COMPANION TO THE AISC *STEEL CONSTRUCTION MANUAL*

Volume 1: Design Examples

For Use in First Semester
Structural Steel Design Classes

Version 15.1

AMERICAN INSTITUTE
OF
STEEL CONSTRUCTION

Printed in the United States of America

Notes: Design Examples For Use in First Semester Structural Steel Design Course

The AISC Committee on Manuals prepares design examples to illustrate the application of the provisions in the AISC *Specification for Structural Steel Buildings*. The complete set of design examples includes 166 example problems totaling 985 pages, and it is a free download that can be found at www.aisc.org/designexamples.

The AISC Partners in Education Committee has condensed the set of Design Examples to include 45 example problems that will be most likely to address topics that are studied in a first semester structural steel design course. These 45 example problems can be found on the following pages.

PREFACE

The primary objective of this Companion is to provide guidance and additional resources of the use of the 2016 AISC *Specification for Structural Steel Buildings* (ANSI/AISC 360-16) and the 15th Edition AISC *Steel Construction Manual*.

The Companion consists of design examples in Parts I, II and III. The design examples provide coverage of all applicable limit states, whether or not a particular limit state controls the design of the member or connection. In addition to the examples that demonstrate the use of the AISC *Manual* tables, design examples are provided for connection designs beyond the scope of the tables in the AISC *Manual*. These design examples are intended to demonstrate an approach to the design, and are not intended to suggest that the approach presented is the only approach. The committee responsible for the development of these design examples recognizes that designers have alternate approaches that work best for them and their projects. Design approaches that differ from those presented in these examples are considered viable as long as the AISC *Specification*, sound engineering, and project specific requirements are satisfied.

Part I of these examples is organized to correspond with the organization of the AISC *Specification*. The Chapter titles match the corresponding chapters in the AISC *Specification*.

Part II is devoted primarily to connection examples that draw on the tables from the AISC *Manual*, recommended design procedures, and the breadth of the AISC *Specification*. The chapters of Part II are labeled II-A, II-B, II-C, etc.

Part III addresses aspects of design that are linked to the performance of a building as a whole. This includes coverage of lateral stability and second-order analysis, illustrated through a four-story braced-frame and moment-frame building.

The Design Examples are arranged with LRFD and ASD designs presented side-by-side, for consistency with the AISC *Manual*. Design with ASD and LRFD are based on the same nominal strength for each element so that the only differences between the approaches are the set of load combinations from ASCE/SEI 7-16 used for design, and whether the resistance factor for LRFD or the safety factor for ASD is used.

CONVENTIONS

The following conventions are used throughout these examples:

1. The 2016 AISC *Specification for Structural Steel Buildings* is referred to as the AISC *Specification* and the 15th Edition AISC *Steel Construction Manual,* is referred to as the AISC *Manual*.

2. The 2016 ASCE *Minimum Design Loads and Associated Criteria for Buildings and Other Structures* is referred to as ASCE/SEI 7.

3. The source of equations or tabulated values taken from the AISC *Specification* or AISC *Manual* is noted along the right-hand edge of the page.

4. When the design process differs between LRFD and ASD, the designs equations are presented side-by-side. This rarely occurs, except when the resistance factor, ϕ, and the safety factor, Ω, are applied.

5. The results of design equations are presented to three significant figures throughout these calculations.

ACKNOWLEDGMENTS

The AISC Committee on Manuals reviewed and approved V15.1 of the AISC *Design Examples*:

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The committee gratefully acknowledges the contributions made to this document by the AISC Committee on Specifications and the following individuals: W. Scott Goodrich, Heath Mitchell, William N. Scott, Marc L. Sorenson and Sriramulu Vinnakota.

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Part I
Examples Based on the AISC *Specification*

This part contains design examples demonstrating select provisions of the AISC *Specification for Structural Steel Buildings.*

Chapter C
Design for Stability

C1. GENERAL STABILITY REQUIREMENTS

The AISC *Specification* requires that the designer account for both the stability of the structural system as a whole and the stability of individual elements. Thus, the lateral analysis used to assess stability must include consideration of the combined effect of gravity and lateral loads, as well as member inelasticity, out-of-plumbness, out-of-straightness, and the resulting second-order effects, P-Δ and P-δ. The effects of "leaning columns" must also be considered, as illustrated in the examples in this chapter and in the four-story building design example in Part III of these *Design Examples*.

P-Δ and P-δ effects are illustrated in AISC *Specification* Commentary Figure C-C2.1. Methods for addressing stability, including P-Δ and P-δ effects, are provided in AISC *Specification* Section C2 and Appendix 7.

C2. CALCULATION OF REQUIRED STRENGTHS

The calculation of required strengths is illustrated in the examples in this chapter and in the four-story building design example in Part III of these *Design Examples*.

C3. CALCULATION OF AVAILABLE STRENGTHS

The calculation of available strengths is illustrated in the four-story building design example in Part III of these *Design Examples*.

EXAMPLE C.1A DESIGN OF A MOMENT FRAME BY THE DIRECT ANALYSIS METHOD

Given:

Determine the required strengths and effective length factors for the columns in the moment frame shown in Figure C.1A-1 for the maximum gravity load combination, using LRFD and ASD. The uniform load, w_D, includes beam self-weight and an allowance for column self-weight. Use the direct analysis method. All members are ASTM A992 material.

Columns are unbraced between the footings and roof in the *x*- and *y*-axes and have pinned bases.

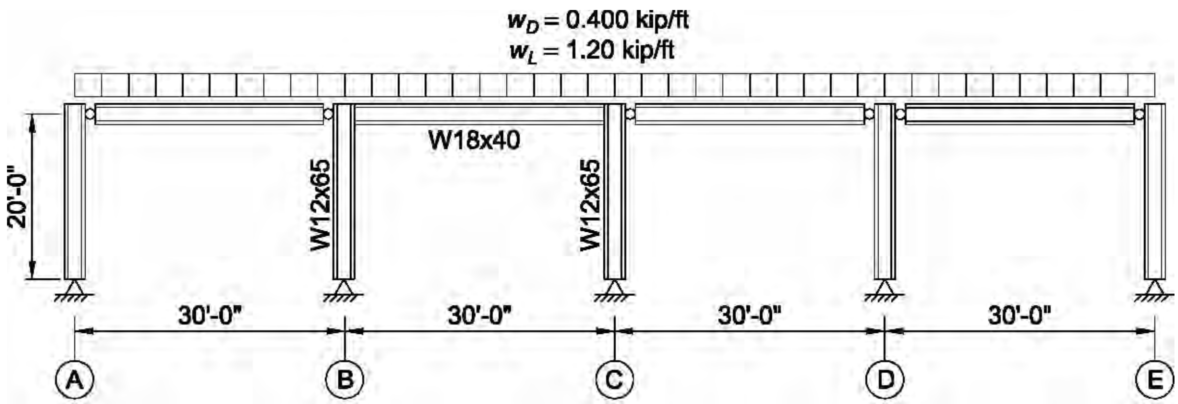

Fig. C.1A-1. Example C.1A moment frame elevation.

Solution:

From AISC *Manual* Table 1-1, the W12×65 has A = 19.1 in.^{2}

The beams from grid lines A to B and C to E and the columns at A, D and E are pinned at both ends and do not contribute to the lateral stability of the frame. There are no P-Δ effects to consider in these members and they may be designed using $L_c = L$.

The moment frame between grid lines B and C is the source of lateral stability and therefore will be evaluated using the provisions of Chapter C of the AISC *Specification*. Although the columns at grid lines A, D and E do not contribute to lateral stability, the forces required to stabilize them must be considered in the moment-frame analysis. The entire frame from grid line A to E could be modeled, but in this case the model is simplified as shown in Figure C.1A-2, in which the stability loads from the three "leaning" columns are combined into a single representative column.

From Chapter 2 of ASCE/SEI 7, the maximum gravity load combinations are:

LRFD	ASD
$w_u = 1.2D + 1.6L$ $= 1.2(0.400 \text{ kip/ft}) + 1.6(1.20 \text{ kip/ft})$ $= 2.40 \text{ kip/ft}$	$w_u = D + L$ $= 0.400 \text{ kip/ft} + 1.20 \text{ kip/ft}$ $= 1.60 \text{ kip/ft}$

Per AISC *Specification* Section C2.1(d), for LRFD, perform a second-order analysis and member strength checks using the LRFD load combinations. For ASD, perform a second-order analysis using 1.6 times the ASD load combinations and divide the analysis results by 1.6 for the ASD member strength checks.

Frame analysis gravity loads

The uniform gravity loads to be considered in a second-order analysis on the beam from B to C are:

LRFD	ASD
$w'_u = 2.40\text{ kip/ft}$	$w'_a = 1.6(1.60\text{ kip/ft})$ $= 2.56\text{ kip/ft}$

Concentrated gravity loads to be considered in a second-order analysis on the columns at B and C contributed by adjacent beams are:

LRFD	ASD
$P'_u = \dfrac{w'_u l}{2}$ $= \dfrac{(2.40\text{ kip/ft})(30.0\text{ ft})}{2}$ $= 36.0\text{ kips}$	$P'_a = \dfrac{w'_a l}{2}$ $= \dfrac{(2.56\text{ kip/ft})(30.0\text{ ft})}{2}$ $= 38.4\text{ kips}$

Concentrated gravity loads on the representative "leaning" column

The load in this column accounts for all gravity loading that is stabilized by the moment frame, but is not directly applied to it.

LRFD	ASD
$P'_{uL} = (60.0\text{ ft})(2.40\text{ kip/ft})$ $= 144\text{ kips}$	$P'_{aL} = (60.0\text{ ft})(2.56\text{ kip/ft})$ $= 154\text{ kips}$

Frame analysis notional loads

Per AISC *Specification* Section C2.2, frame out-of-plumbness must be accounted for either by explicit modeling of the assumed out-of-plumbness or by the application of notional loads. Use notional loads.

From AISC *Specification* Equation C2-1, the notional loads are:

LRFD	ASD
$\alpha = 1.0$ $Y_i = (120\text{ ft})(2.40\text{ kip/ft})$ $= 288\text{ kips}$ $N_i = 0.002\alpha Y_i$ (*Spec*. Eq. C2-1) $= 0.002(1.0)(288\text{ kips})$ $= 0.576\text{ kip}$	$\alpha = 1.6$ $Y_i = (120\text{ ft})(1.60\text{ kip/ft})$ $= 192\text{ kips}$ $N_i = 0.002\alpha Y_i$ (*Spec*. Eq. C2-1) $= 0.002(1.6)(192\text{ kips})$ $= 0.614\text{ kip}$

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Summary of applied frame loads

The applied loads are shown in Figure C.1A-2.

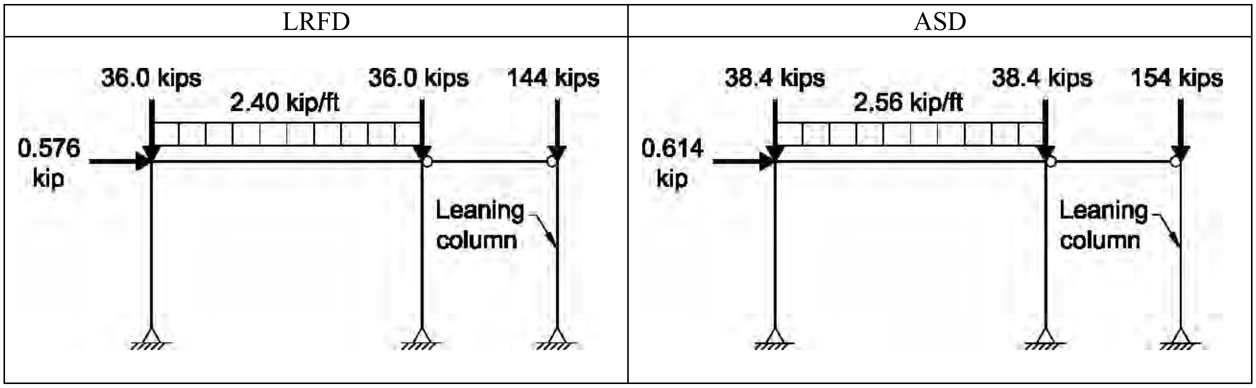


Fig. C.1A-2. Applied loads on the analysis model.

Per AISC *Specification* Section C2.3, conduct the analysis using 80% of the nominal stiffnesses to account for the effects of inelasticity. Assume, subject to verification, that $\alpha P_r / P_{ns}$ is not greater than 0.5; therefore, no additional stiffness reduction is required (τ_b = 1.0).

Half of the gravity load is carried by the columns of the moment-resisting frame. Because the gravity load supported by the moment-resisting frame columns exceeds one-third of the total gravity load tributary to the frame, per AISC *Specification* Section C2.1, the effects of P-δ and P-Δ must be considered in the frame analysis. This example uses analysis software that accounts for both P-Δ and P-δ effects. (If the software used does not account for P-δ effects this may be accomplished by subdividing the columns between the footing and beam.)

Figures C.1A-3 and C.1A-4 show results from a first-order and a second-order analysis. (The first-order analysis is shown for reference only.) In each case, the drift is the average of drifts at grid lines B and C.

First-order results

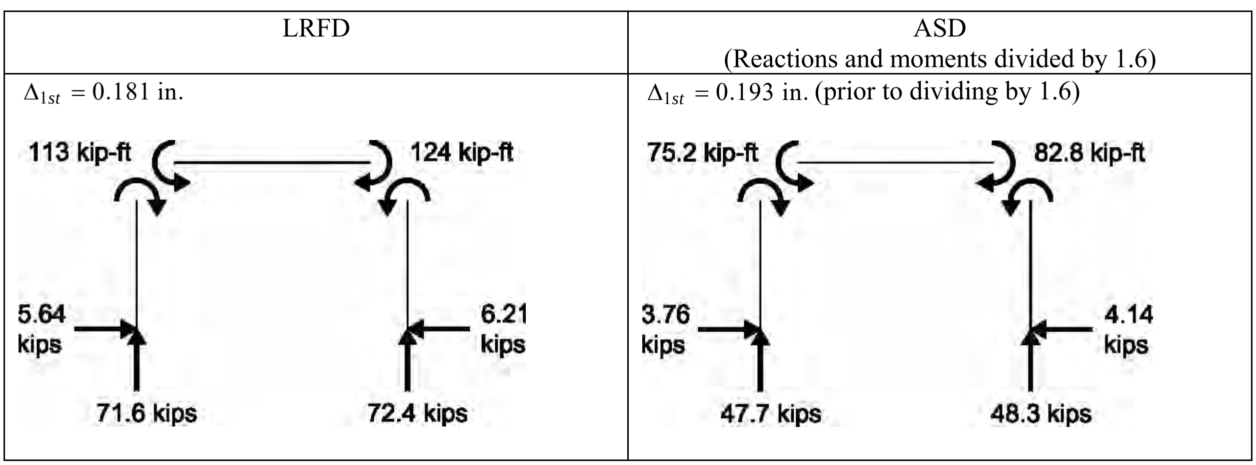


Fig. C.1A-3. Results of first-order analysis.

Second-order results

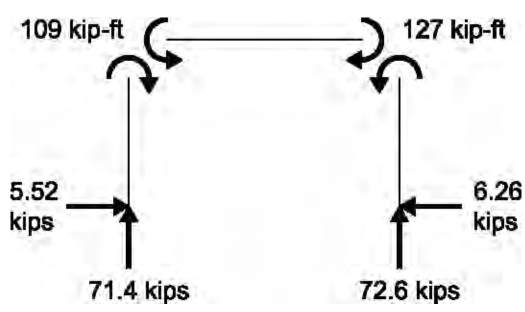
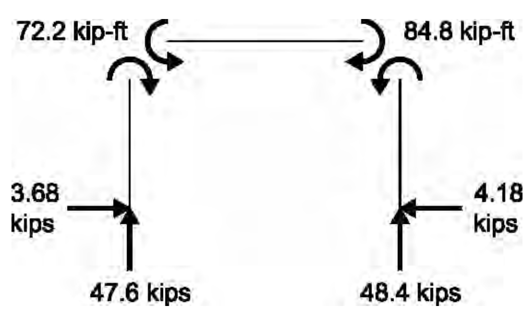
LRFD	ASD (Reactions and moments divided by 1.6)
$\Delta_{2nd} = 0.290$ in. Drift ratio: $\dfrac{\Delta_{2nd}}{\Delta_{1st}} = \dfrac{0.290 \text{ in.}}{0.181 \text{ in.}}$ $= 1.60$ 	$\Delta_{2nd} = 0.321$ in. (prior to dividing by 1.6) Drift ratio: $\dfrac{\Delta_{2nd}}{\Delta_{1st}} = \dfrac{0.321 \text{ in.}}{0.193 \text{ in.}}$ $= 1.66$ 

Fig. C.1A-4. Results of second-order analysis.

Check the assumption that $\alpha P_r / P_{ns} \le 0.5$ on the column on grid line C.

Because a W12×65 column contains no elements that are slender for uniform compression,

$$
\begin{aligned}
P_{ns} &= F_y A_g \\
&= (50 \text{ ksi})(19.1 \text{ in.}^2) \\
&= 955 \text{ kips}
\end{aligned}
$$

LRFD	ASD
$\dfrac{\alpha P_r}{P_{ns}} = \dfrac{1.0(72.6 \text{ kips})}{955 \text{ kips}}$ $= 0.0760 \le 0.5$ **o.k.**	$\dfrac{\alpha P_r}{P_{ns}} = \dfrac{1.6(48.4 \text{ kips})}{955 \text{ kips}}$ $= 0.0811 \le 0.5$ **o.k.**

The stiffness assumption used in the analysis, $\tau_b = 1.0$, is verified.

Note that the drift ratio, 1.60 (LRFD) or 1.66 (ASD), does not exceed the recommended limit of 2.5 from AISC *Specification* Commentary Section C1.

The required axial compressive strength in the columns is 72.6 kips (LRFD) or 48.4 kips (ASD). The required bending moment diagram is linear, varying from zero at the bottom to 127 kip-ft (LRFD) or 84.8 kip-ft (ASD) at the top. These required strengths apply to both columns because the notional load must be applied in each direction.

Although the second-order sway multiplier (drift ratio) is fairly large at 1.60 (LRFD) or 1.66 (ASD), the change in bending moment is small because the only sway moments are those produced by the small notional loads. For load combinations with significant gravity and lateral loadings, the increase in bending moments is larger.

Per AISC *Specification* Section C3, the effective length for flexural buckling of all members is taken as the unbraced length (K = 1.0):

L_{cx} = 20.0 ft
L_{cy} = 20.0 ft

EXAMPLE C.1B DESIGN OF A MOMENT FRAME BY THE EFFECTIVE LENGTH METHOD

Given:

Repeat Example C.1A using the effective length method.

Determine the required strengths and effective length factors for the columns in the moment frame shown in Figure C.1B-1 for the maximum gravity load combination, using LRFD and ASD. Use the effective length method.

Columns are unbraced between the footings and roof in the *x*- and *y*-axes and have pinned bases.

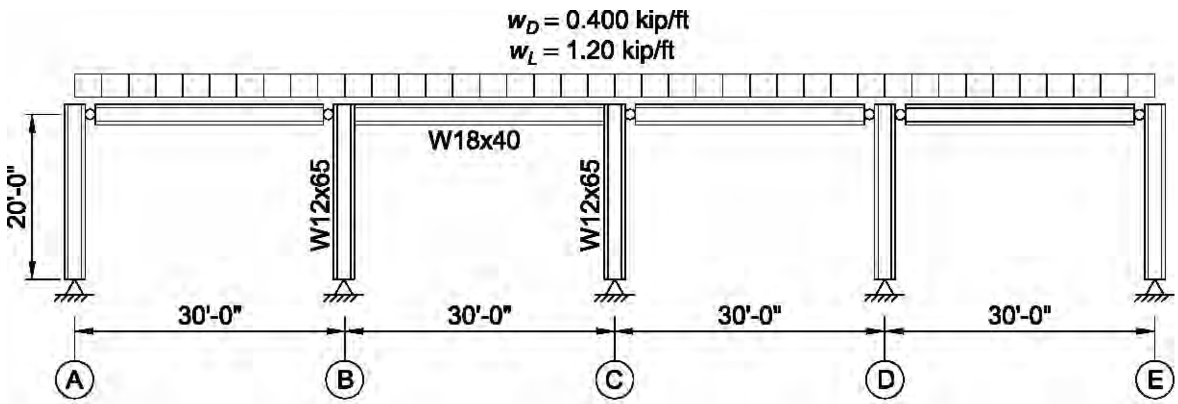


Fig. C.1B-1. Example C.1B moment frame elevation.

Solution:

From AISC *Manual* Table 1-1, the W12×65 has $I_x = 533$ in.^{4}

The beams from grid lines A to B and C to E and the columns at A, D and E are pinned at both ends and do not contribute to the lateral stability of the frame. There are no P-Δ effects to consider in these members and they may be designed using $L_c = L$.

The moment frame between grid lines B and C is the source of lateral stability and therefore will be evaluated using the provisions of Chapter C of the AISC *Specification*. Although the columns at grid lines A, D and E do not contribute to lateral stability, the forces required to stabilize them must be considered in the moment-frame analysis. The entire frame from grid line A to E could be modeled, but in this case the model is simplified as shown in Figure C.1B-2, in which the stability loads from the three "leaning" columns are combined into a single representative column.

Check the limitations for the use of the effective length method given in AISC *Specification* Appendix 7, Section 7.2.1:

(a) The structure supports gravity loads primarily through nominally vertical columns, walls or frames.
(b) The ratio of maximum second-order drift to the maximum first-order drift (both determined for LRFD load combinations or 1.6 times ASD load combinations, with stiffness not adjusted as specified in AISC *Specification* Section C2.3) in all stories will be assumed to be no greater than 1.5, subject to verification in the following.

From Chapter 2 of ASCE/SEI 7, the maximum gravity load combinations are:

LRFD	ASD
$w_u = 1.2D + 1.6L$ $= 1.2(0.400\text{ kip/ft}) + 1.6(1.20\text{ kip/ft})$ $= 2.40\text{ kip/ft}$	$w_u = D + L$ $= 0.400\text{ kip/ft} + 1.20\text{ kip/ft}$ $= 1.60\text{ kip/ft}$

Per AISC *Specification* Appendix 7, Section 7.2.2, the analysis must conform to the requirements of AISC *Specification* Section C2.1, with the exception of the stiffness reduction required by the provisions of Section C2.1(a).

Per AISC *Specification* Section C2.1(d), for LRFD perform a second-order analysis and member strength checks using the LRFD load combinations. For ASD, perform a second-order analysis at 1.6 times the ASD load combinations and divide the analysis results by 1.6 for the ASD member strength checks.

Frame analysis gravity loads

The uniform gravity loads to be considered in a second-order analysis on the beam from B to C are:

LRFD	ASD
$w'_u = 2.40\text{ kip/ft}$	$w'_a = 1.6(1.60\text{ kip/ft})$ $= 2.56\text{ kip/ft}$

Concentrated gravity loads to be considered in a second-order analysis on the columns at B and C contributed by adjacent beams are:

LRFD	ASD
$P'_u = \dfrac{w'_u l}{2}$ $= \dfrac{(2.40\text{ kip/ft})(30.0\text{ ft})}{2}$ $= 36.0\text{ kips}$	$P'_a = \dfrac{w'_a l}{2}$ $= \dfrac{(2.56\text{ kip/ft})(30.0\text{ ft})}{2}$ $= 38.4\text{ kips}$

Concentrated gravity loads on the representative "leaning" column

The load in this column accounts for all gravity loads that is stabilized by the moment frame, but not directly applied to it.

LRFD	ASD
$P'_{uL} = (60.0\text{ ft})(2.40\text{ kip/ft})$ $= 144\text{ kips}$	$P'_{aL} = (60.0\text{ ft})(2.56\text{ kip/ft})$ $= 154\text{ kips}$

Frame analysis notional loads

Per AISC *Specification* Appendix 7, Section 7.2.2, frame out-of-plumbness must be accounted for by the application of notional loads in accordance with AISC *Specification* Section C2.2b. Note that notional loads need to only be applied to the gravity load combinations per AISC *Specification* Section C2.2b(d) when the requirement that $\Delta_{2nd} / \Delta_{1st} \le 1.7$ (using stiffness adjusted as specified in Section C2.3) is satisfied. Per the User Note in AISC *Specification* Appendix 7, Section 7.2.2, Section C2.2b(d) will be satisfied in all cases where the effective length method is applicable, and therefore the notional load need only be applied in gravity-only load cases.
From AISC *Specification* Equation C2-1, the notional loads are:

LRFD	ASD
$\alpha = 1.0$ $Y_i = (120 \text{ ft})(2.40 \text{ kip/ft})$ $= 288 \text{ kips}$ $N_i = 0.002\alpha Y_i$ (*Spec.* Eq. C2-1) $= 0.002(1.0)(288 \text{ kips})$ $= 0.576 \text{ kip}$	$\alpha = 1.6$ $Y_i = (120 \text{ ft})(1.60 \text{ kip/ft})$ $= 192 \text{ kips}$ $N_i = 0.002\alpha Y_i$ (*Spec.* Eq. C2-1) $= 0.002(1.6)(192 \text{ kips})$ $= 0.614 \text{ kip}$

Summary of applied frame loads

The applied loads are shown in Figure C.1B-2.

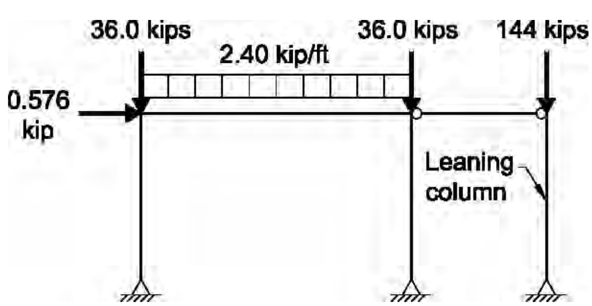
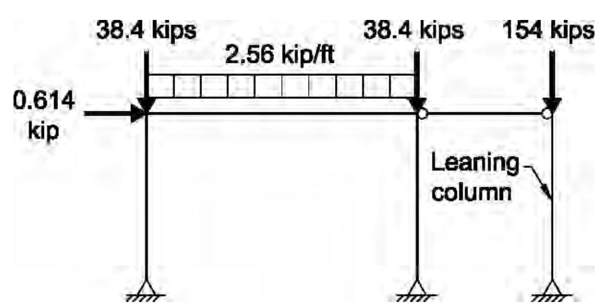
<table>
<tr><th>LRFD</th><th>ASD</th></tr>
<tr><td>
</td><td>
</td></tr>
</table>

Fig. C.1B-2. Applied loads on the analysis model.

Per AISC *Specification* Appendix 7, Section 7.2.2, conduct the analysis using the full nominal stiffnesses.

Half of the gravity load is carried by the columns of the moment-resisting frame. Because the gravity load supported by the moment-resisting frame columns exceeds one-third of the total gravity load tributary to the frame, per AISC *Specification* Section C2.1(b), the effects of P-δ on the response of the structure must be considered in the frame analysis. This example uses analysis software that accounts for both P-Δ and P-δ effects. When using software that does not account for P-δ effects, this could be accomplished by subdividing columns between the footing and beam.

Figures C.1B-3 and C.1B-4 show results from a first-order and second-order analysis. In each case, the drift is the average of drifts at grid lines B and C.

First-order results

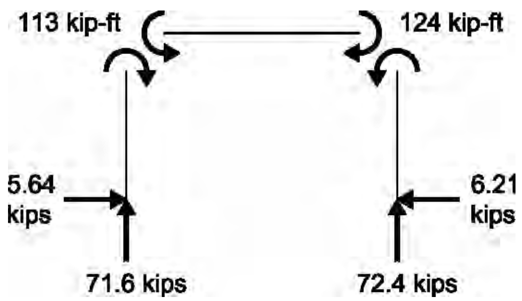
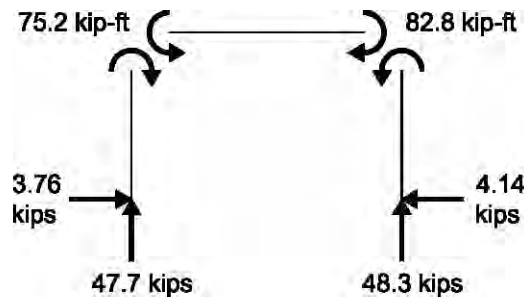
LRFD	ASD (Reactions and moments divided by 1.6)
$\Delta_{1st} = 0.145$ in.	$\Delta_{1st} = 0.155$ in. (prior to dividing by 1.6)
	

Fig. C.1B-3. Results of first-order analysis.

Second-order results

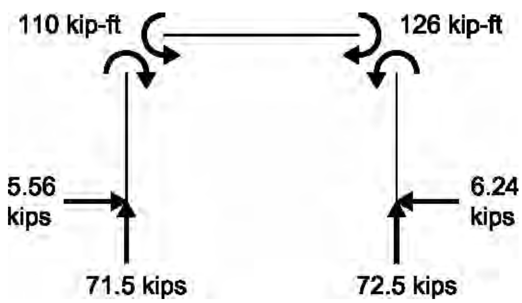
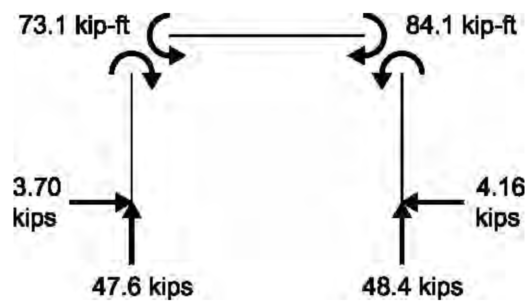
LRFD	ASD
$\Delta_{2nd} = 0.204$ in. Drift ratio: $\dfrac{\Delta_{2nd}}{\Delta_{1st}} = \dfrac{0.204 \text{ in.}}{0.145 \text{ in.}}$ $= 1.41$	$\Delta_{2nd} = 0.223$ in. (prior to dividing by 1.6) Drift ratio: $\dfrac{\Delta_{2nd}}{\Delta_{1st}} = \dfrac{0.223 \text{ in.}}{0.155 \text{ in.}}$ $= 1.44$
	

Fig. C-1B-4. Results of second-order analysis.

The assumption that the ratio of the maximum second-order drift to the maximum first-order drift is no greater than 1.5 is verified; therefore, the effective length method is permitted.

Although the second-order sway multiplier is fairly large at approximately 1.41 (LRFD) or 1.44 (ASD), the change in bending moment is small because the only sway moments for this load combination are those produced by the small notional loads. For load combinations with significant gravity and lateral loadings, the increase in bending moments is larger.

Calculate the in-plane effective length factor, K_x, using the "story stiffness approach" and Equation C-A-7-5 presented in AISC *Specification* Commentary Appendix 7, Section 7.2. With $K_x = K_2$:

$$K_x = \sqrt{\frac{P_{story}}{R_M P_r}\left(\frac{\pi^2 EI}{L^2}\right)\left(\frac{\Delta_H}{HL}\right)} \geq \sqrt{\frac{\pi^2 EI}{L^2}\left(\frac{\Delta_H}{1.7 H_{col} L}\right)}$$ (*Spec*. Eq. C-A-7-5)

Calculate the total load in all columns, P_{story}, as follows:

LRFD	ASD
$P_{story} = (2.40 \text{ kip/ft})(120 \text{ ft})$ $= 288 \text{ kips}$	$P_{story} = (1.60 \text{ kip/ft})(120 \text{ ft})$ $= 192 \text{ kips}$

Calculate the coefficient to account for the influence of P-δ on P-Δ, R_M, as follows, using AISC *Specification* Commentary Appendix 7, Equation C-A-7-6:

LRFD	ASD
$P_{mf} = 71.5 \text{ kips} + 72.5 \text{ kips}$ $= 144 \text{ kips}$ $R_M = 1 - 0.15(P_{mf} / P_{story})$ (*Spec*. Eq. C-A-7-6) $= 1 - 0.15\left(\frac{144 \text{ kips}}{288 \text{ kips}}\right)$ $= 0.925$	$P_{mf} = 47.6 \text{ kips} + 48.4 \text{ kips}$ $= 96.0 \text{ kips}$ $R_M = 1 - 0.15(P_{mf} / P_{story})$ (*Spec*. Eq. C-A-7-6) $= 1 - 0.15\left(\frac{96.0 \text{ kips}}{192 \text{ kips}}\right)$ $= 0.925$

Calculate the Euler buckling strength of one moment frame.

$$\frac{\pi^2 EI}{L^2} = \frac{\pi^2 (29{,}000 \text{ ksi})(533 \text{ in.}^4)}{\left[(20.0 \text{ ft})(12 \text{ in./ft})\right]^2}$$
$$= 2{,}650 \text{ kips}$$

From AISC *Specification* Commentary Equation C-A-7-5, for the column at line C:

LRFD	ASD
$K_x = \sqrt{\left(\dfrac{P_{story}}{R_M P_r}\right)\left(\dfrac{\pi^2 EI}{L^2}\right)\left(\dfrac{\Delta_H}{HL}\right)}$	$K_x = \sqrt{\left(\dfrac{1.6P_{story}}{R_M (1.6) P_r}\right)\left(\dfrac{\pi^2 EI}{L^2}\right)\left(\dfrac{\Delta_H}{HL}\right)}$
$\geq \sqrt{\left(\dfrac{\pi^2 EI}{L^2}\right)\left(\dfrac{\Delta_H}{1.7H_{col}L}\right)}$	$\geq \sqrt{\left(\dfrac{\pi^2 EI}{L^2}\right)\left(\dfrac{\Delta_H}{1.7(1.6)H_{col}L}\right)}$
$= \sqrt{\left[\dfrac{288\text{ kips}}{(0.925)(72.5\text{ kips})}\right](2{,}650\text{ kips}) \times \left[\dfrac{0.145\text{ in.}}{(0.576\text{ kip})(20.0\text{ ft})(12\text{ in./ft})}\right]}$	$= \sqrt{\left[\dfrac{1.6(192\text{ kips})}{0.925(1.6)(48.4\text{ kips})}\right](2{,}650\text{ kips}) \times \left[\dfrac{0.155\text{ in.}}{(0.614\text{ kip})(20.0\text{ ft})(12\text{ in./ft})}\right]}$
$\geq \sqrt{(2{,}650\text{ kips}) \times \left[\dfrac{0.145\text{ in.}}{1.7(6.21\text{ kips})(20.0\text{ ft})(12\text{ in./ft})}\right]}$	$\geq \sqrt{(2{,}650\text{ kips}) \times \left[\dfrac{0.155\text{ in.}}{1.7(1.6)(4.14\text{ kips})(20.0\text{ ft})(12\text{ in./ft})}\right]}$
$= 3.45 \geq 0.389$	$= 3.46 \geq 0.390$
Use K_x = 3.45	Use K_x = 3.46

Note that the column loads are multiplied by 1.6 for ASD in Equation C-A-7-5.

With K_x = 3.45 and K_y = 1.00, the column available strengths can be verified for the given member sizes for the second-order forces (calculations not shown), using the following effective lengths:

$$\begin{aligned} L_{cx} &= K_x L_x \\ &= 3.45(20.0\text{ ft}) \\ &= 69.0\text{ ft} \end{aligned}$$

$$\begin{aligned} L_{cy} &= K_y L_y \\ &= 1.00(20.0\text{ ft}) \\ &= 20.0\text{ ft} \end{aligned}$$

Chapter D
Design of Members for Tension

D1. SLENDERNESS LIMITATIONS

AISC *Specification* Section D1 does not establish a slenderness limit for tension members, but recommends limiting L/r to a maximum of 300. This is not an absolute requirement. Rods and hangers are specifically excluded from this recommendation.

D2. TENSILE STRENGTH

Both tensile yielding strength and tensile rupture strength must be considered for the design of tension members. It is not unusual for tensile rupture strength to govern the design of a tension member, particularly for small members with holes or heavier sections with multiple rows of holes.

For preliminary design, tables are provided in Part 5 of the AISC *Manual* for W-shapes, L-shapes, WT-shapes, rectangular HSS, square HSS, round HSS, Pipe, and 2L-shapes. The calculations in these tables for available tensile rupture strength assume an effective area, A_e, of $0.75A_g$. The gross area, A_g, is the total cross-sectional area of the member. If the actual effective area is greater than $0.75A_g$, the tabulated values will be conservative and calculations can be performed to obtain higher available strengths. If the actual effective area is less than $0.75A_g$, the tabulated values will be unconservative and calculations are necessary to determine the available strength.

D3. EFFECTIVE NET AREA

In computing net area, A_n, AISC *Specification* Section B4.3b requires that an extra 1⁄16 in. be added to the bolt hole diameter. A computation of the effective area for a chain of holes is presented in Example D.9.

Unless all elements of the cross section are connected, $A_e = A_n U$, where U is a reduction factor to account for shear lag. The appropriate values of U can be obtained from AISC *Specification* Table D3.1.

D4. BUILT-UP MEMBERS

The limitations for connections of built-up members are discussed in Section D4 of the AISC *Specification*.

D5. PIN-CONNECTED MEMBERS

An example of a pin-connected member is given in Example D.7.

D6. EYEBARS

An example of an eyebar is given in Example D.8. The strength of an eyebar meeting the dimensional requirements of AISC *Specification* Section D6 is governed by tensile yielding of the body.

EXAMPLE D.1 W-SHAPE TENSION MEMBER

Given:

Select an ASTM A992 W-shape with 8 in. nominal depth to carry a dead load of 30 kips and a live load of 90 kips in tension. The member is 25.0 ft long. Verify the member strength by both LRFD and ASD with the bolted end connection as shown in Figure D.1-1. Verify that the member satisfies the recommended slenderness limit. Assume that connection limit states do not govern.

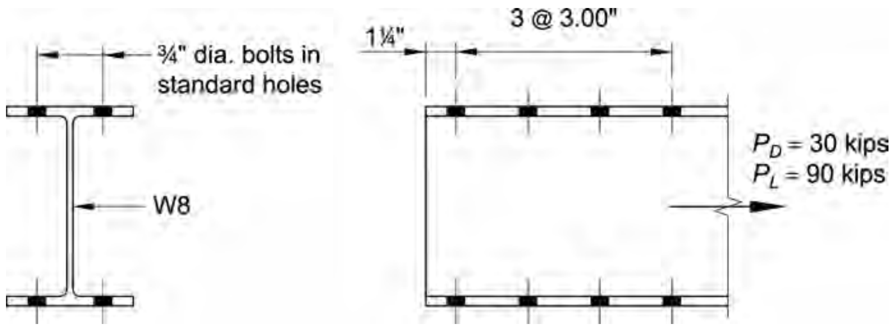


Fig D.1-1. Connection geometry for Example D.1.

