

**COMPANION TO THE
AISC
*STEEL CONSTRUCTION MANUAL***

Volume 1: Design Examples

**For Use in First Semester
Structural Steel Design Classes**

Version 16.0



**AMERICAN INSTITUTE
OF
STEEL CONSTRUCTION**

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by

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Printed in the United States of America

Notes: Design Examples For Use in First Semester Structural Steel Design Course

The AISC Committee on Manuals prepares design examples to illustrate the application of the provisions in the AISC *Specification for Structural Steel Buildings*. The complete set of design examples includes 169 example problems totaling 1054 pages, and it is a free download that can be found at aisc.org/designexamples.

The AISC Partners in Education Committee has condensed the set of Design Examples to include 45 example problems that will be most likely to address topics that are studied in a first semester structural steel design course. These 45 example problems can be found on the following pages.

PREFACE

The primary objective of this Companion is to provide guidance and additional resources in the use of the 2022 AISC *Specification for Structural Steel Buildings* (ANSI/AISC 360-22) and the 16th Edition AISC *Steel Construction Manual*.

The Companion consists of design examples in Parts I, II, and III. The design examples provide coverage of all applicable limit states, whether or not a particular limit state controls the design of the member or connection. In addition to the examples that demonstrate the use of the AISC *Manual* tables, design examples are provided for connection designs beyond the scope of the tables in the AISC *Manual*. These design examples are intended to demonstrate an approach to the design, and are not intended to suggest that the approach presented is the only approach. The committee responsible for the development of these design examples recognizes that designers have alternate approaches that work best for them and their projects. Design approaches that differ from those presented in these examples are considered viable as long as the AISC *Specification*, sound engineering, and project-specific requirements are satisfied.

Part I of these examples is organized to correspond with the organization of the AISC *Specification*. The Chapter titles match the corresponding chapters in the AISC *Specification*.

Part II is devoted primarily to connection examples that draw on the tables from the AISC *Manual*, recommended design procedures, and the breadth of the AISC *Specification*. The chapters of Part II are labeled II-A, II-B, II-C, etc.

Part III addresses aspects of design that are linked to the performance of a building as a whole. This includes coverage of lateral stability and second-order analysis, illustrated through a four-story braced-frame and moment-frame building.

The Design Examples are arranged with LRFD and ASD designs presented side-by-side, for consistency with the AISC *Manual*. Design with LRFD and ASD are based on the same nominal strength for each element so that the only differences between the approaches are the set of load combinations from ASCE/SEI 7-22 used for design, and whether the resistance factor for LRFD or the safety factor for ASD is used.

CONVENTIONS

The following conventions are used throughout these examples:

1. The 2022 AISC *Specification for Structural Steel Buildings* is referred to as the AISC *Specification* and the 16th Edition AISC *Steel Construction Manual* is referred to as the AISC *Manual*.
2. The 2022 ASCE *Minimum Design Loads and Associated Criteria for Buildings and Other Structures* is referred to as ASCE/SEI 7.
3. The source of equations or tabulated values taken from the AISC *Specification* or AISC *Manual* is noted along the right-hand edge of the page.
4. When the design process differs between LRFD and ASD, the designs equations are presented side-by-side. This rarely occurs, except when the resistance factor, ϕ , and the safety factor, Ω , are applied.
5. The results of design equations are presented to three significant figures throughout these calculations.

ACKNOWLEDGMENTS

The AISC Committee on Manuals reviewed and approved V16.0 of the AISC *Design Examples*:

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The committee gratefully acknowledges the contributions made to this Manual by Patrick J. Fortney and William T. Segui who passed away during this cycle, as well as the AISC Committee on Specifications, and the following individuals: Michael Desch, Carlo Lini, and Matt Smith and Duff Zimmerman who both provide AISC Board Oversight.

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Part I

Examples Based on the AISC *Specification*

This part contains design examples demonstrating select provisions of the AISC *Specification for Structural Steel Buildings*.

Chapter C

Design for Stability

C1. GENERAL STABILITY REQUIREMENTS

The AISC *Specification* requires that the designer account for both the stability of the structural system as a whole and the stability of individual elements. Thus, the lateral analysis used to assess stability must include consideration of the combined effect of gravity and lateral loads, as well as member inelasticity, out-of-plumbness, out-of-straightness, and the resulting second-order effects (including $P-\Delta$ and $P-\delta$ effects). The effects of “leaning columns” must also be considered, as illustrated in the examples in this chapter and in the four-story building design example in Part III of this document.

$P-\Delta$ and $P-\delta$ effects are illustrated in AISC *Specification* Commentary Figure C-C2.1. Methods for addressing stability, including $P-\Delta$ and $P-\delta$ effects, are provided in AISC *Specification* Section C2 and Appendix 7.

C2. CALCULATION OF REQUIRED STRENGTHS

The calculation of required strengths is illustrated in the examples in this chapter and in the four-story building design example in Part III of these *Design Examples*.

C3. CALCULATION OF AVAILABLE STRENGTHS

The calculation of available strengths is illustrated in the four-story building design example in Part III of these *Design Examples*.

EXAMPLE C.1A DESIGN OF A MOMENT FRAME BY THE DIRECT ANALYSIS METHOD**Given:**

Determine the required strengths and effective length factors for the columns in the moment frame shown in Figure C.1A-1 for the maximum gravity load combination, using LRFD and ASD. The uniform load, w_D , includes beam self-weight and an allowance for column self-weight. Use the direct analysis method. All members are ASTM A992/A992M material.

Columns are unbraced between the footings and roof in the x - x and y - y axes and have pinned bases.

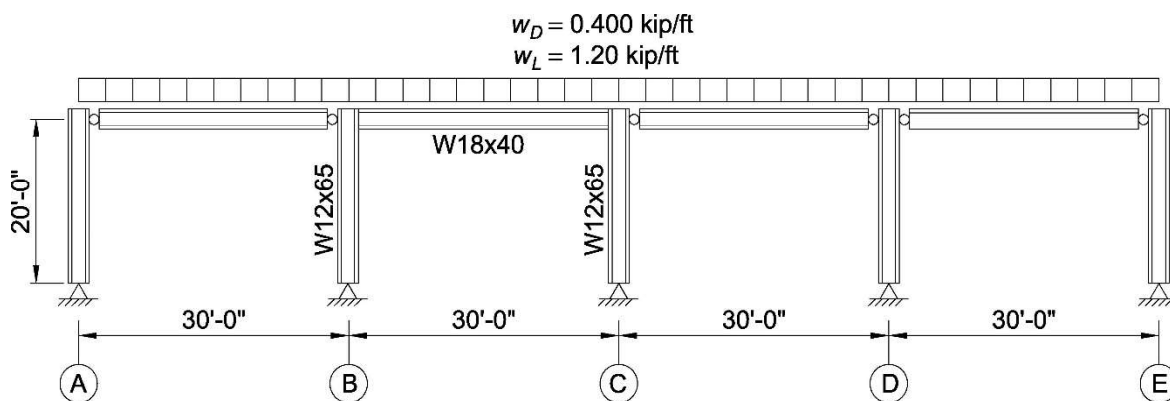


Fig. C.1A-1. Example C.1A moment frame elevation.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992/A992M
 $F_y = 50$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W12x65
 $A_g = 19.1$ in.²

The beams from grid lines A to B and C to E and the columns at A, D, and E are pinned at both ends and do not contribute to the lateral stability of the frame. There are no P - Δ effects to consider in these members and they may be designed using $L_c = L$.

The moment frame between grid lines B and C is the source of lateral stability and therefore will be evaluated using the provisions of Chapter C of the AISC *Specification*. Although the columns at grid lines A, D, and E do not contribute to lateral stability, the forces required to stabilize them must be considered in the moment-frame analysis. The entire frame from grid line A to E could be modeled, but in this case the model is simplified as shown in Figure C.1A-2, in which the stability loads from the three “leaning” columns are combined into a single representative column.

From Chapter 2 of ASCE/SEI 7, the maximum gravity load combinations are:

LRFD	ASD
$w_u = 1.2D + 1.6L$ $= 1.2(0.400 \text{ kip/ft}) + 1.6(1.20 \text{ kip/ft})$ $= 2.40 \text{ kip/ft}$	$w_a = D + L$ $= 0.400 \text{ kip/ft} + 1.20 \text{ kip/ft}$ $= 1.60 \text{ kip/ft}$

Per AISC *Specification* Section C2.1(d), for LRFD, perform a second-order analysis and member strength checks using the LRFD load combinations. For ASD, perform a second-order analysis using 1.6 times the ASD load combinations and divide the analysis results by 1.6 for the ASD member strength checks.

Frame analysis gravity loads

The uniform gravity loads to be considered in a second-order analysis on the beam from B to C are:

LRFD	ASD
$w'_u = 2.40 \text{ kip/ft}$	$w'_a = 1.6(1.60 \text{ kip/ft})$ $= 2.56 \text{ kip/ft}$

Concentrated gravity loads to be considered in a second-order analysis on the columns at B and C contributed by adjacent beams are:

LRFD	ASD
$P'_u = \frac{w'_u l}{2}$ $= \frac{(2.40 \text{ kip/ft})(30.0 \text{ ft})}{2}$ $= 36.0 \text{ kips}$	$P'_a = \frac{w'_a l}{2}$ $= \frac{(2.56 \text{ kip/ft})(30.0 \text{ ft})}{2}$ $= 38.4 \text{ kips}$

Concentrated gravity loads on the representative “leaning” column

The load in this column accounts for all gravity loading that is stabilized by the moment frame, but is not directly applied to it.

LRFD	ASD
$P'_{uL} = (60.0 \text{ ft})(2.40 \text{ kip/ft})$ $= 144 \text{ kips}$	$P'_{aL} = (60.0 \text{ ft})(2.56 \text{ kip/ft})$ $= 154 \text{ kips}$

Frame analysis notional loads

Per AISC *Specification* Section C2.2, frame out-of-plumbness must be accounted for either by explicit modeling of the design out-of-plumbness or by the application of notional loads. Notional loads will be used in this example.

From AISC *Specification* Equation C2-1, the notional loads are:

LRFD	ASD
$\alpha = 1.0$	$\alpha = 1.6$
$Y_i = (120 \text{ ft})(2.40 \text{ kip/ft})$ $= 288 \text{ kips}$	$Y_i = (120 \text{ ft})(1.60 \text{ kip/ft})$ $= 192 \text{ kips}$
$N_i = 0.002\alpha Y_i$ (Spec. Eq. C2-1) $= 0.002(1.0)(288 \text{ kips})$ $= 0.576 \text{ kip}$	$N_i = 0.002\alpha Y_i$ (Spec. Eq. C2-1) $= 0.002(1.6)(192 \text{ kips})$ $= 0.614 \text{ kip}$

Summary of applied frame loads

The applied loads are shown in Figure C.1A-2.

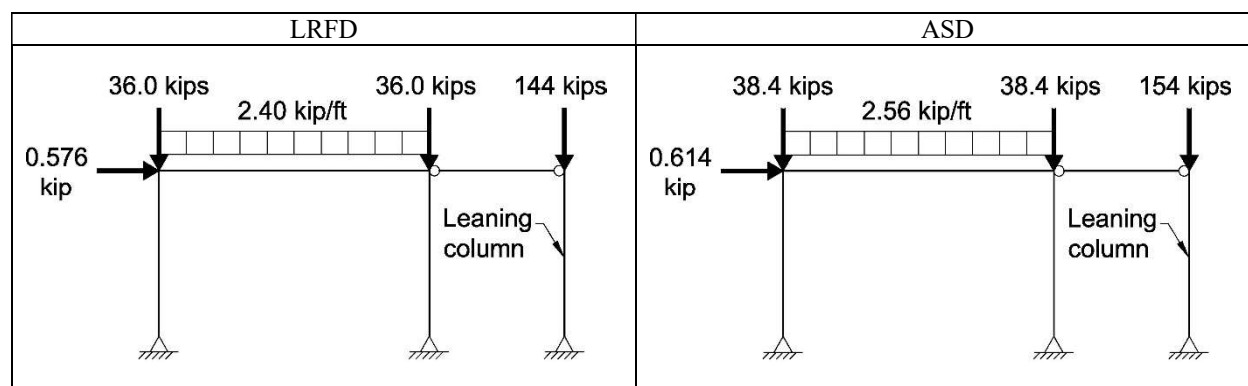


Fig. C.1A-2. Applied loads on the analysis model.

Per AISC *Specification* Section C2.3, conduct the analysis using 80% of the nominal stiffnesses to account for the effects of inelasticity. Assume, subject to verification, that $\alpha P_r/P_{ns}$ is not greater than 0.5; therefore, no additional stiffness reduction is required ($\tau_b = 1.0$).

Half of the gravity load is carried by the columns of the moment-resisting frame. Because the gravity load supported by the moment-resisting frame columns exceeds one-third of the total gravity load tributary to the frame, per AISC *Specification* Section C2.1(b), the effects of $P-\delta$ and $P-\Delta$ must be considered in the frame analysis. This example uses analysis software that accounts for both $P-\Delta$ and $P-\delta$ effects. (If the software used does not account for $P-\delta$ effects, this may be accomplished by subdividing the columns between the footing and beam.)

Figures C.1A-3 and C.1A-4 show results from a first-order and a second-order analysis, respectively. (The first-order analysis is shown for reference only.) In each case, the drift is the average of drifts at grid lines B and C.

First-order results

LRFD	ASD (Reactions and moments divided by 1.6)
$\Delta_{1st} = 0.181$ in. 	$\Delta_{1st} = 0.193$ in. (prior to dividing by 1.6)

Fig. C.1A-3. Results of first-order analysis.

Second-order results

LRFD	ASD (Reactions and moments divided by 1.6)
$\Delta_{2nd} = 0.290$ in. Drift ratio: $\frac{\Delta_{2nd}}{\Delta_{1st}} = \frac{0.290 \text{ in.}}{0.181 \text{ in.}} = 1.60$	$\Delta_{2nd} = 0.321$ in. (prior to dividing by 1.6) Drift ratio: $\frac{\Delta_{2nd}}{\Delta_{1st}} = \frac{0.321 \text{ in.}}{0.193 \text{ in.}} = 1.66$

Fig. C.1A-4. Results of second-order analysis.

Check the assumption that $\alpha P_r / P_{ns} \leq 0.5$ on the column on grid line C.

Because a W12×65 column contains no elements that are slender for uniform compression,

$$\begin{aligned}
 P_{ns} &= F_y A_g \\
 &= (50 \text{ ksi})(19.1 \text{ in.}^2) \\
 &= 955 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\frac{\alpha P_r}{P_{ns}} = \frac{1.0(72.6 \text{ kips})}{955 \text{ kips}}$ $= 0.0760 \leq 0.5 \quad \mathbf{o.k.}$	$\frac{\alpha P_r}{P_{ns}} = \frac{1.6(48.4 \text{ kips})}{955 \text{ kips}}$ $= 0.0811 \leq 0.5 \quad \mathbf{o.k.}$

The stiffness assumption used in the analysis, $\tau_b = 1.0$, is verified.

Note that the drift ratio, 1.60 (LRFD) or 1.66 (ASD), does not exceed the recommended limit of 2.5 from AISC *Specification* Commentary Section C1.

The required axial compressive strength in the columns is 72.6 kips (LRFD) or 48.4 kips (ASD). The required bending moment diagram is linear, varying from zero at the bottom to 127 kip-ft (LRFD) or 84.8 kip-ft (ASD) at the top. These required strengths apply to both columns because the notional load must be applied in each direction.

Although the second-order sway multiplier (drift ratio) is fairly large at 1.60 (LRFD) or 1.66 (ASD), the change in bending moment is small because the only sway moments are those produced by the small notional loads. For load combinations with significant gravity and lateral loadings, the increase in bending moments is larger.

Per AISC *Specification* Section C3, the effective length for flexural buckling of all members is taken as the unbraced length ($K = 1.0$):

$$L_{cx} = 20.0 \text{ ft}$$

$$L_{cy} = 20.0 \text{ ft}$$

EXAMPLE C.1B DESIGN OF A MOMENT FRAME BY THE EFFECTIVE LENGTH METHOD

Given:

Repeat Example C.1A using the effective length method.

Determine the required strengths and effective length factors for the columns in the moment frame shown in Figure C.1B-1 for the maximum gravity load combination, using LRFD and ASD. Use the effective length method.

Columns are unbraced between the footings and roof in the x - x and y - y axes and have pinned bases.

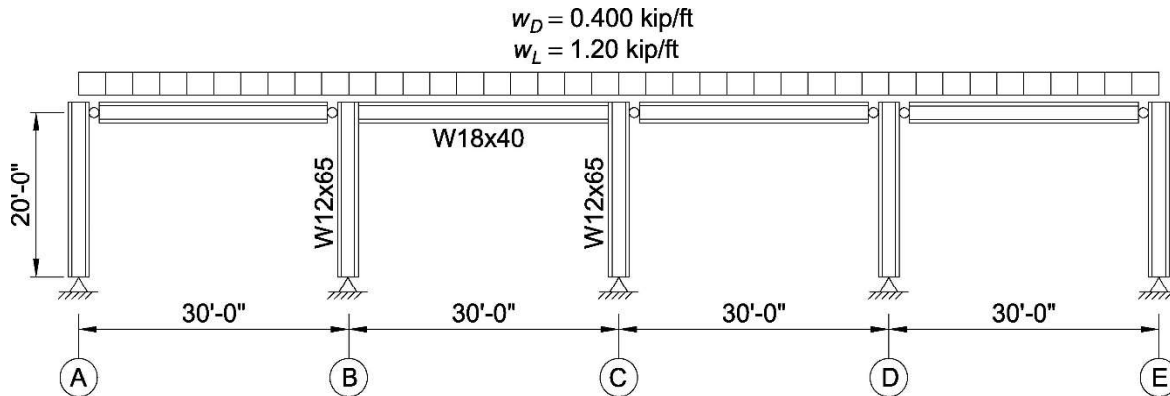


Fig. C.1B-1. Example C.1B moment frame elevation.

Solution:

From AISC *Manual* Table 1-1, the geometric properties are as follows:

$$\begin{aligned} &W12 \times 65 \\ &I_x = 533 \text{ in.}^4 \end{aligned}$$

The beams from grid lines A to B and C to E and the columns at A, D, and E are pinned at both ends and do not contribute to the lateral stability of the frame. There are no P - Δ effects to consider in these members and they may be designed using $L_c = L$.

The moment frame between grid lines B and C is the source of lateral stability and therefore will be evaluated using the provisions of Chapter C of the AISC *Specification*. Although the columns at grid lines A, D, and E do not contribute to lateral stability, the forces required to stabilize them must be considered in the moment-frame analysis. The entire frame from grid line A to E could be modeled, but in this case the model is simplified as shown in Figure C.1B-2, in which the stability loads from the three “leaning” columns are combined into a single representative column.

Check the limitations for the use of the effective length method given in AISC *Specification* Appendix 7, Section 7.2.1:

- The structure supports gravity loads primarily through nominally vertical columns, walls, or frames.
- The ratio of maximum second-order drift to the maximum first-order drift (both determined for LRFD load combinations or 1.6 times ASD load combinations, with stiffness not adjusted as specified in AISC *Specification* Section C2.3) in all stories will be assumed to be no greater than 1.5, subject to verification in the following.

From Chapter 2 of ASCE/SEI 7, the maximum gravity load combinations are:

LRFD	ASD
$w_u = 1.2D + 1.6L$ $= 1.2(0.400 \text{ kip/ft}) + 1.6(1.20 \text{ kip/ft})$ $= 2.40 \text{ kip/ft}$	$w_a = D + L$ $= 0.400 \text{ kip/ft} + 1.20 \text{ kip/ft}$ $= 1.60 \text{ kip/ft}$

Per AISC *Specification* Appendix 7, Section 7.2.2, the analysis must conform to the requirements of AISC *Specification* Section C2.1, with the exception of the stiffness reduction required by the provisions of Section C2.1(a).

Per AISC *Specification* Section C2.1(d), for LRFD perform a second-order analysis and member strength checks using the LRFD load combinations. For ASD, perform a second-order analysis at 1.6 times the ASD load combinations and divide the analysis results by 1.6 for the ASD member strength checks.

Frame analysis gravity loads

The uniform gravity loads to be considered in a second-order analysis on the beam from B to C are:

LRFD	ASD
$w'_u = 2.40 \text{ kip/ft}$	$w'_a = 1.6(1.60 \text{ kip/ft})$ $= 2.56 \text{ kip/ft}$

Concentrated gravity loads to be considered in a second-order analysis on the columns at B and C contributed by adjacent beams are:

LRFD	ASD
$P'_u = \frac{w'_u l}{2}$ $= \frac{(2.40 \text{ kip/ft})(30.0 \text{ ft})}{2}$ $= 36.0 \text{ kips}$	$P'_a = \frac{w'_a l}{2}$ $= \frac{(2.56 \text{ kip/ft})(30.0 \text{ ft})}{2}$ $= 38.4 \text{ kips}$

Concentrated gravity loads on the representative “leaning” column

The load in this column accounts for all gravity loading that is stabilized by the moment frame, but not directly applied to it.

LRFD	ASD
$P'_{uL} = (60.0 \text{ ft})(2.40 \text{ kip/ft})$ $= 144 \text{ kips}$	$P'_{aL} = (60.0 \text{ ft})(2.56 \text{ kip/ft})$ $= 154 \text{ kips}$

Frame analysis notional loads

Per AISC *Specification* Appendix 7, Section 7.2.2, frame out-of-plumbness must be accounted for by the application of notional loads in accordance with AISC *Specification* Section C2.2b. Note that notional loads need to only be applied to the gravity load combinations per AISC *Specification* Section C2.2b(d) when the requirement that $\Delta_{2nd}/\Delta_{1st} \leq 1.7$ (using stiffness adjusted as specified in Section C2.3) is satisfied. Per the User Note in AISC *Specification* Appendix 7, Section 7.2.2, Section C2.2b(d) will be satisfied in all cases where the effective length method is applicable, and therefore the notional load need only be applied in gravity-only load cases.

From AISC *Specification* Equation C2-1, the notional loads are:

LRFD	ASD
$\alpha = 1.0$	$\alpha = 1.6$
$Y_i = (120 \text{ ft})(2.40 \text{ kip/ft})$ $= 288 \text{ kips}$	$Y_i = (120 \text{ ft})(1.60 \text{ kip/ft})$ $= 192 \text{ kips}$
$N_i = 0.002\alpha Y_i$ (Spec. Eq. C2-1) $= 0.002(1.0)(288 \text{ kips})$ $= 0.576 \text{ kip}$	$N_i = 0.002\alpha Y_i$ (Spec. Eq. C2-1) $= 0.002(1.6)(192 \text{ kips})$ $= 0.614 \text{ kip}$

Summary of applied frame loads

The applied loads are shown in Figure C.1B-2.

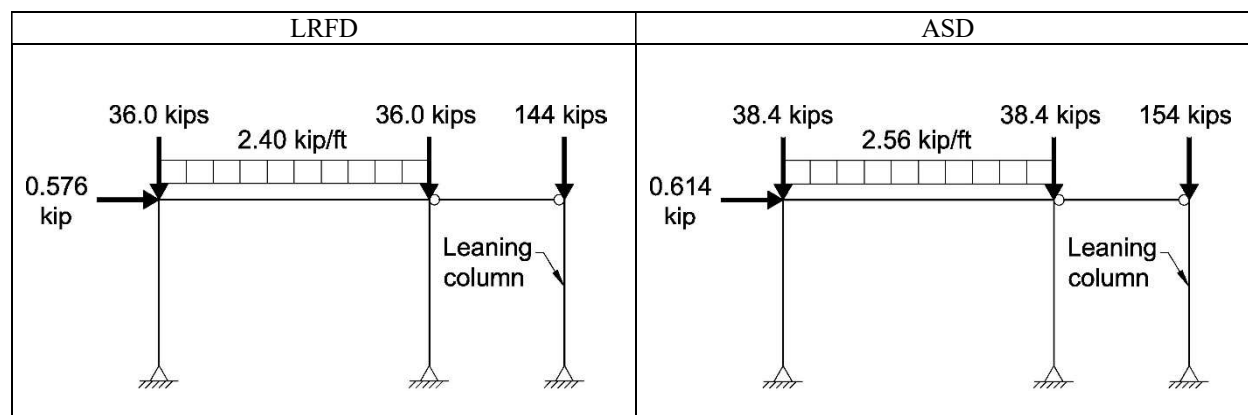


Fig. C.1B-2. Applied loads on the analysis model.

Per AISC *Specification* Appendix 7, Section 7.2.2, conduct the analysis using the full nominal stiffnesses.

Half of the gravity load is carried by the columns of the moment-resisting frame. Because the gravity load supported by the moment-resisting frame columns exceeds one-third of the total gravity load tributary to the frame, per AISC *Specification* Section C2.1(b), the effects of $P-\delta$ on the response of the structure must be considered in the frame analysis. This example uses analysis software that accounts for both $P-\Delta$ and $P-\delta$ effects. (If the software used does not account for $P-\delta$ effects, this may be accomplished by subdividing columns between the footing and beam.)

Figures C.1B-3 and C.1B-4 show results from a first-order and second-order analysis, respectively. In each case, the drift is the average of drifts at grid lines B and C.

First-order results

LRFD	ASD (Reactions and moments divided by 1.6)
$\Delta_{1st} = 0.145$ in. 	$\Delta_{1st} = 0.155$ in. (prior to dividing by 1.6)

Fig. C-1B-3. Results of first-order analysis.

Second-order results

LRFD	ASD (Reactions and moments divided by 1.6)
$\Delta_{2nd} = 0.204$ in. Drift ratio: $\frac{\Delta_{2nd}}{\Delta_{1st}} = \frac{0.204 \text{ in.}}{0.145 \text{ in.}} = 1.41$	$\Delta_{2nd} = 0.223$ in. (prior to dividing by 1.6) Drift ratio: $\frac{\Delta_{2nd}}{\Delta_{1st}} = \frac{0.223 \text{ in.}}{0.155 \text{ in.}} = 1.44$

Fig. C-1B-4. Results of second-order analysis.

The assumption that the ratio of the maximum second-order drift to the maximum first-order drift is no greater than 1.5 is verified; therefore, the effective length method is permitted.

Although the second-order sway multiplier is fairly large at approximately 1.41 (LRFD) or 1.44 (ASD), the change in bending moment is small because the only sway moments for this load combination are those produced by the small notional loads. For load combinations with significant gravity and lateral loadings, the increase in bending moments is larger.

Calculate the in-plane effective length factor, K_x , using the “story stiffness approach” and Equation C-A-7-5 presented in AISC *Specification Commentary* Appendix 7, Section 7.2. With $K_x = K_2$:

$$K_x = \sqrt{\frac{P_{story}}{R_M P_r} \left(\frac{\pi^2 EI}{L^2} \right) \left(\frac{\Delta_H}{HL} \right)} \geq \sqrt{\frac{\pi^2 EI}{L^2} \left(\frac{\Delta_H}{1.7 H_{col} L} \right)} \quad (\text{Spec. Eq. C-A-7-5})$$

Calculate the total load in all columns, P_{story} , as follows:

LRFD	ASD
$P_{story} = (2.40 \text{ kip/ft})(120 \text{ ft})$ $= 288 \text{ kips}$	$P_{story} = (1.60 \text{ kip/ft})(120 \text{ ft})$ $= 192 \text{ kips}$

Calculate the coefficient to account for the influence of $P-\delta$ on $P-\Delta$, R_M , as follows, using AISC *Specification* Commentary Appendix 7, Equation C-A-7-6:

LRFD	ASD
$P_{mf} = 71.5 \text{ kips} + 72.5 \text{ kips}$ $= 144 \text{ kips}$	$P_{mf} = 47.6 \text{ kips} + 48.4 \text{ kips}$ $= 96.0 \text{ kips}$
$R_M = 1 - 0.15(P_{mf}/P_{story}) \quad (\text{Spec. Eq. C-A-7-6})$ $= 1 - 0.15 \left(\frac{144 \text{ kips}}{288 \text{ kips}} \right)$ $= 0.925$	$R_M = 1 - 0.15(P_{mf}/P_{story}) \quad (\text{Spec. Eq. C-A-7-6})$ $= 1 - 0.15 \left(\frac{96.0 \text{ kips}}{192 \text{ kips}} \right)$ $= 0.925$

Calculate the Euler buckling strength of one moment frame.

$$\frac{\pi^2 EI_x}{L^2} = \frac{\pi^2 (29,000 \text{ ksi})(533 \text{ in.}^4)}{[(20.0 \text{ ft})(12 \text{ in./ft})]^2}$$

$$= 2,650 \text{ kips}$$

From AISC *Specification* Commentary Equation C-A-7-5, for the column at line C:

LRFD	ASD
$K_x = \sqrt{\frac{P_{story}}{R_M P_r} \left(\frac{\pi^2 EI}{L^2} \right) \left(\frac{\Delta_H}{HL} \right)}$ $\geq \sqrt{\left(\frac{\pi^2 EI}{L^2} \right) \left(\frac{\Delta_H}{1.7 H_{col} L} \right)}$ $= \sqrt{\left[\frac{288 \text{ kips}}{(0.925)(72.5 \text{ kips})} \right] (2,650 \text{ kips})}$ $\times \left[\frac{0.145 \text{ in.}}{(0.576 \text{ kip})(20.0 \text{ ft})(12 \text{ in./ft})} \right]$ $> \sqrt{\left[\frac{2,650 \text{ kips}}{1.7(6.21 \text{ kips})(20.0 \text{ ft})(12 \text{ in./ft})} \right]}$ $= 3.45 > 0.389$ Use $K_x = 3.45$	$K_x = \sqrt{\frac{1.6 P_{story}}{R_M (1.6) P_r} \left(\frac{\pi^2 EI}{L^2} \right) \left(\frac{\Delta_H}{HL} \right)}$ $\geq \sqrt{\left(\frac{\pi^2 EI}{L^2} \right) \left(\frac{\Delta_H}{1.7(1.6) H_{col} L} \right)}$ $= \sqrt{\left[\frac{1.6(192 \text{ kips})}{0.925(1.6)(48.4 \text{ kips})} \right] (2,650 \text{ kips})}$ $\times \left[\frac{0.155 \text{ in.}}{(0.614 \text{ kip})(20.0 \text{ ft})(12 \text{ in./ft})} \right]$ $> \sqrt{\left[\frac{2,650 \text{ kips}}{1.7(1.6)(4.14 \text{ kips})(20.0 \text{ ft})(12 \text{ in./ft})} \right]}$ $= 3.46 > 0.390$ Use $K_x = 3.46$

Note that the column loads are multiplied by 1.6 for ASD in Equation C-A-7-5. In the calculations above, the H_{col} values used include the shear in the column from both the gravity and lateral loads. To obtain more precise results, the designer may follow the commentary to AISC *Specification* Appendix 7, Section 7.2, which states that the H_{col} term is the shear produced by the lateral forces used to compute Δ_H .

With $K_x = 3.46$ and $K_y = 1.00$, the column available strengths can be verified for the given member sizes for the second-order forces (calculations not shown), using the following effective lengths:

$$\begin{aligned}
 L_{cx} &= K_x L_x \\
 &= 3.46(20.0 \text{ ft}) \\
 &= 69.2 \text{ ft}
 \end{aligned}$$

$$\begin{aligned}
 L_{cy} &= K_y L_y \\
 &= 1.00(20.0 \text{ ft}) \\
 &= 20.0 \text{ ft}
 \end{aligned}$$

Chapter D

Design of Members for Tension

D1. SLENDERNESS LIMITATIONS

AISC *Specification* Section D1 does not establish a slenderness limit for tension members but recommends limiting L/r to a maximum of 300. This is not an absolute requirement. Rods are specifically excluded from this recommendation.

D2. TENSILE STRENGTH

Both tensile yielding strength and tensile rupture strength must be considered for the design of tension members. It is not unusual for tensile rupture strength to govern the design of a tension member, particularly for small members with holes or heavier sections with multiple rows of holes.

For preliminary design, tables are provided in Part 5 of the AISC *Manual* for W-shapes, L-shapes, WT-shapes, rectangular HSS, square HSS, round HSS, Pipe, and 2L-shapes. The calculations in these tables for available tensile rupture strength assume an effective area, A_e , of $0.75A_g$. The gross area, A_g , is the total cross-sectional area of the member. If the actual effective area is greater than $0.75A_g$, the tabulated values will be conservative and calculations can be performed to obtain higher available strengths. If the actual effective area is less than $0.75A_g$, the tabulated values will be unconservative and calculations are necessary to determine the available strength.

D3. EFFECTIVE NET AREA

In computing net area, A_n , AISC *Specification* Section B4.3b requires that an extra $1/16$ in. be added to the bolt hole diameter. A computation of the effective area for a chain of holes is presented in Example D.9.

Unless all elements of the cross section are connected, $A_e = A_n U$, where U is a reduction factor to account for shear lag. The appropriate values of U can be obtained from AISC *Specification* Table D3.1.

D4. BUILT-UP MEMBERS

The limitations for connections of built-up members are discussed in Section D4 of the AISC *Specification*.

D5. PIN-CONNECTED MEMBERS

An example of a pin-connected member is given in Example D.7.

D6. EYEBARS

An example of an eyebar is given in Example D.8. The strength of an eyebar meeting the dimensional requirements of AISC *Specification* Section D6 is governed by tensile yielding of the body.

EXAMPLE D.1 W-SHAPE TENSION MEMBER**Given:**

Select an ASTM A992/A992M W-shape with 8 in. nominal depth to carry a dead load of 30 kips and a live load of 90 kips in tension. The member is 25.0 ft long. Verify the member strength by both LRFD and ASD with the bolted end connection as shown in Figure D.1-1. Verify that the member satisfies the recommended slenderness limit. Assume that connection limit states do not govern.

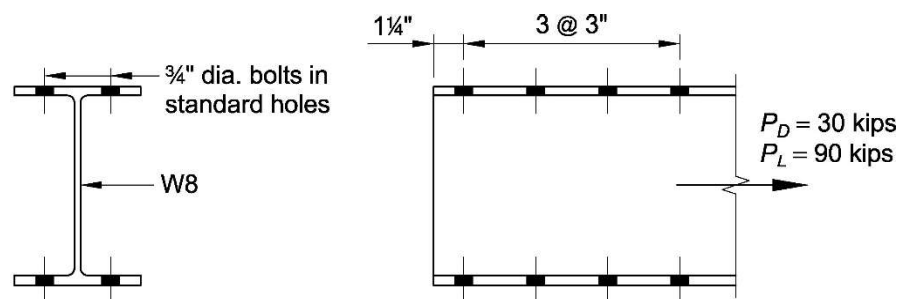


Fig D.1-1. Connection geometry for Example D.1.

Solution:

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
$P_u = 1.2(30 \text{ kips}) + 1.6(90 \text{ kips})$ $= 180 \text{ kips}$	$P_a = 30 \text{ kips} + 90 \text{ kips}$ $= 120 \text{ kips}$

From AISC *Manual* Table 5-1, try a W8×21.

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992/A992M

$F_y = 50 \text{ ksi}$

$F_u = 65 \text{ ksi}$

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W8×21

$A_g = 6.16 \text{ in.}^2$

$b_f = 5.27 \text{ in.}$

$t_f = 0.400 \text{ in.}$

$d = 8.28 \text{ in.}$

$r_y = 1.26 \text{ in.}$

The WT-shape corresponding to a W8×21 is a WT4×10.5. From AISC *Manual* Table 1-8, the geometric properties are as follows:

WT4×10.5

$\bar{y} = 0.831 \text{ in.}$

From AISC *Specification* Table J3.3, the hole diameter for $\frac{3}{4}$ in. diameter bolts in standard holes is:

$$d_h = \frac{13}{16} \text{ in.}$$

Tensile Yielding

From AISC *Manual* Table 5-1, the available tensile yielding strength of a W8×21 is:

LRFD	ASD
$\phi_t P_n = 277 \text{ kips} > 180 \text{ kips} \quad \mathbf{o.k.}$	$\frac{P_n}{\Omega_t} = 184 \text{ kips} > 120 \text{ kips} \quad \mathbf{o.k.}$

Tensile Rupture

Verify the table assumption that $A_e/A_g \geq 0.75$ for this connection.

