left[ \frac{\sum (EI/L)_{\text{col}}}{\sum (EI/L)_{g}} \right] = 1.00
\]

\[
\left[ \frac{(29,000 \text{ ksi})(881 \text{ in.}^4)}{14.0 \text{ ft}} \right] = 1.38
\]

\[
G_{\text{bottom}} = \tau_b \left[ \frac{\sum (EI/L)_{\text{col}}}{\sum (EI/L)_{g}} \right] = 1.00
\]

\[
\left[ \frac{2(29,000 \text{ ksi})(881 \text{ in.}^4)}{35.0 \text{ ft}} \right] = 1.63
\]

From the alignment chart, AISC Specification Commentary Figure C-A-7.2, \( K \) is slightly less than 1.5; therefore use \( K = 1.5 \). Because the column available strength tables are based on the \( L_c \) about the \( y \)-\( y \) axis, the equivalent effective column length of the upper segment for use in the table is:

\[
L_{\text{ex}} = (KL)_x
\]

\[
= 1.5(14 \text{ ft})
\]

\[
= 21.0 \text{ ft}
\]

From AISC Manual Table 4-1a, for a W14×82:
\[
\frac{r_x}{r_y} = 2.44
\]

\[
L_c = \frac{L_{cx}}{\left(\frac{r_x}{r_y}\right)} = \frac{21.0 \text{ ft}}{2.44} = 8.61 \text{ ft}
\]

Take the available strength of the W14×82 from AISC Manual Table 4-1a.

At \(L_c = 9 \text{ ft}\), the available strength in axial compression is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_k P_n = 940 \text{ kips} &gt; 250 \text{ kips}) o.k.</td>
<td>(\frac{P_n}{\Omega_c} = 626 \text{ kips} &gt; 167 \text{ kips}) o.k.</td>
</tr>
</tbody>
</table>

**Column A-B**

From Chapter 2 of ASCE/SEI 7, the required compressive strength for the column between the floor and the foundation is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_u = 1.2(100 \text{ kips}) + 1.6(300 \text{ kips})) = 600 kips</td>
<td>(P_u = 100 \text{ kips} + 300 \text{ kips}) = 400 kips</td>
</tr>
</tbody>
</table>

**Effective Length Factor**

Determine the stiffness reduction parameter, \(\tau_b\), for column A-B using AISC Manual Table 4-13.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{P_u}{A_g} = 600 \text{ kips} \quad 24.0 \text{ in.}^2) = 25.0 ksi</td>
<td>(\frac{P_u}{A_g} = 400 \text{ kips} \quad 24.0 \text{ in.}^2) = 16.7 ksi</td>
</tr>
<tr>
<td>(\tau_b = 1.00)</td>
<td>(\tau_b = 0.994)</td>
</tr>
</tbody>
</table>

Use \(\tau_b = 0.994\).
\[ G_{\text{top}} = \tau_b \left[ \frac{\Sigma (EI / L)_{\text{col}}}{\Sigma (EI / L)_{g}} \right] \]


\[ = 0.994 \left[ \frac{2 \left( 29,000 \text{ ksi} \right) \left( 881 \text{ in}^4 \right)}{14.0 \text{ ft}} \right] \]

\[ = \frac{2 \left( 29,000 \text{ ksi} \right) \left( 1,350 \text{ in}^4 \right)}{35.0 \text{ ft}} \]

\[ = 1.62 \]

\[ G_{\text{bottom}} = 1.0 \text{ (fixed), from AISC Specification Commentary Appendix 7, Section 7.2} \]

From the alignment chart, AISC Specification Commentary Figure C-A-7.2, \( K \) is approximately 1.4. Because the column available strength tables are based on \( L_c \) about the \( y-y \) axis, the effective column length of the lower segment for use in the table is:

\[ L_{cx} = (KL)_x \]

\[ = 1.4(14 \text{ ft}) \]

\[ = 19.6 \text{ ft} \]

\[ L_c = \frac{L_{cx}}{r_x/r_y} \]

\[ = \frac{19.6}{2.44} \]

\[ = 8.03 \text{ ft} \]

Take the available strength of the W14×82 from AISC Manual Table 4-1a.

At \( L_c = 9 \text{ ft} \), (conservative) the available strength in axial compression is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi P_n = 940 \text{ kips} &gt; 600 \text{ kips} ) o.k.</td>
<td>( P_n/\Omega_c = 626 \text{ kips} &gt; 400 \text{ kips} ) o.k.</td>
</tr>
</tbody>
</table>

A more accurate strength could be determined by interpolation from AISC Manual Table 4-1a.
EXAMPLE E.4B  W-SHAPE COMPRESSION MEMBER (MOMENT FRAME)

Given:

Using the effective length method, determine the available strength of the column shown subject to the same gravity loads shown in Example E.4A with the column pinned at the base about the \(x\)-\(x\) axis. All other assumptions remain the same.

Solution:

As determined in Example E.4A, for the column segment B-C between the roof and the floor, the column strength is adequate.

As determined in Example E.4A, for the column segment A-B between the floor and the foundation,

\[ G_{\text{top}} = 1.62 \]

At the base,

\[ G_{\text{bottom}} = 10 \text{ (pinned) from AISC Specification Commentary Appendix 7, Section 7.2} \]

Note: this is the only change in the analysis.

From the alignment chart, AISC Specification Commentary Figure C-A-7.2, \(K\) is approximately equal to 2.0. Because the column available strength tables are based on the effective length, \(L_{ce}\), about the \(y\)-\(y\) axis, the effective column length of the segment A-B for use in the table is:

\[ L_{ce} = (KL)_{e} \]
\[ = 2.0(14 \text{ ft}) \]
\[ = 28.0 \text{ ft} \]

From AISC Manual Table 4-1a, for a W14×82:

\[ \frac{r_x}{r_y} = 2.44 \]
$$L_c = \frac{L_{cs}}{\left(\frac{r_o}{r_y}\right)}$$

$$= \frac{28.0 \text{ ft}}{2.44}$$

$$= 11.5 \text{ ft}$$

Interpolate the available strength of the W14×82 from AISC Manual Table 4-1a.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi \frac{P_n}{\Omega_c} = 861 \text{ kips} &gt; 600 \text{ kips}$ o.k.</td>
<td>$\frac{P_n}{\Omega_c} = 573 \text{ kips} &gt; 400 \text{ kips}$ o.k.</td>
</tr>
</tbody>
</table>
EXAMPLE E.7  WT COMPRESSION MEMBER WITHOUT SLENDER ELEMENTS

Given:

Select an ASTM A992 nonslender WT-shape compression member with a length of 20 ft to support a dead load of 20 kips and live load of 60 kips in axial compression. The ends are pinned. The solution will be provided using:

1. AISC Manual Tables
2. Calculations using AISC Specification provisions

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A992

\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

From ASCE/SEI 7, Chapter 2, the required compressive strength is:

\[
\begin{align*}
P_u &= 1.2(20 \text{ kips}) + 1.6(60 \text{ kips}) \\
&= 120 \text{ kips}
\end{align*}
\]
\[
\begin{align*}
P_u &= 20 \text{ kips} + 60 \text{ kips} \\
&= 80.0 \text{ kips}
\end{align*}
\]

(1) AISC Manual Table Solution

From AISC Specification Commentary Table C-A-7.1, for a pinned-pinned condition, \( K = 1.0 \). Therefore, \( L_{cx} = L_{cy} = KL = 1.0(20 \text{ ft}) = 20.0 \text{ ft} \).

Select the lightest nonslender member from AISC Manual Table 4-7 with sufficient available strength about both the \( x-x \) axis (upper portion of the table) and the \( y-y \) axis (lower portion of the table) to support the required strength.

Try a WT7\times34.

The available strength in compression is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_c P_{nx} = 128 \text{ kips} &gt; 120 \text{ kips} ) <strong>o.k. controls</strong></td>
<td>( \frac{P_{nx}}{\Omega_c} = 85.5 \text{ kips} &gt; 80.0 \text{ kips} ) <strong>o.k. controls</strong></td>
</tr>
<tr>
<td>( \phi_c P_{ny} = 222 \text{ kips} &gt; 120 \text{ kips} ) <strong>o.k.</strong></td>
<td>( \frac{P_{ny}}{\Omega_c} = 147 \text{ kips} &gt; 80.0 \text{ kips} ) <strong>o.k.</strong></td>
</tr>
</tbody>
</table>

Available strength can also be determined by hand calculations, as demonstrated in the following.


From AISC Manual Table 1-8, the geometric properties are as follows.

WT7\times34

\( A_g = 10.0 \text{ in.}^2 \)
\( r_x = 1.81 \text{ in.} \)
\( r_y = 2.46 \text{ in.} \)
\( J = 1.50 \text{ in.}^4 \)
\( \bar{y} = 1.29 \text{ in.} \)
\( I_x = 32.6 \text{ in.}^4 \)
\( I_y = 60.7 \text{ in.}^4 \)
\( d = 7.02 \text{ in.} \)
\( t_w = 0.415 \text{ in.} \)
\( b_y = 10.0 \text{ in.} \)
\( t_f = 0.720 \text{ in.} \)

**Stem Slenderness Check**

\[
\lambda = \frac{d}{t_w} = \frac{7.02 \text{ in.}}{0.415 \text{ in.}} = 16.9
\]

Determine the stem limiting slenderness ratio, \( \lambda_r \), from AISC Specification Table B4.1a, Case 4:

\[
\lambda_r = 0.75 \sqrt{\frac{E}{F_y}} = 0.75 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} = 18.1
\]

\( \lambda < \lambda_r \); therefore, the stem is not slender

**Flange Slenderness Check**

\[
\lambda = \frac{b_f}{2t_f} = \frac{10.0 \text{ in.}}{2(0.720 \text{ in.})} = 6.94
\]

Determine the flange limiting slenderness ratio, \( \lambda_r \), from AISC Specification Table B4.1a, Case 1:

\[
\lambda_r = 0.56 \sqrt{\frac{E}{F_y}} = 0.56 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} = 13.5
\]

\( \lambda < \lambda_r \); therefore, the flange is not slender

There are no slender elements.

For compression members without slender elements, AISC Specification Sections E3 and E4 apply. The nominal compressive strength, \( P_n \), is determined based on the limit states of flexural, torsional and flexural-torsional buckling.
Elastic Flexural Buckling Stress about the x-x Axis

\[ \frac{L_{ex}}{r_x} = \frac{(20.0 \text{ ft})(12 \text{ in./ft})}{1.81 \text{ in.}} = 133 \]

\[ F_{ex} = \frac{\pi^2 E}{\left(\frac{L_{ex}}{r_x}\right)^2} \]

\[ = \frac{\pi^2 (29,000 \text{ ksi})}{(133)^2} \]

\[ = 16.2 \text{ ksi} \quad \text{controls} \]

Elastic Flexural Buckling Stress about the y-y Axis

\[ \frac{L_{ey}}{r_y} = \frac{(20.0 \text{ ft})(12 \text{ in./ft})}{2.46 \text{ in.}} = 97.6 \]

\[ F_{ey} = \frac{\pi^2 E}{\left(\frac{L_{ey}}{r_y}\right)^2} \]

\[ = \frac{\pi^2 (29,000 \text{ ksi})}{(97.6)^2} \]

\[ = 30.0 \text{ ksi} \]

Torsional and Flexural-Torsional Elastic Buckling Stress

Because the WT7×34 section does not have any slender elements, AISC Specification Section E4 will be applicable for torsional and flexural-torsional buckling. \( F_t \) will be calculated using AISC Specification Equation E4-3. Per the User Note for AISC Specification Section E4, the term with \( C_w \) is omitted when computing \( F_{ez} \), and \( x_o \) is taken as zero. The flexural buckling term about the y-y axis, \( F_{ey} \), was computed in the preceding section.

\[ x_o = 0 \]

\[ y_o = \frac{y_f - t_f}{2} = 1.29 \text{ in.} - \frac{0.720 \text{ in.}}{2} = 0.930 \text{ in.} \]
\[
\tau_0^2 = x_o^2 + y_o^2 + \frac{I_x + I_y}{A_g} \\
= 0 + (0.930 \text{ in.})^2 + \frac{32.6 \text{ in.}^4 + 60.7 \text{ in.}^4}{10.0 \text{ in.}^2} \\
= 10.2 \text{ in.}^2
\]  
(Spec. Eq. E4-9)

\[
F_{ez} = \left(\frac{\pi^2 E C_w}{L_{ez}^2} + G J\right) \frac{1}{A_g \tau_0^2} \\
= \left[0 + (11,200 \text{ ksi}) (1.50 \text{ in.}^4)\right] \frac{1}{(10.0 \text{ in.}^2)(10.2 \text{ in.}^2)} \\
= 165 \text{ ksi}
\]  
(Spec. Eq. E4-7)

\[
H = 1 - \frac{x_o^2 + y_o^2}{\tau_0^2} \\
= 1 - \frac{0 + (0.930 \text{ in.})^2}{10.2 \text{ in.}^2} \\
= 0.915
\]  
(Spec. Eq. E4-8)

\[
F_e = \left(\frac{F_{ey} + F_{ez}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}}\right] \\
= \left[\frac{30.0 \text{ ksi} + 165 \text{ ksi}}{2(0.915)}\right]\left[1 - \sqrt{1 - \frac{4(30.0 \text{ ksi})(165 \text{ ksi})(0.915)}{(30.0 \text{ ksi} + 165 \text{ ksi})^2}}\right] \\
= 29.5 \text{ ksi}
\]  
(Spec. Eq. E4-3)

Critical Buckling Stress

The critical buckling stress for the member could be controlled by flexural buckling about either the x-x axis or y-y axis, \(F_{ex}\) or \(F_{ey}\), respectively. Note that AISC Specification Equations E4-5 and E4-6 reflect the same buckling modes as calculated in AISC Specification Equation E3-4. Or, the critical buckling stress for the member could be controlled by torsional or flexural-torsional buckling calculated per AISC Specification Equation E4-3. In this example, \(F_e\) calculated in accordance with AISC Specification Equation E4-5 is less than that calculated in accordance with AISC Specification Equation E4-3 or E4-6 and controls. Therefore:

\[
F_e = 16.2 \text{ ksi}
\]

\[
\frac{F_y}{F_e} = \frac{50 \text{ ksi}}{16.2 \text{ ksi}} = 3.09
\]

Per the AISC Specification User Note for Section E3, the two inequalities for calculating limits of applicability of Sections E3(a) and E3(b) provide the same result for flexural buckling only. When the elastic buckling stress, \(F_e\), is controlled by torsional or flexural-torsional buckling, the \(L_e/r\) limits would not be applicable unless an equivalent \(L_e/r\) ratio is first calculated by substituting the governing \(F_e\) into AISC Specification Equation E3-4 and solving for \(L_e/r\). The \(F_y/F_e\) limits may be used regardless of which buckling mode governs.
Because $\frac{F_y}{F_e} > 2.25$:

$$F_{cr} = 0.877F_e$$  \hspace{1cm} (Spec. Eq. E3-3)

$$= 0.877(16.2 \text{ ksi})$$

$$= 14.2 \text{ ksi}$$

**Available Compressive Strength**

$$P_n = F_{cr}A_g$$  \hspace{1cm} (Spec. Eq. E3-1)

$$= (14.2 \text{ ksi})(10.0 \text{ in.}^2)$$

$$= 142 \text{ kips}$$

From AISC Specification Section E1, the available compressive strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_c = 0.90$</td>
<td>$\Omega_c = 1.67$</td>
</tr>
<tr>
<td>$\phi_c P_n = 0.90(142 \text{ kips})$</td>
<td>$\frac{P_n}{\Omega_c} = \frac{142 \text{ kips}}{1.67}$</td>
</tr>
<tr>
<td>$= 128 \text{ kips} &gt; 120 \text{ kips} \text{ o.k.}$</td>
<td>$= 85.0 \text{ kips} &gt; 80.0 \text{ kips} \text{ o.k.}$</td>
</tr>
</tbody>
</table>
Chapter F
Design of Members for Flexure

INTRODUCTION

This Specification chapter contains provisions for calculating the flexural strength of members subject to simple bending about one principal axis. Included are specific provisions for I-shaped members, channels, HSS, box sections, tees, double angles, single angles, rectangular bars, rounds and unsymmetrical shapes. Also included is a section with proportioning requirements for beams and girders.

There are selection tables in the AISC Manual for standard beams in the commonly available yield strengths. The section property tables for most cross sections provide information that can be used to conveniently identify noncompact and slender element sections. LRFD and ASD information is presented side-by-side.

Most of the formulas from this chapter are illustrated by the following examples. The design and selection techniques illustrated in the examples for both LRFD and ASD will result in similar designs.

F1. GENERAL PROVISIONS

Selection and evaluation of all members is based on deflection requirements and strength, which is determined as the design flexural strength, $\phi_b M_n$, or the allowable flexural strength, $M_n / \Omega_b$,

where

$M_n$ = the lowest nominal flexural strength based on the limit states of yielding, lateral torsional-buckling, and local buckling, where applicable

$\phi_b = 0.90$ (LRFD)

$\Omega_b = 1.67$ (ASD)

This design approach is followed in all examples.

The term $L_b$ is used throughout this chapter to describe the length between points which are either braced against lateral displacement of the compression flange or braced against twist of the cross section. Requirements for bracing systems and the required strength and stiffness at brace points are given in AISC Specification Appendix 6.

The use of $C_b$ is illustrated in several of the following examples. AISC Manual Table 3-1 provides tabulated $C_b$ values for some common situations.

F2. DOUBLY SYMMETRIC COMPACT I-SHAPED MEMBERS AND CHANNELS BENT ABOUT THEIR MAJOR AXIS

AISC Specification Section F2 applies to the design of compact beams and channels. As indicated in the User Note in Section F2 of the AISC Specification, the vast majority of rolled I-shaped beams and channels fall into this category. The curve presented as a solid line in Figure F-1 is a generic plot of the nominal flexural strength, $M_n$, as a function of the unbraced length, $L_b$. The horizontal segment of the curve at the far left, between $L_b = 0$ ft and $L_{p0}$, is the range where the strength is limited by flexural yielding. In this region, the nominal strength is taken as the full plastic moment strength of the section as given by AISC Specification Equation F2-1. In the range of the curve at the far right, starting at $L_{p0}$, the strength is limited by elastic buckling. The strength in this region is given by AISC Specification Equation F2-3. Between these regions, within the linear region of the curve between $M_n = M_p$ at $L_{p0}$ on the left, and $M_n = 0.7M_p = 0.7FyS_x$ at $L_r$ on the right, the strength is limited by inelastic buckling. The strength in this region is provided in AISC Specification Equation F2-2.

The curve plotted as a heavy solid line represents the case where $C_b = 1.0$, while the heavy dashed line represents the case where $C_b$ exceeds 1.0. The nominal strengths calculated in both AISC Specification Equations F2-2 and F2-3 are linearly proportional to $C_b$, but are limited to $M_p$ as shown in the figure.
Fig. F-1. Nominal flexural strength versus unbraced length.

\[ M_n = M_p = F_y Z_x \]  
\[ M_n = C_b \left[ M_p \left( 1 + \frac{0.7 F_y S_x}{L_p} \right) \right] \leq M_p \]  
\[ M_n = F_{cr} S_x \leq M_p \]

where

\[ F_{cr} = C_b \pi^2 E \left[ \frac{L_b}{r_{fs}} \right]^2 \right] \left[ 1 + 0.078 \frac{J_c}{S_x b_o} \left( \frac{L_b}{r_{fs}} \right)^2 \right] \]

The provisions of this section are illustrated in Example F.1 (W-shape beam) and Example F.2 (channel).

Inelastic design provisions are given in AISC Specification Appendix 1. \( L_{pds} \), the maximum unbraced length for prismatic member segments containing plastic hinges is less than \( L_p \).

F3. **DOUBLY SYMMETRIC I-SHAPED MEMBERS WITH COMPACT WEBS AND NONCOMPACT OR SLENDER FLANGES BENT ABOUT THEIR MAJOR AXIS**

The strength of shapes designed according to this section is limited by local buckling of the compression flange. Only a few standard wide-flange shapes have noncompact flanges. For these sections, the strength reduction for \( F_y = 50 \) ksi steel varies. The approximate percentages of \( M_p \) about the strong axis that can be developed by noncompact members when braced such that \( L_b \leq L_p \) are shown as follows:

- W21\times48 = 99%
- W14\times99 = 99%
- W14\times90 = 97%
- W12\times65 = 98%
- W10\times12 = 99%
- W8\times31 = 99%
- W8\times10 = 99%
- W6\times15 = 94%
- W6\times8.5 = 97%

The strength curve for the flange local buckling limit state, shown in Figure F-2, is similar in nature to that of the lateral-torsional buckling curve. The horizontal axis parameter is \( \lambda = b_f / 2 t_f \). The flat portion of the curve to the left of \( \lambda_{pf} \) is the plastic yielding strength, \( M_p \). The curved portion to the right of \( \lambda_{cf} \) is the strength limited by elastic
buckling of the flange. The linear transition between these two regions is the strength limited by inelastic flange buckling.

Fig. F-2. Flange local buckling strength.

\[ M_n = M_p = F_y Z_x \quad \text{(Spec. Eq. F2-1)} \]

\[ M_n = M_p - (M_p - 0.7F_yS_x) \left( \frac{\lambda - \lambda_{pf}}{\lambda_{ref} - \lambda_{pf}} \right) \quad \text{(Spec. Eq. F3-1)} \]

\[ M_n = \frac{0.9E_k c_x S_x}{\lambda^2} \quad \text{(Spec. Eq. F3-2)} \]

where

\[ k_c = \frac{4}{\sqrt{h/t_w}} \] and shall not be taken less than 0.35 nor greater than 0.76 for calculation purposes.

The strength reductions due to flange local buckling of the few standard rolled shapes with noncompact flanges are incorporated into the design tables in Part 3 and Part 6 of the AISC Manual.

There are no standard I-shaped members with slender flanges. The noncompact flange provisions of this section are illustrated in Example F.3.

F4. OTHER I-SHAPED MEMBERS WITH COMPACT OR NONCOMPACT WEBS BENT ABOUT THEIR MAJOR AXIS

This section of the AISC Specification applies to doubly symmetric I-shaped members with noncompact webs and singly symmetric I-shaped members (those having different flanges) with compact or noncompact webs.

F5. DOUBLY SYMMETRIC AND SINGLY SYMMETRIC I-SHAPED MEMBERS WITH SLENDER WEBS BENT ABOUT THEIR MAJOR AXIS

This section applies to doubly symmetric and singly symmetric I-shaped members with slender webs, formerly designated as “plate girders”.

\[ \text{AMERICAN INSTITUTE OF STEEL CONSTRUCTION} \]
F6. I-SHAPED MEMBERS AND CHANNELS BENT ABOUT THEIR MINOR AXIS

I-shaped members and channels bent about their minor axis are not subject to lateral-torsional buckling. Rolled or built-up shapes with noncompact or slender flanges, as determined by AISC Specification Table B4.1b, must be checked for strength based on the limit state of flange local buckling using Equations F6-2 or F6-3 as applicable.

The vast majority of W, M, C and MC shapes have compact flanges, and can therefore develop the full plastic moment, $M_p$, about the minor axis. The provisions of this section are illustrated in Example F.5.

F7. SQUARE AND RECTANGULAR HSS AND BOX SECTIONS

Square and rectangular HSS need to be checked for the limit states of yielding, and flange and web local buckling. Lateral-torsional buckling is also possible for rectangular HSS or box sections bent about the strong axis; however, as indicated in the User Note in AISC Specification Section F7, deflection will usually control the design before there is a significant reduction in flexural strength due to lateral-torsional buckling.

The design and section property tables in the AISC Manual were calculated using a design wall thickness of 93% of the nominal wall thickness (see AISC Specification Section B4.2). Strength reductions due to local buckling have been accounted for in the AISC Manual design tables. The selection of a square HSS with compact flanges is illustrated in Example F.6. The provisions for a rectangular HSS with noncompact flanges is illustrated in Example F.7. The provisions for a square HSS with slender flanges are illustrated in Example F.8. Available flexural strengths of rectangular and square HSS are listed in Tables 3-12 and 3-13, respectively. If HSS members are specified using ASTM A1065 or ASTM A1085 material, the design wall thickness may be taken equal to the nominal wall thickness.

F8. ROUND HSS

The definition of HSS encompasses both tube and pipe products. The lateral-torsional buckling limit state does not apply, but round HSS are subject to strength reductions from local buckling. Available strengths of round HSS and Pipes are listed in AISC Manual Tables 3-14 and 3-15, respectively. The tabulated properties and available flexural strengths of these shapes in the AISC Manual are calculated using a design wall thickness of 93% of the nominal wall thickness. The design of a Pipe is illustrated in Example F.9. If round HSS members are specified using ASTM A1085 material, the design wall thickness may be taken equal to the nominal wall thickness.

F9. TEES AND DOUBLE ANGLES LOADED IN THE PLANE OF SYMMETRY

The AISC Specification provides a check for flange local buckling, which applies only when a noncompact or slender flange is in compression due to flexure. This limit state will seldom govern. A check for local buckling of the tee stem in flexural compression was added in the 2010 edition of the Specification. The provisions were expanded to include local buckling of double-angle web legs in flexural compression in the 2016 edition. Attention should be given to end conditions of tees to avoid inadvertent fixed end moments that induce compression in the web unless this limit state is checked. The design of a WT-shape in bending is illustrated in Example F.10.

F10. SINGLE ANGLES

Section F10 of the AISC Specification permits the flexural design of single angles using either the principal axes or geometric axes ($x$- and $y$-axes). When designing single angles without continuous bracing using the geometric axis design provisions, $M_y$ must be multiplied by 0.80 for use in Equations F10-1, F10-2 and F10-3. The design of a single angle in bending is illustrated in Example F.11.

F11. RECTANGULAR BARS AND ROUNDS

The AISC Manual does not include design tables for these shapes. The local buckling limit state does not apply to any bars. With the exception of rectangular bars bent about the strong axis, solid square, rectangular and round bars are not subject to lateral-torsional buckling and are governed by the yielding limit state only. Rectangular bars bent
about the strong axis are subject to lateral-torsional buckling and are checked for this limit state with Equations F11-2 and F11-3, as applicable.

These provisions can be used to check plates and webs of tees in connections. A design example of a rectangular bar in bending is illustrated in Example F.12. A design example of a round bar in bending is illustrated in Example F.13.

F12. UNSYMMETRICAL SHAPES

Due to the wide range of possible unsymmetrical cross sections, specific lateral-torsional and local buckling provisions are not provided in this Specification section. A general template is provided, but appropriate literature investigation and engineering judgment are required for the application of this section. A design example of a Z-shaped section in bending is illustrated in Example F.14.