Solution:

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
$P_u = 1.2(30\text{ kips}) + 1.6(90\text{ kips})$ $= 180\text{ kips}$	$P_a = 30\text{ kips} + 90\text{ kips}$ $= 120\text{ kips}$

From AISC *Manual* Table 5-1, try a W8×21.

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W8×21
A_g = 6.16 in.^{2}
b_f = 5.27 in.
t_f = 0.400 in.
d = 8.28 in.
r_y = 1.26 in.

The WT-shape corresponding to a W8×21 is a WT4×10.5. From AISC *Manual* Table 1-8, the geometric properties are as follows:

WT4×10.5
$\bar{y}$ = 0.831 in.

Tensile Yielding

From AISC *Manual* Table 5-1, the available tensile yielding strength of a W8×21 is:

LRFD	ASD
$\phi_t P_n = 277 \text{ kips} > 180 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_t} = 184 \text{ kips} > 120 \text{ kips}$ **o.k.**

Tensile Rupture

Verify the table assumption that $A_e/A_g \geq 0.75$ for this connection.

From the description of the element in AISC *Specification* Table D3.1, Case 7, calculate the shear lag factor, U, as the larger of the values from AISC *Specification* Section D3, Table D3.1 Case 2 and Case 7.

From AISC *Specification* Section D3, for open cross sections, U need not be less than the ratio of the gross area of the connected element(s) to the member gross area.

$$
\begin{aligned}
U &= \frac{2b_f t_f}{A_g} \\
&= \frac{2(5.27 \text{ in.})(0.400 \text{ in.})}{6.16 \text{ in.}^2} \\
&= 0.684
\end{aligned}
$$

Case 2: Determine U based on two WT-shapes per AISC *Specification* Commentary Figure C-D3.1, with $\bar{x} = \bar{y} = 0.831$ in. and where l is the length of connection.

$$
\begin{aligned}
U &= 1 - \frac{\bar{x}}{l} \\
&= 1 - \frac{0.831 \text{ in.}}{9.00 \text{ in.}} \\
&= 0.908
\end{aligned}
$$

Case 7:

$$
\begin{aligned}
b_f &= 5.27 \text{ in.} \\
\frac{2}{3}d &= \frac{2}{3}(8.28 \text{ in.}) \\
&= 5.52 \text{ in.}
\end{aligned}
$$

Because the flange is connected with three or more fasteners per line in the direction of loading and $b_f < \frac{2}{3}d$:

$$U = 0.85$$

Therefore, use the larger $U = 0.908$.

Calculate A_n using AISC *Specification* Section B4.3b.

$$A_n = A_g - 4\left(d_h + \tfrac{1}{16}\text{ in.}\right)t_f$$
$$= 6.16\text{ in.}^2 - 4\left(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)(0.400\text{ in.})$$
$$= 4.76\text{ in.}^2$$

Calculate A_e using AISC *Specification* Section D3.

$$A_e = A_n U \qquad (Spec.\text{ Eq. D3-1})$$
$$= \left(4.76\text{ in.}^2\right)(0.908)$$
$$= 4.32\text{ in.}^2$$

$$\frac{A_e}{A_g} = \frac{4.32\text{ in.}^2}{6.16\text{ in.}^2}$$
$$= 0.701 < 0.75$$

Because A_e/A_g < 0.75, the tensile rupture strength from AISC *Manual* Table 5-1 is not valid. The available tensile rupture strength is determined using AISC *Specification* Section D2 as follows:

$$P_n = F_u A_e \qquad (Spec.\text{ Eq. D2-2})$$
$$= (65\text{ ksi})\left(4.32\text{ in.}^2\right)$$
$$= 281\text{ kips}$$

From AISC *Specification* Section D2, the available tensile rupture strength is:

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(281\text{ kips})$ $= 211\text{ kips} > 180\text{ kips}$ **o.k.**	$\Omega_t = 2.00$ $\dfrac{P_n}{\Omega_t} = \dfrac{281\text{ kips}}{2.00}$ $= 141\text{ kips} > 120\text{ kips}$ **o.k.**

Note that the W8×21 available tensile strength is governed by the tensile rupture limit state at the end connection versus the tensile yielding limit state.

See Chapter J for illustrations of connection limit state checks.

Check Recommended Slenderness Limit

$$\frac{L}{r} = \frac{(25.0\text{ ft})(12\text{ in./ft})}{1.26\text{ in.}}$$
$$= 238 < 300 \text{ from AISC } \textit{Specification} \text{ Section D1} \quad \textbf{o.k.}$$

EXAMPLE D.2 SINGLE-ANGLE TENSION MEMBER

Given:

Verify the tensile strength of an ASTM A36 L4×4×½ with one line of four ¾-in.-diameter bolts in standard holes, as shown in Figure D.2-1. The member carries a dead load of 20 kips and a live load of 60 kips in tension. Additionally, calculate at what length this tension member would cease to satisfy the recommended slenderness limit. Assume that connection limit states do not govern.

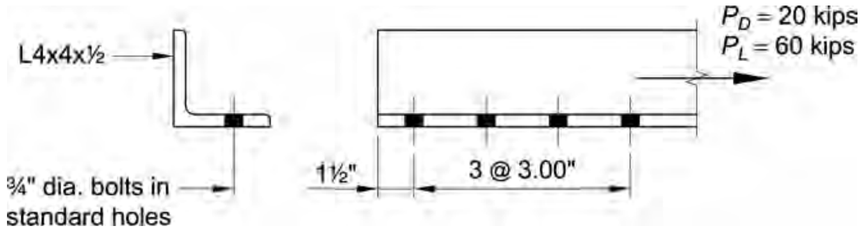


Fig. D.2-1. Connection geometry for Example D.2.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

From AISC *Manual* Table 1-7, the geometric properties are as follows:

L4×4×½
$A_g = 3.75$ in.^{2}
$r_z = 0.776$ in.
$\bar{x} = 1.18$ in.

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
$P_u = 1.2(20 \text{ kips}) + 1.6(60 \text{ kips})$ $= 120 \text{ kips}$	$P_a = 20 \text{ kips} + 60 \text{ kips}$ $= 80.0 \text{ kips}$

Tensile Yielding

$$P_n = F_y A_g \quad (Spec.\ \text{Eq. D2-1})$$
$$= (36 \text{ ksi})(3.75 \text{ in.}^2)$$
$$= 135 \text{ kips}$$

From AISC *Specification* Section D2, the available tensile yielding strength is:

LRFD	ASD
$\phi_t = 0.90$ $\phi_t P_n = 0.90(135 \text{ kips})$ $= 122 \text{ kips} > 120 \text{ kips}$ **o.k.**	$\Omega_t = 1.67$ $\dfrac{P_n}{\Omega_t} = \dfrac{135 \text{ kips}}{1.67}$ $= 80.8 \text{ kips} > 80.0 \text{ kips}$ **o.k.**

Tensile Rupture

From the description of the element in AISC *Specification* Table D3.1 Case 8, calculate the shear lag factor, U, as the larger of the values from AISC *Specification* Section D3, Table D3.1 Case 2 and Case 8.

From AISC *Specification* Section D3, for open cross sections, U need not be less than the ratio of the gross area of the connected element(s) to the member gross area. Half of the member is connected, therefore, the minimum value of U is:

$$U = 0.500$$

Case 2, where l is the length of connection and $\bar{y} = \bar{x}$:

$$\begin{aligned} U &= 1 - \frac{\bar{x}}{l} \\ &= 1 - \frac{1.18 \text{ in.}}{9.00 \text{ in.}} \\ &= 0.869 \end{aligned}$$

Case 8, with four or more fasteners per line in the direction of loading:

$$U = 0.80$$

Therefore, use the larger $U = 0.869$.

Calculate A_n using AISC *Specification* Section B4.3b.

$$\begin{aligned} A_n &= A_g - (d_h + \tfrac{1}{16} \text{ in.})t \\ &= 3.75 \text{ in.} - (\tfrac{13}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(\tfrac{1}{2} \text{ in.}) \\ &= 3.31 \text{ in.}^2 \end{aligned}$$

Calculate A_e using AISC *Specification* Section D3.

$$\begin{aligned} A_e &= A_n U \\ &= (3.31 \text{ in.}^2)(0.869) \\ &= 2.88 \text{ in.}^2 \end{aligned}$$

(*Spec*. Eq. D3-1)

$$\begin{aligned} P_n &= F_u A_e \\ &= (58 \text{ ksi})(2.88 \text{ in.}^2) \\ &= 167 \text{ kips} \end{aligned}$$

(*Spec*. Eq. D2-2)

From AISC *Specification* Section D2, the available tensile rupture strength is:

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(167 \text{ kips})$ $= 125 \text{ kips} > 120 \text{ kips}$ **o.k.**	$\Omega_t = 2.00$ $\dfrac{P_n}{\Omega_t} = \dfrac{167 \text{ kips}}{2.00}$ $= 83.5 \text{ kips} > 80.0 \text{ kips}$ **o.k.**

The L4×4×½ available tensile strength is governed by the tensile yielding limit state.

LRFD	ASD
$\phi_t P_n = 122 \text{ kips} > 120 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_t} = 80.8 \text{ kips} > 80.0 \text{ kips}$ **o.k.**

Recommended L_{max}

Using AISC *Specification* Section D1:

$$L_{max} = 300 r_z$$
$$= 300\left(\frac{0.776 \text{ in.}}{12 \text{ in./ft}}\right)$$
$$= 19.4 \text{ ft}$$

Note: The *L/r* limit is a recommendation, not a requirement.

See Chapter J for illustrations of connection limit state checks.

EXAMPLE D.3 WT-SHAPE TENSION MEMBER

Given:

An ASTM A992 WT6×20 member has a length of 30 ft and carries a dead load of 40 kips and a live load of 120 kips in tension. As shown in Figure D3-1, the end connection is fillet welded on each side for 16 in. Verify the member tensile strength by both LRFD and ASD. Assume that the gusset plate and the weld are satisfactory.

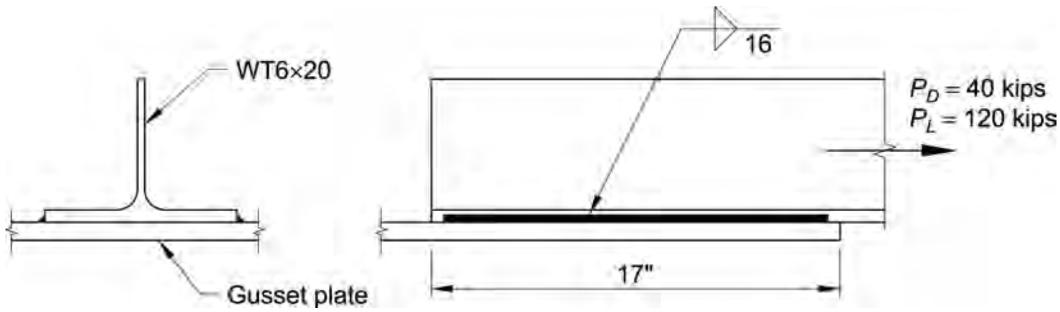


Fig. D.3-1. Connection geometry for Example D.3.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1-8, the geometric properties are as follows:

WT6×20
$A_g = 5.84$ in.^{2}
$b_f = 8.01$ in.
$t_f = 0.515$ in.
$r_x = 1.57$ in.
$\bar{y} = 1.09$ in.

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
$P_u = 1.2(40 \text{ kips}) + 1.6(120 \text{ kips})$ $= 240 \text{ kips}$	$P_a = 40 \text{ kips} + 120 \text{ kips}$ $= 160 \text{ kips}$

Tensile Yielding

Check tensile yielding limit state using AISC *Manual* Table 5-3.

LRFD	ASD
$\phi_t P_n = 263 \text{ kips} > 240 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_t} = 175 \text{ kips} > 160 \text{ kips}$ **o.k.**

Tensile Rupture

Check tensile rupture limit state using AISC *Manual* Table 5-3.

LRFD	ASD
$\phi_t P_n = 214 \text{ kips} < 240 \text{ kips}$ **n.g.**	$\dfrac{P_n}{\Omega_t} = 142 \text{ kips} < 160 \text{ kips}$ **n.g.**

The tabulated available rupture strengths don't work and may be conservative for this case; therefore, calculate the exact solution.

Calculate U as the larger of the values from AISC *Specification* Section D3 and Table D3.1 Case 4.

From AISC *Specification* Section D3, for open cross sections, U need not be less than the ratio of the gross area of the connected element(s) to the member gross area.

$$\begin{aligned} U &= \frac{b_f t_f}{A_g} \\ &= \frac{(8.01 \text{ in.})(0.515 \text{ in.})}{5.84 \text{ in.}^2} \\ &= 0.706 \end{aligned}$$

Case 4, where l is the length of the connection and $\bar{x} = \bar{y}$:

$$\begin{aligned} U &= \frac{3l^2}{3l^2 + w^2}\left(1 - \frac{\bar{x}}{l}\right) \\ &= \left[\frac{3(16.0 \text{ in.})^2}{3(16.0 \text{ in.})^2 + (8.01 \text{ in.})^2}\right]\left(1 - \frac{1.09 \text{ in.}}{16.0 \text{ in.}}\right) \\ &= 0.860 \end{aligned}$$

Therefore, use $U = 0.860$.

Calculate A_n using AISC *Specification* Section B4.3. Because there are no reductions due to bolt holes or notches:

$$\begin{aligned} A_n &= A_g \\ &= 5.84 \text{ in.}^2 \end{aligned}$$

Calculate A_e using AISC *Specification* Section D3.

$$\begin{aligned} A_e &= A_n U \\ &= (5.84 \text{ in.}^2)(0.860) \\ &= 5.02 \text{ in.}^2 \end{aligned}$$ (*Spec*. Eq. D3-1)

Calculate P_n.

$$\begin{aligned} P_n &= F_u A_e \\ &= (65 \text{ ksi})(5.02 \text{ in.}^2) \\ &= 326 \text{ kips} \end{aligned}$$ (*Spec*. Eq. D2-2)

From AISC *Specification* Section D2, the available tensile rupture strength is:

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(326 \text{ kips})$ $= 245 \text{ kips} > 240 \text{ kips}$ **o.k.**	$\Omega_t = 2.00$ $\dfrac{P_n}{\Omega_t} = \dfrac{326 \text{ kips}}{2.00}$ $= 163 \text{ kips} > 160 \text{ kips}$ **o.k.**

Alternately, the available tensile rupture strengths can be determined by modifying the tabulated values. The available tensile rupture strengths published in the tension member selection tables are based on the assumption that $A_e = 0.75A_g$. The actual available strengths can be determined by adjusting the values from AISC *Manual* Table 5-3 as follows:

LRFD	ASD
$\phi_t P_n = (214 \text{ kips})\left(\dfrac{A_e}{0.75A_g}\right)$ $= (214 \text{ kips})\left[\dfrac{5.02 \text{ in.}^2}{0.75(5.84 \text{ in.}^2)}\right]$ $= 245 \text{ kips} > 240 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_t} = (142 \text{ kips})\left(\dfrac{A_e}{0.75A_g}\right)$ $= (142 \text{ kips})\left[\dfrac{5.02 \text{ in.}^2}{0.75(5.84 \text{ in.}^2)}\right]$ $= 163 \text{ kips} > 160 \text{ kips}$ **o.k.**

Recommended Slenderness Limit

$$\frac{L}{r_x} = \frac{(30.0 \text{ ft})(12 \text{ in./ft})}{1.57 \text{ in.}}$$

$= 229 < 300$ from AISC *Specification* Section D1 **o.k.**

Note: The L/r_x limit is a recommendation, not a requirement.

See Chapter J for illustrations of connection limit state checks.

EXAMPLE D.6 DOUBLE-ANGLE TENSION MEMBER

Given:

An ASTM A36 2L4×4×½ (⅜-in. separation) has one line of eight ¾-in.-diameter bolts in standard holes and is 25 ft in length as shown in Figure D.6-1. The double angle is carrying a dead load of 40 kips and a live load of 120 kips in tension. Verify the member tensile strength. Assume that the gusset plate and bolts are satisfactory.

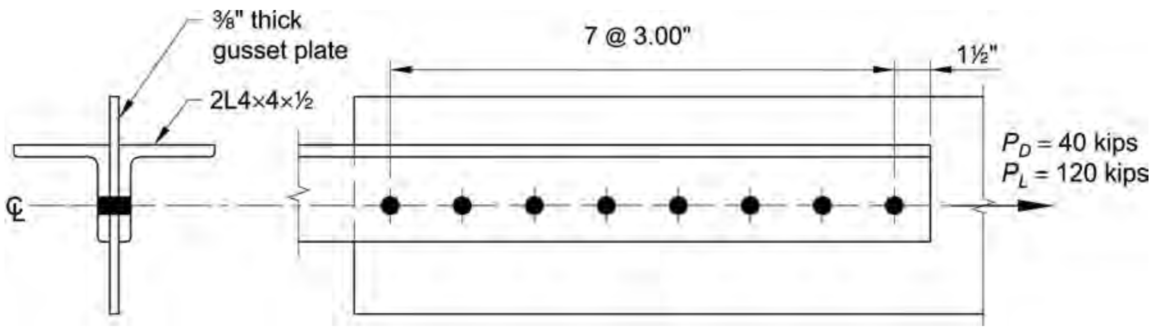


Fig. D.6-1. Connection geometry for Example D.6.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-7 and 1-15, the geometric properties are as follows:

L4×4×½
$\bar{x}$ = 1.18 in.

2L4×4×½ (s = ⅜ in.)
A_g = 7.50 in.^{2}
r_y = 1.83 in.
r_x = 1.21 in.

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
$P_u = 1.2(40\text{ kips}) + 1.6(120\text{ kips})$ $= 240\text{ kips}$	$P_a = 40\text{ kips} + 120\text{ kips}$ $= 160\text{ kips}$

Tensile Yielding

Check tensile yielding limit state using AISC *Manual* Table 5-8.

LRFD	ASD
$\phi_t P_n = 243\text{ kips} > 240\text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_t} = 162\text{ kips} > 160\text{ kips}$ **o.k.**

Tensile Rupture

Determine the available tensile rupture strength using AISC *Specification* Section D2. Calculate U as the larger of the values from AISC *Specification* Section D3, Table D3.1 Case 2 and Case 8.

From AISC *Specification* Section D3, for open cross sections, U need not be less than the ratio of the gross area of the connected element(s) to the member gross area. Half of the member is connected, therefore, the minimum U value is:

$$U = 0.500$$

From Case 2, where l is the length of connection:

$$\begin{aligned} U &= 1 - \frac{\bar{x}}{l} \\ &= 1 - \frac{1.18 \text{ in.}}{21.0 \text{ in.}} \\ &= 0.944 \end{aligned}$$

From Case 8, with four or more fasteners per line in the direction of loading:

$$U = 0.80$$

Therefore, use $U = 0.944$.

Calculate A_n using AISC *Specification* Section B4.3.

$$\begin{aligned} A_n &= A_g - 2\left(d_h + \tfrac{1}{16} \text{ in.}\right)t \\ &= 7.50 \text{ in.}^2 - 2\left(\tfrac{13}{16} \text{ in.} + \tfrac{1}{16} \text{ in.}\right)\left(\tfrac{1}{2} \text{ in.}\right) \\ &= 6.63 \text{ in.}^2 \end{aligned}$$

Calculate A_e using AISC *Specification* Section D3.

$$\begin{aligned} A_e &= A_n U \\ &= \left(6.63 \text{ in.}^2\right)\left(0.944\right) \\ &= 6.26 \text{ in.}^2 \end{aligned} \qquad \text{(}\textit{Spec}\text{. Eq. D3-1)}$$

Calculate P_n.

$$\begin{aligned} P_n &= F_u A_e \\ &= \left(58 \text{ ksi}\right)\left(6.26 \text{ in.}^2\right) \\ &= 363 \text{ kips} \end{aligned} \qquad \text{(}\textit{Spec}\text{. Eq. D2-2)}$$

From AISC *Specification* Section D2, the available tensile rupture strength is:

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(363 \text{ kips})$ $= 272 \text{ kips}$	$\Omega_t = 2.00$ $\dfrac{P_n}{\Omega_t} = \dfrac{363 \text{ kips}}{2.00}$ $= 182 \text{ kips}$

Note that AISC *Manual* Table 5-8 could also be conservatively used since $A_e \geq 0.75A_g$.

The double-angle available tensile strength is governed by the tensile yielding limit state.

LRFD	ASD
$243 \text{ kips} > 240 \text{ kips}$ **o.k.**	$162 \text{ kips} > 160 \text{ kips}$ **o.k.**

Recommended Slenderness Limit

$$\frac{L}{r_x} = \frac{(25.0 \text{ ft})(12 \text{ in./ft})}{1.21 \text{ in.}}$$

$$= 248 < 300 \text{ from AISC } \textit{Specification} \text{ Section D1} \quad \textbf{o.k.}$$

Note: From AISC *Specification* Section D4, the longitudinal spacing of connectors between components of built-up members should preferably limit the slenderness ratio in any component between the connectors to a maximum of 300.

See Chapter J for illustrations of connection limit state checks.

EXAMPLE D.9 PLATE WITH STAGGERED BOLTS

Given:

Compute A_n and A_e for a 14-in.-wide and ½-in.-thick plate subject to tensile loading with staggered holes as shown in Figure D.9-1.

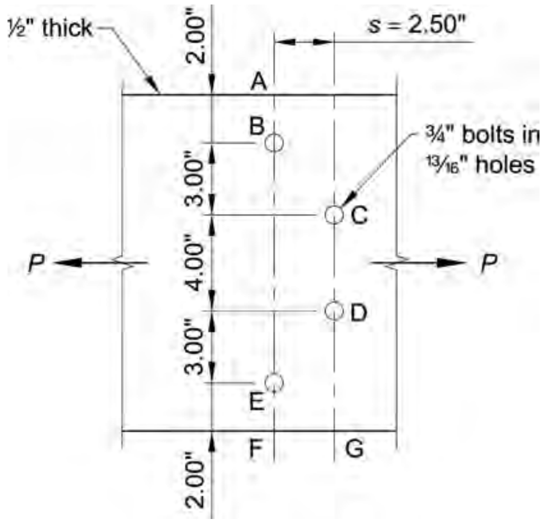


Fig. D.9-1. Connection geometry for Example D.9.

Solution:

Calculate the net hole diameter using AISC *Specification* Section B4.3b.

$$
\begin{aligned}
d_{net} &= d_h + \tfrac{1}{16} \text{ in.} \\
&= \tfrac{13}{16} \text{ in.} + \tfrac{1}{16} \text{ in.} \\
&= 0.875 \text{ in.}
\end{aligned}
$$

Compute the net width for all possible paths across the plate. Because of symmetry, many of the net widths are identical and need not be calculated.

$$w = 14.0 \text{ in.} - \Sigma d_{net} + \Sigma \frac{s^2}{4g} \quad \text{from AISC } \textit{Specification} \text{ Section B4.3b.}$$

Line A-B-E-F:

$$
\begin{aligned}
w &= 14.0 \text{ in.} - 2(0.875 \text{ in.}) \\
&= 12.3 \text{ in.}
\end{aligned}
$$

Line A-B-C-D-E-F:

$$
\begin{aligned}
w &= 14.0 \text{ in.} - 4(0.875 \text{ in.}) + \frac{(2.50 \text{ in.})^2}{4(3.00 \text{ in.})} + \frac{(2.50 \text{ in.})^2}{4(3.00 \text{ in.})} \\
&= 11.5 \text{ in.}
\end{aligned}
$$

Line A-B-C-D-G:

$$w = 14.0\text{ in.} - 3(0.875\text{ in.}) + \frac{(2.50\text{ in.})^2}{4(3.00\text{ in.})}$$
$$= 11.9\text{ in.}$$

Line A-B-D-E-F:

$$w = 14.0\text{ in.} - 3(0.875\text{ in.}) + \frac{(2.50\text{ in.})^2}{4(7.00\text{ in.})} + \frac{(2.50\text{ in.})^2}{4(3.00\text{ in.})}$$
$$= 12.1\text{ in.}$$

Line A-B-C-D-E-F controls the width, *w*, therefore:

$$A_n = wt$$
$$= (11.5\text{ in.})(\tfrac{1}{2}\text{ in.})$$
$$= 5.75\text{ in.}^2$$

Calculate *U*.

From AISC *Specification* Table D3.1 Case 1, because tension load is transmitted to all elements by the fasteners,

$$U = 1.0$$

$$A_e = A_n U \qquad \text{(}Spec\text{. Eq. D3-1)}$$
$$= (5.75\text{ in.}^2)(1.0)$$
$$= 5.75\text{ in.}^2$$

Chapter E
Design of Members for Compression

This chapter covers the design of compression members, the most common of which are columns. The AISC *Manual* includes design tables for the following compression member types in their most commonly available grades:

- W-shapes and HP-shapes
- Rectangular, square and round HSS
- Pipes
- WT-shapes
- Double angles
- Single angles

LRFD and ASD information is presented side-by-side for quick selection, design or verification. All of the tables account for the reduced strength of sections with slender elements.

The design and selection method for both LRFD and ASD is similar to that of previous editions of the AISC *Specification*, and will provide similar designs. In this AISC *Specification*, LRFD and ASD will provide identical designs when the live load is approximately three times the dead load.

The design of built-up shapes with slender elements can be tedious and time consuming, and it is recommended that standard rolled shapes be used whenever possible.

E1. GENERAL PROVISIONS

The design compressive strength, $\phi_c P_n$, and the allowable compressive strength, P_n/Ω_c, are determined as follows:

P_n = nominal compressive strength is the lowest value obtained based on the applicable limit states of flexural buckling, torsional buckling, and flexural-torsional buckling, kips

ϕ_c = 0.90 (LRFD) Ω_c = 1.67 (ASD)

Because the critical stress, F_{cr}, is used extensively in calculations for compression members, it has been tabulated in AISC *Manual* Table 4-14 for all of the common steel yield strengths.

E2. EFFECTIVE LENGTH

In the AISC *Specification*, there is no limit on slenderness, L_c/r. Per the User Note in AISC *Specification* Section E2, it is recommended that L_c/r not exceed 200, as a practical limit based on professional judgment and construction economics.

Although there is no restriction on the unbraced length of columns, the tables of the AISC *Manual* are stopped at common or practical lengths for ordinary usage. For example, a double L3×3×¼, with a ⅜-in. separation has an r_y of 1.38 in. At a L_c/r of 200, this strut would be 23 ft long. This is thought to be a reasonable limit based on fabrication and handling requirements.

Throughout the AISC *Manual*, shapes that contain slender elements for compression when supplied in their most common material grade are footnoted with the letter "c." For example, see a W14×22[c].

E3. FLEXURAL BUCKLING OF MEMBERS WITHOUT SLENDER ELEMENTS

Nonslender-element compression members, including nonslender built-up I-shaped columns and nonslender HSS columns, are governed by these provisions. The general design curve for critical stress versus L_c/r is shown in Figure E-1.

The term L_c is used throughout this chapter to describe the length between points that are braced against lateral and/or rotational displacement.

E4. TORSIONAL AND FLEXURAL-TORSIONAL BUCKLING OF SINGLE ANGLES AND MEMBERS WITHOUT SLENDER ELEMENTS

This section is most commonly applicable to double angles and WT sections, which are singly symmetric shapes subject to torsional and flexural-torsional buckling. The available strengths in axial compression of these shapes are tabulated in AISC *Manual* Part 4 and examples on the use of these tables have been included in this chapter for the shapes.

E5. SINGLE-ANGLE COMPRESSION MEMBERS

The available strength of single-angle compression members is tabulated in AISC *Manual* Part 4.

E6. BUILT-UP MEMBERS

The available strengths in axial compression for built-up double angles with intermediate connectors are tabulated in AISC *Manual* Part 4. There are no tables for other built-up shapes in the AISC *Manual*, due to the number of possible geometries.

E7. MEMBERS WITH SLENDER ELEMENTS

The design of these members is similar to members without slender elements except that a reduced effective area is used in lieu of the gross cross-sectional area.

The tables of AISC *Manual* Part 4 incorporate the appropriate reductions in available strength to account for slender elements.

Design examples have been included in this Chapter for built-up I-shaped members with slender webs and slender flanges**.** Examples have also been included for a double angle, WT and an HSS with slender elements.

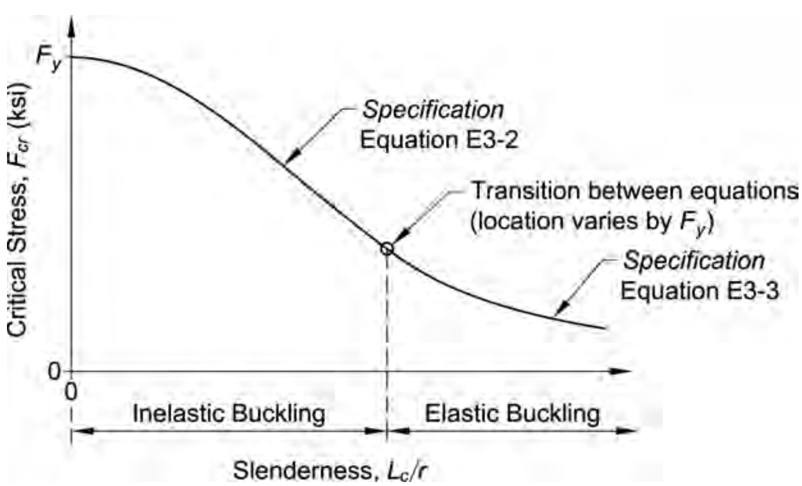


Fig. E-1. Standard column curve.

Table E-1 **Limiting Values of L_c/r and F_e**		
F_y, ksi	**Limiting L_c/r**	**F_e, ksi**
36	134	15.9
50	113	22.4
65	99.5	28.9
70	95.9	31.1

EXAMPLE E.1A W-SHAPE COLUMN DESIGN WITH PINNED ENDS

Given:

Select a W-shape column to carry the loading as shown in Figure E.1A. The column is pinned top and bottom in both axes. Limit the column size to a nominal 14-in. shape. A column is selected for both ASTM A992 and ASTM A913 Grade 65 material.

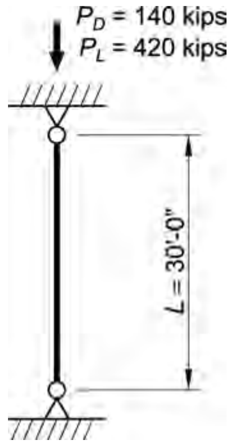


Fig. E.1A. Column loading and bracing.

Solution:

Note that ASTM A913 Grade 70 might also be used in this design. The requirement for higher preheat when welding and the need to use 90-ksi filler metals for complete-joint-penetration (CJP) welds to other 70-ksi pieces offset the advantage of the lighter column and should be considered in the selection of which grade to use.

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi

ASTM A913 Grade 65
F_y = 65 ksi

From ASCE/SEI 7, Chapter 2, the required compressive strength is:

LRFD	ASD
$P_u = 1.2(140 \text{ kips}) + 1.6(420 \text{ kips})$ $= 840 \text{ kips}$	$P_a = 140 \text{ kips} + 420 \text{ kips}$ $= 560 \text{ kips}$

Column Selection—ASTM A992

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, $K_x = K_y = 1.0$. The effective length is:

$$L_c = K_x L_x$$
$$= K_y L_y$$
$$= 1.0(30 \text{ ft})$$
$$= 30.0 \text{ ft}$$

Because the unbraced length is the same in both the *x-x* and *y-y* directions and r_x exceeds r_y for all W-shapes, *y-y* axis bucking will govern.

Enter AISC *Manual* Table 4-1a with an effective length, L_c, of 30 ft, and proceed across the table until reaching the least weight shape with an available strength that equals or exceeds the required strength. Select a W14×132.

From AISC *Manual* Table 4-1a, the available strength for a *y-y* axis effective length of 30 ft is:

LRFD	ASD
$\phi_c P_n = 893 \text{ kips} > 840 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 594 \text{ kips} > 560 \text{ kips}$ **o.k.**

Column Selection–ASTM A913 Grade 65

Enter AISC *Manual* Table 4-1b with an effective length, L_c, of 30 ft, and proceed across the table until reaching the least weight shape with an available strength that equals or exceeds the required strength. Select a W14×120.

From AISC *Manual* Table 4-1b, the available strength for a *y-y* axis effective length of 30 ft is:

LRFD	ASD
$\phi_c P_n = 856 \text{ kips} > 840 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 569 \text{ kips} > 560 \text{ kips}$ **o.k.**

EXAMPLE E.1B W-SHAPE COLUMN DESIGN WITH INTERMEDIATE BRACING

Given:

Verify a W14×90 is adequate to carry the loading as shown in Figure E.1B. The column is pinned top and bottom in both axes and braced at the midpoint about the *y-y* axis and torsionally. The column is verified for both ASTM A992 and ASTM A913 Grade 65 material.

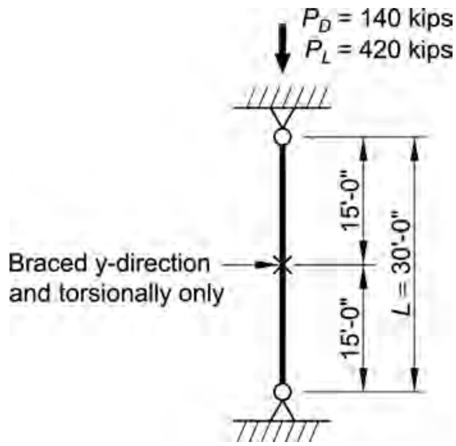


Fig. E.1B. Column loading and bracing.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi

ASTM A913 Grade 65
F_y = 65 ksi

From ASCE/SEI 7, Chapter 2, the required compressive strength is:

LRFD	ASD
$P_u = 1.2(140 \text{ kips}) + 1.6(420 \text{ kips})$ $= 840 \text{ kips}$	$P_a = 140 \text{ kips} + 420 \text{ kips}$ $= 560 \text{ kips}$

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, $K_x = K_y$ = 1.0. The effective length about the *y*-y axis is:

$$L_{cy} = K_y L_y$$
$$= 1.0(15 \text{ ft})$$
$$= 15.0 \text{ ft}$$

The values tabulated in AISC *Manual* Tables 4-1a, 4-1b and 4-1c are provided for buckling in the *y-y* direction. To determine the buckling strength in the *x-x* axis, an equivalent effective length for the *y-y* axis is determined using the r_x/r_y ratio provided at the the bottom of these tables. For a W14×90, r_x/r_y = 1.66, and the equivalent *y-y* axis effective length for *x-x* axis buckling is computed as:

$$L_{cx} = K_x L_x$$
$$= 1.0(30 \text{ ft})$$
$$= 30.0 \text{ ft}$$

$$L_{cy\ eq} = \frac{L_{cx}}{r_x/r_y} \qquad (Manual \text{ Eq. 4-1})$$
$$= \frac{30.0 \text{ ft}}{1.66}$$
$$= 18.1 \text{ ft}$$

Because 18.1 ft > 15.0 ft, the available compressive strength is governed by the *x-x* axis flexural buckling limit state.

Available Compressive Strength—ASTM A992

The available strength of a W14×90 is determined using AISC *Manual* Table 4-1a, conservatively using an unbraced length of L_c = 19.0 ft.

LRFD	ASD
$\phi_c P_n = 903 \text{ kips} > 840 \text{ kips}$ **o.k.**	$\frac{P_n}{\Omega_c} = 601 \text{ kips} > 560 \text{ kips}$ **o.k.**

Available Compressive Strength—ASTM 913 Grade 65

The available strength of a W14×90 is determined using AISC *Manual* Table 4-1b, conservatively using an unbraced length of L_c = 19.0 ft.

LRFD	ASD
$\phi_c P_n = 1{,}080 \text{ kips} > 840 \text{ kips}$ **o.k.**	$\frac{P_n}{\Omega_c} = 719 \text{ kips} > 560 \text{ kips}$ **o.k.**

The available strengths of the columns described in Examples E.1A and E.1B are easily selected directly from the AISC *Manual* Tables. The available strengths can also be determined as shown in the following Examples E.1C and E.1D.

EXAMPLE E.1C W-SHAPE AVAILABLE STRENGTH CALCULATION

Given:

Calculate the available strength of the column sizes selected in Example E.1A with unbraced lengths of 30 ft in both axes. The material properties and loads are as given in Example E.1A.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi

ASTM A913 Grade 65
F_y = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W14×120
A_g = 35.3 in.^{2}
r_x = 6.24 in.
r_y = 3.74 in.

W14×132
A_g = 38.8 in.^{2}
r_x = 6.28 in.
r_y = 3.76 in.

Column Compressive Strength—ASTM A992

Slenderness Check

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, $K_x = K_y$ = 1.0. The effective length about the *y*-y axis is:

$$\begin{aligned} L_{cy} &= K_y L_y \\ &= 1.0(30 \text{ ft}) \\ &= 30.0 \text{ ft} \end{aligned}$$

Because the unbraced length for the W14×132 column is the same for both axes, the *y-y* axis will govern.

$$\begin{aligned} \frac{L_{cy}}{r_y} &= \frac{(30.0 \text{ ft})(12 \text{ in./ft})}{3.76 \text{ in.}} \\ &= 95.7 \end{aligned}$$

Critical Stress

For F_y = 50 ksi, the available critical stresses, $\phi_c F_{cr}$ and F_{cr}/Ω_c for L_c/r = 95.7 are interpolated from AISC *Manual* Table 4-14 as follows. The available critical stress can also be determined as shown in Example E.1D.