From the description of the element in AISC *Specification* Table D3.1, Case 7, calculate the shear lag factor, U , as the larger of the values from AISC *Specification* Section D3, Table D3.1 Case 2 and Case 7.

From AISC *Specification* Section D3, for open cross sections, U need not be less than the ratio of the gross area of the connected element(s) to the member gross area.

$$\begin{aligned}
 U &= \frac{2b_f t_f}{A_g} \\
 &= \frac{2(5.27 \text{ in.})(0.400 \text{ in.})}{6.16 \text{ in.}^2} \\
 &= 0.684
 \end{aligned}$$

Case 2: Determine U based on two WT-shapes per AISC *Specification* Commentary Figure C-D3.1, with $\bar{x} = \bar{y} = 0.831 \text{ in.}$ and where l is the length of connection.

$$\begin{aligned}
 U &= 1 - \frac{\bar{x}}{l} \\
 &= 1 - \frac{0.831 \text{ in.}}{9.00 \text{ in.}} \\
 &= 0.908
 \end{aligned}$$

Case 7:

$$\begin{aligned}
 b_f &= 5.27 \text{ in.} \\
 \frac{2}{3}d &= \frac{2}{3}(8.28 \text{ in.}) \\
 &= 5.52 \text{ in.}
 \end{aligned}$$

Because the flange is connected with three or more fasteners per line in the direction of loading and $b_f < \frac{2}{3}d$:

$$U = 0.85$$

Therefore, use the larger $U = 0.908$.

Calculate A_n using AISC *Specification* Section B4.3b.

$$\begin{aligned}
 A_n &= A_g - 4(d_h + 1/16 \text{ in.})t_f \\
 &= 6.16 \text{ in.}^2 - 4(13/16 \text{ in.} + 1/16 \text{ in.})(0.400 \text{ in.}) \\
 &= 4.76 \text{ in.}^2
 \end{aligned}$$

Calculate A_e using AISC *Specification* Section D3.

$$\begin{aligned}
 A_e &= A_n U && (\text{Spec. Eq. D3-1}) \\
 &= (4.76 \text{ in.}^2)(0.908) \\
 &= 4.32 \text{ in.}^2
 \end{aligned}$$

$$\begin{aligned}
 \frac{A_e}{A_g} &= \frac{4.32 \text{ in.}^2}{6.16 \text{ in.}^2} \\
 &= 0.701 < 0.75
 \end{aligned}$$

Because $A_e/A_g < 0.75$, the tensile rupture strength from AISC *Manual* Table 5-1 is not valid. The available tensile rupture strength is determined using AISC *Specification* Section D2 as follows:

$$\begin{aligned}
 P_n &= F_u A_e && (\text{Spec. Eq. D2-2}) \\
 &= (65 \text{ ksi})(4.32 \text{ in.}^2) \\
 &= 281 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi_t = 0.75$	$\Omega_t = 2.00$
$\phi_t P_n = 0.75(281 \text{ kips})$ $= 211 \text{ kips} > 180 \text{ kips} \quad \mathbf{o.k.}$	$\frac{P_n}{\Omega_t} = \frac{281 \text{ kips}}{2.00}$ $= 141 \text{ kips} > 120 \text{ kips} \quad \mathbf{o.k.}$

Note that the W8×21 available tensile strength is governed by the tensile rupture limit state at the end connection versus the tensile yielding limit state.

See Chapter J for illustrations of connection limit state checks.

Check Recommended Slenderness Limit

$$\begin{aligned}
 \frac{L}{r} &= \frac{(25.0 \text{ ft})(12 \text{ in./ft})}{1.26 \text{ in.}} \\
 &= 238 < 300 \text{ from AISC } \textit{Specification} \text{ Section D1} \quad \mathbf{o.k.}
 \end{aligned}$$

EXAMPLE D.2 SINGLE-ANGLE TENSION MEMBER**Given:**

Verify the tensile strength of an ASTM A572/A572M Gr. 50 L4×4×½ with one line of four ¾ in. diameter bolts in standard holes, as shown in Figure D.2-1. The member carries a dead load of 20 kips and a live load of 60 kips in tension. Additionally, calculate at what length this tension member would cease to satisfy the recommended slenderness limit. Assume that connection limit states do not govern.

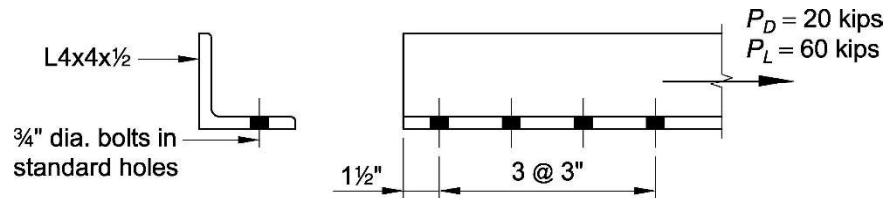


Fig. D.2-1. Connection geometry for Example D.2.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A572/A572M Gr. 50

$F_y = 50$ ksi

$F_u = 65$ ksi

From AISC *Manual* Table 1-7, the geometric properties are as follows:

L4×4×½

$A_g = 3.75$ in.²

$r_z = 0.776$ in.

$\bar{x} = 1.18$ in.

From AISC *Specification* Table J3.3, the hole diameter for ¾ in. diameter bolts in standard holes is:

$$d_h = 13/16 \text{ in.}$$

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
$P_u = 1.2(20 \text{ kips}) + 1.6(60 \text{ kips})$ $= 120 \text{ kips}$	$P_a = 20 \text{ kips} + 60 \text{ kips}$ $= 80.0 \text{ kips}$

Tensile Yielding

The available tensile yielding strength is determined using AISC *Specification* Section D2 as follows:

$$\begin{aligned}
 P_n &= F_y A_g && (\text{Spec. Eq. D2-1}) \\
 &= (50 \text{ ksi})(3.75 \text{ in.}^2) \\
 &= 188 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi_t = 0.90$	$\Omega_t = 1.67$
$\phi_t P_n = 0.90(188 \text{ kips})$ $= 169 \text{ kips} > 120 \text{ kips} \quad \mathbf{o.k.}$	$\frac{P_n}{\Omega_t} = \frac{188 \text{ kips}}{1.67}$ $= 113 \text{ kips} > 80.0 \text{ kips} \quad \mathbf{o.k.}$

Tensile Rupture

From the description of the element in AISC *Specification* Table D3.1 Case 8, calculate the shear lag factor, U , as the larger of the values from AISC *Specification* Section D3, Table D3.1 Case 2 and Case 8.

From AISC *Specification* Section D3, for open cross sections, U need not be less than the ratio of the gross area of the connected element(s) to the member gross area. Half of the member is connected, therefore, the minimum value of U is:

$$U = 0.500$$

Case 2, where l is the length of connection and $\bar{y} = \bar{x}$:

$$\begin{aligned} U &= 1 - \frac{\bar{x}}{l} \\ &= 1 - \frac{1.18 \text{ in.}}{9.00 \text{ in.}} \\ &= 0.869 \end{aligned}$$

Case 8, with four or more fasteners per line in the direction of loading:

$$U = 0.80$$

Therefore, use the larger $U = 0.869$.

Calculate A_n using AISC *Specification* Section B4.3b.

$$\begin{aligned} A_n &= A_g - (d_h + 1/16 \text{ in.})t \\ &= 3.75 \text{ in.} - (13/16 \text{ in.} + 1/16 \text{ in.})(1/2 \text{ in.}) \\ &= 3.31 \text{ in.}^2 \end{aligned}$$

Calculate A_e using AISC *Specification* Section D3.

$$\begin{aligned} A_e &= A_n U \\ &= (3.31 \text{ in.}^2)(0.869) \\ &= 2.88 \text{ in.}^2 \end{aligned} \quad (\text{Spec. Eq. D3-1})$$

The available tensile rupture strength is determined using AISC *Specification* Section D2 as follows:

$$\begin{aligned} P_n &= F_u A_e \\ &= (65 \text{ ksi})(2.88 \text{ in.}^2) \\ &= 187 \text{ kips} \end{aligned} \quad (\text{Spec. Eq. D2-2})$$

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(187 \text{ kips})$ $= 140 \text{ kips} > 120 \text{ kips} \quad \mathbf{o.k.}$	$\Omega_t = 2.00$ $\frac{P_n}{\Omega_t} = \frac{187 \text{ kips}}{2.00}$ $= 93.5 \text{ kips} > 80.0 \text{ kips} \quad \mathbf{o.k.}$

The L4×4×½ available tensile strength is governed by the tensile rupture limit state.

Recommended L_{max}

Using AISC *Specification* Section D1:

$$\begin{aligned}
 L_{max} &= 300r_z \\
 &= 300 \left(\frac{0.776 \text{ in.}}{12 \text{ in./ft}} \right) \\
 &= 19.4 \text{ ft}
 \end{aligned}$$

Note: The L/r limit is a recommendation, not a requirement.

See Chapter J for illustrations of connection limit state checks.

EXAMPLE D.3 WT-SHAPE TENSION MEMBER**Given:**

An ASTM A992/A992M WT6×20 member has a length of 30 ft and carries a dead load of 40 kips and a live load of 120 kips in tension. As shown in Figure D3-1, the end connection is fillet welded on each side for 16 in. Verify the member tensile strength by both LRFD and ASD. Assume that the gusset plate and the weld are satisfactory.

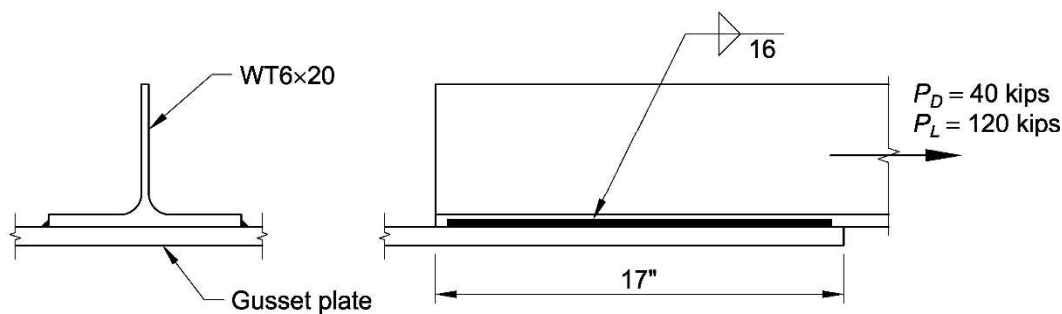


Fig. D.3-1. Connection geometry for Example D.3.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992/A992M

$F_y = 50$ ksi

$F_u = 65$ ksi

From AISC *Manual* Table 1-8, the geometric properties are as follows:

WT6×20

$A_g = 5.84$ in.²

$b_f = 8.01$ in.

$t_f = 0.515$ in.

$r_x = 1.57$ in.

$\bar{y} = 1.09$ in.

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
$P_u = 1.2(40 \text{ kips}) + 1.6(120 \text{ kips})$ $= 240 \text{ kips}$	$P_a = 40 \text{ kips} + 120 \text{ kips}$ $= 160 \text{ kips}$

Tensile Yielding

Check the tensile yielding limit state using AISC *Manual* Table 5-3.

LRFD	ASD
$\phi_t P_n = 263 \text{ kips} > 240 \text{ kips}$ o.k.	$\frac{P_n}{\Omega_t} = 175 \text{ kips} > 160 \text{ kips}$ o.k.

Tensile Rupture

Check the tensile rupture limit state using AISC *Manual* Table 5-3.

LRFD	ASD
$\phi_t P_n = 214 \text{ kips} < 240 \text{ kips} \quad \mathbf{n.g.}$	$\frac{P_n}{\Omega_t} = 142 \text{ kips} < 160 \text{ kips} \quad \mathbf{n.g.}$

The tabulated available rupture strengths don't work and may be conservative for this case; therefore, calculate the exact solution.

Calculate U as the larger of the values from AISC *Specification* Section D3 and Table D3.1 Case 4.

From AISC *Specification* Section D3, for open cross sections, U need not be less than the ratio of the gross area of the connected element(s) to the member gross area.

$$\begin{aligned}
 U &= \frac{b_f t_f}{A_g} \\
 &= \frac{(8.01 \text{ in.})(0.515 \text{ in.})}{5.84 \text{ in.}^2} \\
 &= 0.706
 \end{aligned}$$

Case 4, where l is the length of the connection and $\bar{x} = \bar{y}$:

$$\begin{aligned}
 U &= \frac{3l^2}{3l^2 + w^2} \left(1 - \frac{\bar{x}}{l} \right) \\
 &= \left[\frac{3(16.0 \text{ in.})^2}{3(16.0 \text{ in.})^2 + (8.01 \text{ in.})^2} \right] \left(1 - \frac{1.09 \text{ in.}}{16.0 \text{ in.}} \right) \\
 &= 0.860
 \end{aligned}$$

Therefore, use $U = 0.860$.

Calculate A_n using AISC *Specification* Section B4.3b. Because there are no reductions due to bolt holes or notches:

$$\begin{aligned}
 A_n &= A_g \\
 &= 5.84 \text{ in.}^2
 \end{aligned}$$

Calculate A_e using AISC *Specification* Section D3.

$$\begin{aligned}
 A_e &= A_n U && (\text{Spec. Eq. D3-1}) \\
 &= (5.84 \text{ in.}^2)(0.860) \\
 &= 5.02 \text{ in.}^2
 \end{aligned}$$

The available tensile rupture strength is determined using AISC *Specification* Section D2 as follows:

$$\begin{aligned}
 P_n &= F_u A_e && (\text{Spec. Eq. D2-2}) \\
 &= (65 \text{ ksi})(5.02 \text{ in.}^2) \\
 &= 326 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(326 \text{ kips})$ $= 245 \text{ kips} > 240 \text{ kips} \quad \mathbf{o.k.}$	$\Omega_t = 2.00$ $\frac{P_n}{\Omega_t} = \frac{326 \text{ kips}}{2.00}$ $= 163 \text{ kips} > 160 \text{ kips} \quad \mathbf{o.k.}$

Alternately, the available tensile rupture strengths can be determined by modifying the tabulated values. The available tensile rupture strengths published in the tension member selection tables are based on the assumption that $A_e = 0.75A_g$. The actual available strengths can be determined by adjusting the values from AISC *Manual* Table 5-3 as follows:

LRFD	ASD
$\phi_t P_n = (214 \text{ kips}) \left(\frac{A_e}{0.75 A_g} \right)$ $= (214 \text{ kips}) \left[\frac{5.02 \text{ in.}^2}{0.75(5.84 \text{ in.}^2)} \right]$ $= 245 \text{ kips} > 240 \text{ kips} \quad \mathbf{o.k.}$	$\frac{P_n}{\Omega_t} = (142 \text{ kips}) \left(\frac{A_e}{0.75 A_g} \right)$ $= (142 \text{ kips}) \left[\frac{5.02 \text{ in.}^2}{0.75(5.84 \text{ in.}^2)} \right]$ $= 163 \text{ kips} > 160 \text{ kips} \quad \mathbf{o.k.}$

Recommended Slenderness Limit

$$\begin{aligned}
 \frac{L}{r_x} &= \frac{(30.0 \text{ ft})(12 \text{ in./ft})}{1.57 \text{ in.}} \\
 &= 229 < 300 \text{ from AISC Specification Section D1} \quad \mathbf{o.k.}
 \end{aligned}$$

Note: The L/r_x limit is a recommendation, not a requirement.

See Chapter J for illustrations of connection limit state checks.

EXAMPLE D.6 DOUBLE-ANGLE TENSION MEMBER**Given:**

An ASTM A572/A572M Gr. 50 $2L4 \times 4 \times \frac{1}{2}$ ($\frac{3}{8}$ in. separation) has one line of eight $\frac{3}{4}$ in. diameter bolts in standard holes and is 25 ft in length as shown in Figure D.6-1. The double angle is carrying a dead load of 40 kips and a live load of 120 kips in tension. Verify the member tensile strength. Assume that the gusset plate and bolts are satisfactory.

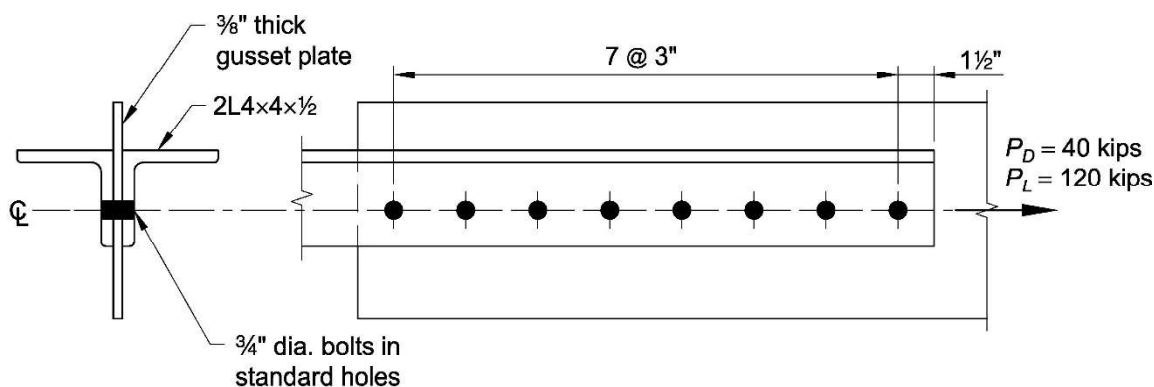


Fig. D.6-1. Connection geometry for Example D.6.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A572/A572M Gr. 50

$F_y = 50$ ksi

$F_u = 65$ ksi

From AISC *Manual* Tables 1-7 and 1-15, the geometric properties are as follows:

$L4 \times 4 \times \frac{1}{2}$

$\bar{x} = 1.18$ in.

$2L4 \times 4 \times \frac{1}{2}$ ($s = \frac{3}{8}$ in.)

$A_g = 7.50$ in.²

$r_y = 1.83$ in.

$r_x = 1.21$ in.

From AISC *Specification* Table J3.3, the hole diameter for $\frac{3}{4}$ in. diameter bolts in standard holes is:

$$d_h = \frac{13}{16} \text{ in.}$$

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
$P_u = 1.2(40 \text{ kips}) + 1.6(120 \text{ kips})$ $= 240 \text{ kips}$	$P_a = 40 \text{ kips} + 120 \text{ kips}$ $= 160 \text{ kips}$

Tensile Yielding

Check the tensile yielding limit state using AISC *Manual* Table 5-8.

LRFD	ASD
$\phi_t P_n = 338 \text{ kips} > 240 \text{ kips} \quad \mathbf{o.k.}$	$\frac{P_n}{\Omega_t} = 225 \text{ kips} > 160 \text{ kips} \quad \mathbf{o.k.}$

Tensile Rupture

Determine the available tensile rupture strength using AISC *Specification* Section D2. Calculate U as the larger of the values from AISC *Specification* Section D3, Table D3.1 Case 2 and Case 8.

From AISC *Specification* Section D3, for open cross sections, U need not be less than the ratio of the gross area of the connected element(s) to the member gross area. Half of the member is connected, therefore, the minimum U value is:

$$U = 0.50$$

From Case 2, where l is the length of connection:

$$\begin{aligned} U &= 1 - \frac{\bar{x}}{l} \\ &= 1 - \frac{1.18 \text{ in.}}{21.0 \text{ in.}} \\ &= 0.944 \end{aligned}$$

From Case 8, with four or more fasteners per line in the direction of loading:

$$U = 0.80$$

Therefore, use $U = 0.944$.

Calculate A_n using AISC *Specification* Section B4.3b.

$$\begin{aligned} A_n &= A_g - 2(d_h + 1/16 \text{ in.})t \\ &= 7.50 \text{ in.}^2 - 2(13/16 \text{ in.} + 1/16 \text{ in.})(1/2 \text{ in.}) \\ &= 6.63 \text{ in.}^2 \end{aligned}$$

Calculate A_e using AISC *Specification* Section D3.

$$\begin{aligned} A_e &= A_n U && (\text{Spec. Eq. D3-1}) \\ &= (6.63 \text{ in.}^2)(0.944) \\ &= 6.26 \text{ in.}^2 \end{aligned}$$

The available tensile rupture strength is determined using AISC *Specification* Section D2 as follows:

$$\begin{aligned} P_n &= F_u A_e && (\text{Spec. Eq. D2-2}) \\ &= (65 \text{ ksi})(6.26 \text{ in.}^2) \\ &= 407 \text{ kips} \end{aligned}$$

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(407 \text{ kips})$ $= 305 \text{ kips} > 240 \text{ kips} \quad \mathbf{o.k.}$	$\Omega_t = 2.00$ $\frac{P_n}{\Omega_t} = \frac{407 \text{ kips}}{2.00}$ $= 204 \text{ kips} > 160 \text{ kips} \quad \mathbf{o.k.}$

Note that AISC *Manual* Table 5-8 could also be conservatively used because $A_e \geq 0.75A_g$.

The double-angle available tensile strength is governed by the tensile rupture limit state.

Recommended Slenderness Limit

$$\frac{L}{r_x} = \frac{(25.0 \text{ ft})(12 \text{ in./ft})}{1.21 \text{ in.}}$$

$$= 248 < 300 \text{ from AISC Specification Section D1} \quad \mathbf{o.k.}$$

Note: From AISC *Specification* Section D4, the longitudinal spacing of connectors between components of built-up members should preferably limit the slenderness ratio in any component between the connectors to a maximum of 300.

See Chapter J for illustrations of connection limit state checks.

EXAMPLE D.9 PLATE WITH STAGGERED BOLTS**Given:**

Compute A_n and A_e for a 14-in.-wide and $\frac{1}{2}$ -in.-thick plate subject to tensile loading with staggered holes as shown in Figure D.9-1.

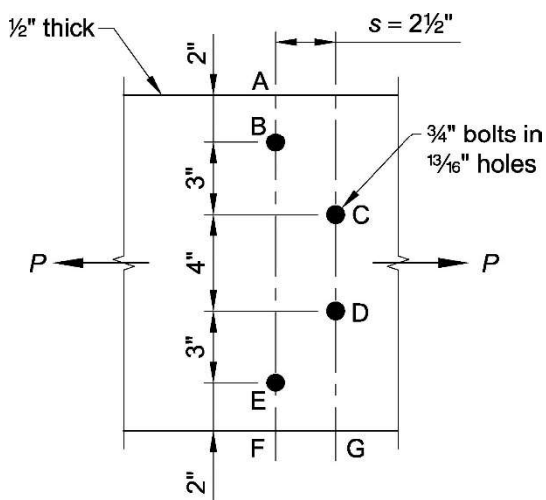


Fig. D.9-1. Connection geometry for Example D.9.

Solution:

Calculate the net hole diameter using AISC *Specification* Section B4.3b.

$$\begin{aligned} d_{net} &= d_h + \frac{1}{16} \text{ in.} \\ &= \frac{13}{16} \text{ in.} + \frac{1}{16} \text{ in.} \\ &= 0.875 \text{ in.} \end{aligned}$$

Compute the net width for all possible paths across the plate. Because of symmetry, many of the net widths are identical and need not be calculated.

$$w = 14.0 \text{ in.} - \sum d_{net} + \sum \frac{s^2}{4g} \text{ from AISC } \textit{Specification} \text{ Section B4.3b.}$$

Line A-B-E-F:

$$\begin{aligned} w &= 14 \text{ in.} - 2(0.875 \text{ in.}) \\ &= 12.3 \text{ in.} \end{aligned}$$

Line A-B-C-D-E-F:

$$\begin{aligned} w &= 14 \text{ in.} - 4(0.875 \text{ in.}) + \frac{(2\frac{1}{2} \text{ in.})^2}{4(3 \text{ in.})} + \frac{(2\frac{1}{2} \text{ in.})^2}{4(3 \text{ in.})} \\ &= 11.5 \text{ in.} \end{aligned}$$

Line A-B-C-D-G:

$$w = 14 \text{ in.} - 3(0.875 \text{ in.}) + \frac{(2\frac{1}{2} \text{ in.})^2}{4(3 \text{ in.})}$$

$$= 11.9 \text{ in.}$$

Line A-B-D-E-F:

$$w = 14 \text{ in.} - 3(0.875 \text{ in.}) + \frac{(2\frac{1}{2} \text{ in.})^2}{4(7 \text{ in.})} + \frac{(2\frac{1}{2} \text{ in.})^2}{4(3 \text{ in.})}$$

$$= 12.1 \text{ in.}$$

Line A-B-C-D-E-F controls the width, w , therefore:

$$A_n = wt$$

$$= (11.5 \text{ in.})(\frac{1}{2} \text{ in.})$$

$$= 5.75 \text{ in.}^2$$

Determine U from AISC *Specification* Table D3.1.

From AISC *Specification* Table D3.1 Case 1, because tension load is transmitted to all elements by the fasteners,

$$U = 1.0$$

$$A_e = A_n U$$

$$= (5.75 \text{ in.}^2)(1.0)$$

$$= 5.75 \text{ in.}^2$$
(Spec. Eq. D3-1)

Chapter E

Design of Members for Compression

This chapter covers the design of compression members, the most common of which are columns. The AISC *Manual* includes design tables for the following compression member types in their most commonly available grades:

- W-shapes and HP-shapes
- Rectangular, square, and round HSS
- Pipes
- WT-shapes
- Double angles
- Single angles

LRFD and ASD information is presented side-by-side for quick selection, design, or verification. All tables account for the reduced strength of sections with slender elements.

The design and selection method for both LRFD and ASD is similar to that of previous editions of the AISC *Specification* and will provide similar designs. In the AISC *Specification*, LRFD and ASD will provide identical designs when the live load is approximately three times the dead load.

The design of built-up shapes with slender elements can be tedious and time consuming, and it is recommended that standard rolled shapes be used whenever possible.

E1. GENERAL PROVISIONS

The design compressive strength, $\phi_c P_n$, and the allowable compressive strength, P_n/Ω_c , are determined as follows:

P_n = nominal compressive strength is the lowest value obtained based on the applicable limit states of flexural buckling, torsional buckling, and flexural-torsional buckling, kips

$\phi_c = 0.90$ (LRFD)

$\Omega_c = 1.67$ (ASD)

Because the available nominal stress, F_n , is used extensively in calculations for compression members, it has been tabulated in AISC *Manual* Table 4-14 for all of the common steel yield strengths.

E2. EFFECTIVE LENGTH

In the AISC *Specification*, there is no limit on slenderness, L_c/r . Per the User Note in AISC *Specification* Section E2, for members designed on the basis of compression it is recommended that L_c/r not exceed 200, as a practical limit based on professional judgment and construction economics.

Although there is no restriction on the unbraced length of columns, AISC *Manual* tables are limited to common or practical lengths for ordinary usage. For example, a double L3×3×¼, with a ¾ in. separation has an r_y of 1.38 in. At an L_c/r of 200, this strut would be 23 ft long. This is thought to be a reasonable limit based on fabrication and handling requirements.

Throughout the AISC *Manual*, shapes that contain slender elements for compression when supplied in their most common material grade are footnoted with the letter “[c].” For example, see a W14×22^[c].

E3. FLEXURAL BUCKLING OF MEMBERS WITHOUT SLENDER ELEMENTS

Nonslender-element compression members, including nonslender built-up I-shaped columns and nonslender HSS columns, are governed by these provisions. The general design curve for critical stress versus L_c/r is shown in Figure E-1. The limit between elastic and inelastic buckling is defined to be $L_c/r = 4.71\sqrt{E/F_y}$ or $F_y/F_e = 2.25$. For convenience, these limits are defined in Table E-1 for the common values of F_y .

The term L_c is used throughout this chapter to describe the length between points that are braced against lateral and/or rotational displacement.

E4. TORSIONAL AND FLEXURAL-TORSIONAL BUCKLING OF SINGLE ANGLES AND MEMBERS WITHOUT SLENDER ELEMENTS

This section is most commonly applicable to double angles and WT sections, which are singly symmetric shapes subject to torsional and flexural-torsional buckling. The available strengths in axial compression of these shapes are tabulated in AISC *Manual* Part 4 and examples on the use of these tables have been included in this chapter for the shapes.

E5. SINGLE-ANGLE COMPRESSION MEMBERS

The available strength of single-angle compression members is tabulated in AISC *Manual* Part 4.

Table E-1. Limiting Values of L_c/r and F_e		
F_y , ksi	Limiting L_c/r	F_e , ksi
36	134	16.0
50	113	22.2
65	99.5	28.9
70	95.9	31.1

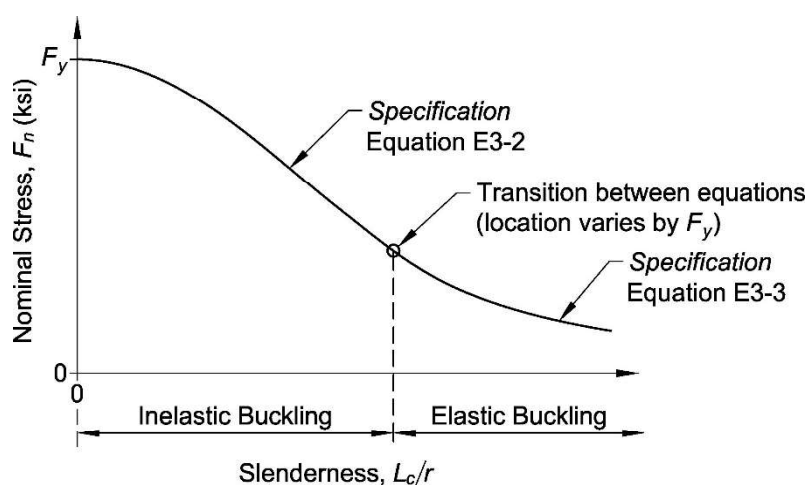


Fig. E-1. Standard column curve.

E6. BUILT-UP MEMBERS

The available strengths in axial compression for built-up double angles with intermediate connectors are tabulated in AISC *Manual* Part 4. There are no tables for other built-up shapes in the AISC *Manual*, due to the number of possible geometries.

E7. MEMBERS WITH SLENDER ELEMENTS

The design of these members is similar to members without slender elements except that a reduced effective area is used in lieu of the gross cross-sectional area.

The tables of AISC *Manual* Part 4 incorporate the appropriate reductions in available strength to account for slender elements.

Design examples have been included in this Chapter for built-up I-shaped members with slender webs and slender flanges. Examples have also been included for a double angle, WT, and a rectangular HSS with slender elements.

EXAMPLE E.1A W-SHAPE COLUMN DESIGN WITH PINNED ENDS**Given:**

Select an ASTM A992/A992M W-shape column to carry the loading shown in Figure E.1A. The column is pinned top and bottom in both axes. Limit the column size to a nominal 14 in. shape.

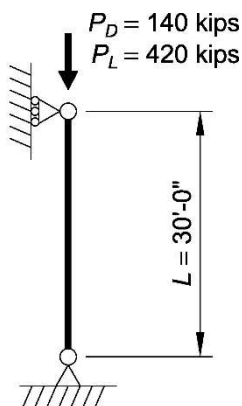


Fig. E.1A. Column loading and bracing.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992/A992M
 $F_y = 50$ ksi

From ASCE/SEI 7, Chapter 2, the required compressive strength is:

LRFD	ASD
$P_u = 1.2(140 \text{ kips}) + 1.6(420 \text{ kips})$ $= 840 \text{ kips}$	$P_a = 140 \text{ kips} + 420 \text{ kips}$ $= 560 \text{ kips}$

Column Selection

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, $K_x = K_y = 1.0$. The effective length is:

$$\begin{aligned}
 L_c &= K_x L_x \\
 &= K_y L_y \\
 &= 1.0(30 \text{ ft}) \\
 &= 30.0 \text{ ft}
 \end{aligned}$$

Because the unbraced length is the same for buckling about the x - x and y - y axes and r_x exceeds r_y for all W-shapes, y - y axis buckling will govern.

Enter AISC *Manual* Table 4-1a with an effective length, $L_c = 30$ ft, and proceed across the table until reaching the least weight shape with an available strength that equals or exceeds the required strength. Select a W14×132.

From AISC *Manual* Table 4-1a, the available strength for a y - y axis effective length of 30 ft is:

LRFD	ASD
$\phi_c P_n = 893 \text{ kips} > 840 \text{ kips} \quad \mathbf{o.k.}$	$\frac{P_n}{\Omega_c} = 594 \text{ kips} > 560 \text{ kips} \quad \mathbf{o.k.}$

EXAMPLE E.1B W-SHAPE COLUMN DESIGN WITH INTERMEDIATE BRACING**Given:**

Verify an ASTM A992/A992M W14×90 is adequate to carry the loading as shown in Figure E.1B. The column is pinned top and bottom in both axes and braced at the midpoint about the y - y axis and torsionally.

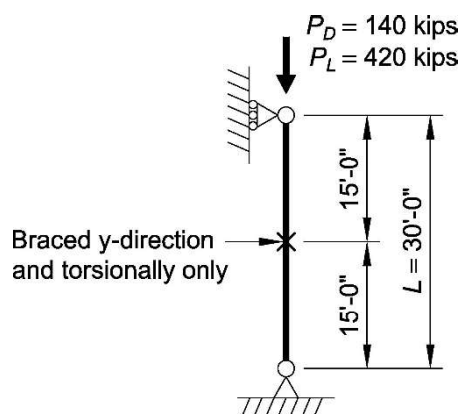


Fig. E.1B. Column loading and bracing.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992/A992M

$F_y = 50$ ksi

From ASCE/SEI 7, Chapter 2, the required compressive strength is:

LRFD	ASD
$P_u = 1.2(140 \text{ kips}) + 1.6(420 \text{ kips})$ $= 840 \text{ kips}$	$P_a = 140 \text{ kips} + 420 \text{ kips}$ $= 560 \text{ kips}$

Column Effective Length

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, $K_x = K_y = 1.0$. The effective length about the y - y axis is:

$$\begin{aligned}
 L_{cy} &= K_y L_y \\
 &= 1.0(15 \text{ ft}) \\
 &= 15.0 \text{ ft}
 \end{aligned}$$

The values tabulated in AISC *Manual* Table 4-1a are provided for buckling in the y - y direction. To determine the buckling strength about the x - x axis, an equivalent effective length for the y - y axis is determined using the r_x/r_y ratio provided at the bottom of this table. For a W14×90, $r_x/r_y = 1.66$, and the equivalent y - y axis effective length for x - x axis buckling is computed as:

$$\begin{aligned}
 L_{cx} &= K_x L_x \\
 &= 1.0(30 \text{ ft}) \\
 &= 30.0 \text{ ft}
 \end{aligned}$$

$$\begin{aligned}
 L_{cy \text{ eq}} &= \frac{L_{cx}}{r_x/r_y} && (\text{Manual Eq. 4-1}) \\
 &= \frac{30.0 \text{ ft}}{1.66} \\
 &= 18.1 \text{ ft}
 \end{aligned}$$

Because 18.1 ft > 15.0 ft, the available compressive strength is governed by the x - x axis flexural buckling limit state.

Available Compressive Strength

The available strength of a W14×90 is determined using AISC *Manual* Table 4-1a, conservatively using an unbraced length of $L_c = 19.0$ ft.