F13. PROPORTIONS OF BEAMS AND GIRDERS

This section of the Specification includes a limit state check for tensile rupture due to holes in the tension flange of beams, proportioning limits for I-shaped members, detail requirements for cover plates and connection requirements for built-up beams connected side-to-side. Also included are unbraced length requirements for beams designed using the moment redistribution provisions of AISC Specification Section B3.3.
EXAMPLE F.1-1A W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR AXIS BENDING, CONTINUOUSLY BRACED

Given:

Select a W-shape beam for span and uniform dead and live loads as shown in Figure F.1-1A. Limit the member to a maximum nominal depth of 18 in. Limit the live load deflection to \( L/360 \). The beam is simply supported and continuously braced. The beam is ASTM A992 material.

![Beam loading and bracing diagram](image)

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

<table>
<thead>
<tr>
<th>Material</th>
<th>( F_y )</th>
<th>( F_u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASTM A992</td>
<td>50 ksi</td>
<td>65 ksi</td>
</tr>
</tbody>
</table>

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

\[
\begin{align*}
\text{LRFD} & : \quad w_u = 1.2 (0.45 \text{ kip/ft}) + 1.6 (0.75 \text{ kip/ft}) \\
& = 1.74 \text{ kip/ft} \\
\text{ASD} & : \quad w_u = 0.45 \text{ kip/ft} + 0.75 \text{ kip/ft} \\
& = 1.20 \text{ kip/ft}
\end{align*}
\]

From AISC Manual Table 3-23, Case 1:

\[
\begin{align*}
M_u & = \frac{w_u L^2}{8} \\
& = \frac{(1.74 \text{ kip/ft})(35 \text{ ft})^2}{8} \\
& = 266 \text{ kip-ft} \\
M_d & = \frac{w_u L^2}{8} \\
& = \frac{(1.20 \text{ kip/ft})(35 \text{ ft})^2}{8} \\
& = 184 \text{ kip-ft}
\end{align*}
\]

\[\text{Required Moment of Inertia for Live-Load Deflection Criterion of } L/360\]

\[
\Delta_{\text{max}} = \frac{L}{360} \\
= \frac{(35 \text{ ft})(12 \text{ in./ft})}{360} \\
= 1.17 \text{ in.}
\]
$$I_{x\,(reqd)} = \frac{5w_{L}L^4}{384E\Delta_{\max}}$$

(from AISC Manual Table 3-23, Case 1)

$$= \frac{5(0.75 \text{ kip}/\text{ft})(35 \text{ ft})^4 (12 \text{ in.}/\text{ft})^3}{384(29,000 \text{ ksi})(1.17 \text{ in.})}$$

$$= 746 \text{ in.}^4$$

**Beam Selection**

Select a W18×50 from AISC Manual Table 3-3.

$$I_x = 800 \text{ in.}^4 > 746 \text{ in.}^4 \quad \text{o.k.}$$

Per the User Note in AISC Specification Section F2, the section is compact. Because the beam is continuously braced and compact, only the yielding limit state applies.

From AISC Manual Table 3-2, the available flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_b M_u = \phi_b M_{px}$</td>
<td>$M_u = M_{px}$</td>
</tr>
<tr>
<td>$= 379 \text{ kip-ft} &gt; 266 \text{ kip-ft} \quad \text{o.k.}$</td>
<td>$\Omega_b = \Omega_b$</td>
</tr>
<tr>
<td>$= 379 \text{ kip-ft} &gt; 266 \text{ kip-ft} \quad \text{o.k.}$</td>
<td>$= 252 \text{ kip-ft} &gt; 184 \text{ kip-ft} \quad \text{o.k.}$</td>
</tr>
</tbody>
</table>
EXAMPLE F.1-1B  W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR AXIS BENDING, CONTINUOUSLY BRACED

Given:

Verify the available flexural strength of the ASTM A992 W18×50 beam selected in Example F.1-1A by directly applying the requirements of the AISC Specification.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

- ASTM A992
  - $F_y = 50$ ksi
  - $F_u = 65$ ksi

From AISC Manual Table 1-1, the geometric properties are as follows:

- W18×50
  - $Z_x = 101$ in.³

The required flexural strength from Example F.1-1A is:

- LRFD
  - $M_u = 266$ kip-ft
- ASD
  - $M_a = 184$ kip-ft

Nominal Flexural Strength

Per the User Note in AISC Specification Section F2, the section is compact. Because the beam is continuously braced and compact, only the yielding limit state applies.

$$M_n = M_p = F_y Z_x$$

$$= (50 \text{ ksi})(101 \text{ in.}³)$$

$$= 5,050 \text{ kip-in. or 421 kip-ft}$$

Available Flexural Strength

From AISC Specification Section F1, the available flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_b M_n = 0.90(421 \text{ kip-ft})$</td>
<td>$\Omega_b = 1.67$</td>
</tr>
<tr>
<td>$= 379 \text{ kip-ft} &gt; 266 \text{ kip-ft}$</td>
<td>$M_n = 252 \text{ kip-ft}$</td>
</tr>
<tr>
<td></td>
<td>$\Omega_b = 1.67$</td>
</tr>
<tr>
<td></td>
<td>$= 252 \text{ kip-ft} &gt; 184 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>
EXAMPLE F.1-2A  W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR AXIS BENDING, BRACED AT THIRD POINTS

Given:

Use the AISC Manual tables to verify the available flexural strength of the W18×50 beam size selected in Example F.1-1A for span and uniform dead and live loads as shown in Figure F.1-2A. The beam is simply supported and braced at the ends and third points. The beam is ASTM A992 material.

Solution:

The required flexural strength at midspan from Example F.1-1A is:

\[ M_u = 266 \text{ kip-ft} \]
\[ M_u = 184 \text{ kip-ft} \]

Unbraced Length

\[ L_b = \frac{35 \text{ ft}}{3} = 11.7 \text{ ft} \]

By inspection, the middle segment will govern. From AISC Manual Table 3-1, for a uniformly loaded beam braced at the ends and third points, \( C_b = 1.01 \) in the middle segment. Conservatively neglect this small adjustment in this case.

Available Flexural Strength

Enter AISC Manual Table 3-10 and find the intersection of the curve for the W18×50 with an unbraced length of 11.7 ft. Obtain the available strength from the appropriate vertical scale to the left.

From AISC Manual Table 3-10, the available flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b M_n \approx 302 \text{ kip-ft} &gt; 266 \text{ kip-ft} )</td>
<td>o.k.</td>
</tr>
<tr>
<td>( M_n \approx 201 \text{ kip-ft} &gt; 184 \text{ kip-ft} )</td>
<td>o.k.</td>
</tr>
</tbody>
</table>
EXAMPLE F.1-2B  W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR AXIS BENDING, BRACED AT THIRD POINTS

Given:

Verify the available flexural strength of the W18×50 beam selected in Example F.1-1A with the beam braced at the ends and third points by directly applying the requirements of the AISC Specification. The beam is ASTM A992 material.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A992  
\[ F_y = 50 \text{ ksi} \]  
\[ F_u = 65 \text{ ksi} \]

From AISC Manual Table 1-1, the geometric properties are as follows:

\[ \text{W18} \times 50 \]  
\[ r_y = 1.65 \text{ in.} \]  
\[ S_x = 88.9 \text{ in.}^3 \]  
\[ J = 1.24 \text{ in.}^4 \]  
\[ r_{yt} = 1.98 \text{ in.} \]  
\[ h_y = 17.4 \text{ in.} \]

The required flexural strength from Example F.1-1A is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ M_u ] = 266 kip-ft</td>
<td>[ M_a ] = 184 kip-ft</td>
<td></td>
</tr>
</tbody>
</table>

Nominal Flexural Strength

Calculate \( C_b \). For the lateral-torsional buckling limit state, the nonuniform moment modification factor can be calculated using AISC Specification Equation F1-1. For the center segment of the beam, the required moments for AISC Specification Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: \( M_{\text{max}} = 1.00 \), \( M_A = 0.972 \), \( M_B = 1.00 \), and \( M_C = 0.972 \).

\[
C_b = \frac{12.5 M_{\text{max}}}{2.5 M_{\text{max}} + 3 M_A + 4 M_B + 3 M_C} \quad (\text{Spec. Eq. F1-1})
\]

\[
= \frac{12.5(1.00)}{2.5(1.00) + 3(0.972) + 4(1.00) + 3(0.972)}
\]

\[
= 1.01
\]

For the end-span beam segments, the required moments for AISC Specification Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: \( M_{\text{max}} = 0.889 \), \( M_A = 0.306 \), \( M_B = 0.556 \), and \( M_C = 0.750 \).

\[
C_b = \frac{12.5 M_{\text{max}}}{2.5 M_{\text{max}} + 3 M_A + 4 M_B + 3 M_C} \quad (\text{Spec. Eq. F1-1})
\]

\[
= \frac{12.5(0.889)}{2.5(0.889) + 3(0.306) + 4(0.556) + 3(0.750)}
\]

\[
= 1.46
\]
Thus, the center span, with the higher required strength and lower \( C_v \), will govern.

The limiting laterally unbraced length for the limit state of yielding is:

\[
L_p = 1.76 r_s \sqrt{\frac{E}{F_y}} \\
= 1.76(1.65 \text{ in.}) \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} \\
= 69.9 \text{ in. or 5.83 ft}
\]

The limiting unbraced length for the limit state of inelastic lateral-torsional buckling, with \( c = 1 \) from AISC Specification Equation F2-8a for doubly symmetric I-shaped members, is:

\[
L_r = 1.95 r_s \frac{E}{0.7 F_y} \left( \frac{J_c}{S_{x,h_o}} + \sqrt{\frac{J_c}{S_{y,h_o}}} \right) + 6.76 \left( \frac{0.7 F_y}{E} \right)^2 \\
= 1.95(1.98 \text{ in.}) \left[ \frac{29,000 \text{ ksi}}{0.7(50 \text{ ksi})} \right] \left[ \frac{(1.24 \text{ in.}^4)(1.0)}{(88.9 \text{ in.}^3)(17.4 \text{ in.})} + \frac{(1.24 \text{ in.}^4)(1.0)}{(88.9 \text{ in.}^3)(17.4 \text{ in.})} \right]^2 + 6.76 \left[ \frac{0.7(50 \text{ ksi})}{29,000 \text{ ksi}} \right]^2 \\
= 203 \text{ in. or 16.9 ft}
\]

For a compact beam with an unbraced length of \( L_p < L_b \leq L_r \), the lesser of either the flexural yielding limit state or the inelastic lateral-torsional buckling limit state controls the nominal strength.

\[
M_p = 5,050 \text{ kip-in. (from Example F.1-1B)}
\]

\[
M_n = C_b \left[ M_p - (M_p - 0.7 F_y S_c) \left( \frac{L_b}{L_r} - \frac{L_p}{L_r} \right) \right] \leq M_p \\
= 1.01 \left[ 5,050 \text{ kip-in.} - 5,050 \text{ kip-in.} - 0.7(50 \text{ ksi})(88.9 \text{ in.}^3)(11.7 \text{ ft} - 5.83 \text{ ft}) \right] \leq 5,050 \text{ kip-in.} \\
= 4,060 \text{ kip-in.} < 5,050 \text{ kip-in.} \\
= 4,060 \text{ kip-in.} \text{ or 339 kip-ft}
\]

Available Flexural Strength

From AISC Specification Section F1, the available flexural strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b )</td>
<td>0.90</td>
<td>( \Omega_b ) = 1.67</td>
</tr>
<tr>
<td>( \phi_b M_n )</td>
<td>0.90(339 kip-ft)</td>
<td>( M_n ) = 339 kip-ft</td>
</tr>
<tr>
<td></td>
<td>= 305 kip-ft &gt; 266 kip-ft o.k.</td>
<td>( \Omega_b \frac{M_n}{\phi_b} ) = 1.67</td>
</tr>
<tr>
<td></td>
<td></td>
<td>= 203 kip-ft &gt; 184 kip-ft o.k.</td>
</tr>
</tbody>
</table>
EXAMPLE F.1-3A  W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR AXIS BENDING, BRACED AT MIDSPAN

Given:

Use the AISC Manual tables to verify the available flexural strength of the W18×50 beam size selected in Example F.1-1A for span and uniform dead and live loads as shown in Figure F.1-3A. The beam is simply supported and braced at the ends and midpoint. The beam is ASTM A992 material.

![Beam loading and bracing diagram](image)

Solution:

The required flexural strength at midspan from Example F.1-1A is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_u$</td>
<td>266 kip-ft</td>
<td>184 kip-ft</td>
</tr>
</tbody>
</table>

Unbraced Length

$$L_b = \frac{35 \text{ ft}}{2} = 17.5 \text{ ft}$$

From AISC Manual Table 3-1, for a uniformly loaded beam braced at the ends and at the center point, $C_b = 1.30$. There are several ways to make adjustments to AISC Manual Table 3-10 to account for $C_b$ greater than 1.0.

Procedure A

Available moments from the sloped and curved portions of the plots from AISC Manual Table 3-10 may be multiplied by $C_b$, but may not exceed the value of the horizontal portion ($\phi M_p$ for LRFD, $M_p/\Omega$ for ASD).

Obtain the available strength of a W18×50 with an unbraced length of 17.5 ft from AISC Manual Table 3-10.

Enter AISC Manual Table 3-10 and find the intersection of the curve for the W18×50 with an unbraced length of 17.5 ft. Obtain the available strength from the appropriate vertical scale to the left.

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_b M_p$</td>
<td>$\geq 222$</td>
<td>$\leq 379$</td>
</tr>
<tr>
<td>$\phi_b M_p$</td>
<td>$\approx 379$</td>
<td>upper limit on $C_b \phi_b M_u$</td>
</tr>
</tbody>
</table>

From AISC Manual Table 3-2:

$$\frac{M_p}{\Omega} = 252 \text{ kip-ft}$$ upper limit on $C_b \frac{M_p}{\Omega}$

$M_p/\Omega$
<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Adjust for</strong>  ( C_b ).</td>
<td><strong>Adjust for</strong>  ( C_b ).</td>
</tr>
<tr>
<td>( 1.30(222 \text{ kip-ft}) = 289 \text{ kip-ft} )</td>
<td>( 1.30(148 \text{ kip-ft}) = 192 \text{ kip-ft} )</td>
</tr>
<tr>
<td>Check limit.</td>
<td>Check limit.</td>
</tr>
<tr>
<td>( 289 \text{ kip-ft} &lt; \phi_b M_p = 379 \text{ kip-ft} \quad \text{o.k.} )</td>
<td>( 192 \text{ kip-ft} &lt; \frac{M_p}{\Omega_b} = 252 \text{ kip-ft} \quad \text{o.k.} )</td>
</tr>
<tr>
<td>Check available versus required strength.</td>
<td>Check available versus required strength.</td>
</tr>
<tr>
<td>( 289 \text{ kip-ft} &gt; 266 \text{ kip-ft} \quad \text{o.k.} )</td>
<td>( 192 \text{ kip-ft} &gt; 184 \text{ kip-ft} \quad \text{o.k.} )</td>
</tr>
</tbody>
</table>

**Procedure B**

For preliminary selection, the required strength can be divided by \( C_b \) and directly compared to the strengths in AISC Manual Table 3-10. Members selected in this way must be checked to ensure that the required strength does not exceed the available plastic moment strength of the section.

Calculate the adjusted required strength.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ M'_u = \frac{266 \text{ kip-ft}}{1.30} = 205 \text{ kip-ft} ]</td>
<td>[ M'_u = \frac{184 \text{ kip-ft}}{1.30} = 142 \text{ kip-ft} ]</td>
</tr>
</tbody>
</table>

Obtain the available strength for a W18×50 with an unbraced length of 17.5 ft from AISC Manual Table 3-10.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b M_u \approx 222 \text{ kip-ft} &gt; 205 \text{ kip-ft} \quad \text{o.k.} )</td>
<td>( \frac{M_n}{\Omega_b} \approx 148 \text{ kip-ft} &gt; 142 \text{ kip-ft} \quad \text{o.k.} )</td>
</tr>
<tr>
<td>( \phi_b M_p = 379 \text{ kip-ft} &gt; 266 \text{ kip-ft} \quad \text{o.k.} )</td>
<td>( \frac{M_p}{\Omega_b} = 252 \text{ kip-ft} &gt; 184 \text{ kip-ft} \quad \text{o.k.} )</td>
</tr>
</tbody>
</table>
EXAMPLE F.1-3B  W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR-AXIS BENDING, BRACED AT MIDSPAN

Given:

Verify the available flexural strength of the W18×50 beam selected in Example F.1-1A with the beam braced at the ends and center point by directly applying the requirements of the AISC Specification. The beam is ASTM A992 material.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A992

\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]

From AISC Manual Table 1-1, the geometric properties are as follows:

\[ W18\times50 \]

\[ r_s = 1.98 \text{ in.} \]
\[ S_x = 88.9 \text{ in.}^3 \]
\[ J = 1.24 \text{ in.}^4 \]
\[ h_o = 17.4 \text{ in.} \]

The required flexural strength from Example F.1-1A is:

\[ M_u = 266 \text{ kip-ft} \]
\[ M_a = 184 \text{ kip-ft} \]

Nominal Flexural Strength

Calculate \( C_b \). The required moments for AISC Specification Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: \( M_{\text{max}} = 1.00, M_A = 0.438, M_B = 0.750, \) and \( M_C = 0.938 \).

\[
C_b = \frac{12.5M_{\text{max}}}{2.5M_{\text{max}} + 3M_A + 4M_B + 3M_C} \quad (\text{Spec. Eq. F1-1})
\]

\[
= \frac{12.5(1.00)}{2.5(1.00) + 3(0.438) + 4(0.750) + 3(0.938)}
\]

\[
= 1.30
\]

From AISC Manual Table 3-2:

\[ L_p = 5.83 \text{ ft} \]
\[ L_r = 16.9 \text{ ft} \]

From Example F.1-3A:

\[ L_c = 17.5 \text{ ft} \]

For a compact beam with an unbraced length \( L_b > L_r \), the limit state of elastic lateral-torsional buckling applies.
Calculate $F_{cr}$, where $c = 1.0$ for doubly symmetric I-shapes.

$$F_{cr} = \frac{C_b\pi^2E}{L_y} \left(1 + \frac{0.078}{S_b} \left(\frac{L_b}{r_t}\right)^2\right) \left[\frac{(17.5 \text{ ft})(12 \text{ in./ft})}{1.98 \text{ in.}}\right]$$

$$= 1.30\pi^2 \left(\frac{29,000 \text{ ksi}}{19.1 \text{ in.}}\right) \left[1 + \frac{0.078}{\frac{1.24 \text{ in.}^4}{88.9 \text{ in.}^3}} \left(\frac{17.4 \text{ in.}}{1.98 \text{ in.}}\right)^2\right]$$

$$= 1.24 \text{ ksi}$$

$M_p = 5,050 \text{ kip-in.} \ (\text{from Example F.1-1B})$

$$M_n = F_{cr}S_x \leq M_p$$

$$= (43.2 \text{ ksi}) \left(88.9 \text{ in.}^3\right) \leq 5,050 \text{ kip-in.}$$

$$= 3,840 \text{ kip-in.} < 5,050 \text{ kip-in.}$$

$$= 3,840 \text{ kip-in. or 320 kip-ft}$$

**Available Flexural Strength**

From AISC Specification Section F1, the available flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_b = 0.90$</td>
<td>$\Omega_b = 1.67$</td>
</tr>
<tr>
<td>$\phi_bM_n = 0.90(320 \text{ kip-ft}) = 288 \text{ kip-ft} &gt; 266 \text{ kip-ft} \ \text{o.k.}$</td>
<td>$M_n = 320 \text{ kip-ft} \ \frac{\Omega_b}{1.67} = 192 \text{ kip-ft} &gt; 184 \text{ kip-ft} \ \text{o.k.}$</td>
</tr>
</tbody>
</table>
EXAMPLE F.3A  W-SHAPE FLEXURAL MEMBER WITH NONCOMPACT FLANGES IN MAJOR AXIS BENDING

Given:

Using the AISC Manual tables, select a W-shape beam for span, uniform dead load, and concentrated live loads as shown in Figure F.3A. The beam is simply supported and continuously braced. Also calculate the deflection. The beam is ASTM A992 material.

![Figure F.3A. Beam loading and bracing diagram.](image)

Note: A beam with noncompact flanges will be selected to demonstrate that the tabulated values of the AISC Manual account for flange compactness.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

- ASTM A992
  - \( F_y = 50 \text{ ksi} \)
  - \( F_u = 65 \text{ ksi} \)

From Chapter 2 of ASCE/SEI 7, the required flexural strength at midspan is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_u = 1.2 \times (0.05 \text{ kip/ft}) )</td>
<td>( w_u = 0.05 \text{ kip/ft} )</td>
</tr>
<tr>
<td>= 0.0600 kip/ft</td>
<td></td>
</tr>
<tr>
<td>( P_a = 1.6 \times (18 \text{ kips}) )</td>
<td>( P_a = 18 \text{ kips} )</td>
</tr>
<tr>
<td>= 28.8 kips</td>
<td></td>
</tr>
</tbody>
</table>

From AISC Manual Table 3-23, Cases 1 and 9:

\[
M_u = \frac{w_u L^2}{8} + P_a a
\]

\[
= \left( \frac{0.0600 \text{ kip/ft}}{8} \right) \left( 40 \text{ ft} \right)^2 + \left( 28.8 \text{ kips} \right) \left( \frac{40 \text{ ft}}{3} \right)
\]

\[
= 396 \text{ kip-ft}
\]

From AISC Manual Table 3-23, Cases 1 and 9:

\[
M_a = \frac{w_a L^2}{8} + P_a a
\]

\[
= \left( \frac{0.05 \text{ kip/ft}}{8} \right) \left( 40 \text{ ft} \right)^2 + \left( 18 \text{ kips} \right) \left( \frac{40 \text{ ft}}{3} \right)
\]

\[
= 250 \text{ kip-ft}
\]

**Beam Selection**

For a continuously braced W-shape, the available flexural strength equals the available plastic flexural strength.
Select the lightest section providing the required strength from the bold entries in AISC Manual Table 3-2.

Try a W21×48.

This beam has a noncompact compression flange at $F_y = 50$ ksi as indicated by footnote “f” in AISC Manual Table 3-2. This shape is also footnoted in AISC Manual Table 1-1.

From AISC Manual Table 3-2, the available flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_b M_a = \phi_b M_{px}$</td>
<td>$M_a = \frac{M_{px}}{\Omega_b}$</td>
</tr>
<tr>
<td>$398 \text{ kip-ft} &gt; 396 \text{ kip-ft}$</td>
<td>$265 \text{ kip-ft} &gt; 250 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

Note: The value $M_{px}$ in AISC Manual Table 3-2 includes the strength reductions due to the shape being noncompact.

**Deflection**

From AISC Manual Table 1-1:

$I_x = 959 \text{ in}^4$

The maximum deflection occurs at the center of the beam.

$$\Delta_{max} = \frac{5wL^4}{384EI} + \frac{23P_LL^3}{648EI}$$

(AISC Manual Table 3-23, Cases 1 and 9)

$$= \frac{5(0.05 \text{ kip/ft})(40 \text{ ft})^4(12 \text{ in./ft})^3}{384(29,000 \text{ ksi})(959 \text{ in.}^4)} + \frac{23(18 \text{ kips})(40 \text{ ft})^3(12 \text{ in./ft})^3}{648(29,000 \text{ ksi})(959 \text{ in.}^4)}$$

$$= 2.64 \text{ in.}$$

This deflection can be compared with the appropriate deflection limit for the application. Deflection will often be more critical than strength in beam design.
EXAMPLE F.3B  W-SHAPE FLEXURAL MEMBER WITH NONCOMPACT FLANGES IN MAJOR AXIS BENDING

Given:

Verify the results from Example F.3A by directly applying the requirements of the AISC Specification.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A992
\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]

From AISC Manual Table 1-1, the geometric properties are as follows:

\[ W21 \times 48 \]
\[ S_x = 93.0 \text{ in.}^3 \]
\[ Z_x = 107 \text{ in.}^3 \]
\[ \frac{b_f}{2t_f} = 9.47 \]

The required flexural strength from Example F.3A is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_u )</td>
<td>396 kip-ft</td>
<td>250 kip-ft</td>
</tr>
</tbody>
</table>

Flange Slenderness

\[ \lambda = \frac{b_f}{2t_f} \]
\[ = 9.47 \]

The limiting width-to-thickness ratios for the compression flange are:

\[ \lambda_{pf} = 0.38 \sqrt{\frac{E}{F_y}} \quad (\text{Spec. Table B4.1b, Case 10}) \]
\[ = 0.38 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} \]
\[ = 9.15 \]

\[ \lambda_{cf} = 1.0 \sqrt{\frac{E}{F_y}} \quad (\text{Spec. Table B4.1b, Case 10}) \]
\[ = 1.0 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} \]
\[ = 24.1 \]

\( \lambda_{pf} < \lambda < \lambda_{cf} \), therefore, the compression flange is noncompact. This could also be determined from the footnote “f” in AISC Manual Table 1-1.
Nominal Flexural Strength

Because the beam is continuously braced, and therefore not subject to lateral-torsional buckling, the available strength is based on the limit state of compression flange local buckling. From AISC Specification Section F3.2:

\[ M_p = F_y Z_x \]
\[ = (50 \text{ ksi}) (107 \text{ in}^2) \]
\[ = 5,350 \text{ kip-in. or 446 kip-ft} \]

\[ M_n = \left[ M_p - (M_p - 0.7F_yS_x) \frac{\lambda - \lambda_{pf}}{\lambda_{pf} - \lambda_{pf}} \right] \]
\[ = \left[ 5,350 \text{ kip-in.} - 5,350 \text{ kip-in.} - 0.7(50 \text{ ksi})(93.0 \text{ in}^3) \left( \frac{9.47 - 9.15}{24.1 - 9.15} \right) \right] \]
\[ = 5,310 \text{ kip-in. or 442 kip-ft} \]

Available Flexural Strength

From AISC Specification Section F1, the available flexural strength is:

\[ \phi_h \sigma_{ho} M_n \]

\[ \phi_h M_n = 0.90(442 \text{ kip-ft}) = 398 \text{ kip-ft} > 396 \text{ kip-ft} \text{ o.k.} \]

\[ \Omega_h = 1.67 \]

Note that these available strengths are identical to the tabulated values in AISC Manual Table 3-2, as shown in Example F.3A, which account for the noncompact flange.
EXAMPLE F.4 W-SHAPE FLEXURAL MEMBER, SELECTION BY MOMENT OF INERTIA FOR MAJOR AXIS BENDING

Given:

Using the AISC Manual tables, select a W-shape using the moment of inertia required to limit the live load deflection to 1.00 in. for span and uniform dead and live loads as shown in Figure F.4. The beam is simply supported and continuously braced. The beam is ASTM A992 material.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A992

\( F_y = 50 \text{ ksi} \)

\( F_u = 65 \text{ ksi} \)

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

\[
\left( F_{LRFD} \right) = 1.2 (0.8 \text{ kip/ft}) + 1.6 (2 \text{ kip/ft}) = 4.16 \text{ kip/ft}
\]

\[
\left( F_{ASD} \right) = 0.8 \text{ kip/ft} + 2 \text{ kip/ft} = 2.80 \text{ kip/ft}
\]

From AISC Manual Table 3-23, Case 1:

\[
M_u = \frac{w_L L^2}{8} = (4.16 \text{ kip/ft})(30 \text{ ft})^2 \left( \frac{1}{8} \right) = 468 \text{ kip-ft}
\]

\[
M_u = \frac{w_L L^2}{8} = \left( 2.80 \text{ kip/ft} \right)(30 \text{ ft})^2 \left( \frac{1}{8} \right) = 315 \text{ kip-ft}
\]

Minimum Required Moment of Inertia

The maximum live load deflection, \( \Delta_{\text{max}} \), occurs at midspan and is calculated as:

\[
\Delta_{\text{max}} = \frac{5w_L L^4}{384EI}
\]

(AISC Manual Table 3-23, Case 1)

Rearranging and substituting \( \Delta_{\text{max}} = 1.00 \text{ in.} \)
$$I_{min} = \frac{5wL^4}{384E\Delta_{max}}$$

$$= \frac{5(2 \text{ kip/ft})(30 \text{ ft})^4(12 \text{ in./ft})^3}{384(29,000 \text{ ksi})(1.00 \text{ in.})}$$

$$= 1,260 \text{ in.}^4$$

**Beam Selection**

Select the lightest section with the required moment of inertia from the bold entries in AISC *Manual* Table 3-3.