LRFD	ASD
$\phi_c F_{cr} = 23.0 \text{ ksi}$	$\dfrac{F_{cr}}{\Omega_c} = 15.4 \text{ ksi}$

From AISC *Specification* Equation E3-1, the available compressive strength of the W14×132 column is:

LRFD	ASD
$\phi_c P_n = (\phi_c F_{cr}) A_g$ $= (23.0 \text{ ksi})(38.8 \text{ in.}^2)$ $= 892 \text{ kips} > 840 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = \left(\dfrac{F_{cr}}{\Omega_c}\right) A_g$ $= (15.4 \text{ ksi})(38.8 \text{ in.}^2)$ $= 598 \text{ kips} > 560 \text{ kips}$ **o.k.**

Column Compressive Strength—ASTM A913 Grade 65

Slenderness Check

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, $K_x = K_y = 1.0$. The effective length about the *y*-y axis is:

$$
\begin{aligned}
L_{cy} &= K_y L_y \\
&= 1.0(30 \text{ ft}) \\
&= 30.0 \text{ ft}
\end{aligned}
$$

Because the unbraced length for the W14×120 column is the same for both axes, the *y-y* axis will govern.

$$
\begin{aligned}
\frac{L_{cy}}{r_y} &= \frac{(30.0 \text{ ft})(12 \text{ in./ft})}{3.74 \text{ in.}} \\
&= 96.3
\end{aligned}
$$

Critical Stress

For $F_y = 65$ ksi, the available critical stresses, $\phi_c F_{cr}$ and F_{cr}/Ω_c for $L_c/r = 96.3$ are interpolated from AISC *Manual* Table 4-14 as follows. The available critical stress can also be determined as shown in Example E.1D.

LRFD	ASD
$\phi_c F_{cr} = 24.3 \text{ ksi}$	$\dfrac{F_{cr}}{\Omega_c} = 16.1 \text{ ksi}$

From AISC *Specification* Equation E3-1, the available compressive strength of the W14×120 column is:

LRFD	ASD
$\phi_c P_n = (\phi_c F_{cr}) A_g$ $= (24.3 \text{ ksi})(35.3 \text{ in.}^2)$ $= 858 \text{ kips} > 840 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = \left(\dfrac{F_{cr}}{\Omega_c}\right) A_g$ $= (16.1 \text{ ksi})(35.3 \text{ in.}^2)$ $= 568 \text{ kips} > 560 \text{ kips}$ **o.k.**

Note that the calculated values are approximately equal to the tabulated values.

EXAMPLE E.1D W-SHAPE AVAILABLE STRENGTH CALCULATION

Given:

Calculate the available strength of a W14×90 with a *x-x* axis unbraced length of 30 ft and *y-y* axis and torsional unbraced lengths of 15 ft. The material properties and loads are as given in Example E.1A.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi

ASTM A913 Grade 65
F_y = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W14×90
A_g = 26.5 in.^{2}
r_x = 6.14 in.
r_y = 3.70 in.

$$\frac{b_f}{2t_f} = 10.2$$

$$\frac{h}{t_w} = 25.9$$

Slenderness Check

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, $K_x = K_y = 1.0$.

$$\begin{aligned} L_{cx} &= K_x L_x \\ &= 1.0(30 \text{ ft}) \\ &= 30.0 \text{ ft} \end{aligned}$$

$$\begin{aligned} \frac{L_{cx}}{r_x} &= \frac{(30.0 \text{ ft})(12 \text{ in./ft})}{6.14 \text{ in.}} \\ &= 58.6 \quad \textbf{governs} \end{aligned}$$

$$\begin{aligned} L_{cy} &= K_y L_y \\ &= 1.0(15 \text{ ft}) \\ &= 15.0 \text{ ft} \end{aligned}$$

$$\begin{aligned} \frac{L_{cy}}{r_y} &= \frac{(15.0 \text{ ft})(12 \text{ in./ft})}{3.70 \text{ in.}} \\ &= 48.6 \end{aligned}$$

Column Compressive Strength—ASTM A992

Width-to-Thickness Ratio

The width-to-thickness ratio of the flanges of the W14×90 is:

$$\frac{b_f}{2t_f} = 10.2$$

From AISC *Specification* Table B4.1a, Case 1, the limiting width-to-thickness ratio of the flanges is:

$$0.56\sqrt{\frac{E}{F_y}} = 0.56\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 13.5 > 10.2; \text{ therefore, the flanges are nonslender}$$

The width-to-thickness ratio of the web of the W14×90 is:

$$\frac{h}{t_w} = 25.9$$

From AISC *Specification* Table B4.1a, Case 5, the limiting width-to-thickness ratio of the web is:

$$1.49\sqrt{\frac{E}{F_y}} = 1.49\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 35.9 > 25.9; \text{ therefore, the web is nonslender}$$

Because the web and flanges are nonslender, the limit state of local buckling does not apply.

Critical Stresses

The available critical stresses may be interpolated from AISC *Manual* Table 4-14 or calculated directly as follows.

Calculate the elastic critical buckling stress, F_e, according to AISC *Specification* Section E3. As noted in AISC *Specification* Commentary Section E4, torsional buckling of symmetric shapes is a failure mode usually not considered in the design of hot-rolled columns. This failure mode generally does not govern unless the section is manufactured from relatively thin plates or a torsional unbraced length significantly larger than the y-y axis flexural unbraced length is present.

$$F_e = \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)^2} \qquad \text{(\textit{Spec}. Eq. E3-4)}$$
$$= \frac{\pi^2 (29{,}000 \text{ ksi})}{(58.6)^2}$$
$$= 83.3 \text{ ksi}$$

Calculate the flexural buckling stress, F_{cr}.

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 113$$

Because $\frac{L_c}{r} = 58.6 < 113,$

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right)F_y \qquad (\textit{Spec}.\ \text{Eq. E3-2})$$

$$= \left(0.658^{\frac{50\text{ ksi}}{83.3\text{ ksi}}}\right)(50\text{ ksi})$$

$$= 38.9\text{ ksi}$$

Nominal Compressive Strength

$$P_n = F_{cr}A_g \qquad (\textit{Spec}.\ \text{Eq. E3-1})$$

$$= (38.9\text{ ksi})(26.5\text{ in.}^2)$$

$$= 1{,}030\text{ kips}$$

From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(1{,}030\text{ kips})$ $= 927\text{ kips} > 840\text{ kips}$ **o.k.**	$\Omega_c = 1.67$ $\frac{P_n}{\Omega_c} = \frac{1{,}030\text{ kips}}{1.67}$ $= 617\text{ kips} > 560\text{ kips}$ **o.k.**

Column Compressive Strength—ASTM A913 Grade 65

Width-to-Thickness Ratio

The width-to-thickness ratio of the flanges of the W14×90 is:

$$\frac{b_f}{2t_f} = 10.2$$

From AISC *Specification* Table B4.1a, Case 1, the limiting width-to-thickness ratio of the flanges is:

$$0.56\sqrt{\frac{E}{F_y}} = 0.56\sqrt{\frac{29{,}000\text{ ksi}}{65\text{ ksi}}}$$

$$= 11.8 > 10.2;\ \text{therefore, the flanges are nonslender}$$

The width-to-thickness ratio of the web of the W14×90 is:

$$\frac{h}{t_w} = 25.9$$

From AISC *Specification* Table B4.1a, Case 5, the limiting width-to-thickness ratio of the web is:

$$1.49\sqrt{\frac{E}{F_y}} = 1.49\sqrt{\frac{29{,}000 \text{ ksi}}{65 \text{ ksi}}}$$
$$= 31.5 > 25.9; \text{ therefore, the web is nonslender}$$

Because the web and flanges are nonslender, the limit state of local buckling does not apply.

Critical Stress

$F_e = 83.3$ ksi (calculated previously)

Calculate the flexural buckling stress, F_{cr}.

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000 \text{ ksi}}{65 \text{ ksi}}}$$
$$= 99.5$$

Because $\frac{L_c}{r} = 58.6 < 99.5$,

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right)F_y \qquad \text{(Spec. Eq. E3-2)}$$
$$= \left(0.658^{\frac{65 \text{ ksi}}{83.3 \text{ ksi}}}\right)(65 \text{ ksi})$$
$$= 46.9 \text{ ksi}$$

Nominal Compressive Strength

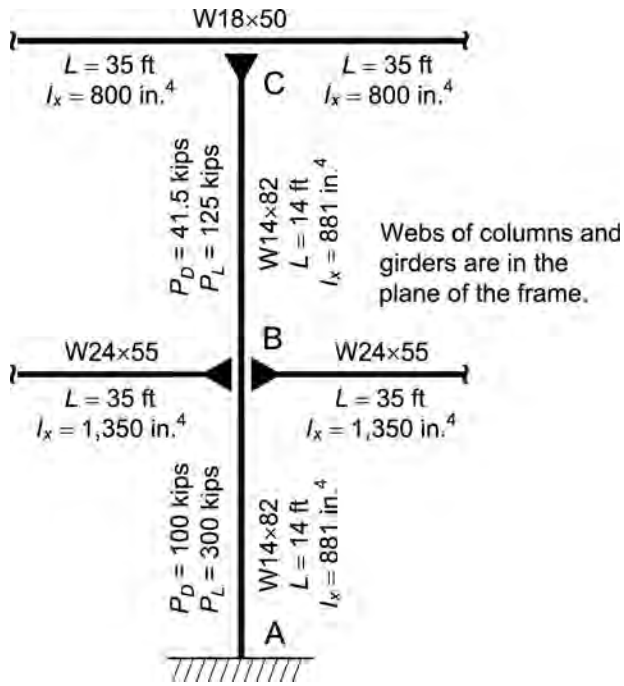
$$P_n = F_{cr}A_g \qquad \text{(Spec. Eq. E3-1)}$$
$$= (46.9 \text{ ksi})(26.5 \text{ in.}^2)$$
$$= 1{,}240 \text{ kips}$$

From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(1{,}240 \text{ kips})$ $= 1{,}120 \text{ kips} > 840 \text{ kips}$ **o.k.**	$\Omega_c = 1.67$ $\frac{P_n}{\Omega_c} = \frac{1{,}240 \text{ kips}}{1.67}$ $= 743 \text{ kips} > 560 \text{ kips}$ **o.k.**

EXAMPLE E.4A W-SHAPE COMPRESSION MEMBER (MOMENT FRAME)

This example is primarily intended to illustrate the use of the alignment chart for sidesway uninhibited columns in conjunction with the effective length method.



Given:

The member sizes shown for the moment frame illustrated here (sidesway uninhibited in the plane of the frame) have been determined to be adequate for lateral loads. The material for both the column and the girders is ASTM A992. The loads shown at each level are the accumulated dead loads and live loads at that story. The column is fixed at the base about the *x-x* axis of the column.

Determine if the column is adequate to support the gravity loads shown. Assume the column is continuously supported in the transverse direction (the *y-y* axis of the column).

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W18×50
I_x = 800 in.^{4}

W24×55
I_x = 1,350 in.^{4}

W14×82
A_g = 24.0 in.^{2}
I_x = 881 in.^{4}

Column B-C

From ASCE/SEI 7, Chapter 2, the required compressive strength for the column between the roof and floor is:

LRFD	ASD
$P_u = 1.2(41.5 \text{ kips}) + 1.6(125 \text{ kips})$ $= 250 \text{ kips}$	$P_a = 41.5 \text{ kips} + 125 \text{ kips}$ $= 167 \text{ kips}$

Effective Length Factor

Using the effective length method, the effective length factor is determined using AISC *Specification* Commentary Appendix 7, Section 7.2. As discussed there, column inelasticity should be addressed by incorporating the stiffness reduction parameter, τ_b. Determine G_{top} and G_{bottom} accounting for column inelasticity by replacing $E_{col}I_{col}$ with $\tau_b(E_{col}I_{col})$. Calculate the stiffness reduction parameter, τ_b, for the column B-C using AISC *Manual* Table 4-13.

LRFD	ASD
$\frac{P_u}{A_g} = \frac{250 \text{ kips}}{24.0 \text{ in.}^2} = 10.4 \text{ ksi}$ $\tau_b = 1.00$	$\frac{P_a}{A_g} = \frac{167 \text{ kips}}{24.0 \text{ in.}^2} = 6.96 \text{ ksi}$ $\tau_b = 1.00$

Therefore, no reduction in stiffness for inelastic buckling will be required.

Determine G_{top} and G_{bottom}.

$$G_{top} = \tau_b \left[\frac{\sum (EI/L)_{col}}{\sum (EI/L)_g} \right] \quad \text{(from } \textit{Spec.} \text{ Comm. Eq. C-A-7-3)}$$

$$= 1.00 \left\{ \frac{\left[\frac{(29{,}000 \text{ ksi})(881 \text{ in.}^4)}{14.0 \text{ ft}} \right]}{2\left[\frac{(29{,}000 \text{ ksi})(800 \text{ in.}^4)}{35.0 \text{ ft}} \right]} \right\}$$

$$= 1.38$$

$$G_{bottom} = \tau_b \left[\frac{\sum (EI/L)_{col}}{\sum (EI/L)_g} \right] \quad \text{(from } \textit{Spec.} \text{ Comm. Eq. C-A-7-3)}$$

$$= 1.00 \left\{ \frac{2\left[\frac{(29{,}000 \text{ ksi})(881 \text{ in.}^4)}{14.0 \text{ ft}} \right]}{2\left[\frac{(29{,}000 \text{ ksi})(1{,}350 \text{ in.}^4)}{35.0 \text{ ft}} \right]} \right\}$$

$$= 1.63$$

From the alignment chart, AISC *Specification* Commentary Figure C-A-7.2, K is slightly less than 1.5; therefore use $K = 1.5$. Because the column available strength tables are based on the L_c about the y-y axis, the equivalent effective column length of the upper segment for use in the table is:

$$L_{cx} = (KL)_x$$
$$= 1.5(14 \text{ ft})$$
$$= 21.0 \text{ ft}$$

From AISC *Manual* Table 4-1a, for a W14×82:

$$\frac{r_x}{r_y} = 2.44$$

$$L_c = \frac{L_{cx}}{\left(\frac{r_x}{r_y}\right)}$$
$$= \frac{21.0 \text{ ft}}{2.44}$$
$$= 8.61 \text{ ft}$$

Take the available strength of the W14×82 from AISC *Manual* Table 4-1a.

At L_c = 9 ft, the available strength in axial compression is:

LRFD	ASD
$\phi_c P_n = 940 \text{ kips} > 250 \text{ kips}$ **o.k.**	$\frac{P_n}{\Omega_c} = 626 \text{ kips} > 167 \text{ kips}$ **o.k.**

Column A-B

From Chapter 2 of ASCE/SEI 7, the required compressive strength for the column between the floor and the foundation is:

LRFD	ASD
$P_u = 1.2(100 \text{ kips}) + 1.6(300 \text{ kips})$ $= 600 \text{ kips}$	$P_a = 100 \text{ kips} + 300 \text{ kips}$ $= 400 \text{ kips}$

Effective Length Factor

Determine the stiffness reduction parameter, τ_b, for column A-B using AISC *Manual* Table 4-13.

LRFD	ASD
$\frac{P_u}{A_g} = \frac{600 \text{ kips}}{24.0 \text{ in.}^2}$ $= 25.0 \text{ ksi}$ $\tau_b = 1.00$	$\frac{P_a}{A_g} = \frac{400 \text{ kips}}{24.0 \text{ in.}^2}$ $= 16.7 \text{ ksi}$ $\tau_b = 0.994$

Use $\tau_b = 0.994$.

$$G_{top} = \tau_b \left[\frac{\Sigma(EI/L)_{col}}{\Sigma(EI/L)_g} \right]$$ (from *Spec*. Comm. Eq. C-A-7-3)

$$= 0.994 \left\{ \frac{2\left[\frac{(29{,}000 \text{ ksi})(881 \text{ in.}^4)}{14.0 \text{ ft}}\right]}{2\left[\frac{(29{,}000 \text{ ksi})(1{,}350 \text{ in.}^4)}{35.0 \text{ ft}}\right]} \right\}$$

$$= 1.62$$

$G_{bottom} = 1.0$ (fixed), from AISC *Specification* Commentary Appendix 7, Section 7.2

From the alignment chart, AISC *Specification* Commentary Figure C-A-7.2, K is approximately 1.4. Because the column available strength tables are based on L_c about the y-y axis, the effective column length of the lower segment for use in the table is:

$$\begin{aligned} L_{cx} &= (KL)_x \\ &= 1.4(14 \text{ ft}) \\ &= 19.6 \text{ ft} \end{aligned}$$

$$\begin{aligned} L_c &= \frac{L_{cx}}{\left(\frac{r_x}{r_y}\right)} \\ &= \frac{19.6 \text{ ft}}{2.44} \\ &= 8.03 \text{ ft} \end{aligned}$$

Take the available strength of the W14×82 from AISC *Manual* Table 4-1a.

At $L_c = 9$ ft, (conservative) the available strength in axial compression is:

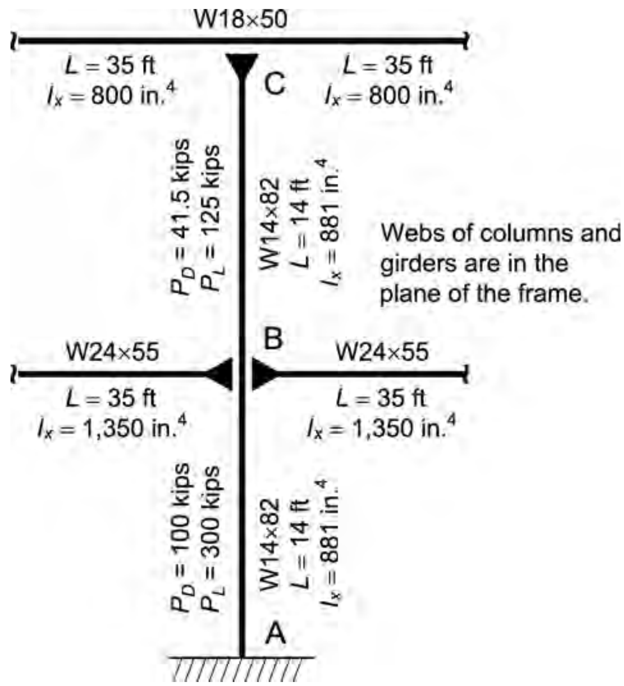
LRFD	ASD
$\phi_c P_n = 940 \text{ kips} > 600 \text{ kips}$ **o.k.**	$\frac{P_n}{\Omega_c} = 626 \text{ kips} > 400 \text{ kips}$ **o.k.**

A more accurate strength could be determined by interpolation from AISC *Manual* Table 4-1a.

EXAMPLE E.4B W-SHAPE COMPRESSION MEMBER (MOMENT FRAME)

Given:

Using the effective length method, determine the available strength of the column shown subject to the same gravity loads shown in Example E.4A with the column pinned at the base about the *x-x* axis. All other assumptions remain the same.



Solution:

As determined in Example E.4A, for the column segment B-C between the roof and the floor, the column strength is adequate.

As determined in Example E.4A, for the column segment A-B between the floor and the foundation,

$$G_{top} = 1.62$$

At the base,

$G_{bottom} = 10$ (pinned) from AISC *Specification* Commentary Appendix 7, Section 7.2

Note: this is the only change in the analysis.

From the alignment chart, AISC *Specification* Commentary Figure C-A-7.2, K is approximately equal to 2.0. Because the column available strength tables are based on the effective length, L_c, about the *y-y* axis, the effective column length of the segment A-B for use in the table is:

$$\begin{aligned} L_{cx} &= (KL)_x \\ &= 2.0(14 \text{ ft}) \\ &= 28.0 \text{ ft} \end{aligned}$$

From AISC *Manual* Table 4-1a, for a W14×82:

$$\frac{r_x}{r_y} = 2.44$$

$$L_c = \frac{L_{cx}}{\left(\dfrac{r_x}{r_y}\right)}$$

$$= \frac{28.0 \text{ ft}}{2.44}$$

$$= 11.5 \text{ ft}$$

Interpolate the available strength of the W14×82 from AISC *Manual* Table 4-1a.

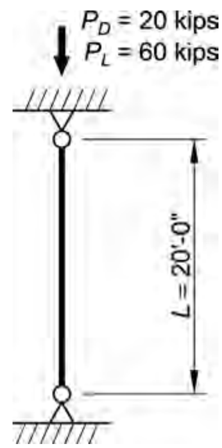
LRFD	ASD
$\phi_c P_n = 861 \text{ kips} > 600 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 573 \text{ kips} > 400 \text{ kips}$ **o.k.**

EXAMPLE E.7 WT COMPRESSION MEMBER WITHOUT SLENDER ELEMENTS

Given:

Select an ASTM A992 nonslender WT-shape compression member with a length of 20 ft to support a dead load of 20 kips and live load of 60 kips in axial compression. The ends are pinned. The solution will be provided using:

(1) AISC *Manual* Tables
(2) Calculations using AISC *Specification* provisions



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From ASCE/SEI 7, Chapter 2, the required compressive strength is:

LRFD	ASD
$P_u = 1.2(20 \text{ kips}) + 1.6(60 \text{ kips})$ $= 120 \text{ kips}$	$P_a = 20 \text{ kips} + 60 \text{ kips}$ $= 80.0 \text{ kips}$

(1) AISC *Manual* Table Solution

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0. Therefore, $L_{cx} = L_{cy} = KL = 1.0(20 \text{ ft}) = 20.0$ ft.

Select the lightest nonslender member from AISC *Manual* Table 4-7 with sufficient available strength about both the *x-x* axis (upper portion of the table) and the *y-y* axis (lower portion of the table) to support the required strength.

Try a WT7×34.

The available strength in compression is:

LRFD	ASD
$\phi_c P_{nx} = 128 \text{ kips} > 120 \text{ kips}$ **o.k. controls**	$\dfrac{P_{nx}}{\Omega_c} = 85.5 \text{ kips} > 80.0 \text{ kips}$ **o.k. controls**
$\phi_c P_{ny} = 222 \text{ kips} > 120 \text{ kips}$ **o.k.**	$\dfrac{P_{ny}}{\Omega_c} = 147 \text{ kips} > 80.0 \text{ kips}$ **o.k.**

Available strength can also be determined by hand calculations, as demonstrated in the following.

(2) Calculation Using AISC *Specification* Provisions

From AISC *Manual* Table 1-8, the geometric properties are as follows.

WT7×34
$A_g = 10.0 \text{ in.}^2$
$r_x = 1.81$ in.
$r_y = 2.46$ in.
$J = 1.50 \text{ in.}^4$

$\bar{y}$ = 1.29 in.
I_x = 32.6 in.^{4}
I_y = 60.7 in.^{4}
d = 7.02 in.
t_w = 0.415 in.
b_f = 10.0 in.
t_f = 0.720 in.

Stem Slenderness Check

$$\lambda = \frac{d}{t_w}$$
$$= \frac{7.02\text{ in.}}{0.415\text{ in.}}$$
$$= 16.9$$

Determine the stem limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a, Case 4:

$$\lambda_r = 0.75\sqrt{\frac{E}{F_y}}$$
$$= 0.75\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}}$$
$$= 18.1$$

$\lambda < \lambda_r$; therefore, the stem is not slender

Flange Slenderness Check

$$\lambda = \frac{b_f}{2t_f}$$
$$= \frac{10.0\text{ in.}}{2(0.720\text{ in.})}$$
$$= 6.94$$

Determine the flange limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a, Case 1:

$$\lambda_r = 0.56\sqrt{\frac{E}{F_y}}$$
$$= 0.56\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}}$$
$$= 13.5$$

$\lambda < \lambda_r$; therefore, the flange is not slender

There are no slender elements.

For compression members without slender elements, AISC *Specification* Sections E3 and E4 apply. The nominal compressive strength, P_n, is determined based on the limit states of flexural, torsional and flexural-torsional buckling.

Elastic Flexural Buckling Stress about the x-x Axis

$$\frac{L_{cx}}{r_x} = \frac{(20.0\text{ ft})(12\text{ in./ft})}{1.81\text{ in.}}$$
$$= 133$$

$$F_{ex} = \frac{\pi^2 E}{\left(\dfrac{L_{cx}}{r_x}\right)^2} \qquad (\textit{Spec}.\text{ Eq. E3-4 or E4-5})$$
$$= \frac{\pi^2 (29{,}000\text{ ksi})}{(133)^2}$$
$$= 16.2\text{ ksi} \quad \textbf{controls}$$

Elastic Flexural Buckling Stress about the y-y Axis

$$\frac{L_{cy}}{r_y} = \frac{(20.0\text{ ft})(12\text{ in./ft})}{2.46\text{ in.}}$$
$$= 97.6$$

$$F_{ey} = \frac{\pi^2 E}{\left(\dfrac{L_{cy}}{r_y}\right)^2} \qquad (\textit{Spec}.\text{ Eq. E3-4 or E4-6})$$
$$= \frac{\pi^2 (29{,}000\text{ ksi})}{(97.6)^2}$$
$$= 30.0\text{ ksi}$$

Torsional and Flexural-Torsional Elastic Buckling Stress

Because the WT7×34 section does not have any slender elements, AISC *Specification* Section E4 will be applicable for torsional and flexural-torsional buckling. F_e will be calculated using AISC *Specification* Equation E4-3. Per the User Note for AISC *Specification* Section E4, the term with C_w is omitted when computing F_{ez}, and x_o is taken as zero. The flexural buckling term about the *y-y* axis, F_{ey}, was computed in the preceding section.

$$x_o = 0$$

$$y_o = \bar{y} - \frac{t_f}{2}$$
$$= 1.29\text{ in.} - \frac{0.720\text{ in.}}{2}$$
$$= 0.930\text{ in.}$$

$$\bar{r}_o^{\,2} = x_o^{\,2} + y_o^{\,2} + \frac{I_x + I_y}{A_g} \qquad \text{(}\textit{Spec}\text{. Eq. E4-9)}$$

$$= 0 + (0.930 \text{ in.})^2 + \frac{32.6 \text{ in.}^4 + 60.7 \text{ in.}^4}{10.0 \text{ in.}^2}$$

$$= 10.2 \text{ in.}^2$$

$$F_{ez} = \left(\frac{\pi^2 EC_w}{L_{cz}^{\,2}} + GJ\right)\frac{1}{A_g \bar{r}_o^{\,2}} \qquad \text{(}\textit{Spec}\text{. Eq. E4-7)}$$

$$= \left[0 + (11{,}200 \text{ ksi})(1.50 \text{ in.}^4)\right]\frac{1}{(10.0 \text{ in.}^2)(10.2 \text{ in.}^2)}$$

$$= 165 \text{ ksi}$$

$$H = 1 - \frac{x_o^{\,2} + y_o^{\,2}}{\bar{r}_o^{\,2}} \qquad \text{(}\textit{Spec}\text{. Eq. E4-8)}$$

$$= 1 - \frac{0 + (0.930 \text{ in.})^2}{10.2 \text{ in.}^2}$$

$$= 0.915$$

$$F_e = \left(\frac{F_{ey} + F_{ez}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}}\right] \qquad \text{(}\textit{Spec}\text{. Eq. E4-3)}$$

$$= \left[\frac{30.0 \text{ ksi} + 165 \text{ ksi}}{2(0.915)}\right]\left[1 - \sqrt{1 - \frac{4(30.0 \text{ ksi})(165 \text{ ksi})(0.915)}{(30.0 \text{ ksi} + 165 \text{ ksi})^2}}\right]$$

$$= 29.5 \text{ ksi}$$

Critical Buckling Stress

The critical buckling stress for the member could be controlled by flexural buckling about either the *x-x* axis or *y-y* axis, F_{ex} or F_{ey}, respectively. Note that AISC *Specification* Equations E4-5 and E4-6 reflect the same buckling modes as calculated in AISC *Specification* Equation E3-4. Or, the critical buckling stress for the member could be controlled by torsional or flexural-torsional buckling calculated per AISC *Specification* Equation E4-3. In this example, F_e calculated in accordance with AISC *Specification* Equation E4-5 is less than that calculated in accordance with AISC *Specification* Equation E4-3 or E4-6 and controls. Therefore:

$$F_e = 16.2 \text{ ksi}$$

$$\frac{F_y}{F_e} = \frac{50 \text{ ksi}}{16.2 \text{ ksi}}$$

$$= 3.09$$

Per the AISC *Specification* User Note for Section E3, the two inequalities for calculating limits of applicability of Sections E3(a) and E3(b) provide the same result for flexural buckling only. When the elastic buckling stress, F_e, is controlled by torsional or flexural-torsional buckling, the L_c/r limits would not be applicable unless an equivalent L_c/r ratio is first calculated by substituting the governing F_e into AISC *Specification* Equation E3-4 and solving for L_c/r. The F_y/F_e limits may be used regardless of which buckling mode governs.

Because $\dfrac{F_y}{F_e} > 2.25$:

$$\begin{aligned} F_{cr} &= 0.877 F_e \\ &= 0.877(16.2 \text{ ksi}) \\ &= 14.2 \text{ ksi} \end{aligned}$$ (*Spec*. Eq. E3-3)

Available Compressive Strength

$$\begin{aligned} P_n &= F_{cr} A_g \\ &= (14.2 \text{ ksi})(10.0 \text{ in.}^2) \\ &= 142 \text{ kips} \end{aligned}$$ (*Spec*. Eq. E3-1)

From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(142 \text{ kips})$ $= 128 \text{ kips} > 120 \text{ kips}$ **o.k.**	$\Omega_c = 1.67$ $\dfrac{P_n}{\Omega_c} = \dfrac{142 \text{ kips}}{1.67}$ $= 85.0 \text{ kips} > 80.0 \text{ kips}$ **o.k.**

Chapter F
Design of Members for Flexure

INTRODUCTION

This *Specification* chapter contains provisions for calculating the flexural strength of members subject to simple bending about one principal axis. Included are specific provisions for I-shaped members, channels, HSS, box sections, tees, double angles, single angles, rectangular bars, rounds and unsymmetrical shapes. Also included is a section with proportioning requirements for beams and girders.

There are selection tables in the AISC *Manual* for standard beams in the commonly available yield strengths. The section property tables for most cross sections provide information that can be used to conveniently identify noncompact and slender element sections. LRFD and ASD information is presented side-by-side.

Most of the formulas from this chapter are illustrated by the following examples. The design and selection techniques illustrated in the examples for both LRFD and ASD will result in similar designs.

F1. GENERAL PROVISIONS

Selection and evaluation of all members is based on deflection requirements and strength, which is determined as the design flexural strength, $\phi_b M_n$, or the allowable flexural strength, M_n/Ω_b,

where

M_n = the lowest nominal flexural strength based on the limit states of yielding, lateral torsional-buckling, and local buckling, where applicable

ϕ_b = 0.90 (LRFD)

Ω_b = 1.67 (ASD)

This design approach is followed in all examples.

The term L_b is used throughout this chapter to describe the length between points which are either braced against lateral displacement of the compression flange or braced against twist of the cross section. Requirements for bracing systems and the required strength and stiffness at brace points are given in AISC *Specification* Appendix 6.

The use of C_b is illustrated in several of the following examples. AISC *Manual* Table 3-1 provides tabulated C_b values for some common situations.

F2. DOUBLY SYMMETRIC COMPACT I-SHAPED MEMBERS AND CHANNELS BENT ABOUT THEIR MAJOR AXIS

AISC *Specification* Section F2 applies to the design of compact beams and channels. As indicated in the User Note in Section F2 of the AISC *Specification*, the vast majority of rolled I-shaped beams and channels fall into this category. The curve presented as a solid line in Figure F-1 is a generic plot of the nominal flexural strength, M_n, as a function of the unbraced length, L_b. The horizontal segment of the curve at the far left, between L_b = 0 ft and L_p, is the range where the strength is limited by flexural yielding. In this region, the nominal strength is taken as the full plastic moment strength of the section as given by AISC *Specification* Equation F2-1. In the range of the curve at the far right, starting at L_r, the strength is limited by elastic buckling. The strength in this region is given by AISC *Specification* Equation F2-3. Between these regions, within the linear region of the curve between $M_n = M_p$ at L_p on the left, and $M_n = 0.7M_y = 0.7F_yS_x$ at L_r on the right, the strength is limited by inelastic buckling. The strength in this region is provided in AISC *Specification* Equation F2-2.

The curve plotted as a heavy solid line represents the case where C_b = 1.0, while the heavy dashed line represents the case where C_b exceeds 1.0. The nominal strengths calculated in both AISC *Specification* Equations F2-2 and F2-3 are linearly proportional to C_b, but are limited to M_p as shown in the figure.

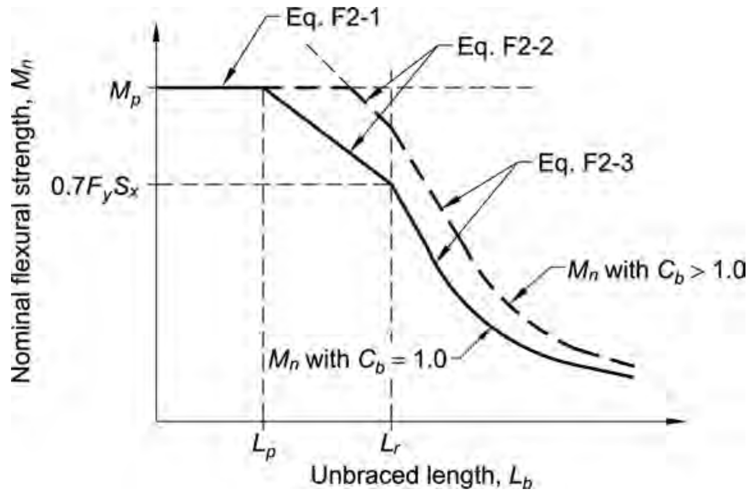


Fig. F-1. Nominal flexural strength versus unbraced length.

$$M_n = M_p = F_y Z_x \quad \text{(}\textit{Spec}\text{. Eq. F2-1)}$$

$$M_n = C_b \left[M_p - \left(M_p - 0.7F_y S_x \right) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \le M_p \quad \text{(}\textit{Spec}\text{. Eq. F2-2)}$$

$$M_n = F_{cr} S_x \le M_p \quad \text{(}\textit{Spec}\text{. Eq. F2-3)}$$

where

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}} \right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_o} \left(\frac{L_b}{r_{ts}} \right)^2} \quad \text{(}\textit{Spec}\text{. Eq. F2-4)}$$

The provisions of this section are illustrated in Example F.1 (W-shape beam) and Example F.2 (channel).

Inelastic design provisions are given in AISC *Specification* Appendix 1. L_{pd}, the maximum unbraced length for prismatic member segments containing plastic hinges is less than L_p.

F3. DOUBLY SYMMETRIC I-SHAPED MEMBERS WITH COMPACT WEBS AND NONCOMPACT OR SLENDER FLANGES BENT ABOUT THEIR MAJOR AXIS

The strength of shapes designed according to this section is limited by local buckling of the compression flange. Only a few standard wide-flange shapes have noncompact flanges. For these sections, the strength reduction for F_y = 50 ksi steel varies. The approximate percentages of M_p about the strong axis that can be developed by noncompact members when braced such that $L_b \le L_p$ are shown as follows:

W21×48 = 99%	W14×99 = 99%	W14×90 = 97%	W12×65 = 98%
W10×12 = 99%	W8×31 = 99%	W8×10 = 99%	W6×15 = 94%
W6×8.5 = 97%			

The strength curve for the flange local buckling limit state, shown in Figure F-2, is similar in nature to that of the lateral-torsional buckling curve. The horizontal axis parameter is $\lambda = b_f/2t_f$. The flat portion of the curve to the left of λ_{pf} is the plastic yielding strength, M_p. The curved portion to the right of λ_{rf} is the strength limited by elastic buckling of the flange. The linear transition between these two regions is the strength limited by inelastic flange buckling.

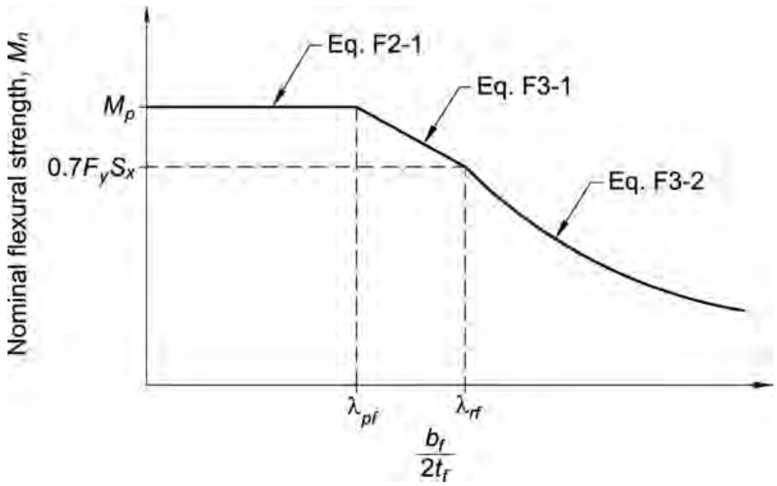


Fig. F-2. Flange local buckling strength.

$$M_n = M_p = F_y Z_x \qquad \text{(}\textit{Spec}\text{. Eq. F2-1)}$$

$$M_n = M_p - \left(M_p - 0.7F_y S_x\right)\left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right) \qquad \text{(}\textit{Spec}\text{. Eq. F3-1)}$$

$$M_n = \frac{0.9Ek_c S_x}{\lambda^2} \qquad \text{(}\textit{Spec}\text{. Eq. F3-2)}$$

where

$k_c = \frac{4}{\sqrt{h/t_w}}$ and shall not be taken less than 0.35 nor greater than 0.76 for calculation purposes.

The strength reductions due to flange local buckling of the few standard rolled shapes with noncompact flanges are incorporated into the design tables in Part 3 and Part 6 of the AISC *Manual*.

There are no standard I-shaped members with slender flanges. The noncompact flange provisions of this section are illustrated in Example F.3.

F4. OTHER I-SHAPED MEMBERS WITH COMPACT OR NONCOMPACT WEBS BENT ABOUT THEIR MAJOR AXIS

This section of the AISC *Specification* applies to doubly symmetric I-shaped members with noncompact webs and singly symmetric I-shaped members (those having different flanges) with compact or noncompact webs.

F5. DOUBLY SYMMETRIC AND SINGLY SYMMETRIC I-SHAPED MEMBERS WITH SLENDER WEBS BENT ABOUT THEIR MAJOR AXIS

This section applies to doubly symmetric and singly symmetric I-shaped members with slender webs, formerly designated as “plate girders”.