LRFD	ASD
$\phi_c P_n = 903 \text{ kips} > 840 \text{ kips} \quad \mathbf{o.k.}$	$\frac{P_n}{\Omega_c} = 601 \text{ kips} > 560 \text{ kips} \quad \mathbf{o.k.}$

EXAMPLE E.1C W-SHAPE AVAILABLE COMPRESSIVE STRENGTH CALCULATION**Given:**

Calculate the available compressive strength of the column size selected in Example E.1A with an unbraced length of 30 ft for both axes. The loads are as given in Example E.1A.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992/A992M

$F_y = 50$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W14×132

$A_g = 38.8$ in.²

$r_x = 6.28$ in.

$r_y = 3.76$ in.

$\frac{b_f}{2t_f} = 7.15$

$\frac{h}{t_w} = 17.7$

Slenderness Check

The width-to-thickness ratio of the flanges of the W14×132 is:

$$\frac{b_f}{2t_f} = 7.15$$

From AISC *Specification* Table B4.1a, Case 1, the limiting width-to-thickness ratio of the flanges is:

$$\begin{aligned}\lambda_r &= 0.56 \sqrt{\frac{E}{F_y}} \\ &= 0.56 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} \\ &= 13.5 > 7.15; \text{ therefore, the flanges are nonslender}\end{aligned}$$

The width-to-thickness ratio of the web of the W14×132 is:

$$\frac{h}{t_w} = 17.7$$

From AISC *Specification* Table B4.1a, Case 5, the limiting width-to-thickness ratio of the web is:

$$\begin{aligned}
 \lambda_r &= 1.49 \sqrt{\frac{E}{F_y}} \\
 &= 1.49 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} \\
 &= 35.9 > 17.7; \text{ therefore, the web is nonslender}
 \end{aligned}$$

Because the web and flanges are nonslender, the limit state of local buckling does not apply.

Column Effective Length

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, $K_x = K_y = 1.0$. The effective length about the y-y axis is:

$$\begin{aligned}
 L_{cy} &= K_y L_y \\
 &= 1.0(30 \text{ ft}) \\
 &= 30.0 \text{ ft}
 \end{aligned}$$

Because the unbraced length for the W14×132 column is the same for both axes, the y-y axis will govern.

$$\begin{aligned}
 \frac{L_{cy}}{r_y} &= \frac{(30.0 \text{ ft})(12 \text{ in./ft})}{3.76 \text{ in.}} \\
 &= 95.7
 \end{aligned}$$

Nominal Stress

For $F_y = 50 \text{ ksi}$, the available nominal stresses, $\phi_c F_n$ or F_n/Ω_c , for $L_c/r = 95.7$ are interpolated from AISC *Manual* Table 4-14 as follows. The available nominal stress can also be determined as shown in Example E.1D.

LRFD	ASD
$\phi_c F_n = 23.0 \text{ ksi}$	$\frac{F_n}{\Omega_c} = 15.4 \text{ ksi}$

Available Compressive Strength

From AISC *Specification* Equation E3-1, the available compressive strength of the W14×132 column is:

LRFD	ASD
$\phi_c P_n = (\phi_c F_n) A_g$ $= (23.0 \text{ ksi})(38.8 \text{ in.}^2)$ $= 892 \text{ kips} > 840 \text{ kips} \quad \text{o.k.}$	$\frac{P_n}{\Omega_c} = \left(\frac{F_n}{\Omega_c} \right) A_g$ $= (15.4 \text{ ksi})(38.8 \text{ in.}^2)$ $= 598 \text{ kips} > 560 \text{ kips} \quad \text{o.k.}$

EXAMPLE E.1D W-SHAPE AVAILABLE COMPRESSIVE STRENGTH CALCULATION**Given:**

Calculate the available compressive strength of a W14×90 with an x - x axis unbraced length of 30 ft and y - y axis and torsional unbraced lengths of 15 ft. The loads are as given in Example E.1A.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992/A992M

$$F_y = 50 \text{ ksi}$$

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W14×90

$$A_g = 26.5 \text{ in.}^2$$

$$r_x = 6.14 \text{ in.}$$

$$r_y = 3.70 \text{ in.}$$

$$\frac{b_f}{2t_f} = 10.2$$

$$\frac{h}{t_w} = 25.9$$

Slenderness Check

The width-to-thickness ratio of the flanges of the W14×90 is:

$$\frac{b_f}{2t_f} = 10.2$$

From AISC *Specification* Table B4.1a, Case 1, the limiting width-to-thickness ratio of the flanges is:

$$\begin{aligned} \lambda_r &= 0.56 \sqrt{\frac{E}{F_y}} \\ &= 0.56 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} \\ &= 13.5 > 10.2; \text{ therefore, the flanges are nonslender} \end{aligned}$$

The width-to-thickness ratio of the web of the W14×90 is:

$$\frac{h}{t_w} = 25.9$$

From AISC *Specification* Table B4.1a, Case 5, the limiting width-to-thickness ratio of the web is:

$$\begin{aligned}
 \lambda_r &= 1.49 \sqrt{\frac{E}{F_y}} \\
 &= 1.49 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} \\
 &= 35.9 > 25.9; \text{ therefore, the web is nonslender}
 \end{aligned}$$

Because the web and flanges are nonslender, the limit state of local buckling does not apply.

Column Effective Length

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, $K_x = K_y = 1.0$.

$$\begin{aligned}
 L_{cx} &= K_x L_x \\
 &= 1.0(30 \text{ ft}) \\
 &= 30.0 \text{ ft} \\
 \frac{L_{cx}}{r_x} &= \frac{(30.0 \text{ ft})(12 \text{ in./ft})}{6.14 \text{ in.}} \\
 &= 58.6 \quad \textbf{governs}
 \end{aligned}$$

$$\begin{aligned}
 L_{cy} &= K_y L_y \\
 &= 1.0(15 \text{ ft}) \\
 &= 15.0 \text{ ft}
 \end{aligned}$$

$$\begin{aligned}
 \frac{L_{cy}}{r_y} &= \frac{(15.0 \text{ ft})(12 \text{ in./ft})}{3.70 \text{ in.}} \\
 &= 48.6
 \end{aligned}$$

Because $\frac{L_{cx}}{r_x} > \frac{L_{cy}}{r_y}$, x - x buckling controls.

Nominal Stress

The available nominal stress may be interpolated from AISC *Manual* Table 4-14 or calculated directly as follows.

Calculate the elastic buckling stress, F_e , according to AISC *Specification* Section E3. As noted in AISC *Specification* Commentary Section E4, torsional buckling of symmetric shapes is a failure mode usually not considered in the design of hot-rolled columns. This failure mode generally does not govern unless the section is manufactured from relatively thin plates or a torsional unbraced length significantly larger than the y - y axis flexural unbraced length is present.

$$\begin{aligned}
 F_e &= \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)^2} && (\text{Spec. Eq. E3-4}) \\
 &= \frac{\pi^2 (29,000 \text{ ksi})}{(58.6)^2} \\
 &= 83.3 \text{ ksi}
 \end{aligned}$$

Calculate the nominal stress, F_n .

$$4.71 \sqrt{\frac{E}{F_y}} = 4.71 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} \\ = 113$$

Because $\frac{L_c}{r} = 58.6 < 113$,

$$F_n = \left(0.658^{\frac{F_y}{F_e}} \right) F_y \quad (\text{Spec. Eq. E3-2}) \\ = \left(0.658^{\frac{50 \text{ ksi}}{83.3 \text{ ksi}}} \right) (50 \text{ ksi}) \\ = 38.9 \text{ ksi}$$

Column Compressive Strength

From AISC *Specification* Section E3, the nominal compressive strength is:

$$P_n = F_n A_g \quad (\text{Spec. Eq. E3-1}) \\ = (38.9 \text{ ksi})(26.5 \text{ in.}^2) \\ = 1,030 \text{ kips}$$

From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$	$\Omega_c = 1.67$
$\phi_c P_n = 0.90(1,030 \text{ kips})$ $= 927 \text{ kips} > 840 \text{ kips} \quad \mathbf{o.k.}$	$\frac{P_n}{\Omega_c} = \frac{1,030 \text{ kips}}{1.67}$ $= 617 \text{ kips} > 560 \text{ kips} \quad \mathbf{o.k.}$

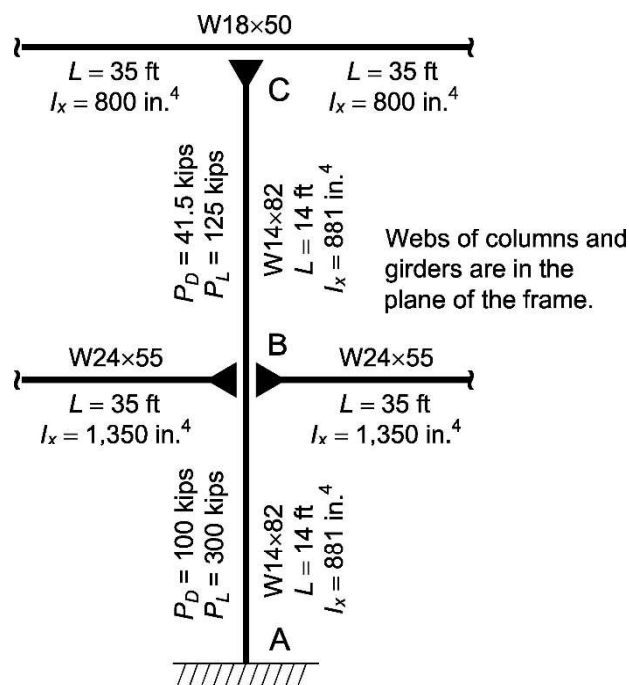
EXAMPLE E.4A W-SHAPE COMPRESSION MEMBER (MOMENT FRAME)

This example is primarily intended to illustrate the use of the alignment chart for sidesway uninhibited columns in conjunction with the effective length method.

Given:

The member sizes shown for the moment frame illustrated here (sidesway uninhibited in the plane of the frame) have been determined to be adequate for lateral loads. The material for both the column and the girders is ASTM A992/A992M. The loads shown at each level are the accumulated dead loads and live loads at that story. The column is fixed at the base about the x - x axis of the column.

Determine if the column is adequate to support the gravity loads shown. Assume the column is continuously supported in the transverse direction (the y - y axis of the column).

**Solution:**

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992/A992M

$F_y = 50$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W18x50

$I_x = 800$ in.⁴

W24x55

$I_x = 1,350$ in.⁴

W14x82

$A_g = 24.0$ in.²

$I_x = 881$ in.⁴

Column B-C

From ASCE/SEI 7, Chapter 2, the required compressive strength for the column between the roof and floor is:

LRFD	ASD
$P_u = 1.2(41.5 \text{ kips}) + 1.6(125 \text{ kips})$ $= 250 \text{ kips}$	$P_a = 41.5 \text{ kips} + 125 \text{ kips}$ $= 167 \text{ kips}$

Effective Length Factor

Using the effective length method, the effective length factor is determined using AISC *Specification* Commentary Appendix 7, Section 7.2. As discussed there, column inelasticity should be addressed by incorporating the stiffness reduction parameter, τ_b . Determine G_{top} and G_{bottom} accounting for column inelasticity by replacing $E_{col}I_{col}$ with $\tau_b(E_{col}I_{col})$. Calculate the stiffness reduction parameter, τ_b , for column B-C using AISC *Manual* Table 4-13.

LRFD	ASD
$\frac{P_u}{A_g} = \frac{250 \text{ kips}}{24.0 \text{ in.}^2}$ $= 10.4 \text{ ksi}$	$\frac{P_a}{A_g} = \frac{167 \text{ kips}}{24.0 \text{ in.}^2}$ $= 6.96 \text{ ksi}$
$\tau_b = 1.00$	$\tau_b = 1.00$

Therefore, no reduction in stiffness for inelastic buckling will be required.

Determine G_{top} and G_{bottom} .

$$G_{top} = \tau_b \left[\frac{\sum (EI/L)_{col}}{\sum (EI/L)_g} \right] \quad (\text{from Spec. Comm. Eq. C-A-7-3})$$

$$= 1.00 \left\{ \frac{\left[\frac{(29,000 \text{ ksi})(881 \text{ in.}^4)}{14.0 \text{ ft}} \right]}{2 \left[\frac{(29,000 \text{ ksi})(800 \text{ in.}^4)}{35.0 \text{ ft}} \right]} \right\}$$

$$= 1.38$$

$$G_{bottom} = \tau_b \left[\frac{\sum (EI/L)_{col}}{\sum (EI/L)_g} \right] \quad (\text{from Spec. Comm. Eq. C-A-7-3})$$

$$= 1.00 \left\{ \frac{2 \left[\frac{(29,000 \text{ ksi})(881 \text{ in.}^4)}{14.0 \text{ ft}} \right]}{2 \left[\frac{(29,000 \text{ ksi})(1,350 \text{ in.}^4)}{35.0 \text{ ft}} \right]} \right\}$$

$$= 1.63$$

From the alignment chart, AISC *Specification* Commentary Figure C-A-7.2, K is slightly less than 1.5; therefore, use $K = 1.5$. Because the column available strength tables are based on the L_c about the y - y axis, the equivalent effective column length of the upper segment for use in the table is:

$$L_{cx} = (KL)_x$$

$$= 1.5(14 \text{ ft})$$

$$= 21.0 \text{ ft}$$

From AISC *Manual* Table 4-1a, for a W14×82:

$$\frac{r_x}{r_y} = 2.44$$

$$\begin{aligned} L_c &= \frac{L_{cx}}{\left(\frac{r_x}{r_y}\right)} \\ &= \frac{21.0 \text{ ft}}{2.44} \\ &= 8.61 \text{ ft} \end{aligned}$$

Using $L_c = 9 \text{ ft}$ in AISC *Manual* Table 4-1a, the available strength in axial compression of the W14×82 is:

LRFD	ASD
$\phi_c P_n = 940 \text{ kips} > 250 \text{ kips} \quad \text{o.k.}$	$\frac{P_n}{\Omega_c} = 626 \text{ kips} > 167 \text{ kips} \quad \text{o.k.}$

Column A-B

From Chapter 2 of ASCE/SEI 7, the required compressive strength for the column between the floor and the foundation is:

LRFD	ASD
$P_u = 1.2(100 \text{ kips}) + 1.6(300 \text{ kips})$ $= 600 \text{ kips}$	$P_a = 100 \text{ kips} + 300 \text{ kips}$ $= 400 \text{ kips}$

Effective Length Factor

Determine the stiffness reduction parameter, τ_b , for column A-B using AISC *Manual* Table 4-13.

LRFD	ASD
$\frac{P_u}{A_g} = \frac{600 \text{ kips}}{24.0 \text{ in.}^2}$ $= 25.0 \text{ ksi}$ $\tau_b = 1.00$	$\frac{P_a}{A_g} = \frac{400 \text{ kips}}{24.0 \text{ in.}^2}$ $= 16.7 \text{ ksi}$ $\tau_b = 0.994$

Use $\tau_b = 0.994$.

$$\begin{aligned}
 G_{top} &= \tau_b \left[\frac{\sum (EI/L)_{col}}{\sum (EI/L)_g} \right] && \text{(from Spec. Comm. Eq. C-A-7-3)} \\
 &= 0.994 \left\{ \frac{2 \left[\frac{(29,000 \text{ ksi})(881 \text{ in.}^4)}{14.0 \text{ ft}} \right]}{2 \left[\frac{(29,000 \text{ ksi})(1,350 \text{ in.}^4)}{35.0 \text{ ft}} \right]} \right\} \\
 &= 1.62
 \end{aligned}$$

$G_{bottom} = 1.0$ (fixed), from AISC *Specification* Commentary Appendix 7, Section 7.2

From the alignment chart, AISC *Specification* Commentary Figure C-A-7.2, K is approximately 1.4. Because the column available strength tables are based on L_c about the y - y axis, the effective column length of the lower segment for use in the table is:

$$\begin{aligned}
 L_{cx} &= (KL)_x \\
 &= 1.4(14 \text{ ft}) \\
 &= 19.6 \text{ ft}
 \end{aligned}$$

$$\begin{aligned}
 L_c &= \frac{L_{cx}}{\left(\frac{r_x}{r_y} \right)} \\
 &= \frac{19.6 \text{ ft}}{2.44} \\
 &= 8.03 \text{ ft}
 \end{aligned}$$

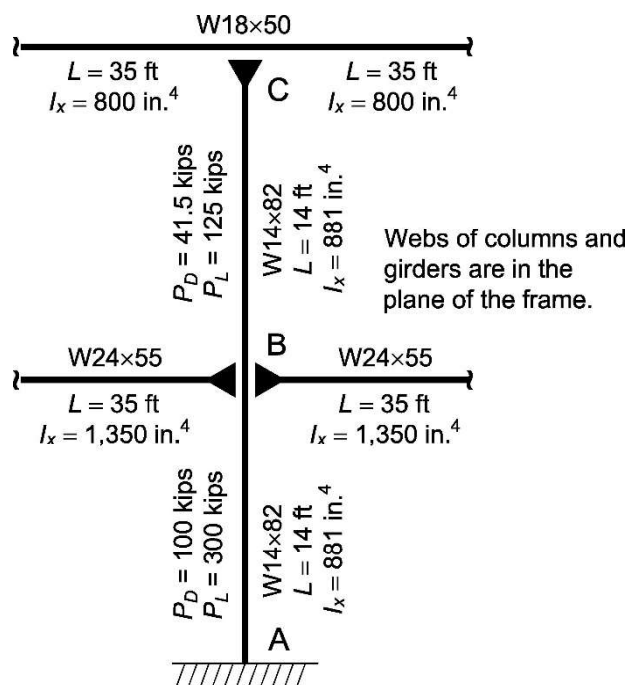
Conservatively using $L_c = 9 \text{ ft}$ in AISC *Manual* Table 4-1a, the available strength in axial compression of the W14×82 is:

LRFD	ASD
$\phi_c P_n = 940 \text{ kips} > 600 \text{ kips}$ o.k.	$\frac{P_n}{\Omega_c} = 626 \text{ kips} > 400 \text{ kips}$ o.k.

A more accurate strength could be determined by interpolation from AISC *Manual* Table 4-1a.

EXAMPLE E.4B W-SHAPE COMPRESSION MEMBER (MOMENT FRAME)**Given:**

Using the effective length method, determine the available strength of the column shown subject to the same gravity loads shown in Example E.4A with the column pinned at the base about the x - x axis. All other assumptions remain the same.

**Solution:**

As determined in Example E.4A, for the column segment B-C between the roof and the floor, the column strength is adequate.

As determined in Example E.4A, for the column segment A-B between the floor and the foundation,

$$G_{top} = 1.62$$

At the base,

$$G_{bottom} = 10 \text{ (pinned) from AISC Specification Commentary Appendix 7, Section 7.2}$$

Note: this is the only change in the analysis.

From the alignment chart, AISC *Specification* Commentary Figure C-A-7.2, K is approximately equal to 2.0. Because the column available strength tables are based on the effective length, L_c , about the y - y axis, the effective column length of the segment A-B for use in the table is:

$$\begin{aligned} L_{cx} &= (KL)_x \\ &= 2.0(14 \text{ ft}) \\ &= 28.0 \text{ ft} \end{aligned}$$

From AISC *Manual* Table 4-1a, for a W14x82:

$$\frac{r_x}{r_y} = 2.44$$

$$\begin{aligned}
 L_c &= \frac{L_{cx}}{\left(\frac{r_x}{r_y} \right)} \\
 &= \frac{28.0 \text{ ft}}{2.44} \\
 &= 11.5 \text{ ft}
 \end{aligned}$$

Interpolate the available strength of the W14×82 from AISC *Manual* Table 4-1a.

LRFD	ASD
$\phi_c P_n = 861 \text{ kips} > 600 \text{ kips} \quad \mathbf{o.k.}$	$\frac{P_n}{\Omega_c} = 573 \text{ kips} > 400 \text{ kips} \quad \mathbf{o.k.}$

EXAMPLE E.7 WT COMPRESSION MEMBER WITHOUT SLENDER ELEMENTS**Given:**

Select an ASTM A992/A992M nonslender WT-shape compression member with a length of 20 ft to support a dead load of 20 kips and live load of 60 kips in axial compression. The ends are pinned. The solution will be provided using:

- (1) AISC *Manual* Tables
- (2) Calculations using AISC *Specification* provisions

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992/A992M
 $F_y = 50$ ksi

From ASCE/SEI 7, Chapter 2, the required compressive strength is:

LRFD	ASD
$P_u = 1.2(20 \text{ kips}) + 1.6(60 \text{ kips})$ = 120 kips	$P_a = 20 \text{ kips} + 60 \text{ kips}$ = 80.0 kips

(1) AISC *Manual* Table Solution

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, $K = 1.0$. Therefore, $L_{cx} = L_{cy} = KL = 1.0(20 \text{ ft}) = 20.0 \text{ ft}$.

Select the lightest nonslender member from AISC *Manual* Table 4-7 with sufficient available strength about both the x - x axis (upper portion of the table) and the y - y axis (lower portion of the table) to support the required strength.

Try a WT7×34.

The available strength in compression is:

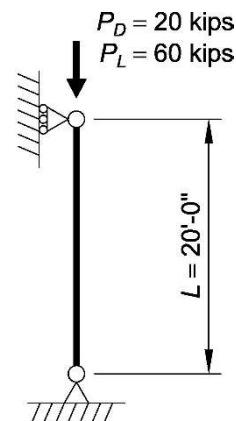
LRFD	ASD
$\phi_c P_{nx} = 128 \text{ kips} > 120 \text{ kips}$ o.k. controls	$\frac{P_{nx}}{\Omega_c} = 85.5 \text{ kips} > 80.0 \text{ kips}$ o.k. controls
$\phi_c P_{ny} = 222 \text{ kips} > 120 \text{ kips}$ o.k.	$\frac{P_{ny}}{\Omega_c} = 147 \text{ kips} > 80.0 \text{ kips}$ o.k.

Available strength can also be determined by hand calculations, as demonstrated in the following.

(2) Calculation Using AISC *Specification* Provisions

From AISC *Manual* Table 1-8, the geometric properties are as follows.

WT7×34
 $A_g = 10.0 \text{ in.}^2$
 $I_x = 32.6 \text{ in.}^4$
 $I_y = 60.7 \text{ in.}^4$



$$J = 1.50 \text{ in.}^4$$

$$r_x = 1.81 \text{ in.}$$

$$r_y = 2.46 \text{ in.}$$

$$t_w = 0.415 \text{ in.}$$

$$t_f = 0.720 \text{ in.}$$

$$\bar{y} = 1.29 \text{ in.}$$

$$C_w = 3.21 \text{ in.}^6$$

$$\frac{b_f}{2t_f} = 6.97$$

$$\frac{d}{t_w} = 16.9$$

Stem Slenderness Check

$$\begin{aligned}\lambda &= \frac{d}{t_w} \\ &= 16.9\end{aligned}$$

Determine the stem limiting slenderness ratio, λ_r , from AISC *Specification* Table B4.1a, Case 4:

$$\begin{aligned}\lambda_r &= 0.75 \sqrt{\frac{E}{F_y}} \\ &= 0.75 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} \\ &= 18.1\end{aligned}$$

$\lambda < \lambda_r$; therefore, the stem is not slender.

Flange Slenderness Check

$$\begin{aligned}\lambda &= \frac{b_f}{2t_f} \\ &= 6.97\end{aligned}$$

Determine the flange limiting slenderness ratio, λ_r , from AISC *Specification* Table B4.1a, Case 1:

$$\begin{aligned}\lambda_r &= 0.56 \sqrt{\frac{E}{F_y}} \\ &= 0.56 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} \\ &= 13.5\end{aligned}$$

$\lambda < \lambda_r$; therefore, the flange is not slender.

There are no slender elements.

For compression members without slender elements, AISC *Specification* Sections E3 and E4 apply. The nominal compressive strength, P_n , is determined based on the limit states of flexural, torsional, and flexural-torsional buckling.

Elastic Flexural Buckling Stress about the x-x Axis

$$\frac{L_{cx}}{r_x} = \frac{(20.0 \text{ ft})(12 \text{ in./ft})}{1.81 \text{ in.}}$$

$$= 133$$

$$F_{ex} = \frac{\pi^2 E}{\left(\frac{L_{cx}}{r_x}\right)^2} \quad (\text{Spec. Eq. E3-4 or E4-5})$$

$$= \frac{\pi^2 (29,000 \text{ ksi})}{(133)^2}$$

$$= 16.2 \text{ ksi} \quad \textbf{controls}$$

Elastic Flexural Buckling Stress about the y-y Axis

$$\frac{L_{cy}}{r_y} = \frac{(20.0 \text{ ft})(12 \text{ in./ft})}{2.46 \text{ in.}}$$

$$= 97.6$$

$$F_{ey} = \frac{\pi^2 E}{\left(\frac{L_{cy}}{r_y}\right)^2} \quad (\text{Spec. Eq. E3-4 or E4-6})$$

$$= \frac{\pi^2 (29,000 \text{ ksi})}{(97.6)^2}$$

$$= 30.0 \text{ ksi}$$

Torsional and Flexural-Torsional Elastic Buckling Stress

Because the WT7×34 section does not have any slender elements, AISC *Specification* Section E4 will be applicable for torsional and flexural-torsional buckling. F_e will be calculated using AISC *Specification* Equation E4-3. The flexural buckling term about the y-y axis, F_{ey} , was computed in the preceding section.

$$x_o = 0$$

$$y_o = \bar{y} - \frac{t_f}{2}$$

$$= 1.29 \text{ in.} - \frac{0.720 \text{ in.}}{2}$$

$$= 0.930 \text{ in.}$$

$$\bar{r}_o^2 = x_o^2 + y_o^2 + \frac{I_x + I_y}{A_g} \quad (\text{Spec. Eq. E4-9})$$

$$= 0 + (0.930 \text{ in.})^2 + \frac{32.6 \text{ in.}^4 + 60.7 \text{ in.}^4}{10.0 \text{ in.}^2}$$

$$= 10.2 \text{ in.}^2$$

$$\begin{aligned}
 F_{ez} &= \left(\frac{\pi^2 EC_w}{L_{cz}^2} + GJ \right) \frac{1}{A_g \bar{r}_o^2} & (\text{Spec. Eq. E4-7}) \\
 &= \left\{ \frac{\pi^2 (29,000 \text{ ksi})(3.21 \text{ in.}^6)}{[(20.0 \text{ ft})(12 \text{ in./ft})]^2} + (11,200 \text{ ksi})(1.50 \text{ in.}^4) \right\} \frac{1}{(10.0 \text{ in.}^2)(10.2 \text{ in.}^2)} \\
 &= 165 \text{ ksi}
 \end{aligned}$$

$$\begin{aligned}
 H &= 1 - \frac{x_o^2 + y_o^2}{\bar{r}_o^2} & (\text{Spec. Eq. E4-8}) \\
 &= 1 - \frac{0 + (0.930 \text{ in.})^2}{10.2 \text{ in.}^2} \\
 &= 0.915
 \end{aligned}$$

$$\begin{aligned}
 F_e &= \left(\frac{F_{ey} + F_{ez}}{2H} \right) \left[1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}} \right] & (\text{Spec. Eq. E4-3}) \\
 &= \left[\frac{30.0 \text{ ksi} + 165 \text{ ksi}}{2(0.915)} \right] \left[1 - \sqrt{1 - \frac{4(30.0 \text{ ksi})(165 \text{ ksi})(0.915)}{(30.0 \text{ ksi} + 165 \text{ ksi})^2}} \right] \\
 &= 29.5 \text{ ksi}
 \end{aligned}$$

Nominal Stress

The nominal stress for the member could be controlled by flexural buckling about either the x - x axis or y - y axis, F_{ex} or F_{ey} , respectively. Note that AISC *Specification* Equations E4-5 and E4-6 reflect the same buckling modes as calculated in AISC *Specification* Equation E3-4. Or, the nominal buckling stress for the member could be controlled by torsional or flexural-torsional buckling calculated per AISC *Specification* Equation E4-3. In this example, F_e calculated in accordance with AISC *Specification* Equation E4-5 is less than that calculated in accordance with AISC *Specification* Equation E4-3 or E4-6 and controls. Therefore:

$$F_e = 16.2 \text{ ksi}$$

$$\begin{aligned}
 \frac{F_y}{F_e} &= \frac{50 \text{ ksi}}{16.2 \text{ ksi}} \\
 &= 3.09
 \end{aligned}$$

Per the AISC *Specification* Section E3 User Note for, the two inequalities for calculating limits of applicability of Sections E3(a) and E3(b) provide the same result for flexural buckling only. When the elastic buckling stress, F_e , is controlled by torsional or flexural-torsional buckling, the L_c/r limits would not be applicable unless an equivalent L_c/r ratio is first calculated by substituting the governing F_e into AISC *Specification* Equation E3-4 and solving for L_c/r . The F_y/F_e limits may be used regardless of which buckling mode governs.

Because $\frac{F_y}{F_e} > 2.25$:

$$\begin{aligned}
 F_n &= 0.877 F_e \\
 &= 0.877(16.2 \text{ ksi}) \\
 &= 14.2 \text{ ksi}
 \end{aligned}
 \tag{Spec. Eq. E3-3}$$

Compressive Strength

From AISC *Specification* Section E3, the nominal compressive strength is:

$$\begin{aligned}
 P_n &= F_n A_g \\
 &= (14.2 \text{ ksi})(10.0 \text{ in.}^2) \\
 &= 142 \text{ kips}
 \end{aligned}
 \tag{Spec. Eq. E3-1}$$

From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(142 \text{ kips})$ $= 128 \text{ kips} > 120 \text{ kips} \quad \mathbf{o.k.}$	$\Omega_c = 1.67$ $\frac{P_n}{\Omega_c} = \frac{142 \text{ kips}}{1.67}$ $= 85.0 \text{ kips} > 80.0 \text{ kips} \quad \mathbf{o.k.}$

Chapter F

Design of Members for Flexure

INTRODUCTION

This *Specification* chapter contains provisions for calculating the flexural strength of members subject to simple bending about one principal axis. Included are specific provisions for I-shaped members, channels, HSS, box sections, tees, double angles, single angles, rectangular bars, rounds, and unsymmetrical shapes. Also included is a section with proportioning requirements for beams and girders.

There are selection tables in Parts 3 and 6 of the AISC *Manual* for standard beams in the commonly available yield strengths. The section property tables for most cross sections provide information that can be used to conveniently identify noncompact and slender element sections. LRFD and ASD information is presented side-by-side.

Most of the formulas from this chapter are illustrated in the following examples. The design and selection techniques illustrated in the examples for both LRFD and ASD will result in similar designs.

F1. GENERAL PROVISIONS

Selection and evaluation of all members is based on deflection requirements and strength, which is determined as the design flexural strength, $\phi_b M_n$, or the allowable flexural strength, M_n/Ω_b ,

where

M_n = the lowest nominal flexural strength based on the limit states of yielding, lateral torsional-buckling, and local buckling, where applicable

$\phi_b = 0.90$ (LRFD)

$\Omega_b = 1.67$ (ASD)

This design approach is followed in all examples.

The term L_b is used throughout this chapter to describe the length between points which are either braced against lateral displacement of the compression flange or braced against twist of the cross section. Requirements for bracing systems and the required strength and stiffness at brace points are given in AISC *Specification* Appendix 6.

The use of C_b is illustrated in several of the following examples. AISC *Manual* Table 3-1 provides tabulated C_b values for some common situations.

F2. DOUBLY SYMMETRIC COMPACT I-SHAPED MEMBERS AND CHANNELS BENT ABOUT THEIR MAJOR AXIS

AISC *Specification* Section F2 applies to the design of compact beams and channels. As indicated in the User Note in Section F2 of the AISC *Specification*, the vast majority of rolled I-shaped beams and channels fall into this category. The curve presented as a solid line in Figure F-1 is a generic plot of the nominal flexural strength, M_n , as a function of the unbraced length, L_b . The horizontal segment of the curve at the far left, between $L_b = 0$ ft and L_p , is the range where the strength is limited by flexural yielding. In this region, the nominal strength is taken as the full plastic moment strength of the section as given by AISC *Specification* Equation F2-1. In the range of the curve at the far right, starting at L_r , the strength is limited by elastic buckling. The strength in this region is given by AISC *Specification* Equation F2-3. Between these regions, within the linear region of the curve between $M_n = M_p$ at L_p on the left, and $M_n = 0.7M_y = 0.7F_y S_x$ at L_r on the right, the strength is limited by inelastic buckling. The strength in this region is provided in AISC *Specification* Equation F2-2.

The curve plotted in Figure F-1 as a heavy solid line represents the case where $C_b = 1.0$, while the heavy dashed line represents the case where C_b exceeds 1.0. The nominal strengths calculated in both AISC *Specification* Equations F2-2 and F2-3 are linearly proportional to C_b , but are limited to M_p as shown in the figure.

$$M_n = M_p = F_y Z_x \quad (\text{Spec. Eq. F2-1})$$

$$M_n = C_b \left[M_p - (M_p - 0.7 F_y S_x) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_p \quad (\text{Spec. Eq. F2-2})$$

$$M_n = F_{cr} S_x \leq M_p \quad (\text{Spec. Eq. F2-3})$$

where

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}} \right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_o} \left(\frac{L_b}{r_{ts}} \right)^2} \quad (\text{Spec. Eq. F2-4})$$

The provisions of this section are illustrated in Examples F.1-1A to F.1-3B (W-shape beam) and Examples F.2-1A to F.2-2B (channel).

Inelastic design provisions are given in AISC *Specification* Appendix 1. L_{pd} , the maximum unbraced length for prismatic member segments containing plastic hinges is less than L_p .

F3. DOUBLY SYMMETRIC I-SHAPED MEMBERS WITH COMPACT WEBS AND NONCOMPACT OR SLENDER FLANGES BENT ABOUT THEIR MAJOR AXIS

The strength of shapes designed according to this section is limited by local buckling of the compression flange. Only a few standard wide-flange shapes have noncompact flanges. For these sections, the strength reduction for $F_y = 50$ ksi steel varies. The approximate percentages of M_p about the strong axis that can be developed by noncompact members when braced such that $L_b \leq L_p$ are shown as follows:

W21×48 = 99%	W14×99 = 99%	W14×90 = 97%	W12×65 = 98%
W10×12 = 99%	W8×31 = 99%	W8×10 = 99%	W6×15 = 94%
W6×8.5 = 98%			

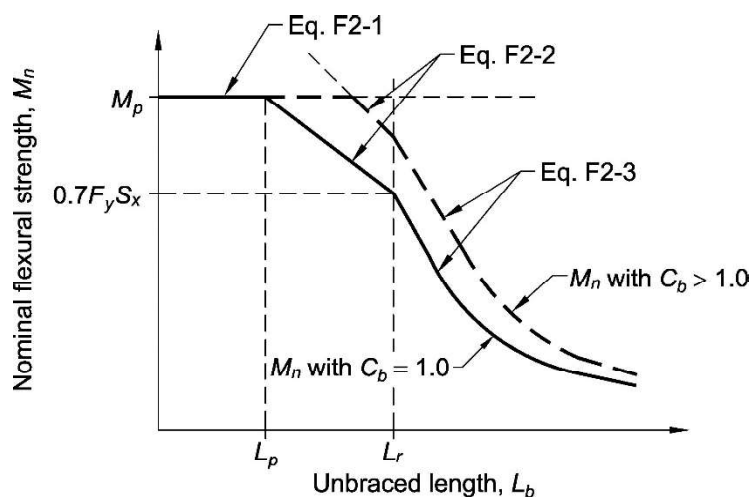


Fig. F-1. Nominal flexural strength versus unbraced length.