Try a W24×55.

$$I_x = 1,350 \text{ in.}^4 > 1,260 \text{ in.}^4 \quad \text{o.k.}$$

Because the W24×55 is continuously braced and compact, its strength is governed by the yielding limit state and AISC *Specification* Section F2.1.

From AISC *Manual* Table 3-2, the available flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_bM_a = \phi_bM_{px}$</td>
<td>$M_a = M_{px}$</td>
</tr>
<tr>
<td>$= 503 \text{ kip-ft} &gt; 468 \text{ kip-ft}$</td>
<td>$\Omega_b = \Omega_b$</td>
</tr>
<tr>
<td></td>
<td>$= 334 \text{ kip-ft} &gt; 315 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>
EXAMPLE F.5    I-SHAPED FLEXURAL MEMBER IN MINOR AXIS BENDING

Given:

Using the AISC Manual tables, select a W-shape beam loaded on its minor axis for span and uniform dead and live loads as shown in Figure F.5. Limit the live load deflection to $L/240$. The beam is simply supported and braced only at the ends. The beam is ASTM A992 material.

![Fig. F.5. Beam loading and bracing diagram.](image)

Note: Although not a common design case, this example is being used to illustrate AISC Specification Section F6 (I-shaped members and channels bent about their minor axis).

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

<table>
<thead>
<tr>
<th>ASTM A992</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_y = 50$ ksi</td>
</tr>
<tr>
<td>$F_u = 65$ ksi</td>
</tr>
</tbody>
</table>

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

$$w_u = 1.2(0.667 \text{ kip/ft}) + 1.6(2 \text{ kip/ft}) = 4.00 \text{ kip/ft}$$

From AISC Manual Table 3-23, Case 1:

$$M_u = \frac{w_u L^2}{8} = \frac{(4.00 \text{ kip/ft})(15 \text{ ft})^2}{8} = 113 \text{ kip-ft}$$

Minimum Required Moment of Inertia

The maximum live load deflection permitted is:

$$\Delta_{max} = \frac{L}{240} = \frac{(15 \text{ ft})(12 \text{ in./ft})}{240} = 0.750 \text{ in.}$$
Try a W12×58.

From AISC Manual Table 1-1, the geometric properties are as follows:

\[
\begin{align*}
W12\times58 \\
S_y &= 21.4 \text{ in.}^3 \\
Z_y &= 32.5 \text{ in.}^3 \\
I_y &= 107 \text{ in.}^4 > 105 \text{ in.}^4 \quad \text{o.k. (for deflection requirement)}
\end{align*}
\]

Nominal Flexural Strength

AISC Specification Section F6 applies. Because the W12×58 has compact flanges per the User Note in this Section, the yielding limit state governs the design.

\[
M_n = M_p = F_y Z_y \leq 1.6 F_y S_y \quad \text{(Spec. Eq. F6-1)}
\]

\[
= (50 \text{ ksi})(32.5 \text{ in.}^3) \leq 1.6(50 \text{ ksi})(21.4 \text{ in.}^3)
\]

\[
= 1,630 \text{ kip-in.} < 1,710 \text{ kip-in.}
\]

\[
= 1,630 \text{ kip-in. or 136 kip-ft}
\]

Available Flexural Strength

From AISC Specification Section F1, the available flexural strength is:

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
\phi_b M_n = 0.90(136 \text{ kip-ft}) & \Omega_b = 1.67 \\
\hline
\phi_b M_n = 0.90(136 \text{ kip-ft}) = 122 \text{ kip-ft} > 113 \text{ kip-ft} \quad \text{o.k.} & M_n = 136 \text{ kip-ft} \\
\hline
\end{array}
\]
CHAPTER F DESIGN EXAMPLE REFERENCES


Chapter G
Design of Members for Shear

INTRODUCTION

This Specification chapter addresses webs of singly or doubly symmetric members subject to shear in the plane of the web, single angles and HSS subject to shear, and shear in the weak direction of singly or doubly symmetric shapes.

G1. GENERAL PROVISIONS

The design shear strength, $\phi V_n$, and the allowable shear strength, $V_n/\Omega_v$, are determined as follows:

$$V_n = \text{nominal shear strength based on shear yielding or shear buckling}$$

$$\phi_v = 0.90 \text{ (LRFD)}$$

$$\Omega_v = 1.67 \text{ (ASD)}$$

Exception: For all current ASTM A6, W, S and HP shapes except $W44 \times 230$, $W40 \times 149$, $W36 \times 135$, $W33 \times 118$, $W30 \times 90$, $W24 \times 55$, $W16 \times 26$ and $W12 \times 14$ for $F_y = 50 \text{ ksi}$:

$$\phi_v = 1.00 \text{ (LRFD)}$$

$$\Omega_v = 1.50 \text{ (ASD)}$$

Strong axis shear values are tabulated for W-shapes in AISC Manual Tables 3-2, 3-6 and 6-2, for S-shapes in AISC Manual Table 3-7, for C-shapes in AISC Manual Table 3-8, and for MC-shapes in AISC Manual Table 3-9. Strong axis shear values are tabulated for rectangular HSS, round HSS and pipe in Part IV. Weak axis shear values for W-shapes, S-shapes, C-shapes and MC-shapes, and shear values for angles, rectangular HSS and box members are not tabulated.

G2. I-SHAPED MEMBERS AND CHANNELS

This section includes provisions for shear strength of webs without the use of tension field action and for interior web panels considering tension field action. Provisions for the design of transverse stiffeners are also included in Section G2.

As indicated in the User Note of this section, virtually all W, S and HP shapes are not subject to shear buckling and are also eligible for the more liberal safety and resistance factors, $\phi_v = 1.00$ (LRFD) and $\Omega_v = 1.50$ (ASD). This is presented in Example G.1 for a W-shape. A channel shear strength design is presented in Example G.2. A built-up girder with a thin web and transverse stiffeners is presented in Example G.8.

G3. SINGLE ANGLES AND TEES

A single angle example is illustrated in Example G.3.
G4. RECTANGULAR HSS, BOX SECTIONS, AND OTHER SINGLY AND DOUBLY SYMMETRIC MEMBERS

The shear height for HSS, \( h \), is taken as the clear distance between the flanges less the inside corner radius on each side. If the corner radii are unknown, \( h \) shall be taken as the corresponding outside dimension minus 3 times the design thickness. A rectangular HSS example is provided in Example G.4.

G5. ROUND HSS

For all round HSS of ordinary length listed in the AISC Manual, \( F_{cr} \) can be taken as \( 0.6 F_y \) in AISC Specification Equation G5-1. A round HSS example is illustrated in Example G.5.

G6. WEAK AXIS SHEAR IN DOUBLY SYMMETRIC AND SINGLY SYMMETRIC SHAPES

For examples of weak axis shear, see Example G.6 and Example G.7.

G7. BEAMS AND GIRDERS WITH WEB OPENINGS

For a beam and girder with web openings example, see AISC Design Guide 2, Design of Steel and Composite Beams with Web Openings (Darwin, 1990).
EXAMPLE G.1A  W-SHAPE IN STRONG AXIS SHEAR

Given:

Using AISC Manual tables, determine the available shear strength and adequacy of an ASTM A992 W24×62 with end shears of 48 kips from dead load and 145 kips from live load.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A992

\[ F_y = 50 \text{ ksi} \]

\[ F_u = 65 \text{ ksi} \]

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

\[
\begin{align*}
V_u &= 1.2(48 \text{ kips}) + 1.6(145 \text{ kips}) \\
    &= 290 \text{ kips}
\end{align*}
\]

\[
\begin{align*}
V_a &= 48 \text{ kips} + 145 \text{ kips} \\
    &= 193 \text{ kips}
\end{align*}
\]

From AISC Manual Table 3-2, the available shear strength is:

\[
\begin{align*}
\phi_a V_a &= 306 \text{ kips} > 290 \text{ kips} \quad \textbf{o.k.} \\
\frac{V_n}{\Omega_v} &= 204 \text{ kips} > 193 \text{ kips} \quad \textbf{o.k.}
\end{align*}
\]
EXAMPLE G.1B  W-SHAPE IN STRONG AXIS SHEAR

Given:

The available shear strength of the W-shape in Example G.1A was easily determined using tabulated values in the AISC Manual. This example demonstrates the calculation of the available strength by directly applying the provisions of the AISC Specification.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]

From AISC Manual Table 1-1, the geometric properties are as follows:

\[ W24 \times 62 \]
\[ d = 23.7 \text{ in.} \]
\[ t_w = 0.430 \text{ in.} \]

Nominal Shear Strength

Except for very few sections, which are listed in the User Note, AISC Specification Section G2.1(a) is applicable to the I-shaped beams published in the AISC Manual for \( F_y = 50 \text{ ksi} \). The W-shape sections that do not meet the criteria of AISC Specification Section G2.1(a) are indicated with footnote “v” in Tables 1-1, 3-2 and 6-2.

\[ C_{v1} = 1.0 \]  \hspace{1cm} (Spec. Eq. G2-2)

From AISC Specification Section G2.1, area of the web, \( A_w \), is determined as follows:

\[ A_w = d t_w \]
\[ = (23.7 \text{ in.})(0.430 \text{ in.}) \]
\[ = 10.2 \text{ in.}^2 \]

From AISC Specification Section G2.1, the nominal shear strength is:

\[ V_n = 0.6 F_y A_w C_{v1} \]  \hspace{1cm} (Spec. Eq. G2-1)
\[ = 0.6 (50 \text{ ksi})(10.2 \text{ in.}^2)(1.0) \]
\[ = 306 \text{ kips} \]

Available Shear Strength

From AISC Specification Section G2.1, the available shear strength is:

| \( \phi_v = 1.00 \) | \( \Omega_v = 1.50 \) |
| \( \phi_v V_n = 1.00(306 \text{ kips}) \) | \( \frac{V_n}{\Omega_v} = \frac{306 \text{ kips}}{1.50} \) |
| \( = 306 \text{ kips} \) | \( = 204 \text{ kips} \) |
EXAMPLE G.2A  CHANNEL IN STRONG AXIS SHEAR

Given:

Using AISC Manual tables, verify the available shear strength and adequacy of an ASTM A36 C15×33.9 channel with end shears of 17.5 kips from dead load and 52.5 kips from live load.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A36
\( F_y = 36 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

\[
V_r = 1.2(17.5 \text{ kips}) + 1.6(52.5 \text{ kips}) = 105 \text{ kips}
\]
\[
V_r = 17.5 \text{ kips} + 52.5 \text{ kips} = 70.0 \text{ kips}
\]

From AISC Manual Table 3-8, the available shear strength is:

\[
\phi V_n = 117 \text{ kips} > 105 \text{ kips} \quad \text{o.k.}
\]
\[
\frac{V_n}{\Omega_v} = 77.6 \text{ kips} > 70.0 \text{ kips} \quad \text{o.k.}
\]
EXAMPLE G.2B  CHANNEL IN STRONG AXIS SHEAR

Given:

The available shear strength of the channel in Example G.2A was easily determined using tabulated values in the AISC Manual. This example demonstrates the calculation of the available strength by directly applying the provisions of the AISC Specification.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

- ASTM A36
  - $F_y = 36$ ksi
  - $F_u = 58$ ksi

From AISC Manual Table 1-5, the geometric properties are as follows:

- C15x33.9
  - $d = 15.0$ in.
  - $t_w = 0.400$ in.

Nominal Shear Strength

All ASTM A36 channels listed in the AISC Manual have $h/t_w \leq 1.10 \sqrt{E / F_y}$; therefore,

$$C_{v1} = 1.0$$  \hfill (Spec. Eq. G2-3)

From AISC Specification Section G2.1, the area of the web, $A_w$, is determined as follows:

$$A_w = dt_w$$
$$= (15.0 \text{ in.})(0.400 \text{ in.})$$
$$= 6.00 \text{ in.}^2$$

From AISC Specification Section G2.1, the nominal shear strength is:

$$V_n = 0.6 F_y A_w C_{v1}$$  \hfill (Spec. Eq. G2-1)
$$= 0.6 (36 \text{ ksi})(6.00 \text{ in.}^2)(1.0)$$
$$= 130 \text{ kips}$$

Available Shear Strength

Because AISC Specification Section G2.1(a) does not apply for channels, the values of $\phi_v = 1.00$ (LRFD) and $\Omega_v = 1.50$ (ASD) may not be used. Instead $\phi_v = 0.90$ (LRFD) and $\Omega_v = 1.67$ (ASD) from AISC Specification Section G1(a) must be used.
<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_v = 0.90$</td>
<td>$\Omega_v = 1.67$</td>
</tr>
<tr>
<td>$\phi_v V_n = 0.90(130 \text{ kips})$</td>
<td>$V_n = \frac{130 \text{ kips}}{1.67}$</td>
</tr>
<tr>
<td>$= 117 \text{ kips}$</td>
<td>$= 77.8 \text{ kips}$</td>
</tr>
</tbody>
</table>
EXAMPLE G.6 DOUBLY SYMMETRIC SHAPE IN WEAK AXIS SHEAR

Given:

Verify the available shear strength and adequacy of an ASTM A992 W21×48 beam with end shears of 20.0 kips from dead load and 60.0 kips from live load in the weak direction.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A992

\( F_y = 50 \text{ ksi} \)

\( F_u = 65 \text{ ksi} \)

From AISC Manual Table 1-1, the geometric properties are as follows:

W21×48

\( b_f = 8.14 \text{ in.} \)

\( t_f = 0.430 \text{ in.} \)

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

\[
V_a = 1.2 \times 20.0 \text{ kips} + 1.6 \times 60.0 \text{ kips} = 120 \text{ kips}
\]

Nominal Shear Strength

From AISC Specification Section G6, for weak axis shear, use AISC Specification Equation G6-1.

Calculate \( C_{v2} \) using AISC Specification Section G2.2 with \( h/t_w = b_f / 2t_f \) and \( k_v = 1.2 \).

\[
\frac{h}{t_w} = \frac{b_f}{2t_f} = \frac{8.14 \text{ in.}}{2(0.430 \text{ in.})} = 9.47
\]

\[
1.10 \sqrt{\frac{k_v E}{F_y}} = 1.10 \sqrt{\frac{1.2(29,000 \text{ ksi})}{50 \text{ ksi}}} = 29.0 > 9.47
\]

Therefore, use AISC Specification Equation G2-9:

\( C_{v2} = 1.0 \)

Note: From the User Note in AISC Specification Section G6, \( C_{v2} = 1.0 \) for all ASTM A6 W-, S-, M- and HP-shapes when \( F_y \leq 70 \text{ ksi} \).

Calculate \( V_n \). (Multiply the flange area by two to account for both shear resisting elements.)
\[ V_n = 0.6F_v b f t_f C_v \chi \ (2) \]
\[ = 0.6(50 \text{ ksi})(8.14 \text{ in.})(0.430 \text{ in.})(1.0)(2) \]
\[ = 210 \text{ kips} \]

Available Shear Strength

From AISC Specification Section G1, the available shear strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \phi_v = 0.90 ]</td>
<td>[ \Omega_v = 1.67 ]</td>
</tr>
<tr>
<td>[ \phi_v V_n = 0.90(210 \text{ kips}) ]</td>
<td>[ \frac{V_n}{\Omega_v} = \frac{210 \text{ kips}}{1.67} ]</td>
</tr>
<tr>
<td>= 189 \text{ kips &gt; 120 kips o.k.}</td>
<td>= 126 \text{ kips &gt; 80.0 kips o.k.}</td>
</tr>
</tbody>
</table>
CHAPTER G DESIGN EXAMPLE REFERENCES

Chapter H
Design of Members for Combined Forces and Torsion

For all interaction equations in AISC Specification Chapter H, the required forces and moments must include second-order effects, as required by Chapter C of the AISC Specification. ASD users of the 1989 AISC Specification are accustomed to using an interaction equation that includes a partial second-order amplification. Second-order effects are now addressed in the analysis and are not included in these interaction equations.
EXAMPLE H.1A  W-SHAPE SUBJECT TO COMBINED COMPRESSION AND BENDING ABOUT BOTH AXES (BRACED FRAME)

Given:

Using Table IV-5 (located in this document), determine if an ASTM A992 W14 x 99 has sufficient available strength to support the axial forces and moments listed as follows, obtained from a second-order analysis that includes \( P-\delta \) effects. The unbraced length is 14 ft and the member has pinned ends.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_a = 400 \text{ kips} )</td>
<td>( P_a = 267 \text{ kips} )</td>
</tr>
<tr>
<td>( M_{ax} = 250 \text{ kip-ft} )</td>
<td>( M_{ax} = 167 \text{ kip-ft} )</td>
</tr>
<tr>
<td>( M_{av} = 80.0 \text{ kip-ft} )</td>
<td>( M_{av} = 53.3 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

The effective length of the member is:

\[
L_{cx} = L_{cy} = KL = 1.0(14 \text{ ft}) = 14.0 \text{ ft}
\]

For \( L_c = 14 \text{ ft} \), the combined strength parameters from Table IV-5 are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p = \frac{0.887}{10^3} \text{ kips} )</td>
<td>( p = \frac{1.33}{10^3} \text{ kips} )</td>
</tr>
<tr>
<td>( b_x = \frac{1.38}{10^3} \text{ kip-ft} )</td>
<td>( b_x = \frac{2.08}{10^3} \text{ kip-ft} )</td>
</tr>
<tr>
<td>( b_y = \frac{2.85}{10^3} \text{ kip-ft} )</td>
<td>( b_y = \frac{4.29}{10^3} \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

Check \( P/P_c \) limit for AISC Specification Equation H1-1a.

\[
\frac{P_u}{\phi_c P_c} = \frac{p P_u}{P_a} = \left( \frac{0.887}{10^3} \text{ kips} \right) (400 \text{ kips}) = 0.355
\]

Check \( P/P_c \) limit for AISC Specification Equation H1-1a.

\[
\frac{P_a}{P_a/\Omega_c} = \frac{p P_a}{P_a} = \left( \frac{1.33}{10^3} \text{ kips} \right) (267 \text{ kips}) = 0.355
\]
Because $p_{Pu} \geq 0.2$,

$$p_{Pu} + b_x M_{ax} + b_y M_{ay} \leq 1.0 \quad \text{(from Part IV, Eq. IV-8)}$$

$$= 0.355 + \left(\frac{1.38}{10^{3} \text{ kip-ft}}\right)(250 \text{ kip-ft})$$  
$$+ \left(\frac{2.85}{10^{3} \text{ kip-ft}}\right)(80.0 \text{ kip-ft}) \leq 1.0$$  
$$= 0.928 < 1.0 \quad \text{o.k.}$$

Because $p_{Pu} \geq 0.2$,

$$p_{Pu} + b_x M_{ax} + b_y M_{ay} \leq 1.0 \quad \text{(from Part IV, Eq. IV-8)}$$

$$= 0.355 + \left(\frac{2.08}{10^{3} \text{ kip-ft}}\right)(167 \text{ kip-ft})$$  
$$+ \left(\frac{4.29}{10^{3} \text{ kip-ft}}\right)(53.3 \text{ kip-ft}) \leq 1.0$$  
$$= 0.931 < 1.0 \quad \text{o.k.}$$

Table IV-5 simplifies the calculation of AISC Specification Equations H1-1a and H1-1b. A direct application of these equations is shown in Example H.1B.
EXAMPLE H.1B  W-SHAPE SUBJECT TO COMBINED COMPRESSION AND BENDING MOMENT ABOUT BOTH AXES (BRACED FRAME)

Given:

Using AISC Manual tables to determine the available compressive and flexural strengths, determine if an ASTM A992 W14×99 has sufficient available strength to support the axial forces and moments listed as follows, obtained from a second-order analysis that includes P-δ effects. The unbraced length is 14 ft and the member has pinned ends.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_a = 400$ kips</td>
<td>$P_a = 267$ kips</td>
</tr>
<tr>
<td>$M_{ax} = 250$ kip-ft</td>
<td>$M_{ax} = 167$ kip-ft</td>
</tr>
<tr>
<td>$M_{ay} = 80$ kip-ft</td>
<td>$M_{ay} = 53.3$ kip-ft</td>
</tr>
</tbody>
</table>

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

The effective length of the member is:

$$L_{cx} = L_{cy} = KL = 1.0(14 \text{ ft}) = 14.0 \text{ ft}$$

For $L_c = 14.0$ ft, the available axial and flexural strengths from AISC Manual Table 6-2 are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_c = \phi_c P_a$</td>
<td>$P_c = \frac{P_a}{\Omega_c}$</td>
</tr>
<tr>
<td>$= 1,130$ kips</td>
<td>$= 750$ kips</td>
</tr>
<tr>
<td>$M_{cx} = \phi_b M_{ax}$</td>
<td>$M_{cx} = \frac{M_{ax}}{\Omega_b}$</td>
</tr>
<tr>
<td>$= 642$ kip-ft</td>
<td>$= 427$ kip-ft</td>
</tr>
<tr>
<td>$M_{cy} = \phi_b M_{ay}$</td>
<td>$M_{cy} = \frac{M_{ay}}{\Omega_b}$</td>
</tr>
<tr>
<td>$= 311$ kip-ft</td>
<td>$= 207$ kip-ft</td>
</tr>
<tr>
<td>$\frac{P_a}{\phi_c P_a}$</td>
<td>$P_a / \Omega_c$</td>
</tr>
<tr>
<td>$= 0.354$</td>
<td>$= 0.356$</td>
</tr>
</tbody>
</table>
Because $\frac{P_u}{\phi_c P_n} \geq 0.2$, 

\[
\frac{P_c}{P_c} + \frac{8}{9} \left( \frac{M_{cx}}{M_{cx}} + \frac{M_{cy}}{M_{cy}} \right) \leq 1.0 \quad \text{(Spec. Eq. H1-1a)}
\]

\[
= \frac{400 \text{ kips}}{1,130 \text{ kips}} + \frac{8}{9} \left( \frac{250 \text{ kip-ft}}{642 \text{ kip-ft}} + \frac{80.0 \text{ kip-ft}}{311 \text{ kip-ft}} \right) \leq 1.0
\]

\[
= 0.928 < 1.0 \quad \text{o.k.}
\]

Because $\frac{P_u}{\phi_c P_n} \geq 0.2$, 

\[
\frac{P_c}{P_c} + \frac{8}{9} \left( \frac{M_{cx}}{M_{cx}} + \frac{M_{cy}}{M_{cy}} \right) \leq 1.0 \quad \text{(Spec. Eq. H1-1a)}
\]

\[
= \frac{267 \text{ kips}}{750 \text{ kips}} + \frac{8}{9} \left( \frac{167 \text{ kip-ft}}{427 \text{ kip-ft}} + \frac{53.3 \text{ kip-ft}}{207 \text{ kip-ft}} \right)
\]

\[
= 0.932 < 1.0 \quad \text{o.k.}
\]
EXAMPLE H.2  W-SHAPE SUBJECT TO COMBINED COMPRESSION AND BENDING MOMENT ABOUT BOTH AXES (BY AISC SPECIFICATION SECTION H2)

Given:

Using AISC Specification Section H2, determine if an ASTM A992 W14×99 has sufficient available strength to support the axial forces and moments listed as follows, obtained from a second-order analysis that includes P-δ effects. The unbraced length is 14 ft and the member has pinned ends. This example is included primarily to illustrate the use of AISC Specification Section H2.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_u = 360$ kips</td>
<td>$P_u = 240$ kips</td>
</tr>
<tr>
<td>$M_{ux} = 250$ kip-ft</td>
<td>$M_{ux} = 167$ kip-ft</td>
</tr>
<tr>
<td>$M_{uy} = 80$ kip-ft</td>
<td>$M_{uy} = 53.3$ kip-ft</td>
</tr>
</tbody>
</table>

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A992

$F_y = 50$ ksi

$F_u = 65$ ksi

From AISC Manual Table 1-1, the geometric properties are as follows:

W14×99

$A = 29.1$ in.$^2$

$S_x = 157$ in.$^3$

$S_y = 55.2$ in.$^3$

The required flexural and axial stresses are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{ra} = \frac{P_u}{A}$</td>
<td>$f_{ra} = \frac{P_u}{A}$</td>
</tr>
<tr>
<td>$= \frac{360}{29.1}$ kpsi</td>
<td>$= \frac{240}{29.1}$ kpsi</td>
</tr>
<tr>
<td>$= 12.4$ kpsi</td>
<td>$= 8.25$ kpsi</td>
</tr>
<tr>
<td>$f_{rbx} = \frac{M_{ux}}{S_x}$</td>
<td>$f_{rbx} = \frac{M_{ux}}{S_x}$</td>
</tr>
<tr>
<td>$= \frac{(250 \text{ kip-ft})(12 \text{ in./ft})}{157 \text{ in.}^3}$</td>
<td>$= \frac{(167 \text{ kip-ft})(12 \text{ in./ft})}{157 \text{ in.}^3}$</td>
</tr>
<tr>
<td>$= 19.1$ kpsi</td>
<td>$= 12.8$ kpsi</td>
</tr>
<tr>
<td>$f_{rby} = \frac{M_{uy}}{S_y}$</td>
<td>$f_{rby} = \frac{M_{uy}}{S_y}$</td>
</tr>
<tr>
<td>$= \frac{(80 \text{ kip-ft})(12 \text{ in./ft})}{55.2 \text{ in.}^3}$</td>
<td>$= \frac{(53.3 \text{ kip-ft})(12 \text{ in./ft})}{55.2 \text{ in.}^3}$</td>
</tr>
<tr>
<td>$= 17.4$ kpsi</td>
<td>$= 11.6$ kpsi</td>
</tr>
</tbody>
</table>
The effective length of the member is:

\[ L_{cx} = L_{cy} = KL = 1.0 \times (14 \text{ ft}) = 14.0 \text{ ft} \]

For \( L_c = 14.0 \text{ ft} \), calculate the available axial and flexural stresses using the available strengths from AISC Manual Table 6-2.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \[ F_{ca} = \phi_c F_{cx} \]  
\[ = \frac{\phi_c P_a}{A} = \frac{1,130 \text{ kips}}{29.1 \text{ in.}^2} = 38.8 \text{ ksi} \]  
\[ F_{cbx} = \frac{\phi_b M_{nx}}{S_x} = \frac{(642 \text{ kip-ft})(12 \text{ in./ft})}{157 \text{ in.}^3} = 49.1 \text{ ksi} \]  
\[ F_{cby} = \frac{\phi_b M_{ny}}{S_y} = \frac{(311 \text{ kip-ft})(12 \text{ in./ft})}{55.2 \text{ in.}^3} = 67.6 \text{ ksi} \]  
| \[ F_{ca} = \frac{F_{cx}}{\Omega_c} = \frac{P_a}{\Omega_c A} = \frac{750 \text{ kips}}{29.1 \text{ in.}^2} = 25.8 \text{ ksi} \]  
\[ F_{cbx} = \frac{M_{nx}}{\Omega_b S_x} = \frac{(427 \text{ kip-ft})(12 \text{ in./ft})}{157 \text{ in.}^3} = 32.6 \text{ ksi} \]  
\[ F_{cby} = \frac{M_{ny}}{\Omega_b S_y} = \frac{(207 \text{ kip-ft})(12 \text{ in./ft})}{55.2 \text{ in.}^3} = 45.0 \text{ ksi} \] |

As shown in the LRFD calculation of \( F_{cbx} \) in the preceding text, the available flexural stresses can exceed the yield stress in cases where the available strength is governed by yielding and the yielding strength is calculated using the plastic section modulus.