F6. I-SHAPED MEMBERS AND CHANNELS BENT ABOUT THEIR MINOR AXIS

I-shaped members and channels bent about their minor axis are not subject to lateral-torsional buckling. Rolled or built-up shapes with noncompact or slender flanges, as determined by AISC *Specification* Table B4.1b, must be checked for strength based on the limit state of flange local buckling using Equations F6-2 or F6-3 as applicable.

The vast majority of W, M, C and MC shapes have compact flanges, and can therefore develop the full plastic moment, M_p, about the minor axis. The provisions of this section are illustrated in Example F.5.

F7. SQUARE AND RECTANGULAR HSS AND BOX SECTIONS

Square and rectangular HSS need to be checked for the limit states of yielding, and flange and web local buckling. Lateral-torsional buckling is also possible for rectangular HSS or box sections bent about the strong axis; however, as indicated in the User Note in AISC *Specification* Section F7, deflection will usually control the design before there is a significant reduction in flexural strength due to lateral-torsional buckling.

The design and section property tables in the AISC *Manual* were calculated using a design wall thickness of 93% of the nominal wall thickness (see AISC *Specification* Section B4.2). Strength reductions due to local buckling have been accounted for in the AISC *Manual* design tables. The selection of a square HSS with compact flanges is illustrated in Example F.6. The provisions for a rectangular HSS with noncompact flanges is illustrated in Example F.7. The provisions for a square HSS with slender flanges are illustrated in Example F.8. Available flexural strengths of rectangular and square HSS are listed in Tables 3-12 and 3-13, respectively. If HSS members are specified using ASTM A1065 or ASTM A1085 material, the design wall thickness may be taken equal to the nominal wall thickness.

F8. ROUND HSS

The definition of HSS encompasses both tube and pipe products. The lateral-torsional buckling limit state does not apply, but round HSS are subject to strength reductions from local buckling. Available strengths of round HSS and Pipes are listed in AISC *Manual* Tables 3-14 and 3-15, respectively. The tabulated properties and available flexural strengths of these shapes in the AISC *Manual* are calculated using a design wall thickness of 93% of the nominal wall thickness. The design of a Pipe is illustrated in Example F.9. If round HSS members are specified using ASTM A1085 material, the design wall thickness may be taken equal to the nominal wall thickness.

F9. TEES AND DOUBLE ANGLES LOADED IN THE PLANE OF SYMMETRY

The AISC *Specification* provides a check for flange local buckling, which applies only when a noncompact or slender flange is in compression due to flexure. This limit state will seldom govern. A check for local buckling of the tee stem in flexural compression was added in the 2010 edition of the *Specification*. The provisions were expanded to include local buckling of double-angle web legs in flexural compression in the 2016 edition. Attention should be given to end conditions of tees to avoid inadvertent fixed end moments that induce compression in the web unless this limit state is checked. The design of a WT-shape in bending is illustrated in Example F.10.

F10. SINGLE ANGLES

Section F10 of the AISC *Specification* permits the flexural design of single angles using either the principal axes or geometric axes (*x*- and *y*-axes). When designing single angles without continuous bracing using the geometric axis design provisions, M_y must be multiplied by 0.80 for use in Equations F10-1, F10-2 and F10-3. The design of a single angle in bending is illustrated in Example F.11.

F11. RECTANGULAR BARS AND ROUNDS

The AISC *Manual* does not include design tables for these shapes. The local buckling limit state does not apply to any bars. With the exception of rectangular bars bent about the strong axis, solid square, rectangular and round bars are not subject to lateral-torsional buckling and are governed by the yielding limit state only. Rectangular bars bent about the strong axis are subject to lateral-torsional buckling and are checked for this limit state with Equations F11-2 and F11-3, as applicable.

These provisions can be used to check plates and webs of tees in connections. A design example of a rectangular bar in bending is illustrated in Example F.12. A design example of a round bar in bending is illustrated in Example F.13.

F12. UNSYMMETRICAL SHAPES

Due to the wide range of possible unsymmetrical cross sections, specific lateral-torsional and local buckling provisions are not provided in this *Specification* section. A general template is provided, but appropriate literature investigation and engineering judgment are required for the application of this section. A design example of a Z-shaped section in bending is illustrated in Example F.14.

F13. PROPORTIONS OF BEAMS AND GIRDERS

This section of the *Specification* includes a limit state check for tensile rupture due to holes in the tension flange of beams, proportioning limits for I-shaped members, detail requirements for cover plates and connection requirements for built-up beams connected side-to-side. Also included are unbraced length requirements for beams designed using the moment redistribution provisions of AISC *Specification* Section B3.3.

EXAMPLE F.1-1A W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR AXIS BENDING, CONTINUOUSLY BRACED

Given:

Select a W-shape beam for span and uniform dead and live loads as shown in Figure F.1-1A. Limit the member to a maximum nominal depth of 18 in. Limit the live load deflection to *L*/360. The beam is simply supported and continuously braced. The beam is ASTM A992 material.

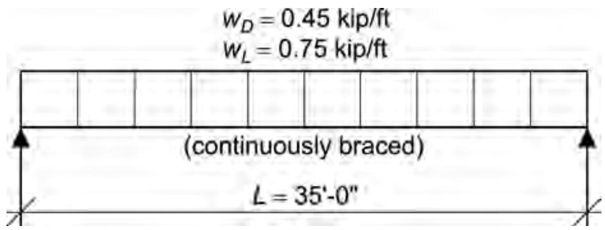


Fig. F.1-1A. Beam loading and bracing diagram.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.45\text{ kip/ft}) + 1.6(0.75\text{ kip/ft})$ $= 1.74\text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_u = \dfrac{w_u L^2}{8}$ $= \dfrac{(1.74\text{ kip/ft})(35\text{ ft})^2}{8}$ $= 266\text{ kip-ft}$	$w_a = 0.45\text{ kip/ft} + 0.75\text{ kip/ft}$ $= 1.20\text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_a = \dfrac{w_a L^2}{8}$ $= \dfrac{(1.20\text{ kip/ft})(35\text{ ft})^2}{8}$ $= 184\text{ kip-ft}$

Required Moment of Inertia for Live-Load Deflection Criterion of L/360

$$\Delta_{max} = \frac{L}{360}$$
$$= \frac{(35\text{ ft})(12\text{ in./ft})}{360}$$
$$= 1.17\text{ in.}$$

$$I_{x(reqd)} = \frac{5w_L L^4}{384E\Delta_{max}}$$ (from AISC *Manual* Table 3-23, Case 1)

$$= \frac{5(0.75 \text{ kip/ft})(35 \text{ ft})^4 (12 \text{ in./ft})^3}{384(29{,}000 \text{ ksi})(1.17 \text{ in.})}$$

$$= 746 \text{ in.}^4$$

Beam Selection

Select a W18×50 from AISC *Manual* Table 3-3.

$I_x = 800 \text{ in.}^4 > 746 \text{ in.}^4$ **o.k.**

Per the User Note in AISC *Specification* Section F2, the section is compact. Because the beam is continuously braced and compact, only the yielding limit state applies.

From AISC *Manual* Table 3-2, the available flexural strength is:

LRFD	ASD
$\phi_b M_n = \phi_b M_{px}$ $= 379 \text{ kip-ft} > 266 \text{ kip-ft}$ **o.k.**	$\frac{M_n}{\Omega_b} = \frac{M_{px}}{\Omega_b}$ $= 252 \text{ kip-ft} > 184 \text{ kip-ft}$ **o.k.**

EXAMPLE F.1-1B W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR AXIS BENDING, CONTINUOUSLY BRACED

Given:

Verify the available flexural strength of the ASTM A992 W18×50 beam selected in Example F.1-1A by directly applying the requirements of the AISC *Specification*.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W18×50
$Z_x = 101$ in.^{3}

The required flexural strength from Example F.1-1A is:

LRFD	ASD
$M_u = 266$ kip-ft	$M_a = 184$ kip-ft

Nominal Flexural Strength

Per the User Note in AISC *Specification* Section F2, the section is compact. Because the beam is continuously braced and compact, only the yielding limit state applies.

$$M_n = M_p = F_y Z_x \qquad \text{(}\textit{Spec}\text{. Eq. F2-1)}$$
$$= (50 \text{ ksi})(101 \text{ in.}^3)$$
$$= 5{,}050 \text{ kip-in. or } 421 \text{ kip-ft}$$

Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(421 \text{ kip-ft})$ $= 379 \text{ kip-ft} > 266 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{421 \text{ kip-ft}}{1.67}$ $= 252 \text{ kip-ft} > 184 \text{ kip-ft}$ **o.k.**

EXAMPLE F.1-2A W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR AXIS BENDING, BRACED AT THIRD POINTS

Given:

Use the AISC *Manual* tables to verify the available flexural strength of the W18×50 beam size selected in Example F.1-1A for span and uniform dead and live loads as shown in Figure F.1-2A. The beam is simply supported and braced at the ends and third points. The beam is ASTM A992 material.

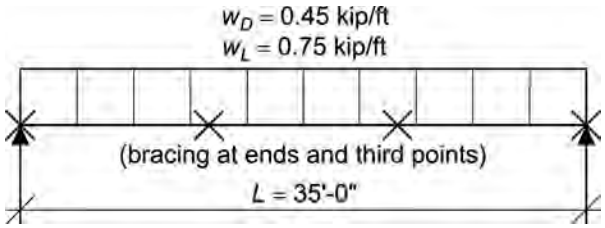


Fig. F.1-2A. Beam loading and bracing diagram.

Solution:

The required flexural strength at midspan from Example F.1-1A is:

LRFD	ASD
$M_u = 266$ kip-ft	$M_a = 184$ kip-ft

Unbraced Length

$$L_b = \frac{35\text{ ft}}{3} = 11.7\text{ ft}$$

By inspection, the middle segment will govern. From AISC *Manual* Table 3-1, for a uniformly loaded beam braced at the ends and third points, C_b = 1.01 in the middle segment. Conservatively neglect this small adjustment in this case.

Available Flexural Strength

Enter AISC *Manual* Table 3-10 and find the intersection of the curve for the W18×50 with an unbraced length of 11.7 ft. Obtain the available strength from the appropriate vertical scale to the left.

From AISC *Manual* Table 3-10, the available flexural strength is:

LRFD	ASD
$\phi_b M_n \approx 302\text{ kip-ft} > 266\text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} \approx 201\text{ kip-ft} > 184\text{ kip-ft}$ **o.k.**

EXAMPLE F.1-2B W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR AXIS BENDING, BRACED AT THIRD POINTS

Given:

Verify the available flexural strength of the W18×50 beam selected in Example F.1-1A with the beam braced at the ends and third points by directly applying the requirements of the AISC *Specification*. The beam is ASTM A992 material.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W18×50
r_y = 1.65 in.
S_x = 88.9 in.^{3}
J = 1.24 in.^{4}
r_{ts} = 1.98 in.
h_o = 17.4 in.

The required flexural strength from Example F.1-1A is:

LRFD	ASD
$M_u = 266$ kip-ft	$M_a = 184$ kip-ft

Nominal Flexural Strength

Calculate C_b. For the lateral-torsional buckling limit state, the nonuniform moment modification factor can be calculated using AISC *Specification* Equation F1-1. For the center segment of the beam, the required moments for AISC *Specification* Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: M_{max} = 1.00, M_A = 0.972, M_B = 1.00, and M_C = 0.972.

$$C_b = \frac{12.5M_{max}}{2.5M_{max}+3M_A+4M_B+3M_C} \qquad \text{(Spec. Eq. F1-1)}$$
$$= \frac{12.5(1.00)}{2.5(1.00)+3(0.972)+4(1.00)+3(0.972)}$$
$$= 1.01$$

For the end-span beam segments, the required moments for AISC *Specification* Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: M_{max} = 0.889, M_A = 0.306, M_B = 0.556, and M_C = 0.750.

$$C_b = \frac{12.5M_{max}}{2.5M_{max}+3M_A+4M_B+3M_C} \qquad \text{(Spec. Eq. F1-1)}$$
$$= \frac{12.5(0.889)}{2.5(0.889)+3(0.306)+4(0.556)+3(0.750)}$$
$$= 1.46$$

Thus, the center span, with the higher required strength and lower C_b, will govern.

The limiting laterally unbraced length for the limit state of yielding is:

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} \quad \text{(\textit{Spec}. Eq. F2-5)}$$

$$= 1.76(1.65 \text{ in.})\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$

$$= 69.9 \text{ in. or } 5.83 \text{ ft}$$

The limiting unbraced length for the limit state of inelastic lateral-torsional buckling, with c = 1 from AISC *Specification* Equation F2-8a for doubly symmetric I-shaped members, is:

$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y}\sqrt{\frac{Jc}{S_x h_o} + \sqrt{\left(\frac{Jc}{S_x h_o}\right)^2 + 6.76\left(\frac{0.7F_y}{E}\right)^2}} \quad \text{(\textit{Spec}. Eq. F2-6)}$$

$$= 1.95(1.98 \text{ in.})\left[\frac{29{,}000 \text{ ksi}}{0.7(50 \text{ ksi})}\right]\sqrt{\frac{(1.24 \text{ in.}^4)(1.0)}{(88.9 \text{ in.}^3)(17.4 \text{ in.})} + \sqrt{\left[\frac{(1.24 \text{ in.}^4)(1.0)}{(88.9 \text{ in.}^3)(17.4 \text{ in.})}\right]^2 + 6.76\left[\frac{0.7(50 \text{ ksi})}{29{,}000 \text{ ksi}}\right]^2}}$$

$$= 203 \text{ in. or } 16.9 \text{ ft}$$

For a compact beam with an unbraced length of $L_p < L_b \le L_r$, the lesser of either the flexural yielding limit state or the inelastic lateral-torsional buckling limit state controls the nominal strength.

M_p = 5,050 kip-in. (from Example F.1-1B)

$$M_n = C_b\left[M_p - (M_p - 0.7F_y S_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \quad \text{(\textit{Spec}. Eq. F2-2)}$$

$$= 1.01\left\{5{,}050 \text{ kip-in.} - \left[5{,}050 \text{ kip-in.} - 0.7(50 \text{ ksi})(88.9 \text{ in.}^3)\right]\left(\frac{11.7 \text{ ft} - 5.83 \text{ ft}}{16.9 \text{ ft} - 5.83 \text{ ft}}\right)\right\} \le 5{,}050 \text{ kip-in.}$$

$$= 4{,}060 \text{ kip-in.} < 5{,}050 \text{ kip-in.}$$

$$= 4{,}060 \text{ kip-in. or } 339 \text{ kip-ft}$$

Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(339 \text{ kip-ft})$ $= 305 \text{ kip-ft} > 266 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\frac{M_n}{\Omega_b} = \frac{339 \text{ kip-ft}}{1.67}$ $= 203 \text{ kip-ft} > 184 \text{ kip-ft}$ **o.k.**

EXAMPLE F.1-3A W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR AXIS BENDING, BRACED AT MIDSPAN

Given:

Use the AISC *Manual* tables to verify the available flexural strength of the W18×50 beam size selected in Example F.1-1A for span and uniform dead and live loads as shown in Figure F.1-3A. The beam is simply supported and braced at the ends and midpoint. The beam is ASTM A992 material.

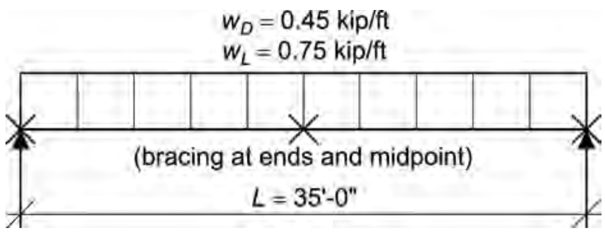


Fig. F.1-3A. Beam loading and bracing diagram.

Solution:

The required flexural strength at midspan from Example F.1-1A is:

LRFD	ASD
$M_u = 266$ kip-ft	$M_a = 184$ kip-ft

Unbraced Length

$$L_b = \frac{35 \text{ ft}}{2} = 17.5 \text{ ft}$$

From AISC *Manual* Table 3-1, for a uniformly loaded beam braced at the ends and at the center point, C_b = 1.30. There are several ways to make adjustments to AISC *Manual* Table 3-10 to account for C_b greater than 1.0.

Procedure A

Available moments from the sloped and curved portions of the plots from AISC *Manual* Table 3-10 may be multiplied by C_b, but may not exceed the value of the horizontal portion (ϕM_p for LRFD, M_p/Ω for ASD).

Obtain the available strength of a W18×50 with an unbraced length of 17.5 ft from AISC *Manual* Table 3-10.

Enter AISC *Manual* Table 3-10 and find the intersection of the curve for the W18×50 with an unbraced length of 17.5 ft. Obtain the available strength from the appropriate vertical scale to the left.

LRFD	ASD
$\phi_b M_n \approx 222$ kip-ft From AISC *Manual* Table 3-2: $\phi_b M_p = 379$ kip-ft (upper limit on $C_b \phi_b M_n$)	$\frac{M_n}{\Omega_b} \approx 148$ kip-ft From AISC *Manual* Table 3-2: $\frac{M_p}{\Omega_b} = 252$ kip-ft (upper limit on $C_b \frac{M_n}{\Omega_b}$)

LRFD	ASD
Adjust for C_b. $1.30(222\text{ kip-ft}) = 289\text{ kip-ft}$ Check limit. $289\text{ kip-ft} < \phi_b M_p = 379\text{ kip-ft}$ **o.k.** Check available versus required strength. $289\text{ kip-ft} > 266\text{ kip-ft}$ **o.k.**	Adjust for C_b. $1.30(148\text{ kip-ft}) = 192\text{ kip-ft}$ Check limit. $192\text{ kip-ft} < \dfrac{M_p}{\Omega_b} = 252\text{ kip-ft}$ **o.k.** Check available versus required strength. $192\text{ kip-ft} > 184\text{ kip-ft}$ **o.k.**

Procedure B

For preliminary selection, the required strength can be divided by C_b and directly compared to the strengths in AISC *Manual* Table 3-10. Members selected in this way must be checked to ensure that the required strength does not exceed the available plastic moment strength of the section.

Calculate the adjusted required strength.

LRFD	ASD
$M'_u = \dfrac{266\text{ kip-ft}}{1.30}$ $= 205\text{ kip-ft}$	$M'_a = \dfrac{184\text{ kip-ft}}{1.30}$ $= 142\text{ kip-ft}$

Obtain the available strength for a W18×50 with an unbraced length of 17.5 ft from AISC *Manual* Table 3-10.

LRFD	ASD
$\phi_b M_n \approx 222\text{ kip-ft} > 205\text{ kip-ft}$ **o.k.** $\phi_b M_p = 379\text{ kip-ft} > 266\text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} \approx 148\text{ kip-ft} > 142\text{ kip-ft}$ **o.k.** $\dfrac{M_p}{\Omega_b} = 252\text{ kip-ft} > 184\text{ kip-ft}$ **o.k.**

EXAMPLE F.1-3B W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR-AXIS BENDING, BRACED AT MIDSPAN

Given:

Verify the available flexural strength of the W18×50 beam selected in Example F.1-1A with the beam braced at the ends and center point by directly applying the requirements of the AISC *Specification*. The beam is ASTM A992 material.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W18×50
r_{ts} = 1.98 in.
S_x = 88.9 in.^{3}
J = 1.24 in.^{4}
h_o = 17.4 in.

The required flexural strength from Example F.1-1A is:

LRFD	ASD
$M_u = 266$ kip-ft	$M_a = 184$ kip-ft

Nominal Flexural Strength

Calculate C_b. The required moments for AISC *Specification* Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: M_{max} = 1.00, M_A = 0.438, M_B = 0.750, and M_C = 0.938.

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C}$$ (*Spec*. Eq. F1-1)

$$= \frac{12.5(1.00)}{2.5(1.00) + 3(0.438) + 4(0.750) + 3(0.938)}$$

$$= 1.30$$

From AISC *Manual* Table 3-2:

L_p = 5.83 ft
L_r = 16.9 ft

From Example F.1-3A:

L_b = 17.5 ft

For a compact beam with an unbraced length $L_b > L_r$, the limit state of elastic lateral-torsional buckling applies.

Calculate F_{cr}, where c = 1.0 for doubly symmetric I-shapes.

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_o} \left(\frac{L_b}{r_{ts}}\right)^2} \qquad \text{(}\textit{Spec}\text{. Eq. F2-4)}$$

$$= \frac{1.30\pi^2 \left(29{,}000 \text{ ksi}\right)}{\left[\frac{(17.5 \text{ ft})(12 \text{ in./ft})}{1.98 \text{ in.}}\right]^2} \sqrt{1 + 0.078 \frac{\left(1.24 \text{ in.}^4\right)(1.0)}{\left(88.9 \text{ in.}^3\right)(17.4 \text{ in.})} \left[\frac{(17.5 \text{ ft})(12 \text{ in./ft})}{1.98 \text{ in.}}\right]^2}$$

$$= 43.2 \text{ ksi}$$

$$M_p = 5{,}050 \text{ kip-in. (from Example F.1-1B)}$$

$$M_n = F_{cr} S_x \le M_p \qquad \text{(}\textit{Spec}\text{. Eq. F2-3)}$$

$$= \left(43.2 \text{ ksi}\right)\left(88.9 \text{ in.}^3\right) \le 5{,}050 \text{ kip-in.}$$

$$= 3{,}840 \text{ kip-in.} < 5{,}050 \text{ kip-in.}$$

$$= 3{,}840 \text{ kip-in. or } 320 \text{ kip-ft}$$

Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(320 \text{ kip-ft})$ $= 288 \text{ kip-ft} > 266 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\frac{M_n}{\Omega_b} = \frac{320 \text{ kip-ft}}{1.67}$ $= 192 \text{ kip-ft} > 184 \text{ kip-ft}$ **o.k.**

EXAMPLE F.3A W-SHAPE FLEXURAL MEMBER WITH NONCOMPACT FLANGES IN MAJOR AXIS BENDING

Given:

Using the AISC *Manual* tables, select a W-shape beam for span, uniform dead load, and concentrated live loads as shown in Figure F.3A. The beam is simply supported and continuously braced. Also calculate the deflection. The beam is ASTM A992 material.

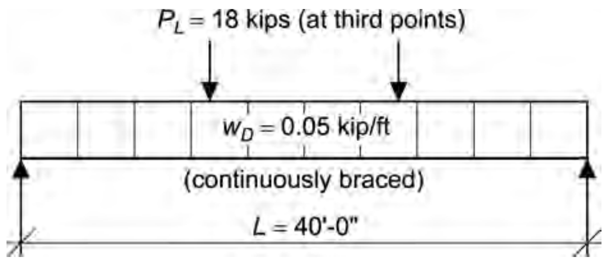


Fig. F.3A. Beam loading and bracing diagram.

Note: A beam with noncompact flanges will be selected to demonstrate that the tabulated values of the AISC *Manual* account for flange compactness.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength at midspan is:

LRFD	ASD
$w_u = 1.2(0.05\text{ kip/ft})$ $= 0.0600\text{ kip/ft}$	$w_a = 0.05\text{ kip/ft}$
$P_u = 1.6(18\text{ kips})$ $= 28.8\text{ kips}$	$P_a = 18\text{ kips}$
From AISC *Manual* Table 3-23, Cases 1 and 9:	From AISC *Manual* Table 3-23, Cases 1 and 9:
$M_u = \dfrac{w_u L^2}{8} + P_u a$ $= \dfrac{(0.0600\text{ kip/ft})(40\text{ ft})^2}{8} + (28.8\text{ kips})\left(\dfrac{40\text{ ft}}{3}\right)$ $= 396\text{ kip-ft}$	$M_a = \dfrac{w_a L^2}{8} + P_a a$ $= \dfrac{(0.05\text{ kip/ft})(40\text{ ft})^2}{8} + (18\text{ kips})\left(\dfrac{40\text{ ft}}{3}\right)$ $= 250\text{ kip-ft}$

Beam Selection

For a continuously braced W-shape, the available flexural strength equals the available plastic flexural strength.

Select the lightest section providing the required strength from the bold entries in AISC *Manual* Table 3-2.

Try a W21×48.

This beam has a noncompact compression flange at F_y = 50 ksi as indicated by footnote "f" in AISC *Manual* Table 3-2. This shape is also footnoted in AISC *Manual* Table 1-1.

From AISC *Manual* Table 3-2, the available flexural strength is:

LRFD	ASD
$\phi_b M_n = \phi_b M_{px}$ $= 398 \text{ kip-ft} > 396 \text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = \dfrac{M_{px}}{\Omega_b}$ $= 265 \text{ kip-ft} > 250 \text{ kip-ft}$ **o.k.**

Note: The value M_{px} in AISC *Manual* Table 3-2 includes the strength reductions due to the shape being noncompact.

Deflection

From AISC *Manual* Table 1-1:

$I_x = 959 \text{ in.}^4$

The maximum deflection occurs at the center of the beam.

$$\Delta_{max} = \frac{5w_D L^4}{384EI} + \frac{23P_L L^3}{648EI} \quad \text{(AISC } \textit{Manual} \text{ Table 3-23, Cases 1 and 9)}$$

$$= \frac{5(0.05 \text{ kip/ft})(40 \text{ ft})^4(12 \text{ in./ft})^3}{384(29{,}000 \text{ ksi})(959 \text{ in.}^4)} + \frac{23(18 \text{ kips})(40 \text{ ft})^3(12 \text{ in./ft})^3}{648(29{,}000 \text{ ksi})(959 \text{ in.}^4)}$$

$$= 2.64 \text{ in.}$$

This deflection can be compared with the appropriate deflection limit for the application. Deflection will often be more critical than strength in beam design.

EXAMPLE F.3B W-SHAPE FLEXURAL MEMBER WITH NONCOMPACT FLANGES IN MAJOR AXIS BENDING

Given:

Verify the results from Example F.3A by directly applying the requirements of the AISC *Specification*.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W21×48
$S_x = 93.0 \text{ in.}^3$
$Z_x = 107 \text{ in.}^3$
$\dfrac{b_f}{2t_f} = 9.47$

The required flexural strength from Example F.3A is:

LRFD	ASD
$M_u = 396$ kip-ft	$M_a = 250$ kip-ft

Flange Slenderness

$$\lambda = \frac{b_f}{2t_f} = 9.47$$

The limiting width-to-thickness ratios for the compression flange are:

$$\lambda_{pf} = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}} = 9.15$$ (*Spec*. Table B4.1b, Case 10)

$$\lambda_{rf} = 1.0\sqrt{\frac{E}{F_y}} = 1.0\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}} = 24.1$$ (*Spec*. Table B4.1b, Case 10)

$\lambda_{pf} < \lambda < \lambda_{rf}$, therefore, the compression flange is noncompact. This could also be determined from the footnote "f" in AISC *Manual* Table 1-1.

Nominal Flexural Strength

Because the beam is continuously braced, and therefore not subject to lateral-torsional buckling, the available strength is based on the limit state of compression flange local buckling. From AISC *Specification* Section F3.2:

$$
\begin{aligned}
M_p &= F_y Z_x \\
&= (50 \text{ ksi})(107 \text{ in.}^3) \\
&= 5{,}350 \text{ kip-in. or } 446 \text{ kip-ft}
\end{aligned}
$$ (*Spec*. Eq. F2-1)

$$
\begin{aligned}
M_n &= \left[M_p - \left(M_p - 0.7F_yS_x\right)\left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right)\right] \\
&= \left\{5{,}350 \text{ kip-in.} - \left[5{,}350 \text{ kip-in.} - 0.7(50 \text{ ksi})(93.0 \text{ in.}^3)\right]\left(\frac{9.47 - 9.15}{24.1 - 9.15}\right)\right\} \\
&= 5{,}310 \text{ kip-in. or } 442 \text{ kip-ft}
\end{aligned}
$$ (*Spec*. Eq. F3-1)

Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(442 \text{ kip-ft})$ $= 398 \text{ kip-ft} > 396 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\frac{M_n}{\Omega_b} = \frac{442 \text{ kip-ft}}{1.67}$ $= 265 \text{ kip-ft} > 250 \text{ kip-ft}$ **o.k.**

Note that these available strengths are identical to the tabulated values in AISC *Manual* Table 3-2, as shown in Example F.3A, which account for the noncompact flange.

EXAMPLE F.4 W-SHAPE FLEXURAL MEMBER, SELECTION BY MOMENT OF INERTIA FOR MAJOR AXIS BENDING

Given:

Using the AISC *Manual* tables, select a W-shape using the moment of inertia required to limit the live load deflection to 1.00 in. for span and uniform dead and live loads as shown in Figure F.4. The beam is simply supported and continuously braced. The beam is ASTM A992 material.

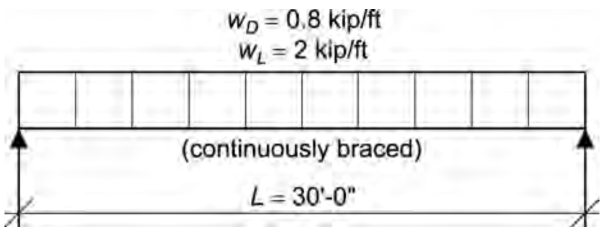


Fig. F.4. Beam loading and bracing diagram.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.8\text{ kip/ft}) + 1.6(2\text{ kip/ft})$ $= 4.16\text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_u = \dfrac{w_u L^2}{8}$ $= \dfrac{(4.16\text{ kip/ft})(30\text{ ft})^2}{8}$ $= 468\text{ kip-ft}$	$w_a = 0.8\text{ kip/ft} + 2\text{ kip/ft}$ $= 2.80\text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_a = \dfrac{w_a L^2}{8}$ $= \dfrac{(2.80\text{ kip/ft})(30\text{ ft})^2}{8}$ $= 315\text{ kip-ft}$

Minimum Required Moment of Inertia

The maximum live load deflection, Δ_{max}, occurs at midspan and is calculated as:

$$\Delta_{max} = \frac{5 w_L L^4}{384 EI}$$ (AISC *Manual* Table 3-23, Case 1)

Rearranging and substituting Δ_{max} = 1.00 in.,

$$I_{min} = \frac{5w_L L^4}{384E\Delta_{max}}$$
$$= \frac{5(2 \text{ kip/ft})(30 \text{ ft})^4 (12 \text{ in./ft})^3}{384(29{,}000 \text{ ksi})(1.00 \text{ in.})}$$
$$= 1{,}260 \text{ in.}^4$$

Beam Selection

Select the lightest section with the required moment of inertia from the bold entries in AISC *Manual* Table 3-3.

Try a W24×55.

I_x = 1,350 in.^{4} > 1,260 in.^{4} **o.k.**

Because the W24×55 is continuously braced and compact, its strength is governed by the yielding limit state and AISC *Specification* Section F2.1.

From AISC *Manual* Table 3-2, the available flexural strength is:

LRFD	ASD
$\phi_b M_n = \phi_b M_{px}$ $= 503 \text{ kip-ft} > 468 \text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = \dfrac{M_{px}}{\Omega_b}$ $= 334 \text{ kip-ft} > 315 \text{ kip-ft}$ **o.k.**

EXAMPLE F.5 I-SHAPED FLEXURAL MEMBER IN MINOR AXIS BENDING

Given:

Using the AISC *Manual* tables, select a W-shape beam loaded on its minor axis for span and uniform dead and live loads as shown in Figure F.5. Limit the live load deflection to L/240. The beam is simply supported and braced only at the ends. The beam is ASTM A992 material.

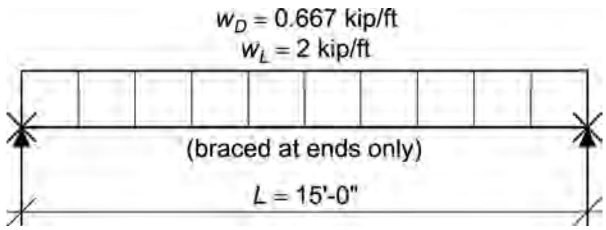


Fig. F.5. Beam loading and bracing diagram.

Note: Although not a common design case, this example is being used to illustrate AISC *Specification* Section F6 (I-shaped members and channels bent about their minor axis).

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.667 \text{ kip/ft}) + 1.6(2 \text{ kip/ft})$ $= 4.00 \text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_u = \dfrac{w_u L^2}{8}$ $= \dfrac{(4.00 \text{ kip/ft})(15 \text{ ft})^2}{8}$ $= 113 \text{ kip-ft}$	$w_a = 0.667 \text{ kip/ft} + 2 \text{ kip/ft}$ $= 2.67 \text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_a = \dfrac{w_a L^2}{8}$ $= \dfrac{(2.67 \text{ kip/ft})(15 \text{ ft})^2}{8}$ $= 75.1 \text{ kip-ft}$

Minimum Required Moment of Inertia

The maximum live load deflection permitted is:

$$\Delta_{max} = \frac{L}{240}$$
$$= \frac{(15 \text{ ft})(12 \text{ in./ft})}{240}$$
$$= 0.750 \text{ in.}$$

$$I_{y,reqd} = \frac{5w_L L^4}{384E\Delta_{max}}$$ (modified AISC *Manual* Table 3-23, Case 1)

$$= \frac{5(2\text{ kip/ft})(15\text{ ft})^4(12\text{ in./ft})^3}{384(29{,}000\text{ ksi})(0.750\text{ in.})}$$

$$= 105\text{ in.}^4$$

Beam Selection

Select the lightest section from the bold entries in AISC *Manual* Table 3-5.

Try a W12×58.

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W12×58
S_y = 21.4 in.^{3}
Z_y = 32.5 in.^{3}
I_y = 107 in.^{4} > 105 in.^{4} **o.k.** (for deflection requirement)

Nominal Flexural Strength

AISC *Specification* Section F6 applies. Because the W12×58 has compact flanges per the User Note in this Section, the yielding limit state governs the design.

$$M_n = M_p = F_y Z_y \le 1.6 F_y S_y$$ (*Spec*. Eq. F6-1)

$$= (50\text{ ksi})(32.5\text{ in.}^3) \le 1.6(50\text{ ksi})(21.4\text{ in.}^3)$$

$$= 1{,}630\text{ kip-in.} < 1{,}710\text{ kip-in.}$$

$$= 1{,}630\text{ kip-in or 136 kip-ft}$$

Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(136\text{ kip-ft})$ $= 122\text{ kip-ft} > 113\text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\frac{M_n}{\Omega_b} = \frac{136\text{ kip-ft}}{1.67}$ $= 81.4\text{ kip-ft} > 75.1\text{ kip-ft}$ **o.k.**

CHAPTER F DESIGN EXAMPLE REFERENCES

AISI (2016), *North American Specification for the Design of Cold-Formed Steel Structural Members*, ANSI/AISI Standard S100, American Iron and Steel Institute, Washington D.C.

Seaburg, P.A. and Carter, C.J. (1997), *Torsional Analysis of Structural Steel Members*, Design Guide 9, AISC, Chicago, IL.

Chapter G
Design of Members for Shear

INTRODUCTION

This *Specification* chapter addresses webs of singly or doubly symmetric members subject to shear in the plane of the web, single angles and HSS subject to shear, and shear in the weak direction of singly or doubly symmetric shapes.

G1. GENERAL PROVISIONS

The design shear strength, $\phi_v V_n$, and the allowable shear strength, V_n/Ω_v, are determined as follows:

V_n = nominal shear strength based on shear yielding or shear buckling
ϕ_v = 0.90 (LRFD)
Ω_v = 1.67 (ASD)

Exception: For all current ASTM A6, W, S and HP shapes except W44×230, W40×149, W36×135, W33×118, W30×90, W24×55, W16×26 and W12×14 for F_y = 50 ksi:

ϕ_v = 1.00 (LRFD)
Ω_v = 1.50 (ASD)

Strong axis shear values are tabulated for W-shapes in AISC *Manual* Tables 3-2, 3-6 and 6-2, for S-shapes in AISC *Manual* Table 3-7, for C-shapes in AISC *Manual* Table 3-8, and for MC-shapes in AISC *Manual* Table 3-9. Strong axis shear values are tabulated for rectangular HSS, round HSS and pipe in Part IV. Weak axis shear values for W-shapes, S-shapes, C-shapes and MC-shapes, and shear values for angles, rectangular HSS and box members are not tabulated.

G2. I-SHAPED MEMBERS AND CHANNELS

This section includes provisions for shear strength of webs without the use of tension field action and for interior web panels considering tension field action. Provisions for the design of transverse stiffeners are also included in Section G2.

As indicated in the User Note of this section, virtually all W, S and HP shapes are not subject to shear buckling and are also eligible for the more liberal safety and resistance factors, ϕ_v = 1.00 (LRFD) and Ω_v = 1.50 (ASD). This is presented in Example G.1 for a W-shape. A channel shear strength design is presented in Example G.2. A built-up girder with a thin web and transverse stiffeners is presented in Example G.8.

G3. SINGLE ANGLES AND TEES

A single angle example is illustrated in Example G.3.

G4. RECTANGULAR HSS, BOX SECTIONS, AND OTHER SINGLY AND DOUBLY SYMMETRIC MEMBERS

The shear height for HSS, h, is taken as the clear distance between the flanges less the inside corner radius on each side. If the corner radii are unknown, h shall be taken as the corresponding outside dimension minus 3 times the design thickness. A rectangular HSS example is provided in Example G.4.

G5. ROUND HSS

For all round HSS of ordinary length listed in the AISC *Manual*, F_{cr} can be taken as $0.6F_y$ in AISC *Specification* Equation G5-1. A round HSS example is illustrated in Example G.5.

G6. WEAK AXIS SHEAR IN DOUBLY SYMMETRIC AND SINGLY SYMMETRIC SHAPES

For examples of weak axis shear, see Example G.6 and Example G.7.

G7. BEAMS AND GIRDERS WITH WEB OPENINGS

For a beam and girder with web openings example, see AISC Design Guide 2, *Design of Steel and Composite Beams with Web Openings* (Darwin, 1990).