The strength curve for the flange local buckling limit state, shown in Figure F-2, is similar in nature to that of the lateral-torsional buckling curve. The horizontal axis parameter is $\lambda = b_f / 2t_f$. The flat portion of the curve to the left of λ_{pf} is the plastic yielding strength, M_p . The curved portion to the right of λ_{rf} is the strength limited by elastic buckling of the flange. The linear transition between these two regions is the strength limited by inelastic flange buckling.

$$M_n = M_p = F_y Z_x \quad (\text{Spec. Eq. F2-1})$$

$$M_n = M_p - (M_p - 0.7F_y S_x) \left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}} \right) \quad (\text{Spec. Eq. F3-1})$$

$$M_n = \frac{0.9Ek_c S_x}{\lambda^2} \quad (\text{Spec. Eq. F3-2})$$

where

$$k_c = \frac{4}{\sqrt{h/t_w}} \text{ and shall not be taken less than 0.35 nor greater than 0.76 for calculation purposes.}$$

The strength reductions due to flange local buckling of the few standard rolled shapes with noncompact flanges are incorporated into the design tables in Part 3 and Part 6 of the AISC *Manual*.

There are no standard I-shaped members with slender flanges. The noncompact flange provisions of this section are illustrated in Examples F.3A and F.3B.

F4. OTHER I-SHAPED MEMBERS WITH COMPACT OR NONCOMPACT WEBS BENT ABOUT THEIR MAJOR AXIS

This section of the AISC *Specification* applies to doubly symmetric I-shaped members bent about their major axis with noncompact webs and singly symmetric I-shaped members (those having different flanges) with compact or noncompact webs.

F5. DOUBLY SYMMETRIC AND SINGLY SYMMETRIC I-SHAPED MEMBERS WITH SLENDER WEBS BENT ABOUT THEIR MAJOR AXIS

This section applies to doubly symmetric and singly symmetric I-shaped members bent about their major axis with slender webs, formerly designated as “plate girders”.

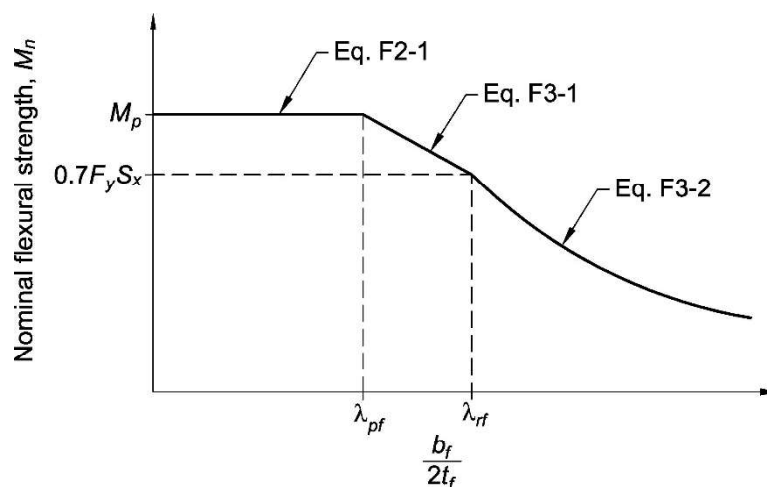


Fig. F-2. Flange local buckling strength.

F6. I-SHAPED MEMBERS AND CHANNELS BENT ABOUT THEIR MINOR AXIS

I-shaped members and channels bent about their minor axis are not subject to lateral-torsional buckling. Rolled or built-up shapes with noncompact or slender flanges, as determined by AISC *Specification* Table B4.1b, must be checked for strength based on the limit state of flange local buckling using Equations F6-2 or F6-3 as applicable.

The vast majority of W, M, C, and MC shapes have compact flanges, and can therefore develop the full plastic moment, M_p , about the minor axis. The provisions of this section are illustrated in Example F.5.

F7. SQUARE AND RECTANGULAR HSS AND BOX SECTIONS

Square and rectangular HSS need to be checked for the limit states of yielding, and flange and web local buckling. Lateral-torsional buckling is also possible for rectangular HSS or box sections bent about the strong axis; however, as indicated in the User Note in AISC *Specification* Section F7, deflection will usually control the design before there is a significant reduction in flexural strength due to lateral-torsional buckling.

The design and section property tables in the AISC *Manual* were calculated using a design wall thickness of 93% of the nominal wall thickness (see AISC *Specification* Section B4.2). Strength reductions due to local buckling have been accounted for in the AISC *Manual* design tables. The selection of a square HSS with compact flanges is illustrated in Example F.6. The provisions for a rectangular HSS with noncompact flanges are illustrated in Examples F.7A and F.7B. The provisions for a square HSS with slender flanges are illustrated in Examples F.8A and F.8B. Available flexural strengths of rectangular and square HSS are listed in Tables 3-12 and 3-13, respectively. If HSS members are specified using ASTM A1065/A1065M or ASTM A1085/A1085M material, the design wall thickness may be taken equal to the nominal wall thickness.

F8. ROUND HSS

The definition of HSS encompasses both tube and pipe products. The lateral-torsional buckling limit state does not apply, but round HSS are subject to strength reductions from local buckling. Available strengths of round HSS and Pipes are listed in AISC *Manual* Tables 3-14 and 3-15, respectively. The tabulated properties and available flexural strengths of these shapes in the AISC *Manual* are calculated using a design wall thickness of 93% of the nominal wall thickness. The design of a Pipe is illustrated in Examples F.9A and F.9B. If round HSS members are specified using ASTM A1085/A1085M material, the design wall thickness may be taken equal to the nominal wall thickness.

F9. TEES AND DOUBLE ANGLES LOADED IN THE PLANE OF SYMMETRY

The AISC *Specification* provides a check for flange local buckling, which applies only when a noncompact or slender flange is in compression due to flexure. This limit state will seldom govern. A check for local buckling of the tee stem in flexural compression was added in the 2010 edition of the *Specification*. The provisions were expanded to include local buckling of double-angle web legs in flexural compression in the 2016 edition. Attention should be given to end conditions of tees to avoid inadvertent fixed end moments that induce compression in the web unless this limit state is checked. The design of a WT-shape in bending is illustrated in Example F.10.

F10. SINGLE ANGLES

Section F10 of the AISC *Specification* permits the flexural design of single angles using either the principal axes (w - w and z - z axes) or geometric axes (x - x and y - y axes). When designing single angles with no lateral-torsional restraint using the geometric axis design provisions, M_y must be multiplied by 0.80 for use in Equations F10-1, F10-2, and F10-3. The design of a single angle in bending is illustrated in Examples F.11A to F.11C.

F11. RECTANGULAR BARS AND ROUNDS

The AISC *Manual* does not include design tables for these shapes. The local buckling limit state does not apply to any bars. With the exception of rectangular bars bent about the major axis, solid square, rectangular, and round bars are not subject to lateral-torsional buckling and are governed by the yielding limit state only. Rectangular bars bent about the major axis are subject to lateral-torsional buckling and are checked for this limit state with Equations F11-3 and F11-4, as applicable.

These provisions can be used to check plates and webs of tees in connections. A design example of a rectangular bar in bending is illustrated in Example F.12. A design example of a round bar in bending is illustrated in Example F.13.

F12. UNSYMMETRICAL SHAPES

Due to the wide range of possible unsymmetrical cross sections, specific lateral-torsional and local buckling provisions are not provided in this *Specification* section. A general template is provided, but appropriate literature investigation and engineering judgment are required for the application of this section.

F13. PROPORTIONS OF BEAMS AND GIRDERS

This section of the *Specification* includes a limit state check for tensile rupture due to bolt holes in the tension flange of beams, proportioning limits for I-shaped members, detail requirements for cover plates and connection requirements for built-up beams connected side-to-side.

EXAMPLE F.1-1A W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR-AXIS BENDING, CONTINUOUSLY BRACED**Given:**

Select a W-shape beam for the span and uniform dead and live loads shown in Figure F.1-1A. Limit the member to a maximum nominal depth of 18 in. Limit the live load deflection to $L/360$. The beam is simply supported and continuously braced. The beam is ASTM A992/A992M material.

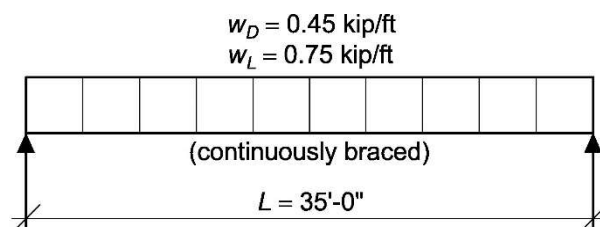


Fig. F.1-1A. Beam loading and bracing diagram.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992/A992M
 $F_y = 50$ ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.45 \text{ kip/ft}) + 1.6(0.75 \text{ kip/ft})$ $= 1.74 \text{ kip/ft}$ From AISC <i>Manual</i> Table 3-22, Case 1: $M_u = \frac{w_u L^2}{8}$ $= \frac{(1.74 \text{ kip/ft})(35 \text{ ft})^2}{8}$ $= 266 \text{ kip-ft}$	$w_a = 0.45 \text{ kip/ft} + 0.75 \text{ kip/ft}$ $= 1.20 \text{ kip/ft}$ From AISC <i>Manual</i> Table 3-22, Case 1: $M_a = \frac{w_a L^2}{8}$ $= \frac{(1.20 \text{ kip/ft})(35 \text{ ft})^2}{8}$ $= 184 \text{ kip-ft}$

Required Moment of Inertia for Live-Load Deflection Criterion of $L/360$

$$\begin{aligned} \Delta_{max} &= \frac{L}{360} \\ &= \frac{(35 \text{ ft})(12 \text{ in./ft})}{360} \\ &= 1.17 \text{ in.} \end{aligned}$$

Determine the minimum required moment of inertia from AISC *Manual* Table 3-22, Case 1:

$$\begin{aligned}
 I_{x, req} &= \frac{5w_L L^4}{384E\Delta_{max}} \\
 &= \frac{5(0.75 \text{ kip/ft})(35 \text{ ft})^4 (12 \text{ in./ft})^3}{384(29,000 \text{ ksi})(1.17 \text{ in.})} \\
 &= 746 \text{ in.}^4
 \end{aligned}$$

Beam Selection

Select a W18×50 from AISC *Manual* Table 3-3.

$$I_x = 800 \text{ in.}^4 > 746 \text{ in.}^4 \quad \mathbf{o.k.}$$

Per the User Note in AISC *Specification* Section F2, the section is compact. Because the beam is continuously braced and compact, only the yielding limit state applies.

From AISC *Manual* Table 3-2, the available flexural strength is:

LRFD	ASD
$\phi_b M_n = \phi_b M_{px}$ $= 379 \text{ kip-ft} > 266 \text{ kip-ft} \quad \mathbf{o.k.}$	$\frac{M_n}{\Omega_b} = \frac{M_{px}}{\Omega_b}$ $= 252 \text{ kip-ft} > 184 \text{ kip-ft} \quad \mathbf{o.k.}$

EXAMPLE F.1-1B W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR-AXIS BENDING, CONTINUOUSLY BRACED**Given:**

Verify the available flexural strength of the ASTM A992/A992M W18×50 beam selected in Example F.1-1A by directly applying the requirements of the AISC *Specification*.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

$$\begin{aligned} &\text{ASTM A992/A992M} \\ &F_y = 50 \text{ ksi} \end{aligned}$$

From AISC *Manual* Table 1-1, the geometric properties are as follows:

$$\begin{aligned} &\text{W18} \times 50 \\ &Z_x = 101 \text{ in.}^3 \end{aligned}$$

The required flexural strength from Example F.1-1A is:

LRFD	ASD
$M_u = 266 \text{ kip-ft}$	$M_a = 184 \text{ kip-ft}$

Nominal Flexural Strength

Per the User Note in AISC *Specification* Section F2, the section is compact. Because the beam is continuously braced and compact, only the yielding limit state applies.

$$\begin{aligned} M_n &= M_p = F_y Z_x && (\text{Spec. Eq. F2-1}) \\ &= (50 \text{ ksi})(101 \text{ in.}^3) \\ &= 5,050 \text{ kip-in. or } 421 \text{ kip-ft} \end{aligned}$$

Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$	$\Omega_b = 1.67$
$\phi_b M_n = 0.90(421 \text{ kip-ft})$	$\frac{M_n}{\Omega_b} = \frac{421 \text{ kip-ft}}{1.67}$
$= 379 \text{ kip-ft} > 266 \text{ kip-ft} \quad \mathbf{o.k.}$	$= 252 \text{ kip-ft} > 184 \text{ kip-ft} \quad \mathbf{o.k.}$

EXAMPLE F.1-2A W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR-AXIS BENDING, BRACED AT THIRD POINTS

Given:

Use the AISC *Manual* tables to verify the available flexural strength of the W18×50 beam size selected in Example F.1-1A for the span and uniform dead and live loads shown in Figure F.1-2A. The beam is simply supported and braced at the ends and third points. The beam is ASTM A992/A992M material.

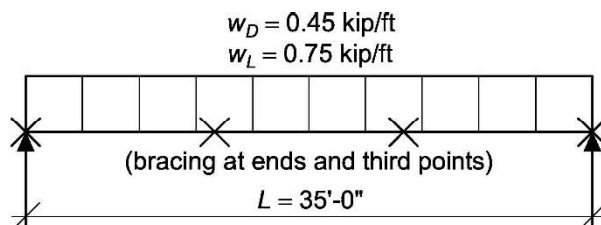


Fig. F.1-2A. Beam loading and bracing diagram.

Solution:

The required flexural strength at midspan from Example F.1-1A is:

LRFD	ASD
$M_u = 266 \text{ kip-ft}$	$M_a = 184 \text{ kip-ft}$

Unbraced Length

$$L_b = \frac{35 \text{ ft}}{3} = 11.7 \text{ ft}$$

By inspection, the middle segment will govern. From AISC *Manual* Table 3-1, for a uniformly loaded beam braced at the ends and third points, $C_b = 1.01$ in the middle segment. Conservatively, neglect this small adjustment in this case.

Available Flexural Strength

Enter AISC *Manual* Table 3-10 and find the intersection of the curve for the W18×50 with an unbraced length of 11.7 ft. Obtain the available strength from the appropriate vertical scale to the left.

From AISC *Manual* Table 3-10, the available flexural strength is:

LRFD	ASD
$\phi_b M_n \approx 302 \text{ kip-ft} > 266 \text{ kip-ft} \quad \text{o.k.}$	$\frac{M_n}{\Omega_b} \approx 201 \text{ kip-ft} > 184 \text{ kip-ft} \quad \text{o.k.}$

EXAMPLE F.1-2B W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR-AXIS BENDING, BRACED AT THIRD POINTS**Given:**

Verify the available flexural strength of the W18×50 beam selected in Example F.1-1A with the beam braced at the ends and third points by directly applying the requirements of the AISC *Specification*. The beam is ASTM A992/A992M material.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992/A992M

$F_y = 50$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W18×50

$r_y = 1.65$ in.

$S_x = 88.9$ in.³

$J = 1.24$ in.⁴

$r_{ts} = 1.98$ in.

$h_o = 17.4$ in.

The required flexural strength from Example F.1-1A is:

LRFD	ASD
$M_u = 266$ kip-ft	$M_a = 184$ kip-ft

Nominal Flexural Strength

Calculate C_b . For the lateral-torsional buckling limit state, the nonuniform moment modification factor can be calculated using AISC *Specification* Equation F1-1. For the center segment of the beam, the required moments for AISC *Specification* Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: $M_{max} = 1.00$, $M_A = 0.972$, $M_B = 1.00$, and $M_C = 0.972$.

$$\begin{aligned}
 C_b &= \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} && (\text{Spec. Eq. F1-1}) \\
 &= \frac{12.5(1.00)}{2.5(1.00) + 3(0.972) + 4(1.00) + 3(0.972)} \\
 &= 1.01
 \end{aligned}$$

For the end-span beam segments, the required moments for AISC *Specification* Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: $M_{max} = 0.889$, $M_A = 0.306$, $M_B = 0.556$, and $M_C = 0.750$.

$$\begin{aligned}
 C_b &= \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} && (\text{Spec. Eq. F1-1}) \\
 &= \frac{12.5(0.889)}{2.5(0.889) + 3(0.306) + 4(0.556) + 3(0.750)} \\
 &= 1.46
 \end{aligned}$$

Thus, the center span, with the higher required strength and lower C_b , will govern.

The limiting laterally unbraced length for the limit state of yielding is:

$$\begin{aligned}
 L_p &= 1.76r_y \sqrt{\frac{E}{F_y}} \\
 &= 1.76(1.65 \text{ in.}) \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} \\
 &= 69.9 \text{ in. or } 5.83 \text{ ft}
 \end{aligned}
 \quad (\text{Spec. Eq. F2-5})$$

The limiting unbraced length for the limit state of inelastic lateral-torsional buckling, with $c = 1$ from AISC *Specification* Equation F2-8a for doubly symmetric I-shaped members, is:

$$\begin{aligned}
 L_r &= 1.95r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{Jc}{S_x h_o} + \sqrt{\left(\frac{Jc}{S_x h_o}\right)^2 + 6.76 \left(\frac{0.7F_y}{E}\right)^2}} \\
 &= 1.95(1.98 \text{ in.}) \left[\frac{29,000 \text{ ksi}}{0.7(50 \text{ ksi})} \right] \sqrt{\frac{(1.24 \text{ in.}^4)(1)}{(88.9 \text{ in.}^3)(17.4 \text{ in.})} + \sqrt{\left[\frac{(1.24 \text{ in.}^4)(1)}{(88.9 \text{ in.}^3)(17.4 \text{ in.})} \right]^2 + 6.76 \left[\frac{0.7(50 \text{ ksi})}{29,000 \text{ ksi}} \right]^2}} \\
 &= 203 \text{ in. or } 16.9 \text{ ft}
 \end{aligned}
 \quad (\text{Spec. Eq. F2-6})$$

$$L_b = 11.7 \text{ ft (from Example F.1-2A)}$$

For a compact beam with an unbraced length of $L_p < L_b \leq L_r$, the lesser of either the flexural yielding limit state or the inelastic lateral-torsional buckling limit state controls the nominal strength.

$$M_p = 5,050 \text{ kip-in. (from Example F.1-1B)}$$

$$\begin{aligned}
 M_n &= C_b \left[M_p - (M_p - 0.7F_y S_x) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_p \\
 &= 1.01 \left\{ 5,050 \text{ kip-in.} - \left[5,050 \text{ kip-in.} - 0.7(50 \text{ ksi})(88.9 \text{ in.}^3) \right] \left(\frac{11.7 \text{ ft} - 5.83 \text{ ft}}{16.9 \text{ ft} - 5.83 \text{ ft}} \right) \right\} < 5,050 \text{ kip-in.} \\
 &= 4,060 \text{ kip-in.} < 5,050 \text{ kip-in.} \\
 &= 4,060 \text{ kip-in. or } 338 \text{ kip-ft}
 \end{aligned}
 \quad (\text{Spec. Eq. F2-2})$$

Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$	$\Omega_b = 1.67$
$\phi_b M_n = 0.90(338 \text{ kip-ft})$	$\frac{M_n}{\Omega_b} = \frac{338 \text{ kip-ft}}{1.67}$
$= 304 \text{ kip-ft} > 266 \text{ kip-ft} \quad \mathbf{o.k.}$	$= 202 \text{ kip-ft} > 184 \text{ kip-ft} \quad \mathbf{o.k.}$

EXAMPLE F.1-3A W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR-AXIS BENDING, BRACED AT MIDSPAN
Given:

Use the AISC *Manual* tables to verify the available flexural strength of the W18×50 beam size selected in Example F.1-1A for the span and uniform dead and live loads shown in Figure F.1-3A. The beam is simply supported and braced at the ends and midpoint. The beam is ASTM A992/A992M material.

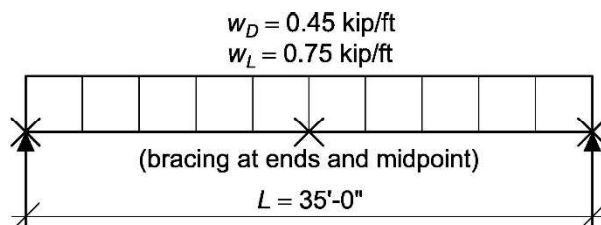


Fig. F.1-3A. Beam loading and bracing diagram.

Solution:

The required flexural strength at midspan from Example F.1-1A is:

LRFD	ASD
$M_u = 266 \text{ kip-ft}$	$M_a = 184 \text{ kip-ft}$

Unbraced Length

$$L_b = \frac{35 \text{ ft}}{2} = 17.5 \text{ ft}$$

From AISC *Manual* Table 3-1, for a uniformly loaded beam braced at the ends and at the center point, $C_b = 1.30$. There are several ways to make adjustments to AISC *Manual* Table 3-10 to account for C_b greater than 1.0.

Procedure A

Available moments from the sloped and curved portions of the plots from AISC *Manual* Table 3-10 may be multiplied by C_b , but may not exceed the value of the horizontal portion (ϕM_p for LRFD, M_p/Ω for ASD).

Obtain the available strength of a W18×50 with an unbraced length of 17.5 ft from AISC *Manual* Table 3-10.

Enter AISC *Manual* Table 3-10 and find the intersection of the curve for the W18×50 with an unbraced length of 17.5 ft. Obtain the available strength from the appropriate vertical scale to the left.

LRFD	ASD
$\phi_b M_n \approx 222 \text{ kip-ft}$	$\frac{M_n}{\Omega_b} \approx 148 \text{ kip-ft}$
From AISC <i>Manual</i> Table 3-2:	From AISC <i>Manual</i> Table 3-2:
$\phi_b M_p = 379 \text{ kip-ft}$ (upper limit on $C_b \phi_b M_n$)	$\frac{M_p}{\Omega_b} = 252 \text{ kip-ft}$ (upper limit on $C_b \frac{M_n}{\Omega_b}$)

LRFD	ASD
Adjust for C_b .	Adjust for C_b .
$1.30(222 \text{ kip-ft}) = 289 \text{ kip-ft}$	$1.30(148 \text{ kip-ft}) = 192 \text{ kip-ft}$
Check limit.	Check limit.
$289 \text{ kip-ft} < \phi_b M_p = 379 \text{ kip-ft} \quad \mathbf{o.k.}$	$192 \text{ kip-ft} < \frac{M_p}{\Omega_b} = 252 \text{ kip-ft} \quad \mathbf{o.k.}$
Check available versus required strength.	Check available versus required strength.
$289 \text{ kip-ft} > 266 \text{ kip-ft} \quad \mathbf{o.k.}$	$192 \text{ kip-ft} > 184 \text{ kip-ft} \quad \mathbf{o.k.}$

Procedure B

For preliminary selection, the required strength can be divided by C_b and directly compared to the strengths in AISC *Manual* Table 3-10. Members selected in this way must be checked to ensure that the required strength does not exceed the available plastic moment strength of the section.

Calculate the adjusted required strength.

LRFD	ASD
$M'_u = \frac{266 \text{ kip-ft}}{1.30}$ $= 205 \text{ kip-ft}$	$M'_a = \frac{184 \text{ kip-ft}}{1.30}$ $= 142 \text{ kip-ft}$

Obtain the available strength for a W18×50 with an unbraced length of 17.5 ft from AISC *Manual* Table 3-10.

LRFD	ASD
$\phi_b M_n \approx 222 \text{ kip-ft} > 205 \text{ kip-ft} \quad \mathbf{o.k.}$	$\frac{M_n}{\Omega_b} \approx 148 \text{ kip-ft} > 142 \text{ kip-ft} \quad \mathbf{o.k.}$
$\phi_b M_p = 379 \text{ kip-ft} > 266 \text{ kip-ft} \quad \mathbf{o.k.}$	$\frac{M_p}{\Omega_b} = 252 \text{ kip-ft} > 184 \text{ kip-ft} \quad \mathbf{o.k.}$

EXAMPLE F.1-3B W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR-AXIS BENDING, BRACED AT MIDSPAN**Given:**

Verify the available flexural strength of the W18×50 beam selected in Example F.1-1A with the beam braced at the ends and center point by directly applying the requirements of the AISC *Specification*. The beam is ASTM A992/A992M material.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992/A992M

$F_y = 50$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W18×50

$r_{ts} = 1.98$ in.

$S_x = 88.9$ in.³

$J = 1.24$ in.⁴

$h_o = 17.4$ in.

The required flexural strength from Example F.1-1A is:

LRFD	ASD
$M_u = 266$ kip-ft	$M_a = 184$ kip-ft

Nominal Flexural Strength

Calculate C_b . The required moments for AISC *Specification* Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: $M_{max} = 1.00$, $M_A = 0.438$, $M_B = 0.750$, and $M_C = 0.938$.

$$\begin{aligned}
 C_b &= \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} && (\text{Spec. Eq. F1-1}) \\
 &= \frac{12.5(1.00)}{2.5(1.00) + 3(0.438) + 4(0.750) + 3(0.938)} \\
 &= 1.30
 \end{aligned}$$

From AISC *Manual* Table 3-2:

$L_p = 5.83$ ft

$L_r = 16.9$ ft

From Example F.1-3A:

$L_b = 17.5$ ft

For a compact beam with an unbraced length $L_b > L_r$, the limit state of elastic lateral-torsional buckling applies.

Calculate F_{cr} , where $c = 1$ for doubly symmetric I-shapes.

$$\begin{aligned}
 F_{cr} &= \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_o} \left(\frac{L_b}{r_{ts}}\right)^2} && (\text{Spec. Eq. F2-4}) \\
 &= \frac{1.30(\pi^2)(29,000 \text{ ksi})}{\left[\frac{(17.5 \text{ ft})(12 \text{ in./ft})}{1.98 \text{ in.}}\right]^2} \sqrt{1 + 0.078 \frac{(1.24 \text{ in.}^4)(1)}{(88.9 \text{ in.}^3)(17.4 \text{ in.}) \left[\frac{(17.5 \text{ ft})(12 \text{ in./ft})}{1.98 \text{ in.}}\right]^2}} \\
 &= 43.2 \text{ ksi}
 \end{aligned}$$

$$M_p = 5,050 \text{ kip-in. (from Example F.1-1B)}$$

$$\begin{aligned}
 M_n &= F_{cr} S_x \leq M_p && (\text{Spec. Eq. F2-3}) \\
 &= (43.2 \text{ ksi})(88.9 \text{ in.}^3) < 5,050 \text{ kip-in.} \\
 &= 3,840 \text{ kip-in.} < 5,050 \text{ kip-in.} \\
 &= 3,840 \text{ kip-in. or } 320 \text{ kip-ft}
 \end{aligned}$$

Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$	$\Omega_b = 1.67$
$\phi_b M_n = 0.90(320 \text{ kip-ft})$ $= 288 \text{ kip-ft} > 266 \text{ kip-ft} \quad \mathbf{o.k.}$	$\frac{M_n}{\Omega_b} = \frac{320 \text{ kip-ft}}{1.67}$ $= 192 \text{ kip-ft} > 184 \text{ kip-ft} \quad \mathbf{o.k.}$

EXAMPLE F.3A W-SHAPE FLEXURAL MEMBER WITH NONCOMPACT FLANGES IN MAJOR-AXIS BENDING

Given:

Using the AISC *Manual* tables, select a W-shape beam for the span, uniform dead load, and concentrated live loads shown in Figure F.3A. The beam is simply supported and continuously braced. Also calculate the deflection. The beam is ASTM A992/A992M material.

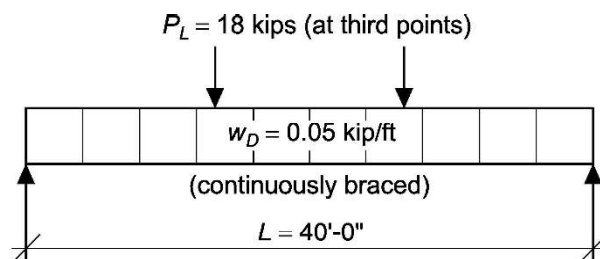


Fig. F.3A. Beam loading and bracing diagram.

Note: A beam with noncompact flanges will be selected to demonstrate that the tabulated values of the AISC *Manual* account for flange compactness.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992/A992M
 $F_y = 50$ ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength at midspan is:

LRFD	ASD
$w_u = 1.2(0.05 \text{ kip/ft})$ $= 0.0600 \text{ kip/ft}$	$w_a = 0.05 \text{ kip/ft}$
$P_u = 1.6(18 \text{ kips})$ $= 28.8 \text{ kips}$	$P_a = 18 \text{ kips}$
From AISC <i>Manual</i> Table 3-22, Cases 1 and 9:	From AISC <i>Manual</i> Table 3-22, Cases 1 and 9:
$M_u = \frac{w_u L^2}{8} + P_u a$ $= \frac{(0.0600 \text{ kip/ft})(40 \text{ ft})^2}{8} + (28.8 \text{ kips})\left(\frac{40 \text{ ft}}{3}\right)$ $= 396 \text{ kip-ft}$	$M_a = \frac{w_a L^2}{8} + P_a a$ $= \frac{(0.05 \text{ kip/ft})(40 \text{ ft})^2}{8} + (18 \text{ kips})\left(\frac{40 \text{ ft}}{3}\right)$ $= 250 \text{ kip-ft}$

Beam Selection

For a continuously braced W-shape, the available flexural strength equals the available plastic flexural strength.

Select the lightest section providing the required strength from the bold entries in AISC *Manual* Table 3-2.

Try a W21×48.

This beam has a noncompact compression flange at $F_y = 50$ ksi as indicated by footnote “[f]” in AISC *Manual* Table 3-2. This shape is also footnoted in AISC *Manual* Table 1-1.

From AISC *Manual* Table 3-2, the available flexural strength is:

LRFD	ASD
$\phi_b M_n = \phi_b M_{px}$ $= 398 \text{ kip-ft} > 396 \text{ kip-ft} \quad \text{o.k.}$	$\frac{M_n}{\Omega_b} = \frac{M_{px}}{\Omega_b}$ $= 265 \text{ kip-ft} > 250 \text{ kip-ft} \quad \text{o.k.}$

Note: The value M_{px} in AISC *Manual* Table 3-2 includes the strength reductions due to the shape being noncompact.

Deflection

From AISC *Manual* Table 3-2:

$$I_x = 959 \text{ in.}^4$$

The maximum deflection occurs at the center of the beam. From AISC *Manual* Table 3-22, Cases 1 and 9:

$$\begin{aligned} \Delta &= \frac{5w_D L^4}{384EI} + \frac{23P_L L^3}{648EI} \\ &= \frac{5(0.05 \text{ kip/ft})(40 \text{ ft})^4 (12 \text{ in./ft})^3}{384(29,000 \text{ ksi})(959 \text{ in.}^4)} + \frac{23(18 \text{ kips})(40 \text{ ft})^3 (12 \text{ in./ft})^3}{648(29,000 \text{ ksi})(959 \text{ in.}^4)} \\ &= 2.64 \text{ in.} \end{aligned}$$

This deflection can be compared with the appropriate deflection limit for the application. Deflection will often be more critical than strength in beam design.

EXAMPLE F.3B W-SHAPE FLEXURAL MEMBER WITH NONCOMPACT FLANGES IN MAJOR-AXIS BENDING**Given:**

Verify the results from Example F.3A by directly applying the requirements of the AISC *Specification*.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992/A992M

$F_y = 50$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W21×48

$S_x = 93.0$ in.³

$Z_x = 107$ in.³

$\frac{b_f}{2t_f} = 9.47$

The required flexural strength from Example F.3A is:

LRFD	ASD
$M_u = 396$ kip-ft	$M_a = 250$ kip-ft

Flange Slenderness

$$\lambda = \frac{b_f}{2t_f}$$

$$= 9.47$$

The limiting width-to-thickness ratios for the compression flange are determined from AISC *Specification* Table B4.1b, Case 10:

$$\lambda_{pf} = 0.38 \sqrt{\frac{E}{F_y}}$$

$$= 0.38 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}}$$

$$= 9.15$$

$$\lambda_{rf} = 1.0 \sqrt{\frac{E}{F_y}}$$

$$= 1.0 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}}$$

$$= 24.1$$

Because $\lambda_{pf} < \lambda < \lambda_{rf}$, the compression flange is noncompact. This could also be determined from the footnote “[f]” in AISC *Manual* Table 1-1.

Nominal Flexural Strength

Because the beam is continuously braced, and therefore not subject to lateral-torsional buckling, the available strength is based on the limit state of compression flange local buckling. From AISC *Specification* Section F3.2:

$$\begin{aligned}
 M_p &= F_y Z_x && (\text{Spec. Eq. F2-1}) \\
 &= (50 \text{ ksi})(107 \text{ in.}^3) \\
 &= 5,350 \text{ kip-in. or } 446 \text{ kip-ft}
 \end{aligned}$$

$$\begin{aligned}
 M_n &= M_p - (M_p - 0.7 F_y S_x) \left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}} \right) && (\text{Spec. Eq. F3-1}) \\
 &= 5,350 \text{ kip-in.} - \left[5,350 \text{ kip-in.} - 0.7(50 \text{ ksi})(93.0 \text{ in.}^3) \right] \left(\frac{9.47 - 9.15}{24.1 - 9.15} \right) \\
 &= 5,310 \text{ kip-in. or } 442 \text{ kip-ft}
 \end{aligned}$$

Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$	$\Omega_b = 1.67$
$\phi_b M_n = 0.90(442 \text{ kip-ft})$ $= 398 \text{ kip-ft} > 396 \text{ kip-ft} \quad \mathbf{o.k.}$	$\frac{M_n}{\Omega_b} = \frac{442 \text{ kip-ft}}{1.67}$ $= 265 \text{ kip-ft} > 250 \text{ kip-ft} \quad \mathbf{o.k.}$

Note that these available strengths are identical to the tabulated values in AISC *Manual* Table 3-2, as shown in Example F.3A, which account for the noncompact flange.

EXAMPLE F.4 W-SHAPE FLEXURAL MEMBER, SELECTION BY MOMENT OF INERTIA FOR MAJOR-AXIS BENDING

Given:

Using the AISC *Manual* tables, select a W-shape using the moment of inertia required to limit the live load deflection to 1.00 in. for the span and uniform dead and live loads shown in Figure F.4. The beam is simply supported and continuously braced. The beam is ASTM A992/A992M material.

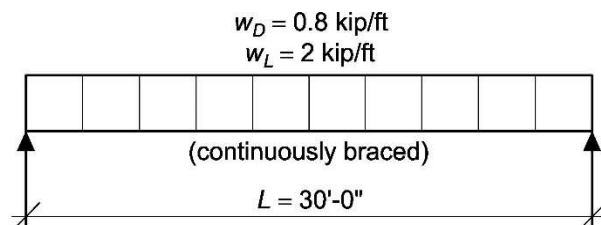


Fig. F.4. Beam loading and bracing diagram.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992/A992M

$F_y = 50$ ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.8 \text{ kip/ft}) + 1.6(2 \text{ kip/ft})$ $= 4.16 \text{ kip/ft}$	$w_a = 0.8 \text{ kip/ft} + 2 \text{ kip/ft}$ $= 2.80 \text{ kip/ft}$
From AISC <i>Manual</i> Table 3-22, Case 1:	From AISC <i>Manual</i> Table 3-22, Case 1:
$M_u = \frac{w_u L^2}{8}$ $= \frac{(4.16 \text{ kip/ft})(30 \text{ ft})^2}{8}$ $= 468 \text{ kip-ft}$	$M_a = \frac{w_a L^2}{8}$ $= \frac{(2.80 \text{ kip/ft})(30 \text{ ft})^2}{8}$ $= 315 \text{ kip-ft}$

Minimum Required Moment of Inertia

From AISC *Manual* Table 3-22, Case 1, the maximum live load deflection, Δ_{max} , occurs at midspan and is calculated as:

$$\Delta_{max} = \frac{5w_L L^4}{384EI}$$

Rearranging and substituting $\Delta_{max} = 1.00$ in.,

$$\begin{aligned}
 I_{req} &= \frac{5w_L L^4}{384E\Delta_{max}} \\
 &= \frac{5(2 \text{ kip/ft})(30 \text{ ft})^4 (12 \text{ in./ft})^3}{384(29,000 \text{ ksi})(1.00 \text{ in.})} \\
 &= 1,260 \text{ in.}^4
 \end{aligned}$$

Beam Selection

Select the lightest section with the required moment of inertia from the bold entries in AISC *Manual* Table 3-3.