**Combined Stress Ratio**

From AISC Specification Section H2, check the combined stress ratios as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \[ \frac{f_a + f_{bhx} + f_{bby}}{F_{ca} + F_{cbx} + F_{cby}} \leq 1.0 \] (from Spec. Eq. H2-1)  
\[ \begin{array}{c} 
12.4 \text{ ksi} \\
38.8 \text{ ksi} 
\end{array} + 
\begin{array}{c} 
19.1 \text{ ksi} \\
49.1 \text{ ksi} 
\end{array} + 
\begin{array}{c} 
17.4 \text{ ksi} \\
67.6 \text{ ksi} 
\end{array} = 0.966 < 1.0 \text{ o.k.} \]  
| \[ \frac{f_a + f_{bhx} + f_{bby}}{F_{ca} + F_{cbx} + F_{cby}} \leq 1.0 \] (from Spec. Eq. H2-1)  
\[ \begin{array}{c} 
8.25 \text{ ksi} \\
25.8 \text{ ksi} 
\end{array} + 
\begin{array}{c} 
12.8 \text{ ksi} \\
32.6 \text{ ksi} 
\end{array} + 
\begin{array}{c} 
11.6 \text{ ksi} \\
45.0 \text{ ksi} 
\end{array} = 0.970 < 1.0 \text{ o.k.} \] |

A comparison of these results with those from Example H.1B shows that AISC Specification Equation H1-1a will produce less conservative results than AISC Specification Equation H2-1 when its use is permitted.
Note: This check is made at a point on the cross section (extreme fiber, in this example). The designer must therefore determine which point on the cross section is critical, or check multiple points if the critical point cannot be readily determined.
EXAMPLE H.3  W-SHAPE SUBJECT TO COMBINED AXIAL TENSION AND FLEXURE

Given:

Select an ASTM A992 W-shape with a 14-in.-nominal-depth to carry forces of 29 kips from dead load and 87 kips from live load in axial tension, as well as the following moments due to uniformly distributed loads:

\[ M_{xD} = 32 \text{ kip-ft} \]
\[ M_{xL} = 96 \text{ kip-ft} \]
\[ M_{yD} = 11.3 \text{ kip-ft} \]
\[ M_{yL} = 33.8 \text{ kip-ft} \]

The unbraced length is 30 ft and the ends are pinned. Assume the connections are made with no holes.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A992
\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]

From ASCE/SEI 7, Chapter 2, the required strengths are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_u = 1.2(29 \text{ kips}) + 1.6(87 \text{ kips}) )</td>
<td>( P_u = 29 \text{ kips} + 87 \text{ kips} )</td>
</tr>
<tr>
<td>= 174 kips</td>
<td>= 116 kips</td>
</tr>
<tr>
<td>( M_{ax} = 1.2(32 \text{ kip-ft}) + 1.6(96 \text{ kip-ft}) )</td>
<td>( M_{ax} = 32 \text{ kip-ft} + 96 \text{ kip-ft} )</td>
</tr>
<tr>
<td>= 192 kip-ft</td>
<td>= 128 kip-ft</td>
</tr>
<tr>
<td>( M_{ay} = 1.2(11.3 \text{ kip-ft}) + 1.6(33.8 \text{ kip-ft}) )</td>
<td>( M_{ay} = 11.3 \text{ kip-ft} + 33.8 \text{ kip-ft} )</td>
</tr>
<tr>
<td>= 67.6 kip-ft</td>
<td>= 45.1 kip-ft</td>
</tr>
</tbody>
</table>

Try a W14×82.

From AISC Manual Tables 1-1 and 3-2, the properties are as follows:

\[ W14\times82 \]
\[ A_g = 24.0 \text{ in.}^2 \]
\[ S_x = 123 \text{ in.}^3 \]
\[ Z_x = 139 \text{ in.}^3 \]
\[ S_y = 29.3 \text{ in.}^3 \]
\[ Z_y = 44.8 \text{ in.}^3 \]
\[ I_y = 148 \text{ in.}^4 \]
\[ L_p = 8.76 \text{ ft} \]
\[ L_p = 33.2 \text{ ft} \]

Nominal Tensile Strength

From AISC Specification Section D2(a), the nominal tensile strength due to tensile yielding in the gross section is:
Note that for a member with holes, the rupture strength of the member would also have to be computed using AISC Specification Equation D2-2.

**Nominal Flexural Strength for Bending About the Major Axis**

**Yielding**

From AISC Specification Section F2.1, the nominal flexural strength due to yielding (plastic moment) is:

\[
M_{nc} = M_p = F_y Z_x \\
= (50 \text{ ksi})(139 \text{ in.}^3) \\
= 6,950 \text{ kip-in.}
\]

**Lateral-Torsional Buckling**

From AISC Specification Section F2.2, the nominal flexural strength due to lateral-torsional buckling is determined as follows:

Because \( L_p < L_b \leq L_c \), i.e., 8.76 ft < 30 ft < 33.2 ft, AISC Specification Equation F2-2 applies.

**Lateral-Torsional Buckling Modification Factor, \( C_b \)**

From AISC Manual Table 3-1, \( C_b = 1.14 \), without considering the beneficial effects of the tension force. However, per AISC Specification Section H1.2, \( C_b \) may be modified because the column is in axial tension concurrently with flexure.

\[
P_{cy} = \frac{\pi^2 EI_x}{L_b^2} \\
= \frac{\pi^2 (29,000 \text{ ksi})(148 \text{ in.}^4)}{[(30 \text{ ft})(12.0 \text{ in./ft})]^2} \\
= 327 \text{ kips}
\]

<table>
<thead>
<tr>
<th>( P_{cy} )</th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1 + \frac{\alpha P_a}{P_{cy}} )</td>
<td>( 1 + \frac{1.0(174 \text{ kips})}{327 \text{ kips}} )</td>
<td>( 1 + \frac{1.6(116 \text{ kips})}{327 \text{ kips}} )</td>
</tr>
<tr>
<td>( = 1.24 )</td>
<td></td>
<td>( = 1.25 )</td>
</tr>
</tbody>
</table>

\( C_b = 1.24(1.14) \)

\( = 1.41 \)
\[ M_n = C_b \left[ M_p - \left( M_p - 0.7 F_y S_y \left( \frac{L_h - L_p}{L_w - L_p} \right) \right) \right] \leq M_p \]  
\[ = 1.41 \left[ 6,950 \text{ kip-in.} - \left( 6,950 \text{ kip-in.} - 0.7(50 \text{ ksi})(123 \text{ in}^3) \right) \right] \leq 6,950 \text{ kip-in.} \]
\[ = 6,560 \text{ kip-in. or 547 kip-ft} \text{ controls} \]

**Local Buckling**

Per AISC Manual Table 1-1, the cross section is compact at \( F_y = 50 \text{ ksi} \); therefore, the local buckling limit state does not apply.

**Nominal Flexural Strength for Bending About the Minor Axis and the Interaction of Flexure and Tension**

Because a W14×82 has compact flanges, only the limit state of yielding applies for bending about the minor axis.

\[ M_{ny} = M_p = F_y Z_y \leq 1.6 F_y S_y \]
\[ = (50 \text{ ksi})(44.8 \text{ in}^3) \leq 1.6(50 \text{ ksi})(29.3 \text{ in}^3) \]
\[ = 2,240 \text{ kip-in.} < 2,340 \text{ kip-in.} \]
\[ = 2,240 \text{ kip-in. or 187 kip-ft} \]

**Available Strength**

From AISC Specification Sections D2 and F1, the available strengths are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b = \phi_t = 0.90 )</td>
<td>( \Omega_b = \Omega_t = 1.67 )</td>
</tr>
<tr>
<td>( P_c = \phi_t P_n )</td>
<td>( P_c = \frac{P_n}{\Omega_t} )</td>
</tr>
<tr>
<td>( = 0.90(1,200 \text{ kips}) )</td>
<td>( = \frac{1,200 \text{ kips}}{1.67} )</td>
</tr>
<tr>
<td>( = 1,080 \text{ kips} )</td>
<td>( = 719 \text{ kips} )</td>
</tr>
<tr>
<td>( M_{cx} = \phi_b M_{nx} )</td>
<td>( M_{cx} = \frac{M_{nx}}{\Omega_b} )</td>
</tr>
<tr>
<td>( = 0.90(547 \text{ kip-ft}) )</td>
<td>( = \frac{547 \text{ kip-ft}}{1.67} )</td>
</tr>
<tr>
<td>( = 492 \text{ kip-ft} )</td>
<td>( = 328 \text{ kip-ft} )</td>
</tr>
<tr>
<td>( M_{cy} = \phi_b M_{ny} )</td>
<td>( M_{cy} = \frac{M_{ny}}{\Omega_b} )</td>
</tr>
<tr>
<td>( = 0.90(187 \text{ kip-ft}) )</td>
<td>( = \frac{187 \text{ kip-ft}}{1.67} )</td>
</tr>
<tr>
<td>( = 168 \text{ kip-ft} )</td>
<td>( = 112 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

**Interaction of Tension and Flexure**
Check limit for AISC Specification Equation H1-1a.

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{P_r}{P_c} = \frac{P_u}{\phi_r P_n} )</td>
<td>( \frac{P_r}{P_c} = \frac{P_u}{P_n / \Omega_r} )</td>
</tr>
<tr>
<td></td>
<td>( P_r = 174 \text{ kips} )</td>
<td>( P_r = 116 \text{ kips} )</td>
</tr>
<tr>
<td></td>
<td>( P_c = 1,080 \text{ kips} )</td>
<td>( P_c = 719 \text{ kips} )</td>
</tr>
<tr>
<td></td>
<td>( = 0.161 &lt; 0.2 )</td>
<td>( = 0.161 &lt; 0.2 )</td>
</tr>
<tr>
<td><strong>Because</strong></td>
<td>( \frac{P_r}{P_c} &lt; 0.2, )</td>
<td><strong>Because</strong> ( \frac{P_r}{P_c} &lt; 0.2, )</td>
</tr>
<tr>
<td>( P_r )</td>
<td>( \frac{174 \text{ kips}}{2 \times 1,080 \text{ kips}} + \frac{192 \text{ kip-ft}}{492 \text{ kip-ft}} + \frac{67.6 \text{ kip-ft}}{168 \text{ kip-ft}} \leq 1.0 )</td>
<td>( \frac{116 \text{ kips}}{2 \times 719 \text{ kips}} + \frac{128 \text{ kip-ft}}{328 \text{ kip-ft}} + \frac{45.1 \text{ kip-ft}}{112 \text{ kip-ft}} \leq 1.0 )</td>
</tr>
<tr>
<td>( = 0.873 &lt; 1.0 )</td>
<td><strong>o.k.</strong></td>
<td>( = 0.874 &lt; 1.0 ) <strong>o.k.</strong></td>
</tr>
</tbody>
</table>
EXAMPLE H.4  W-SHAPE SUBJECT TO COMBINED AXIAL COMPRESSION AND FLEXURE

Given:

Select an ASTM A992 W-shape with a 10-in.-nominal-depth to carry axial compression forces of 5 kips from dead load and 15 kips from live load. The unbraced length is 14 ft and the ends are pinned. The member also has the following required moment strengths due to uniformly distributed loads, not including second-order effects:

\[ M_{sD} = 15 \text{ kip-ft} \]
\[ M_{sL} = 45 \text{ kip-ft} \]
\[ M_{yD} = 2 \text{ kip-ft} \]
\[ M_{yL} = 6 \text{ kip-ft} \]

The member is not subject to sidesway (no lateral translation).

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

ASTM A992
\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]

From Chapter 2 of ASCE/SEI 7, the required strength (not considering second-order effects) is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_a = 1.2 \times (5 \text{ kips}) + 1.6 \times (15 \text{ kips}) ) = 30.0 kips</td>
<td>( P_a = 5 \text{ kips} + 15 \text{ kips} ) = 20.0 kips</td>
</tr>
<tr>
<td>( M_{ux} = 1.2 \times (15 \text{ kip-ft}) + 1.6 \times (45 \text{ kip-ft}) ) = 90.0 kip-ft</td>
<td>( M_{ux} = 15 \text{ kip-ft} + 45 \text{ kip-ft} ) = 60.0 kip-ft</td>
</tr>
<tr>
<td>( M_{uy} = 1.2 \times (2 \text{ kip-ft}) + 1.6 \times (6 \text{ kip-ft}) ) = 12.0 kip-ft</td>
<td>( M_{uy} = 2 \text{ kip-ft} + 6 \text{ kip-ft} ) = 8.00 kip-ft</td>
</tr>
</tbody>
</table>

Try a W10×33.

From AISC Manual Tables 1-1 and 3-2, the properties are as follows:

W10×33
\[ A = 9.71 \text{ in.}^2 \]
\[ S_x = 35.0 \text{ in.}^3 \]
\[ Z_x = 38.8 \text{ in.}^3 \]
\[ I_x = 171 \text{ in.}^4 \]
\[ r_x = 4.19 \text{ in.} \]
\[ S_y = 9.20 \text{ in.}^3 \]
\[ Z_y = 14.0 \text{ in.}^3 \]
\[ I_y = 36.6 \text{ in.}^4 \]
\[ r_y = 1.94 \text{ in.} \]
\[ L_p = 6.85 \text{ ft} \]
\[ L_r = 21.8 \text{ ft} \]
Available Axial Strength

From AISC Specification Commentary Table C-A-7.1, for a pinned-pinned condition, \( K = 1.0 \). Because \( L_c = KL_x = KL_y = 14.0 \) ft and \( r_x > r_y \), the \( y-y \) axis will govern.

From AISC Manual Table 6-2, the available axial strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_c = \phi_c P_n )</td>
<td>253 kips</td>
<td>( P_c = \frac{P_n}{\Omega_c} ) = 168 kips</td>
</tr>
</tbody>
</table>

Required Flexural Strength (including second-order amplification)

Use the approximate method of second-order analysis procedure from AISC Specification Appendix 8. Because the member is not subject to sidesway, only \( P-\delta \) amplifiers need to be added.

\[
B_l = \frac{C_m}{1 - \alpha P_r / P_{elx}} \geq 1 \quad (Spec. \ Eq. \ A-8-3)
\]

where \( C_m \) is conservatively taken per AISC Specification A-8.2.1(b):

\( C_m = 1.0 \)

The \( x-x \) axis flexural magnifier is:

\[
P_{elx} = \frac{\pi^2 EI_x}{(L_{elx})^2} \quad (from \ Spec. \ Eq. \ A-8-5)
\]

\[
= \frac{\pi^2 (29,000 \text{ ksi})(171 \text{ in.}^4)}{[(14 \text{ ft})(12 \text{ in./ft})]^2} = 1,730 \text{ kips}
\]

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha = 1.0 )</td>
<td>( B_{1x} = \frac{C_m}{1 - \alpha P_r / P_{elx}} \geq 1.0 )</td>
<td>( B_{1x} = \frac{C_m}{1 - \alpha P_r / P_{elx}} \geq 1.0 )</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>( = \frac{1.0}{1 - 1.0(30 \text{ kips}/1,730 \text{ kips})} \geq 1.0 )</td>
</tr>
<tr>
<td></td>
<td>= 1.02</td>
<td>( = \frac{1.0}{1 - 1.6(20 \text{ kips}/1,730 \text{ kips})} \geq 1.0 )</td>
</tr>
<tr>
<td></td>
<td>91.8 kip-ft</td>
<td>( M_{ax} = 1.02(60 \text{ kip-ft}) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( = 61.2 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

The \( y-y \) axis flexural magnifier is:
\[ P_{el_y} = \frac{\pi^2 E I_y}{(L_{el_y})^2} \]  
\[ = \frac{\pi^2 (29,000 \text{ ksi}) (36.6 \text{ in.}^4)}{[(14 \text{ ft})(12 \text{ in./ft})]^2} \]  
\[ = 371 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha = 1.0)</td>
<td>(\alpha = 1.6)</td>
</tr>
<tr>
<td>(B_{ly} = \frac{C_m}{1-\alpha P_r/P_{el_y}} \geq 1.0)</td>
<td>(B_{ly} = \frac{C_m}{1-\alpha P_r/P_{el_y}} \geq 1.0)</td>
</tr>
<tr>
<td>[=\frac{1.0}{1-1.0(30\text{ kips}/371\text{ kips})} \geq 1.0]</td>
<td>[=\frac{1.0}{1-1.6(20\text{ kips}/371\text{ kips})} \geq 1.0]</td>
</tr>
<tr>
<td>(= 1.09)</td>
<td>(= 1.09)</td>
</tr>
<tr>
<td>(M_{u_y} = 1.09(12\text{ kip-ft}))</td>
<td>(M_{u_y} = 1.09(8\text{ kip-ft}))</td>
</tr>
<tr>
<td>(= 13.1\text{ kip-ft})</td>
<td>(= 8.72\text{ kip-ft})</td>
</tr>
</tbody>
</table>

Nominal Flexural Strength about the Major Axis

Yielding

\[ M_{mx} = M_p = F_y Z_x \]  
\[ = (50 \text{ ksi})(38.8 \text{ in.}^3) \]  
\[ = 1,940 \text{ kip-in.} \]

Lateral-Torsional Buckling

Because \(L_p < L_b < L_r\), i.e., 6.85 ft < 14.0 ft < 21.8 ft, AISC Specification Equation F2-2 applies.

From AISC Manual Table 3-1, \(C_b = 1.14\)

\[ M_{mx} = C_b \left[ M_p - \left( M_p - 0.7F_yS_x \right) \left( \frac{L_b - L_p}{L_r - L_p} \right) \right] \leq \allowbreak M_p \]  
\[ = 1.14 \left[ 1,940 \text{ kip-in.} - \left( 1,940 \text{ kip-in.} - 0.7(50 \text{ ksi})(35.0 \text{ in.}^3) \right) \left( \frac{14 \text{ ft} - 6.85 \text{ ft}}{21.8 \text{ ft} - 6.85 \text{ ft}} \right) \right] \]  
\[ = 1,820 \text{ kip-in.} < 1,940 \text{ kip-in.} \]  
\[ = 1,820 \text{ kip-in.} \text{ or 152 kip-ft controls} \]

Local Buckling

Per AISC Manual Table 1-1, the member is compact for \(F_y = 50 \text{ ksi}\), so the local buckling limit state does not apply.

Nominal Flexural Strength about the Minor Axis
Determine the nominal flexural strength for bending about the minor axis from AISC Specification Section F6. Because a W10×33 has compact flanges, only the yielding limit state applies.

From AISC Specification Section F6.1:

\[ M_{nx} = M_p = F_f Z_x \leq 1.6 F_f S_y \]

\[ = (50 \text{ ksi})(14.0 \text{ in.}^3) \leq 1.6(50 \text{ ksi})(9.20 \text{ in.}^3) \]

\[ = 700 \text{ kip-in.} < 736 \text{ kip-in.} \]

\[ = 700 \text{ kip-in. or 58.3 kip-ft} \]

From AISC Specification Section F1, the available flexural strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b = 0.90 )</td>
<td>( \Omega_b = 1.67 )</td>
</tr>
<tr>
<td>( M_{cx} = \phi_b M_{nx} )</td>
<td>( M_{cx} = \frac{M_{nx}}{\Omega_b} )</td>
</tr>
<tr>
<td>( = 0.90(152 \text{ kip-ft}) )</td>
<td>( = \frac{152 \text{ kip-ft}}{1.67} )</td>
</tr>
<tr>
<td>( = 137 \text{ kip-ft} )</td>
<td>( = 91.0 \text{ kip-ft} )</td>
</tr>
<tr>
<td>( M_{cy} = \phi_b M_{ny} )</td>
<td>( M_{cy} = \frac{M_{ny}}{\Omega_b} )</td>
</tr>
<tr>
<td>( = 0.90(58.3 \text{ kip-ft}) )</td>
<td>( = \frac{58.3 \text{ kip-ft}}{1.67} )</td>
</tr>
<tr>
<td>( = 52.5 \text{ kip-ft} )</td>
<td>( = 34.9 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

Check limit for AISC Specification Equations H1-1a and H1-1b.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_r ) = \frac{P_u}{\phi_b P_n} )</td>
<td>( P_r ) = \frac{P_u}{P_n / \Omega_c}</td>
</tr>
<tr>
<td>( = \frac{30 \text{ kips}}{253 \text{ kips}} )</td>
<td>( = \frac{20 \text{ kips}}{168 \text{ kips}} )</td>
</tr>
<tr>
<td>( = 0.119 &lt; 0.2 )</td>
<td>( = 0.119 &lt; 0.2 )</td>
</tr>
</tbody>
</table>

Because \( \frac{P_r}{P_c} < 0.2 \),

\[ \frac{P_r}{2P_c} + \left( \frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \right) \leq 1.0 \quad (\text{Spec. Eq. H1-1b}) \]

\[ = \frac{30 \text{ kips}}{2(253 \text{ kips})} + \left( \frac{91.8 \text{ kip-ft}}{137 \text{ kip-ft}} + \frac{13.1 \text{ kip-ft}}{52.5 \text{ kip-ft}} \right) \leq 1.0 \]

\[ = 0.979 < 1.0 \quad \text{o.k.} \]

Because \( \frac{P_r}{P_c} < 0.2 \),

\[ \frac{P_r}{2P_c} + \left( \frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \right) \leq 1.0 \quad (\text{Spec. Eq. H1-1b}) \]

\[ = \frac{20 \text{ kips}}{2(168 \text{ kips})} + \left( \frac{61.2 \text{ kip-ft}}{91.0 \text{ kip-ft}} + \frac{8.72 \text{ kip-ft}}{34.9 \text{ kip-ft}} \right) \]

\[ = 0.982 < 1.0 \quad \text{o.k.} \]
CHAPTER H DESIGN EXAMPLE REFERENCES

Chapter J
Design of Connections

AISC Specification Chapter J addresses the design of connections. The chapter’s primary focus is the design of welded and bolted connections. Design requirements for fillers, splices, column bases, concentrated forces, anchors rods and other threaded parts are also covered. See AISC Specification Appendix 3 for special requirements for connections subject to fatigue.
EXAMPLE J.1  FILLET WELD IN LONGITUDINAL SHEAR

Given:

As shown in Figure J.1-1, a $\frac{1}{4}$-in.-thick $\times$ 18-in. wide plate is fillet welded to a $\frac{3}{8}$-in.-thick plate. The plates are ASTM A572 Grade 50 and have been properly sized. Use 70-ksi electrodes. Note that the plates could be specified as ASTM A36, but $F_y = 50$ ksi plate has been used here to demonstrate the requirements for long welds.

Confirm that the size and length of the welds shown are adequate to resist the applied loading.

![Figure J.1-1. Geometry and loading for Example J.1.](image)

Solution:

From AISC Manual Table 2-5, the material properties are as follows:

- ASTM A572 Grade 50
- $F_y = 50$ ksi
- $F_u = 65$ ksi

From ASCE/SEI 7, Chapter 2, the required strength is:

$$P_{u} = 1.2(33 \text{ kips}) + 1.6(100 \text{ kips})$$
$$= 200 \text{ kips}$$

$$P_{d} = 33 \text{ kips} + 100 \text{ kips}$$
$$= 133 \text{ kips}$$

Maximum and Minimum Weld Size

Because the thickness of the overlapping plate is $\frac{1}{4}$ in., the maximum fillet weld size that can be used without special notation per AISC Specification Section J2.2b, is a $\frac{3}{16}$-in. fillet weld. A $\frac{3}{16}$-in. fillet weld can be deposited in the flat or horizontal position in a single pass (true up to $\frac{3}{16}$-in.).

From AISC Specification Table J2.4, the minimum size of the fillet weld, based on a material thickness of $\frac{1}{4}$ in. is $\frac{1}{8}$ in.
Weld Strength

The nominal weld strength per inch of \( \frac{\gamma_{16}}{\text{in.}} \) weld, determined from AISC Specification Section J2.4(b) is:

\[
R_n = F_{ew}A_{ew}
\]

\[
= \left( 0.60F_{EXX} \right) A_{ew}
\]

\[
= 0.60(70 \text{ ksi}) \left( \frac{\gamma_{16}}{\text{in.}} \right) \sqrt{2}
\]

\[
= 5.57 \text{ kip/in.}
\]

From AISC Specification Section J2.2b, check the weld length to weld size ratio, because this is an end-loaded fillet weld.

\[
\frac{l}{w} = \frac{27.0 \text{ in.}}{\frac{\gamma_{16}}{\text{in.}}}
\]

\[
= 144 > 100; \text{ therefore, AISC Specification Equation J2-1 must be applied}
\]

\[
\beta = 1.2 - 0.002(\frac{l}{w}) \leq 1.0
\]

\[
= 1.2 - 0.002(144) \leq 1.0
\]

\[
= 0.912
\]

The nominal weld shear rupture strength is:

\[
R_n = 0.912(5.57 \text{ kip/in.})(2 \text{ welds})(27 \text{ in.})
\]

\[
= 274 \text{ kips}
\]

From AISC Specification Section J2.4, the available shear rupture strength is:

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
\phi = 0.75 & \Omega = 2.00 \\
\hline
\phi R_n = 0.75(274 \text{ kips}) & \frac{R_n}{\Omega} = \frac{274 \text{ kips}}{2.00} \\
& = 206 \text{ kips} > 200 \text{ kips} \quad \text{o.k.} \\
\hline
\end{array}
\]

The base metal strength is determined from AISC Specification Section J2.4(a). The \( \frac{1}{4} \)-in.-thick plate controls:

\[
R_n = F_{nBM}A_{BM}
\]

\[
= 0.60F_{u}t_{p}l_{\text{weld}}
\]

\[
= 0.60(65 \text{ ksi})(\frac{1}{4} \text{ in.})(2 \text{ welds})(27 \text{ in.})
\]

\[
= 527 \text{ kips}
\]

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
\phi = 0.75 & \Omega = 2.00 \\
\hline
\phi R_n = 0.75(527 \text{ kips}) & \frac{R_n}{\Omega} = \frac{527 \text{ kips}}{2.00} \\
& = 264 \text{ kips} > 133 \text{ kips} \quad \text{o.k.} \\
\hline
\end{array}
\]
EXAMPLE J.2  FILLET WELD LOADED AT AN ANGLE

Given:

Verify a fillet weld at the edge of a gusset plate is adequate to resist a force of 50 kips due to dead load and 150 kips due to live load, at an angle of 60° relative to the weld, as shown in Figure J.2-1. Assume the beam and the gusset plate thickness and length have been properly sized. Use a 70-ksi electrode.