EXAMPLE G.1A W-SHAPE IN STRONG AXIS SHEAR

Given:

Using AISC *Manual* tables, determine the available shear strength and adequacy of an ASTM A992 W24×62 with end shears of 48 kips from dead load and 145 kips from live load.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

LRFD	ASD
$V_u = 1.2(48\text{ kips}) + 1.6(145\text{ kips})$ $= 290\text{ kips}$	$V_a = 48\text{ kips} + 145\text{ kips}$ $= 193\text{ kips}$

From AISC *Manual* Table 3-2, the available shear strength is:

LRFD	ASD
$\phi_v V_n = 306\text{ kips} > 290\text{ kips}$ **o.k.**	$\dfrac{V_n}{\Omega_v} = 204\text{ kips} > 193\text{ kips}$ **o.k.**

EXAMPLE G.1B W-SHAPE IN STRONG AXIS SHEAR

Given:

The available shear strength of the W-shape in Example G.1A was easily determined using tabulated values in the AISC *Manual*. This example demonstrates the calculation of the available strength by directly applying the provisions of the AISC *Specification*.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W24×62
$d = 23.7$ in.
$t_w = 0.430$ in.

Nominal Shear Strength

Except for very few sections, which are listed in the User Note, AISC *Specification* Section G2.1(a) is applicable to the I-shaped beams published in the AISC *Manual* for F_y = 50 ksi. The W-shape sections that do not meet the criteria of AISC *Specification* Section G2.1(a) are indicated with footnote "v" in Tables 1-1, 3-2 and 6-2.

$C_{v1} = 1.0$ (*Spec*. Eq. G2-2)

From AISC *Specification* Section G2.1, area of the web, A_w, is determined as follows:

$$\begin{aligned} A_w &= dt_w \\ &= (23.7 \text{ in.})(0.430 \text{ in.}) \\ &= 10.2 \text{ in.}^2 \end{aligned}$$

From AISC *Specification* Section G2.1, the nominal shear strength is:

$$\begin{aligned} V_n &= 0.6F_y A_w C_{v1} \\ &= 0.6(50 \text{ ksi})(10.2 \text{ in.}^2)(1.0) \\ &= 306 \text{ kips} \end{aligned}$$ (*Spec*. Eq. G2-1)

Available Shear Strength

From AISC *Specification* Section G2.1, the available shear strength is:

LRFD	ASD
$\phi_v = 1.00$ $\phi_v V_n = 1.00(306 \text{ kips})$ $= 306 \text{ kips}$	$\Omega_v = 1.50$ $\dfrac{V_n}{\Omega_v} = \dfrac{306 \text{ kips}}{1.50}$ $= 204 \text{ kips}$

EXAMPLE G.2A CHANNEL IN STRONG AXIS SHEAR

Given:

Using AISC *Manual* tables, verify the available shear strength and adequacy of an ASTM A36 C15×33.9 channel with end shears of 17.5 kips from dead load and 52.5 kips from live load.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

LRFD	ASD
$V_u = 1.2(17.5 \text{ kips}) + 1.6(52.5 \text{ kips})$ $= 105 \text{ kips}$	$V_a = 17.5 \text{ kips} + 52.5 \text{ kips}$ $= 70.0 \text{ kips}$

From AISC *Manual* Table 3-8, the available shear strength is:

LRFD	ASD
$\phi_v V_n = 117 \text{ kips} > 105 \text{ kips}$ **o.k.**	$\dfrac{V_n}{\Omega_v} = 77.6 \text{ kips} > 70.0 \text{ kips}$ **o.k.**

EXAMPLE G.2B CHANNEL IN STRONG AXIS SHEAR

Given:

The available shear strength of the channel in Example G.2A was easily determined using tabulated values in the AISC *Manual*. This example demonstrates the calculation of the available strength by directly applying the provisions of the AISC *Specification*.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

From AISC *Manual* Table 1-5, the geometric properties are as follows:

C15×33.9
$d = 15.0$ in.
$t_w = 0.400$ in.

Nominal Shear Strength

All ASTM A36 channels listed in the AISC *Manual* have $h/t_w \le 1.10\sqrt{k_v E / F_y}$; therefore,

$$C_{v1} = 1.0 \qquad \text{(}Spec\text{. Eq. G2-3)}$$

From AISC *Specification* Section G2.1, the area of the web, A_w, is determined as follows:

$$\begin{aligned} A_w &= dt_w \\ &= (15.0\text{ in.})(0.400\text{ in.}) \\ &= 6.00\text{ in.}^2 \end{aligned}$$

From AISC *Specification* Section G2.1, the nominal shear strength is:

$$\begin{aligned} V_n &= 0.6F_y A_w C_{v1} \qquad \text{(}Spec\text{. Eq. G2-1)} \\ &= 0.6(36\text{ ksi})(6.00\text{ in.}^2)(1.0) \\ &= 130\text{ kips} \end{aligned}$$

Available Shear Strength

Because AISC *Specification* Section G2.1(a) does not apply for channels, the values of $\phi_v = 1.00$ (LRFD) and $\Omega_v = 1.50$ (ASD) may not be used. Instead $\phi_v = 0.90$ (LRFD) and $\Omega_v = 1.67$ (ASD) from AISC *Specification* Section G1(a) must be used.

LRFD	ASD
$\phi_v = 0.90$ $\phi_v V_n = 0.90(130\text{ kips})$ $= 117\text{ kips}$	$\Omega_v = 1.67$ $\dfrac{V_n}{\Omega_v} = \dfrac{130\text{ kips}}{1.67}$ $= 77.8\text{ kips}$

EXAMPLE G.6 DOUBLY SYMMETRIC SHAPE IN WEAK AXIS SHEAR

Given:

Verify the available shear strength and adequacy of an ASTM A992 W21×48 beam with end shears of 20.0 kips from dead load and 60.0 kips from live load in the weak direction.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W21×48
b_f = 8.14 in.
t_f = 0.430 in.

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

LRFD	ASD
$V_u = 1.2(20.0 \text{ kips}) + 1.6(60.0 \text{ kips})$ $= 120 \text{ kips}$	$V_a = 20.0 \text{ kips} + 60.0 \text{ kips}$ $= 80.0 \text{ kips}$

Nominal Shear Strength

From AISC *Specification* Section G6, for weak axis shear, use AISC *Specification* Equation G6-1.

Calculate C_{v2} using AISC *Specification* Section G2.2 with $h/t_w = b_f/2t_f$ and k_v = 1.2.

$$\begin{aligned}\frac{h}{t_w} &= \frac{b_f}{2t_f} \\ &= \frac{8.14 \text{ in.}}{2(0.430 \text{ in.})} \\ &= 9.47\end{aligned}$$

$$\begin{aligned}1.10\sqrt{\frac{k_v E}{F_y}} &= 1.10\sqrt{\frac{1.2(29{,}000 \text{ ksi})}{50 \text{ ksi}}} \\ &= 29.0 > 9.47\end{aligned}$$

Therefore, use AISC *Specification* Equation G2-9:

$$C_{v2} = 1.0$$

Note: From the User Note in AISC *Specification* Section G6, C_{v2} = 1.0 for all ASTM A6 W-, S-, M- and HP-shapes when $F_y \le 70$ ksi.

Calculate V_n. (Multiply the flange area by two to account for both shear resisting elements.)

$$V_n = 0.6F_y b_f t_f C_{v2}(2) \quad \text{(from } \textit{Spec}\text{. Eq. G6-1)}$$
$$= 0.6(50 \text{ ksi})(8.14 \text{ in.})(0.430 \text{ in.})(1.0)(2)$$
$$= 210 \text{ kips}$$

Available Shear Strength

From AISC *Specification* Section G1, the available shear strength is:

LRFD	ASD
$\phi_v = 0.90$ $\phi_v V_n = 0.90(210 \text{ kips})$ $= 189 \text{ kips} > 120 \text{ kips}$ **o.k.**	$\Omega_v = 1.67$ $\dfrac{V_n}{\Omega_v} = \dfrac{210 \text{ kips}}{1.67}$ $= 126 \text{ kips} > 80.0 \text{ kips}$ **o.k.**

CHAPTER G DESIGN EXAMPLE REFERENCES

Darwin, D. (1990), *Steel and Composite Beams with Web Openings,* Design Guide 2, AISC, Chicago, IL.

Chapter H
Design of Members for Combined Forces and Torsion

For all interaction equations in AISC *Specification* Chapter H, the required forces and moments must include second-order effects, as required by Chapter C of the AISC *Specification*. ASD users of the 1989 AISC *Specification* are accustomed to using an interaction equation that includes a partial second-order amplification. Second-order effects are now addressed in the analysis and are not included in these interaction equations.

EXAMPLE H.1A W-SHAPE SUBJECT TO COMBINED COMPRESSION AND BENDING ABOUT BOTH AXES (BRACED FRAME)

Given:

Using Table IV-5 (located in this document), determine if an ASTM A992 W14×99 has sufficient available strength to support the axial forces and moments listed as follows, obtained from a second-order analysis that includes P-δ effects. The unbraced length is 14 ft and the member has pinned ends.

LRFD	ASD
$P_u = 400$ kips $M_{ux} = 250$ kip-ft $M_{uy} = 80.0$ kip-ft	$P_a = 267$ kips $M_{ax} = 167$ kip-ft $M_{ay} = 53.3$ kip-ft

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

The effective length of the member is:

$$\begin{aligned} L_{cx} &= L_{cy} \\ &= KL \\ &= 1.0(14 \text{ ft}) \\ &= 14.0 \text{ ft} \end{aligned}$$

For $L_c = 14$ ft, the combined strength parameters from Table IV-5 are:

LRFD	ASD
$p = \dfrac{0.887}{10^3 \text{ kips}}$	$p = \dfrac{1.33}{10^3 \text{ kips}}$
$b_x = \dfrac{1.38}{10^3 \text{ kip-ft}}$	$b_x = \dfrac{2.08}{10^3 \text{ kip-ft}}$
$b_y = \dfrac{2.85}{10^3 \text{ kip-ft}}$	$b_y = \dfrac{4.29}{10^3 \text{ kip-ft}}$
Check P_r/P_c limit for AISC *Specification* Equation H1-1a.	Check P_r/P_c limit for AISC *Specification* Equation H1-1a.
$\dfrac{P_u}{\phi_c P_n} = pP_u = \left(\dfrac{0.887}{10^3 \text{ kips}}\right)(400 \text{ kips}) = 0.355$	$\dfrac{P_a}{P_n / \Omega_c} = pP_a = \left(\dfrac{1.33}{10^3 \text{ kips}}\right)(267 \text{ kips}) = 0.355$

LRFD	ASD
Because $pP_u \geq 0.2$,	Because $pP_a \geq 0.2$,
$pP_u + b_x M_{ux} + b_y M_{uy} \leq 1.0$ (from Part IV, Eq. IV-8)	$pP_a + b_x M_{ax} + b_y M_{ay} \leq 1.0$ (from Part IV, Eq. IV-8)
$= 0.355 + \left(\dfrac{1.38}{10^3 \text{ kip-ft}}\right)(250 \text{ kip-ft})$	$= 0.355 + \left(\dfrac{2.08}{10^3 \text{ kip-ft}}\right)(167 \text{ kip-ft})$
$+ \left(\dfrac{2.85}{10^3 \text{ kip-ft}}\right)(80.0 \text{ kip-ft}) \leq 1.0$	$+ \left(\dfrac{4.29}{10^3 \text{ kip-ft}}\right)(53.3 \text{ kip-ft}) \leq 1.0$
$= 0.928 < 1.0$ **o.k.**	$= 0.931 < 1.0$ **o.k.**

Table IV-5 simplifies the calculation of AISC *Specification* Equations H1-1a and H1-1b. A direct application of these equations is shown in Example H.1B.

EXAMPLE H.1B W-SHAPE SUBJECT TO COMBINED COMPRESSION AND BENDING MOMENT ABOUT BOTH AXES (BRACED FRAME)

Given:

Using AISC *Manual* tables to determine the available compressive and flexural strengths, determine if an ASTM A992 W14×99 has sufficient available strength to support the axial forces and moments listed as follows, obtained from a second-order analysis that includes P-δ effects. The unbraced length is 14 ft and the member has pinned ends.

LRFD	ASD
$P_u = 400$ kips $M_{ux} = 250$ kip-ft $M_{uy} = 80$ kip-ft	$P_a = 267$ kips $M_{ax} = 167$ kip-ft $M_{ay} = 53.3$ kip-ft

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

The effective length of the member is:

$$\begin{aligned} L_{cx} &= L_{cy} \\ &= KL \\ &= 1.0(14 \text{ ft}) \\ &= 14.0 \text{ ft} \end{aligned}$$

For $L_c = 14.0$ ft, the available axial and flexural strengths from AISC *Manual* Table 6-2 are:

LRFD	ASD
$P_c = \phi_c P_n$ $= 1{,}130$ kips	$P_c = \dfrac{P_n}{\Omega_c}$ $= 750$ kips
$M_{cx} = \phi_b M_{nx}$ $= 642$ kip-ft	$M_{cx} = \dfrac{M_{nx}}{\Omega_b}$ $= 427$ kip-ft
$M_{cy} = \phi_b M_{ny}$ $= 311$ kip-ft	$M_{cy} = \dfrac{M_{ny}}{\Omega_b}$ $= 207$ kip-ft
$\dfrac{P_u}{\phi_c P_n} = \dfrac{400 \text{ kips}}{1{,}130 \text{ kips}}$ $= 0.354$	$\dfrac{P_a}{P_n / \Omega_c} = \dfrac{267 \text{ kips}}{750 \text{ kips}}$ $= 0.356$

LRFD	ASD
Because $\dfrac{P_u}{\phi_c P_n} \geq 0.2$,	Because $\dfrac{P_a}{P_n / \Omega_c} \geq 0.2$,
$\dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \leq 1.0$ (*Spec*. Eq. H1-1a)	$\dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \leq 1.0$ (*Spec*. Eq. H1-1a)
$= \dfrac{400 \text{ kips}}{1{,}130 \text{ kips}} + \dfrac{8}{9}\left(\dfrac{250 \text{ kip-ft}}{642 \text{ kip-ft}} + \dfrac{80.0 \text{ kip-ft}}{311 \text{ kip-ft}}\right) \leq 1.0$	$= \dfrac{267 \text{ kips}}{750 \text{ kips}} + \dfrac{8}{9}\left(\dfrac{167 \text{ kip-ft}}{427 \text{ kip-ft}} + \dfrac{53.3 \text{ kip-ft}}{207 \text{ kip-ft}}\right)$
$= 0.928 < 1.0$ **o.k.**	$= 0.932 < 1.0$ **o.k.**

EXAMPLE H.2 W-SHAPE SUBJECT TO COMBINED COMPRESSION AND BENDING MOMENT ABOUT BOTH AXES (BY AISC *SPECIFICATION* SECTION H2)

Given:

Using AISC *Specification* Section H2, determine if an ASTM A992 W14×99 has sufficient available strength to support the axial forces and moments listed as follows, obtained from a second-order analysis that includes P-δ effects. The unbraced length is 14 ft and the member has pinned ends. This example is included primarily to illustrate the use of AISC *Specification* Section H2.

LRFD	ASD
$P_u = 360$ kips $M_{ux} = 250$ kip-ft $M_{uy} = 80$ kip-ft	$P_a = 240$ kips $M_{ax} = 167$ kip-ft $M_{ay} = 53.3$ kip-ft

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W14×99
$A = 29.1$ in.^{2}
$S_x = 157$ in.^{3}
$S_y = 55.2$ in.^{3}

The required flexural and axial stresses are:

LRFD	ASD
$f_{ra} = \dfrac{P_u}{A} = \dfrac{360\text{ kips}}{29.1\text{ in.}^2} = 12.4\text{ ksi}$	$f_{ra} = \dfrac{P_a}{A} = \dfrac{240\text{ kips}}{29.1\text{ in.}^2} = 8.25\text{ ksi}$
$f_{rbx} = \dfrac{M_{ux}}{S_x} = \dfrac{(250\text{ kip-ft})(12\text{ in./ft})}{157\text{ in.}^3} = 19.1\text{ ksi}$	$f_{rbx} = \dfrac{M_{ax}}{S_x} = \dfrac{(167\text{ kip-ft})(12\text{ in./ft})}{157\text{ in.}^3} = 12.8\text{ ksi}$
$f_{rby} = \dfrac{M_{uy}}{S_y} = \dfrac{(80\text{ kip-ft})(12\text{ in./ft})}{55.2\text{ in.}^3} = 17.4\text{ ksi}$	$f_{rby} = \dfrac{M_{ay}}{S_y} = \dfrac{(53.3\text{ kip-ft})(12\text{ in./ft})}{55.2\text{ in.}^3} = 11.6\text{ ksi}$

The effective length of the member is:

$$\begin{aligned} L_{cx} &= L_{cy} \\ &= KL \\ &= 1.0(14 \text{ ft}) \\ &= 14.0 \text{ ft} \end{aligned}$$

For L_c = 14.0 ft, calculate the available axial and flexural stresses using the available strengths from AISC *Manual* Table 6-2.

LRFD	ASD
$F_{ca} = \phi_c F_{cr}$ $= \dfrac{\phi_c P_n}{A}$ $= \dfrac{1{,}130 \text{ kips}}{29.1 \text{ in.}^2}$ $= 38.8 \text{ ksi}$	$F_{ca} = \dfrac{F_{cr}}{\Omega_c}$ $= \dfrac{P_n}{\Omega_c A}$ $= \dfrac{750 \text{ kips}}{29.1 \text{ in.}^2}$ $= 25.8 \text{ ksi}$
$F_{cbx} = \dfrac{\phi_b M_{nx}}{S_x}$ $= \dfrac{(642 \text{ kip-ft})(12 \text{ in./ft})}{157 \text{ in.}^3}$ $= 49.1 \text{ ksi}$	$F_{cbx} = \dfrac{M_{nx}}{\Omega_b S_x}$ $= \dfrac{(427 \text{ kip-ft})(12 \text{ in./ft})}{157 \text{ in.}^3}$ $= 32.6 \text{ ksi}$
$F_{cby} = \dfrac{\phi_b M_{ny}}{S_y}$ $= \dfrac{(311 \text{ kip-ft})(12 \text{ in./ft})}{55.2 \text{ in.}^3}$ $= 67.6 \text{ ksi}$	$F_{cby} = \dfrac{M_{ny}}{\Omega_b S_y}$ $= \dfrac{(207 \text{ kip-ft})(12 \text{ in./ft})}{55.2 \text{ in.}^3}$ $= 45.0 \text{ ksi}$

As shown in the LRFD calculation of F_{cby} in the preceding text, the available flexural stresses can exceed the yield stress in cases where the available strength is governed by yielding and the yielding strength is calculated using the plastic section modulus.

Combined Stress Ratio

From AISC *Specification* Section H2, check the combined stress ratios as follows:

LRFD	ASD
$\left\| \dfrac{f_{ra}}{F_{ca}} + \dfrac{f_{rbx}}{F_{cbx}} + \dfrac{f_{rby}}{F_{cby}} \right\| \leq 1.0$ (from *Spec*. Eq. H2-1) $\left\| \dfrac{12.4 \text{ ksi}}{38.8 \text{ ksi}} + \dfrac{19.1 \text{ ksi}}{49.1 \text{ ksi}} + \dfrac{17.4 \text{ ksi}}{67.6 \text{ ksi}} \right\| = 0.966 < 1.0$ **o.k.**	$\left\| \dfrac{f_{ra}}{F_{ca}} + \dfrac{f_{rbx}}{F_{cbx}} + \dfrac{f_{rby}}{F_{cby}} \right\| \leq 1.0$ (from *Spec*. Eq. H2-1) $\left\| \dfrac{8.25 \text{ ksi}}{25.8 \text{ ksi}} + \dfrac{12.8 \text{ ksi}}{32.6 \text{ ksi}} + \dfrac{11.6 \text{ ksi}}{45.0 \text{ ksi}} \right\| = 0.970 < 1.0$ **o.k.**

A comparison of these results with those from Example H.1B shows that AISC *Specification* Equation H1-1a will produce less conservative results than AISC *Specification* Equation H2-1 when its use is permitted.

Note: This check is made at a point on the cross section (extreme fiber, in this example). The designer must therefore determine which point on the cross section is critical, or check multiple points if the critical point cannot be readily determined.

EXAMPLE H.3 W-SHAPE SUBJECT TO COMBINED AXIAL TENSION AND FLEXURE

Given:

Select an ASTM A992 W-shape with a 14-in.-nominal-depth to carry forces of 29 kips from dead load and 87 kips from live load in axial tension, as well as the following moments due to uniformly distributed loads:

$$M_{xD} = 32 \text{ kip-ft}$$
$$M_{xL} = 96 \text{ kip-ft}$$

$$M_{yD} = 11.3 \text{ kip-ft}$$
$$M_{yL} = 33.8 \text{ kip-ft}$$

The unbraced length is 30 ft and the ends are pinned. Assume the connections are made with no holes.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From ASCE/SEI 7, Chapter 2, the required strengths are:

LRFD	ASD
$P_u = 1.2(29 \text{ kips}) + 1.6(87 \text{ kips})$ $= 174 \text{ kips}$	$P_a = 29 \text{ kips} + 87 \text{ kips}$ $= 116 \text{ kips}$
$M_{ux} = 1.2(32 \text{ kip-ft}) + 1.6(96 \text{ kip-ft})$ $= 192 \text{ kip-ft}$	$M_{ax} = 32 \text{ kip-ft} + 96 \text{ kip-ft}$ $= 128 \text{ kip-ft}$
$M_{uy} = 1.2(11.3 \text{ kip-ft}) + 1.6(33.8 \text{ kip-ft})$ $= 67.6 \text{ kip-ft}$	$M_{ay} = 11.3 \text{ kip-ft} + 33.8 \text{ kip-ft}$ $= 45.1 \text{ kip-ft}$

Try a W14×82.

From AISC *Manual* Tables 1-1 and 3-2, the properties are as follows:

W14×82
A_g = 24.0 in.^{2}
S_x = 123 in.^{3}
Z_x = 139 in.^{3}
S_y = 29.3 in.^{3}
Z_y = 44.8 in.^{3}
I_y = 148 in.^{4}
L_p = 8.76 ft
L_r = 33.2 ft

Nominal Tensile Strength

From AISC *Specification* Section D2(a), the nominal tensile strength due to tensile yielding in the gross section is:

$$P_n = F_y A_g \qquad (Spec.\ \text{Eq. D2-1})$$
$$= (50\text{ ksi})(24.0\text{ in.}^2)$$
$$= 1{,}200\text{ kips}$$

Note that for a member with holes, the rupture strength of the member would also have to be computed using AISC *Specification* Equation D2-2.

Nominal Flexural Strength for Bending About the Major Axis

Yielding

From AISC *Specification* Section F2.1, the nominal flexural strength due to yielding (plastic moment) is:

$$M_{nx} = M_p = F_y Z_x \qquad (Spec.\ \text{Eq. F2-1})$$
$$= (50\text{ ksi})(139\text{ in.}^3)$$
$$= 6{,}950\text{ kip-in.}$$

Lateral-Torsional Buckling

From AISC *Specification* Section F2.2, the nominal flexural strength due to lateral-torsional buckling is determined as follows:

Because $L_p < L_b \le L_r$, i.e., 8.76 ft < 30 ft < 33.2 ft, AISC *Specification* Equation F2-2 applies.

Lateral-Torsional Buckling Modification Factor, C_b

From AISC *Manual* Table 3-1, C_b = 1.14, without considering the beneficial effects of the tension force. However, per AISC *Specification* Section H1.2, C_b may be modified because the column is in axial tension concurrently with flexure.

$$P_{ey} = \frac{\pi^2 E I_y}{L_b^2}$$
$$= \frac{\pi^2 (29{,}000\text{ ksi})(148\text{ in.}^4)}{\left[(30\text{ ft})(12.0\text{ in./ft})\right]^2}$$
$$= 327\text{ kips}$$

LRFD	ASD
$\sqrt{1+\dfrac{\alpha P_u}{P_{ey}}} = \sqrt{1+\dfrac{1.0(174\text{ kips})}{327\text{ kips}}}$ $= 1.24$	$\sqrt{1+\dfrac{\alpha P_a}{P_{ey}}} = \sqrt{1+\dfrac{1.6(116\text{ kips})}{327\text{ kips}}}$ $= 1.25$

$$C_b = 1.24(1.14)$$
$$= 1.41$$

$$M_n = C_b\left[M_p - \left(M_p - 0.7F_yS_x\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad (\textit{Spec}.\ \text{Eq. F2-2})$$

$$= 1.41\left\{6{,}950\text{ kip-in.} - \left[6{,}950\text{ kip-in.} - 0.7\left(50\text{ ksi}\right)\left(123\text{ in.}^3\right)\right]\left(\frac{30\text{ ft} - 8.76\text{ ft}}{33.2\text{ ft} - 8.76\text{ ft}}\right)\right\} \le 6{,}950\text{ kip-in.}$$

$$= 6{,}560\text{ kip-in. or } 547\text{ kip-ft} \quad \textbf{controls}$$

Local Buckling

Per AISC *Manual* Table 1-1, the cross section is compact at F_y = 50 ksi; therefore, the local buckling limit state does not apply.

Nominal Flexural Strength for Bending About the Minor Axis and the Interaction of Flexure and Tension

Because a W14×82 has compact flanges, only the limit state of yielding applies for bending about the minor axis.

$$M_{ny} = M_p = F_yZ_y \le 1.6F_yS_y \qquad (\textit{Spec}.\ \text{Eq. F6-1})$$

$$= \left(50\text{ ksi}\right)\left(44.8\text{ in.}^3\right) \le 1.6\left(50\text{ ksi}\right)\left(29.3\text{ in.}^3\right)$$

$$= 2{,}240\text{ kip-in.} < 2{,}340\text{ kip-in.}$$

$$= 2{,}240\text{ kip-in. or } 187\text{ kip-ft}$$

Available Strength

From AISC *Specification* Sections D2 and F1, the available strengths are:

LRFD	ASD
$\phi_b = \phi_t = 0.90$	$\Omega_b = \Omega_t = 1.67$
$P_c = \phi_t P_n$ $= 0.90(1{,}200\text{ kips})$ $= 1{,}080\text{ kips}$	$P_c = \dfrac{P_n}{\Omega_t}$ $= \dfrac{1{,}200\text{ kips}}{1.67}$ $= 719\text{ kips}$
$M_{cx} = \phi_b M_{nx}$ $= 0.90(547\text{ kip-ft})$ $= 492\text{ kip-ft}$	$M_{cx} = \dfrac{M_{nx}}{\Omega_b}$ $= \dfrac{547\text{ kip-ft}}{1.67}$ $= 328\text{ kip-ft}$
$M_{cy} = \phi_b M_{ny}$ $= 0.90(187\text{ kip-ft})$ $= 168\text{ kip-ft}$	$M_{cy} = \dfrac{M_{ny}}{\Omega_b}$ $= \dfrac{187\text{ kip-ft}}{1.67}$ $= 112\text{ kip-ft}$

Interaction of Tension and Flexure

Check limit for AISC *Specification* Equation H1-1a.

LRFD	ASD
$\frac{P_r}{P_c} = \frac{P_u}{\phi_t P_n}$ $= \frac{174 \text{ kips}}{1{,}080 \text{ kips}}$ $= 0.161 < 0.2$ Because $\frac{P_r}{P_c} < 0.2$, $\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0$ (*Spec*. Eq. H1-1b) $= \frac{174 \text{ kips}}{2(1{,}080 \text{ kips})} + \frac{192 \text{ kip-ft}}{492 \text{ kip-ft}} + \frac{67.6 \text{ kip-ft}}{168 \text{ kip-ft}} \leq 1.0$ $= 0.873 < 1.0$ **o.k.**	$\frac{P_r}{P_c} = \frac{P_a}{P_n / \Omega_t}$ $= \frac{116 \text{ kips}}{719 \text{ kips}}$ $= 0.161 < 0.2$ Because $\frac{P_r}{P_c} < 0.2$, $\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0$ (*Spec*. Eq. H1-1b) $= \frac{116 \text{ kips}}{2(719 \text{ kips})} + \frac{128 \text{ kip-ft}}{328 \text{ kip-ft}} + \frac{45.1 \text{ kip-ft}}{112 \text{ kip-ft}} \leq 1.0$ $= 0.874 < 1.0$ **o.k.**

EXAMPLE H.4 W-SHAPE SUBJECT TO COMBINED AXIAL COMPRESSION AND FLEXURE

Given:

Select an ASTM A992 W-shape with a 10-in.-nominal-depth to carry axial compression forces of 5 kips from dead load and 15 kips from live load. The unbraced length is 14 ft and the ends are pinned. The member also has the following required moment strengths due to uniformly distributed loads, not including second-order effects:

$M_{xD} = 15$ kip-ft
$M_{xL} = 45$ kip-ft

$M_{yD} = 2$ kip-ft
$M_{yL} = 6$ kip-ft

The member is not subject to sidesway (no lateral translation).

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From Chapter 2 of ASCE/SEI 7, the required strength (not considering second-order effects) is:

LRFD	ASD
$P_u = 1.2(5\text{ kips}) + 1.6(15\text{ kips})$ $= 30.0\text{ kips}$ $M_{ux} = 1.2(15\text{ kip-ft}) + 1.6(45\text{ kip-ft})$ $= 90.0\text{ kip-ft}$ $M_{uy} = 1.2(2\text{ kip-ft}) + 1.6(6\text{ kip-ft})$ $= 12.0\text{ kip-ft}$	$P_a = 5\text{ kips} + 15\text{ kips}$ $= 20.0\text{ kips}$ $M_{ax} = 15\text{ kip-ft} + 45\text{ kip-ft}$ $= 60.0\text{ kip-ft}$ $M_{ay} = 2\text{ kip-ft} + 6\text{ kip-ft}$ $= 8.00\text{ kip-ft}$

Try a W10×33.

From AISC *Manual* Tables 1-1 and 3-2, the properties are as follows:

W10×33
$A = 9.71\text{ in.}^2$
$S_x = 35.0\text{ in.}^3$
$Z_x = 38.8\text{ in.}^3$
$I_x = 171\text{ in.}^4$
$r_x = 4.19$ in.
$S_y = 9.20\text{ in.}^3$
$Z_y = 14.0\text{ in.}^3$
$I_y = 36.6\text{ in.}^4$
$r_y = 1.94$ in.
$L_p = 6.85$ ft
$L_r = 21.8$ ft

Available Axial Strength

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0. Because $L_c = KL_x = KL_y$ = 14.0 ft and $r_x > r_y$, the *y-y* axis will govern.

From AISC *Manual* Table 6-2, the available axial strength is:

LRFD	ASD
$P_c = \phi_c P_n$ $= 253\text{ kips}$	$P_c = \dfrac{P_n}{\Omega_c}$ $= 168\text{ kips}$

Required Flexural Strength (including second-order amplification)

Use the approximate method of second-order analysis procedure from AISC *Specification* Appendix 8. Because the member is not subject to sidesway, only P-δ amplifiers need to be added.

$$B_1 = \frac{C_m}{1 - \alpha P_r / P_{e1}} \geq 1 \qquad \text{(}\textit{Spec}\text{. Eq. A-8-3)}$$

where C_m is conservatively taken per AISC *Specification* A-8.2.1(b):

$$C_m = 1.0$$

The *x-x* axis flexural magnifier is:

$$P_{e1x} = \frac{\pi^2 EI_x}{(L_{c1x})^2} \qquad \text{(from }\textit{Spec}\text{. Eq. A-8-5)}$$

$$= \frac{\pi^2 (29{,}000\text{ ksi})(171\text{ in.}^4)}{\left[(14\text{ ft})(12\text{ in./ft})\right]^2}$$

$$= 1{,}730\text{ kips}$$

LRFD	ASD
$\alpha = 1.0$ $B_{1x} = \dfrac{C_m}{1 - \alpha P_r / P_{e1x}} \geq 1.0$ $= \dfrac{1.0}{1 - 1.0(30\text{ kips}/1{,}730\text{ kips})} \geq 1.0$ $= 1.02$ $M_{ux} = 1.02(90\text{ kip-ft})$ $= 91.8\text{ kip-ft}$	$\alpha = 1.6$ $B_{1x} = \dfrac{C_m}{1 - \alpha P_r / P_{e1x}} \geq 1.0$ $= \dfrac{1.0}{1 - 1.6(20\text{ kips}/1{,}730\text{ kips})} \geq 1.0$ $= 1.02$ $M_{ax} = 1.02(60\text{ kip-ft})$ $= 61.2\text{ kip-ft}$

The *y-y* axis flexural magnifier is:

$$P_{e1y} = \frac{\pi^2 EI_y}{(L_{c1y})^2} \qquad \text{(modified } \textit{Spec.} \text{ Eq. A-8-5)}$$

$$= \frac{\pi^2 (29{,}000 \text{ ksi})(36.6 \text{ in.}^4)}{[(14 \text{ ft})(12 \text{ in./ft})]^2}$$

$$= 371 \text{ kips}$$

LRFD	ASD
$\alpha = 1.0$ $B_{1y} = \dfrac{C_m}{1 - \alpha P_r / P_{e1y}} \geq 1.0$ $= \dfrac{1.0}{1 - 1.0(30 \text{ kips} / 371 \text{ kips})} \geq 1.0$ $= 1.09$ $M_{uy} = 1.09(12 \text{ kip-ft})$ $= 13.1 \text{ kip-ft}$	$\alpha = 1.6$ $B_{1y} = \dfrac{C_m}{1 - \alpha P_r / P_{e1y}} \geq 1.0$ $= \dfrac{1.0}{1 - 1.6(20 \text{ kips} / 371 \text{ kips})} \geq 1.0$ $= 1.09$ $M_{ay} = 1.09(8 \text{ kip-ft})$ $= 8.72 \text{ kip-ft}$

Nominal Flexural Strength about the Major Axis

Yielding

$$M_{nx} = M_p = F_y Z_x \qquad (\textit{Spec.} \text{ Eq. F2-1})$$

$$= (50 \text{ ksi})(38.8 \text{ in.}^3)$$

$$= 1{,}940 \text{ kip-in.}$$

Lateral-Torsional Buckling

Because $L_p < L_b \leq L_r$, i.e., 6.85 ft < 14.0 ft < 21.8 ft, AISC *Specification* Equation F2-2 applies.

From AISC *Manual* Table 3-1, C_b = 1.14

$$M_{nx} = C_b \left[M_p - (M_p - 0.7 F_y S_x) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_p \qquad (\textit{Spec.} \text{ Eq. F2-2})$$

$$= 1.14 \left\{ 1{,}940 \text{ kip-in.} - \left[1{,}940 \text{ kip-in.} - 0.7(50 \text{ ksi})(35.0 \text{ in.}^3) \right] \left(\frac{14 \text{ ft} - 6.85 \text{ ft}}{21.8 \text{ ft} - 6.85 \text{ ft}} \right) \right\}$$

$$= 1{,}820 \text{ kip-in.} < 1{,}940 \text{ kip-in.}$$

$$= 1{,}820 \text{ kip-in. or } 152 \text{ kip-ft} \quad \textbf{controls}$$

Local Buckling

Per AISC *Manual* Table 1-1, the member is compact for F_y = 50 ksi, so the local buckling limit state does not apply.

Nominal Flexural Strength about the Minor Axis

Determine the nominal flexural strength for bending about the minor axis from AISC *Specification* Section F6. Because a W10×33 has compact flanges, only the yielding limit state applies.

From AISC *Specification* Section F6.1:

$$M_{nx} = M_p = F_y Z_x \le 1.6 F_y S_y \qquad \text{(Spec. Eq. F6-1)}$$
$$= (50 \text{ ksi})(14.0 \text{ in.}^3) \le 1.6(50 \text{ ksi})(9.20 \text{ in.}^3)$$
$$= 700 \text{ kip-in.} < 736 \text{ kip-in.}$$
$$= 700 \text{ kip-in. or } 58.3 \text{ kip-ft}$$

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$	$\Omega_b = 1.67$
$M_{cx} = \phi_b M_{nx}$ $= 0.90(152 \text{ kip-ft})$ $= 137 \text{ kip-ft}$	$M_{cx} = \dfrac{M_{nx}}{\Omega_b}$ $= \dfrac{152 \text{ kip-ft}}{1.67}$ $= 91.0 \text{ kip-ft}$
$M_{cy} = \phi_b M_{ny}$ $= 0.90(58.3 \text{ kip-ft})$ $= 52.5 \text{ kip-ft}$	$M_{cy} = \dfrac{M_{ny}}{\Omega_b}$ $= \dfrac{58.3 \text{ kip-ft}}{1.67}$ $= 34.9 \text{ kip-ft}$

Check limit for AISC *Specification* Equations H1-1a and H1-1b.

LRFD	ASD
$\dfrac{P_r}{P_c} = \dfrac{P_u}{\phi_c P_n}$ $= \dfrac{30 \text{ kips}}{253 \text{ kips}}$ $= 0.119 < 0.2$	$\dfrac{P_r}{P_c} = \dfrac{P_a}{P_n / \Omega_c}$ $= \dfrac{20 \text{ kips}}{168 \text{ kips}}$ $= 0.119 < 0.2$
Because $\dfrac{P_r}{P_c} < 0.2$,	Because $\dfrac{P_r}{P_c} < 0.2$,
$\dfrac{P_r}{2P_c} + \left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \le 1.0$ (*Spec*. Eq. H1-1b) $= \dfrac{30 \text{ kips}}{2(253 \text{ kips})} + \left(\dfrac{91.8 \text{ kip-ft}}{137 \text{ kip-ft}} + \dfrac{13.1 \text{ kip-ft}}{52.5 \text{ kip-ft}}\right) \le 1.0$ $= 0.979 < 1.0$ **o.k.**	$\dfrac{P_r}{2P_c} + \left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \le 1.0$ (*Spec*. Eq. H1-1b) $= \dfrac{20 \text{ kips}}{2(168 \text{ kips})} + \left(\dfrac{61.2 \text{ kip-ft}}{91.0 \text{ kip-ft}} + \dfrac{8.72 \text{ kip-ft}}{34.9 \text{ kip-ft}}\right) \le 1.0$ $= 0.982 < 1.0$ **o.k.**

CHAPTER H DESIGN EXAMPLE REFERENCES

Seaburg, P.A. and Carter, C.J. (1997), *Torsional Analysis of Structural Steel Members,* Design Guide 9, AISC, Chicago, IL.

Chapter J
Design of Connections

AISC *Specification* Chapter J addresses the design of connections. The chapter's primary focus is the design of welded and bolted connections. Design requirements for fillers, splices, column bases, concentrated forces, anchors rods and other threaded parts are also covered. See AISC *Specification* Appendix 3 for special requirements for connections subject to fatigue.