Try a W24×55.

$$I_x = 1,350 \text{ in.}^4 > 1,260 \text{ in.}^4 \quad \text{o.k.}$$

Because the W24×55 is continuously braced and compact, its strength is governed by the yielding limit state and AISC *Specification* Section F2.1.

From AISC *Manual* Table 3-2, the available flexural strength is:

LRFD	ASD
$\phi_b M_n = \phi_b M_{px}$ $= 503 \text{ kip-ft} > 468 \text{ kip-ft} \quad \text{o.k.}$	$\frac{M_n}{\Omega_b} = \frac{M_{px}}{\Omega_b}$ $= 334 \text{ kip-ft} > 315 \text{ kip-ft} \quad \text{o.k.}$

EXAMPLE F.5 I-SHAPED FLEXURAL MEMBER IN MINOR-AXIS BENDING**Given:**

Using the AISC *Manual* tables, select a W-shape beam loaded on its minor axis for the span and uniform dead and live loads shown in Figure F.5. Limit the live load deflection to $L/240$. The beam is simply supported and braced only at the ends. The beam is ASTM A992/A992M material.

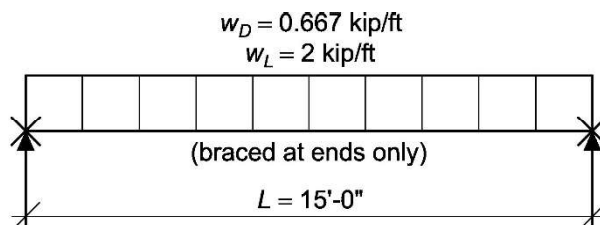


Fig. F.5. Beam loading and bracing diagram.

Note: Although not a common design case, this example is being used to illustrate AISC *Specification* Section F6 (I-shaped members and channels bent about their minor axis).

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992/A992M

$F_y = 50$ ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.667 \text{ kip/ft}) + 1.6(2 \text{ kip/ft})$ $= 4.00 \text{ kip/ft}$ From AISC <i>Manual</i> Table 3-22, Case 1: $M_u = \frac{w_u L^2}{8}$ $= \frac{(4.00 \text{ kip/ft})(15 \text{ ft})^2}{8}$ $= 113 \text{ kip-ft}$	$w_a = 0.667 \text{ kip/ft} + 2 \text{ kip/ft}$ $= 2.67 \text{ kip/ft}$ From AISC <i>Manual</i> Table 3-22, Case 1: $M_a = \frac{w_a L^2}{8}$ $= \frac{(2.67 \text{ kip/ft})(15 \text{ ft})^2}{8}$ $= 75.1 \text{ kip-ft}$

Minimum Required Moment of Inertia

The maximum live load deflection permitted is:

$$\begin{aligned}
 \Delta_{max} &= \frac{L}{240} \\
 &= \frac{(15 \text{ ft})(12 \text{ in./ft})}{240} \\
 &= 0.750 \text{ in.}
 \end{aligned}$$

Determine the minimum required moment of inertia from AISC *Manual* Table 3-22, Case 1:

$$\begin{aligned}
 I_{y, req} &= \frac{5w_L L^4}{384E\Delta_{max}} \\
 &= \frac{5(2 \text{ kip/ft})(15 \text{ ft})^4 (12 \text{ in./ft})^3}{384(29,000 \text{ ksi})(0.750 \text{ in.})} \\
 &= 105 \text{ in.}^4
 \end{aligned}$$

Beam Selection

Select the lightest section from the bold entries in AISC *Manual* Table 3-5.

Try a W12×58.

From AISC *Manual* Table 1-1, the geometric properties are as follows:

$$\begin{aligned}
 &\text{W12}\times\text{58} \\
 &S_y = 21.4 \text{ in.}^3 \\
 &Z_y = 32.5 \text{ in.}^3 \\
 &I_y = 107 \text{ in.}^4 > 105 \text{ in.}^4 \quad \text{o.k. (for deflection requirement)}
 \end{aligned}$$

Nominal Flexural Strength

AISC *Specification* Section F6 applies. Because the W12×58 has compact flanges per the User Note in this Section, the yielding limit state governs the design.

$$\begin{aligned}
 M_n = M_p = F_y Z_y &\leq 1.6 F_y S_y && (\text{Spec. Eq. F6-1}) \\
 &= (50 \text{ ksi})(32.5 \text{ in.}^3) < 1.6(50 \text{ ksi})(21.4 \text{ in.}^3) \\
 &= 1,630 \text{ kip-in.} < 1,710 \text{ kip-in.} \\
 &= 1,630 \text{ kip-in. or } 136 \text{ kip-ft}
 \end{aligned}$$

Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$	$\Omega_b = 1.67$
$\phi_b M_n = 0.90(136 \text{ kip-ft})$ $= 122 \text{ kip-ft} > 113 \text{ kip-ft} \quad \text{o.k.}$	$\frac{M_n}{\Omega_b} = \frac{136 \text{ kip-ft}}{1.67}$ $= 81.4 \text{ kip-ft} > 75.1 \text{ kip-ft} \quad \text{o.k.}$

Chapter G

Design of Members for Shear

INTRODUCTION

This *Specification* chapter addresses webs of singly or doubly symmetric members subjected to shear in the plane of the web, single angles and HSS subjected to shear, and shear in the weak direction of singly or doubly symmetric shapes.

G1. GENERAL PROVISIONS

The design shear strength, $\phi_v V_n$, and the allowable shear strength, V_n/Ω_v , are determined as follows:

V_n = nominal shear strength based on shear yielding or shear buckling

$\phi_v = 0.90$ (LRFD)

$\Omega_v = 1.67$ (ASD)

Exception: For all current ASTM A6/A6M, W-, S-, and HP-shapes except W44×230, W40×149, W36×135, W33×118, W30×90, W24×55, W16×26, and W12×14 for $F_y = 50$ ksi:

$\phi_v = 1.00$ (LRFD)

$\Omega_v = 1.50$ (ASD)

Major-axis shear values are tabulated for W-shapes in AISC *Manual* Tables 3-2, 3-6, and 6-1, for S-shapes in AISC *Manual* Table 3-7, for C-shapes in AISC *Manual* Table 3-8, and for MC-shapes in AISC *Manual* Table 3-9. Major- and minor-axis shear values are tabulated for rectangular HSS in AISC *Manual* Table 3-12. The shear values for square HSS are tabulated in AISC *Manual* Table 3-13. The shear values for round HSS and pipe are tabulated in the *Companion to the AISC Steel Construction Manual, Volume 2: Design Tables* (AISC, 2023). Minor-axis shear values for W-shapes, S-shapes, C-shapes, and MC-shapes, and shear values for angles and box members are not tabulated.

G2. I-SHAPED MEMBERS AND CHANNELS

This section includes provisions for shear strength of webs without the use of tension field action, for interior web panels considering tension field action, and for end web panels considering tension field action. Provisions for the design of transverse stiffeners are also included in Section G2.

As indicated in the User Note of this section, virtually all W-, S-, and HP-shapes are not subject to shear buckling and are also eligible for the more liberal safety and resistance factors, $\phi_v = 1.00$ (LRFD) and $\Omega_v = 1.50$ (ASD). This is presented in Examples G.1A and G.1B for a W-shape. A channel shear strength design is presented in Examples G.2A and G.2B. A built-up girder with a thin web and transverse stiffeners is presented in Examples G.8A and G.8B.

G3. SINGLE ANGLES AND TEES

A single angle example is illustrated in Example G.3.

G4. RECTANGULAR HSS, BOX S/ECTIONS, AND OTHER SINGLY AND DOUBLY SYMMETRIC MEMBERS

The shear height for HSS, h , is taken as the clear distance between the flanges less the inside corner radius on each side. If the corner radii are unknown, h shall be taken as the corresponding outside dimension minus 3 times the design wall thickness. A rectangular HSS example is provided in Example G.4.

G5. ROUND HSS

For all round HSS of ordinary length listed in the *AISC Manual*, F_{cr} can be taken as $0.6F_y$ in *AISC Specification* Equation G5-1. A round HSS example is illustrated in Example G.5.

G6. DOUBLY SYMMETRIC AND SINGLY SYMMETRIC MEMBERS SUBJECTED TO MINOR-AXIS SHEAR

For examples of minor-axis shear, see Example G.6 and Example G.7.

G7. BEAMS AND GIRDERS WITH WEB OPENINGS

For a beam and girder with web openings example, see *AISC Design Guide 2, Design of Steel and Composite Beams with Web Openings* (Darwin, 1990).

EXAMPLE G.1A W-SHAPE IN MAJOR-AXIS SHEAR**Given:**

Using AISC *Manual* tables, determine the available shear strength and adequacy of an ASTM A992/A992M W24×62 with end shears of 48 kips from dead load and 145 kips from live load.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992/A992M

$F_y = 50$ ksi

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

LRFD	ASD
$V_u = 1.2(48 \text{ kips}) + 1.6(145 \text{ kips})$ $= 290 \text{ kips}$	$V_a = 48 \text{ kips} + 145 \text{ kips}$ $= 193 \text{ kips}$

From AISC *Manual* Table 3-2, the available shear strength is:

LRFD	ASD
$\phi_v V_{nx} = 306 \text{ kips} > 290 \text{ kips} \quad \text{o.k.}$	$\frac{V_{nx}}{\Omega_v} = 204 \text{ kips} > 193 \text{ kips} \quad \text{o.k.}$

EXAMPLE G.1B W-SHAPE IN MAJOR-AXIS SHEAR**Given:**

The available shear strength of the W-shape in Example G.1A was easily determined using tabulated values in the *AISC Manual*. This example demonstrates the calculation of the available strength by directly applying the provisions of the *AISC Specification*.

Solution:

From *AISC Manual* Table 2-4, the material properties are as follows:

$$\begin{aligned} &\text{ASTM A992/A992M} \\ &F_y = 50 \text{ ksi} \end{aligned}$$

From *AISC Manual* Table 1-1, the geometric properties are as follows:

$$\begin{aligned} &W24 \times 62 \\ &d = 23.7 \text{ in.} \\ &t_w = 0.430 \text{ in.} \end{aligned}$$

Nominal Shear Strength

Except for very few sections, which are listed in the User Note, *AISC Specification* Section G2.1(a) is applicable to the I-shaped beams published in the *AISC Manual* for $F_y = 50$ ksi. The W-shape sections that do not meet the criteria of *AISC Specification* Section G2.1(a) are indicated with footnote “[v]” in Tables 1-1, 3-2, and 6-1.

$$C_{v1} = 1.0 \quad (\text{Spec. Eq. G2-2})$$

From *AISC Specification* Section G2.1, area of the web, A_w , is determined as follows:

$$\begin{aligned} A_w &= dt_w \\ &= (23.7 \text{ in.})(0.430 \text{ in.}) \\ &= 10.2 \text{ in.}^2 \end{aligned}$$

From *AISC Specification* Section G2.1, the nominal shear strength is:

$$\begin{aligned} V_n &= 0.6F_y A_w C_{v1} \\ &= 0.6(50 \text{ ksi})(10.2 \text{ in.}^2)(1.0) \\ &= 306 \text{ kips} \end{aligned} \quad (\text{Spec. Eq. G2-1})$$

Available Shear Strength

From *AISC Specification* Section G2.1, the available shear strength is:

LRFD	ASD
$\phi_v = 1.00$	$\Omega_v = 1.50$
$\phi_v V_n = 1.00(306 \text{ kips})$ $= 306 \text{ kips}$	$\frac{V_n}{\Omega_v} = \frac{306 \text{ kips}}{1.50}$ $= 204 \text{ kips}$

EXAMPLE G.2A CHANNEL IN MAJOR-AXIS SHEAR**Given:**

Using AISC *Manual* tables, verify the available shear strength and adequacy of an ASTM A992/A992M C15×33.9 channel with end shears of 25 kips from dead load and 75 kips from live load.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992/A992M

$F_y = 50$ ksi

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

LRFD	ASD
$V_u = 1.2(25 \text{ kips}) + 1.6(75 \text{ kips})$ $= 150 \text{ kips}$	$V_a = 25 \text{ kips} + 75 \text{ kips}$ $= 100 \text{ kips}$

From AISC *Manual* Table 3-8, the available shear strength is:

LRFD	ASD
$\phi_v V_n = 162 \text{ kips} > 150 \text{ kips} \quad \text{o.k.}$	$\frac{V_n}{\Omega_v} = 108 \text{ kips} > 100 \text{ kips} \quad \text{o.k.}$

EXAMPLE G.2B CHANNEL IN MAJOR-AXIS SHEAR**Given:**

The available shear strength of the channel in Example G.2A was easily determined using tabulated values in the *AISC Manual*. This example demonstrates the calculation of the available strength by directly applying the provisions of the *AISC Specification*.

Solution:

From *AISC Manual* Table 2-4, the material properties are as follows:

$$\begin{aligned} &\text{ASTM A992/A992M} \\ &F_y = 50 \text{ ksi} \end{aligned}$$

From *AISC Manual* Table 1-5, the geometric properties are as follows:

$$\begin{aligned} &\text{C15} \times 33.9 \\ &d = 15.0 \text{ in.} \\ &t_w = 0.400 \text{ in.} \end{aligned}$$

Nominal Shear Strength

All ASTM A992/A992M channels listed in the *AISC Manual* have $h/t_w \leq 1.10\sqrt{k_v E/F_y}$; therefore,

$$C_{v1} = 1.0 \quad (\text{Spec. Eq. G2-3})$$

From *AISC Specification* Section G2.1, the area of the web, A_w , is determined as follows:

$$\begin{aligned} A_w &= dt_w \\ &= (15.0 \text{ in.})(0.400 \text{ in.}) \\ &= 6.00 \text{ in.}^2 \end{aligned}$$

From *AISC Specification* Section G2.1, the nominal shear strength is:

$$\begin{aligned} V_n &= 0.6F_y A_w C_{v1} \\ &= 0.6(50 \text{ ksi})(6.00 \text{ in.}^2)(1.0) \\ &= 180 \text{ kips} \end{aligned} \quad (\text{Spec. Eq. G2-1})$$

Available Shear Strength

Because *AISC Specification* Section G2.1(a) does not apply for channels, the values of $\phi_v = 1.00$ (LRFD) and $\Omega_v = 1.50$ (ASD) may not be used. Instead $\phi_v = 0.90$ (LRFD) and $\Omega_v = 1.67$ (ASD) from *AISC Specification* Section G1(a) must be used.

LRFD	ASD
$\phi_v = 0.90$	$\Omega_v = 1.67$
$\phi_v V_n = 0.90(180 \text{ kips})$ $= 162 \text{ kips}$	$\frac{V_n}{\Omega_v} = \frac{180 \text{ kips}}{1.67}$ $= 108 \text{ kips}$

EXAMPLE G.6 DOUBLY SYMMETRIC SHAPE IN MINOR-AXIS SHEAR**Given:**

Verify the available shear strength and adequacy of an ASTM A992/A992M W21×48 beam with end shears of 20.0 kips from dead load and 60.0 kips from live load in the weak direction.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992/A992M

$F_y = 50$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W21×48

$b_f = 8.14$ in.

$t_f = 0.430$ in.

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

LRFD	ASD
$V_u = 1.2(20.0 \text{ kips}) + 1.6(60.0 \text{ kips})$ $= 120 \text{ kips}$	$V_a = 20.0 \text{ kips} + 60.0 \text{ kips}$ $= 80.0 \text{ kips}$

Nominal Shear Strength

From AISC *Specification* Section G6, for minor axis shear, use AISC *Specification* Equation G6-1.

Calculate C_{v2} using AISC *Specification* Section G2.2 with $h/t_w = b_f/2t_f$ and $k_v = 1.2$.

$$\begin{aligned}\frac{h}{t_w} &= \frac{b_f}{2t_f} \\ &= \frac{8.14 \text{ in.}}{2(0.430 \text{ in.})} \\ &= 9.47\end{aligned}$$

$$\begin{aligned}1.10 \sqrt{\frac{k_v E}{F_y}} &= 1.10 \sqrt{\frac{1.2(29,000 \text{ ksi})}{50 \text{ ksi}}} \\ &= 29.0 > 9.47\end{aligned}$$

Therefore, use AISC *Specification* Equation G2-9:

$$C_{v2} = 1.0 \quad (\text{Spec. Eq. G2-9})$$

Note: From the User Note in AISC *Specification* Section G6, $C_{v2} = 1.0$ for all ASTM A6/A6M W-, S-, M-, and HP-shapes when $F_y \leq 70$ ksi.

Calculate V_n using AISC *Specification* Section G6, multiplying the flange area by two to account for both shear resisting elements:

$$\begin{aligned}
 V_n &= 0.6F_y b_f t_f C_{v2} (2) && \text{(from Spec. Eq. G6-1)} \\
 &= 0.6(50 \text{ ksi})(8.14 \text{ in.})(0.430 \text{ in.})(1.0)(2) \\
 &= 210 \text{ kips}
 \end{aligned}$$

Available Shear Strength

From AISC *Specification* Section G1, the available shear strength is:

LRFD	ASD
$\phi_v = 0.90$ $\phi_v V_n = 0.90(210 \text{ kips})$ $= 189 \text{ kips} > 120 \text{ kips} \quad \mathbf{o.k.}$	$\Omega_v = 1.67$ $\frac{V_n}{\Omega_v} = \frac{210 \text{ kips}}{1.67}$ $= 126 \text{ kips} > 80.0 \text{ kips} \quad \mathbf{o.k.}$

CHAPTER G DESIGN EXAMPLE REFERENCES

- AISC (2023), *Companion to the AISC Steel Construction Manual, Volume 2: Design Tables*, V16.0, American Institute of Steel Construction, Chicago, Ill.
- Darwin, D. (1990), *Steel and Composite Beams with Web Openings*, Design Guide 2, AISC, Chicago, Ill.

Chapter H

Design of Members for Combined Forces and Torsion

For all interaction equations in AISC *Specification* Chapter H, the required forces and moments must include second-order effects, as required by Chapter C of the AISC *Specification*. ASD users of the 1989 AISC *Specification* are accustomed to using an interaction equation that includes a partial second-order amplification. Second-order effects are now addressed in the analysis and are not included in these interaction equations.

EXAMPLE H.1A W-SHAPE SUBJECT TO COMBINED COMPRESSION AND BENDING ABOUT BOTH AXES (BRACED FRAME)**Given:**

Using Table 6-J (located in Volume 2 of this document), determine if an ASTM A992/A992M W14×99 has sufficient available strength to support the axial forces and moments listed as follows, obtained from a second-order analysis that includes P - δ effects. The unbraced length is 14 ft and the member has pinned ends.

LRFD	ASD
$P_u = 400$ kips	$P_a = 267$ kips
$M_{ux} = 250$ kip-ft	$M_{ax} = 167$ kip-ft
$M_{uy} = 80.0$ kip-ft	$M_{ay} = 53.3$ kip-ft

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992/A992M

$F_y = 50$ ksi

The effective length of the member is:

$$\begin{aligned}
 L_{cx} &= L_{cy} \\
 &= KL \\
 &= 1.0(14 \text{ ft}) \\
 &= 14.0 \text{ ft}
 \end{aligned}$$

For $L_c = 14$ ft, the combined strength parameters from Table 6-J are:

LRFD	ASD
$p = \frac{0.887}{10^3} \text{ kips}$	$p = \frac{1.33}{10^3} \text{ kips}$
$b_x = \frac{1.38}{10^3} \text{ kip-ft}$	$b_x = \frac{2.08}{10^3} \text{ kip-ft}$
$b_y = \frac{2.85}{10^3} \text{ kip-ft}$	$b_y = \frac{4.29}{10^3} \text{ kip-ft}$
Check P_r/P_c limit for AISC <i>Specification</i> Equation H1-1a.	Check P_r/P_c limit for AISC <i>Specification</i> Equation H1-1a.
$ \begin{aligned} \frac{P_u}{\phi_c P_n} &= p P_u \\ &= \left(\frac{0.887}{10^3} \text{ kips} \right) (400 \text{ kips}) \\ &= 0.355 \end{aligned} $	$ \begin{aligned} \frac{P_a}{P_n / \Omega_c} &= p P_a \\ &= \left(\frac{1.33}{10^3} \text{ kips} \right) (267 \text{ kips}) \\ &= 0.355 \end{aligned} $

LRFD	ASD
Because $pP_u \geq 0.2$, $pP_u + b_x M_{ux} + b_y M_{uy} \leq 1.0 \quad (\text{from Vol. 2, Eq. 9})$ $= 0.355 + \left(\frac{1.38}{10^3 \text{ kip-ft}} \right) (250 \text{ kip-ft})$ $+ \left(\frac{2.85}{10^3 \text{ kip-ft}} \right) (80.0 \text{ kip-ft}) \leq 1.0$ $= 0.928 < 1.0 \quad \mathbf{o.k.}$	Because $pP_a \geq 0.2$, $pP_a + b_x M_{ax} + b_y M_{ay} \leq 1.0 \quad (\text{from Vol. 2, Eq. 9})$ $= 0.355 + \left(\frac{2.08}{10^3 \text{ kip-ft}} \right) (167 \text{ kip-ft})$ $+ \left(\frac{4.29}{10^3 \text{ kip-ft}} \right) (53.3 \text{ kip-ft}) \leq 1.0$ $= 0.931 < 1.0 \quad \mathbf{o.k.}$

Table 6-J simplifies the calculation of AISC *Specification* Equations H1-1a and H1-1b. A direct application of these equations is shown in Example H.1B.

EXAMPLE H.1B W-SHAPE SUBJECT TO COMBINED COMPRESSION AND BENDING MOMENT ABOUT BOTH AXES (BRACED FRAME)
Given:

Using AISC *Manual* tables to determine the available compressive and flexural strengths, determine if an ASTM A992/A992M W14×99 has sufficient available strength to support the axial forces and moments listed as follows, obtained from a second-order analysis that includes P - δ effects. The unbraced length is 14 ft and the member has pinned ends.

LRFD	ASD
$P_u = 400$ kips $M_{ux} = 250$ kip-ft $M_{uy} = 80.0$ kip-ft	$P_a = 267$ kips $M_{ax} = 167$ kip-ft $M_{ay} = 53.3$ kip-ft

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992/A992M

$F_y = 50$ ksi

The effective length of the member is:

$$\begin{aligned}
 L_{cx} &= L_{cy} \\
 &= KL \\
 &= 1.0(14 \text{ ft}) \\
 &= 14.0 \text{ ft}
 \end{aligned}$$

For $L_c = 14.0$ ft, the available axial and flexural strengths from AISC *Manual* Table 6-1 are:

LRFD	ASD
$P_c = \phi_c P_n$ $= 1,130$ kips	$P_c = \frac{P_n}{\Omega_c}$ $= 750$ kips
$M_{cx} = \phi_b M_{nx}$ $= 642$ kip-ft	$M_{cx} = \frac{M_{nx}}{\Omega_b}$ $= 427$ kip-ft
$M_{cy} = \phi_b M_{ny}$ $= 311$ kip-ft	$M_{cy} = \frac{M_{ny}}{\Omega_b}$ $= 207$ kip-ft
$\frac{P_u}{\phi_c P_n} = \frac{400 \text{ kips}}{1,130 \text{ kips}}$ $= 0.354$	$\frac{P_a}{P_n/\Omega_c} = \frac{267 \text{ kips}}{750 \text{ kips}}$ $= 0.356$

LRFD	ASD
Because $\frac{P_u}{\phi_c P_n} \geq 0.2$, $\frac{P_r}{P_c} + \frac{8}{9} \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \right) \leq 1.0 \quad (\text{Spec. Eq. H1-1a})$ $= \frac{400 \text{ kips}}{1,130 \text{ kips}} + \frac{8}{9} \left(\frac{250 \text{ kip-ft}}{642 \text{ kip-ft}} + \frac{80.0 \text{ kip-ft}}{311 \text{ kip-ft}} \right) \leq 1.0$ $= 0.929 < 1.0 \quad \mathbf{o.k.}$	Because $\frac{P_a}{P_n / \Omega_c} \geq 0.2$, $\frac{P_r}{P_c} + \frac{8}{9} \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \right) \leq 1.0 \quad (\text{Spec. Eq. H1-1a})$ $= \frac{267 \text{ kips}}{750 \text{ kips}} + \frac{8}{9} \left(\frac{167 \text{ kip-ft}}{427 \text{ kip-ft}} + \frac{53.3 \text{ kip-ft}}{207 \text{ kip-ft}} \right)$ $= 0.933 < 1.0 \quad \mathbf{o.k.}$

EXAMPLE H.2 W-SHAPE SUBJECT TO COMBINED COMPRESSION AND BENDING MOMENT ABOUT BOTH AXES (BY AISC SPECIFICATION SECTION H2)
Given:

Using AISC *Specification* Section H2, determine if an ASTM A992/A992M W14×99 has sufficient available strength to support the axial forces and moments listed as follows, obtained from a second-order analysis that includes $P-\delta$ effects. The unbraced length is 14 ft and the member has pinned ends. This example is included primarily to illustrate the use of AISC *Specification* Section H2.

LRFD	ASD
$P_u = 360$ kips	$P_a = 240$ kips
$M_{ux} = 250$ kip-ft	$M_{ax} = 167$ kip-ft
$M_{uy} = 80.0$ kip-ft	$M_{ay} = 53.3$ kip-ft

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992/A992M

$F_y = 50$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W14×99

$A = 29.1$ in.²

$S_x = 157$ in.³

$S_y = 55.2$ in.³

The required flexural and axial stresses are:

LRFD	ASD
$f_{ra} = \frac{P_u}{A}$ $= \frac{360 \text{ kips}}{29.1 \text{ in.}^2}$ $= 12.4 \text{ ksi}$	$f_{ra} = \frac{P_a}{A}$ $= \frac{240 \text{ kips}}{29.1 \text{ in.}^2}$ $= 8.25 \text{ ksi}$
$f_{rbx} = \frac{M_{ux}}{S_x}$ $= \frac{(250 \text{ kip-ft})(12 \text{ in./ft})}{157 \text{ in.}^3}$ $= 19.1 \text{ ksi}$	$f_{rbx} = \frac{M_{ax}}{S_x}$ $= \frac{(167 \text{ kip-ft})(12 \text{ in./ft})}{157 \text{ in.}^3}$ $= 12.8 \text{ ksi}$
$f_{rby} = \frac{M_{uy}}{S_y}$ $= \frac{(80.0 \text{ kip-ft})(12 \text{ in./ft})}{55.2 \text{ in.}^3}$ $= 17.4 \text{ ksi}$	$f_{rby} = \frac{M_{ay}}{S_y}$ $= \frac{(53.3 \text{ kip-ft})(12 \text{ in./ft})}{55.2 \text{ in.}^3}$ $= 11.6 \text{ ksi}$

The effective length of the member is:

$$\begin{aligned}
 L_{cx} &= L_{cy} \\
 &= KL \\
 &= 1.0(14 \text{ ft}) \\
 &= 14.0 \text{ ft}
 \end{aligned}$$

For $L_c = 14.0$ ft, calculate the available axial and flexural stresses using the available strengths from AISC *Manual* Table 6-1.

LRFD	ASD
$F_{ca} = \frac{\phi_c P_n}{A}$ $= \frac{1,130 \text{ kips}}{29.1 \text{ in.}^2}$ $= 38.8 \text{ ksi}$	$F_{ca} = \frac{P_n}{\Omega_c A}$ $= \frac{750 \text{ kips}}{29.1 \text{ in.}^2}$ $= 25.8 \text{ ksi}$
$F_{cbx} = \frac{\phi_b M_{nx}}{S_x}$ $= \frac{(642 \text{ kip-ft})(12 \text{ in./ft})}{157 \text{ in.}^3}$ $= 49.1 \text{ ksi}$	$F_{cbx} = \frac{M_{nx}}{\Omega_b S_x}$ $= \frac{(427 \text{ kip-ft})(12 \text{ in./ft})}{157 \text{ in.}^3}$ $= 32.6 \text{ ksi}$
$F_{cby} = \frac{\phi_b M_{ny}}{S_y}$ $= \frac{(311 \text{ kip-ft})(12 \text{ in./ft})}{55.2 \text{ in.}^3}$ $= 67.6 \text{ ksi}$	$F_{cby} = \frac{M_{ny}}{\Omega_b S_y}$ $= \frac{(207 \text{ kip-ft})(12 \text{ in./ft})}{55.2 \text{ in.}^3}$ $= 45.0 \text{ ksi}$

As shown in the LRFD calculation of F_{cby} in the preceding text, the available flexural stresses can exceed the yield stress in cases where the available strength is governed by yielding and the yielding strength is calculated using the plastic section modulus.

Combined Stress Ratio

From AISC *Specification* Section H2, check the combined stress ratios as follows:

LRFD	ASD
$\left \frac{f_{ra}}{F_{ca}} + \frac{f_{rbx}}{F_{cbx}} + \frac{f_{rby}}{F_{cby}} \right \leq 1.0 \quad (\text{from Spec. Eq. H2-1})$ $\left \frac{12.4 \text{ ksi}}{38.8 \text{ ksi}} + \frac{19.1 \text{ ksi}}{49.1 \text{ ksi}} + \frac{17.4 \text{ ksi}}{67.6 \text{ ksi}} \right = 0.966 < 1.0 \quad \text{o.k.}$	$\left \frac{f_{ra}}{F_{ca}} + \frac{f_{rbx}}{F_{cbx}} + \frac{f_{rby}}{F_{cby}} \right \leq 1.0 \quad (\text{from Spec. Eq. H2-1})$ $\left \frac{8.25 \text{ ksi}}{25.8 \text{ ksi}} + \frac{12.8 \text{ ksi}}{32.6 \text{ ksi}} + \frac{11.6 \text{ ksi}}{45.0 \text{ ksi}} \right = 0.970 < 1.0 \quad \text{o.k.}$

A comparison of these results with those from Example H.1B shows that AISC *Specification* Equation H1-1a will produce less conservative results than AISC *Specification* Equation H2-1 when its use is permitted.

Note: This check is made at a point on the cross section (extreme fiber, in this example). The designer must therefore determine which point on the cross section is critical or check multiple points if the critical point cannot be readily determined.

EXAMPLE H.3 W-SHAPE SUBJECT TO COMBINED AXIAL TENSION AND FLEXURE**Given:**

Select an ASTM A992/A992M W-shape with a 14 in. nominal depth to carry forces of 29 kips from dead load and 87 kips from live load in axial tension, as well as the following moments due to uniformly distributed loads:

$$M_{xD} = 32 \text{ kip-ft}$$

$$M_{xL} = 96 \text{ kip-ft}$$

$$M_{yD} = 11.3 \text{ kip-ft}$$

$$M_{yL} = 33.8 \text{ kip-ft}$$

The unbraced length is 30 ft and the ends are pinned. Assume the connections are made with no holes.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992/A992M

$$F_y = 50 \text{ ksi}$$

From ASCE/SEI 7, Chapter 2, the required strengths are:

LRFD	ASD
$P_u = 1.2(29 \text{ kips}) + 1.6(87 \text{ kips})$ $= 174 \text{ kips}$	$P_a = 29 \text{ kips} + 87 \text{ kips}$ $= 116 \text{ kips}$
$M_{ux} = 1.2(32 \text{ kip-ft}) + 1.6(96 \text{ kip-ft})$ $= 192 \text{ kip-ft}$	$M_{ax} = 32 \text{ kip-ft} + 96 \text{ kip-ft}$ $= 128 \text{ kip-ft}$
$M_{uy} = 1.2(11.3 \text{ kip-ft}) + 1.6(33.8 \text{ kip-ft})$ $= 67.6 \text{ kip-ft}$	$M_{ay} = 11.3 \text{ kip-ft} + 33.8 \text{ kip-ft}$ $= 45.1 \text{ kip-ft}$

Try a W14×82.

From AISC *Manual* Tables 1-1 and 3-2, the properties are as follows:

W14×82

$$A_g = 24.0 \text{ in.}^2$$

$$S_x = 123 \text{ in.}^3$$

$$Z_x = 139 \text{ in.}^3$$

$$S_y = 29.3 \text{ in.}^3$$

$$Z_y = 44.8 \text{ in.}^3$$

$$I_y = 148 \text{ in.}^4$$

$$L_p = 8.76 \text{ ft}$$

$$L_r = 33.2 \text{ ft}$$

Nominal Tensile Strength

From AISC *Specification* Section D2(a), the nominal tensile strength due to tensile yielding in the gross section is:

$$\begin{aligned}
 P_n &= F_y A_g && (\text{Spec. Eq. D2-1}) \\
 &= (50 \text{ ksi})(24.0 \text{ in.}^2) \\
 &= 1,200 \text{ kips}
 \end{aligned}$$

Note that for a member with holes, the rupture strength of the member would also have to be computed using AISC *Specification* Equation D2-2.

Nominal Flexural Strength for Bending About the Major Axis

Yielding

From AISC *Specification* Section F2.1, the nominal flexural strength due to yielding (plastic moment) is:

$$\begin{aligned}
 M_{nx} &= M_p = F_y Z_x && (\text{Spec. Eq. F2-1}) \\
 &= (50 \text{ ksi})(139 \text{ in.}^3) \\
 &= 6,950 \text{ kip-in. or } 579 \text{ kip-ft}
 \end{aligned}$$

Lateral-Torsional Buckling

From AISC *Specification* Section F2.2, the nominal flexural strength due to lateral-torsional buckling is determined as follows:

Because $L_p < L_b \leq L_r$, i.e., 8.76 ft < 30 ft < 33.2 ft, AISC *Specification* Equation F2-2 applies.

Lateral-Torsional Buckling Modification Factor, C_b

From AISC *Manual* Table 3-1, $C_b = 1.14$, without considering the beneficial effects of the tension force. However, per AISC *Specification* Section H1.2, C_b may be modified because the column is in axial tension concurrently with flexure.

$$\begin{aligned}
 P_{ey} &= \frac{\pi^2 EI_y}{L_b^2} && (\text{Spec. Eq. H1-2}) \\
 &= \frac{\pi^2 (29,000 \text{ ksi})(148 \text{ in.}^4)}{[(30 \text{ ft})(12.0 \text{ in./ft})]^2} \\
 &= 327 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$ \sqrt{1 + \frac{\alpha P_u}{P_{ey}}} = \sqrt{1 + \frac{1.0(174 \text{ kips})}{327 \text{ kips}}} $ $= 1.24$	$ \sqrt{1 + \frac{\alpha P_a}{P_{ey}}} = \sqrt{1 + \frac{1.6(116 \text{ kips})}{327 \text{ kips}}} $ $= 1.25$

$$\begin{aligned}
 C_b &= 1.24(1.14) \\
 &= 1.41
 \end{aligned}$$

$$\begin{aligned}
 M_n &= C_b \left[M_p - (M_p - 0.7F_y S_x) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_p \quad (\text{Spec. Eq. F2-2}) \\
 &= 1.41 \left\{ 6,950 \text{ kip-in.} - \left[6,950 \text{ kip-in.} - 0.7(50 \text{ ksi})(123 \text{ in.}^3) \right] \left(\frac{30 \text{ ft} - 8.76 \text{ ft}}{33.2 \text{ ft} - 8.76 \text{ ft}} \right) \right\} \leq 6,950 \text{ kip-in.} \\
 &= 6,560 \text{ kip-in. or } 547 \text{ kip-ft} \quad \textbf{controls}
 \end{aligned}$$

Local Buckling

Per AISC *Manual* Table 1-1, a W14×82 does not have an “[f]” footnote, indicating the cross section is compact at $F_y = 50$ ksi; therefore, the local buckling limit state does not apply.