Solution:

From ASCE/SEI 7, Chapter 2, the required tensile strength is:

\[
P_{u} = 1.2(50 \text{ kips}) + 1.6(150 \text{ kips})
\]
\[
= 300 \text{ kips}
\]
\[
P_{a} = 50 \text{ kips} + 150 \text{ kips}
\]
\[
= 200 \text{ kips}
\]

Assume a \(\frac{3}{16}\)-in. fillet weld is used on each side of the plate.

Note that from AISC Specification Table J2.4, the minimum size of fillet weld, based on a material thickness of \(\frac{3}{4}\) in. is \(\frac{3}{8}\) in. (assuming the beam flange thickness exceeds \(\frac{3}{4}\) in.).

Available Shear Strength of the Fillet Weld Per Inch of Length

From AISC Specification Section J2.4(b), the nominal strength of the fillet weld is determined as follows:

\[
R_{n} = F_{nw}A_{we}
\]
\[
= 0.60F_{XX} \left(1.0 + 0.50\sin^{1.5} 60^\circ\right)A_{we}
\]
\[
= 0.60(70 \text{ ksi})(1.0 + 0.50\sin^{1.5} 60^\circ)\left(\frac{\frac{3}{16} \text{ in.}}{\sqrt{2}}\right)
\]
\[
= 13.0 \text{ kip/in.}
\]

From AISC Specification Section J2.4(b), the available shear strength per inch of weld for fillet welds on two sides is:
### Required Length of Weld

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_n = 0.75(13.0 \text{ kip/in.})(2 \text{ sides}) )</td>
<td>( \frac{R_n}{\Omega} = \frac{13.0 \text{ kip/in.}}{2.00} (2 \text{ sides}) )</td>
</tr>
<tr>
<td>= 19.5 kip/in.</td>
<td>= 13.0 kip/in.</td>
</tr>
</tbody>
</table>

- Use 16 in. on each side of the plate.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l = \frac{300 \text{ kips}}{19.5 \text{ kip/in.}} ) = 15.4 in.</td>
<td>( l = \frac{200 \text{ kips}}{13.0 \text{ kip/in.}} ) = 15.4 in.</td>
</tr>
<tr>
<td>Use 16 in. on each side of the plate.</td>
<td>Use 16 in. on each side of the plate.</td>
</tr>
</tbody>
</table>
EXAMPLE J.3  COMBINED TENSION AND SHEAR IN BEARING-TYPE CONNECTIONS

Given:

A $\frac{3}{8}$-in.-diameter, Group A bolt with threads not excluded from the shear plane (thread condition N) is subjected to a tension force of 3.5 kips due to dead load and 12 kips due to live load, and a shear force of 1.33 kips due to dead load and 4 kips due to live load. Check the combined stresses according to AISC Specification Equations J3-3a and J3-3b.

Solution:

From ASCE/SEI 7, Chapter 2, the required tensile and shear strengths are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tension:</strong></td>
<td><strong>Tension:</strong></td>
</tr>
<tr>
<td>$T_u = 1.2(3.5\text{kips}) + 1.6(12\text{kips})$</td>
<td>$T_a = 3.5\text{kips} + 12\text{kips}$</td>
</tr>
<tr>
<td>= 23.4 kips</td>
<td>= 15.5 kips</td>
</tr>
<tr>
<td><strong>Shear:</strong></td>
<td><strong>Shear:</strong></td>
</tr>
<tr>
<td>$V_u = 1.2(1.33\text{kips}) + 1.6(4\text{kips})$</td>
<td>$V_a = 1.33\text{kips} + 4\text{kips}$</td>
</tr>
<tr>
<td>= 8.00 kips</td>
<td>= 5.33 kips</td>
</tr>
</tbody>
</table>

**Available Tensile Strength**

When a bolt is subject to combined tension and shear, the available tensile strength is determined according to the limit states of tension and shear rupture, from AISC Specification Section J3.7 as follows.

From AISC Specification Table J3.2, Group A bolts:

- $F_{nt} = 90 \text{ksi}$
- $F_{nv} = 54 \text{ksi}$

From AISC Manual Table 7-2, for a $\frac{3}{8}$-in.-diameter bolt:

- $A_b = 0.442 \text{in.}^2$

The available shear stress is determined as follows and must equal or exceed the required shear stress.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi F_{nv} = 0.75(54 \text{ksi})$</td>
<td>$\frac{F_{nv}}{\Omega} = \frac{54 \text{ksi}}{2.00} = 27.0 \text{ksi}$</td>
</tr>
<tr>
<td>= 40.5 ksi</td>
<td></td>
</tr>
<tr>
<td>$f_{rv} = \frac{V_u}{A_b}$</td>
<td>$f_{rv} = \frac{V_a}{A_b}$</td>
</tr>
<tr>
<td>= 8.00 kips</td>
<td>= 5.33 kips</td>
</tr>
<tr>
<td>$\frac{0.442 \text{ in.}^2}{0.442 \text{ in.}^2}$</td>
<td>$\frac{0.442 \text{ in.}^2}{0.442 \text{ in.}^2}$</td>
</tr>
<tr>
<td>= 18.1 ksi &lt; 40.5 ksi o.k.</td>
<td>= 12.1 ksi &lt; 27.0 ksi o.k.</td>
</tr>
</tbody>
</table>
The available tensile strength of a bolt subject to combined tension and shear is as follows:

![Image](image-url)

The effects of combined shear and tensile stresses need not be investigated if either the required shear or tensile stress is less than or equal to 30% of the corresponding available stress per the User Note at the end of AISC Specification Section J3.7. In the example herein, both the required shear and tensile stresses exceeded the 30% threshold and evaluation of combined stresses was necessary.

AISC Specification Equations J3-3a and J3-3b may be rewritten so as to find a nominal shear stress, $F_{nv}'$, as a function of the required tensile stress as is shown in AISC Specification Commentary Equations C-J3-7a and C-J3-7b.
EXAMPLE J.4A  SLIP-CRITICAL CONNECTION WITH SHORT-SLOTTED HOLES

Slip-critical connections shall be designed to prevent slip and for the limit states of bearing-type connections.

Given:

Refer to Figure J.4A-1 and select the number of bolts that are required to support the loads shown when the connection plates have short slots transverse to the load and no fillers are provided. Select the number of bolts required for slip resistance only.

Solution:

From ASCE/SEI 7, Chapter 2, the required strength is:

\[
P_u = 1.2(17 \text{ kips}) + 1.6(51 \text{ kips}) = 102 \text{ kips}
\]

From AISC Specification Section J3.8(a), the available slip resistance for the limit state of slip for standard size and short-slotted holes perpendicular to the direction of the load is determined as follows:

\[
\phi = 1.00 \\
\Omega = 1.50 \\
\mu = 0.30 \text{ for Class A surface} \\
D_a = 1.13 \\
h_f = 1.0, \text{ no filler is provided} \\
T_b = 28 \text{ kips, from AISC Specification Table J3.1, Group A} \\
n_s = 2, \text{ number of slip planes}
\]

\[
R_u = \mu D_a h_f T_b n_s 
\]

\[
= 0.30(1.13)(1.0)(28 \text{ kips})(2) = 19.0 \text{ kips/bolt}
\]

The available slip resistance is:
<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_a = 1.00(19.0 \text{ kips/bolt})$</td>
<td>$R_a = \frac{19.0 \text{ kips/bolt}}{\Omega}$</td>
</tr>
<tr>
<td>$= 19.0 \text{ kips/bolt}$</td>
<td>$= \frac{12.7 \text{ kips/bolt}}{1.50}$</td>
</tr>
</tbody>
</table>

**Required Number of Bolts**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_b = \frac{P_a}{\phi R_a}$</td>
<td>$n_b = \frac{P_a}{\left(\frac{R_a}{\Omega}\right)}$</td>
</tr>
<tr>
<td>$= 102 \text{ kips}$</td>
<td>$= 68.0 \text{ kips}$</td>
</tr>
<tr>
<td>$19.0 \text{ kips/bolt}$</td>
<td>$= \frac{12.7 \text{ kips/bolt}}{1.50}$</td>
</tr>
<tr>
<td>$= 5.37 \text{ bolts}$</td>
<td>$= 5.35 \text{ bolts}$</td>
</tr>
</tbody>
</table>

Use 6 bolts                                      Use 6 bolts

Note: To complete the verification of this connection, the limit states of bolt shear, bearing, tearout, tensile yielding, tensile rupture, and block shear rupture must also be checked.
EXAMPLE J.4B  SLIP-CRITICAL CONNECTION WITH LONG-SLOTTED HOLES

Given:

Repeat Example J.4A with the same loads, but assuming that the connection plates have long-slotted holes in the direction of the load, as shown in Figure J.4B-1.

Solution:

The required strength from Example J.4A is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_u$</td>
<td>102 kips</td>
<td>$P_u = 68.0$ kips</td>
</tr>
</tbody>
</table>

From AISC Specification Section J3.8(c), the available slip resistance for the limit state of slip for long-slotted holes is determined as follows:

- $\phi = 0.70$
- $\Omega = 2.14$
- $\mu = 0.30$ for Class A surface
- $D_u = 1.13$
- $h_f = 1.0$, no filler is provided
- $T_b = 28$ kips, from AISC Specification Table J3.1, Group A
- $n_s = 2$, number of slip planes

\[
R_n = \mu D_u h_f T_b n_s
\]

\[
= 0.30(1.13)(1.0)(28 \text{ kips})(2)
\]

\[
= 19.0 \text{ kips/bolt}
\]

The available slip resistance is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_n$</td>
<td>0.70(19.0 kips/bolt)</td>
<td>$R_n = 19.0$ kips/bolt</td>
</tr>
<tr>
<td>$= 13.3$ kips/bolt</td>
<td>[\Omega = \frac{19.0 \text{ kips/bolt}}{2.14} = 8.88 \text{ kips/bolt}]</td>
<td></td>
</tr>
</tbody>
</table>
### Required Number of Bolts

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_b = \frac{P_n}{\phi R_n} )</td>
<td>( n_b = \frac{P_n}{\left(\frac{R_n}{\Omega}\right)} )</td>
</tr>
<tr>
<td>( = \frac{102 \text{ kips}}{13.3 \text{ kips/bolt}} )</td>
<td>( = \frac{68.0 \text{ kips}}{8.88 \text{ kips/bolt}} )</td>
</tr>
<tr>
<td>( = 7.67 \text{ bolts} )</td>
<td>( = 7.66 \text{ bolts} )</td>
</tr>
</tbody>
</table>

Use 8 bolts

Use 8 bolts

Note: To complete the verification of this connection, the limit states of bolt shear, bearing, tearout, tensile yielding, tensile rupture, and block shear rupture must be determined.
EXAMPLE J.5  COMBINED TENSION AND SHEAR IN A SLIP-CRITICAL CONNECTION

Because the pretension of a bolt in a slip-critical connection is used to create the clamping force that produces the shear strength of the connection, the available shear strength must be reduced for any load that produces tension in the connection.

Given:

The slip-critical bolt group shown in Figure J.5-1 is subjected to tension and shear. This example shows the design for bolt slip resistance only, and assumes that the beams and plates are adequate to transmit the loads. Determine if the bolts are adequate.

Solution:

From ASCE/SEI 7, Chapter 2, the required strength is:

\[
P_{u} = 1.2(15 \text{ kips}) + 1.6(45 \text{ kips})
\]

\[
= 90.0 \text{ kips}
\]

By geometry:

\[
T_{u} = \frac{4}{5}(90.0 \text{ kips})
\]

\[
= 72.0 \text{ kips}
\]

\[
V_{u} = \frac{3}{5}(90.0 \text{ kips})
\]

\[
= 54.0 \text{ kips}
\]

\[
P_{a} = 15 \text{ kips} + 45 \text{ kips}
\]

\[
= 60.0 \text{ kips}
\]

By geometry:

\[
T_{a} = \frac{4}{5}(60.0 \text{ kips})
\]

\[
= 48.0 \text{ kips}
\]

\[
V_{a} = \frac{3}{5}(60.0 \text{ kips})
\]

\[
= 36.0 \text{ kips}
\]

**Available Bolt Tensile Strength**

The available tensile strength is determined from AISC Specification Section J3.6.

From AISC Specification Table J3.2 for Group A bolts, the nominal tensile strength in ksi is, \( F_{nt} = 90 \text{ ksi} \). From AISC Manual Table 7-1, for a \( \frac{3}{8} \)-in.-diameter bolt:
The nominal tensile strength is:

\[ R_n = F_{u} A_b \]

\[ = (90 \text{ ksi})(0.442 \text{ in.}^2) \]

\[ = 39.8 \text{ kips} \]

The available tensile strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_n )</td>
<td>( R_u )</td>
</tr>
<tr>
<td>0.75(39.8 kips/bolt)</td>
<td>39.8 kips/bolt</td>
</tr>
<tr>
<td>&gt; 72.0 kips/8 bolts</td>
<td>&gt; 48.0 kips/8 bolts</td>
</tr>
<tr>
<td>= 29.9 kips/bolt &gt; 9.00 kips/bolt</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Note that the available tensile strength per bolt can also be taken from AISC Manual Table 7-2.

**Available Slip Resistance per Bolt**

The available slip resistance for one bolt in standard size holes is determined using AISC Specification Section J3.8(a):

\[ \phi = 1.00 \]

\[ \Omega = 1.50 \]

\( \mu = 0.30 \) for Class A surface

\( D_u = 1.13 \)

\( h_f = 1.0, \) factor for fillers, assuming no more than one filler

\( T_b = 28 \text{ kips}, \) from AISC Specification Table J3.1, Group A

\( n_s = 1, \) number of slip planes

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Determine the available slip resistance ((T_u = 0)) of a bolt:</td>
<td>Determine the available slip resistance ((T_u = 0)) of a bolt:</td>
</tr>
<tr>
<td>( \phi R_n = \phi \mu D_u h_f T_b n_s )</td>
<td>( R_u = \frac{\mu D_u h_f T_b n_s}{\Omega} )</td>
</tr>
<tr>
<td>( = 1.00(0.30)(1.13)(1.0)(28 \text{ kips})(1) )</td>
<td>( = 0.30(1.13)(1.0)(28 \text{ kips})(1) )</td>
</tr>
<tr>
<td>( = 9.49 \text{ kips/bolt} )</td>
<td>( = \frac{1.50}{1.50} = 6.33 \text{ kips/bolt} )</td>
</tr>
</tbody>
</table>

Note that the available slip resistance for one bolt with a Class A faying surface can also be taken from AISC Manual Table 7-3.

**Available Slip Resistance of the Connection**

Because the slip-critical connection is subject to combined tension and shear, the available slip resistance is multiplied by a reduction factor provided in AISC Specification Section J3.9.
Slip-critical combined tension and shear factor:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{sc} = 1 - \frac{T_s}{D_a T_b n_b} \geq 0$ (Spec. Eq. J3-5a)</td>
<td>$k_{sc} = 1 - \frac{1.5T_s}{D_a T_b n_b} \geq 0$ (Spec. Eq. J3-5b)</td>
</tr>
<tr>
<td>$= 1 - \frac{72.0 \text{ kips}}{1.13(28 \text{ kips})(8)} &gt; 0$</td>
<td>$= 1 - \frac{1.5(48.0 \text{ kips})}{1.13(28 \text{ kips})(8)} &gt; 0$</td>
</tr>
<tr>
<td>$= 0.716$</td>
<td>$= 0.716$</td>
</tr>
</tbody>
</table>

\[
\phi R_n = \frac{\phi R_u k_{sc} n_b}{\Omega} = (9.49 \text{ kips/bolt})(0.716)(8 \text{ bolts}) = 54.4 \text{ kips} > 54.0 \text{ kips} \text{ o.k.}
\]

\[
\frac{R_n}{\Omega} = \frac{R_u k_{sc} n_b}{\Omega} = (6.33 \text{ kips/bolt})(0.716)(8 \text{ bolts}) = 36.3 \text{ kips} > 36.0 \text{ kips} \text{ o.k.}
\]

Note: The bolt group must still be checked for all applicable strength limit states for a bearing-type connection.
Chapter IIA
Simple Shear Connections

EXAMPLE II.A-1A ALL-BOLTED DOUBLE-ANGLE CONNECTION

Given:

Using the tables in AISC Manual Part 10, verify the available strength of an all-bolted double-angle shear connection between an ASTM A992 W36×231 beam and an ASTM A992 W14×90 column flange, as shown in Figure IIA-1A-1, supporting the following beam end reactions:

\[ R_D = 37.5 \text{ kips} \]
\[ R_L = 113 \text{ kips} \]

Use ASTM A36 angles.

![Connection diagram](image)

* This dimension (see sketch, Section A) is determined as one-half of the decimal web thickness rounded to the next higher \( \frac{3}{16} \) in. Example: 0.760\%/2 = 0.380\%; use \( \frac{3}{16} \) in. This will produce spacing of holes in the supporting beam slightly larger than detailed in the angles to permit spreading of angles (angles can be spread but not closed) at time of erection to supporting member. Alternatively, consider using horizontal short slots in the support legs of the angles.

**See AISC Manual Tables 7-15 and 7-16 for driving clearance.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

Beam and column
ASTM A992
\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]

Angles
ASTM A36
\[ F_y = 36 \text{ ksi} \]
\[ F_u = 58 \text{ ksi} \]
From AISC Manual Table 1-1, the geometric properties are as follows:

Beam
W36×231
t_w = 0.760 in.

Column
W14×90
t_f = 0.710 in.

From AISC Specification Table J3.3, the hole diameter for a 3⁄8-in.-diameter bolt with standard holes is:

\[ d_h = \frac{3}{16} \text{ in.} \]

From ASCE/SEI 7, Chapter 2, the required strength is:

\[
\begin{align*}
\text{LRFD} & : R_v = 1.2(37.5 \text{ kips}) + 1.6(113 \text{ kips}) = 226 \text{ kips} \\
\text{ASD} & : R_v = 37.5 \text{ kips} + 113 \text{ kips} = 151 \text{ kips}
\end{align*}
\]

Connection Selection

AISC Manual Table 10-1 includes checks for the limit states of bolt shear, bolt bearing and tearout on the angles, shear yielding of the angles, shear rupture of the angles, and block shear rupture of the angles.

Try 8 rows of bolts and 2L5×3½×3⁄8 (SLBB). From AISC Manual Table 10-1:

\[
\begin{align*}
\text{LRFD} & : \phi R_n = 248 \text{ kips} > 226 \text{ kips} \quad \text{O.K.} \\
\text{ASD} & : \frac{R_n}{\Omega} = 165 \text{ kips} > 151 \text{ kips} \quad \text{O.K.}
\end{align*}
\]

Available Beam Web Strength

The available beam web strength is the lesser of the limit states of block shear rupture, shear yielding, shear rupture, and the sum of the effective strengths of the individual fasteners. Because the beam is not coped, the only applicable limit state is the effective strength of the individual fasteners, which is the lesser of the bolt shear strength per AISC Specification Section J3.6, and the bolt bearing and tearout strength per AISC Specification Section J3.10.

Bolt Shear

From AISC Manual Table 7-1, the available shear strength per bolt for 3⁄8-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) in double shear is:

\[
\begin{align*}
\text{LRFD} & : \phi R_n = 35.8 \text{ kips/bolt} \\
\text{ASD} & : \frac{R_n}{\Omega} = 23.9 \text{ kips/bolt}
\end{align*}
\]

Bolt Bearing on Beam Web

The nominal bearing strength of the beam web per bolt is determined from AISC Specification Section J3.10, assuming deformation at service load is a design consideration:
From AISC Specification Section J3.10, the available bearing strength of the beam web per bolt is:

\[ r_p = 2.4d_t F_u \]

\[ = 2.4 \left( \frac{3}{8} \text{ in.} \right) \left( 0.760 \text{ in.} \right) \left( 65 \text{ ksi} \right) \]

\[ = 88.9 \text{ kips/bolt} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi r_p = 0.75(88.9 \text{ kips/bolt}) )</td>
<td>( r_p = \frac{88.9 \text{ kips/bolt}}{2.00} = 44.5 \text{ kips/bolt} )</td>
</tr>
</tbody>
</table>

**Bolt Tearout on Beam Web**

The available tearout strength of the beam web per bolt is determined from AISC Specification Section J3.10, assuming deformation at service load is a design consideration:

\[ l_c = 3.00 \text{ in.} - \frac{1}{10} \text{ in.} \]

\[ = 2.19 \text{ in.} \]

The available tearout strength is:

\[ r_t = 1.2l_t F_u \]

\[ = 1.2 \left( 2.19 \text{ in.} \right) \left( 0.760 \text{ in.} \right) \left( 65 \text{ ksi} \right) \]

\[ = 130 \text{ kips/bolt} \]

From AISC Specification Section J3.10, the available tearout strength of the beam web per bolt is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi r_t = 0.75(130 \text{ kips/bolt}) )</td>
<td>( r_t = \frac{130 \text{ kips/bolt}}{2.00} = 65.0 \text{ kips/bolt} )</td>
</tr>
</tbody>
</table>

Bolt shear strength is the governing limit state for all bolts at the beam web. Bolt shear strength is one of the limit states included in the capacities shown in Table 10-1 as used above; thus, the effective strength of the fasteners is adequate.

**Available Strength at the Column Flange**

Since the thickness of the column flange, \( t_f = 0.710 \text{ in.} \), is greater than the thickness of the angles, \( t = \frac{5}{6} \text{ in.} \), bolt bearing will control for the angles, which was previously checked. The column flange is adequate for the required loading.

**Conclusion**

The connection is found to be adequate as given for the applied loads.
EXAMPLE II.A-1B  ALL-BOLTED DOUBLE-ANGLE CONNECTION SUBJECT TO AXIAL AND SHEAR LOADING

Given:

Verify the available strength of an all-bolted double-angle connection for an ASTM A992 W18×50 beam, as shown in Figure II.A-1B-1, to support the following beam end reactions:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear, $V_u = 75$ kips</td>
<td>Shear, $V_a = 50$ kips</td>
</tr>
<tr>
<td>Axial tension, $N_u = 60$ kips</td>
<td>Axial tension, $N_a = 40$ kips</td>
</tr>
</tbody>
</table>

Use ASTM A36 double angles that will be shop-bolted to the beam.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

Beam
ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

Angles
ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi
From AISC Manual Table 1-1, the geometric properties are as follows:

Beam
W18×50
\( A_g = 14.7 \text{ in.}^2 \)
\( d = 18.0 \text{ in.} \)
\( t_w = 0.355 \text{ in.} \)
\( t_f = 0.570 \text{ in.} \)

From AISC Specification Table J3.3, the hole diameter for \( \frac{3}{8} \)-in.-diameter bolts with standard holes is:

\( d_h = \frac{1}{16} \text{ in.} \)

The resultant load is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_u = \sqrt{V_u^2 + N_u^2} ) ( = \sqrt{(75 \text{ kips})^2 + (60 \text{ kips})^2} ) ( = 96.0 \text{ kips} )</td>
<td>( R_a = \sqrt{V_a^2 + N_a^2} ) ( = \sqrt{(50 \text{ kips})^2 + (40 \text{ kips})^2} ) ( = 64.0 \text{ kips} )</td>
</tr>
</tbody>
</table>

Try 5 rows of bolts and 2L5×3\( \frac{3}{8} \)×\%in (SLBB).

Strength of the Bolted Connection—Angles

From the Commentary to AISC Specification Section J3.6, the strength of the bolt group is taken as the sum of the individual strengths of the individual fasteners, which may be taken as the lesser of the fastener shear strength per AISC Specification Section J3.6, the bearing strength at the bolt hole per AISC Specification Section J3.10, or the tearout strength at the bolt hole per AISC Specification Section J3.10.

Bolt shear

From AISC Manual Table 7-1, the available shear strength for \( \frac{3}{8} \)-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) in double shear (or pair of bolts) is:

\( \phi r_u = 48.7 \text{ kips/bolt (or per pair of bolts)} \)

Bolt bearing on angles

The available bearing strength of the angles per bolt in double shear is determined from AISC Specification Section J3.10, assuming deformation at service load is a design consideration:

\( r_n = \frac{(2 \text{ angles})2.4dtF_u}{\Omega} \) (from Spec. Eq. J3-6a)
\( = \frac{(2 \text{ angles})(2.4)(\frac{3}{8} \text{ in.})(\frac{3}{4} \text{ in.})(58 \text{ ksi})}{152 \text{ kips/bolt}} \)
\( = 152 \text{ kips/bolt} \)
**Bolt tearout on angles**

From AISC *Specification* Section J3.10, the available tearout strength of the angles per bolt in double shear is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration.

As shown in Figures II.A-1B-2(a) and II.A-1B-2(b), the tearout dimensions on the angle differ between the edge bolt and the other bolts.