EXAMPLE J.1 FILLET WELD IN LONGITUDINAL SHEAR

Given:

As shown in Figure J.1-1, a ¼-in.-thick × 18-in. wide plate is fillet welded to a ⅜-in.-thick plate. The plates are ASTM A572 Grade 50 and have been properly sized. Use 70-ksi electrodes. Note that the plates could be specified as ASTM A36, but F_y = 50 ksi plate has been used here to demonstrate the requirements for long welds.

Confirm that the size and length of the welds shown are adequate to resist the applied loading.

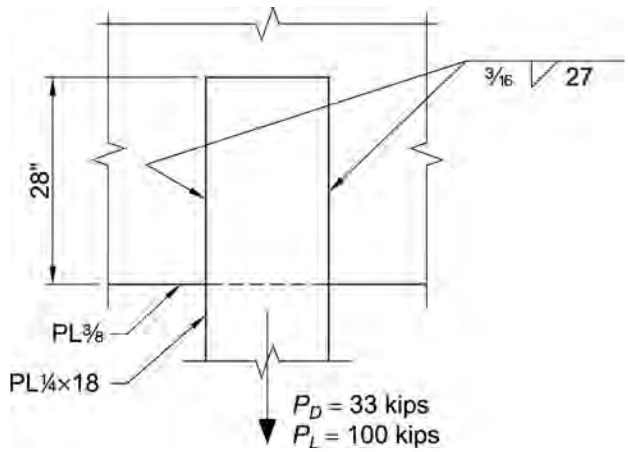


Fig. J.1-1. Geometry and loading for Example J.1.

Solution:

From AISC *Manual* Table 2-5, the material properties are as follows:

ASTM A572 Grade 50
F_y = 50 ksi
F_u = 65 ksi

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$P_u = 1.2(33 \text{ kips}) + 1.6(100 \text{ kips})$ $= 200 \text{ kips}$	$P_a = 33 \text{ kips} + 100 \text{ kips}$ $= 133 \text{ kips}$

Maximum and Minimum Weld Size

Because the thickness of the overlapping plate is ¼ in., the maximum fillet weld size that can be used without special notation per AISC *Specification* Section J2.2b, is a 3⁄16-in. fillet weld. A 3⁄16-in. fillet weld can be deposited in the flat or horizontal position in a single pass (true up to 5⁄16-in.).

From AISC *Specification* Table J2.4, the minimum size of the fillet weld, based on a material thickness of ¼ in. is ⅛ in.

Weld Strength

The nominal weld strength per inch of 3⁄16-in. weld, determined from AISC *Specification* Section J2.4(b) is:

$$R_n = F_{nw} A_{we} \qquad (Spec.\ \text{Eq. J2-4})$$
$$= (0.60F_{EXX})A_{we}$$
$$= 0.60(70\text{ ksi})\left(\frac{3/16\text{ in.}}{\sqrt{2}}\right)$$
$$= 5.57\text{ kip/in.}$$

From AISC *Specification* Section J2.2b, check the weld length to weld size ratio, because this is an end-loaded fillet weld.

$$\frac{l}{w} = \frac{27.0\text{ in.}}{3/16\text{ in.}}$$
$$= 144 > 100;\ \text{therefore, AISC } \textit{Specification} \text{ Equation J2-1 must be applied}$$

$$\beta = 1.2 - 0.002(l/w) \le 1.0 \qquad (Spec.\ \text{Eq. J2-1})$$
$$= 1.2 - 0.002(144) \le 1.0$$
$$= 0.912$$

The nominal weld shear rupture strength is:

$$R_n = 0.912(5.57\text{ kip/in.})(2\text{ welds})(27\text{ in.})$$
$$= 274\text{ kips}$$

From AISC *Specification* Section J2.4, the available shear rupture strength is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(274\text{ kips})$ $= 206\text{ kips} > 200\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{274\text{ kips}}{2.00}$ $= 137\text{ kips} > 133\text{ kips}$ **o.k.**

The base metal strength is determined from AISC *Specification* Section J2.4(a). The ¼-in.-thick plate controls:

$$R_n = F_{nBM} A_{BM} \qquad (Spec.\ \text{Eq. J2-2})$$
$$= 0.60F_u t_p l_{weld}$$
$$= 0.60(65\text{ ksi})(1/4\text{ in.})(2\text{ welds})(27\text{ in.})$$
$$= 527\text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(527\text{ kips})$ $= 395\text{ kips} > 200\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{527\text{ kips}}{2.00}$ $= 264\text{ kips} > 133\text{ kips}$ **o.k.**

EXAMPLE J.2 FILLET WELD LOADED AT AN ANGLE

Given:

Verify a fillet weld at the edge of a gusset plate is adequate to resist a force of 50 kips due to dead load and 150 kips due to live load, at an angle of 60° relative to the weld, as shown in Figure J.2-1. Assume the beam and the gusset plate thickness and length have been properly sized. Use a 70-ksi electrode.

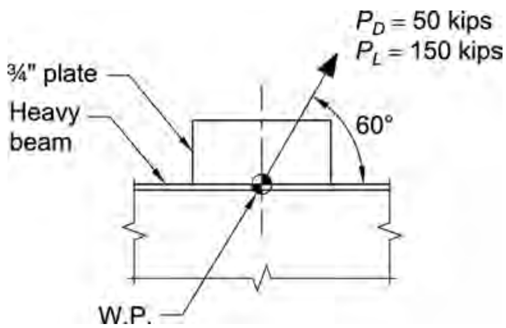


Fig. J.2-1. Geometry and loading for Example J.2.

Solution:

From ASCE/SEI 7, Chapter 2, the required tensile strength is:

LRFD	ASD
$P_u = 1.2(50\text{ kips}) + 1.6(150\text{ kips})$ $= 300\text{ kips}$	$P_a = 50\text{ kips} + 150\text{ kips}$ $= 200\text{ kips}$

Assume a $\frac{5}{16}$-in. fillet weld is used on each side of the plate.

Note that from AISC *Specification* Table J2.4, the minimum size of fillet weld, based on a material thickness of ¾ in. is ¼ in. (assuming the beam flange thickness exceeds ¾ in.).

Available Shear Strength of the Fillet Weld Per Inch of Length

From AISC *Specification* Section J2.4(b), the nominal strength of the fillet weld is determined as follows:

$$R_n = F_{nw}A_{we} \qquad (\textit{Spec}.\text{ Eq. J2-4})$$

$$= 0.60F_{EXX}\left(1.0 + 0.50\sin^{1.5}60°\right)A_{we}$$

$$= 0.60(70\text{ ksi})\left(1.0 + 0.50\sin^{1.5}60°\right)\left(\frac{\frac{5}{16}\text{ in.}}{\sqrt{2}}\right)$$

$$= 13.0\text{ kip/in.}$$

From AISC *Specification* Section J2.4(b), the available shear strength per inch of weld for fillet welds on two sides is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(13.0 \text{ kip/in.})(2 \text{ sides})$ $= 19.5 \text{ kip/in.}$	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{13.0 \text{ kip/in.}}{2.00}(2 \text{ sides})$ $= 13.0 \text{ kip/in.}$

Required Length of Weld

LRFD	ASD
$l = \dfrac{300 \text{ kips}}{19.5 \text{ kip/in.}}$ $= 15.4 \text{ in.}$ Use 16 in. on each side of the plate.	$l = \dfrac{200 \text{ kips}}{13.0 \text{ kip/in.}}$ $= 15.4 \text{ in.}$ Use 16 in. on each side of the plate.

EXAMPLE J.3 COMBINED TENSION AND SHEAR IN BEARING-TYPE CONNECTIONS

Given:

A ¾-in.-diameter, Group A bolt with threads not excluded from the shear plane (thread condition N) is subjected to a tension force of 3.5 kips due to dead load and 12 kips due to live load, and a shear force of 1.33 kips due to dead load and 4 kips due to live load. Check the combined stresses according to AISC *Specification* Equations J3-3a and J3-3b.

Solution:

From ASCE/SEI 7, Chapter 2, the required tensile and shear strengths are:

LRFD	ASD
Tension: $T_u = 1.2(3.5\text{ kips}) + 1.6(12\text{ kips})$ $= 23.4\text{ kips}$ Shear: $V_u = 1.2(1.33\text{ kips}) + 1.6(4\text{ kips})$ $= 8.00\text{ kips}$	Tension: $T_a = 3.5\text{ kips} + 12\text{ kips}$ $= 15.5\text{ kips}$ Shear: $V_a = 1.33\text{ kips} + 4\text{ kips}$ $= 5.33\text{ kips}$

Available Tensile Strength

When a bolt is subject to combined tension and shear, the available tensile strength is determined according to the limit states of tension and shear rupture, from AISC *Specification* Section J3.7 as follows.

From AISC *Specification* Table J3.2, Group A bolts:

$F_{nt} = 90\text{ ksi}$
$F_{nv} = 54\text{ ksi}$

From AISC *Manual* Table 7-2, for a ¾-in.-diameter bolt:

$A_b = 0.442\text{ in.}^2$

The available shear stress is determined as follows and must equal or exceed the required shear stress.

LRFD	ASD
$\phi = 0.75$ $\phi F_{nv} = 0.75(54\text{ ksi})$ $= 40.5\text{ ksi}$ $f_{rv} = \dfrac{V_u}{A_b}$ $= \dfrac{8.00\text{ kips}}{0.442\text{ in.}^2}$ $= 18.1\text{ ksi} < 40.5\text{ ksi}$ **o.k.**	$\Omega = 2.00$ $\dfrac{F_{nv}}{\Omega} = \dfrac{54\text{ ksi}}{2.00}$ $= 27.0\text{ ksi}$ $f_{rv} = \dfrac{V_a}{A_b}$ $= \dfrac{5.33\text{ kips}}{0.442\text{ in.}^2}$ $= 12.1\text{ ksi} < 27.0\text{ ksi}$ **o.k.**

The available tensile strength of a bolt subject to combined tension and shear is as follows:

LRFD	ASD
$F'_{nt} = 1.3F_{nt} - \dfrac{F_{nt}}{\phi F_{nv}} f_{rv} \le F_{nt}$ (*Spec*. Eq. J3-3a) $= 1.3(90\text{ ksi}) - \dfrac{90\text{ ksi}}{40.5\text{ ksi}}(18.1\text{ ksi}) \le 90\text{ ksi}$ $= 76.8\text{ ksi}$ For combined tension and shear, ϕ = 0.75, from AISC *Specification* Section J3.7. $\phi R_n = \phi F'_{nt} A_b$ (*Spec*. Eq. J3-2) $= 0.75(76.8\text{ ksi})(0.442\text{ in.}^2)$ $= 25.5\text{ kips} > 23.4\text{ kips}$ **o.k.**	$F'_{nt} = 1.3F_{nt} - \dfrac{\Omega F_{nt}}{F_{nv}} f_{rv} \le F_{nt}$ (*Spec*. Eq. J3-3b) $= 1.3(90\text{ ksi}) - \dfrac{90\text{ ksi}}{27.0\text{ ksi}}(12.1\text{ ksi}) \le 90\text{ ksi}$ $= 76.7\text{ ksi}$ For combined tension and shear, Ω = 2.00, from AISC *Specification* Section J3.7. $\dfrac{R_n}{\Omega} = \dfrac{F'_{nt} A_b}{\Omega}$ (*Spec*. Eq. J3-2) $= \dfrac{(76.7\text{ ksi})(0.442\text{ in.}^2)}{2.00}$ $= 17.0\text{ kips} > 15.5\text{ kips}$ **o.k.**

The effects of combined shear and tensile stresses need not be investigated if either the required shear or tensile stress is less than or equal to 30% of the corresponding available stress per the User Note at the end of AISC *Specification* Section J3.7. In the example herein, both the required shear and tensile stresses exceeded the 30% threshold and evaluation of combined stresses was necessary.

AISC *Specification* Equations J3-3a and J3-3b may be rewritten so as to find a nominal shear stress, F'_{nv}, as a function of the required tensile stress as is shown in AISC *Specification* Commentary Equations C-J3-7a and C-J3-7b.

EXAMPLE J.4A SLIP-CRITICAL CONNECTION WITH SHORT-SLOTTED HOLES

Slip-critical connections shall be designed to prevent slip and for the limit states of bearing-type connections.

Given:

Refer to Figure J.4A-1 and select the number of bolts that are required to support the loads shown when the connection plates have short slots transverse to the load and no fillers are provided. Select the number of bolts required for slip resistance only.

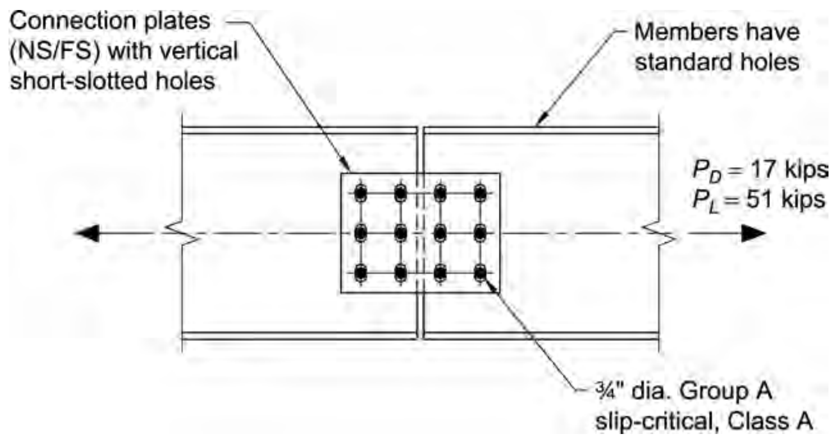


Fig. J.4A-1. Geometry and loading for Example J.4A.

Solution:

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$P_u = 1.2(17\text{ kips}) + 1.6(51\text{ kips})$ $= 102\text{ kips}$	$P_a = 17\text{ kips} + 51\text{ kips}$ $= 68.0\text{ kips}$

From AISC *Specification* Section J3.8(a), the available slip resistance for the limit state of slip for standard size and short-slotted holes perpendicular to the direction of the load is determined as follows:

ϕ = 1.00
Ω = 1.50
μ = 0.30 for Class A surface
D_u = 1.13
h_f = 1.0, no filler is provided
T_b = 28 kips, from AISC *Specification* Table J3.1, Group A
n_s = 2, number of slip planes

$$R_n = \mu D_u h_f T_b n_s \quad \text{(Spec. Eq. J3-4)}$$
$$= 0.30(1.13)(1.0)(28\text{ kips})(2)$$
$$= 19.0\text{ kips/bolt}$$

The available slip resistance is:

LRFD	ASD
$\phi R_n = 1.00(19.0 \text{ kips/bolt})$ $= 19.0 \text{ kips/bolt}$	$\dfrac{R_n}{\Omega} = \dfrac{19.0 \text{ kips/bolt}}{1.50}$ $= 12.7 \text{ kips/bolt}$

Required Number of Bolts

LRFD	ASD
$n_b = \dfrac{P_u}{\phi R_n}$ $= \dfrac{102 \text{ kips}}{19.0 \text{ kips/bolt}}$ $= 5.37 \text{ bolts}$ Use 6 bolts	$n_b = \dfrac{P_a}{\left(\dfrac{R_n}{\Omega}\right)}$ $= \dfrac{68.0 \text{ kips}}{12.7 \text{ kips/bolt}}$ $= 5.35 \text{ bolts}$ Use 6 bolts

Note: To complete the verification of this connection, the limit states of bolt shear, bearing, tearout, tensile yielding, tensile rupture, and block shear rupture must also be checked.

EXAMPLE J.4B SLIP-CRITICAL CONNECTION WITH LONG-SLOTTED HOLES

Given:

Repeat Example J.4A with the same loads, but assuming that the connection plates have long-slotted holes in the direction of the load, as shown in Figure J.4B-1.

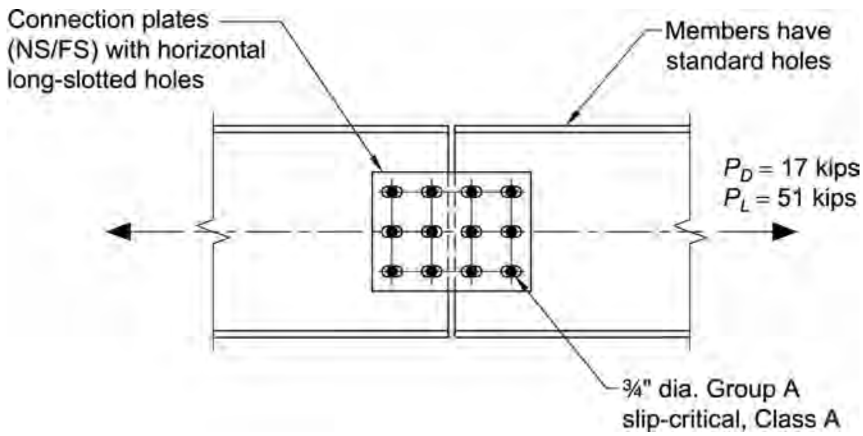


Fig. J.4B-1. Geometry and loading for Example J.4B.

Solution:

The required strength from Example J.4A is:

LRFD	ASD
$P_u = 102$ kips	$P_a = 68.0$ kips

From AISC *Specification* Section J3.8(c), the available slip resistance for the limit state of slip for long-slotted holes is determined as follows:

ϕ = 0.70
Ω = 2.14
μ = 0.30 for Class A surface
D_u = 1.13
h_f = 1.0, no filler is provided
T_b = 28 kips, from AISC *Specification* Table J3.1, Group A
n_s = 2, number of slip planes

$$R_n = \mu D_u h_f T_b n_s \qquad \text{(Spec. Eq. J3-4)}$$
$$= 0.30(1.13)(1.0)(28 \text{ kips})(2)$$
$$= 19.0 \text{ kips/bolt}$$

The available slip resistance is:

LRFD	ASD
$\phi R_n = 0.70(19.0 \text{ kips/bolt})$ $= 13.3 \text{ kips/bolt}$	$\dfrac{R_n}{\Omega} = \dfrac{19.0 \text{ kips/bolt}}{2.14}$ $= 8.88 \text{ kips/bolt}$

Required Number of Bolts

LRFD	ASD
$n_b = \dfrac{P_u}{\phi R_n}$ $= \dfrac{102 \text{ kips}}{13.3 \text{ kips/bolt}}$ $= 7.67 \text{ bolts}$ Use 8 bolts	$n_b = \dfrac{P_a}{\left(\dfrac{R_n}{\Omega}\right)}$ $= \dfrac{68.0 \text{ kips}}{8.88 \text{ kips/bolt}}$ $= 7.66 \text{ bolts}$ Use 8 bolts

Note: To complete the verification of this connection, the limit states of bolt shear, bearing, tearout, tensile yielding, tensile rupture, and block shear rupture must be determined.

EXAMPLE J.5 COMBINED TENSION AND SHEAR IN A SLIP-CRITICAL CONNECTION

Because the pretension of a bolt in a slip-critical connection is used to create the clamping force that produces the shear strength of the connection, the available shear strength must be reduced for any load that produces tension in the connection.

Given:

The slip-critical bolt group shown in Figure J.5-1 is subjected to tension and shear. This example shows the design for bolt slip resistance only, and assumes that the beams and plates are adequate to transmit the loads. Determine if the bolts are adequate.

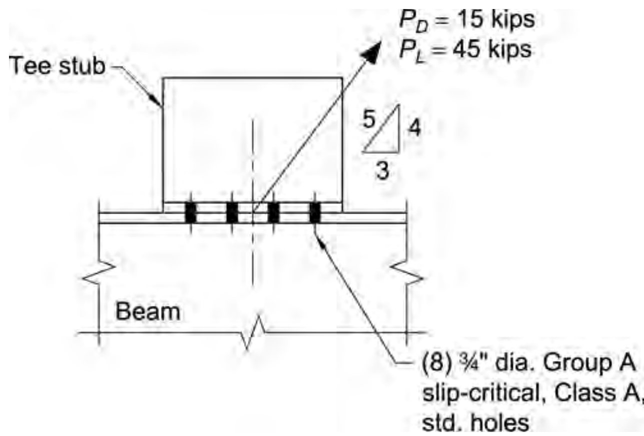


Fig. J.5-1. Geometry and loading for Example J.5.

Solution:

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$P_u = 1.2(15\text{ kips}) + 1.6(45\text{ kips})$ $= 90.0\text{ kips}$ By geometry: $T_u = \frac{4}{5}(90.0\text{ kips})$ $= 72.0\text{ kips}$ $V_u = \frac{3}{5}(90.0\text{ kips})$ $= 54.0\text{ kips}$	$P_a = 15\text{ kips} + 45\text{ kips}$ $= 60.0\text{ kips}$ By geometry: $T_a = \frac{4}{5}(60.0\text{ kips})$ $= 48.0\text{ kips}$ $V_a = \frac{3}{5}(60.0\text{ kips})$ $= 36.0\text{ kips}$

Available Bolt Tensile Strength

The available tensile strength is determined from AISC *Specification* Section J3.6.

From AISC *Specification* Table J3.2 for Group A bolts, the nominal tensile strength in ksi is, F_{nt} = 90 ksi. From AISC *Manual* Table 7-1, for a ¾-in.-diameter bolt:

$A_b = 0.442 \text{ in.}^2$

The nominal tensile strength is:

$$R_n = F_{nt} A_b \qquad \text{(from } \textit{Spec}\text{. Eq. J3-1)}$$
$$= (90 \text{ ksi})(0.442 \text{ in.}^2)$$
$$= 39.8 \text{ kips}$$

The available tensile strength is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(39.8 \text{ kips/bolt}) > \dfrac{72.0 \text{ kips}}{8 \text{ bolts}}$ $= 29.9 \text{ kips/bolt} > 9.00 \text{ kips/bolt}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{39.8 \text{ kips/bolt}}{2.00} > \dfrac{48.0 \text{ kips}}{8 \text{ bolts}}$ $= 19.9 \text{ kips/bolt} > 6.00 \text{ kips/bolt}$ **o.k.**

Note that the available tensile strength per bolt can also be taken from AISC *Manual* Table 7-2.

Available Slip Resistance per Bolt

The available slip resistance for one bolt in standard size holes is determined using AISC *Specification* Section J3.8(a):

$\phi = 1.00$
$\Omega = 1.50$
$\mu = 0.30$ for Class A surface
$D_u = 1.13$
$h_f = 1.0$, factor for fillers, assuming no more than one filler
$T_b = 28$ kips, from AISC *Specification* Table J3.1, Group A
$n_s = 1$, number of slip planes

LRFD	ASD
Determine the available slip resistance ($T_u = 0$) of a bolt: $\phi R_n = \phi \mu D_u h_f T_b n_s$ (from *Spec*. Eq. J3-4) $= 1.00(0.30)(1.13)(1.0)(28 \text{ kips})(1)$ $= 9.49 \text{ kips/bolt}$	Determine the available slip resistance ($T_a = 0$) of a bolt: $\dfrac{R_n}{\Omega} = \dfrac{\mu D_u h_f T_b n_s}{\Omega}$ (from *Spec*. Eq. J3-4) $= \dfrac{0.30(1.13)(1.0)(28 \text{ kips})(1)}{1.50}$ $= 6.33 \text{ kips/bolt}$

Note that the available slip resistance for one bolt with a Class A faying surface can also be taken from AISC *Manual* Table 7-3.

Available Slip Resistance of the Connection

Because the slip-critical connection is subject to combined tension and shear, the available slip resistance is multiplied by a reduction factor provided in AISC *Specification* Section J3.9.

LRFD	ASD
Slip-critical combined tension and shear factor: $k_{sc} = 1 - \dfrac{T_u}{D_u T_b n_b} \geq 0$ (*Spec*. Eq. J3-5a) $= 1 - \dfrac{72.0\text{ kips}}{1.13(28\text{ kips})(8)} > 0$ $= 0.716$ $\phi R_n = \phi R_n k_{sc} n_b$ $= (9.49\text{ kips/bolt})(0.716)(8\text{ bolts})$ $= 54.4\text{ kips} > 54.0\text{ kips}$ **o.k.**	Slip-critical combined tension and shear factor: $k_{sc} = 1 - \dfrac{1.5T_a}{D_u T_b n_b} \geq 0$ (*Spec*. Eq. J3-5b) $= 1 - \dfrac{1.5(48.0\text{ kips})}{1.13(28\text{ kips})(8)} > 0$ $= 0.716$ $\dfrac{R_n}{\Omega} = \dfrac{R_n}{\Omega} k_{sc} n_b$ $= (6.33\text{ kips/bolt})(0.716)(8\text{ bolts})$ $= 36.3\text{ kips} > 36.0\text{ kips}$ **o.k.**

Note: The bolt group must still be checked for all applicable strength limit states for a bearing-type connection.

Chapter IIA
Simple Shear Connections

The design of connecting elements are covered in Part 9 of the AISC *Manual*. The design of simple shear connections is covered in Part 10 of the AISC *Manual*.

EXAMPLE II.A-1A ALL-BOLTED DOUBLE-ANGLE CONNECTION

Given:

Using the tables in AISC *Manual* Part 10, verify the available strength of an all-bolted double-angle shear connection between an ASTM A992 W36×231 beam and an ASTM A992 W14×90 column flange, as shown in Figure IIA-1A-1, supporting the following beam end reactions:

R_D = 37.5 kips
R_L = 113 kips

Use ASTM A36 angles.

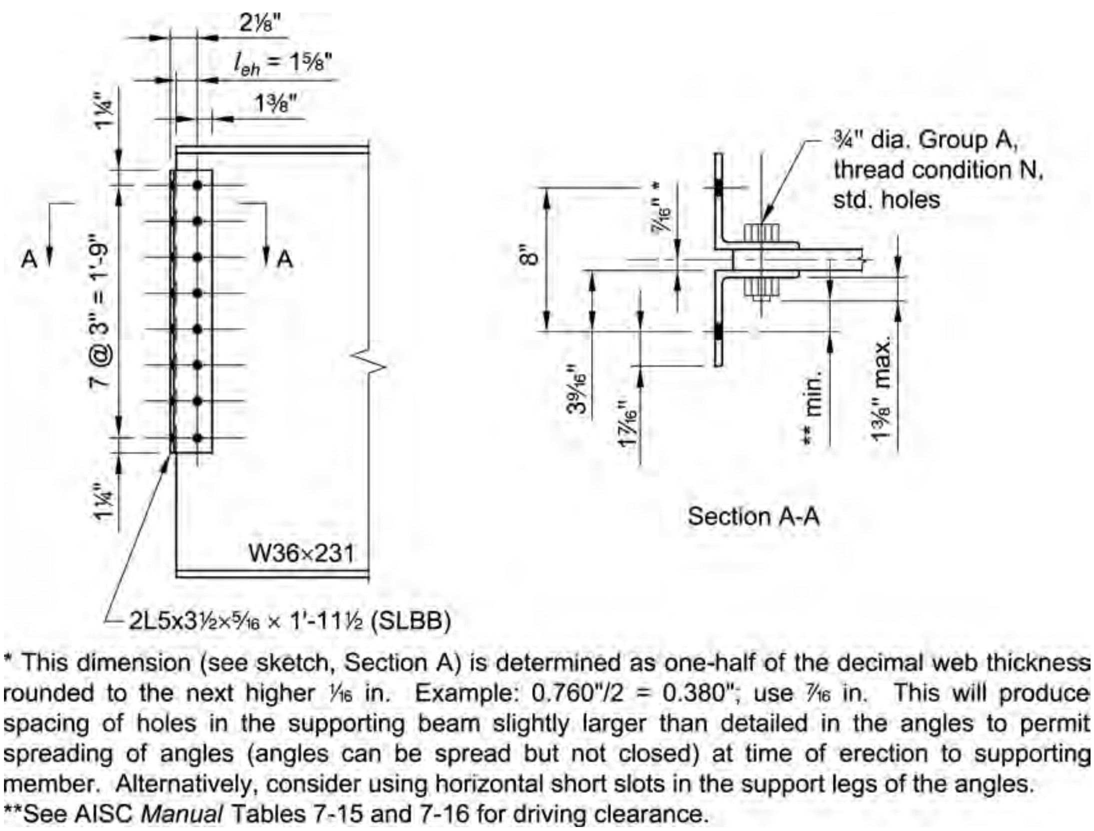


Fig. IIA-1A-1. Connection geometry for Example II.A-1A.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam and column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W36×231
t_w = 0.760 in.

Column
W14×90
t_f = 0.710 in.

From AISC *Specification* Table J3.3, the hole diameter for a ¾-in.-diameter bolt with standard holes is:

$$d_h = {}^{13}\!/_{16} \text{ in.}$$

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(37.5 \text{ kips}) + 1.6(113 \text{ kips})$ $= 226 \text{ kips}$	$R_a = 37.5 \text{ kips} + 113 \text{ kips}$ $= 151 \text{ kips}$

Connection Selection

AISC *Manual* Table 10-1 includes checks for the limit states of bolt shear, bolt bearing and tearout on the angles, shear yielding of the angles, shear rupture of the angles, and block shear rupture of the angles.

Try 8 rows of bolts and 2L5×3½×5⁄16 (SLBB). From AISC *Manual* Table 10-1:

LRFD	ASD
$\phi R_n = 248 \text{ kips} > 226 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 165 \text{ kips} > 151 \text{ kips}$ **o.k.**

Available Beam Web Strength

The available beam web strength is the lesser of the limit states of block shear rupture, shear yielding, shear rupture, and the sum of the effective strengths of the individual fasteners. Because the beam is not coped, the only applicable limit state is the effective strength of the individual fasteners, which is the lesser of the bolt shear strength per AISC *Specification* Section J3.6, and the bolt bearing and tearout strength per AISC *Specification* Section J3.10.

Bolt Shear

From AISC *Manual* Table 7-1, the available shear strength per bolt for ¾-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) in double shear is:

LRFD	ASD
$\phi R_n = 35.8 \text{ kips/bolt}$	$\dfrac{R_n}{\Omega} = 23.9 \text{ kips/bolt}$

Bolt Bearing on Beam Web

The nominal bearing strength of the beam web per bolt is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration:

$$r_n = 2.4dtF_u \qquad \text{(}Spec\text{. Eq. J3-6a)}$$
$$= 2.4(\tfrac{3}{4}\text{ in.})(0.760\text{ in.})(65\text{ ksi})$$
$$= 88.9\text{ kips/bolt}$$

From AISC *Specification* Section J3.10, the available bearing strength of the beam web per bolt is:

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(88.9\text{ kips/bolt})$ $= 66.7\text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{88.9\text{ kips/bolt}}{2.00}$ $= 44.5\text{ kips/bolt}$

Bolt Tearout on Beam Web

The available tearout strength of the beam web per bolt is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration:

$$l_c = 3.00\text{ in.} - \tfrac{13}{16}\text{ in.}$$
$$= 2.19\text{ in.}$$

The available tearout strength is:

$$r_n = 1.2l_ctF_u \qquad \text{(}Spec\text{. Eq. J3-6c)}$$
$$= 1.2(2.19\text{ in.})(0.760\text{ in.})(65\text{ ksi})$$
$$= 130\text{ kips/bolt}$$

From AISC *Specification* Section J3.10, the available tearout strength of the beam web per bolt is:

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(130\text{ kips/bolt})$ $= 97.5\text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{130\text{ kips/bolt}}{2.00}$ $= 65.0\text{ kips/bolt}$

Bolt shear strength is the governing limit state for all bolts at the beam web. Bolt shear strength is one of the limit states included in the capacities shown in Table 10-1 as used above; thus, the effective strength of the fasteners is adequate.

Available Strength at the Column Flange

Since the thickness of the column flange, t_f = 0.710 in., is greater than the thickness of the angles, t = $\tfrac{5}{16}$ in., bolt bearing will control for the angles, which was previously checked. The column flange is adequate for the required loading.

Conclusion

The connection is found to be adequate as given for the applied loads.

EXAMPLE II.A-1B ALL-BOLTED DOUBLE-ANGLE CONNECTION SUBJECT TO AXIAL AND SHEAR LOADING

Given:

Verify the available strength of an all-bolted double-angle connection for an ASTM A992 W18×50 beam, as shown in Figure II.A-1B-1, to support the following beam end reactions:

LRFD	ASD
Shear, V_u = 75 kips Axial tension, N_u = 60 kips	Shear, V_a = 50 kips Axial tension, N_a = 40 kips

Use ASTM A36 double angles that will be shop-bolted to the beam.

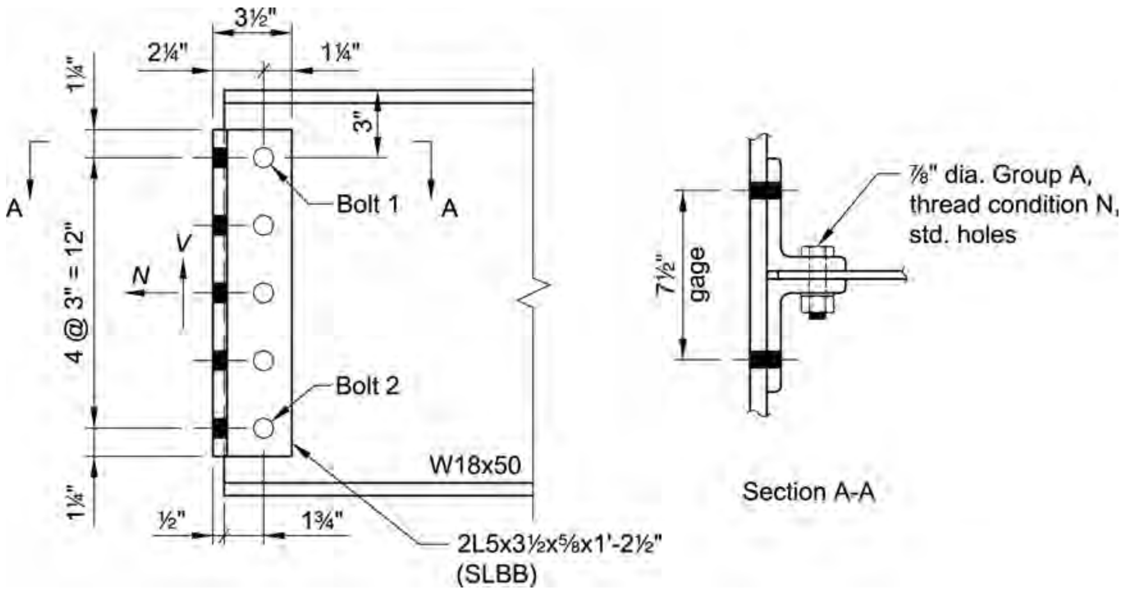


Fig. II.A-1B-1. Connection geometry for Example II.A-1B.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W18×50
A_g = 14.7 in.^{2}
d = 18.0 in.
t_w = 0.355 in.
t_f = 0.570 in.

From AISC *Specification* Table J3.3, the hole diameter for ⅞-in.-diameter bolts with standard holes is:

d_h = 15⁄16 in.

The resultant load is:

LRFD	ASD
$R_u = \sqrt{V_u^2 + N_u^2}$ $= \sqrt{(75 \text{ kips})^2 + (60 \text{ kips})^2}$ $= 96.0 \text{ kips}$	$R_a = \sqrt{V_a^2 + N_a^2}$ $= \sqrt{(50 \text{ kips})^2 + (40 \text{ kips})^2}$ $= 64.0 \text{ kips}$

Try 5 rows of bolts and 2L5×3½×⅝ (SLBB).

Strength of the Bolted Connection—Angles

From the Commentary to AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the invividual strengths of the individual fasteners, which may be taken as the lesser of the fastener shear strength per AISC *Specification* Section J3.6, the bearing strength at the bolt hole per AISC *Specification* Section J3.10, or the tearout strength at the bolt hole per AISC *Specification* Section J3.10.

Bolt shear

From AISC *Manual* Table 7-1, the available shear strength for ⅞-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) in double shear (or pair of bolts) is:

LRFD	ASD
$\phi r_n = 48.7$ kips/bolt (or per pair of bolts)	$\dfrac{r_n}{\Omega} = 32.5$ kips/bolt (or per pair of bolts)

Bolt bearing on angles

The available bearing strength of the angles per bolt in double shear is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration:

$$r_n = (2 \text{ angles})2.4dtF_u$$ (from *Spec.* Eq. J3-6a)

$$= (2 \text{ angles})(2.4)(\tfrac{7}{8} \text{ in.})(\tfrac{5}{8} \text{ in.})(58 \text{ ksi})$$

$$= 152 \text{ kips/bolt}$$

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(152 \text{ kips/bolt})$ $= 114 \text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{152 \text{ kips/bolt}}{2.00}$ $= 76.0 \text{ kips/bolt}$

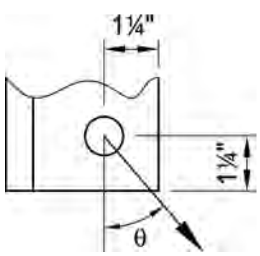
Bolt tearout on angles

From AISC *Specification* Section J3.10, the available tearout strength of the angles per bolt in double shear is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration.

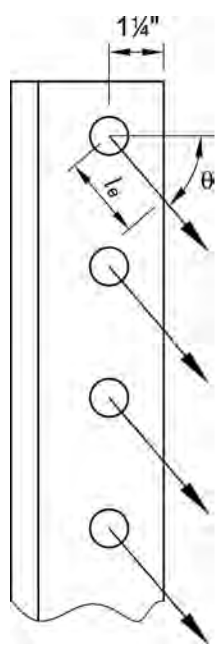
As shown in Figures II.A-1B-2(a) and II.A-1B-2(b), the tearout dimensions on the angle differ between the edge bolt and the other bolts.

The angle θ, as shown in Figure II.A-1B-2(a), of the resultant force on the edge bolt is:

LRFD	ASD
$\theta = \tan^{-1}\left(\dfrac{N_u}{V_u}\right)$ $= \tan^{-1}\left(\dfrac{60 \text{ kips}}{75 \text{ kips}}\right)$ $= 38.7°$	$\theta = \tan^{-1}\left(\dfrac{N_a}{V_a}\right)$ $= \tan^{-1}\left(\dfrac{40 \text{ kips}}{50 \text{ kips}}\right)$ $= 38.7°$



(a) Edge bolt



(b) Other bolts

Fig. II.A-1B-2. Bolt tearout on angles.