Nominal Flexural Strength for Bending About the Minor Axis and the Interaction of Flexure and Tension

Because a W14×82 has compact flanges, only the limit state of yielding applies for bending about the minor axis.

$$\begin{aligned}
 M_{ny} &= M_p = F_y Z_y \leq 1.6 F_y S_y \quad (\text{Spec. Eq. F6-1}) \\
 &= (50 \text{ ksi})(44.8 \text{ in.}^3) \leq 1.6(50 \text{ ksi})(29.3 \text{ in.}^3) \\
 &= 2,240 \text{ kip-in.} < 2,340 \text{ kip-in.} \\
 &= 2,240 \text{ kip-in. or } 187 \text{ kip-ft}
 \end{aligned}$$

Available Strength

From AISC *Specification* Sections D2 and F1, the available strengths are:

LRFD	ASD
$\phi_b = \phi_t = 0.90$	$\Omega_b = \Omega_t = 1.67$
$P_c = \phi_t P_n$ $= 0.90(1,200 \text{ kips})$ $= 1,080 \text{ kips}$	$P_c = \frac{P_n}{\Omega_t}$ $= \frac{1,200 \text{ kips}}{1.67}$ $= 719 \text{ kips}$
$M_{cx} = \phi_b M_{nx}$ $= 0.90(547 \text{ kip-ft})$ $= 492 \text{ kip-ft}$	$M_{cx} = \frac{M_{nx}}{\Omega_b}$ $= \frac{547 \text{ kip-ft}}{1.67}$ $= 328 \text{ kip-ft}$
$M_{cy} = \phi_b M_{ny}$ $= 0.90(187 \text{ kip-ft})$ $= 168 \text{ kip-ft}$	$M_{cy} = \frac{M_{ny}}{\Omega_b}$ $= \frac{187 \text{ kip-ft}}{1.67}$ $= 112 \text{ kip-ft}$

Interaction of Tension and Flexure

Check the limit for AISC *Specification* Equation H1-1a.

LRFD	ASD
$\frac{P_r}{P_c} = \frac{P_u}{\phi_t P_n}$ $= \frac{174 \text{ kips}}{1,080 \text{ kips}}$ $= 0.161 < 0.2$ Because $\frac{P_r}{P_c} < 0.2$, $\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \right) \leq 1.0 \quad (\text{Spec. Eq. H1-1b})$ $= \frac{174 \text{ kips}}{2(1,080 \text{ kips})} + \left(\frac{192 \text{ kip-ft}}{492 \text{ kip-ft}} + \frac{67.6 \text{ kip-ft}}{168 \text{ kip-ft}} \right) \leq 1.0$ $= 0.873 < 1.0 \quad \mathbf{o.k.}$	$\frac{P_r}{P_c} = \frac{P_a}{P_n / \Omega_t}$ $= \frac{116 \text{ kips}}{719 \text{ kips}}$ $= 0.161 < 0.2$ Because $\frac{P_r}{P_c} < 0.2$, $\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \right) \leq 1.0 \quad (\text{Spec. Eq. H1-1b})$ $= \frac{116 \text{ kips}}{2(719 \text{ kips})} + \left(\frac{128 \text{ kip-ft}}{328 \text{ kip-ft}} + \frac{45.1 \text{ kip-ft}}{112 \text{ kip-ft}} \right) \leq 1.0$ $= 0.874 < 1.0 \quad \mathbf{o.k.}$

EXAMPLE H.4 W-SHAPE SUBJECT TO COMBINED AXIAL COMPRESSION AND FLEXURE**Given:**

Select an ASTM A992/A992M W-shape with a 10 in. nominal depth to carry axial compression forces of 5 kips from dead load and 15 kips from live load. The unbraced length is 14 ft and the ends are pinned. The member also has the following required moment strengths due to uniformly distributed loads, not including second-order effects:

$$M_{xD} = 15 \text{ kip-ft}$$

$$M_{xL} = 45 \text{ kip-ft}$$

$$M_{yD} = 2 \text{ kip-ft}$$

$$M_{yL} = 6 \text{ kip-ft}$$

The member is not subject to sidesway (no lateral translation).

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992/A992M

$$F_y = 50 \text{ ksi}$$

From Chapter 2 of ASCE/SEI 7, the required strength (not considering second-order effects) is:

LRFD	ASD
$P_u = 1.2(5 \text{ kips}) + 1.6(15 \text{ kips})$ $= 30.0 \text{ kips}$	$P_a = 5 \text{ kips} + 15 \text{ kips}$ $= 20.0 \text{ kips}$
$M_{ux} = 1.2(15 \text{ kip-ft}) + 1.6(45 \text{ kip-ft})$ $= 90.0 \text{ kip-ft}$	$M_{ax} = 15 \text{ kip-ft} + 45 \text{ kip-ft}$ $= 60.0 \text{ kip-ft}$
$M_{uy} = 1.2(2 \text{ kip-ft}) + 1.6(6 \text{ kip-ft})$ $= 12.0 \text{ kip-ft}$	$M_{ay} = 2 \text{ kip-ft} + 6 \text{ kip-ft}$ $= 8.00 \text{ kip-ft}$

Try a W10×33.

From AISC *Manual* Tables 1-1 and 3-2, the properties are as follows:

W10×33

$$S_x = 35.0 \text{ in.}^3$$

$$Z_x = 38.8 \text{ in.}^3$$

$$I_x = 171 \text{ in.}^4$$

$$r_x = 4.19 \text{ in.}$$

$$S_y = 9.20 \text{ in.}^3$$

$$Z_y = 14.0 \text{ in.}^3$$

$$I_y = 36.6 \text{ in.}^4$$

$$r_y = 1.94 \text{ in.}$$

$$L_p = 6.85 \text{ ft}$$

$$L_r = 21.8 \text{ ft}$$

Available Axial Strength

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, $K = 1.0$. Because $L_c = KL_x = KL_y = 14.0$ ft and $r_x > r_y$, the y - y axis will govern.

From AISC *Manual* Table 6-1, the available axial strength is:

LRFD	ASD
$P_c = \phi_c P_n$ $= 253$ kips	$P_c = \frac{P_n}{\Omega_c}$ $= 168$ kips

Required Flexural Strength (including second-order amplification)

Use the approximate method of second-order analysis procedure from AISC *Specification* Appendix 8. Because the member is not subject to sidesway, only P - δ amplifiers need to be added.

$$B_1 = \frac{C_m}{1 - \alpha P_r / P_{el}} \geq 1 \quad (\text{Spec. Eq. A-8-3})$$

where C_m is conservatively taken per AISC *Specification* Appendix 8, Section 8.1.2(b):

$$C_m = 1.0$$

The x - x axis flexural magnifier is:

$$\begin{aligned}
 P_{elx} &= \frac{\pi^2 EI_x}{(L_{clx})^2} && (\text{from Spec. Eq. A-8-5}) \\
 &= \frac{\pi^2 (29,000 \text{ ksi})(171 \text{ in.}^4)}{[(14 \text{ ft})(12 \text{ in./ft})]^2} \\
 &= 1,730 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\alpha = 1.0$ $B_{1x} = \frac{C_m}{1 - \alpha P_r / P_{elx}} \geq 1.0$ $= \frac{1.0}{1 - 1.0(30.0 \text{ kips}/1,730 \text{ kips})} \geq 1.0$ $= 1.02$ $M_{ux} = 1.02(90 \text{ kip-ft})$ $= 91.8 \text{ kip-ft}$	$\alpha = 1.6$ $B_{1x} = \frac{C_m}{1 - \alpha P_r / P_{elx}} \geq 1.0$ $= \frac{1.0}{1 - 1.6(20.0 \text{ kips}/1,730 \text{ kips})} \geq 1.0$ $= 1.02$ $M_{ax} = 1.02(60 \text{ kip-ft})$ $= 61.2 \text{ kip-ft}$

The y - y axis flexural magnifier is:

$$\begin{aligned}
 P_{ely} &= \frac{\pi^2 EI_y}{(L_{ely})^2} && \text{(from Spec. Eq. A-8-5)} \\
 &= \frac{\pi^2 (29,000 \text{ ksi})(36.6 \text{ in.}^4)}{[(14 \text{ ft})(12 \text{ in./ft})]^2} \\
 &= 371 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\alpha = 1.0$ $B_{1y} = \frac{C_m}{1 - \alpha P_r / P_{ely}} \geq 1.0$ $= \frac{1.0}{1 - 1.0(30.0 \text{ kips}/371 \text{ kips})} \geq 1.0$ $= 1.09$ $M_{uy} = 1.09(12.0 \text{ kip-ft})$ $= 13.1 \text{ kip-ft}$	$\alpha = 1.6$ $B_{1y} = \frac{C_m}{1 - \alpha P_r / P_{ely}} \geq 1.0$ $= \frac{1.0}{1 - 1.6(20.0 \text{ kips}/371 \text{ kips})} \geq 1.0$ $= 1.09$ $M_{ay} = 1.09(8.00 \text{ kip-ft})$ $= 8.72 \text{ kip-ft}$

Nominal Flexural Strength about the Major Axis

Yielding

$$\begin{aligned}
 M_{nx} &= M_p = F_y Z_x && \text{(Spec. Eq. F2-1)} \\
 &= (50 \text{ ksi})(38.8 \text{ in.}^3) \\
 &= 1,940 \text{ kip-in.}
 \end{aligned}$$

Lateral-Torsional Buckling

Because $L_p < L_b \leq L_r$, i.e., 6.85 ft < 14.0 ft < 21.8 ft, AISC *Specification* Equation F2-2 applies.

From AISC *Manual* Table 3-1, $C_b = 1.14$.

$$\begin{aligned}
 M_{nx} &= C_b \left[M_p - (M_p - 0.7F_y S_x) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_p && \text{(Spec. Eq. F2-2)} \\
 &= 1.14 \left\{ 1,940 \text{ kip-in.} - \left[1,940 \text{ kip-in.} - 0.7(50 \text{ ksi})(35.0 \text{ in.}^3) \right] \left(\frac{14 \text{ ft} - 6.85 \text{ ft}}{21.8 \text{ ft} - 6.85 \text{ ft}} \right) \right\} < 1,940 \text{ kip-in.} \\
 &= 1,820 \text{ kip-in.} < 1,940 \text{ kip-in.} \\
 &= 1,820 \text{ kip-in. or } 152 \text{ kip-ft} \quad \textbf{controls}
 \end{aligned}$$

Local Buckling

Per AISC *Manual* Table 1-1, a W10×33 does not have an “[f]” footnote, indicating the member is compact for $F_y = 50$ ksi, so the local buckling limit state does not apply.

Nominal Flexural Strength about the Minor Axis

Determine the nominal flexural strength for bending about the minor axis from AISC *Specification* Section F6. Because a W10×33 has compact flanges, only the yielding limit state applies.

From AISC *Specification* Section F6.1:

$$\begin{aligned}
 M_{ny} &= M_p = F_y Z_y \leq 1.6 F_y S_y && (\text{Spec. Eq. F6-1}) \\
 &= (50 \text{ ksi})(14.0 \text{ in.}^3) \leq 1.6(50 \text{ ksi})(9.20 \text{ in.}^3) \\
 &= 700 \text{ kip-in.} < 736 \text{ kip-in.} \\
 &= 700 \text{ kip-in. or } 58.3 \text{ kip-ft}
 \end{aligned}$$

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$	$\Omega_b = 1.67$
$M_{cx} = \phi_b M_{nx}$ $= 0.90(152 \text{ kip-ft})$ $= 137 \text{ kip-ft}$	$M_{cx} = \frac{M_{nx}}{\Omega_b}$ $= \frac{152 \text{ kip-ft}}{1.67}$ $= 91.0 \text{ kip-ft}$
$M_{cy} = \phi_b M_{ny}$ $= 0.90(58.3 \text{ kip-ft})$ $= 52.5 \text{ kip-ft}$	$M_{cy} = \frac{M_{ny}}{\Omega_b}$ $= \frac{58.3 \text{ kip-ft}}{1.67}$ $= 34.9 \text{ kip-ft}$

Check the limit for AISC *Specification* Equations H1-1a and H1-1b.

LRFD	ASD
$\frac{P_r}{P_c} = \frac{P_u}{\phi_c P_n}$ $= \frac{30.0 \text{ kips}}{253 \text{ kips}}$ $= 0.119 < 0.2$	$\frac{P_r}{P_c} = \frac{P_a}{P_n / \Omega_c}$ $= \frac{20.0 \text{ kips}}{168 \text{ kips}}$ $= 0.119 < 0.2$
Because $\frac{P_r}{P_c} < 0.2$,	Because $\frac{P_r}{P_c} < 0.2$,
$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \right) \leq 1.0 \quad (\text{Spec. Eq. H1-1b})$ $= \frac{30.0 \text{ kips}}{2(253 \text{ kips})} + \left(\frac{91.8 \text{ kip-ft}}{137 \text{ kip-ft}} + \frac{13.1 \text{ kip-ft}}{52.5 \text{ kip-ft}} \right) \leq 1.0$ $= 0.979 < 1.0 \quad \text{o.k.}$	$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \right) \leq 1.0 \quad (\text{Spec. Eq. H1-1b})$ $= \frac{20.0 \text{ kips}}{2(168 \text{ kips})} + \left(\frac{61.2 \text{ kip-ft}}{91.0 \text{ kip-ft}} + \frac{8.72 \text{ kip-ft}}{34.9 \text{ kip-ft}} \right)$ $= 0.982 < 1.0 \quad \text{o.k.}$

CHAPTER H DESIGN EXAMPLE REFERENCES

Seaburg, P.A. and Carter, C.J. (1997), *Torsional Analysis of Structural Steel Members*, Design Guide 9, AISC, Chicago, Ill.

Chapter J

Design of Connections

AISC Specification Chapter J addresses the design of connections. The chapter's primary focus is the design of welded and bolted connections. Design requirements for fillers, splices, column bases, concentrated forces, anchor rods, and other threaded parts are also covered. See *AISC Specification* Appendix 3 for special requirements for connections subject to fatigue.

EXAMPLE J.1 FILLET WELD IN LONGITUDINAL SHEAR**Given:**

As shown in Figure J.1-1, a $\frac{1}{4}$ -in.-thick $\times$ 18-in.-wide plate is fillet welded to a $\frac{3}{8}$ -in.-thick plate. The plates are ASTM A572/A572M Grade 50 and have been properly sized. Use 70-ksi electrodes.

Confirm that the size and length of the welds shown are adequate to resist the applied loading.

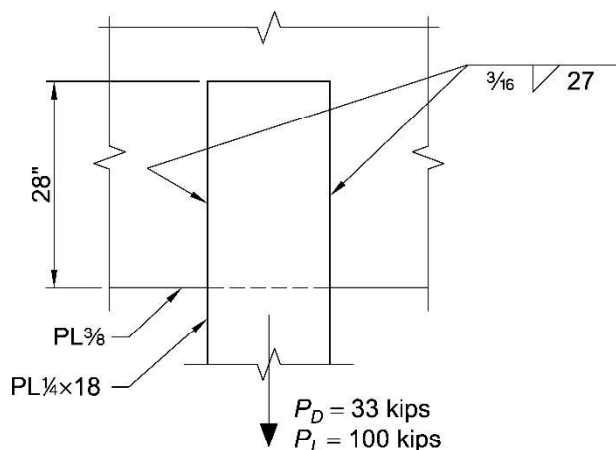


Fig. J.1-1. Geometry and loading for Example J.1.

Solution:

From AISC *Manual* Table 2-5, the material properties are as follows:

ASTM A572/A572M Grade 50

$F_y = 50$ ksi

$F_u = 65$ ksi

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$P_u = 1.2(33 \text{ kips}) + 1.6(100 \text{ kips})$ $= 200 \text{ kips}$	$P_a = 33 \text{ kips} + 100 \text{ kips}$ $= 133 \text{ kips}$

Maximum and Minimum Weld Size

Because the thickness of the overlapping plate is $\frac{1}{4}$ in., the maximum fillet weld size that can be used without special notation per AISC *Specification* Section J2.2b(b)(2), is a $\frac{3}{16}$ in. fillet weld. A $\frac{3}{16}$ in. fillet weld can be deposited in the flat or horizontal position in a single pass (true up to $\frac{5}{16}$ in.).

From AISC *Specification* Table J2.4, the minimum size of the fillet weld, based on a material thickness of $\frac{1}{4}$ in. is $\frac{1}{8}$ in.

Weld Strength

The nominal weld strength per inch of a $\frac{3}{16}$ in. weld is determined from AISC *Specification* Section J2.4(a). From the User Note in AISC *Specification* Section J2.4, the requirements for strain compatibility are satisfied for this weld and the directional strength increase factor may be used.

$$\begin{aligned} k_{ds} &= 1.0 + 0.50 \sin^{1.5} \theta \\ &= 1.0 + 0.50 \sin^{1.5} 0^\circ \\ &= 1.0 \end{aligned} \quad (\text{Spec. Eq. J2-5})$$

$$\begin{aligned} R_n &= F_{nw} A_{we} k_{ds} \\ &= (0.60 F_{EXX}) A_{we} k_{ds} \\ &= 0.60 (70 \text{ ksi}) \left(\frac{\frac{3}{16} \text{ in.}}{\sqrt{2}} \right) (1.0) \\ &= 5.57 \text{ kip/in.} \end{aligned} \quad (\text{Spec. Eq. J2-4})$$

From AISC *Specification* Section J2.2b(d), check the weld length to weld size ratio, because this is an end-loaded fillet weld.

$$\begin{aligned} \frac{l}{w} &= \frac{27.0 \text{ in.}}{\frac{3}{16} \text{ in.}} \\ &= 144 > 100 \end{aligned}$$

Therefore, AISC *Specification* Equation J2.1 must be applied.

$$\begin{aligned} \beta &= 1.2 - 0.002(l/w) \leq 1.0 \\ &= 1.2 - 0.002(144) \leq 1.0 \\ &= 0.912 < 1.0 \quad \mathbf{o.k.} \end{aligned} \quad (\text{Spec. Eq. J2-1})$$

The nominal weld shear rupture strength is:

$$\begin{aligned} R_n &= 0.912 (5.57 \text{ kip/in.}) (2 \text{ welds}) (27 \text{ in.}) \\ &= 274 \text{ kips} \end{aligned}$$

From AISC *Specification* Section J2.4, the available shear rupture strength is:

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi R_n = 0.75 (274 \text{ kips})$	$\frac{R_n}{\Omega} = \frac{274 \text{ kips}}{2.00}$
$= 206 \text{ kips} > 200 \text{ kips} \quad \mathbf{o.k.}$	$= 137 \text{ kips} > 133 \text{ kips} \quad \mathbf{o.k.}$

The base metal strength is determined from AISC *Specification* Section J2.4(a). The $\frac{1}{4}$ -in.-thick plate controls:

$$\begin{aligned}
 R_n &= F_{nBM} A_{BM} && (\text{Spec. Eq. J2-2}) \\
 &= 0.60 F_u t_p l_{weld} \\
 &= 0.60 (65 \text{ ksi}) (\tfrac{1}{4} \text{ in.}) (2 \text{ welds}) (27 \text{ in.}) \\
 &= 527 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75 (527 \text{ kips})$ $= 395 \text{ kips} > 200 \text{ kips} \quad \mathbf{o.k.}$	$\Omega = 2.00$ $\frac{R_n}{\Omega} = \frac{527 \text{ kips}}{2.00}$ $= 264 \text{ kips} > 133 \text{ kips} \quad \mathbf{o.k.}$

EXAMPLE J.2A FILLET WELD LOADED AT AN ANGLE**Given:**

Determine the required length, l , of a two-sided fillet weld between a gusset plate and beam flange to resist loads as shown in Figure J.2A-1. The weld is loaded at an angle of 60° relative to the weld longitudinal axis. Assume the beam and the gusset plate thickness and length have been properly sized. Use a 70-ksi electrode.

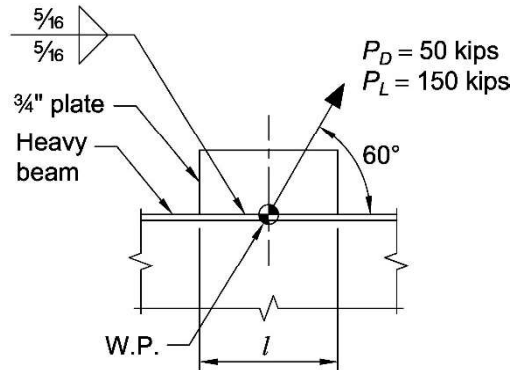


Fig. J.2A-1. Geometry and loading for Example J.2A.

Solution:

From ASCE/SEI 7, Chapter 2, the required tensile strength is:

LRFD	ASD
$P_u = 1.2(50 \text{ kips}) + 1.6(150 \text{ kips})$ $= 300 \text{ kips}$	$P_a = 50 \text{ kips} + 150 \text{ kips}$ $= 200 \text{ kips}$

Note that from AISC *Specification* Table J2.4, the minimum size of fillet weld, based on a material thickness of $\frac{3}{4}$ in. is $\frac{1}{4}$ in. (assuming the beam flange thickness exceeds $\frac{3}{4}$ in.).

Available Shear Strength of the Fillet Weld Per Inch of Length

The nominal strength of the fillet weld is determined from AISC *Specification* Section J2.4(a). From the User Note in AISC *Specification* Section J2.4, the requirements for strain compatibility are satisfied for this weld and the directional strength increase factor may be used.

$$\begin{aligned}
 k_{ds} &= 1.0 + 0.50 \sin^{1.5} \theta \\
 &= 1.0 + 0.50 \sin^{1.5} 60^\circ \\
 &= 1.40
 \end{aligned}
 \tag{Spec. Eq. J2-5}$$

$$\begin{aligned}
 R_n &= F_{nw} A_{we} k_{ds} \\
 &= 0.60 F_{EXX} A_{we} k_{ds} \\
 &= 0.60 (70 \text{ ksi}) \left(\frac{\frac{5}{16} \text{ in.}}{\sqrt{2}} \right) (1.40) \\
 &= 13.0 \text{ kip/in.}
 \end{aligned}
 \tag{Spec. Eq. J2-4}$$

The available shear strength per inch of weld for fillet welds on both sides of the plate is:

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi R_n = 0.75(13.0 \text{ kip/in.})(2 \text{ sides})$ $= 19.5 \text{ kip/in.}$	$\frac{R_n}{\Omega} = \left(\frac{13.0 \text{ kip/in.}}{2.00} \right)(2 \text{ sides})$ $= 13.0 \text{ kip/in.}$

Required Length of Weld

LRFD	ASD
$l = \frac{300 \text{ kips}}{19.5 \text{ kip/in.}}$ $= 15.4 \text{ in.}$	$l = \frac{200 \text{ kips}}{13.0 \text{ kip/in.}}$ $= 15.4 \text{ in.}$
Use $l = 16 \text{ in.}$	Use $l = 16 \text{ in.}$

EXAMPLE J.3 COMBINED TENSION AND SHEAR IN BEARING-TYPE CONNECTIONS**Given:**

A $\frac{3}{4}$ in. diameter, Group 120 bolt with threads not excluded from the shear plane (thread condition N) is subjected to a tension force of 3.5 kips due to dead load and 12 kips due to live load, and a shear force of 1.33 kips due to dead load and 4 kips due to live load. Check the combined stresses according to AISC *Specification* Equations J3-3a and J3-3b.

Solution:

From ASCE/SEI 7, Chapter 2, the required tensile and shear strengths are:

LRFD	ASD
Tension: $T_u = 1.2(3.5 \text{ kips}) + 1.6(12 \text{ kips})$ $= 23.4 \text{ kips}$ Shear: $V_u = 1.2(1.33 \text{ kips}) + 1.6(4 \text{ kips})$ $= 8.00 \text{ kips}$	Tension: $T_a = 3.5 \text{ kips} + 12 \text{ kips}$ $= 15.5 \text{ kips}$ Shear: $V_a = 1.33 \text{ kips} + 4 \text{ kips}$ $= 5.33 \text{ kips}$

Available Tensile Strength

When a bolt is subject to combined tension and shear, the available tensile strength is determined according to the limit states of tension and shear rupture, from AISC *Specification* Section J3.8 as follows.

From AISC *Specification* Table J3.2, Group 120 bolts:

$$F_{nt} = 90 \text{ ksi}$$

$$F_{nv} = 54 \text{ ksi}$$

From AISC *Manual* Table 7-2, for a $\frac{3}{4}$ in. diameter bolt:

$$A_b = 0.442 \text{ in.}^2$$

The available shear stress is determined as follows and must equal or exceed the required shear stress.

LRFD	ASD
$\phi = 0.75$ $\phi F_{nv} = 0.75(54 \text{ ksi})$ $= 40.5 \text{ ksi}$ $f_{rv} = \frac{V_u}{A_b}$ $= \frac{8.00 \text{ kips}}{0.442 \text{ in.}^2}$ $= 18.1 \text{ ksi} < 40.5 \text{ ksi} \quad \text{o.k.}$	$\Omega = 2.00$ $\frac{F_{nv}}{\Omega} = \frac{54 \text{ ksi}}{2.00}$ $= 27.0 \text{ ksi}$ $f_{rv} = \frac{V_a}{A_b}$ $= \frac{5.33 \text{ kips}}{0.442 \text{ in.}^2}$ $= 12.1 \text{ ksi} < 27.0 \text{ ksi} \quad \text{o.k.}$

The available tensile strength of a bolt subject to combined tension and shear is as follows:

LRFD	ASD
$F'_nt = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_{rv} \leq F_{nt} \quad (\text{Spec. Eq. J3-3a})$ $= 1.3(90 \text{ ksi}) - \frac{90 \text{ ksi}}{40.5 \text{ ksi}} (18.1 \text{ ksi}) \leq 90 \text{ ksi}$ $= 76.8 \text{ ksi}$ For combined tension and shear, $\phi = 0.75$, from AISC <i>Specification</i> Section J3.8. $\phi R_n = \phi F'_nt A_b \quad (\text{Spec. Eq. J3-2})$ $= 0.75(76.8 \text{ ksi})(0.442 \text{ in.}^2)$ $= 25.5 \text{ kips} > 23.4 \text{ kips} \quad \mathbf{o.k.}$	$F'_nt = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_{rv} \leq F_{nt} \quad (\text{Spec. Eq. J3-3b})$ $= 1.3(90 \text{ ksi}) - \frac{90 \text{ ksi}}{27.0 \text{ ksi}} (12.1 \text{ ksi}) \leq 90 \text{ ksi}$ $= 76.7 \text{ ksi}$ For combined tension and shear, $\Omega = 2.00$, from AISC <i>Specification</i> Section J3.8. $\frac{R_n}{\Omega} = \frac{F'_nt A_b}{\Omega} \quad (\text{Spec. Eq. J3-2})$ $= \frac{(76.7 \text{ ksi})(0.442 \text{ in.}^2)}{2.00}$ $= 17.0 \text{ kips} > 15.5 \text{ kips} \quad \mathbf{o.k.}$

The effects of combined shear and tensile stresses need not be investigated if either the required shear or tensile stress is less than or equal to 30% of the corresponding available stress per the User Note at the end of AISC *Specification* Section J3.8. In the example herein, both the required shear and tensile stresses exceeded the 30% threshold and evaluation of combined stresses was necessary.

AISC *Specification* Equations J3-3a and J3-3b may be rewritten so as to find a nominal shear stress, F'_{nv} , as a function of the required tensile stress, as is shown in AISC *Specification* Commentary Equations C-J3-6a and C-J3-6b.

EXAMPLE J.4A SLIP-CRITICAL CONNECTION WITH SHORT-SLOTTED HOLES**Given:**

Refer to Figure J.4A-1 and select the number of bolts that are required to support the loads shown when the connection plates have short slots transverse to the load and no fillers are provided. Select the number of bolts required for slip resistance only. Washers have been provided in accordance with the RCSC *Specification* Section 6.

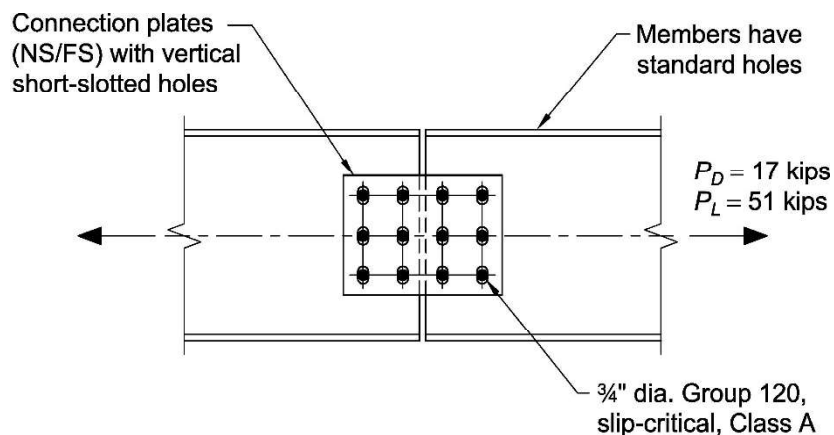


Fig. J.4A-1. Geometry and loading for Example J.4A.

Solution:

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$P_u = 1.2(17 \text{ kips}) + 1.6(51 \text{ kips})$ $= 102 \text{ kips}$	$P_a = 17 \text{ kips} + 51 \text{ kips}$ $= 68.0 \text{ kips}$

From AISC *Specification* Section J3.9(a), the available slip resistance for the limit state of slip for standard size and short-slotted holes perpendicular to the direction of the load is determined as follows:

$$\begin{aligned}\phi &= 1.00 \\ \Omega &= 1.50 \\ \mu &= 0.30 \text{ for Class A surface} \\ D_u &= 1.13 \\ h_f &= 1.0, \text{ no filler is provided} \\ T_b &= 28 \text{ kips, from AISC } \textit{Specification} \text{ Table J3.1, Group 120} \\ n_s &= 2, \text{ number of slip planes}\end{aligned}$$

$$\begin{aligned}R_n &= 0.30(1.13)(1.0)(28 \text{ kips})(2) \\ &= 19.0 \text{ kips/bolt}\end{aligned} \quad (\text{Spec. Eq. J3-4})$$

The available slip resistance per bolt is:

LRFD	ASD
$\phi R_n = 1.00(19.0 \text{ kips/bolt})$ $= 19.0 \text{ kips/bolt}$	$\frac{R_n}{\Omega} = \frac{19.0 \text{ kips/bolt}}{1.50}$ $= 12.7 \text{ kips/bolt}$

Note that the available slip resistance per bolt can also be determined using *AISC Manual* Table 7-3.

Required Number of Bolts

LRFD	ASD
$n_b = \frac{P_u}{\phi R_n}$ $= \frac{102 \text{ kips}}{19.0 \text{ kips/bolt}}$ $= 5.37 \text{ bolts}$	$n_b = \frac{P_a}{\left(\frac{R_n}{\Omega} \right)}$ $= \frac{68.0 \text{ kips}}{12.7 \text{ kips/bolt}}$ $= 5.35 \text{ bolts}$
Use 6 bolts	Use 6 bolts

Note: Slip-critical connections shall be designed to prevent slip and for the limit states of bearing-type connections. To complete the verification of this connection, the limit states of bolt shear, bearing, tearout, tensile yielding, tensile rupture, and block shear rupture must also be checked.

EXAMPLE J.4B SLIP-CRITICAL CONNECTION WITH LONG-SLOTTED HOLES**Given:**

Repeat Example J.4A with the same loads, but assuming that the connection plates have long-slotted holes in the direction of the load, as shown in Figure J.4B-1. Washers have been provided in accordance with the RCSC *Specification* Section 6.

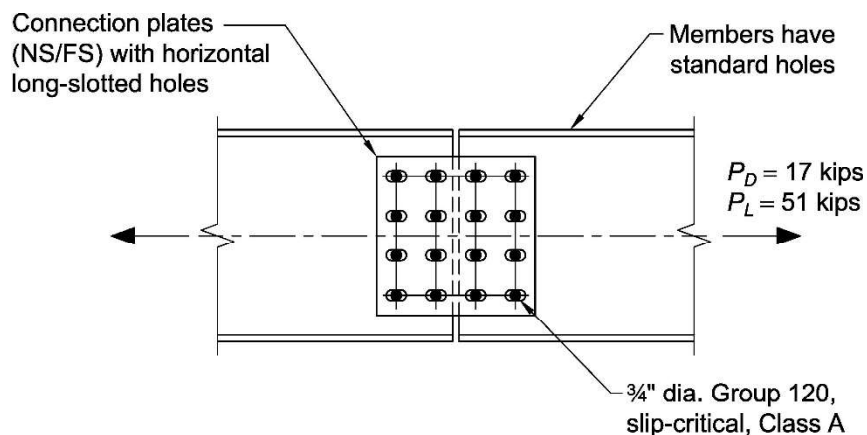


Fig. J.4B-1. Geometry and loading for Example J.4B.

Solution:

The required strength from Example J.4A is:

LRFD	ASD
$P_u = 102$ kips	$P_a = 68.0$ kips

From AISC *Specification* Section J3.9(c), the available slip resistance for the limit state of slip for long-slotted holes is determined as follows:

$$\begin{aligned}
 \phi &= 0.70 \\
 \Omega &= 2.14 \\
 \mu &= 0.30 \text{ for Class A surface} \\
 D_u &= 1.13 \\
 h_f &= 1.0, \text{ no filler is provided} \\
 T_b &= 28 \text{ kips, from AISC } \textit{Specification} \text{ Table J3.1, Group 120} \\
 n_s &= 2, \text{ number of slip planes}
 \end{aligned}$$

$$\begin{aligned}
 R_n &= \mu D_u h_f T_b n_s \\
 &= 0.30(1.13)(1.0)(28 \text{ kips})(2) \\
 &= 19.0 \text{ kips/bolt}
 \end{aligned}
 \tag{Spec. Eq. J3-4}$$

The available slip resistance per bolt is:

LRFD	ASD
$\phi R_n = 0.70(19.0 \text{ kips/bolt})$ $= 13.3 \text{ kips/bolt}$	$\frac{R_n}{\Omega} = \frac{19.0 \text{ kips/bolt}}{2.14}$ $= 8.88 \text{ kips/bolt}$

Note that the available slip resistance per bolt can also be taken from AISC *Manual* Table 7-3.

Required Number of Bolts

LRFD	ASD
$n_b = \frac{P_u}{\phi R_n}$ $= \frac{102 \text{ kips}}{13.3 \text{ kips/bolt}}$ $= 7.67 \text{ bolts}$ Use 8 bolts	$n_b = \frac{P_a}{\left(\frac{R_n}{\Omega} \right)}$ $= \frac{68.0 \text{ kips}}{8.88 \text{ kips/bolt}}$ $= 7.66 \text{ bolts}$ Use 8 bolts

Note: To complete the verification of this connection, the limit states of bolt shear, bearing, tearout, tensile yielding, tensile rupture, and block shear rupture must be determined.

EXAMPLE J.5 COMBINED TENSION AND SHEAR IN A SLIP-CRITICAL CONNECTION

Because the pretension of a bolt in a slip-critical connection is used to create the clamping force that produces the shear strength of the connection, the available shear strength must be reduced for any load that produces tension in the connection.

Given:

The slip-critical bolt group shown in Figure J.5-1 is subjected to tension and shear. This example shows the design for bolt slip resistance only and assumes that the beams and plates are adequate to transmit the loads. Determine if the bolts are adequate.