The angle \( \theta \), as shown in Figure II.A-1B-2(a), of the resultant force on the edge bolt is:

\[
\begin{align*}
\theta &= \tan^{-1} \left( \frac{N_a}{V_a} \right) \\
&= \tan^{-1} \left( \frac{60 \text{ kips}}{75 \text{ kips}} \right) \\
&= 38.7^\circ
\end{align*}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>LRFD</td>
<td>ASD</td>
</tr>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi r_n = 0.75(152 \text{ kips/bolt}) )</td>
<td>( r_n )</td>
</tr>
<tr>
<td>= 114 kips/bolt</td>
<td>( \frac{r_n}{\Omega} = \frac{152 \text{ kips/bolt}}{2.00} )</td>
</tr>
<tr>
<td>= 76.0 kips/bolt</td>
<td></td>
</tr>
</tbody>
</table>

\( (a) \) Edge bolt
\( (b) \) Other bolts

*Fig. II.A-1B-2. Bolt tearout on angles.*
The length from the center of the bolt hole to the edge of the angle along the line of action of the force is:

\[ l_c = \frac{1\frac{1}{4}\text{ in.}}{\cos 38.7^\circ} = 1.60 \text{ in.} \]

The clear distance, along the line of action of the force, between the edge of the hole and the edge of the angle is:

\[ l_c = l_c - 0.5d_h = 1.60 \text{ in.} - 0.5(\frac{1}{16} \text{ in.}) = 1.13 \text{ in.} \]

The available tearout strength of the pair of angles at the edge bolt is:

\[ r_n = (2 \text{ angles})1.2F_u = (2 \text{ angles})(1.2)(1.13 \text{ in.})(\frac{3}{8} \text{ in.})(58 \text{ ksi}) = 98.3 \text{ kips/bolt} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi r_n = 0.75(98.3 \text{ kips/bolt}) )</td>
<td>( \frac{r_n}{\Omega} = \frac{98.3 \text{ kips/bolt}}{2.00} = 49.2 \text{ kips/bolt} )</td>
</tr>
</tbody>
</table>

Therefore, bolt shear controls over bearing or tearout of the angles at the edge bolt.

The angle \( \theta \), as shown in Figure II.A-1B-2(b), of the resultant force on the other bolts is:

\[ \theta = \tan^{-1}\left(\frac{V_n}{N_n}\right) = \tan^{-1}\left(\frac{75 \text{ kips}}{60 \text{ kips}}\right) = 51.3^\circ \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta = \tan^{-1}\left(\frac{V_n}{N_n}\right) )</td>
<td>( \theta = \tan^{-1}\left(\frac{V_n}{N_n}\right) )</td>
</tr>
<tr>
<td>( = \tan^{-1}\left(\frac{75 \text{ kips}}{60 \text{ kips}}\right) )</td>
<td>( = \tan^{-1}\left(\frac{50 \text{ kips}}{40 \text{ kips}}\right) = 51.3^\circ )</td>
</tr>
</tbody>
</table>

The length from the center of the bolt hole to the edge of the angle along the line of action of the force is:

\[ l_c = \frac{1\frac{1}{4}\text{ in.}}{\cos 51.3^\circ} = 2.00 \text{ in.} \]

The clear distance, along the line of action of the force, between the edge of the hole and the edge of the angle is:

\[ l_c = l_c - 0.5d_h = 2.00 \text{ in.} - 0.5(\frac{1}{16} \text{ in.}) = 1.53 \text{ in.} \]

The available tearout strength of the pair of angles at the other bolts is:
\( r_n = (2 \text{ angles})1.2l_F u \) 
\( = (2 \text{ angles})(1.2)(1.53 \text{ in.})(\frac{3}{8} \text{ in.})(58 \text{ ksi}) \)
\( = 133 \text{ kips/bolt} \)

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
\phi = 0.75 & \Omega = 2.00 \\
\phi r_n = 0.75(133 \text{ kips/bolt}) & r_n = 133 \text{ kips/bolt} \\
= 99.8 \text{ kips/bolt} & \frac{r_n}{\Omega} = 2.00 \\
& = 66.5 \text{ kips/bolt} \\
\hline
\end{array}
\]

Therefore, bolt shear controls over bearing or tearout of the angles at the other bolt.

The effective strength for the bolted connection at the angles is determined by summing the effective strength for each bolt using the minimum available strength calculated for bolt shear, bearing on the angles, and tearout on the angles.

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
\phi R_n = n\phi r_n & R_n = \Omega \frac{r_n}{\Omega} \\
= (5 \text{ bolts})(48.7 \text{ kips/bolt}) & = (5 \text{ bolts})(32.5 \text{ kips/bolt}) \\
= 244 \text{ kips} > 96.0 \text{ kips} & = 163 \text{ kips} > 64.0 \text{ kips} \text{ o.k.} \\
\hline
\end{array}
\]

**Strength of the Bolted Connection—Beam Web**

**Bolt bearing on beam web**

The available bearing strength of the beam web per bolt is determined from AISC Specification Section J3.10, assuming deformation at service load is a design consideration:

\( r_n = 2.4d_F u \) 
\( = 2.4(\frac{3}{8} \text{ in.})(0.355 \text{ in.})(65 \text{ ksi}) \)
\( = 48.5 \text{ kips/bolt} \)

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
\phi = 0.75 & \Omega = 2.00 \\
\phi r_n = 0.75(48.5 \text{ kips/bolt}) & r_n = 48.5 \text{ kips/bolt} \\
= 36.4 \text{ kips/bolt} & \frac{r_n}{\Omega} = 2.00 \\
& = 24.3 \text{ kips/bolt} \\
\hline
\end{array}
\]

**Bolt tearout on beam web**

From AISC Specification Section J3.10, the available tearout strength of the beam web is determined from AISC Specification Equation J3-6a, assuming deformation at the bolt hole is a design consideration, where the edge distance, \( l_e \), is based on the angle of the resultant load. As shown in Figure II.A-1B-3, a horizontal edge distance of 1½ in. is used which includes a ¼ in. tolerance to account for possible mill underrun.

The angle, \( \theta \), of the resultant force is:
The length from the center of the bolt hole to the edge of the web along the line of action of the force is:

\[ l_e = \frac{1\frac{1}{2} \text{ in.}}{\cos 51.3^\circ} \]
\[ = 2.40 \text{ in.} \]

The clear distance, along the line of action of the force, between the edge of the hole and the edge of the web is:

\[ l_c = l_e - 0.5d_h \]
\[ = 2.40 \text{ in.} - 0.5 \left( \frac{1}{8} \text{ in.} \right) \]
\[ = 1.93 \text{ in.} \]

The available tearout strength of the beam web is determined as follows:

\[ r_p = 1.2l_e F_u \]
\[ = 1.2(1.93 \text{ in.})(0.355 \text{ in.})(65 \text{ ksi}) \]
\[ = 53.4 \text{ kips/bolt} \]

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
\theta = \tan^{-1} \left( \frac{V_u}{N_u} \right) & \theta = \tan^{-1} \left( \frac{V_u}{N_u} \right) \\
= \tan^{-1} \left( \frac{75 \text{ kips}}{60 \text{ kips}} \right) & = \tan^{-1} \left( \frac{50 \text{ kips}}{40 \text{ kips}} \right) \\
= 51.3^\circ & = 51.3^\circ \\
\hline
\end{array}
\]

\[
\begin{align*}
\phi &= 0.75 \\
\phi r_n &= 0.75 \left( \frac{53.4 \text{ kips/bolt}}{} \right) \\
&= 40.1 \text{ kips/bolt}
\end{align*}
\]

\[
\begin{align*}
\Omega &= 2.00 \\
\frac{r_n}{\Omega} &= \frac{53.4 \text{ kips/bolt}}{2.00} \\
&= 26.7 \text{ kips/bolt}
\end{align*}
\]

Fig. II.A-1B-3. Bolt tearout on beam web.
Therefore, bolt bearing on the beam web is the controlling limit state for all bolts.

The effective strength for the bolted connection at the beam web is determined by summing the effective strength for each bolt using the minimum available strength calculated for bolt shear, bearing on the beam web, and tearout on the beam web.

\[
\begin{align*}
\phi R_n &= n\phi r_n \\
&= (5 \text{ bolts})(36.4 \text{ kips/bolt}) \\
&= 182 \text{ kips} > 96.0 \text{ kips } \text{ o.k.}
\end{align*}
\]

\[
R_n = n\frac{r_n}{\Omega} = (5 \text{ bolts})(24.3 \text{ kips/bolt}) \\
= 122 \text{ kips} > 64.0 \text{ kips } \text{ o.k.}
\]

**Bolt Shear and Tension Interaction—Outstanding Angle Legs**

The available tensile strength of the bolts due to the effect of combined tension and shear is determined from AISC Specification Section J3.7.

The required shear stress is:

\[
f_{rv} = \frac{V_r}{nA_b}
\]

where

\[
A_b = 0.601 \text{ in.}^2 \text{ (from AISC Manual Table 7-1)}
\]

\[
n = 10
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \[
\begin{align*}
f_{rv} &= \frac{V_u}{nA_b} \\
&= \frac{75 \text{ kips}}{10(0.601 \text{ in.}^2)} \\
&= 12.5 \text{ ksi}
\end{align*}
\] | \[
\begin{align*}
f_{rv} &= \frac{V_u}{nA_b} \\
&= \frac{50 \text{ kips}}{10(0.601 \text{ in.}^2)} \\
&= 8.32 \text{ ksi}
\end{align*}
\] |

The nominal tensile strength modified to include the effects of shear stress is determined from AISC Specification Section J3.7 as follows. From AISC Specification Table J3.2:

\[
F_{nt} = 90 \text{ ksi} \\
F_{nv} = 54 \text{ ksi}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \[
\begin{align*}
F_{nt}' &= 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_{rv} \leq F_{nt} \\
&= 1.3(90 \text{ ksi}) - \frac{90}{0.75(54 \text{ ksi})}(12.5 \text{ ksi}) < 90 \text{ ksi} \\
&= 89.2 \text{ ksi} < 90 \text{ ksi}
\end{align*}
\] | \[
\begin{align*}
F_{nt}' &= 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_{rv} \leq F_{nt} \\
&= 1.3(90 \text{ ksi}) - \frac{2.00(90 \text{ ksi})}{54 \text{ ksi}}(8.32 \text{ ksi}) < 90 \text{ ksi} \\
&= 89.3 \text{ ksi} < 90 \text{ ksi}
\end{align*}
\] |
Using the value of $F'_{u}^{t}$ determined for LRFD, the nominal tensile strength of one bolt is:

\[
\begin{align*}
  r_{n} & = F'_{u}^{t} A_{b} \\
       & = (89.2 \text{ ksi})(0.601 \text{ in.}^{2}) \\
       & = 53.6 \text{ kips}
\end{align*}
\]

The available tensile strength of the bolts due to combined tension and shear is:

\[
\begin{align*}
  \phi & = 0.75 \\
  \phi r_{n} & = 0.75(53.6 \text{ kips/bolt}) \\
           & = 40.2 \text{ kips} \\
  \phi R_{n} & = n \phi r_{n} \\
           & = (10 \text{ bolts})(40.2 \text{ kips/bolt}) \\
           & = 402 \text{ kips} > 60 \text{ kips} \quad \text{o.k.}
\end{align*}
\]

\[
\begin{align*}
  \Omega & = 2.00 \\
  r_{n} & = \frac{53.6 \text{ kips/bolt}}{2.00} \\
        & = 26.8 \text{ kips} \\
  R_{n} & = \frac{r_{n}}{\Omega} \\
        & = \frac{268 \text{ kips} > 40 \text{ kips}}{60} \quad \text{o.k.}
\end{align*}
\]

**Prying Action**

From AISC *Manual* Part 9, the available tensile strength of the bolts in the outstanding angle legs taking prying action into account is determined as follows:

\[
\begin{align*}
  a & = \frac{2(\text{angle leg} + t_{w} - \text{gage})}{2} \\
   & = \frac{2(5 \text{ in.} + 0.355 \text{ in.} - 7\frac{1}{2} \text{ in.})}{2} \\
   & = 1.43 \text{ in.} \\
  b & = \frac{\text{gage} - t_{w} - t}{2} \\
   & = \frac{7\frac{1}{2} \text{ in.} - 0.355 \text{ in.} - \frac{1}{8} \text{ in.}}{2} \\
   & = 3.26 \text{ in.} \\
  a' & = \left( \frac{a + \frac{dh}{2}}{2} \right) \leq 1.25b + \frac{dh}{2} \\
  & = 1.43 \text{ in.} + \frac{\frac{1}{8} \text{ in.}}{2} \leq 1.25(3.26 \text{ in.}) + \frac{\frac{1}{8} \text{ in.}}{2} \\
  & = 1.87 \text{ in.} < 4.51 \text{ in.} \quad \text{o.k.}
\end{align*}
\]
\[ b' = \left( b - \frac{d_p}{2} \right) \]
\[ = 3.26 \text{ in.} - \frac{\frac{1}{8} \text{ in.}}{2} \]
\[ = 2.82 \text{ in.} \]

\[ \rho = \frac{b'}{a'} \]
\[ = \frac{2.82 \text{ in.}}{1.87 \text{ in.}} \]
\[ = 1.51 \]

Note that end distances of 1\(\frac{1}{4}\) in. are used on the angles, so \(p\) is the average pitch of the bolts:

\[ p = \frac{l}{n} \]
\[ = \frac{14\frac{1}{2} \text{ in.}}{5 \text{ rows}} \]
\[ = 2.90 \text{ in.} \]

Check:

\[ p < s = 3.00 \text{ in.} \quad \text{o.k.} \]

\[ \delta = 1 - \frac{d'}{p} \]
\[ = 1 - \frac{15\frac{1}{8} \text{ in.}}{2.90 \text{ in.}} \]
\[ = 0.677 \]

The angle thickness required to develop the available strength of the bolt with no prying action is determined as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td>( B_c = 40.2 \text{ kips/bolt (calculated previously)} )</td>
<td>( B_c = 26.8 \text{ kips/bolt (calculated previously)} )</td>
</tr>
<tr>
<td>( t_c = \sqrt[4]{\frac{4B_c b'}{\phi p F_u}} \quad (\text{Manual Eq. 9-26a}) )</td>
<td>( t_c = \sqrt[4]{\frac{\Omega 4B_c b'}{p F_u}} \quad (\text{Manual Eq. 9-26b}) )</td>
</tr>
<tr>
<td>( = \sqrt[4]{\frac{4(40.2 \text{ kips/bolt}) (2.82 \text{ in.})}{0.90 (2.90 \text{ in.})(58 \text{ ksi})}} )</td>
<td>( = \sqrt[4]{\frac{1.67 (4)(26.8 \text{ kips/bolt}) (2.82 \text{ in.})}{(2.90 \text{ in.})(58 \text{ ksi})}} )</td>
</tr>
<tr>
<td>( = 1.73 \text{ in.} )</td>
<td>( = 1.73 \text{ in.} )</td>
</tr>
</tbody>
</table>
\[ \alpha' = \frac{1}{\delta(1 + \rho)} \left[ \left( \frac{t_c}{t} \right)^2 - 1 \right] \]  
\[ = \frac{1}{0.677(1 + 1.51)} \left[ \left( \frac{1.73 \text{ in.}}{\% \text{ in.}} \right)^2 - 1 \right] \]
\[ = 3.92 \]

Because \( \alpha' > 1 \), the angles have insufficient strength to develop the bolt strength, therefore:

\[ Q = \left( \frac{t}{t_c} \right)^2 (1 + \delta) \]
\[ = \left( \frac{\% \text{ in.}}{1.73 \text{ in.}} \right)^2 (1 + 0.677) \]
\[ = 0.219 \]

The available tensile strength of the bolts, taking prying action into account, is determined using AISC Manual Equation 9-27, as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi r_n = B,Q )</td>
<td>( \frac{r_n}{\Omega} = B,Q )</td>
</tr>
<tr>
<td>( = (40.2 \text{kips/bolt}) (0.219) )</td>
<td>( = (26.8 \text{kips/bolt}) (0.219) )</td>
</tr>
<tr>
<td>( = 8.80 \text{kips/bolt} )</td>
<td>( = 5.87 \text{kips/bolt} )</td>
</tr>
<tr>
<td>( \phi R_n = n\phi r_n )</td>
<td>( R_n = n\frac{r_n}{\Omega} )</td>
</tr>
<tr>
<td>( = (10 \text{bolts}) (8.80 \text{kips/bolt}) )</td>
<td>( = (10 \text{bolts}) (5.87 \text{kips/bolt}) )</td>
</tr>
<tr>
<td>( = 88.0 \text{kips} &gt; 60 \text{kips} \text{ o.k.} )</td>
<td>( = 58.7 \text{kips} &gt; 40 \text{kips} \text{ o.k.} )</td>
</tr>
</tbody>
</table>

**Shear Strength of Angles**

From AISC Specification Section J4.2(a), the available shear yielding strength of the angles is determined as follows:

\[ A_{gv} = (2 \text{angles}) lt \]
\[ = (2 \text{angles}) (14\frac{1}{2} \text{ in.}) (\% \text{ in.}) \]
\[ = 18.1 \text{in.}^2 \]

\[ R_n = 0.60F_y A_{gv} \]  \( \text{Spec. Eq. J4-3} \)
\[ = 0.60(36 \text{ksi})(18.1 \text{in.}^2) \]
\[ = 391 \text{kips} \]
<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 1.00$</td>
<td>$\Omega = 1.50$</td>
</tr>
<tr>
<td>$\phi R_n = 1.00(391 \text{ kips})$</td>
<td>$R_n = 391 \text{ kips}$</td>
</tr>
<tr>
<td>$= 391 \text{ kips} &gt; 96.0 \text{ kips}$ o.k.</td>
<td>$\frac{\Omega}{\phi} = 1.50$</td>
</tr>
<tr>
<td></td>
<td>$= 261 \text{ kips} &gt; 64.0 \text{ kips}$ o.k.</td>
</tr>
</tbody>
</table>

From AISC Specification Section J4.2, the available shear rupture strength of the angle is determined using the net area determined in accordance with AISC Specification Section B4.3b.

$$A_{nv} = (2 \text{ angles})\left[1 - n(d_h + \frac{1}{16} \text{ in.})\right]t$$

$$= (2 \text{ angles})\left[14.5 \text{ in.} - 5(\frac{1}{16} \text{ in.} + \frac{1}{16} \text{ in.})\right](\frac{3}{8} \text{ in.})$$

$$= 11.9 \text{ in.}^2$$

$$R_n = 0.60F_u A_{nv}$$

$$= 0.60(58 \text{ ksi})(11.9 \text{ in.}^2)$$

$$= 414 \text{ kips}$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi R_n = 0.75(414 \text{ kips})$</td>
<td>$R_n = 414 \text{ kips}$</td>
</tr>
<tr>
<td>$= 311 \text{ kips} &gt; 96.0 \text{ kips}$ o.k.</td>
<td>$\frac{\Omega}{\phi} = 2.00$</td>
</tr>
<tr>
<td></td>
<td>$= 207 \text{ kips} &gt; 64.0 \text{ kips}$ o.k.</td>
</tr>
</tbody>
</table>

**Tensile Strength of Angles**

From AISC Specification Section J4.1(a), the available tensile yielding strength of the angles is determined as follows:

$$A_g = (2 \text{ angles})lt$$

$$= (2 \text{ angles})(14.5 \text{ in.})(\frac{3}{8} \text{ in.})$$

$$= 18.1 \text{ in.}^2$$

$$R_n = F_y A_g$$

$$= (36 \text{ ksi})(18.1 \text{ in.}^2)$$

$$= 652 \text{ kips}$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.90$</td>
<td>$\Omega = 1.67$</td>
</tr>
<tr>
<td>$\phi R_n = 0.90(652 \text{ kips})$</td>
<td>$R_n = 652 \text{ kips}$</td>
</tr>
<tr>
<td>$= 587 \text{ kips} &gt; 60 \text{ kips}$ o.k.</td>
<td>$\frac{\Omega}{\phi} = 1.67$</td>
</tr>
<tr>
<td></td>
<td>$= 390 \text{ kips} &gt; 40 \text{ kips}$ o.k.</td>
</tr>
</tbody>
</table>

From AISC Specification Sections J4.1, the available tensile rupture strength of the angles is determined from AISC Specification Equation J4-2. Table D3.1, Case 1 applies in this case because the tension load is transmitted directly
to the cross-sectional element by fasteners; therefore, \( U = 1.00 \). With \( A_{nt} = A_{nv} \) (calculated previously), the effective net area is:

\[
A_e = A_{nt} U = (11.9 \text{ in.}^2)(1.00) = 11.9 \text{ in.}^2
\]

\[
R_n = F_u A_e = (58 \text{ ksi})(11.9 \text{ in.}^2) = 690 \text{ kips}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_n = 0.75(690 \text{ kips}) )</td>
<td>( R_n \leq \frac{690 \text{ kips}}{2.00} = 345 \text{ kips} &gt; 40 \text{ kips} \text{ o.k.} )</td>
</tr>
<tr>
<td>( = 518 \text{ kips} &gt; 60 \text{ kips} \text{ o.k.} )</td>
<td>( = 345 \text{ kips} &gt; 40 \text{ kips} \text{ o.k.} )</td>
</tr>
</tbody>
</table>

**Block Shear Rupture of Angles—Beam Web Side**

The nominal strength for the limit state of block shear rupture of the angles, assuming an L-shaped tearout due the shear load only, is determined as follows. The tearout pattern is shown in Figure II.A-1B-4.

\[
R_{bsv} = 0.60 F_u A_{nv} + U_{bs} F_u A_{nt} \leq 0.60 F_y A_{gv} + U_{bs} F_u A_{nt}
\]

(Spec. Eq. J4-5)

where

\[
A_{gv} = (2 \text{ angles})(l - l_{ev})t = (2 \text{ angles})(14\frac{1}{2} \text{ in.} - 1\frac{1}{4} \text{ in.})(\frac{3}{8} \text{ in.}) = 16.6 \text{ in.}^2
\]

Fig. II.A-1B-4. Block shear rupture of angles for shear load only.
\[ A_{nv} = A_{ge} - (2 \text{ angles})(n - 0.5)(d_h + 0.06 \text{ in.})t \]
\[ = 16.6 \text{ in.}^2 - (2 \text{ angles})(5 - 0.5)(1\frac{3}{6} \text{ in.} + 0.06 \text{ in.})(\frac{5}{8} \text{ in.}) \]
\[ = 11.0 \text{ in.}^2 \]

\[ A_{nt} = (2 \text{ angles})[I_{eh} - 0.5(d_h + 0.06 \text{ in.})]t \]
\[ = (2 \text{ angles})[1\frac{1}{4} \text{ in.} - 0.5(1\frac{3}{6} \text{ in.} + 0.06 \text{ in.})](\frac{5}{8} \text{ in.}) \]
\[ = 0.938 \text{ in.}^2 \]

\[ U_{bs} = 1.0 \]

and

\[ R_{bsv} = 0.60(58 \text{ ksi})(11.0 \text{ in.}^2) + 1.0(58 \text{ ksi})(0.938 \text{ in.}^2) \leq 0.60(36 \text{ ksi})(16.6 \text{ in.}^2) + 1.0(58 \text{ ksi})(0.938 \text{ in.}^2) \]
\[ = 437 \text{ kips} > 413 \text{ kips} \]

Therefore:

\[ R_{bsv} = 413 \text{ kips} \]

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture on the angles is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_{bsv} = 0.75(413 \text{ kips}) )</td>
<td>( R_{bsv} = 413 \text{ kips} )</td>
</tr>
<tr>
<td>= 310 kips &gt; 75 kips o.k.</td>
<td>( \Omega = \frac{2.00}{207 \text{ kips} &gt; 50 \text{ kips}} ) o.k.</td>
</tr>
</tbody>
</table>

The block shear rupture failure path due to axial load only could occur as an L- or U-shape. Assuming an L-shaped tearout relative to the axial load on the angles, the nominal block shear rupture strength in the angles is determined as follows. The tearout pattern is shown in Figure II.A-1B-5.