The length from the center of the bolt hole to the edge of the angle along the line of action of the force is:

$$l_e = \frac{1\tfrac{1}{4}\text{ in.}}{\cos 38.7°}$$
$$= 1.60\text{ in.}$$

The clear distance, along the line of action of the force, between the edge of the hole and the edge of the angle is:

$$l_c = l_e - 0.5d_h$$
$$= 1.60\text{ in.} - 0.5\left(\tfrac{15}{16}\text{ in.}\right)$$
$$= 1.13\text{ in.}$$

The available tearout strength of the pair of angles at the edge bolt is:

$$r_n = (2\text{ angles})1.2l_c tF_u \qquad \text{(from } \textit{Spec}\text{. Eq. J3-6c)}$$
$$= (2\text{ angles})(1.2)(1.13\text{ in.})\left(\tfrac{5}{8}\text{ in.}\right)(58\text{ ksi})$$
$$= 98.3\text{ kips/bolt}$$

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(98.3\text{ kips/bolt})$ $= 73.7\text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{98.3\text{ kips/bolt}}{2.00}$ $= 49.2\text{ kips/bolt}$

Therefore, bolt shear controls over bearing or tearout of the angles at the edge bolt.

The angle θ, as shown in Figure II.A-1B-2(b), of the resultant force on the other bolts is:

LRFD	ASD
$\theta = \tan^{-1}\left(\dfrac{V_u}{N_u}\right)$ $= \tan^{-1}\left(\dfrac{75\text{ kips}}{60\text{ kips}}\right)$ $= 51.3°$	$\theta = \tan^{-1}\left(\dfrac{V_a}{N_a}\right)$ $= \tan^{-1}\left(\dfrac{50\text{ kips}}{40\text{ kips}}\right)$ $= 51.3°$

The length from the center of the bolt hole to the edge of the angle along the line of action of the force is:

$$l_e = \frac{1\tfrac{1}{4}\text{ in.}}{\cos 51.3°}$$
$$= 2.00\text{ in.}$$

The clear distance, along the line of action of the force, between the edge of the hole and the edge of the angle is:

$$l_c = l_e - 0.5d_h$$
$$= 2.00\text{ in.} - 0.5\left(\tfrac{15}{16}\text{ in.}\right)$$
$$= 1.53\text{ in.}$$

The available tearout strength of the pair of angles at the other bolts is:

$$r_n = (2 \text{ angles})1.2 l_c t F_u$$ (from *Spec.* Eq. J3-6c)

$$= (2 \text{ angles})(1.2)(1.53 \text{ in.})(\tfrac{5}{8} \text{ in.})(58 \text{ ksi})$$

$$= 133 \text{ kips/bolt}$$

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(133 \text{ kips/bolt})$ $= 99.8 \text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{133 \text{ kips/bolt}}{2.00}$ $= 66.5 \text{ kips/bolt}$

Therefore, bolt shear controls over bearing or tearout of the angles at the other bolt.

The effective strength for the bolted connection at the angles is determined by summing the effective strength for each bolt using the minimum available strength calculated for bolt shear, bearing on the angles, and tearout on the angles.

LRFD	ASD
$\phi R_n = n\phi r_n$ $= (5 \text{ bolts})(48.7 \text{ kips/bolt})$ $= 244 \text{ kips} > 96.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = n\dfrac{r_n}{\Omega}$ $= (5 \text{ bolts})(32.5 \text{ kips/bolt})$ $= 163 \text{ kips} > 64.0 \text{ kips}$ **o.k.**

Strength of the Bolted Connection—Beam Web

Bolt bearing on beam web

The available bearing strength of the beam web per bolt is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration:

$$r_n = 2.4 d t F_u$$ (*Spec.* Eq. J3-6a)

$$= 2.4(\tfrac{7}{8} \text{ in.})(0.355 \text{ in.})(65 \text{ ksi})$$

$$= 48.5 \text{ kips/bolt}$$

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(48.5 \text{ kips/bolt})$ $= 36.4 \text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{48.5 \text{ kips/bolt}}{2.00}$ $= 24.3 \text{ kips/bolt}$

Bolt tearout on beam web

From AISC *Specification* Section J3.10, the available tearout strength of the beam web is determined from AISC *Specification* Equation J3-6a, assuming deformation at the bolt hole is a design consideration, where the edge distance, l_c, is based on the angle of the resultant load. As shown in Figure II.A-1B-3, a horizontal edge distance of 1½ in. is used which includes a ¼ in. tolerance to account for possible mill underrun.

The angle, θ, of the resultant force is:

LRFD	ASD
$\theta = \tan^{-1}\left(\frac{V_u}{N_u}\right)$ $= \tan^{-1}\left(\frac{75 \text{ kips}}{60 \text{ kips}}\right)$ $= 51.3°$	$\theta = \tan^{-1}\left(\frac{V_a}{N_a}\right)$ $= \tan^{-1}\left(\frac{50 \text{ kips}}{40 \text{ kips}}\right)$ $= 51.3°$

The length from the center of the bolt hole to the edge of the web along the line of action of the force is:

$$
\begin{aligned}
l_e &= \frac{1\frac{1}{2} \text{ in.}}{\cos 51.3°} \\
&= 2.40 \text{ in.}
\end{aligned}
$$

The clear distance, along the line of action of the force, between the edge of the hole and the edge of the web is:

$$
\begin{aligned}
l_c &= l_e - 0.5d_h \\
&= 2.40 \text{ in.} - 0.5\left(\tfrac{15}{16} \text{ in.}\right) \\
&= 1.93 \text{ in.}
\end{aligned}
$$

The available tearout strength of the beam web is determined as follows:

$$
\begin{aligned}
r_n &= 1.2 l_c t F_u \qquad \text{(}\textit{Spec.}\text{ Eq. J3-6c)} \\
&= 1.2(1.93 \text{ in.})(0.355 \text{ in.})(65 \text{ ksi}) \\
&= 53.4 \text{ kips/bolt}
\end{aligned}
$$

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(53.4 \text{ kips/bolt})$ $= 40.1 \text{ kips/bolt}$	$\Omega = 2.00$ $\frac{r_n}{\Omega} = \frac{53.4 \text{ kips/bolt}}{2.00}$ $= 26.7 \text{ kips/bolt}$

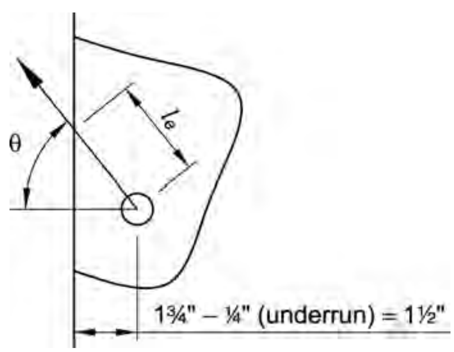


Fig. II.A-1B-3. Bolt tearout on beam web.

Therefore, bolt bearing on the beam web is the controlling limit state for all bolts.

The effective strength for the bolted connection at the beam web is determined by summing the effective strength for each bolt using the minimum available strength calculated for bolt shear, bearing on the beam web, and tearout on the beam web.

LRFD	ASD
$\phi R_n = n\phi r_n$ $= (5\text{ bolts})(36.4\text{ kips/bolt})$ $= 182\text{ kips} > 96.0\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = n\dfrac{r_n}{\Omega}$ $= (5\text{ bolts})(24.3\text{ kips/bolt})$ $= 122\text{ kips} > 64.0\text{ kips}$ **o.k.**

Bolt Shear and Tension Interaction—Outstanding Angle Legs

The available tensile strength of the bolts due to the effect of combined tension and shear is determined from AISC *Specification* Section J3.7.

The required shear stress is:

$$f_{rv} = \frac{V_r}{nA_b}$$

where

$A_b = 0.601\text{ in.}^2$ (from AISC *Manual* Table 7-1)

$n = 10$

LRFD	ASD
$f_{rv} = \dfrac{V_u}{nA_b}$ $= \dfrac{75\text{ kips}}{10\left(0.601\text{ in.}^2\right)}$ $= 12.5\text{ ksi}$	$f_{rv} = \dfrac{V_a}{nA_b}$ $= \dfrac{50\text{ kips}}{10\left(0.601\text{ in.}^2\right)}$ $= 8.32\text{ ksi}$

The nominal tensile strength modified to include the effects of shear stress is determined from AISC *Specification* Section J3.7 as follows. From AISC *Specification* Table J3.2:

$F_{nt} = 90\text{ ksi}$

$F_{nv} = 54\text{ ksi}$

LRFD	ASD
$\phi = 0.75$ $F'_{nt} = 1.3F_{nt} - \dfrac{F_{nt}}{\phi F_{nv}} f_{rv} \le F_{nt}$ (*Spec*. Eq. J3-3a) $= 1.3(90\text{ ksi}) - \dfrac{90\text{ ksi}}{0.75(54\text{ ksi})}(12.5\text{ ksi}) < 90\text{ ksi}$ $= 89.2\text{ ksi} < 90\text{ ksi}$	$\Omega = 2.00$ $F'_{nt} = 1.3F_{nt} - \dfrac{\Omega F_{nt}}{F_{nv}} f_{rv} \le F_{nt}$ (*Spec*. Eq. J3-3b) $= 1.3(90\text{ ksi}) - \dfrac{2.00(90\text{ ksi})}{54\text{ ksi}}(8.32\text{ ksi}) < 90\text{ ksi}$ $= 89.3\text{ ksi} < 90\text{ ksi}$

LRFD	ASD
Therefore: $F'_{nt} = 89.2$ ksi	Therefore: $F'_{nt} = 89.3$ ksi

Using the value of F'_{nt} determined for LRFD, the nominal tensile strength of one bolt is:

$$\begin{aligned} r_n &= F'_{nt} A_b \\ &= (89.2 \text{ ksi})(0.601 \text{ in.}^2) \\ &= 53.6 \text{ kips} \end{aligned}$$ (from *Spec*. Eq. J3-2)

The available tensile strength of the bolts due to combined tension and shear is:

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi r_n = 0.75(53.6 \text{ kips/bolt})$ $= 40.2$ kips	$\dfrac{r_n}{\Omega} = \dfrac{53.6 \text{ kips/bolt}}{2.00}$ $= 26.8$ kips
$\phi R_n = n\phi r_n$ $= (10 \text{ bolts})(40.2 \text{ kips/bolt})$ $= 402 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = n\dfrac{r_n}{\Omega}$ $= (10 \text{ bolts})(26.8 \text{ kips/bolt})$ $= 268 \text{ kips} > 40 \text{ kips}$ **o.k.**

Prying Action

From AISC *Manual* Part 9, the available tensile strength of the bolts in the outstanding angle legs taking prying action into account is determined as follows:

$$\begin{aligned} a &= \frac{2(angle\ leg) + t_w - gage}{2} \\ &= \frac{2(5 \text{ in.}) + 0.355 \text{ in.} - 7\tfrac{1}{2} \text{ in.}}{2} \\ &= 1.43 \text{ in.} \end{aligned}$$

$$\begin{aligned} b &= \frac{gage - t_w - t}{2} \\ &= \frac{7\tfrac{1}{2} \text{ in.} - 0.355 \text{ in.} - \tfrac{5}{8} \text{ in.}}{2} \\ &= 3.26 \text{ in.} \end{aligned}$$

$$\begin{aligned} a' &= \left(a + \frac{d_b}{2}\right) \le \left(1.25b + \frac{d_b}{2}\right) \\ &= 1.43 \text{ in.} + \frac{\tfrac{7}{8} \text{ in.}}{2} \le 1.25(3.26 \text{ in.}) + \frac{\tfrac{7}{8} \text{ in.}}{2} \\ &= 1.87 \text{ in.} < 4.51 \text{ in.} \quad \textbf{o.k.} \end{aligned}$$ (*Manual* Eq. 9-23)

$$b' = \left(b - \frac{d_b}{2}\right) \qquad \text{(\textit{Manual} Eq. 9-18)}$$

$$= 3.26 \text{ in.} - \frac{7/8 \text{ in.}}{2}$$

$$= 2.82 \text{ in.}$$

$$\rho = \frac{b'}{a'} \qquad \text{(\textit{Manual} Eq. 9-22)}$$

$$= \frac{2.82 \text{ in.}}{1.87 \text{ in.}}$$

$$= 1.51$$

Note that end distances of 1¼ in. are used on the angles, so p is the average pitch of the bolts:

$$p = \frac{l}{n}$$

$$= \frac{14\tfrac{1}{2} \text{ in.}}{5 \text{ rows}}$$

$$= 2.90 \text{ in.}$$

Check:

$p < s = 3.00$ in. **o.k.**

$$\delta = 1 - \frac{d'}{p} \qquad \text{(\textit{Manual} Eq. 9-20)}$$

$$= 1 - \frac{15/16 \text{ in.}}{2.90 \text{ in.}}$$

$$= 0.677$$

The angle thickness required to develop the available strength of the bolt with no prying action is determined as follows:

LRFD	ASD
$\phi = 0.90$ $B_c = 40.2$ kips/bolt (calculated previously) $t_c = \sqrt{\dfrac{4B_c b'}{\phi p F_u}}$ (*Manual* Eq. 9-26a) $= \sqrt{\dfrac{4(40.2 \text{ kips/bolt})(2.82 \text{ in.})}{0.90(2.90 \text{ in.})(58 \text{ ksi})}}$ $= 1.73$ in.	$\Omega = 1.67$ $B_c = 26.8$ kips/bolt (calculated previously) $t_c = \sqrt{\dfrac{\Omega 4B_c b'}{p F_u}}$ (*Manual* Eq. 9-26b) $= \sqrt{\dfrac{1.67(4)(26.8 \text{ kips/bolt})(2.82 \text{ in.})}{(2.90 \text{ in.})(58 \text{ ksi})}}$ $= 1.73$ in.

$$\alpha' = \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2 - 1\right] \qquad \text{(\textit{Manual} Eq. 9-28)}$$

$$= \frac{1}{0.677(1+1.51)}\left[\left(\frac{1.73 \text{ in.}}{5/8 \text{ in.}}\right)^2 - 1\right]$$

$$= 3.92$$

Because $\alpha' > 1$, the angles have insufficient strength to develop the bolt strength, therefore:

$$Q = \left(\frac{t}{t_c}\right)^2 (1+\delta)$$

$$= \left(\frac{5/8 \text{ in.}}{1.73 \text{ in.}}\right)^2 (1+0.677)$$

$$= 0.219$$

The available tensile strength of the bolts, taking prying action into account, is determined using AISC *Manual* Equation 9-27, as follows:

LRFD	ASD
$\phi r_n = B_c Q$ $= (40.2 \text{ kips/bolt})(0.219)$ $= 8.80 \text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = B_c Q$ $= (26.8 \text{ kips/bolt})(0.219)$ $= 5.87 \text{ kips/bolt}$
$\phi R_n = n\phi r_n$ $= (10 \text{ bolts})(8.80 \text{ kips/bolt})$ $= 88.0 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = n\dfrac{r_n}{\Omega}$ $= (10 \text{ bolts})(5.87 \text{ kips/bolt})$ $= 58.7 \text{ kips} > 40 \text{ kips}$ **o.k.**

Shear Strength of Angles

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the angles is determined as follows:

$$A_{gv} = (2 \text{ angles})lt$$

$$= (2 \text{ angles})(14\tfrac{1}{2} \text{ in.})(5/8 \text{ in.})$$

$$= 18.1 \text{ in.}^2$$

$$R_n = 0.60 F_y A_{gv} \qquad \text{(\textit{Spec.} Eq. J4-3)}$$

$$= 0.60(36 \text{ ksi})(18.1 \text{ in.}^2)$$

$$= 391 \text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(391 \text{ kips})$ $= 391 \text{ kips} > 96.0 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{391 \text{ kips}}{1.50}$ $= 261 \text{ kips} > 64.0 \text{ kips}$ **o.k.**

From AISC *Specification* Section J4.2, the available shear rupture strength of the angle is determined using the net area determined in accordance with AISC *Specification* Section B4.3b.

$$
\begin{aligned}
A_{nv} &= (2 \text{ angles})\left[l - n\left(d_h + \tfrac{1}{16} \text{ in.}\right)\right]t \\
&= (2 \text{ angles})\left[14\tfrac{1}{2} \text{ in.} - 5\left(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.}\right)\right]\left(\tfrac{5}{8} \text{ in.}\right) \\
&= 11.9 \text{ in.}^2
\end{aligned}
$$

$$
\begin{aligned}
R_n &= 0.60 F_u A_{nv} \qquad \text{(}\textit{Spec}\text{. Eq. J4-4)} \\
&= 0.60(58 \text{ ksi})\left(11.9 \text{ in.}^2\right) \\
&= 414 \text{ kips}
\end{aligned}
$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(414 \text{ kips})$ $= 311 \text{ kips} > 96.0 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{414 \text{ kips}}{2.00}$ $= 207 \text{ kips} > 64.0 \text{ kips}$ **o.k.**

Tensile Strength of Angles

From AISC Specification Section J4.1(a), the available tensile yielding strength of the angles is determined as follows:

$$
\begin{aligned}
A_g &= (2 \text{ angles})lt \\
&= (2 \text{ angles})\left(14\tfrac{1}{2} \text{ in.}\right)\left(\tfrac{5}{8} \text{ in.}\right) \\
&= 18.1 \text{ in.}^2
\end{aligned}
$$

$$
\begin{aligned}
R_n &= F_y A_g \qquad \text{(}\textit{Spec}\text{. Eq. J4-1)} \\
&= (36 \text{ ksi})\left(18.1 \text{ in.}^2\right) \\
&= 652 \text{ kips}
\end{aligned}
$$

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(652 \text{ kips})$ $= 587 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{652 \text{ kips}}{1.67}$ $= 390 \text{ kips} > 40 \text{ kips}$ **o.k.**

From AISC *Specification* Sections J4.1, the available tensile rupture strength of the angles is determined from AISC *Specification* Equation J4-2. Table D3.1, Case 1 applies in this case because the tension load is transmitted directly to the cross-sectional element by fasteners; therefore, U = 1.00. With $A_{nt} = A_{nv}$ (calculated previously), the effective net area is:

$$A_e = A_{nt}U \qquad \text{(Spec. Eq. D3-1)}$$
$$= (11.9\text{ in.}^2)(1.00)$$
$$= 11.9\text{ in.}^2$$

$$R_n = F_u A_e \qquad \text{(Spec. Eq. J4-2)}$$
$$= (58\text{ ksi})(11.9\text{ in.}^2)$$
$$= 690\text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(690\text{ kips})$ $= 518\text{ kips} > 60\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{690\text{ kips}}{2.00}$ $= 345\text{ kips} > 40\text{ kips}$ **o.k.**

Block Shear Rupture of Angles—Beam Web Side

The nominal strength for the limit state of block shear rupture of the angles, assuming an L-shaped tearout due the shear load only, is determined as follows. The tearout pattern is shown in Figure II.A-1B-4.

$$R_{bsv} = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \qquad \text{(Spec. Eq. J4-5)}$$

where

$$A_{gv} = (2\text{ angles})(l - l_{ev})t$$
$$= (2\text{ angles})(14\tfrac{1}{2}\text{ in.} - 1\tfrac{1}{4}\text{ in.})(\tfrac{5}{8}\text{ in.})$$
$$= 16.6\text{ in.}^2$$

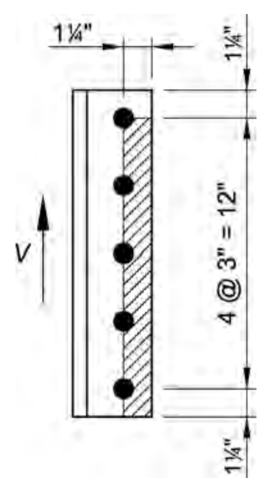


Fig. II.A-1B-4. Block shear rupture of angles for shear load only.

$$A_{nv} = A_{gv} - (2\text{ angles})(n - 0.5)(d_h + \tfrac{1}{16}\text{ in.})t$$
$$= 16.6\text{ in.}^2 - (2\text{ angles})(5 - 0.5)(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(\tfrac{5}{8}\text{ in.})$$
$$= 11.0\text{ in.}^2$$

$$A_{nt} = (2\text{ angles})\left[l_{eh} - 0.5(d_h + \tfrac{1}{16}\text{ in.})\right]t$$
$$= (2\text{ angles})\left[1\tfrac{1}{4}\text{ in.} - 0.5(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right](\tfrac{5}{8}\text{ in.})$$
$$= 0.938\text{ in.}^2$$

$$U_{bs} = 1.0$$

and

$$R_{bsv} = 0.60(58\text{ ksi})(11.0\text{ in.}^2) + 1.0(58\text{ ksi})(0.938\text{ in.}^2) \le 0.60(36\text{ ksi})(16.6\text{ in.}^2) + 1.0(58\text{ ksi})(0.938\text{ in.}^2)$$
$$= 437\text{ kips} > 413\text{ kips}$$

Therefore:

$$R_{bsv} = 413\text{ kips}$$

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture on the angles is:

LRFD	ASD
$\phi = 0.75$ $\phi R_{bsv} = 0.75(413\text{ kips})$ $= 310\text{ kips} > 75\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_{bsv}}{\Omega} = \dfrac{413\text{ kips}}{2.00}$ $= 207\text{ kips} > 50\text{ kips}$ **o.k.**

The block shear rupture failure path due to axial load only could occur as an L- or U-shape. Assuming an L-shaped tearout relative to the axial load on the angles, the nominal block shear rupture strength in the angles is determined as follows. The tearout pattern is shown in Figure II.A-1B-5.

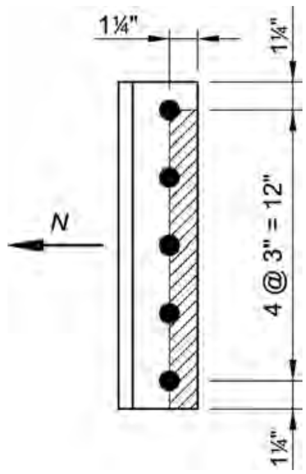


Fig. II.A-1B-5. Block shear rupture of angles for axial load only—L-shape.

$$R_{bsn} = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \quad \text{(}\textit{Spec}\text{. Eq. J4-5)}$$

where

$$\begin{aligned} A_{gv} &= (2 \text{ angles}) l_{eh} t \\ &= (2 \text{ angles})(1\tfrac{1}{4} \text{ in.})(\tfrac{5}{8} \text{ in.}) \\ &= 1.56 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{nv} &= A_{gv} - (2 \text{ angles})(0.5)(d_h + \tfrac{1}{16} \text{ in.}) t \\ &= 1.56 \text{ in.}^2 - (2 \text{ angles})(0.5)(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(\tfrac{5}{8} \text{ in.}) \\ &= 0.935 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{nt} &= (2 \text{ angles})\left[(l - l_{ev}) - (n - 0.5)(d_h + \tfrac{1}{16} \text{ in.})\right] t \\ &= (2 \text{ angles})\left[(14\tfrac{1}{2} \text{ in.} - 1\tfrac{1}{4} \text{ in.}) - (5 - 0.5)(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})\right](\tfrac{5}{8} \text{ in.}) \\ &= 10.9 \text{ in.}^2 \end{aligned}$$

$$U_{bs} = 1.0$$

and

$$\begin{aligned} R_{bsn} &= 0.60(58 \text{ ksi})(0.935 \text{ in.}^2) + 1.0(58 \text{ ksi})(10.9 \text{ in.}^2) \le 0.60(36 \text{ ksi})(1.56 \text{ in.}^2) + 1.0(58 \text{ ksi})(10.9 \text{ in.}^2) \\ &= 665 \text{ kips} < 666 \text{ kips} \end{aligned}$$

Therefore:

$$R_{bsn} = 665 \text{ kips}$$

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture on the angles is:

LRFD	ASD
$\phi = 0.75$ $\phi R_{bsn} = 0.75(665 \text{ kips})$ $= 499 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_{bsn}}{\Omega} = \dfrac{665 \text{ kips}}{2.00}$ $= 333 \text{ kips} > 40 \text{ kips}$ **o.k.**

The nominal strength for the limit state of block shear rupture assuming an U-shaped tearout relative to the axial load on the angles is determined as follows. The tearout pattern is shown in Figure II.A-1B-6.

$$R_{bsn} = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \quad \text{(}\textit{Spec}\text{. Eq. J4-5)}$$

where

$$\begin{aligned} A_{gv} &= (2 \text{ angles})(2 \text{ planes}) l_{eh} t \\ &= (2 \text{ angles})(2 \text{ planes})(1\tfrac{1}{4} \text{ in.})(\tfrac{5}{8} \text{ in.}) \\ &= 3.13 \text{ in.}^2 \end{aligned}$$

$$A_{nv} = (2 \text{ angles})(2 \text{ planes})\left[l_{eh} - 0.5\left(d_h + \tfrac{1}{16}\text{ in.}\right)\right]t$$
$$= (2 \text{ angles})(2 \text{ planes})\left[1\tfrac{1}{4}\text{ in.} - 0.5\left(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right]\left(\tfrac{5}{8}\text{ in.}\right)$$
$$= 1.88 \text{ in.}^2$$

$$A_{nt} = (2 \text{ angles})\left[12.0\text{ in.} - (n-1)\left(d_h + \tfrac{1}{16}\text{ in.}\right)\right]t$$
$$= (2 \text{ angles})\left[12.0\text{ in.} - (5-1)\left(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right]\left(\tfrac{5}{8}\text{ in.}\right)$$
$$= 10.0 \text{ in.}^2$$

$$U_{bs} = 1.0$$

and

$$R_{bsn} = 0.60(58 \text{ ksi})\left(1.88 \text{ in.}^2\right) + 1.0(58 \text{ ksi})\left(10.0 \text{ in.}^2\right) \le 0.60(36 \text{ ksi})\left(3.13 \text{ in.}^2\right) + 1.0(58 \text{ ksi})\left(10.0 \text{ in.}^2\right)$$
$$= 645 \text{ kips} < 648 \text{ kips}$$

Therefore:

$$R_{bsn} = 645 \text{ kips}$$

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture on the angles is:

LRFD	ASD
$\phi = 0.75$ $\phi R_{bsn} = 0.75(645 \text{ kips})$ $= 484 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_{bsn}}{\Omega} = \dfrac{645 \text{ kips}}{2.00}$ $= 323 \text{ kips} > 40 \text{ kips}$ **o.k.**

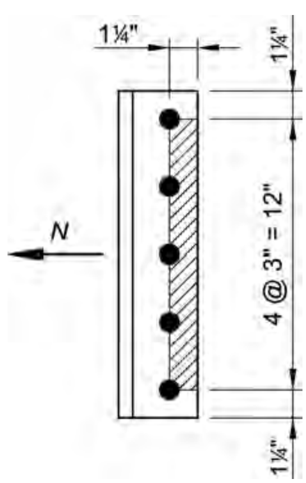


Fig. II.A-1B-6. Block shear rupture of angles for axial load only—U-shape.

Considering the interaction of shear and axial loads, apply a formulation that is similar to AISC *Manual* Equation 10-5:

LRFD	ASD
$\left(\frac{V_u}{\phi R_{bsv}}\right)^2 + \left(\frac{N_u}{\phi R_{bsn}}\right)^2 \leq 1$	$\left(\frac{V_u}{R_{bsv}/\Omega}\right)^2 + \left(\frac{N_u}{R_{bsn}/\Omega}\right)^2 \leq 1$
$\left(\frac{75\text{ kips}}{310\text{ kips}}\right)^2 + \left(\frac{60\text{ kips}}{484\text{ kips}}\right)^2 = 0.0739 \leq 1$ **o.k.**	$\left(\frac{50\text{ kips}}{207\text{ kips}}\right)^2 + \left(\frac{40\text{ kips}}{323\text{ kips}}\right)^2 = 0.0737 \leq 1$ **o.k.**

Block Shear Rupture of Angles–Outstanding Legs

The nominal strength for the limit state of block shear rupture relative to the shear load on the angles is determined as follows. The tearout pattern is shown in Figure II.A-1B-7.

$$R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \leq 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \qquad (Spec.\ \text{Eq. J4-5})$$

where

$$\begin{aligned} A_{gv} &= (2\text{ angles})(l - l_{ev})t \\ &= (2\text{ angles})(14\tfrac{1}{2}\text{ in.} - 1\tfrac{1}{4}\text{ in.})(\tfrac{5}{8}\text{ in.}) \\ &= 16.6\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{nv} &= A_{gv} - (2\text{ angles})(n - 0.5)(d_h + \tfrac{1}{16}\text{ in.})t \\ &= 16.6\text{ in.}^2 - (2\text{ angles})(5 - 0.5)(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(\tfrac{5}{8}\text{ in.}) \\ &= 11.0\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{nt} &= (2\text{ angles})\left[l_{eh} - 0.5(d_h + \tfrac{1}{16}\text{ in.})\right]t \\ &= (2\text{ angles})\left[1\tfrac{7}{16}\text{ in.} - 0.5(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right](\tfrac{5}{8}\text{ in.}) \\ &= 1.17\text{ in.}^2 \end{aligned}$$

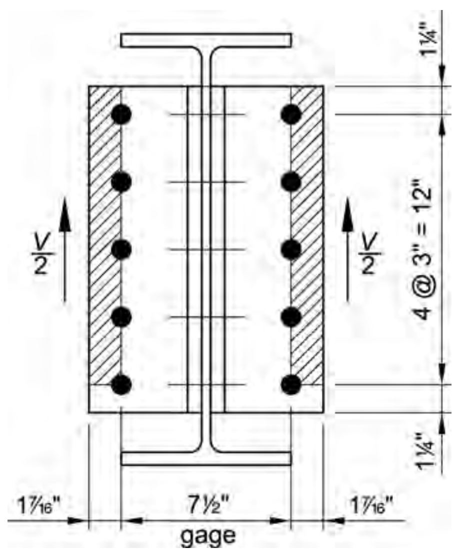


Fig. II.A-1B-7. Block shear rupture of outstanding legs of angles.

$$U_{bs} = 1.0$$

and

$$R_n = 0.60(58\text{ ksi})(11.0\text{ in.}^2) + 1.0(58\text{ ksi})(1.17\text{ in.}^2) \leq 0.60(36\text{ ksi})(16.6\text{ in.}^2) + 1.0(58\text{ ksi})(1.17\text{ in.}^2)$$
$$= 451\text{ kips} > 426\text{ kips}$$

Therefore:

$$R_n = 426\text{ kips}$$

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture on the angles is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(426\text{ kips})$ $= 320\text{ kips} > 75\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{426\text{ kips}}{2.00}$ $= 213\text{ kips} > 50\text{ kips}$ **o.k.**

Shear Strength of Beam Web

From AISC *Specification* Section J4.2(a), the available shear yield strength of the beam web is determined as follows:

$$A_{gv} = dt_w$$
$$= (18.0\text{ in.})(0.355\text{ in.})$$
$$= 6.39\text{ in.}^2$$

$$R_n = 0.60F_yA_{gv} \qquad (Spec.\text{ Eq. J4-3})$$
$$= 0.60(50\text{ ksi})(6.39\text{ in.}^2)$$
$$= 192\text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(192\text{ kips})$ $= 192\text{ kips} > 75\text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{192\text{ kips}}{1.50}$ $= 128\text{ kips} > 50\text{ kips}$ **o.k.**

The limit state of shear rupture of the beam web does not apply in this example because the beam is uncoped.

Tensile Strength of Beam

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the beam is determined as follows:

$$R_n = F_y A_g$$ (*Spec.* Eq. J4-1)

$$= (50 \text{ ksi})(14.7 \text{ in.}^2)$$

$$= 735 \text{ kips}$$

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(735 \text{ kips})$ $= 662 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{735 \text{ kips}}{1.67}$ $= 440 \text{ kips} > 40 \text{ kips}$ **o.k.**

From AISC *Specification* Section J4.1(b), determine the available tensile rupture strength of the beam. The effective net area is $A_e = A_n U$. No cases in AISC *Specification* Table D3.1 apply to this configuration; therefore, U is determined from AISC *Specification* Section D3.

$$A_n = A_g - n(d_h + \tfrac{1}{16} \text{ in.})(t_w)$$

$$= 14.7 \text{ in.}^2 - 5(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(0.355 \text{ in.})$$

$$= 12.9 \text{ in.}^2$$

As stated in AISC *Specification* Section D3, the value of U can be determined as the ratio of the gross area of the connected element (beam web) to the member gross area.

$$U = \frac{(d - 2t_f)(t_w)}{A_g}$$

$$= \frac{[18.0 \text{ in.} - 2(0.570 \text{ in.})](0.355 \text{ in.})}{14.7 \text{ in.}^2}$$

$$= 0.407$$

$$A_e = A_n U$$ (*Spec.* Eq. D3-1)

$$= (12.9 \text{ in.}^2)(0.407)$$

$$= 5.25 \text{ in.}^2$$

$$R_n = F_u A_e$$ (*Spec.* Eq. J4-2)

$$= (65 \text{ ksi})(5.25 \text{ in.}^2)$$

$$= 341 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(341 \text{ kips})$ $= 256 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{341 \text{ kips}}{2.00}$ $= 171 \text{ kips} > 40 \text{ kips}$ **o.k.**

Block Shear Rupture Strength of Beam Web

Block shear rupture is only applicable in the direction of the axial load, because the beam is uncoped and the limit state is not applicable for an uncoped beam subject to vertical shear. Assuming a U-shaped tearout relative to the axial load, and assuming a horizontal edge distance of l_{eh} = 1¾ in. − ¼ in. = 1½ in. to account for a possible beam underrun of ¼ in., the block shear rupture strength is:

$$R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \qquad \text{(Spec. Eq. J4-5)}$$

where

$$\begin{aligned} A_{gv} &= (2) l_{eh} t_w \\ &= (2)(1\tfrac{1}{2}\text{ in.})(0.355\text{ in.}) \\ &= 1.07\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{nv} &= A_{gv} - (2)(0.5)(d_h + \tfrac{1}{16}\text{ in.}) t_w \\ &= 1.07\text{ in.}^2 - (2)(0.5)(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(0.355\text{ in.}) \\ &= 0.715\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{nt} &= \left[12.0\text{ in.} - (n-1)(d_h + \tfrac{1}{16}\text{ in.})\right] t_w \\ &= \left[12.0\text{ in.} - (5-1)(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right](0.355\text{ in.}) \\ &= 2.84\text{ in.}^2 \end{aligned}$$

$$U_{bs} = 1.0$$

and

$$\begin{aligned} R_n &= 0.60(65\text{ ksi})(0.710\text{ in.}^2) + 1.0(65\text{ ksi})(2.84\text{ in.}^2) \le 0.60(50\text{ ksi})(1.07\text{ in.}^2) + 1.0(65\text{ ksi})(2.84\text{ in.}^2) \\ &= 212\text{ kips} < 217\text{ kips} \end{aligned}$$

Therefore:

$$R_n = 212\text{ kips}$$

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture of the beam web is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(212\text{ kips})$ $= 159\text{ kips} > 60\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{212\text{ kips}}{2.00}$ $= 106\text{ kips} > 40\text{ kips}$ **o.k.**

Conclusion

The connection is found to be adequate as given for the applied loads.

EXAMPLE II.A-2A BOLTED/WELDED DOUBLE-ANGLE CONNECTION

Given:

Using the tables in AISC *Manual* Part 10, verify the available strength of a double-angle shear connection with welds in the support legs (welds B) and bolts in the supported-beam-web legs, as shown in Figure II.A-2A-1. The ASTM A992 W36×231 beam is attached to an ASTM A992 W14×90 column flange supporting the following beam end reactions:

R_D = 37.5 kips
R_L = 113 kips

Use ASTM A36 angles and 70-ksi weld electrodes.

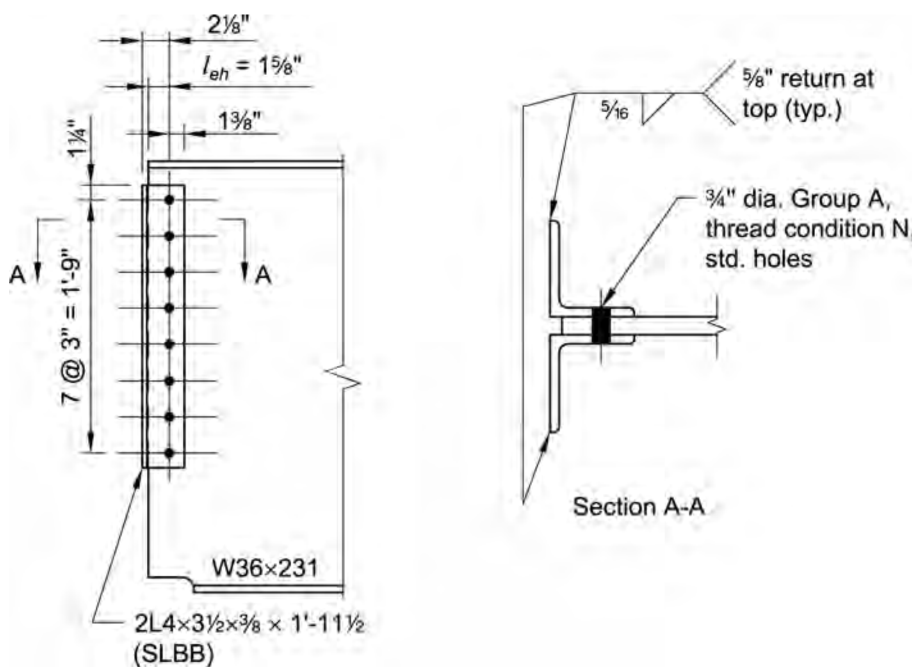


Fig. II.A-2A-1. Connection geometry for Example II.A-2A.

Note: Bottom flange coped for erection.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam and column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W36×231
t_w = 0.760 in.

Column
W14×90
t_f = 0.710 in.