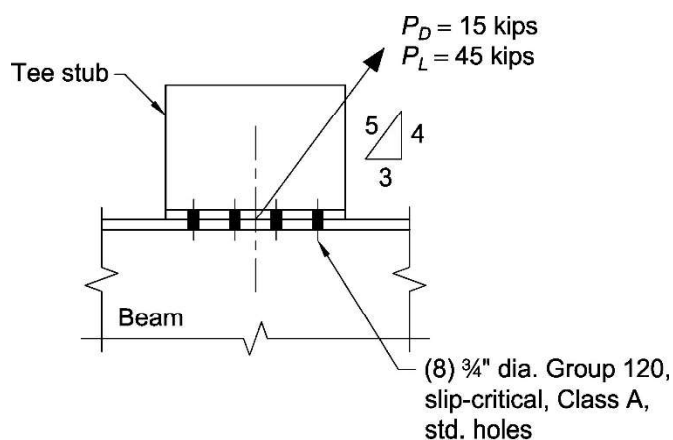


Fig. J.5-1. Geometry and loading for Example J.5.

Solution:

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$P_u = 1.2(15 \text{ kips}) + 1.6(45 \text{ kips})$ $= 90.0 \text{ kips}$	$P_a = 15 \text{ kips} + 45 \text{ kips}$ $= 60.0 \text{ kips}$
By geometry:	By geometry:
$T_u = \frac{4}{5}(90.0 \text{ kips})$ $= 72.0 \text{ kips}$	$T_a = \frac{4}{5}(60.0 \text{ kips})$ $= 48.0 \text{ kips}$
$V_u = \frac{3}{5}(90.0 \text{ kips})$ $= 54.0 \text{ kips}$	$V_a = \frac{3}{5}(60.0 \text{ kips})$ $= 36.0 \text{ kips}$

Available Bolt Tensile Strength

The available tensile strength is determined from AISC *Specification* Section J3.7.

From AISC *Specification* Table J3.2 for Group 120 bolts, the nominal tensile stress is $F_{nt} = 90$ ksi. From AISC *Manual* Table 7-2, for a $\frac{3}{4}$ in. diameter bolt:

$$A_b = 0.442 \text{ in.}^2$$

The nominal tensile strength is:

$$\begin{aligned} R_n &= F_{nt} A_b && \text{(from Spec. Eq. J3-1)} \\ &= (90 \text{ ksi})(0.442 \text{ in.}^2) \\ &= 39.8 \text{ kips} \end{aligned}$$

The available tensile strength is:

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi R_n = 0.75(39.8 \text{ kips/bolt}) > \frac{72.0 \text{ kips}}{8 \text{ bolts}}$ $= 29.9 \text{ kips/bolt} > 9.00 \text{ kips/bolt} \quad \mathbf{o.k.}$	$\frac{R_n}{\Omega} = \frac{39.8 \text{ kips/bolt}}{2.00} > \frac{48.0 \text{ kips}}{8 \text{ bolts}}$ $= 19.9 \text{ kips/bolt} > 6.00 \text{ kips/bolt} \quad \mathbf{o.k.}$

Note that the available tensile strength per bolt can also be determined using AISC *Manual* Table 7-2.

Available Slip Resistance per Bolt

The available slip resistance for one bolt in standard size holes is determined using AISC *Specification* Section J3.9(a):

$$\begin{aligned} \phi &= 1.00 \\ \Omega &= 1.50 \\ \mu &= 0.30 \text{ for Class A surface} \\ D_u &= 1.13 \\ h_f &= 1.0, \text{ assuming no more than one filler} \\ T_b &= 28 \text{ kips, from AISC Specification Table J3.1, Group 120} \\ n_s &= 1, \text{ number of slip planes} \end{aligned}$$

LRFD	ASD
Determine the available slip resistance ($T_u = 0$) of a bolt:	Determine the available slip resistance ($T_a = 0$) of a bolt:
$\phi R_n = \phi \mu D_u h_f T_b n_s \quad \text{(from Spec. Eq. J3-4)}$ $= 1.00(0.30)(1.13)(1.0)(28 \text{ kips})(1)$ $= 9.49 \text{ kips/bolt}$	$\frac{R_n}{\Omega} = \frac{\mu D_u h_f T_b n_s}{\Omega} \quad \text{(from Spec. Eq. J3-4)}$ $= \frac{0.30(1.13)(1.0)(28 \text{ kips})(1)}{1.50}$ $= 6.33 \text{ kips/bolt}$

Note that the available slip resistance for one bolt with a Class A faying surface can also be taken from AISC *Manual* Table 7-3.

Available Slip Resistance of the Connection

Because the slip-critical connection is subject to combined tension and shear, the available slip resistance is multiplied by a reduction factor provided in AISC *Specification* Section J3.10.

LRFD	ASD
Slip-critical combined tension and shear factor:	Slip-critical combined tension and shear factor:
$k_{sc} = 1 - \frac{T_u}{D_u T_b n_b} \geq 0 \quad (\text{Spec. Eq. J3-5a})$ $= 1 - \frac{72.0 \text{ kips}}{1.13(28 \text{ kips})(8)} > 0$ $= 0.716$ $\phi R_n = \phi R_n k_{sc} n_b$ $= (9.49 \text{ kips/bolt})(0.716)(8 \text{ bolts})$ $= 54.4 \text{ kips} > 54.0 \text{ kips} \quad \mathbf{o.k.}$	$k_{sc} = 1 - \frac{1.5T_a}{D_u T_b n_b} \geq 0 \quad (\text{Spec. Eq. J3-5b})$ $= 1 - \frac{1.5(48.0 \text{ kips})}{1.13(28 \text{ kips})(8)} > 0$ $= 0.716$ $\frac{R_n}{\Omega} = \frac{R_n}{\Omega} k_{sc} n_b$ $= (6.33 \text{ kips/bolt})(0.716)(8 \text{ bolts})$ $= 36.3 \text{ kips} > 36.0 \text{ kips} \quad \mathbf{o.k.}$

Note: The bolt group must still be checked for all applicable strength limit states for a bearing-type connection.

Part II

Examples Based on the AISC *Steel Construction Manual*

This part contains design examples demonstrating procedures and use of design aids from the AISC *Steel Construction Manual*.

Chapter IIA

Simple Shear Connections

The design of connecting elements is covered in Part 9 of the *AISC Manual*. The design of simple shear connections is covered in Part 10 of the *AISC Manual*. The design of simple connections for combined forces is covered in Part 12 of the *AISC Manual*.

EXAMPLE IIA-1A ALL-BOLTED DOUBLE-ANGLE CONNECTION**Given:**

Verify the available strength of an all-bolted double-angle shear connection between an ASTM A992/A992M W36×231 beam and an ASTM A992/A992M W14×90 column flange, as shown in Figure IIA-1A-1, supporting the following beam end reactions:

$$R_D = 37.5 \text{ kips}$$

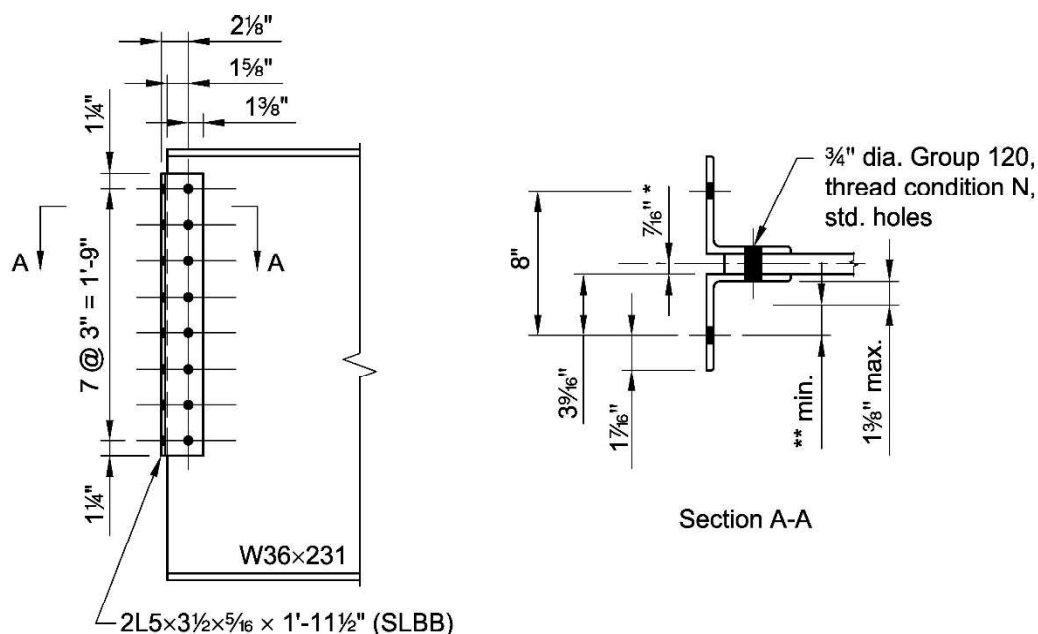
$$R_L = 113 \text{ kips}$$

Use ASTM A572/A572M Grade 50 angles.

This example is repeated using the following two procedures:

Part A: Determine the available connection strength using the tables in *Manual* Part 10.

Part B: Determine the available connection strength by checking individual limit states.



* This dimension (see sketch, Section A-A) is determined as one-half of the decimal web thickness rounded to the next higher $\frac{1}{16}$ in. Example: $0.760"/2 = 0.380"$; use $\frac{7}{16}$ in. This will produce spacing of holes in the supporting beam slightly larger than detailed in the angles to permit spreading of angles (angles can be spread but not closed) at time of erection to supporting member. Alternatively, consider using horizontal short slots in the support legs of the angles.

**See AISC *Manual* Tables 7-15 and 7-16 for driving clearance.

Fig. IIA-1A-1. Connection geometry for Example IIA-1A.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam and column
ASTM A992/A992M

$$F_y = 50 \text{ ksi}$$

$$F_u = 65 \text{ ksi}$$

Angles
 ASTM A572/A572M Grade 50
 $F_y = 50$ ksi
 $F_u = 65$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
 W36×231
 $t_w = 0.760$ in.

Column
 W14×90
 $t_f = 0.710$ in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(37.5 \text{ kips}) + 1.6(113 \text{ kips})$ $= 226 \text{ kips}$	$R_a = 37.5 \text{ kips} + 113 \text{ kips}$ $= 151 \text{ kips}$

Part A—Determine the Available Connection Strength Using the Tables in Manual Part 10

Available Angle Strength

AISC *Manual* Table 10-1a includes checks for the limit states of shear yielding, shear rupture, and block shear rupture of the angles.

Use 8 rows of ¾-in.-diameter bolts in standard holes and 2L5×3½×5/16 (SLBB). From AISC *Manual* Table 10-1a:

LRFD	ASD
$\phi R_n = 302 \text{ kips} > 226 \text{ kips}$ o.k.	$\frac{R_n}{\Omega} = 201 \text{ kips} > 151 \text{ kips}$ o.k.

Available Shear Transfer Strength at Bolt Holes at Beam Web

The available shear transfer strength at bolt holes is the sum of the effective strength of individual bolts per the User Note in AISC *Specification* Section J3.7, which is the least of (1) the available bolt shear strength determined in accordance with AISC *Specification* Section J3.7, (2) the available bearing or tearout strength of the angles at the bolt hole determined in accordance with AISC *Specification* Section J3.11a, and (3) the available bearing or tearout strength of the beam web or support element at the bolt hole determined in accordance with AISC *Specification* Section J3.11a.

From AISC *Manual* Table 10-1b, the available bolt shear strength per bolt for ¾-in.-diameter, Group 120 bolts with threads not excluded from the shear plane (thread condition N) is:

LRFD	ASD
$\phi r_n = 17.9 \text{ kips}$	$\frac{r_n}{\Omega} = 11.9 \text{ kips}$

From AISC *Manual* Table 10-1b, the available bearing and tearout strength of the angle per bolt for ¾-in.-diameter bolts in standard holes is:

LRFD	ASD
For the edge bolt ($l_{ev} = 1\frac{1}{4}$ in.): $\phi r_n = (49.4 \text{ kip/in.})(\frac{5}{16} \text{ in.})$ $= 15.4 \text{ kips}$	For the edge bolt ($l_{ev} = 1\frac{1}{4}$ in.): $\frac{r_n}{\Omega} = (32.9 \text{ kip/in.})(\frac{5}{16} \text{ in.})$ $= 10.3 \text{ kips}$
For the non-edge bolts ($s = 3$ in.): $\phi r_n = (87.8 \text{ kip/in.})(\frac{5}{16} \text{ in.})$ $= 27.4 \text{ kips}$	For the non-edge bolts ($s = 3$ in.): $\frac{r_n}{\Omega} = (58.5 \text{ kip/in.})(\frac{5}{16} \text{ in.})$ $= 18.3 \text{ kips}$

From AISC *Manual* Table 10-1b, the available bearing and tearout strength of the beam web per bolt for $\frac{3}{4}$ -in.-diameter bolts in standard holes is:

LRFD	ASD
For the non-edge bolts ($s = 3$ in.): $\phi r_n = (87.8 \text{ kip/in.})(0.760 \text{ in.})$ $= 66.7 \text{ kips}$	For the non-edge bolts ($s = 3$ in.): $\frac{r_n}{\Omega} = (58.5 \text{ kip/in.})(0.760 \text{ in.})$ $= 44.5 \text{ kips}$

At the top connection bolt, the available shear transfer strength is the minimum of the available bolt shear strength (multiplied by two because the bolts are in double shear), the available bearing and tearout strength of the angles for a non-edge bolt (multiplied by two because there are two angles), and available bearing and tearout strength of the beam web for a non-edge bolt:

LRFD	ASD
$\phi r_{n,top} = \min \begin{cases} 17.9 \text{ kips}(2) = 35.8 \text{ kips,} \\ 27.4 \text{ kips}(2) = 54.8 \text{ kips,} \\ 66.7 \text{ kips} \end{cases}$ $= 35.8 \text{ kips}$	$\frac{r_{n,top}}{\Omega} = \min \begin{cases} 11.9 \text{ kips}(2) = 23.8 \text{ kips,} \\ 18.3 \text{ kips}(2) = 36.6 \text{ kips,} \\ 44.5 \text{ kips} \end{cases}$ $= 23.8 \text{ kips}$

At the middle connection bolts, the available shear transfer strength is the minimum of the available bolt shear strength (multiplied by two because the bolts are in double shear), the available bearing and tearout strength of the angles for a non-edge bolt (multiplied by two because there are two angles), and available bearing and tearout strength of the beam web for a non-edge bolt:

LRFD	ASD
$\phi r_{n,mid} = \min \begin{cases} 17.9 \text{ kips}(2) = 35.8 \text{ kips,} \\ 27.4 \text{ kips}(2) = 54.8 \text{ kips,} \\ 66.7 \text{ kips} \end{cases}$ $= 35.8 \text{ kips}$	$\frac{r_{n,mid}}{\Omega} = \min \begin{cases} 11.9 \text{ kips}(2) = 23.8 \text{ kips,} \\ 18.3 \text{ kips}(2) = 36.6 \text{ kips,} \\ 44.5 \text{ kips} \end{cases}$ $= 23.8 \text{ kips}$

At the bottom connection bolt, the available shear transfer strength is the minimum of the available bolt shear strength (multiplied by two because the bolts are in double shear), the available bearing and tearout strength of the angles for an edge bolt (multiplied by two because there are two angles), and available bearing and tearout strength of the beam web for a non-edge bolt:

LRFD	ASD
$\phi r_{n,bot} = \min \begin{cases} 17.9 \text{ kips (2)} = 35.8 \text{ kips,} \\ 15.4 \text{ kips (2)} = 30.8 \text{ kips,} \\ 66.7 \text{ kips} \end{cases}$ $= 30.8 \text{ kips}$	$\frac{r_{n,bot}}{\Omega} = \min \begin{cases} 11.9 \text{ kips (2)} = 23.8 \text{ kips,} \\ 10.3 \text{ kips (2)} = 20.6 \text{ kips,} \\ 44.5 \text{ kips} \end{cases}$ $= 20.6 \text{ kips}$

The available shear transfer strength at the bolt holes is:

LRFD	ASD
$\phi R_n = \phi r_{n,top} + \phi r_{n,mid} (n-2) + \phi r_{n,bot}$ $= 35.8 \text{ kips} + (35.8 \text{ kips})(8-2) + 30.8 \text{ kips}$ $= 281 \text{ kips} > 226 \text{ kips} \quad \mathbf{o.k.}$	$\frac{R_n}{\Omega} = \frac{r_{n,top}}{\Omega} + \frac{r_{n,mid}}{\Omega} (n-2) + \frac{r_{n,bot}}{\Omega}$ $= 23.8 \text{ kips} + (23.8 \text{ kips})(8-2) + 20.6 \text{ kips}$ $= 187 \text{ kips} > 151 \text{ kips} \quad \mathbf{o.k.}$

Available Beam Web Strength

Because the beam is not coped, the limit states of block shear rupture and shear rupture of the beam are not applicable. The beam web is adequate for the required loading.

Available Shear Transfer Strength at Bolt Holes at Column Flange

The available bolt shear strength and available bearing and tearout strength of the angles is calculated in the previous section.

From AISC *Manual* Table 10-1b, the available bearing and tearout strength of the column flange per bolt for 3/4-in.-diameter bolts in standard holes is:

LRFD	ASD
For the non-edge bolts ($s = 3$ in.): $\phi r_n = (87.8 \text{ kip/in.})(0.710 \text{ in.})$ $= 62.3 \text{ kips}$	For the non-edge bolts ($s = 3$ in.): $\frac{r_n}{\Omega} = (58.5 \text{ kip/in.})(0.710 \text{ in.})$ $= 41.5 \text{ kips}$

At the top connection bolt, the available shear transfer strength is the minimum of the available bolt shear strength (multiplied by 2 because there are two bolts per row), the available bearing and tearout strength of the angles for an edge bolt (multiplied by 2 because there are two angles), and available bearing and tearout strength of the column flange (multiplied by 2 because there are two bolts per row):

LRFD	ASD
$\phi r_{n,top} = \min \begin{cases} 17.9 \text{ kips (2)} = 35.8 \text{ kips,} \\ 15.4 \text{ kips (2)} = 30.8 \text{ kips,} \\ 62.3 \text{ kips (2)} = 125 \text{ kips} \end{cases}$ $= 30.8 \text{ kips}$	$\frac{r_{n,top}}{\Omega} = \min \begin{cases} 11.9 \text{ kips (2)} = 23.8 \text{ kips,} \\ 10.3 \text{ kips (2)} = 20.6 \text{ kips,} \\ 41.5 \text{ kips (2)} = 83.0 \text{ kips} \end{cases}$ $= 20.6 \text{ kips}$

At the middle connection bolts, the available shear transfer strength is the minimum of the available bolt shear strength (multiplied by 2 because there are two bolts per row), the available bearing and tearout strength of the angles for a non-edge bolt (multiplied by 2 because there are two angles), and available bearing and tearout strength of the column flange (multiplied by 2 because there are two bolts per row):

LRFD	ASD
$\phi r_{n,mid} = \min \begin{cases} 17.9 \text{ kips}(2) = 35.8 \text{ kips,} \\ 27.4 \text{ kips}(2) = 54.8 \text{ kips,} \\ 62.3 \text{ kips}(2) = 125 \text{ kips} \end{cases}$ $= 35.8 \text{ kips}$	$\frac{r_{n,mid}}{\Omega} = \min \begin{cases} 11.9 \text{ kips}(2) = 23.8 \text{ kips,} \\ 18.3 \text{ kips}(2) = 36.6 \text{ kips,} \\ 41.5 \text{ kips}(2) = 83.0 \text{ kips} \end{cases}$ $= 23.8 \text{ kips}$

At the bottom connection bolt, the available shear transfer strength is the minimum of the available bolt shear strength (multiplied by 2 because there are two bolts per row), the available bearing and tearout strength of the angles for a non-edge bolt (multiplied by 2 because there are two angles), and the available bearing and tearout strength of the column flange (multiplied by 2 because there are two bolts per row):

LRFD	ASD
$\phi r_{n,bot} = \min \begin{cases} 17.9 \text{ kips}(2) = 35.8 \text{ kips,} \\ 27.4 \text{ kips}(2) = 54.8 \text{ kips,} \\ 62.3 \text{ kips}(2) = 125 \text{ kips} \end{cases}$ $= 35.8 \text{ kips}$	$\frac{r_{n,bot}}{\Omega} = \min \begin{cases} 11.9 \text{ kips}(2) = 23.8 \text{ kips,} \\ 18.3 \text{ kips}(2) = 36.6 \text{ kips,} \\ 41.5 \text{ kips}(2) = 83.0 \text{ kips} \end{cases}$ $= 23.8 \text{ kips}$

The available shear transfer strength at the bolt holes is:

LRFD	ASD
$\phi R_n = \phi r_{n,top} + \phi r_{n,mid} (n-2) + \phi r_{n,bot}$ $= 30.8 \text{ kips} + (35.8 \text{ kips})(8-2) + 35.8 \text{ kips}$ $= 281 \text{ kips} > 226 \text{ kips} \quad \mathbf{o.k.}$	$\frac{R_n}{\Omega} = \frac{r_{n,top}}{\Omega} + \frac{r_{n,mid}}{\Omega} (n-2) + \frac{r_{n,bot}}{\Omega}$ $= 20.6 \text{ kips} + (23.8 \text{ kips})(8-2) + 23.8 \text{ kips}$ $= 187 \text{ kips} > 151 \text{ kips} \quad \mathbf{o.k.}$

Summary

The available shear strength of the connection is controlled by the available shear transfer strength at the bolt holes.

LRFD	ASD
$\phi R_n = 281 \text{ kips} > 226 \text{ kips} \quad \mathbf{o.k.}$	$\frac{R_n}{\Omega} = 187 \text{ kips} > 151 \text{ kips} \quad \mathbf{o.k.}$

The connection is found to be adequate as given for the applied loads.

Part B—Verify the Available Connection Strength by Checking Individual Limit States

Shear Strength of Angles

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the angles is determined as follows:

$$\begin{aligned}
 A_{gv} &= (2 \text{ angles}) l t \\
 &= (2 \text{ angles}) (23 \frac{1}{2} \text{ in.}) (\frac{5}{16} \text{ in.}) \\
 &= 14.7 \text{ in.}^2
 \end{aligned}$$

$$\begin{aligned}
 R_n &= 0.60F_y A_{gv} & (\text{Spec. Eq. J4-3}) \\
 &= 0.60(50 \text{ ksi})(14.7 \text{ in.}^2) \\
 &= 441 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(441 \text{ kips})$ $= 441 \text{ kips} > 226 \text{ kips} \quad \mathbf{o.k.}$	$\Omega = 1.50$ $\frac{R_n}{\Omega} = \frac{441 \text{ kips}}{1.50}$ $= 294 \text{ kips} > 151 \text{ kips} \quad \mathbf{o.k.}$

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the angle is determined using the net area determined in accordance with AISC *Specification* Section B4.3b.

$$\begin{aligned}
 A_{nv} &= (2 \text{ angles})[l - n(d_h + \frac{1}{16} \text{ in.})]t \\
 &= (2 \text{ angles})[23\frac{1}{2} \text{ in.} - 8(\frac{13}{16} \text{ in.} + \frac{1}{16} \text{ in.})](\frac{5}{16} \text{ in.}) \\
 &= 10.3 \text{ in.}^2
 \end{aligned}$$

$$\begin{aligned}
 R_n &= 0.60F_u A_{nv} & (\text{Spec. Eq. J4-4}) \\
 &= 0.60(65 \text{ ksi})(10.3 \text{ in.}^2) \\
 &= 402 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(402 \text{ kips})$ $= 302 \text{ kips} > 226 \text{ kips} \quad \mathbf{o.k.}$	$\Omega = 2.00$ $\frac{R_n}{\Omega} = \frac{402 \text{ kips}}{2.00}$ $= 201 \text{ kips} > 151 \text{ kips} \quad \mathbf{o.k.}$

Block Shear Rupture of Angles

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture for the angles is determined as follows. By inspection, block shear rupture of the angles will control at the beam web side of the connection because the horizontal edge distance at the beam web ($1\frac{3}{8}$ in.) is lesser than the horizontal edge distance at the column flange side of the connection ($1\frac{7}{16}$ in.).

$$R_{bsv} = 0.60F_u A_{nv} + U_{bs}F_u A_{nt} \leq 0.60F_y A_{gv} + U_{bs}F_u A_{nt} \quad (\text{Spec. Eq. J4-5})$$

where

$$\begin{aligned}
 A_{gv} &= (2 \text{ angles})(l - l_{ev})t \\
 &= (2 \text{ angles})(23\frac{1}{2} \text{ in.} - 1\frac{1}{4} \text{ in.})(\frac{5}{16} \text{ in.}) \\
 &= 13.9 \text{ in.}^2
 \end{aligned}$$

$$\begin{aligned}
 A_{nv} &= A_{gv} - (2 \text{ angles})(n - 0.5)(d_h + \frac{1}{16} \text{ in.})t \\
 &= 13.9 \text{ in.}^2 - (2 \text{ angles})(8 - 0.5)(\frac{13}{16} \text{ in.} + \frac{1}{16} \text{ in.})(\frac{5}{16} \text{ in.}) \\
 &= 9.80 \text{ in.}^2
 \end{aligned}$$

$$\begin{aligned}
 A_n &= (2 \text{ angles}) \left[l_{eh} - 0.5(d_h + 1/16 \text{ in.}) \right] t \\
 &= (2 \text{ angles}) \left[1\frac{3}{8} \text{ in.} - 0.5(1\frac{3}{16} \text{ in.} + 1/16 \text{ in.}) \right] (5/16 \text{ in.}) \\
 &= 0.586 \text{ in.}^2
 \end{aligned}$$

$$U_{bs} = 1.0$$

and

$$\begin{aligned}
 R_{bsv} &= 0.60(65 \text{ ksi})(9.80 \text{ in.}^2) + 1.0(65 \text{ ksi})(0.586 \text{ in.}^2) \leq 0.60(50 \text{ ksi})(13.9 \text{ in.}^2) + 1.0(65 \text{ ksi})(0.586 \text{ in.}^2) \\
 &= 420 \text{ kips} < 455 \text{ kips}
 \end{aligned}$$

Therefore:

$$R_{bsv} = 420 \text{ kips}$$

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture of the angles is:

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi R_{bsv} = 0.75(420 \text{ kips})$ $= 315 \text{ kips} > 226 \text{ kips} \quad \mathbf{o.k.}$	$\frac{R_{bsv}}{\Omega} = \frac{420 \text{ kips}}{2.00}$ $= 210 \text{ kips} > 151 \text{ kips} \quad \mathbf{o.k.}$

Available Shear Transfer Strength at Bolt Holes at Beam Web

The available shear transfer strength at bolt holes is the sum of the effective strength of individual bolts per the User Note in AISC *Specification* Section J3.7, which is the least of (1) the available bolt shear strength determined in accordance with AISC *Specification* Section J3.7, (2) the available bearing or tearout strength of the angles at the bolt hole determined in accordance with AISC *Specification* Section J3.11a, and (3) the available bearing or tearout strength of the beam web or support element at the bolt hole determined in accordance with AISC *Specification* Section J3.11a.

From AISC *Manual* Table 7-1, the available shear strength for 3/4-in.-diameter, Group 120 bolts with threads not excluded from the shear plane (thread condition N) is:

LRFD	ASD
$\phi r_n = 17.9 \text{ kips}$	$\frac{r_n}{\Omega} = 11.9 \text{ kips}$

The available bearing strength of the angles is determined from AISC *Specification* Section J3.11, assuming deformation at service load is a design consideration:

$$\begin{aligned}
 r_n &= 2.4dtF_u && \text{(from Spec. Eq. J3-6a)} \\
 &= (2.4)(3/4 \text{ in.})(5/16 \text{ in.})(65 \text{ ksi}) \\
 &= 36.6 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi r_n = 0.75(36.6 \text{ kips})$ $= 27.5 \text{ kips}$	$\frac{r_n}{\Omega} = \frac{36.6 \text{ kips}}{2.00}$ $= 18.3 \text{ kips}$

The available tearout strength of the angles is determined from AISC *Specification* Section J3.11, assuming deformation at service load is a design consideration.

For edge bolt tearout, the clear distance along the line of action of the force, between the edge of the hole and the edge of the angle is:

$$\begin{aligned}
 l_c &= l_{ev} - 0.5d_h \\
 &= 1\frac{1}{4} \text{ in.} - 0.5(1\frac{3}{16} \text{ in.}) \\
 &= 0.844 \text{ in.}
 \end{aligned}$$

The available tearout strength of the angles at the edge bolt is:

$$\begin{aligned}
 r_n &= 1.2l_c t F_u && \text{(from Spec. Eq. J3-6c)} \\
 &= (1.2)(0.844 \text{ in.})(\frac{5}{16} \text{ in.})(65 \text{ ksi}) \\
 &= 20.6 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi r_n = 0.75(20.6 \text{ kips})$ $= 15.5 \text{ kips}$	$\frac{r_n}{\Omega} = \frac{20.6 \text{ kips}}{2.00}$ $= 10.3 \text{ kips}$

For non-edge bolt tearout in the angles, the clear distance is between bolt holes:

$$\begin{aligned}
 l_c &= s - d_h \\
 &= 3 \text{ in.} - 1\frac{3}{16} \text{ in.} \\
 &= 2.19 \text{ in.}
 \end{aligned}$$

The available tearout strength of the angles at non-edge bolts is:

$$\begin{aligned}
 r_n &= 1.2l_c t F_u && \text{(from Spec. Eq. J3-6c)} \\
 &= (1.2)(2.19 \text{ in.})(\frac{5}{16} \text{ in.})(65 \text{ ksi}) \\
 &= 53.4 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi r_n = 0.75(53.4 \text{ kips})$ $= 40.1 \text{ kips}$	$\frac{r_n}{\Omega} = \frac{53.4 \text{ kips}}{2.00}$ $= 26.7 \text{ kips}$

The available bearing strength of the beam web is determined from AISC *Specification* Section J3.11, assuming deformation at service load is a design consideration:

$$\begin{aligned}
 r_n &= 2.4dtF_u && \text{(from Spec. Eq. J3-6a)} \\
 &= (2.4)(\frac{3}{4} \text{ in.})(0.760 \text{ in.})(65 \text{ ksi}) \\
 &= 88.9 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi r_n = 0.75(88.9 \text{ kips})$ $= 66.7 \text{ kips}$	$\frac{r_n}{\Omega} = \frac{88.9 \text{ kips}}{2.00}$ $= 44.5 \text{ kips}$

The available tearout strength of the beam web is determined from AISC *Specification* Section J3.11, assuming deformation at service load is a design consideration.

There is no edge bolt tearout for the beam web. For non-edge bolt tearout in the beam web, the clear distance is between bolt holes:

$$\begin{aligned}
 l_c &= s - d_h \\
 &= 3 \text{ in.} - \frac{13}{16} \text{ in.} \\
 &= 2.19 \text{ in.}
 \end{aligned}$$

The available tearout strength of the beam web at non-edge bolts is:

$$\begin{aligned}
 r_n &= 1.2l_c t F_u && \text{(from Spec. Eq. J3-6c)} \\
 &= (1.2)(2.19 \text{ in.})(0.760 \text{ in.})(65 \text{ ksi}) \\
 &= 130 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi r_n = 0.75(130 \text{ kips})$ $= 97.5 \text{ kips}$	$\frac{r_n}{\Omega} = \frac{130 \text{ kips}}{2.00}$ $= 65.0 \text{ kips}$

At the top connection bolt, the available shear transfer strength is the minimum of the available bolt shear strength (multiplied by 2 because the bolts are in double shear), the available bearing and tearout strength of the angles for a non-edge bolt (multiplied by 2 because there are two angles), and the available bearing and tearout strength of the beam web for a non-edge bolt:

LRFD	ASD
$ \phi r_{n,top} = \min \left\{ \begin{array}{l} 17.9 \text{ kips}(2) = 35.8 \text{ kips,} \\ 27.5 \text{ kips}(2) = 55.0 \text{ kips,} \\ 40.1 \text{ kips}(2) = 80.2 \text{ kips,} \\ 66.7 \text{ kips,} \\ 97.5 \text{ kips} \end{array} \right\} $ $= 35.8 \text{ kips}$	$ \frac{r_{n,top}}{\Omega} = \min \left\{ \begin{array}{l} 11.9 \text{ kips}(2) = 23.8 \text{ kips,} \\ 18.3 \text{ kips}(2) = 36.6 \text{ kips,} \\ 26.7 \text{ kips}(2) = 53.4 \text{ kips,} \\ 44.5 \text{ kips,} \\ 65.0 \text{ kips} \end{array} \right\} $ $= 23.8 \text{ kips}$

At the middle connection bolts, the available shear transfer strength is the minimum of the available bolt shear strength (multiplied by 2 because the bolts are in double shear), the available bearing and tearout strength of the angles for a non-edge bolt (multiplied by 2 because there are two angles), and the available bearing and tearout strength of the beam web for a non-edge bolt:

LRFD	ASD
$\phi r_{n,mid} = \min \left\{ \begin{array}{l} 17.9 \text{ kips}(2) = 35.8 \text{ kips,} \\ 27.5 \text{ kips}(2) = 55.0 \text{ kips,} \\ 40.1 \text{ kips}(2) = 80.2 \text{ kips,} \\ 66.7 \text{ kips,} \\ 97.5 \text{ kips} \end{array} \right\}$ $= 35.8 \text{ kips}$	$\frac{r_{n,mid}}{\Omega} = \min \left\{ \begin{array}{l} 11.9 \text{ kips}(2) = 23.8 \text{ kips,} \\ 18.3 \text{ kips}(2) = 36.6 \text{ kips,} \\ 26.7 \text{ kips}(2) = 53.4 \text{ kips,} \\ 44.5 \text{ kips,} \\ 65.0 \text{ kips} \end{array} \right\}$ $= 23.8 \text{ kips}$

At the bottom connection bolt, the available shear transfer strength is the minimum of the available bolt shear strength (multiplied by 2 because the bolts are in double shear), the available bearing and tearout strength of the angles for an edge bolt (multiplied by 2 because there are two angles), and the available bearing and tearout strength of the beam web for a non-edge bolt:

LRFD	ASD
$\phi r_{n,bot} = \min \left\{ \begin{array}{l} 17.9 \text{ kips}(2) = 35.8 \text{ kips,} \\ 27.5 \text{ kips}(2) = 55.0 \text{ kips,} \\ 15.5 \text{ kips}(2) = 31.0 \text{ kips,} \\ 66.7 \text{ kips,} \\ 97.5 \text{ kips} \end{array} \right\}$ $= 31.0 \text{ kips}$	$\frac{r_{n,bot}}{\Omega} = \min \left\{ \begin{array}{l} 11.9 \text{ kips}(2) = 23.8 \text{ kips,} \\ 18.3 \text{ kips}(2) = 36.6 \text{ kips,} \\ 10.3 \text{ kips}(2) = 20.6 \text{ kips,} \\ 44.5 \text{ kips,} \\ 65.0 \text{ kips} \end{array} \right\}$ $= 20.6 \text{ kips}$

The available shear transfer strength at the bolt holes is:

LRFD	ASD
$\phi R_n = \phi r_{n,top} + \phi r_{n,mid} (n - 2) + \phi r_{n,bot}$ $= 35.8 \text{ kips} + (35.8 \text{ kips})(8 - 2) + 31.0 \text{ kips}$ $= 282 \text{ kips} > 226 \text{ kips} \quad \mathbf{o.k.}$	$\frac{R_n}{\Omega} = \frac{r_{n,top}}{\Omega} + \frac{r_{n,mid}}{\Omega} (n - 2) + \frac{r_{n,bot}}{\Omega}$ $= 23.8 \text{ kips} + (23.8 \text{ kips})(8 - 2) + 20.6 \text{ kips}$ $= 187 \text{ kips} > 151 \text{ kips} \quad \mathbf{o.k.}$

Available Shear Transfer Strength at Bolt Holes at Column Flange

The available bolt shear strength and available bearing and tearout strength of the angles is calculated in the previous section.