![Fig. II.A-1B-5. Block shear rupture of angles for axial load only—L-shape.](image-url)
\[ R_{bs} = 0.60F_uA_{nv} + U_{bs}F_uA_{nt} \leq 0.60F_yA_{gv} + U_{bs}F_uA_{nt} \quad (\text{Spec. Eq. J4-5}) \]

where
\[ A_{gv} = (2 \text{ angles})(2 \text{ planes})l_{el}t \]
\[ = (2 \text{ angles})(2\frac{1}{4} \text{ in.})(\frac{3}{8} \text{ in.}) \]
\[ = 3.13 \text{ in.}^2 \]
\[ A_{nv} = A_{gv} - (2 \text{ angles})(0.5)(d_h + \frac{h_1}{2} \text{ in.})t \]
\[ = 1.56 \text{ in.}^2 - (2 \text{ angles})(0.5)(1.56 \text{ in.} + \frac{0.5}{2} \text{ in.})(\frac{3}{8} \text{ in.}) \]
\[ = 0.935 \text{ in.}^2 \]
\[ A_{nt} = (2 \text{ angles})\left[ \left( l - l_{el} \right) - (n - 0.5)(d_h + \frac{h_1}{2} \text{ in.}) \right]t \]
\[ = (2 \text{ angles})\left[ \left( 14\frac{1}{2} \text{ in.} - 1\frac{1}{4} \text{ in.} \right) - (5 - 0.5)(1.56 \text{ in.} + \frac{0.5}{2} \text{ in.}) \right](\frac{3}{8} \text{ in.}) \]
\[ = 10.9 \text{ in.}^2 \]
\[ U_{bs} = 1.0 \]

and
\[ R_{bs} = 0.60(58 \text{ ksi})(0.935 \text{ in.}^2) + 1.0(58 \text{ ksi})(10.9 \text{ in.}^2) \leq 0.60(36 \text{ ksi})(1.56 \text{ in.}^2) + 1.0(58 \text{ ksi})(10.9 \text{ in.}^2) \]
\[ = 665 \text{ kips} < 666 \text{ kips} \]

Therefore:

\[ R_{bs} = 665 \text{ kips} \]

From AISC Specification Section J4.3, the available strength for the limit state of block shear rupture on the angles is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \phi = 0.75 ]</td>
<td>[ \Omega = 2.00 ]</td>
</tr>
<tr>
<td>[ \phi R_{bs} = 0.75(665 \text{ kips}) ]</td>
<td>[ \frac{R_{bs}}{\Omega} = \frac{665 \text{ kips}}{2.00} ]</td>
</tr>
<tr>
<td>= 499 \text{ kips} &gt; 60 \text{ kips} o.k.</td>
<td>= 333 \text{ kips} &gt; 40 \text{ kips} o.k.</td>
</tr>
</tbody>
</table>

The nominal strength for the limit state of block shear rupture assuming an U-shaped tearout relative to the axial load on the angles is determined as follows. The tearout pattern is shown in Figure II.A-1B-6.

\[ R_{bs} = 0.60F_uA_{nv} + U_{bs}F_uA_{nt} \leq 0.60F_yA_{gv} + U_{bs}F_uA_{nt} \quad (\text{Spec. Eq. J4-5}) \]

where
\[ A_{gv} = (2 \text{ angles})(2 \text{ planes})l_{el}t \]
\[ = (2 \text{ angles})(2 \text{ planes})(1\frac{1}{4} \text{ in.})(\frac{3}{8} \text{ in.}) \]
\[ = 3.13 \text{ in.}^2 \]
\[ A_{nv} = (2 \text{ angles})(2 \text{ planes})\left[ l_{ch} - 0.5\left( d_k + \frac{1}{16} \text{ in.} \right) \right] t \]
\[ = (2 \text{ angles})(2 \text{ planes})\left[ 1.25 \text{ in.} - 0.5\left( 0.625 \text{ in.} + 0.0625 \text{ in.} \right) \right](\frac{1}{16} \text{ in.}) \]
\[ = 1.88 \text{ in.}^2 \]

\[ A_{nt} = (2 \text{ angles})\left[ 12.0 \text{ in.} - (n-1)(d_k + \frac{1}{16} \text{ in.}) \right] t \]
\[ = (2 \text{ angles})\left[ 12.0 \text{ in.} - (5-1)(0.625 \text{ in.} + 0.0625 \text{ in.}) \right](\frac{1}{16} \text{ in.}) \]
\[ = 10.0 \text{ in.}^2 \]

\[ U_{bs} = 1.0 \]

and

\[ R_{bsn} = 0.60(58 \text{ ksi})(1.88 \text{ in.}^2) + 1.0(58 \text{ ksi})(10.0 \text{ in.}^2) \leq 0.60(36 \text{ ksi})(3.13 \text{ in.}^2) + 1.0(58 \text{ ksi})(10.0 \text{ in.}^2) \]
\[ = 645 \text{ kips} < 648 \text{ kips} \]

Therefore:

\[ R_{bsn} = 645 \text{ kips} \]

From AISC Specification Section J4.3, the available strength for the limit state of block shear rupture on the angles is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
<td>( \frac{R_{bsn}}{\Omega} = 645 \text{ kips} ) ( \frac{2.00}{323 \text{ kips} &gt; 40 \text{ kips} \text{ o.k.}} )</td>
</tr>
<tr>
<td>( \phi R_{bsn} = 0.75(645 \text{ kips}) )</td>
<td>( \Omega = 2.00 )</td>
<td>( = 323 \text{ kips} &gt; 40 \text{ kips} \text{ o.k.} )</td>
</tr>
</tbody>
</table>

Fig. II.A-1B-6. Block shear rupture of angles for axial load only—U-shape.
Considering the interaction of shear and axial loads, apply a formulation that is similar to AISC Manual Equation 10-5:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \left( \frac{V_u}{\phi R_{svy}} \right)^2 + \left( \frac{N_u}{\phi R_{tan}} \right)^2 \leq 1 ]</td>
<td>[ \left( \frac{V_u}{R_{svy}/\Omega} \right)^2 + \left( \frac{N_u}{R_{tan}/\Omega} \right)^2 \leq 1 ]</td>
</tr>
<tr>
<td>[ \frac{75 \text{ kips}}{310 \text{ kips}}^2 + \left( \frac{60 \text{ kips}}{484 \text{ kips}} \right)^2 = 0.0739 \leq 1 \text{ o.k.} ]</td>
<td>[ \frac{50 \text{ kips}}{207 \text{ kips}}^2 + \left( \frac{40 \text{ kips}}{323 \text{ kips}} \right)^2 = 0.0737 \leq 1 \text{ o.k.} ]</td>
</tr>
</tbody>
</table>

**Block Shear Rupture of Angles–Outstanding Legs**

The nominal strength for the limit state of block shear rupture relative to the shear load on the angles is determined as follows. The tearout pattern is shown in Figure II.A-1B-7.

\[ R_n = 0.60 F_u A_{ev} + U_{ls} F_u A_{nt} \leq 0.60 F_y A_{ev} + U_{ls} F_u A_{nt} \]  
(Spec. Eq. J4-5)

where

\[ A_{ev} = (2 \text{ angles}) (l - l_{ev}) t \]
\[ = (2 \text{ angles}) (14\frac{1}{2} \text{ in.} - 1\frac{1}{4} \text{ in.}) (\frac{3}{8} \text{ in.}) \]
\[ = 16.6 \text{ in.}^2 \]

\[ A_{nv} = A_{ev} - (2 \text{ angles}) (n - 0.5) (d + \frac{1}{8} \text{ in.}) t \]
\[ = 16.6 \text{ in.}^2 - (2 \text{ angles}) (5 - 0.5) (\frac{15}{32} \text{ in.} + \frac{1}{8} \text{ in.}) (\frac{3}{8} \text{ in.}) \]
\[ = 11.0 \text{ in.}^2 \]

\[ A_{nt} = (2 \text{ angles}) [l_{ch} - 0.5 (d + \frac{1}{8} \text{ in.})] t \]
\[ = (2 \text{ angles}) [\frac{1}{8} \text{ in.} - 0.5 (\frac{15}{32} \text{ in.} + \frac{1}{8} \text{ in.})] (\frac{3}{8} \text{ in.}) \]
\[ = 1.17 \text{ in.}^2 \]

**Fig. II.A-1B-7. Block shear rupture of outstanding legs of angles.**
\( U_{bs} = 1.0 \)

and

\[
R_a = 0.60 \times (58 \text{ ksi})(11.0 \text{ in.}^2) + 1.0 \times (58 \text{ ksi})(1.17 \text{ in.}^2) \leq 0.60 \times (36 \text{ ksi})(16.6 \text{ in.}^2) + 1.0 \times (58 \text{ ksi})(1.17 \text{ in.}^2)
\]

\[= 451 \text{ kips} > 426 \text{ kips} \]

Therefore:

\[ R_a = 426 \text{ kips} \]

From AISC Specification Section J4.3, the available strength for the limit state of block shear rupture on the angles is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_a = 0.75 \times 426 \text{ kips} )</td>
<td>( R_a = 426 \text{ kips} )</td>
</tr>
<tr>
<td>= 320 kips &gt; 75 kips o.k.</td>
<td>( \frac{R_a}{\Omega} = \frac{426}{2.00} )</td>
</tr>
<tr>
<td></td>
<td>= 213 kips &gt; 50 kips o.k.</td>
</tr>
</tbody>
</table>

**Shear Strength of Beam Web**

From AISC Specification Section J4.2(a), the available shear yield strength of the beam web is determined as follows:

\[ A_{gy} = d t_w \]

\[= (18.0 \text{ in.})(0.355 \text{ in.}) \]

\[= 6.39 \text{ in.}^2 \]

\[ R_n = 0.60 F_y A_{gy} \]

\[= 0.60 \times (50 \text{ ksi})(6.39 \text{ in.}^2) \]

\[= 192 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 1.00 )</td>
<td>( \Omega = 1.50 )</td>
</tr>
<tr>
<td>( \phi R_n = 1.00 \times 192 \text{ kips} )</td>
<td>( R_n = 192 \text{ kips} )</td>
</tr>
<tr>
<td>= 192 kips &gt; 75 kips o.k.</td>
<td>( \frac{R_n}{\Omega} = \frac{192}{1.50} )</td>
</tr>
<tr>
<td></td>
<td>= 128 kips &gt; 50 kips o.k.</td>
</tr>
</tbody>
</table>

The limit state of shear rupture of the beam web does not apply in this example because the beam is uncoped.

**Tensile Strength of Beam**

From AISC Specification Section J4.1(a), the available tensile yielding strength of the beam is determined as follows:
\[ R_n = F_y A_g \]  
\[ = (50 \text{ ksi}) (14.7 \text{ in.}^2) \]  
\[ = 735 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td>( \phi R_n = 0.90(735 \text{ kips}) )</td>
<td>( R_n = 735 \text{ kips} )</td>
</tr>
<tr>
<td>( = 662 \text{ kips} &gt; 60 \text{ kips} )</td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td>o.k.</td>
<td>( = 440 \text{ kips} &gt; 40 \text{ kips} )</td>
</tr>
</tbody>
</table>

From AISC Specification Section J4.1(b), determine the available tensile rupture strength of the beam. The effective net area is \( A_e = A_n U \). No cases in AISC Specification Table D3.1 apply to this configuration; therefore, \( U \) is determined from AISC Specification Section D3.

\[ A_n = A_g - n (d_b + \frac{1}{16} \text{ in.})(t_w) \]  
\[ = 14.7 \text{ in.}^2 - 5(15.625 \text{ in.} + \frac{1}{16} \text{ in.})(0.355 \text{ in.}) \]  
\[ = 12.9 \text{ in.}^2 \]

As stated in AISC Specification Section D3, the value of \( U \) can be determined as the ratio of the gross area of the connected element (beam web) to the member gross area.

\[ U = \frac{(d - 2t_f)(t_w)}{A_g} \]  
\[ = \frac{[18.0 \text{ in.} - 2(0.570 \text{ in.})](0.355 \text{ in.})}{14.7 \text{ in.}^2} \]  
\[ = 0.407 \]

\[ A_e = A_n U \]  
\[ = (12.9 \text{ in.}^2)(0.407) \]  
\[ = 5.25 \text{ in.}^2 \]

\[ R_n = F_y A_e \]  
\[ = (65 \text{ ksi})(5.25 \text{ in.}^2) \]  
\[ = 341 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_n = 0.75(341 \text{ kips}) )</td>
<td>( R_n = 341 \text{ kips} )</td>
</tr>
<tr>
<td>( = 256 \text{ kips} &gt; 60 \text{ kips} )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>o.k.</td>
<td>( = 171 \text{ kips} &gt; 40 \text{ kips} )</td>
</tr>
</tbody>
</table>

**Block Shear Rupture Strength of Beam Web**

Block shear rupture is only applicable in the direction of the axial load, because the beam is uncoped and the limit state is not applicable for an uncoped beam subject to vertical shear. Assuming a U-shaped tearout relative to the
axial load, and assuming a horizontal edge distance of \( l_{eh} = 1\frac{1}{4} \text{ in.} - \frac{1}{4} \text{ in.} = 1\frac{1}{2} \text{ in.} \) to account for a possible beam underrun of \( \frac{1}{4} \text{ in.} \), the block shear rupture strength is:

\[
R_u = 0.60F_u A_{nv} + U_{nt} F_u A_{nt} \leq 0.60F_y A_{gv} + U_{nt} F_u A_{nt} \quad \text{(Spec. Eq. J4-5)}
\]

where

\[
A_{gv} = (2)l_{eh} t_w = (2)(1\frac{1}{2} \text{ in.})(0.355 \text{ in.}) = 1.07 \text{ in.}^2
\]

\[
A_{nv} = A_{gv} - (2)(0.5)(d_h + \frac{1}{16} \text{ in.}) t_w = 1.07 \text{ in.}^2 - (2)(0.5)(1\frac{1}{16} \text{ in.} + \frac{1}{16} \text{ in.})(0.355 \text{ in.}) = 0.715 \text{ in.}^2
\]

\[
A_{nt} = \left[12.0 \text{ in.} - (n - 1)(d_h + \frac{1}{16} \text{ in.})\right] t_w = \left[12.0 \text{ in.} - (5 - 1)(1\frac{1}{16} \text{ in.} + \frac{1}{16} \text{ in.})\right](0.355 \text{ in.}) = 2.84 \text{ in.}^2
\]

\[
U_{bs} = 1.0
\]

and

\[
R_u = 0.60(65 \text{ ksi})(0.710 \text{ in.}^2) + 1.0(65 \text{ ksi})(2.84 \text{ in.}^2) \leq 0.60(50 \text{ ksi})(1.07 \text{ in.}^2) + 1.0(65 \text{ ksi})(2.84 \text{ in.}^2)
\]

\[
= 212 \text{ kips} < 217 \text{ kips}
\]

Therefore:

\[
R_u = 212 \text{ kips}
\]

From AISC Specification Section J4.3, the available strength for the limit state of block shear rupture of the beam web is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_u = 0.75(212 \text{ kips}) )</td>
<td>( R_u = \frac{212 \text{ kips}}{2.00} = 106 \text{ kips} &gt; 40 \text{ kips} )</td>
</tr>
<tr>
<td>= 159 kips &gt; 60 kips o.k.</td>
<td>= 106 kips &gt; 40 kips o.k.</td>
</tr>
</tbody>
</table>

**Conclusion**

The connection is found to be adequate as given for the applied loads.
EXAMPLE II.A-2A BOLTED/WELDED DOUBLE-ANGLE CONNECTION

Given:

Using the tables in AISC Manual Part 10, verify the available strength of a double-angle shear connection with welds in the support legs (welds B) and bolts in the supported-beam-web legs, as shown in Figure II.A-2A-1. The ASTM A992 W36×231 beam is attached to an ASTM A992 W14×90 column flange supporting the following beam end reactions:

\[ R_D = 37.5 \text{ kips} \]
\[ R_L = 113 \text{ kips} \]

Use ASTM A36 angles and 70-ksi weld electrodes.

**Solution:**

From AISC Manual Table 2-4, the material properties are as follows:

Beam and column
ASTM A992
\[ F_y = 50 \text{ ksi} \]
\[ F_u = 65 \text{ ksi} \]

Angles
ASTM A36
\[ F_y = 36 \text{ ksi} \]
\[ F_u = 58 \text{ ksi} \]

From AISC Manual Table 1-1, the geometric properties are as follows:

*(Figure II.A-2A-1. Connection geometry for Example II.A-2A.)*

Note: Bottom flange coped for erection.
Beam
W36×231
$t_w = 0.760$ in.

Column
W14×90
$t_f = 0.710$ in.

From AISC Specification Table J3.3, the hole diameter for $\frac{3}{16}$-in.-diameter bolts with standard holes is:

$$d_h = \frac{3}{16} \text{ in.}$$

From ASCE/SEI 7, Chapter 2, the required strength is:

$$\begin{align*}
R_n &= 1.2(37.5 \text{ kips}) + 1.6(113 \text{ kips}) \\
&= 226 \text{ kips} \\
R_n &= 37.5 \text{ kips} + 113 \text{ kips} \\
&= 151 \text{ kips}
\end{align*}$$

Weld Design

Use AISC Manual Table 10-2 (welds B) with $n = 8$. Try $\frac{3}{16}$-in. weld size, $l = 23\frac{1}{2}$ in. From AISC Manual Table 10-2, the minimum support thickness is:

$$t_{min} = 0.238 \text{ in.} < 0.710 \text{ in.} \quad \text{o.k.}$$

Angle Thickness

From AISC Specification Section J2.2b, the minimum angle thickness for a $\frac{3}{16}$-in. fillet weld is:

$$t = w + \frac{3}{16} \text{ in.}$$

$$= \frac{3}{16} \text{ in.} + \frac{3}{16} \text{ in.}$$

$$= \frac{3}{8} \text{ in.}$$

Try 2L4×3\%×\% (SLBB).

Angle and Bolt Design

AISC Manual Table 10-1 includes checks for bolt shear, bolt bearing and tearout on the angles, shear yielding of the angles, shear rupture of the angles, and block shear rupture of the angles.

Check 8 rows of bolts and $\frac{3}{8}$-in. angle thickness.

$$\begin{align*}
\phi R_n &= 284 \text{ kips} > 226 \text{ kips} \quad \text{o.k.} \\
\frac{R_n}{\Omega} &= 189 \text{ kips} > 151 \text{ kips} \quad \text{o.k.}
\end{align*}$$
**Beam Web Strength**

The available beam web strength is the lesser of the limit states of block shear rupture, shear yielding, shear rupture, and the sum of the effective strengths of the individual fasteners. In this example, because of the relative size of the cope to the overall beam size, the coped section will not control, therefore, the strength of the bolt group will control (When this cannot be determined by inspection, see AISC Manual Part 9 for the design of the coped section). From the Commentary to AISC Specification Section J3.6, the strength of the bolt group is taken as the sum of the effective strengths of the individual fasteners. The effective strength of an individual fastener is the lesser of the shear strength, the bearing strength at the bolt holes, and the tearout strength at the bolt holes.

**Bolt Shear**

From AISC Manual Table 7-1, the available shear strength per bolt for ¼-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) in double shear is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_n = 35.8 \text{ kips/bolt}$</td>
<td>$R_n = 23.9 \text{ kips/bolt}$</td>
</tr>
</tbody>
</table>

**Bolt Bearing on Beam Web**

The nominal bearing strength of the beam web per bolt is determined from AISC Specification Section J3.10, assuming deformation at service load is a design consideration:

$$r_n = 2.4 \text{d}t F_u$$

$$= 2.4 \left( \frac{3}{8} \text{ in.} \right) \left( 0.760 \text{ in.} \right) (65 \text{ ksi})$$

$$= 88.9 \text{ kips/bolt}$$

From AISC Specification Section J3.10, the available bearing strength of the beam web per bolt is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi r_n = 0.75(88.9 \text{ kips/bolt})$</td>
<td>$r_n = 88.9 \text{ kips/bolt}$</td>
</tr>
<tr>
<td>$= 66.7 \text{ kips/bolt}$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td></td>
<td>$= 44.5 \text{ kips/bolt}$</td>
</tr>
</tbody>
</table>

**Bolt Tearout on Beam Web**

The available tearout strength of the beam web per bolt is determined from AISC Specification Section J3.10, assuming deformation at service load is a design consideration:

$$l_c = 3.00 \text{ in.} - \frac{1}{16} \text{ in.}$$

$$= 2.19 \text{ in.}$$

$$r_n = 1.2 l_c t F_u$$

$$= 1.2(2.19 \text{ in.}) \left( 0.760 \text{ in.} \right) (65 \text{ ksi})$$

$$= 130 \text{ kips/bolt}$$

From AISC Specification Section J3.10, the available tearout strength of the beam web per bolt is:
Bolt shear strength is the governing limit state for all bolts at the beam web. Bolt shear strength is one of the limit states included in the capacities shown in Table 10-1 as used above; thus, the effective strength of the fasteners is adequate.

**Available strength at the column flange**

Since the thickness of the column flange, $t_f = 0.710$ in., is greater than the thickness of the angles, $t = \frac{3}{8}$ in., shear will control for the angles. The column flange is adequate for the required loading.

**Summary**

The connection is found to be adequate as given for the applied loads.
EXAMPLE II.A-2B BOLTED/WELDED DOUBLE-ANGLE CONNECTION SUBJECT TO AXIAL AND SHEAR LOADING

Given:

Verify the available strength of a double-angle connection with welds in the supported-beam-web legs and bolts in the outstanding legs for an ASTM A992 W18×50 beam, as shown in Figure II.A-2B-1, to support the following beam end reactions:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear, (V_u = 75) kips</td>
<td>Shear, (V_a = 50) kips</td>
</tr>
<tr>
<td>Axial tension, (N_u = 60) kips</td>
<td>Axial tension, (N_a = 40) kips</td>
</tr>
</tbody>
</table>

Use ASTM A36 angles and 70-ksi electrodes.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

- Beam
  - ASTM A992
  - \(F_y = 50\) ksi
  - \(F_u = 65\) ksi

- Angles
  - ASTM A36
  - \(F_y = 36\) ksi
  - \(F_u = 58\) ksi

From AISC Manual Table 1-1, the geometric properties are as follows:
Beam
W18×50

\( A_g = 14.7 \text{ in.}^2 \)
\( d = 18.0 \text{ in.} \)
\( t_w = 0.355 \text{ in.} \)
\( b_f = 7.50 \text{ in.} \)
\( t_f = 0.570 \text{ in.} \)

From AISC Specification Table J3.3, the hole diameter for \( \frac{3}{8}\)-in.-diameter bolts with standard holes is:

\( d_h = \frac{3}{16} \text{ in.} \)

The resultant load is:

\[
\begin{align*}
\text{LRFD} & \quad \text{ASD} \\
R_u &= \sqrt{V_u^2 + N_u^2} \\
&= \sqrt{(75 \text{ kips})^2 + (60 \text{ kips})^2} \\
&= 96.0 \text{ kips} \\
\frac{V_a}{V_u} &= \sqrt{V_a^2 + N_a^2} \\
&= \sqrt{(50 \text{ kips})^2 + (40 \text{ kips})^2} \\
&= 64.0 \text{ kips}
\end{align*}
\]

The following bolt shear, bearing and tearout calculations are for a pair of bolts.

**Bolt Shear**

From AISC Manual Table 7-1, the available shear strength for \( \frac{3}{8}\)-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) in double shear (or pair of bolts):

\[
\begin{align*}
\text{LRFD} & \quad \text{ASD} \\
\phi r_u &= 48.7 \text{ kips (for pair of bolts)} \\
\frac{r_u}{\Omega} &= 32.5 \text{ kips (for pair of bolts)}
\end{align*}
\]

**Bolt Bearing on Angles**

The available bearing strength of the double angle is determined from AISC Specification Section J3.10, assuming deformation at the bolt hole is a design consideration:

\[
r_u = (2 \text{ bolts}) 2.4dtF_u
\]

\[
= (2 \text{ bolts}) (2.4) \left( \frac{3}{8} \text{ in.} \right) \left( \frac{1}{2} \text{ in.} \right) (58 \text{ ksi})
\]

\[
= 122 \text{ kips (for pair of bolts)}
\]

The available bearing strength for a pair of bolts is:

\[
\begin{align*}
\text{LRFD} & \quad \text{ASD} \\
\phi &= 0.75 \\
\phi r_u &= 0.75 (122 \text{ kips}) \\
&= 91.5 \text{ kips (for pair of bolts)} \\
\frac{r_u}{\Omega} &= \frac{122 \text{ kips}}{2.00}
\end{align*}
\]

\[
= 61.0 \text{ kips (for pair of bolts)}
\]

The bolt shear strength controls over bearing in the angles.
Bolt Tearout on Angles

The available tearout strength of the angle is determined from AISC Specification Section J3.10, assuming deformation at the bolt hole is a design consideration:

For the edge bolt:

\[ l_c = l_e - 0.5d_h \]
\[ = 1\frac{1}{4} \text{ in.} - 0.5(\frac{5}{8} \text{ in.}) \]
\[ = 0.781 \text{ in.} \]

\[ r_n = (2 \text{ bolts})1.2l_cF_{tu} \]
\[ = (2 \text{ bolts})(1.2)(0.781 \text{ in.})(\frac{1}{2} \text{ in.})(58 \text{ ksi}) \]
\[ = 54.4 \text{ kips (for pair of bolts)} \]

The available tearout strength of the angles for a pair of edge bolts is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi r_n = 0.75(54.4 \text{ kips}) )</td>
<td>( \frac{r_n}{\Omega} = \frac{54.4 \text{ kips}}{2.00} = 27.2 \text{ kips} )</td>
</tr>
<tr>
<td>( = 40.8 \text{ kips} )</td>
<td></td>
</tr>
</tbody>
</table>

The tearout strength controls over bolt shear and bearing for the edge bolts in the angles.

For the other bolts:

\[ l_c = s - d_h \]
\[ = 3 \text{ in.} - \frac{15}{8} \text{ in.} \]
\[ = 2.06 \text{ in.} \]

\[ r_n = (2 \text{ bolts})1.2l_cF_{tu} \]
\[ = (2 \text{ bolts})(1.2)(2.06 \text{ in.})(\frac{1}{2} \text{ in.})(58 \text{ ksi}) \]
\[ = 143 \text{ kips (for pair of bolts)} \]

The available tearout strength for a pair of other bolts is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi r_n = 0.75(143 \text{ kips}) )</td>
<td>( \frac{r_n}{\Omega} = \frac{143 \text{ kips}}{2.00} = 71.5 \text{ kips (for pair of bolts)} )</td>
</tr>
<tr>
<td>( = 107 \text{ kips (for pair of bolts)} )</td>
<td></td>
</tr>
</tbody>
</table>

Bolt shear strength controls over tearout and bearing strength for the other bolts in the angles.
**Strength of Bolted Connection**

The effective strength for the bolted connection at the angles is determined by summing the effective strength for each bolt using the minimum available strength calculated for bolt shear, bearing on the angles, and tearout on the angles.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_n = (1 \text{ bolt})(40.8 \text{ kips}) + (4 \text{ bolts})(48.7 \text{ kips}) = 236 \text{ kips} &gt; 75 \text{ kips \ o.k.} )</td>
<td>( R_n = (1 \text{ bolt})(27.2 \text{ kips}) + (4 \text{ bolts})(32.5 \text{ kips}) = 157 \text{ kips} &gt; 50 \text{ kips \ o.k.} )</td>
</tr>
</tbody>
</table>

**Shear and Tension Interaction in Bolts**

The required shear stress for each bolt is determined as follows:

\[
f_{rv} = \frac{V_r}{nA_b}
\]

where

\( A_b = 0.601 \text{ in}^2 \) (from AISC Manual Table 7-1)

\( n = 10 \) bolts

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{rv} = \frac{75 \text{ kips}}{(10 \text{ bolts})(0.601 \text{ in}^2)} = 12.5 \text{ ksi} )</td>
<td>( f_{rv} = \frac{50 \text{ kips}}{(10 \text{ bolts})(0.601 \text{ in}^2)} = 8.32 \text{ ksi} )</td>
</tr>
</tbody>
</table>

The nominal tensile stress modified to include the effects of shear stress is determined from AISC Specification Section J3.7 as follows. From AISC Specification Table J3.2:

\( F_{nt} = 90 \text{ ksi} \)

\( F_{nv} = 54 \text{ ksi} \)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( F_{nt}' = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_{rv} \leq F_{nt} ) (Spec. Eq. J3-3a)</td>
<td>( F_{nt}' = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_{rv} \leq F_{nt} ) (Spec. Eq. J3-3b)</td>
</tr>
<tr>
<td>( = 1.3(90 \text{ ksi}) - \frac{90 \text{ ksi}}{0.75(54 \text{ ksi})}(12.5 \text{ ksi}) \leq 90 \text{ ksi} ) = 89.2 ksi &lt; 90 ksi ( \text{o.k.} )</td>
<td>( = 1.3(90 \text{ ksi}) - \frac{2.00(90 \text{ ksi})}{54 \text{ ksi}}(8.32 \text{ ksi}) \leq 90 \text{ ksi} ) = 89.3 ksi &lt; 90 ksi ( \text{o.k.} )</td>
</tr>
</tbody>
</table>

Using the value of \( F_{nt}' = 89.2 \text{ ksi} \) determined for LRFD, the nominal tensile strength of one bolt is:

\[
r_n = \frac{F_{nt}' A_b}{\phi} = (89.2 \text{ ksi})(0.601 \text{ in}^2) = 53.6 \text{ kips}
\]
The available tensile strength due to combined tension and shear is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \phi R_n = n \phi r_n )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( (10 \text{ bolts})(0.75)(53.6 \text{ kips}) )</td>
<td>( n = \frac{n}{\Omega} )</td>
<td>( (10 \text{ bolts}) \left( \frac{53.6 \text{ kips}}{2.00} \right) )</td>
</tr>
<tr>
<td>( = 402 \text{ kips} &gt; 60 \text{ kips} \text{ o.k.} )</td>
<td>( = 268 \text{ kips} &gt; 40 \text{ kips} \text{ o.k.} )</td>
<td></td>
</tr>
</tbody>
</table>

**Prying Action on Bolts**

From AISC *Manual* Part 9, the available tensile strength of the bolts in the outstanding angle legs taking prying action into account is determined as follows:

\[
a = \frac{\text{angle leg} + t_w - \text{gage}}{2}
\]

\[
= \frac{(4.00 \text{ in.})(2) + 0.355 \text{ in.} - 5/2 \text{ in.}}{2}
\]

\[
= 1.43 \text{ in.}
\]