From AISC *Specification* Table J3.3, the hole diameter for ¾-in.-diameter bolts with standard holes is:

d_h = 13⁄16 in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(37.5 \text{ kips}) + 1.6(113 \text{ kips})$ $= 226 \text{ kips}$	$R_a = 37.5 \text{ kips} + 113 \text{ kips}$ $= 151 \text{ kips}$

Weld Design

Use AISC *Manual* Table 10-2 (welds B) with n = 8. Try 5⁄16-in. weld size, l = 23½ in. From AISC *Manual* Table 10-2, the minimum support thickness is:

t_{min} = 0.238 in. < 0.710 in. **o.k.**

LRFD	ASD
$\phi R_n = 279 \text{ kips} > 226 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 186 \text{ kips} > 151 \text{ kips}$ **o.k.**

Angle Thickness

From AISC *Specification* Section J2.2b, the minimum angle thickness for a 5⁄16-in. fillet weld is:

$$t = w + \tfrac{1}{16} \text{ in.}$$
$$= \tfrac{5}{16} \text{ in.} + \tfrac{1}{16} \text{ in.}$$
$$= \tfrac{3}{8} \text{ in.}$$

Try 2L4×3½×⅜ (SLBB).

Angle and Bolt Design

AISC *Manual* Table 10-1 includes checks for bolt shear, bolt bearing and tearout on the angles, shear yielding of the angles, shear rupture of the angles, and block shear rupture of the angles.

Check 8 rows of bolts and ⅜-in. angle thickness.

LRFD	ASD
$\phi R_n = 284 \text{ kips} > 226 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 189 \text{ kips} > 151 \text{ kips}$ **o.k.**

Beam Web Strength

The available beam web strength is the lesser of the limit states of block shear rupture, shear yielding, shear rupture, and the sum of the effective strengths of the individual fasteners. In this example, because of the relative size of the cope to the overall beam size, the coped section will not control, therefore, the strength of the bolt group will control (When this cannot be determined by inspection, see AISC *Manual* Part 9 for the design of the coped section). From the Commentary to AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the effective strengths of the individual fasteners. The effective strength of an individual fastener is the lesser of the shear strength, the bearing strength at the bolt holes, and the tearout strength at the bolt holes.

Bolt Shear

From AISC *Manual* Table 7-1, the available shear strength per bolt for ¾-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) in double shear is:

LRFD	ASD
$\phi R_n = 35.8 \text{ kips/bolt}$	$\dfrac{R_n}{\Omega} = 23.9 \text{ kips/bolt}$

Bolt Bearing on Beam Web

The nominal bearing strength of the beam web per bolt is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration:

$$\begin{aligned} r_n &= 2.4dtF_u \\ &= 2.4(\tfrac{3}{4} \text{ in.})(0.760 \text{ in.})(65 \text{ ksi}) \\ &= 88.9 \text{ kips/bolt} \end{aligned}$$ (*Spec*. Eq. J3-6a)

From AISC *Specification* Section J3.10, the available bearing strength of the beam web per bolt is:

LRFD	ASD
$\phi = 0.75$ $\begin{aligned} \phi r_n &= 0.75(88.9 \text{ kips/bolt}) \\ &= 66.7 \text{ kips/bolt} \end{aligned}$	$\Omega = 2.00$ $\begin{aligned} \frac{r_n}{\Omega} &= \frac{88.9 \text{ kips/bolt}}{2.00} \\ &= 44.5 \text{ kips/bolt} \end{aligned}$

Bolt Tearout on Beam Web

The available tearout strength of the beam web per bolt is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration:

$$\begin{aligned} l_c &= 3.00 \text{ in.} - \tfrac{13}{16} \text{ in.} \\ &= 2.19 \text{ in.} \end{aligned}$$

$$\begin{aligned} r_n &= 1.2 l_c t F_u \\ &= 1.2(2.19 \text{ in.})(0.760 \text{ in.})(65 \text{ ksi}) \\ &= 130 \text{ kips/bolt} \end{aligned}$$ (*Spec*. Eq. J3-6c)

From AISC *Specification* Section J3.10, the available tearout strength of the beam web per bolt is:

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(130 \text{ kips/bolt})$ $= 97.5 \text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{130 \text{ kips/bolt}}{2.00}$ $= 65.0 \text{ kips/bolt}$

Bolt shear strength is the governing limit state for all bolts at the beam web. Bolt shear strength is one of the limit states included in the capacities shown in Table 10-1 as used above; thus, the effective strength of the fasteners is adequate.

Available strength at the column flange

Since the thickness of the column flange, t_f = 0.710 in., is greater than the thickness of the angles, t = ⅜ in., shear will control for the angles. The column flange is adequate for the required loading.

Summary

The connection is found to be adequate as given for the applied loads.

EXAMPLE II.A-2B BOLTED/WELDED DOUBLE-ANGLE CONNECTION SUBJECT TO AXIAL AND SHEAR LOADING

Given:

Verify the available strength of a double-angle connection with welds in the supported-beam-web legs and bolts in the outstanding legs for an ASTM A992 W18×50 beam, as showin in Figure II.A-2B-1, to support the following beam end reactions:

LRFD	ASD
Shear, V_u = 75 kips Axial tension, N_u = 60 kips	Shear, V_a = 50 kips Axial tension, N_a = 40 kips

Use ASTM A36 angles and 70-ksi electrodes.

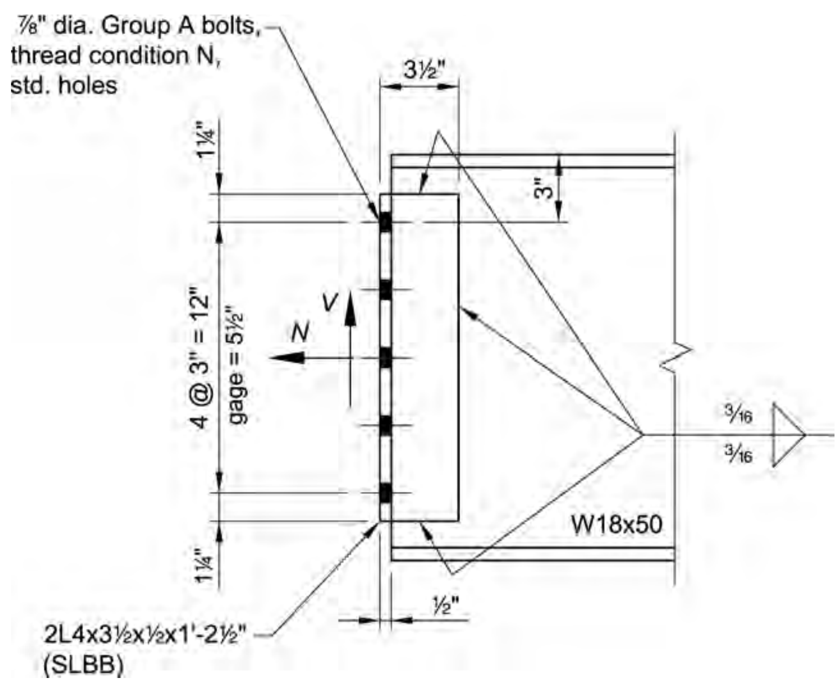


Fig. II.A-2B-1. Connection geometry for Example II.A-2B.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W18×50
A_g = 14.7 in.^{2}
d = 18.0 in.
t_w = 0.355 in.
b_f = 7.50 in.
t_f = 0.570 in.

From AISC *Specification* Table J3.3, the hole diameter for ⅞-in.-diameter bolts with standard holes is:

d_h = 15⁄16 in.

The resultant load is:

LRFD	ASD
$R_u = \sqrt{V_u^2 + N_u^2}$ $= \sqrt{(75 \text{ kips})^2 + (60 \text{ kips})^2}$ $= 96.0$ kips	$R_a = \sqrt{V_a^2 + N_a^2}$ $= \sqrt{(50 \text{ kips})^2 + (40 \text{ kips})^2}$ $= 64.0$ kips

The following bolt shear, bearing and tearout calculations are for a pair of bolts.

Bolt Shear

From AISC *Manual* Table 7-1, the available shear strength for ⅞-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) in double shear (or pair of bolts):

LRFD	ASD
$\phi r_n = 48.7$ kips (for pair of bolts)	$\dfrac{r_n}{\Omega} = 32.5$ kips (for pair of bolts)

Bolt Bearing on Angles

The available bearing strength of the double angle is determined from AISC *Specification* Section J3.10, assuming deformation at the bolt hole is a design consideration:

$r_n = (2 \text{ bolts})2.4dtF_u$ (from *Spec*. Eq. J3-6a)
$= (2 \text{ bolts})(2.4)(⅞ \text{ in.})(½ \text{ in.})(58 \text{ ksi})$
$= 122$ kips (for pair of bolts)

The available bearing strength for a pair of bolts is:

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(122 \text{ kips})$ $= 91.5$ kips (for pair of bolts)	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{122 \text{ kips}}{2.00}$ $= 61.0$ kips (for pair of bolts)

The bolt shear strength controls over bearing in the angles.

Bolt Tearout on Angles

The available tearout strength of the angle is determined from AISC *Specification* Section J3.10, assuming deformation at the bolt hole is a design consideration:

For the edge bolt:

$$
\begin{aligned}
l_c &= l_e - 0.5d_h \\
&= 1\tfrac{1}{4}\text{ in.} - 0.5\left(\tfrac{15}{16}\text{ in.}\right) \\
&= 0.781\text{ in.}
\end{aligned}
$$

$$
\begin{aligned}
r_n &= (2\text{ bolts})1.2l_ctF_u \\
&= (2\text{ bolts})(1.2)(0.781\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi}) \\
&= 54.4\text{ kips (for pair of bolts)}
\end{aligned}
$$

(from *Spec*. Eq. J3-6c)

The available tearout strength of the angles for a pair of edge bolts is:

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(54.4\text{ kips})$ $= 40.8\text{ kips}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{54.4\text{ kips}}{2.00}$ $= 27.2\text{ kips}$

The tearout strength controls over bolt shear and bearing for the edge bolts in the angles.

For the other bolts:

$$
\begin{aligned}
l_c &= s - d_h \\
&= 3\text{ in.} - \tfrac{15}{16}\text{ in.} \\
&= 2.06\text{ in.}
\end{aligned}
$$

$$
\begin{aligned}
r_n &= (2\text{ bolts})1.2l_ctF_u \\
&= (2\text{ bolts})(1.2)(2.06\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi}) \\
&= 143\text{ kips (for pair of bolts)}
\end{aligned}
$$

(*Spec*. Eq. J3-6c)

The available tearout strength for a pair of other bolts is:

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(143\text{ kips})$ $= 107\text{ kips (for pair of bolts)}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{143\text{ kips}}{2.00}$ $= 71.5\text{ kips (for pair of bolts)}$

Bolt shear strength controls over tearout and bearing strength for the other bolts in the angles.

Strength of Bolted Connection

The effective strength for the bolted connection at the angles is determined by summing the effective strength for each bolt using the minimum available strength calculated for bolt shear, bearing on the angles, and tearout on the angles.

LRFD	ASD
$\phi R_n = (1 \text{ bolt})(40.8 \text{ kips}) + (4 \text{ bolts})(48.7 \text{ kips})$ $= 236 \text{ kips} > 75 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (1 \text{ bolt})(27.2 \text{ kips}) + (4 \text{ bolts})(32.5 \text{ kips})$ $= 157 \text{ kips} > 50 \text{ kips}$ **o.k.**

Shear and Tension Interaction in Bolts

The required shear stress for each bolt is determined as follows:

$$f_{rv} = \frac{V_r}{nA_b}$$

where

$A_b = 0.601 \text{ in.}^2$ (from AISC *Manual* Table 7-1)

$n = 10 \text{ bolts}$

LRFD	ASD
$f_{rv} = \dfrac{75 \text{ kips}}{(10 \text{ bolts})(0.601 \text{ in.}^2)}$ $= 12.5 \text{ ksi}$	$f_{rv} = \dfrac{50 \text{ kips}}{(10 \text{ bolts})(0.601 \text{ in.}^2)}$ $= 8.32 \text{ ksi}$

The nominal tensile stress modified to include the effects of shear stress is determined from AISC *Specification* Section J3.7 as follows. From AISC *Specification* Table J3.2:

$F_{nt} = 90 \text{ ksi}$

$F_{nv} = 54 \text{ ksi}$

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$F'_{nt} = 1.3F_{nt} - \dfrac{F_{nt}}{\phi F_{nv}} f_{rv} \le F_{nt}$ (*Spec*. Eq. J3-3a) $= 1.3(90 \text{ ksi}) - \dfrac{90 \text{ ksi}}{0.75(54 \text{ ksi})}(12.5 \text{ ksi}) \le 90 \text{ ksi}$ $= 89.2 \text{ ksi} < 90 \text{ ksi}$ **o.k.**	$F'_{nt} = 1.3F_{nt} - \dfrac{\Omega F_{nt}}{F_{nv}} f_{rv} \le F_{nt}$ (*Spec*. Eq. J3-3b) $= 1.3(90 \text{ ksi}) - \dfrac{2.00(90 \text{ ksi})}{54 \text{ ksi}}(8.32 \text{ ksi}) \le 90 \text{ ksi}$ $= 89.3 \text{ ksi} < 90 \text{ ksi}$ **o.k.**

Using the value of $F'_{nt} = 89.2$ ksi determined for LRFD, the nominal tensile strength of one bolt is:

$$r_n = F'_{nt} A_b$$ (*Spec*. Eq. J3-2)

$$= (89.2 \text{ ksi})(0.601 \text{ in.}^2)$$

$$= 53.6 \text{ kips}$$

The available tensile strength due to combined tension and shear is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = n\phi r_n$ $= (10\text{ bolts})(0.75)(53.6\text{ kips})$ $= 402\text{ kips} > 60\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\frac{R_n}{\Omega} = n\frac{r_n}{\Omega}$ $= (10\text{ bolts})\left(\frac{53.6\text{ kips}}{2.00}\right)$ $= 268\text{ kips} > 40\text{ kips}$ **o.k.**

Prying Action on Bolts

From AISC *Manual* Part 9, the available tensile strength of the bolts in the outstanding angle legs taking prying action into account is determined as follows:

$$a = \frac{angle\ leg(2) + t_w - gage}{2}$$
$$= \frac{(4.00\text{ in.})(2) + 0.355\text{ in.} - 5\tfrac{1}{2}\text{ in.}}{2}$$
$$= 1.43\text{ in.}$$

Note: If the distance from the bolt centerline to the edge of the supporting element is smaller than a = 1.43 in., use the smaller a in the following calculation.

$$b = \frac{gage - t_w - t}{2}$$
$$= \frac{5\tfrac{1}{2}\text{ in.} - 0.355\text{ in.} - \tfrac{1}{2}\text{ in.}}{2}$$
$$= 2.32\text{ in.}$$

$$a' = \left(a + \frac{d_b}{2}\right) \le \left(1.25b + \frac{d_b}{2}\right) \qquad \text{(\textit{Manual} Eq. 9-23)}$$
$$= 1.43\text{ in.} + \frac{\tfrac{7}{8}\text{ in.}}{2} \le 1.25(2.32\text{ in.}) + \frac{\tfrac{7}{8}\text{ in.}}{2}$$
$$= 1.87\text{ in.} < 3.34\text{ in.}$$
$$= 1.87\text{ in.}$$

$$b' = \left(b - \frac{d_b}{2}\right) \qquad \text{(\textit{Manual} Eq. 9-18)}$$
$$= 2.32\text{ in.} - \frac{\tfrac{7}{8}\text{ in.}}{2}$$
$$= 1.88\text{ in.}$$

$$\rho = \frac{b'}{a'} \qquad \text{(\textit{Manual} Eq. 9-22)}$$
$$= \frac{1.88\text{ in.}}{1.87\text{ in.}}$$
$$= 1.01$$

Note that end distances of 1¼ in. are used on the angles, so p is the average pitch of the bolts:

$$
\begin{aligned}
p &= \frac{l}{n} \\
&= \frac{14\tfrac{1}{2}\text{ in.}}{5} \\
&= 2.90\text{ in.}
\end{aligned}
$$

Check:

$$p \le s$$

$2.90\text{ in.} < 3\text{ in.}$ **o.k.**

$$
\begin{aligned}
d' &= d_h \\
&= \tfrac{15}{16}\text{ in.}
\end{aligned}
$$

$$
\begin{aligned}
\delta &= 1 - \frac{d'}{p} \\
&= 1 - \frac{\tfrac{15}{16}\text{ in.}}{2.90\text{ in.}} \\
&= 0.677
\end{aligned}
$$
(*Manual* Eq. 9-20)

The angle thickness required to develop the available strength of the bolt with no prying action as follows:

LRFD	ASD
$B_c = 40.2$ kips/bolt (calculated previously) $\phi = 0.90$ $t_c = \sqrt{\dfrac{4B_c b'}{\phi p F_u}}$ (*Manual* Eq. 9-26a) $= \sqrt{\dfrac{4(40.2\text{ kips/bolt})(1.88\text{ in.})}{0.90(2.90\text{ in.})(58\text{ ksi})}}$ $= 1.41\text{ in.}$	$B_c = 26.8$ kips/bolt (calculated previously) $\Omega = 1.67$ $t_c = \sqrt{\dfrac{\Omega 4B_c b'}{pF_u}}$ (*Manual* Eq. 9-26b) $= \sqrt{\dfrac{1.67(4)(26.8\text{ kips/bolt})(1.88\text{ in.})}{(2.90\text{ in.})(58\text{ ksi})}}$ $= 1.41\text{ in.}$

$$
\begin{aligned}
\alpha' &= \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2 - 1\right] \\
&= \frac{1}{0.677(1+1.01)}\left[\left(\frac{1.41\text{ in.}}{\tfrac{1}{2}\text{ in.}}\right)^2 - 1\right] \\
&= 5.11
\end{aligned}
$$
(*Manual* Eq. 9-28)

Because $\alpha' > 1$, the angles have insufficient strength to develop the bolt strength, therefore:

$$
\begin{aligned}
Q &= \left(\frac{t}{t_c}\right)^2 (1+\delta) \\
&= \left(\frac{\tfrac{1}{2}\text{ in.}}{1.41\text{ in.}}\right)^2 (1+0.677) \\
&= 0.211
\end{aligned}
$$

The available tensile strength of the bolts, taking prying action into account is determined from AISC *Manual* Equation 9-27, as follows:

LRFD	ASD
$\phi r_n = B_c Q$ $= (40.2\text{ kips/bolt})(0.211)$ $= 8.48\text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = B_c Q$ $= (26.8\text{ kips/bolt})(0.211)$ $= 5.65\text{ kips/bolt}$
$\phi R_n = n\phi r_n$ $= (10\text{ bolts})(8.48\text{ kips/bolt})$ $= 84.8\text{ kips} > 60\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = n\dfrac{r_n}{\Omega}$ $= (10\text{ bolts})(5.65\text{ kips/bolt})$ $= 56.5\text{ kips} > 40\text{ kips}$ **o.k.**

Weld Design

The resultant load angle on the weld is:

LRFD	ASD
$\theta = \tan^{-1}\left(\dfrac{N_u}{V_u}\right)$ $= \tan^{-1}\left(\dfrac{60\text{ kips}}{75\text{ kips}}\right)$ $= 38.7°$	$\theta = \tan^{-1}\left(\dfrac{N_a}{V_a}\right)$ $= \tan^{-1}\left(\dfrac{40\text{ kips}}{50\text{ kips}}\right)$ $= 38.7°$

From AISC *Manual* Table 8-8 for Angle = 30° (which will lead to a conservative result), using total beam setback of ½ in. + ¼ in. = ¾ in. (the ¼ in. is included to account for mill underrun):

$$l = 14\tfrac{1}{2}\text{ in.}$$

$$\begin{aligned} kl &= 3\tfrac{1}{2}\text{ in.} - \tfrac{3}{4}\text{ in.} \\ &= 2.75\text{ in.} \end{aligned}$$

$$\begin{aligned} k &= \frac{kl}{l} \\ &= \frac{2.75\text{ in.}}{14\tfrac{1}{2}\text{ in.}} \\ &= 0.190 \end{aligned}$$

$x = 0.027$ by interpolation

$$\begin{aligned} al &= 3\tfrac{1}{2}\text{ in.} - xl \\ &= 3\tfrac{1}{2}\text{ in.} - 0.027(14\tfrac{1}{2}\text{ in.}) \\ &= 3.11\text{ in.} \end{aligned}$$

$$a = \frac{al}{l} = \frac{3.11\text{ in.}}{14\tfrac{1}{2}\text{ in.}} = 0.214$$

$C = 2.69$ by interpolation

The required weld size is determined using AISC *Manual* Equation 8-21, as follows:

LRFD	ASD
$D_{min} = \frac{R_u}{\phi C C_1 l}$ $= \frac{96.0\text{ kips}}{0.75(2.69)(1)(14\tfrac{1}{2}\text{ in.})(2\text{ sides})}$ $= 1.64$ sixteenths	$D_{min} = \frac{\Omega R_a}{C C_1 l}$ $= \frac{2.00(64.0\text{ kips})}{2.69(1)(14\tfrac{1}{2}\text{ in.})(2\text{ sides})}$ $= 1.64$ sixteenths

Use a 3⁄16-in. fillet weld (minimum size from AISC *Specification* Table J2.4).

Beam Web Strength at Fillet Weld

The minimum beam web thickness required to match the shear rupture strength of a weld both sides to that of the base metal is:

$$t_{min} = \frac{6.19 D_{min}}{F_u} = \frac{6.19(1.64)}{65\text{ ksi}} = 0.156\text{ in.} < 0.355\text{ in.} \quad \textbf{o.k.}$$ (from *Manual* Eq. 9-3)

Shear Strength of Angles

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the angles is determined as follows:

$$A_{gv} = (2\text{ angles})lt = (2\text{ angles})(14\tfrac{1}{2}\text{ in.})(\tfrac{1}{2}\text{ in.}) = 14.5\text{ in.}^2$$

$$R_n = 0.60 F_y A_{gv} = 0.60(36\text{ ksi})(14.5\text{ in.}^2) = 313\text{ kips}$$ (*Spec*. Eq. J4-3)

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(313 \text{ kips})$ $= 313 \text{ kips} > 96.0 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{313 \text{ kips}}{1.50}$ $= 209 \text{ kips} > 64.0 \text{ kips}$ **o.k.**

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the angle is determined as follows. The effective net area is determined in accordance with AISC *Specification* Section B4.3b.

$$\begin{aligned} A_{nv} &= (2 \text{ angles})\left[l - n\left(d_h + \tfrac{1}{16}\text{ in.}\right)\right]t \\ &= (2 \text{ angles})\left[14\tfrac{1}{2}\text{ in.} - 5\left(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right]\left(\tfrac{1}{2}\text{ in.}\right) \\ &= 9.50 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} R_n &= 0.60F_uA_{nv} \\ &= 0.60(58 \text{ ksi})\left(9.50 \text{ in.}^2\right) \\ &= 331 \text{ kips} \end{aligned}$$ (*Spec*. Eq. J4-4)

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(331 \text{ kips})$ $= 248 \text{ kips} > 96.0 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{331 \text{ kips}}{2.00}$ $= 166 \text{ kips} > 64.0 \text{ kips}$ **o.k.**

Tensile Strength of Angles—Beam Web Side

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the angles is determined as follows:

$$\begin{aligned} A_g &= (2 \text{ angles})lt \\ &= (2 \text{ angles})\left(14\tfrac{1}{2}\text{ in.}\right)\left(\tfrac{1}{2}\text{ in.}\right) \\ &= 14.5 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} R_n &= F_yA_g \\ &= (36 \text{ ksi})\left(14.5 \text{ in.}^2\right) \\ &= 522 \text{ kips} \end{aligned}$$ (*Spec*. Eq. J4-1)

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(522 \text{ kips})$ $= 470 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{522 \text{ kips}}{1.67}$ $= 313 \text{ kips} > 40 \text{ kips}$ **o.k.**

From AISC *Specification* Sections J4.1(b), the available tensile rupture strength of the angles is determined as follows:

$$R_n = F_u A_e$$ (*Spec*. Eq. J4-2)

Because the angle legs are welded to the beam web there is no bolt hole reduction and $A_e = A_g$; therefore, tensile rupture will not control.

Block Shear Rupture Strength of Angles–Outstanding Legs

The nominal strength for the limit state of block shear rupture of the angles assuming an L-shaped tearout relative to shear load, is determined as follows. The tearout pattern is shown in Figure II.A-2B-2.

$$R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \leq 0.60F_y A_{gv} + U_{bs} F_u A_{nt}$$ (*Spec*. Eq. J4-5)

where

$$\begin{aligned} l_{eh} &= \frac{2(angle\ leg) + t_w - gage}{2} \\ &= \frac{2(4\text{ in.}) + 0.355\text{ in.} - 5\tfrac{1}{2}\text{ in.}}{2} \\ &= 1.43\text{ in.} \end{aligned}$$

$$\begin{aligned} A_{nt} &= (2\text{ angles})\left[l_{eh} - 0.5\left(d_h + \tfrac{1}{16}\text{ in.}\right)\right](t) \\ &= (2\text{ angles})\left[1.43\text{ in.} - 0.5\left(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right]\left(\tfrac{1}{2}\text{ in.}\right) \\ &= 0.930\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{gv} &= (2\text{ angles})\left[l_{ev} + (n-1)s\right](t) \\ &= (2\text{ angles})\left[1\tfrac{1}{4}\text{ in.} + (5-1)(3\text{ in.})\right]\left(\tfrac{1}{2}\text{ in.}\right) \\ &= 13.3\text{ in.}^2 \end{aligned}$$

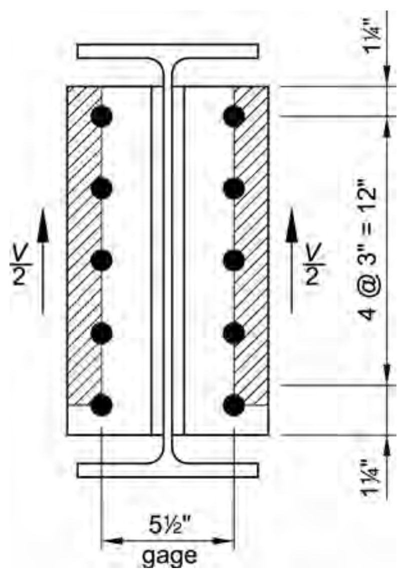


Fig. II.A-2B-2. Block shear rupture of outstanding legs of angles.

$$A_{nv} = A_{gv} - (2\text{ angles})(n - 0.5)(d_h + \tfrac{1}{16}\text{ in.})(t)$$
$$= 13.3\text{ in.}^2 - (2\text{ angles})(5 - 0.5)(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(\tfrac{1}{2}\text{ in.})$$
$$= 8.80\text{ in.}^2$$

$$U_{bs} = 1.0$$

and

$$R_n = 0.60(58\text{ ksi})(8.80\text{ in.}^2) + 1.0(58\text{ ksi})(0.930\text{ in.}^2) \le 0.60(36\text{ ksi})(13.3\text{ in.}^2) + 1.0(58\text{ ksi})(0.930\text{ in.}^2)$$
$$= 360\text{ kips} > 341\text{ kips}$$

Therefore:

$$R_n = 341\text{ kips}$$

The available block shear rupture strength of the angles is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(341\text{ kips})$ $= 256\text{ kips} > 75\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{341\text{ kips}}{2.00}$ $= 171\text{ kips} > 50\text{ kips}$ **o.k.**

Shear Strength of Beam

From AISC *Specification* Section J4.2(a), the available shear yield strength of the beam web is determined as follows:

$$A_{gv} = dt_w$$
$$= (18.0\text{ in.})(0.355\text{ in.})$$
$$= 6.39\text{ in.}^2$$

$$R_n = 0.60F_y A_{gv} \qquad (\textit{Spec}.\text{ Eq. J4-3})$$
$$= 0.60(50\text{ ksi})(6.39\text{ in.}^2)$$
$$= 192\text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(192\text{ kips})$ $= 192\text{ kips} > 75\text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{192\text{ kips}}{1.50}$ $= 128\text{ kips} > 50\text{ kips}$ **o.k.**

The limit state of shear rupture of the beam web does not apply in this example because the beam is uncoped.

Block Shear Rupture Strength of Beam Web

Assuming a U-shaped tearout along the weld relative to the axial load, and a total beam setback of ¾ in. (includes ¼ in. tolerance to account for possible mill underrun), the nominal block shear rupture strength is determined as follows.

$$R_n = 0.60F_uA_{nv} + U_{bs}F_uA_{nt} \leq 0.60F_yA_{gv} + U_{bs}F_uA_{nt}$$ (*Spec*. Eq. J4-5)

where

$$A_{nt} = lt_w$$
$$= (14\tfrac{1}{2}\text{ in.})(0.355\text{ in.})$$
$$= 5.15\text{ in.}^2$$

$$A_{gv} = (2)(3\tfrac{1}{2}\text{ in.} - setback)t_w$$
$$= (2)(3\tfrac{1}{2}\text{ in.} - \tfrac{3}{4}\text{ in.})(0.355\text{ in.})$$
$$= 1.95\text{ in.}^2$$

Because the angles are welded and there is no reduction for bolt holes:

$$A_{nv} = A_{gv}$$
$$= 1.95\text{ in.}^2$$

$$U_{bs} = 1$$

and

$$R_n = 0.60(65\text{ ksi})(1.95\text{ in.}^2) + 1.0(65\text{ ksi})(5.15\text{ in.}^2) \leq 0.60(50\text{ ksi})(1.95\text{ in.}^2) + 1.0(65\text{ ksi})(5.15\text{ in.}^2)$$
$$= 411\text{ kips} > 393\text{ kips}$$

Therefore:

$$R_n = 393\text{ kips}$$

The available block shear rupture strength of the web is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(393\text{ kips})$ $= 295\text{ kips} > 60\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{393\text{ kips}}{2.00}$ $= 197\text{ kips} > 40\text{ kips}$ **o.k.**

Tensile Strength of Beam

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the beam is determined from AISC *Specification* Equation J4-1:

$$R_n = F_yA_g$$ (*Spec*. Eq. J4-1)
$$= (50\text{ ksi})(14.7\text{ in.}^2)$$
$$= 735\text{ kips}$$

The available tensile yielding strength of the beam is:

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(735 \text{ kips})$ $= 662 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{735 \text{ kips}}{1.67}$ $= 440 \text{ kips} > 40 \text{ kips}$ **o.k.**

From AISC *Specification* Section J4.1(b), determine the available tensile rupture strength of the beam. The effective net area is $A_e = A_nU$, where U is determined from AISC *Specification* Table D3.1, Case 2. The value of $\bar{x}$ is determined by treating the W-shape as two channels back-to-back and finding the horizontal distance to the center of gravity of one of the channels from the centerline of the beam. (Note that the fillets are ignored.)

$$\begin{aligned}\bar{x} &= \frac{\Sigma(A\bar{x})}{\Sigma A}\\ &= \frac{(0.178 \text{ in.})\left[18.0 \text{ in.} - 2(0.570 \text{ in.})\right]\left(\dfrac{0.178 \text{ in.}}{2}\right) + 2(0.570 \text{ in.})\left(\dfrac{7.50 \text{ in.}}{2}\right)\left(\dfrac{7.50 \text{ in.}/2}{2}\right)}{\left(\dfrac{14.7 \text{ in.}^2}{2}\right)}\\ &= 1.13 \text{ in.}\end{aligned}$$

The connection length, l, used in the determination of U will be reduced by ¼ in. to account for possible mill underrun. The shear lag factor, U, is:

$$\begin{aligned}U &= 1 - \frac{\bar{x}}{l}\\ &= 1 - \frac{1.13 \text{ in.}}{(3 \text{ in.} - \tfrac{1}{4} \text{ in.})}\\ &= 0.589\end{aligned}$$

The minimum value of U can be determined from AISC *Specification* Section D3, where U is the ratio of the gross area of the connected element to the member gross area.

$$\begin{aligned}U &= \frac{A_{nt}}{A_g}\\ &= \frac{(d - 2t_f)t_w}{A_g}\\ &= \frac{\left[18.0 \text{ in.} - 2(0.570 \text{ in.})\right](0.355 \text{ in.})}{14.7 \text{ in.}^2}\\ &= 0.407\end{aligned}$$

AISC *Specification* Table D3.1, Case 2 controls, use $U = 0.589$. Because the angles are welded and there is no reduction for bolt holes:

$$\begin{aligned}A_n &= A_g\\ &= 14.7 \text{ in.}^2\end{aligned}$$

$$A_e = A_n U \quad \text{(}Spec\text{. Eq. D3-1)}$$
$$= \left(14.7 \text{ in.}^2\right)\left(0.589\right)$$
$$= 8.66 \text{ in.}^2$$

$$R_n = F_u A_e \quad \text{(}Spec\text{. Eq. J4-2)}$$
$$= \left(65 \text{ ksi}\right)\left(8.66 \text{ in.}^2\right)$$
$$= 563 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75\left(563 \text{ kips}\right)$ $= 422 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{563 \text{ kips}}{2.00}$ $= 282 \text{ kips} > 40 \text{ kips}$ **o.k.**

Conclusion

The connection is found to be adequate as given for the applied loads.

EXAMPLE II.A-3 ALL-WELDED DOUBLE-ANGLE CONNECTION

Given:

Repeat Example II.A-1A using AISC *Manual* Table 10-3 and applicable provisions from the AISC *Specification* to verify the strength of an all-welded double-angle connection between an ASTM A992 W36×231 beam and an ASTM A992 W14×90 column flange, as shown in Figure II.A-3-1. Use 70-ksi electrodes and ASTM A36 angles.

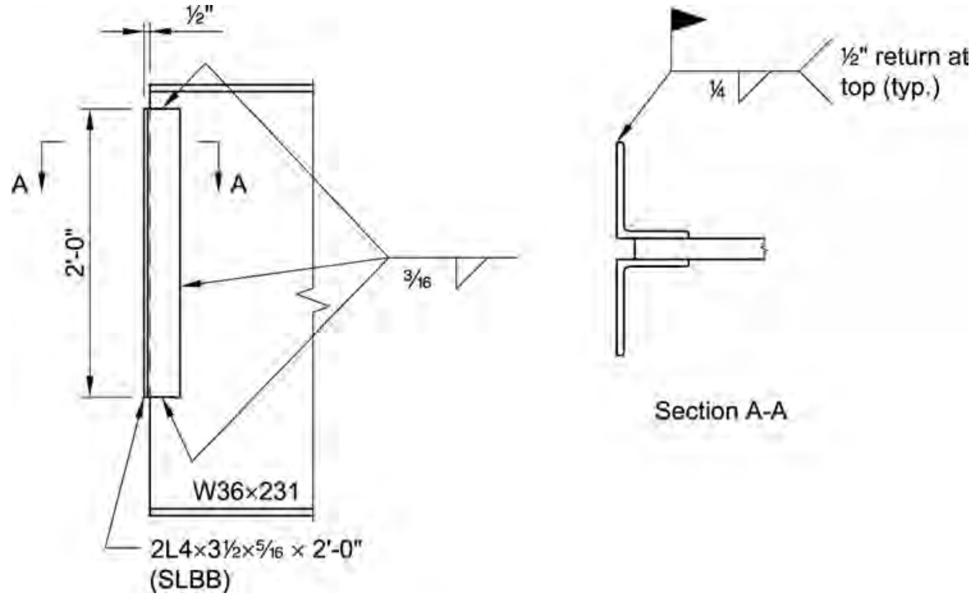


Fig. II.A-3-1. Connection geometry for Example II.A-3.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam and column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W36×231
t_w = 0.760 in.

Column
W14×90
t_f = 0.710 in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(37.5\text{ kips}) + 1.6(113\text{ kips})$ $= 226\text{ kips}$	$R_a = 37.5\text{ kips} + 113\text{ kips}$ $= 151\text{ kips}$

Design of Weld between Beam Web and Angles

Use AISC *Manual* Table 10-3 (Welds A). Try 3⁄16-in. weld size, l = 24 in.

LRFD	ASD
$\phi R_n = 257\text{ kips} > 226\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 171\text{ kips} > 151\text{ kips}$ **o.k.**

From AISC *Manual* Table 10-3, the minimum beam web thickness is:

$$t_{w\,min} = 0.286\text{ in.} < 0.760\text{ in.} \quad \textbf{o.k.}$$

Design of Weld between Column Flange and Angles

Use AISC *Manual* Table 10-3 (Welds B). Try ¼-in. weld size, l = 24 in.

LRFD	ASD
$\phi R_n = 229\text{ kips} > 226\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 153\text{ kips} > 151\text{ kips}$ **o.k.**

From AISC *Manual* Table 10-3, the minimum column flange thickness is:

$$t_{f\,min} = 0.190\text{ in.} < 0.710\text{ in.} \quad \textbf{o.k.}$$

Angle Thickness

Minimum angle thickness for weld from AISC *Specification* Section J2.2b:

$$\begin{aligned} t_{min} &= w + \tfrac{1}{16}\text{ in.} \\ &= \tfrac{1}{4}\text{ in.} + \tfrac{1}{16}\text{ in.} \\ &= \tfrac{5}{16}\text{ in.} \end{aligned}$$

Try 2L4×3½×5⁄16 (SLBB).

Shear Strength of Angles

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the angles is determined as follows:

$$\begin{aligned} A_{gv} &= (2\text{ angles})lt \\ &= (2\text{ angles})(24\text{ in.})(\tfrac{5}{16}\text{ in.}) \\ &= 15.0\text{ in.}^2 \end{aligned}$$

$$R_n = 0.60F_y A_{gv} \qquad \text{(Spec. Eq. J4-3)}$$
$$= 0.60(36 \text{ ksi})(15.0 \text{ in.}^2)$$
$$= 324 \text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(324 \text{ kips})$ $= 324 \text{ kips} > 226 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{324 \text{ kips}}{1.50}$ $= 216 \text{ kips} > 151 \text{ kips}$ **o.k.**

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the angles is determined as follows:

$$A_{nv} = (2 \text{ angles})lt$$
$$= (2 \text{ angles})(24 \text{ in.})(5/16 \text{ in.})$$
$$= 15.0 \text{ in.}^2$$

$$R_n = 0.60F_u A_{nv} \qquad \text{(Spec. Eq. J4-4)}$$
$$= 0.60(58 \text{ ksi})(15.0 \text{ in.}^2)$$
$$= 522 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(522 \text{ kips})$ $= 392 \text{ kips} > 226 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{522 \text{ kips}}{2.00}$ $= 261 \text{ kips} > 151 \text{ kips}$ **o.k.**

Conclusion

The connection is found to be adequate as given for the applied loads.