Note: If the distance from the bolt centerline to the edge of the supporting element is smaller than \( a = 1.43 \text{ in.} \), use the smaller \( a \) in the following calculation.

\[
b = \frac{\text{gage} - t_w - t}{2}
\]

\[
= \frac{5/2 \text{ in.} - 0.355 \text{ in.} - \frac{1}{2} \text{ in.}}{2}
\]

\[
= 2.32 \text{ in.}
\]

\[
a' = \left( a + \frac{d_b}{2} \right) \leq \left( 1.25b + \frac{d_b}{2} \right)
\]

\[
= 1.43 \text{ in.} + \frac{\frac{3}{4} \text{ in.}}{2} \leq 1.25(2.32 \text{ in.}) + \frac{\frac{3}{4} \text{ in.}}{2}
\]

\[
= 1.87 \text{ in.} < 3.34 \text{ in.}
\]

\[
= 1.87 \text{ in.}
\]

\[
b' = \left( b - \frac{d_b}{2} \right)
\]

\[
= 2.32 \text{ in.} - \frac{\frac{3}{4} \text{ in.}}{2}
\]

\[
= 1.88 \text{ in.}
\]

\[
\rho = \frac{b'}{a'}
\]

\[
= \frac{1.88 \text{ in.}}{1.87 \text{ in.}}
\]

\[
= 1.01
\]
Note that end distances of 1¼ in. are used on the angles, so \( p \) is the average pitch of the bolts:

\[
p = \frac{l}{n} = \frac{14\frac{1}{2}}{5} \text{ in.} = 2.90 \text{ in.}
\]

Check:

\[
p \leq s = 3 \text{ in.} \quad \text{O.K.}
\]

\[
d' = d_b = 15/64 \text{ in.}
\]

\[
\delta = 1 - \frac{d'}{p} = 1 - \frac{15/64}{2.90} = 0.677
\]

The angle thickness required to develop the available strength of the bolt with no prying action as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_c = 40.2 \text{ kips/bolt (calculated previously)} )</td>
<td>( B_c = 26.8 \text{ kips/bolt (calculated previously)} )</td>
</tr>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td>( t_c = \sqrt{\frac{4B_c b'}{\phi p F_u}} )</td>
<td>( t_c = \sqrt{\frac{\Omega 4B_c b'}{p F_u}} )</td>
</tr>
<tr>
<td>= ( \sqrt{\frac{4(40.2 \text{ kips/bolt }) (1.88 \text{ in.})}{0.90 (2.90 \text{ in.}) (58 \text{ ksi})}} )</td>
<td>( = \sqrt{\frac{1.67 (4) (26.8 \text{ kips/bolt }) (1.88 \text{ in.})}{(2.90 \text{ in.}) (58 \text{ ksi})}} )</td>
</tr>
<tr>
<td>= 1.41 in.</td>
<td>= 1.41 in.</td>
</tr>
</tbody>
</table>

\[
\alpha' = \frac{1}{\delta (1 + \rho)} \left[ \left( \frac{t_c}{t} \right)^2 - 1 \right] = \frac{1}{0.677 (1 + 1.01)} \left[ \left( \frac{1.41 \text{ in.}}{\frac{1}{2} \text{ in.}} \right)^2 - 1 \right]
\]

\[
= 5.11
\]

Because \( \alpha' > 1 \), the angles have insufficient strength to develop the bolt strength, therefore:

\[
Q = \left( \frac{t}{t_c} \right)^2 (1 + \delta) = \left( \frac{\frac{1}{2} \text{ in.}}{1.41 \text{ in.}} \right)^2 (1 + 0.677) = 0.211
\]
The available tensile strength of the bolts, taking prying action into account is determined from AISC Manual Equation 9-27, as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi r_n = B_c Q$</td>
<td>$r_n = B_c Q$</td>
</tr>
<tr>
<td>= (40.2 kips/bolt)(0.211)</td>
<td>= (26.8 kips/bolt)(0.211)</td>
</tr>
<tr>
<td>= 8.48 kips/bolt</td>
<td>= 5.65 kips/bolt</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\phi R_n = n\phi r_n$</th>
<th>$R_n = n\frac{r_n}{\Omega}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>= (10 bolts)(8.48 kips/bolt)</td>
<td>= (10 bolts)(5.65 kips/bolt)</td>
</tr>
<tr>
<td>= 84.8 kips &gt; 60 kips o.k.</td>
<td>= 56.5 kips &gt; 40 kips o.k.</td>
</tr>
</tbody>
</table>

**Weld Design**

The resultant load angle on the weld is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = \tan^{-1}\left(\frac{N_u}{V_u}\right)$</td>
<td>$\theta = \tan^{-1}\left(\frac{N_u}{V_a}\right)$</td>
</tr>
<tr>
<td>= $\tan^{-1}\left(\frac{60 \text{ kips}}{75 \text{ kips}}\right)$</td>
<td>= $\tan^{-1}\left(\frac{40 \text{ kips}}{50 \text{ kips}}\right)$</td>
</tr>
<tr>
<td>= 38.7°</td>
<td>= 38.7°</td>
</tr>
</tbody>
</table>

From AISC Manual Table 8-8 for Angle = 30° (which will lead to a conservative result), using total beam setback of ½ in. + ¼ in. = ¾ in. (the ¼ in. is included to account for mill underrun):

$l = 14\frac{1}{2} \text{ in.}$

$kl = 3\frac{1}{2} \text{ in.} - \frac{3}{4} \text{ in.}$

$= 2.75 \text{ in.}$

$k = \frac{kl}{l}$

$= \frac{2.75 \text{ in.}}{14\frac{1}{2} \text{ in.}}$

$= 0.190$

$x = 0.027$ by interpolation

$al = 3\frac{1}{2} \text{ in.} - xl$

$= 3\frac{1}{2} \text{ in.} - 0.027 \left(14\frac{1}{2} \text{ in.}\right)$

$= 3.11 \text{ in.}$
\[ a = \frac{a}{l} \]
\[ = \frac{3.11 \text{ in.}}{14\frac{1}{2} \text{ in.}} \]
\[ = 0.214 \]

\[ C = 2.69 \text{ by interpolation} \]

The required weld size is determined using AISC Manual Equation 8-21, as follows:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_{\text{min}} )</td>
<td>( \frac{R_u}{\phi C C_l l} )</td>
<td>( \frac{\Omega R_u}{C C_l l} )</td>
</tr>
<tr>
<td></td>
<td>96.0 kips</td>
<td>2.00 ((64.0 \text{ kips}))</td>
</tr>
<tr>
<td></td>
<td>( 0.75(2.69)(1)(14\frac{1}{2} \text{ in.})(2 \text{ sides}) )</td>
<td>( 2.69(1)(14\frac{1}{2} \text{ in.})(2 \text{ sides}) )</td>
</tr>
<tr>
<td></td>
<td>1.64 sixteenths</td>
<td>1.64 sixteenths</td>
</tr>
</tbody>
</table>

Use a \( \frac{3}{16} \)-in. fillet weld (minimum size from AISC Specification Table J2.4).

**Beam Web Strength at Fillet Weld**

The minimum beam web thickness required to match the shear rupture strength of a weld both sides to that of the base metal is:

\[ t_{\text{min}} = \frac{6.19 D_{\text{min}}}{F_u} \]
\[ = \frac{6.19(1.64)}{65 \text{ ksi}} \]
\[ = 0.156 \text{ in.} < 0.355 \text{ in.} \text{ ok.} \]

**Shear Strength of Angles**

From AISC Specification Section J4.2(a), the available shear yielding strength of the angles is determined as follows:

\[ A_{gy} = (2 \text{ angles})lt \]
\[ = (2 \text{ angles})(14\frac{1}{2} \text{ in.})(\frac{1}{2} \text{ in.}) \]
\[ = 14.5 \text{ in}^2 \]

\[ R_u = 0.60F_y A_{gy} \]
\[ = 0.60(36 \text{ ksi})(14.5 \text{ in}^2) \]
\[ = 313 \text{ kips} \]
From AISC *Specification* Section J4.2(b), the available shear rupture strength of the angle is determined as follows. The effective net area is determined in accordance with AISC *Specification* Section B4.3b.

\[
A_{nv} = (2 \text{ angles}) \left[ I - n (d_{b} + \frac{1}{8} \text{ in.}) \right] t \\
= (2 \text{ angles}) \left[ 14\frac{1}{2} \text{ in.} - 5(\frac{1}{8} \text{ in.} + \frac{1}{8} \text{ in.}) \right] (\frac{1}{2} \text{ in.}) \\
= 9.50 \text{ in.}^2
\]

\[
R_n = 0.60 F_u A_{nv} \\
= 0.60 (58 \text{ ksi}) (9.50 \text{ in.}^2) \\
= 331 \text{ kips}
\]

### LRFD

<table>
<thead>
<tr>
<th>( \phi = 1.00 )</th>
<th>( \Omega = 1.50 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_n = 1.00(313 \text{ kips}) )</td>
<td>( \frac{R_n}{\Omega} = \frac{313 \text{ kips}}{1.50} )</td>
</tr>
<tr>
<td>= 313 kips &gt; 96.0 kips <strong>o.k.</strong></td>
<td>= 209 kips &gt; 64.0 kips <strong>o.k.</strong></td>
</tr>
</tbody>
</table>

### ASD

<table>
<thead>
<tr>
<th>( R_n = 313 \text{ kips} )</th>
<th>( \Omega = 1.50 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{R_n}{\Omega} = \frac{313 \text{ kips}}{1.50} )</td>
<td>= 209 kips &gt; 64.0 kips <strong>o.k.</strong></td>
</tr>
</tbody>
</table>

### Tensile Strength of Angles—Beam Web Side

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the angles is determined as follows:

\[
A_g = (2 \text{ angles}) l t \\
= (2 \text{ angles}) (14\frac{1}{2} \text{ in.}) (\frac{1}{2} \text{ in.}) \\
= 14.5 \text{ in.}^2
\]

\[
R_n = F_y A_g \\
= (36 \text{ ksi}) (14.5 \text{ in.}^2) \\
= 522 \text{ kips}
\]

### LRFD

<table>
<thead>
<tr>
<th>( \phi = 0.75 )</th>
<th>( \Omega = 2.00 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_n = 0.75(331 \text{ kips}) )</td>
<td>( \frac{R_n}{\Omega} = \frac{331 \text{ kips}}{2.00} )</td>
</tr>
<tr>
<td>= 248 kips &gt; 96.0 kips <strong>o.k.</strong></td>
<td>= 166 kips &gt; 64.0 kips <strong>o.k.</strong></td>
</tr>
</tbody>
</table>

### ASD

<table>
<thead>
<tr>
<th>( R_n = 331 \text{ kips} )</th>
<th>( \Omega = 2.00 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{R_n}{\Omega} = \frac{331 \text{ kips}}{2.00} )</td>
<td>= 166 kips &gt; 64.0 kips <strong>o.k.</strong></td>
</tr>
</tbody>
</table>

From AISC *Specification* Sections J4.1(b), the available tensile rupture strength of the angles is determined as follows:

### LRFD

<table>
<thead>
<tr>
<th>( \phi = 0.90 )</th>
<th>( \Omega = 1.67 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_n = 0.90(522 \text{ kips}) )</td>
<td>( \frac{R_n}{\Omega} = \frac{522 \text{ kips}}{1.67} )</td>
</tr>
<tr>
<td>= 470 kips &gt; 60 kips <strong>o.k.</strong></td>
<td>= 313 kips &gt; 40 kips <strong>o.k.</strong></td>
</tr>
</tbody>
</table>

### ASD

<table>
<thead>
<tr>
<th>( R_n = 522 \text{ kips} )</th>
<th>( \Omega = 1.67 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{R_n}{\Omega} = \frac{522 \text{ kips}}{1.67} )</td>
<td>= 313 kips &gt; 40 kips <strong>o.k.</strong></td>
</tr>
</tbody>
</table>
\[ R_n = F_u A_e \]  

(Spec. Eq. J4-2)

Because the angle legs are welded to the beam web there is no bolt hole reduction and \( A_e = A_{eg} \); therefore, tensile rupture will not control.

**Block Shear Rupture Strength of Angles–Outstanding Legs**

The nominal strength for the limit state of block shear rupture of the angles assuming an L-shaped tearout relative to shear load, is determined as follows. The tearout pattern is shown in Figure II.A-2B-2.

\[ R_n = 0.60 F_u A_{nv} + U_{in} F_u A_{nt} \leq 0.60 F_y A_{eg} + U_{in} F_u A_{nt} \]  

(Spec. Eq. J4-5)

where

\[ l_{ch} = \frac{2(\text{angle leg}) + t_{ng} - \text{gage}}{2} \]

\[ = \frac{2(\text{4 in.}) + 0.355 \text{ in.} - 5/2 \text{ in.}}{2} \]

\[ = 1.43 \text{ in.} \]

\[ A_{nt} = (2 \text{ angles}) \left[ l_{ch} - 0.5 \left( d_h + \frac{\text{gage}}{2} \right) \right] (t) \]

\[ = (2 \text{ angles}) \left[ 1.43 \text{ in.} - 0.5 \left( \frac{3/4 \text{ in.} + 1/6 \text{ in.}}{2} \right) \right] (1/2 \text{ in.}) \]

\[ = 0.930 \text{ in.}^2 \]

\[ A_{eg} = (2 \text{ angles}) \left[ l_{ev} + (n - 1) s \right] (t) \]

\[ = (2 \text{ angles}) \left[ 1/4 \text{ in.} + (5 - 1)(3 \text{ in.}) \right] (1/2 \text{ in.}) \]

\[ = 13.3 \text{ in.}^2 \]

![Fig. II.A-2B-2. Block shear rupture of outstanding legs of angles.](image_url)
\[ A_{nv} = A_{gv} - (2 \text{ angles})(n - 0.5)(d + \frac{1}{16} \text{ in.})(t) \]
\[ = 13.3 \text{ in.}^2 - (2 \text{ angles})(5 - 0.5)(\frac{1}{16} \text{ in.} + \frac{1}{16} \text{ in.})(\frac{1}{2} \text{ in.}) \]
\[ = 8.80 \text{ in.}^2 \]

\[ U_{bs} = 1.0 \]

and
\[ R_n = 0.60(58 \text{ ksi})(8.80 \text{ in.}^2) + 1.0(58 \text{ ksi})(0.930 \text{ in.}^2) \leq 0.60(36 \text{ ksi})(13.3 \text{ in.}^2) + 1.0(58 \text{ ksi})(0.930 \text{ in.}^2) \]
\[ = 360 \text{ kips} > 341 \text{ kips} \]

Therefore:
\[ R_n = 341 \text{ kips} \]

The available block shear rupture strength of the angles is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_n = 0.75(341 \text{ kips}) )</td>
<td>( R_n = 341 \text{ kips} )</td>
</tr>
<tr>
<td>= 256 kips &gt; 75 kips \textbf{ o.k.}</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( R_n )</td>
<td>( R_n = 341 \text{ kips} )</td>
</tr>
<tr>
<td>( \Omega = 2.00 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( = 171 \text{ kips} &gt; 50 \text{ kips} \textbf{ o.k.} )</td>
<td>( = 171 \text{ kips} &gt; 50 \text{ kips} \textbf{ o.k.} )</td>
</tr>
</tbody>
</table>

**Shear Strength of Beam**

From AISC Specification Section J4.2(a), the available shear yield strength of the beam web is determined as follows:

\[ A_{gv} = dt_w \]
\[ = (18.0 \text{ in.})(0.355 \text{ in.}) \]
\[ = 6.39 \text{ in.}^2 \]

\[ R_n = 0.60F_yA_{gv} \quad (\text{Spec. Eq. J4-3}) \]
\[ = 0.60(50 \text{ ksi})(6.39 \text{ in.}^2) \]
\[ = 192 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 1.00 )</td>
<td>( \Omega = 1.50 )</td>
</tr>
<tr>
<td>( \phi R_n = 1.00(192 \text{ kips}) )</td>
<td>( R_n = 192 \text{ kips} )</td>
</tr>
<tr>
<td>= 192 kips &gt; 75 kips \textbf{ o.k.}</td>
<td>( \Omega = 1.50 )</td>
</tr>
<tr>
<td>( \Omega = 1.50 )</td>
<td>( \Omega = 1.50 )</td>
</tr>
<tr>
<td>( = 128 \text{ kips} &gt; 50 \text{ kips} \textbf{ o.k.} )</td>
<td>( = 128 \text{ kips} &gt; 50 \text{ kips} \textbf{ o.k.} )</td>
</tr>
</tbody>
</table>

The limit state of shear rupture of the beam web does not apply in this example because the beam is uncoped.

**Block Shear Rupture Strength of Beam Web**
Assuming a U-shaped tearout along the weld relative to the axial load, and a total beam setback of \( \frac{3}{4} \) in. (includes \( \frac{1}{4} \) in. tolerance to account for possible mill underrun), the nominal block shear rupture strength is determined as follows.

\[
R_n = 0.60F_uA_{nt} + U_{ht}F_uA_{nt} \leq 0.60F_yA_{gv} + U_{ht}F_uA_{nt} \tag{Spec. Eq. J4-5}
\]

where

\[
A_{nt} = h_w
\]

\[
= (14\frac{1}{2} \text{ in.})(0.355 \text{ in.})
\]

\[
= 5.15 \text{ in.}^2
\]

\[
A_{gv} = \left(2 \cdot 3\frac{1}{2} \text{ in.} - \text{setback}\right) t_w
\]

\[
= (2)(3\frac{1}{2} \text{ in.} - \frac{3}{4} \text{ in.})(0.355 \text{ in.})
\]

\[
= 1.95 \text{ in.}^2
\]

Because the angles are welded and there is no reduction for bolt holes:

\[
A_{nv} = A_{gv}
\]

\[
= 1.95 \text{ in.}^2
\]

\[
U_{bs} = 1
\]

and

\[
R_n = 0.60(65 \text{ ksi})(1.95 \text{ in.}^2) + 1.0(65 \text{ ksi})(5.15 \text{ in.}^2) \leq 0.60(50 \text{ ksi})(1.95 \text{ in.}^2) + 1.0(65 \text{ ksi})(5.15 \text{ in.}^2)
\]

\[
= 411 \text{ kips} > 393 \text{ kips}
\]

Therefore:

\[
R_n = 393 \text{ kips}
\]

The available block shear rupture strength of the web is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi )</td>
<td>0.75</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_n )</td>
<td>0.75(393 kips)</td>
<td>( R_n = 393 \text{ kips} )</td>
</tr>
<tr>
<td></td>
<td>= 295 kips &gt; 60 kips ( \text{o.k.} )</td>
<td>( \frac{R_n}{\Omega} = 2.00 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>= 197 kips &gt; 40 kips ( \text{o.k.} )</td>
</tr>
</tbody>
</table>

**Tensile Strength of Beam**

From AISC Specification Section J4.1(a), the available tensile yielding strength of the beam is determined from AISC Specification Equation J4-1:

\[
R_n = F_yA_g
\]

\[
= (50 \text{ ksi})(14.7 \text{ in.}^2)
\]

\[
= 735 \text{ kips}
\]
The available tensile yielding strength of the beam is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>$\phi R_n = 0.90(735 \text{ kips})$</td>
<td>662 kips &gt; 60 kips <strong>ok.</strong></td>
<td>$R_n = 735 \text{ kips}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{R_n}{\Omega} = 440 \text{ kips} &gt; 40 \text{ kips} \text{ ok.}$</td>
</tr>
</tbody>
</table>

From AISC Specification Section J4.1(b), determine the available tensile rupture strength of the beam. The effective net area is $A_e = A_n U$, where $U$ is determined from AISC Specification Table D3.1, Case 2. The value of $\bar{x}$ is determined by treating the W-shape as two channels back-to-back and finding the horizontal distance to the center of gravity of one of the channels from the centerline of the beam. (Note that the fillets are ignored.)

$$
\bar{x} = \frac{\sum (A \bar{x})}{\sum A}
= \frac{(0.178 \text{ in.})[18.0 \text{ in.} - 2(0.570 \text{ in.})]}{2} + 2(0.570 \text{ in.})\left(\frac{7.50 \text{ in.}}{2}\right) - \left(\frac{14.7 \text{ in.}^2}{2}\right)
= 1.13 \text{ in.}
$$

The connection length, $l$, used in the determination of $U$ will be reduced by $\frac{1}{4}$ in. to account for possible mill underrun. The shear lag factor, $U$, is:

$$
U = 1 - \frac{\bar{x}}{l}
= 1 - \frac{1.13 \text{ in.}}{(3 \text{ in.} - \frac{1}{4} \text{ in.})}
= 0.589
$$

The minimum value of $U$ can be determined from AISC Specification Section D3, where $U$ is the ratio of the gross area of the connected element to the member gross area.

$$
U = \frac{A_{nt}}{A_g}
= \frac{(d - 2t_f) t_w}{A_g}
= \frac{[18.0 \text{ in.} - 2(0.570 \text{ in.})][0.355 \text{ in.}]}{14.7 \text{ in.}^2}
= 0.407
$$

AISC Specification Table D3.1, Case 2 controls, use $U = 0.589$. Because the angles are welded and there is no reduction for bolt holes:

$$
A_{nt} = A_g
= 14.7 \text{ in.}^2
$$
\[ A_e = A_n U \]
\[ = (14.7 \text{ in.}^2)(0.589) \]
\[ = 8.66 \text{ in.}^2 \]

\[ R_n = F_u A_e \]
\[ = (65 \text{ ksi})(8.66 \text{ in.}^2) \]
\[ = 563 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi R_n = 0.75(563 \text{ kips})$</td>
<td>$\frac{R_n}{\Omega} = 563 \text{ kips}$</td>
</tr>
<tr>
<td>$= 422 \text{ kips} &gt; 60 \text{ kips}$ ok.</td>
<td>$= 282 \text{ kips} &gt; 40 \text{ kips}$ ok.</td>
</tr>
</tbody>
</table>

**Conclusion**

The connection is found to be adequate as given for the applied loads.
EXAMPLE II.A-3  ALL-WELDED DOUBLE-ANGLE CONNECTION

Given:

Repeat Example II.A-1A using AISC Manual Table 10-3 and applicable provisions from the AISC Specification to verify the strength of an all-welded double-angle connection between an ASTM A992 W36×231 beam and an ASTM A992 W14×90 column flange, as shown in Figure II.A-3-1. Use 70-ksi electrodes and ASTM A36 angles.

Solution:

From AISC Manual Table 2-4, the material properties are as follows:

Beam and column  
ASTM A992  
\( F_y = 50 \text{ ksi} \)  
\( F_u = 65 \text{ ksi} \)

Angles  
ASTM A36  
\( F_y = 36 \text{ ksi} \)  
\( F_u = 58 \text{ ksi} \)

From AISC Manual Table 1-1, the geometric properties are as follows:

Beam  
W36×231  
\( t_w = 0.760 \text{ in.} \)

Column  
W14×90  
\( t_f = 0.710 \text{ in.} \)

From ASCE/SEI 7, Chapter 2, the required strength is:
Design of Weld between Beam Web and Angles

Use AISC Manual Table 10-3 (Welds A). Try \( \frac{3}{16}\)-in. weld size, \( l = 24 \) in.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \( R_u = 1.2(37.5 \text{ kips}) + 1.6(113 \text{ kips}) \)  
  \( = 226 \text{ kips} \) | \( R_u = 37.5 \text{ kips} + 113 \text{ kips} \)  
  \( = 151 \text{ kips} \) |

\[ \phi R_u = 257 \text{ kips} > 226 \text{ kips} \quad \text{o.k.} \]

\[ \frac{R_u}{\Omega} = 171 \text{ kips} > 151 \text{ kips} \quad \text{o.k.} \]

From AISC Manual Table 10-3, the minimum beam web thickness is:

\[ t_{w,\text{min}} = 0.286 \text{ in.} < 0.760 \text{ in.} \quad \text{o.k.} \]

Design of Weld between Column Flange and Angles

Use AISC Manual Table 10-3 (Welds B). Try \( \frac{3}{8}\)-in. weld size, \( l = 24 \) in.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_u = 229 \text{ kips} &gt; 226 \text{ kips} \quad \text{o.k.} )</td>
<td>( \frac{R_u}{\Omega} = 153 \text{ kips} &gt; 151 \text{ kips} \quad \text{o.k.} )</td>
</tr>
</tbody>
</table>

From AISC Manual Table 10-3, the minimum column flange thickness is:

\[ t_{f,\text{min}} = 0.190 \text{ in.} < 0.710 \text{ in.} \quad \text{o.k.} \]

Angle Thickness

Minimum angle thickness for weld from AISC Specification Section J2.2b:

\[ t_{\text{min}} = w + \frac{1}{6} \text{ in.} \]

\[ = \frac{3}{8} \text{ in.} + \frac{1}{6} \text{ in.} \]

\[ = \frac{7}{12} \text{ in.} \]

Try \( 2L4 \times 3\frac{3}{8} \times \frac{5}{8} \) (SLBB).

Shear Strength of Angles

From AISC Specification Section J4.2(a), the available shear yielding strength of the angles is determined as follows:

\[ A_{gy} = (2 \text{ angles})l \]

\[ = (2 \times 24 \text{ in.})(\frac{7}{12} \text{ in.}) \]

\[ = 15.0 \text{ in.}^2 \]
\[ R_n = 0.60F_y A_{P\!V} \quad (\text{Spec. Eq. J4-3}) \]
\[ = 0.60(36 \text{ ksi})(15.0 \text{ in.}^2) \]
\[ = 324 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 1.00 )</td>
<td>( \Omega = 1.50 )</td>
</tr>
<tr>
<td>( \phi R_n = 1.00(324 \text{ kips}) )</td>
<td>( R_n )</td>
</tr>
</tbody>
</table>
| \( = 324 \text{ kips} > 226 \text{ kips} \quad \text{o.k.} \) | \( \frac{R_n}{\Omega} = \frac{324 \text{ kips}}{1.50} \)
| \( = 216 \text{ kips} > 151 \text{ kips} \quad \text{o.k.} \) |

From AISC Specification Section J4.2(b), the available shear rupture strength of the angles is determined as follows:

\[ A_{nv} = (2 \text{ angles})lt \]
\[ = (2 \text{ angles})(24 \text{ in.})(\frac{5}{8} \text{ in.}) \]
\[ = 15.0 \text{ in.}^2 \]

\[ R_n = 0.60F_y A_{nv} \quad (\text{Spec. Eq. J4-4}) \]
\[ = 0.60(58 \text{ ksi})(15.0 \text{ in.}^2) \]
\[ = 522 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_n = 0.75(522 \text{ kips}) )</td>
<td>( R_n )</td>
</tr>
</tbody>
</table>
| \( = 392 \text{ kips} > 226 \text{ kips} \quad \text{o.k.} \) | \( \frac{R_n}{\Omega} = \frac{522 \text{ kips}}{2.00} \)
| \( = 261 \text{ kips} > 151 \text{ kips} \quad \text{o.k.} \) |

**Conclusion**

The connection is found to be adequate as given for the applied loads.