The available bearing strength of the column flange is determined from AISC *Specification* Section J3.11, assuming deformation at service load is a design consideration:

$$\begin{aligned}
 r_n &= 2.4dtF_u && \text{(from Spec. Eq. J3-6a)} \\
 &= (2.4)(\frac{3}{4} \text{ in.})(0.710 \text{ in.})(65 \text{ ksi}) \\
 &= 83.1 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi r_n = 0.75(83.1 \text{ kips})$ $= 62.3 \text{ kips}$	$\frac{r_n}{\Omega} = \frac{83.1 \text{ kips}}{2.00}$ $= 41.6 \text{ kips}$

The available tearout strength of the column flange is determined from AISC *Specification* Section J3.11, assuming deformation at service load is a design consideration.

There is no edge bolt tearout for the column flange. For non-edge bolt tearout in the column flange, the clear distance between bolt holes is:

$$\begin{aligned}
 l_c &= s - d_h \\
 &= 3 \text{ in.} - \frac{13}{16} \text{ in.} \\
 &= 2.19 \text{ in.}
 \end{aligned}$$

The available tearout strength of the beam web at non-edge bolts is:

$$\begin{aligned}
 r_n &= 1.2 l_c t F_u && \text{(from Spec. Eq. J3-6c)} \\
 &= (1.2)(2.19 \text{ in.})(0.710 \text{ in.})(65 \text{ ksi}) \\
 &= 121 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi r_n = 0.75(121 \text{ kips})$ $= 90.8 \text{ kips}$	$\frac{r_n}{\Omega} = \frac{121 \text{ kips}}{2.00}$ $= 60.5 \text{ kips}$

At the top connection bolt, the available shear transfer strength is the minimum of the available bolt shear strength (multiplied by 2 because there are two bolts per row), the available bearing and tearout strength of the angles for an edge bolt (multiplied by 2 because there are two angles), and the available bearing and tearout strength of the column flange (multiplied by 2 because there are two bolts per row):

LRFD	ASD
$ \phi r_{n,top} = \min \left\{ \begin{array}{l} 17.9 \text{ kips}(2) = 35.8 \text{ kips,} \\ 27.5 \text{ kips}(2) = 55.0 \text{ kips,} \\ 15.5 \text{ kips}(2) = 31.0 \text{ kips,} \\ 62.3 \text{ kips} \\ 90.8 \text{ kips} \end{array} \right\} $ $= 31.0 \text{ kips}$	$ \frac{r_{n,top}}{\Omega} = \min \left\{ \begin{array}{l} 11.9 \text{ kips}(2) = 23.8 \text{ kips,} \\ 18.3 \text{ kips}(2) = 36.6 \text{ kips,} \\ 10.3 \text{ kips}(2) = 20.6 \text{ kips,} \\ 41.6 \text{ kips,} \\ 60.5 \text{ kips} \end{array} \right\} $ $= 20.6 \text{ kips}$

At the middle connection bolts, the available shear transfer strength is the minimum of the available bolt shear strength (multiplied by 2 because the bolts are in double shear), the available bearing and tearout strength of the angles for a non-edge bolt (multiplied by 2 because there are two angles), and the available bearing and tearout strength of the column flange (multiplied by 2 because there are two bolts per row):

LRFD	ASD
$\phi r_{n,mid} = \min \left\{ \begin{array}{l} 17.9 \text{ kips (2)} = 35.8 \text{ kips,} \\ 27.5 \text{ kips (2)} = 55.0 \text{ kips,} \\ 40.1 \text{ kips (2)} = 80.2 \text{ kips,} \\ 62.3 \text{ kips,} \\ 90.8 \text{ kips} \end{array} \right\}$ $= 35.8 \text{ kips}$	$\frac{r_{n,mid}}{\Omega} = \min \left\{ \begin{array}{l} 11.9 \text{ kips (2)} = 23.8 \text{ kips,} \\ 18.3 \text{ kips (2)} = 36.6 \text{ kips,} \\ 26.7 \text{ kips (2)} = 53.4 \text{ kips,} \\ 41.6 \text{ kips,} \\ 60.5 \text{ kips} \end{array} \right\}$ $= 23.8 \text{ kips}$

At the bottom connection bolt, the available shear transfer strength is the minimum of the available bolt shear strength (multiplied by 2 because the bolts are in double shear), the available bearing and tearout strength of the angles for a non-edge bolt (multiplied by 2 because there are two angles), and available bearing strength of the column flange (multiplied by 2 because there are two bolts per row):

LRFD	ASD
$\phi r_{n,bot} = \min \left\{ \begin{array}{l} 17.9 \text{ kips (2)} = 35.8 \text{ kips,} \\ 27.5 \text{ kips (2)} = 55.0 \text{ kips,} \\ 40.1 \text{ kips (2)} = 80.2 \text{ kips,} \\ 62.3 \text{ kips} \end{array} \right\}$ $= 35.8 \text{ kips}$	$\frac{r_{n,bot}}{\Omega} = \min \left\{ \begin{array}{l} 11.9 \text{ kips (2)} = 23.8 \text{ kips,} \\ 18.3 \text{ kips (2)} = 36.6 \text{ kips,} \\ 26.7 \text{ kips (2)} = 53.4 \text{ kips,} \\ 41.6 \text{ kips} \end{array} \right\}$ $= 23.8 \text{ kips}$

The available shear transfer strength at the bolt holes is:

LRFD	ASD
$\phi R_n = \phi r_{n,top} + \phi r_{n,mid} (n - 2) + \phi r_{n,bot}$ $= 31.0 \text{ kips} + (35.8 \text{ kips})(8 - 2) + 35.8 \text{ kips}$ $= 282 \text{ kips} > 226 \text{ kips} \quad \mathbf{o.k.}$	$\frac{R_n}{\Omega} = \frac{r_{n,top}}{\Omega} + \frac{r_{n,mid}}{\Omega} (n - 2) + \frac{r_{n,bot}}{\Omega}$ $= 20.6 \text{ kips} + (23.8 \text{ kips})(8 - 2) + 23.8 \text{ kips}$ $= 187 \text{ kips} > 151 \text{ kips} \quad \mathbf{o.k.}$

Available Beam Web Strength

Because the beam is not coped, the limit states of block shear rupture and shear rupture of the beam are not applicable. The beam web is adequate for the required loading.

Conclusion

The available shear strength of the connection is controlled by the available shear transfer strength at the bolt holes.

LRFD	ASD
$\phi R_n = 282 \text{ kips} > 226 \text{ kips} \quad \mathbf{o.k.}$	$\frac{R_n}{\Omega} = 187 \text{ kips} > 151 \text{ kips} \quad \mathbf{o.k.}$

The connection is found to be adequate as given for the applied loads.

EXAMPLE IIA-1B ALL-BOLTED DOUBLE-ANGLE CONNECTION SUBJECT TO AXIAL AND SHEAR LOADING

Given:

Verify the available strength of an all-bolted double-angle connection for an ASTM A992/A992M W18×50 beam, as shown in Figure IIA-1B-1, to support the following beam end reactions:

LRFD	ASD
Shear, $V_u = 75$ kips Axial tension, $N_u = 60$ kips	Shear, $V_a = 50$ kips Axial tension, $N_a = 40$ kips

Use ASTM A572/A572M Grade 50 double angles that will be shop-bolted to the beam.

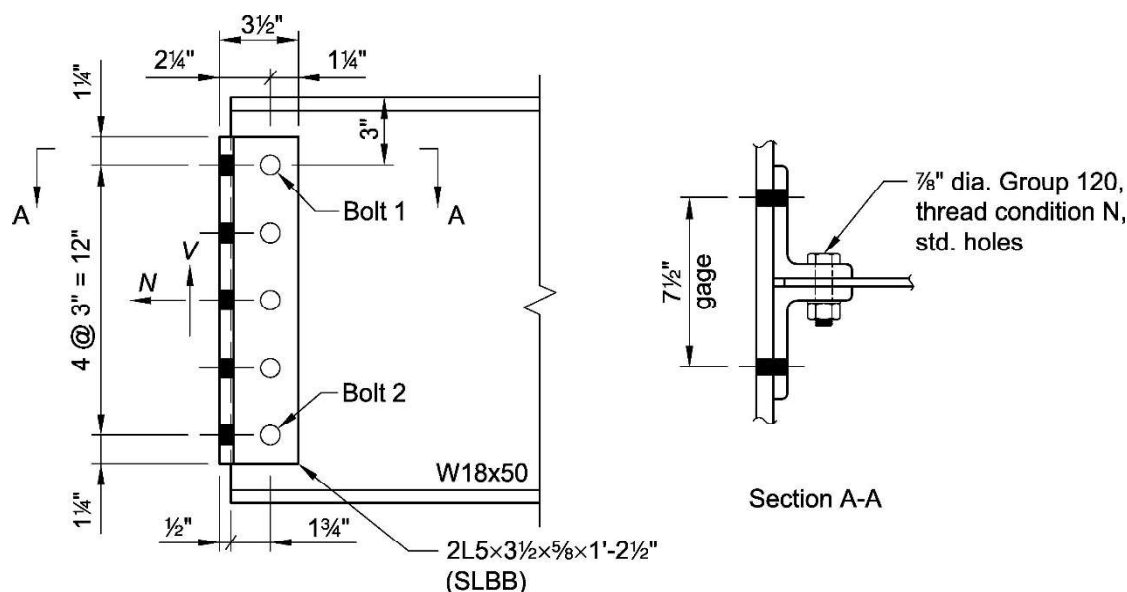


Fig. IIA-1B-1. Connection geometry for Example IIA-1B.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992/A992M
 $F_y = 50$ ksi
 $F_u = 65$ ksi

Angles
ASTM A572/A572M Grade 50
 $F_y = 50$ ksi
 $F_u = 65$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W18×50
 $A_g = 14.7$ in.²
 $d = 18.0$ in.

$$t_w = 0.355 \text{ in.}$$

$$t_f = 0.570 \text{ in.}$$

From AISC *Specification* Table J3.3, the hole diameter for 7/8-in.-diameter bolts with standard holes is:

$$d_h = 15/16 \text{ in.}$$

The resultant load is:

LRFD	ASD
$R_u = \sqrt{V_u^2 + N_u^2}$ $= \sqrt{(75 \text{ kips})^2 + (60 \text{ kips})^2}$ $= 96.0 \text{ kips}$	$R_a = \sqrt{V_a^2 + N_a^2}$ $= \sqrt{(50 \text{ kips})^2 + (40 \text{ kips})^2}$ $= 64.0 \text{ kips}$

Try 5 rows of bolts and 2L5×3½×5/8 (SLBB).

Strength of the Bolted Connection—Angles

From the User Note in AISC *Specification* Section J3.7, the strength of the bolt group is taken as the sum of the individual strengths of the individual fasteners, which may be taken as the lesser of the fastener shear strength per AISC *Specification* Section J3.7, the bearing strength at the bolt hole per AISC *Specification* Section J3.11a, or the tearout strength at the bolt hole per AISC *Specification* Section J3.11a.

Bolt shear

From AISC *Manual* Table 7-1, the available shear strength for 7/8-in.-diameter, Group 120 bolts with threads not excluded from the shear plane (thread condition N) in double shear (or pair of bolts) is:

LRFD	ASD
$\phi r_n = 48.7 \text{ kips/bolt (or per pair of bolts)}$	$\frac{r_n}{\Omega} = 32.5 \text{ kips/bolt (or per pair of bolts)}$

Bolt bearing on angles

The available bearing strength of the angles per bolt in double shear is determined from AISC *Specification* Section J3.11a, assuming deformation at service load is a design consideration:

$$\begin{aligned}
 r_n &= (2 \text{ angles}) 2.4 d t F_u && \text{(from Spec. Eq. J3-6a)} \\
 &= (2 \text{ angles}) (2.4) \left(\frac{7}{8} \text{ in.}\right) \left(\frac{5}{8} \text{ in.}\right) (65 \text{ ksi}) \\
 &= 171 \text{ kips/bolt}
 \end{aligned}$$

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi r_n = 0.75 (171 \text{ kips/bolt})$ $= 128 \text{ kips/bolt}$	$\frac{r_n}{\Omega} = \frac{171 \text{ kips/bolt}}{2.00}$ $= 85.5 \text{ kips/bolt}$

Bolt tearout on angles

From AISC *Specification* Section J3.11a, the available tearout strength of the angles per bolt in double shear is determined from AISC *Specification* Section J3.11a, assuming deformation at service load is a design consideration.

As shown in Figures II.A-1B-2(a) and II.A-1B-2(b), the tearout dimensions on the angle differ between the edge bolt and the other bolts.

The angle θ , as shown in Figure II.A-1B-2(a), of the resultant force on the edge bolt is:

LRFD	ASD
$\theta = \tan^{-1} \left(\frac{N_u}{V_u} \right)$ $= \tan^{-1} \left(\frac{60 \text{ kips}}{75 \text{ kips}} \right)$ $= 38.7^\circ$	$\theta = \tan^{-1} \left(\frac{N_a}{V_a} \right)$ $= \tan^{-1} \left(\frac{40 \text{ kips}}{50 \text{ kips}} \right)$ $= 38.7^\circ$

The length from the center of the bolt hole to the edge of the angle along the line of action of the force is:

$$l_e = \frac{1\frac{1}{4} \text{ in.}}{\cos 38.7^\circ}$$

$$= 1.60 \text{ in.}$$

The clear distance along the line of action of the force, between the edge of the hole and the edge of the angle is:

$$l_c = l_e - 0.5d_h$$

$$= 1.60 \text{ in.} - 0.5 \left(1\frac{5}{16} \text{ in.} \right)$$

$$= 1.13 \text{ in.}$$

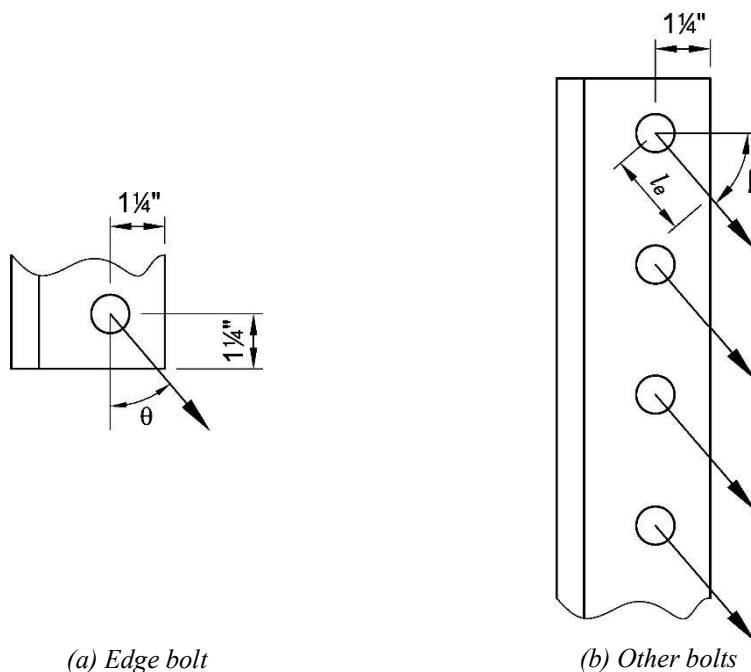


Fig. II.A-1B-2. Bolt tearout on angles.

The available tearout strength of the pair of angles at the edge bolt is:

$$\begin{aligned}
 r_n &= (2 \text{ angles})1.2l_c t F_u && \text{(from Spec. Eq. J3-6c)} \\
 &= (2 \text{ angles})(1.2)(1.13 \text{ in.})(\frac{5}{8} \text{ in.})(65 \text{ ksi}) \\
 &= 110 \text{ kips/bolt}
 \end{aligned}$$

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi r_n = 0.75(110 \text{ kips/bolt})$ $= 82.5 \text{ kips/bolt}$	$\frac{r_n}{\Omega} = \frac{110 \text{ kips/bolt}}{2.00}$ $= 55.0 \text{ kips/bolt}$

Therefore, bolt shear controls over bearing or tearout of the angles at the edge bolt.

The angle β , as shown in Figure II.A-1B-2(b), of the resultant force on the other bolts is:

LRFD	ASD
$\beta = \tan^{-1} \left(\frac{V_u}{N_u} \right)$ $= \tan^{-1} \left(\frac{75 \text{ kips}}{60 \text{ kips}} \right)$ $= 51.3^\circ$	$\beta = \tan^{-1} \left(\frac{V_a}{N_a} \right)$ $= \tan^{-1} \left(\frac{50 \text{ kips}}{40 \text{ kips}} \right)$ $= 51.3^\circ$

The length from the center of the bolt hole to the edge of the angle along the line of action of the force is:

$$\begin{aligned}
 l_e &= \frac{1\frac{1}{4} \text{ in.}}{\cos 51.3^\circ} \\
 &= 2.00 \text{ in.}
 \end{aligned}$$

The clear distance along the line of action of the force, between the edge of the hole and the edge of the angle is:

$$\begin{aligned}
 l_c &= l_e - 0.5d_h \\
 &= 2.00 \text{ in.} - 0.5(1\frac{5}{16} \text{ in.}) \\
 &= 1.53 \text{ in.}
 \end{aligned}$$

The available tearout strength of the pair of angles at the other bolts is:

$$\begin{aligned}
 r_n &= (2 \text{ angles})1.2l_c t F_u && \text{(from Spec. Eq. J3-6c)} \\
 &= (2 \text{ angles})(1.2)(1.53 \text{ in.})(\frac{5}{8} \text{ in.})(65 \text{ ksi}) \\
 &= 149 \text{ kips/bolt}
 \end{aligned}$$

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi r_n = 0.75(149 \text{ kips/bolt})$ $= 112 \text{ kips/bolt}$	$\frac{r_n}{\Omega} = \frac{149 \text{ kips/bolt}}{2.00}$ $= 74.5 \text{ kips/bolt}$

Therefore, bolt shear controls over bearing or tearout of the angles at the other bolt.

The effective strength for the bolted connection at the angles is determined by summing the effective strength for each bolt using the minimum available strength calculated for bolt shear, bearing on the angles, and tearout on the angles.

LRFD	ASD
$\phi R_n = n\phi r_n$ $= (5 \text{ bolts})(48.7 \text{ kips/bolt})$ $= 244 \text{ kips} > 96.0 \text{ kips} \quad \mathbf{o.k.}$	$\frac{R_n}{\Omega} = n \frac{r_n}{\Omega}$ $= (5 \text{ bolts})(32.5 \text{ kips/bolt})$ $= 163 \text{ kips} > 64.0 \text{ kips} \quad \mathbf{o.k.}$

Strength of the Bolted Connection—Beam Web

Bolt bearing on beam web

The available bearing strength of the beam web per bolt is determined from AISC *Specification* Section J3.11a, assuming deformation at service load is a design consideration:

$$\begin{aligned}
 r_n &= 2.4dtF_u && (\text{Spec. Eq. J3-6a}) \\
 &= 2.4\left(\frac{7}{8} \text{ in.}\right)(0.355 \text{ in.})(65 \text{ ksi}) \\
 &= 48.5 \text{ kips/bolt}
 \end{aligned}$$

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(48.5 \text{ kips/bolt})$ $= 36.4 \text{ kips/bolt}$	$\Omega = 2.00$ $\frac{r_n}{\Omega} = \frac{48.5 \text{ kips/bolt}}{2.00}$ $= 24.3 \text{ kips/bolt}$

Bolt tearout on beam web

From AISC *Specification* Section J3.11a, the available tearout strength of the beam web is determined from AISC *Specification* Equation J3-6c, assuming deformation at the bolt hole is a design consideration, where the edge distance, l_c , is based on the angle of the resultant load. As shown in Figure II.A-1B-3, a horizontal edge distance of $1\frac{1}{2}$ in. is used, which includes a $\frac{1}{4}$ in. tolerance to account for possible mill underrun.

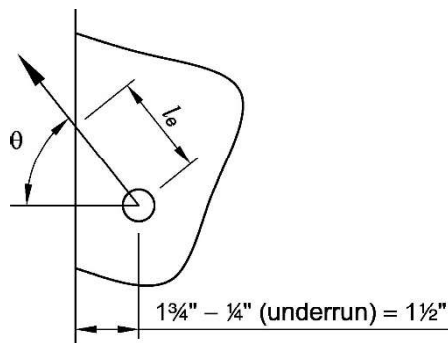


Fig. II.A-1B-3. Bolt tearout on beam web.

The angle, θ , of the resultant force is:

LRFD	ASD
$\theta = \tan^{-1} \left(\frac{V_u}{N_u} \right)$ $= \tan^{-1} \left(\frac{75 \text{ kips}}{60 \text{ kips}} \right)$ $= 51.3^\circ$	$\theta = \tan^{-1} \left(\frac{V_a}{N_a} \right)$ $= \tan^{-1} \left(\frac{50 \text{ kips}}{40 \text{ kips}} \right)$ $= 51.3^\circ$

The length from the center of the bolt hole to the edge of the web along the line of action of the force is:

$$l_e = \frac{1\frac{1}{2} \text{ in.}}{\cos 51.3^\circ}$$

$$= 2.40 \text{ in.}$$

The clear distance along the line of action of the force, between the edge of the hole and the edge of the web is:

$$l_c = l_e - 0.5d_h$$

$$= 2.40 \text{ in.} - 0.5 \left(1\frac{5}{16} \text{ in.} \right)$$

$$= 1.93 \text{ in.}$$

The available tearout strength of the beam web is determined as follows:

$$r_n = 1.2l_c t F_u \quad (\text{Spec. Eq. J3-6c})$$

$$= 1.2(1.93 \text{ in.})(0.355 \text{ in.})(65 \text{ ksi})$$

$$= 53.4 \text{ kips/bolt}$$

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(53.4 \text{ kips/bolt})$ $= 40.1 \text{ kips/bolt}$	$\Omega = 2.00$ $\frac{r_n}{\Omega} = \frac{53.4 \text{ kips/bolt}}{2.00}$ $= 26.7 \text{ kips/bolt}$

Therefore, bolt bearing on the beam web is the controlling limit state for all bolts.

The effective strength for the bolted connection at the beam web is determined by summing the effective strength for each bolt using the minimum available strength calculated for bolt shear, bearing on the beam web, and tearout on the beam web.

LRFD	ASD
$\phi R_n = n\phi r_n$ $= (5 \text{ bolts})(40.1 \text{ kips/bolt})$ $= 182 \text{ kips} > 96.0 \text{ kips} \quad \mathbf{o.k.}$	$\frac{R_n}{\Omega} = n \frac{r_n}{\Omega}$ $= (5 \text{ bolts})(26.7 \text{ kips/bolt})$ $= 122 \text{ kips} > 64.0 \text{ kips} \quad \mathbf{o.k.}$

Bolt Shear and Tension Interaction—Outstanding Angle Legs

The available tensile strength of the bolts due to the effect of combined tension and shear is determined from AISC *Specification* Section J3.8.

The required shear stress is:

$$f_{rv} = \frac{V_r}{nA_b}$$

where

$$A_b = 0.601 \text{ in.}^2 \text{ (from AISC Manual Table 7-1)}$$

$$n = 10$$

LRFD	ASD
$f_{rv} = \frac{V_u}{nA_b}$ $= \frac{75 \text{ kips}}{10(0.601 \text{ in.}^2)}$ $= 12.5 \text{ ksi}$	$f_{rv} = \frac{V_a}{nA_b}$ $= \frac{50 \text{ kips}}{10(0.601 \text{ in.}^2)}$ $= 8.32 \text{ ksi}$

The nominal tensile strength modified to include the effects of shear stress is determined from AISC *Specification* Section J3.8 as follows. From AISC *Specification* Table J3.2:

$$F_{nt} = 90 \text{ ksi}$$

$$F_{nv} = 54 \text{ ksi}$$

LRFD	ASD
$\phi = 0.75$ $F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_{rv} \leq F_{nt} \quad (\text{Spec. Eq. J3-3a})$ $= 1.3(90 \text{ ksi}) - \frac{90 \text{ ksi}}{0.75(54 \text{ ksi})}(12.5 \text{ ksi}) < 90 \text{ ksi}$ $= 89.2 \text{ ksi} < 90 \text{ ksi}$ Therefore: $F'_{nt} = 89.2 \text{ ksi}$	$\Omega = 2.00$ $F'_{nt} = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_{rv} \leq F_{nt} \quad (\text{Spec. Eq. J3-3b})$ $= 1.3(90 \text{ ksi}) - \frac{2.00(90 \text{ ksi})}{54 \text{ ksi}}(8.32 \text{ ksi}) < 90 \text{ ksi}$ $= 89.3 \text{ ksi} < 90 \text{ ksi}$ Therefore: $F'_{nt} = 89.3 \text{ ksi}$

Using the value of F'_{nt} determined for LRFD, the nominal tensile strength of one bolt is:

$$r_n = F'_{nt} A_b \quad (\text{from Spec. Eq. J3-2})$$

$$= (89.2 \text{ ksi})(0.601 \text{ in.}^2)$$

$$= 53.6 \text{ kips}$$

The available tensile strength of the bolts due to combined tension and shear is:

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$

$\phi r_n = 0.75(53.6 \text{ kips/bolt})$ $= 40.2 \text{ kips}$	$\frac{r_n}{\Omega} = \frac{53.6 \text{ kips/bolt}}{2.00}$ $= 26.8 \text{ kips}$
LRFD	ASD
$\phi R_n = n\phi r_n$ $= (10 \text{ bolts})(40.2 \text{ kips/bolt})$ $= 402 \text{ kips} > 60 \text{ kips} \quad \text{o.k.}$	$\frac{R_n}{\Omega} = n \frac{r_n}{\Omega}$ $= (10 \text{ bolts})(26.8 \text{ kips/bolt})$ $= 268 \text{ kips} > 40 \text{ kips} \quad \text{o.k.}$

Prying Action

From AISC *Manual* Part 9, the available tensile strength of the bolts in the outstanding angle legs taking prying action into account is determined as follows:

$$b = \frac{gage - t_w - t}{2}$$

$$= \frac{7\frac{1}{2} \text{ in.} - 0.355 \text{ in.} - \frac{5}{8} \text{ in.}}{2}$$

$$= 3.26 \text{ in.}$$

$$a = \frac{2(\text{angle leg}) + t_w - gage}{2} \leq 1.25b$$

$$= \frac{2(5 \text{ in.}) + 0.355 \text{ in.} - 7\frac{1}{2} \text{ in.}}{2} \leq 1.25(3.26 \text{ in.})$$

$$= 1.43 \text{ in.} \leq 4.08 \text{ in.}$$

$$= 1.43 \text{ in.}$$

$$a' = a + \frac{d}{2} \quad (\text{Manual Eq. 9-23})$$

$$= 1.43 \text{ in.} + \frac{\frac{7}{8} \text{ in.}}{2}$$

$$= 1.87 \text{ in.}$$

$$b' = b - \frac{d}{2} \quad (\text{Manual Eq. 9-24})$$

$$= 3.26 \text{ in.} - \frac{\frac{7}{8} \text{ in.}}{2}$$

$$= 2.82 \text{ in.}$$

$$\rho = \frac{b'}{a'} \quad (\text{Manual Eq. 9-29})$$

$$= \frac{2.82 \text{ in.}}{1.87 \text{ in.}}$$

$$= 1.51$$

Note that end distances of $1\frac{1}{4}$ in. are used on the angles, so p is the average pitch of the bolts:

$$\begin{aligned}
 p &= \frac{l}{n} \\
 &= \frac{14\frac{1}{2} \text{ in.}}{5 \text{ rows}} \\
 &= 2.90 \text{ in.}
 \end{aligned}$$

Check that $p \leq s$:

$$p < s = 3.00 \text{ in.} \quad \mathbf{o.k.}$$

$$\begin{aligned}
 d' &= d_h \\
 &= \frac{15}{16} \text{ in.}
 \end{aligned}$$

$$\begin{aligned}
 \delta &= 1 - \frac{d'}{p} && (\text{Manual Eq. 9-28}) \\
 &= 1 - \frac{\frac{15}{16} \text{ in.}}{2.90 \text{ in.}} \\
 &= 0.677
 \end{aligned}$$

The angle thickness required to develop the available strength of the bolt with no prying action is determined as follows:

LRFD	ASD
$\phi = 0.90$	$\Omega = 1.67$
$T_c = 40.2 \text{ kips/bolt}$ (calculated previously)	$T_c = 26.8 \text{ kips/bolt}$ (calculated previously)
$t_c = \sqrt{\frac{4T_c b'}{\phi_b p F_u}} \quad (\text{Manual Eq. 9-30a})$ $= \sqrt{\frac{4(40.2 \text{ kips/bolt})(2.82 \text{ in.})}{0.90(2.90 \text{ in.})(65 \text{ ksi})}}$ $= 1.63 \text{ in.}$	$t_c = \sqrt{\frac{4\Omega_b T_c b'}{p F_u}} \quad (\text{Manual Eq. 9-30b})$ $= \sqrt{\frac{4(1.67)(26.8 \text{ kips/bolt})(2.82 \text{ in.})}{(2.90 \text{ in.})(65 \text{ ksi})}}$ $= 1.64 \text{ in.}$

Conservatively using the ASD value for t_c :

$$\begin{aligned}
 \alpha' &= \frac{1}{\delta(1+\rho)} \left[\left(\frac{t_c}{t} \right)^2 - 1 \right] && (\text{Manual Eq. 9-38}) \\
 &= \frac{1}{0.677(1+1.51)} \left[\left(\frac{1.64 \text{ in.}}{\frac{5}{8} \text{ in.}} \right)^2 - 1 \right] \\
 &= 3.46
 \end{aligned}$$

Because $\alpha' > 1$, the angles have insufficient strength to develop the bolt strength, therefore:

$$\begin{aligned}
 Q &= \left(\frac{t}{t_c} \right)^2 (1 + \delta) && (\text{Manual Eq. 9-39c}) \\
 &= \left(\frac{\frac{5}{8} \text{ in.}}{1.64 \text{ in.}} \right)^2 (1 + 0.677) \\
 &= 0.244
 \end{aligned}$$

The available tensile strength of the bolts taking prying action into account is determined using AISC *Manual* Equation 9-40 as follows:

LRFD	ASD
$T_{c, adj} = QT_c$ $= (0.244)(40.2 \text{ kips/bolt})$ $= 9.81 \text{ kips/bolt}$	$T_{c, adj} = QT_c$ $= (0.244)(26.8 \text{ kips/bolt})$ $= 6.54 \text{ kips/bolt}$
$\phi R_n = nT_{c, adj}$ $= (10 \text{ bolts})(9.81 \text{ kips/bolt})$ $= 98.1 \text{ kips} > 60 \text{ kips} \quad \text{o.k.}$	$\frac{R_n}{\Omega} = nT_{c, adj}$ $= (10 \text{ bolts})(6.54 \text{ kips/bolt})$ $= 65.4 \text{ kips} > 40 \text{ kips} \quad \text{o.k.}$

Shear Strength of Angles

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the angles is determined as follows:

$$\begin{aligned}
 A_{gv} &= (2 \text{ angles})lt \\
 &= (2 \text{ angles})(14\frac{1}{2} \text{ in.})(\frac{5}{8} \text{ in.}) \\
 &= 18.1 \text{ in.}^2
 \end{aligned}$$

$$\begin{aligned}
 R_n &= 0.60F_y A_{gv} && (\text{Spec. Eq. J4-3}) \\
 &= 0.60(50 \text{ ksi})(18.1 \text{ in.}^2) \\
 &= 543 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(543 \text{ kips})$ $= 543 \text{ kips} > 96.0 \text{ kips} \quad \text{o.k.}$	$\Omega = 1.50$ $\frac{R_n}{\Omega} = \frac{543 \text{ kips}}{1.50}$ $= 362 \text{ kips} > 64.0 \text{ kips} \quad \text{o.k.}$

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the angle is determined using the net area determined in accordance with AISC *Specification* Section B4.3b.

$$\begin{aligned}
 A_{nv} &= (2 \text{ angles})[l - n(d_h + \frac{1}{16} \text{ in.})]t \\
 &= (2 \text{ angles})[14\frac{1}{2} \text{ in.} - 5(\frac{15}{16} \text{ in.} + \frac{1}{16} \text{ in.})](\frac{5}{8} \text{ in.}) \\
 &= 11.9 \text{ in.}^2
 \end{aligned}$$

$$\begin{aligned}
 R_n &= 0.60F_u A_{nv} && (\text{Spec. Eq. J4-4}) \\
 &= 0.60(65 \text{ ksi})(11.9 \text{ in.}^2) \\
 &= 464 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi R_n = 0.75(464 \text{ kips})$ $= 348 \text{ kips} > 96.0 \text{ kips} \quad \mathbf{o.k.}$	$\frac{R_n}{\Omega} = \frac{464 \text{ kips}}{2.00}$ $= 232 \text{ kips} > 64.0 \text{ kips} \quad \mathbf{o.k.}$

Tensile Strength of Angles

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the angles is determined as follows:

$$\begin{aligned}
 A_g &= (2 \text{ angles})lt \\
 &= (2 \text{ angles})(14\frac{1}{2} \text{ in.})(\frac{5}{8} \text{ in.}) \\
 &= 18.1 \text{ in.}^2
 \end{aligned}$$

$$\begin{aligned}
 R_n &= F_y A_g && (\text{Spec. Eq. J4-1}) \\
 &= (50 \text{ ksi})(18.1 \text{ in.}^2) \\
 &= 905 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi = 0.90$	$\Omega = 1.67$
$\phi R_n = 0.90(905 \text{ kips})$ $= 815 \text{ kips} > 60 \text{ kips} \quad \mathbf{o.k.}$	$\frac{R_n}{\Omega} = \frac{905 \text{ kips}}{1.67}$ $= 542 \text{ kips} > 40 \text{ kips} \quad \mathbf{o.k.}$

From AISC *Specification* Section J4.1(b), the available tensile rupture strength of the angles is determined from AISC *Specification* Equation J4-2. AISC *Specification* Table D3.1, Case 1, applies in this case because the tension load is transmitted directly to the cross-sectional element by the fasteners; therefore, $U = 1.00$. With $A_{nt} = A_{nv}$ (calculated previously), the effective net area is:

$$\begin{aligned}
 A_e &= A_{nt}U && (\text{Spec. Eq. D3-1}) \\
 &= (11.9 \text{ in.}^2)(1.00) \\
 &= 11.9 \text{ in.}^2
 \end{aligned}$$

$$\begin{aligned}
 R_n &= F_u A_e && (\text{Spec. Eq. J4-2}) \\
 &= (65 \text{ ksi})(11.9 \text{ in.}^2) \\
 &= 774 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi R_n = 0.75(774 \text{ kips})$ $= 581 \text{ kips} > 60 \text{ kips} \quad \mathbf{o.k.}$	$\frac{R_n}{\Omega} = \frac{774 \text{ kips}}{2.00}$ $= 387 \text{ kips} > 40 \text{ kips} \quad \mathbf{o.k.}$

Block Shear Rupture of Angles—Beam Web Side

The nominal strength for the limit state of block shear rupture of the angles, assuming an L-shaped tearout due to shear load only, is determined as follows. The tearout pattern is shown in Figure II.A-1B-4.

$$R_{bsv} = 0.60F_u A_{nv} + U_{bs}F_u A_{nt} \leq 0.60F_y A_{gv} + U_{bs}F_u A_{nt} \quad (\text{from Spec. Eq. J4-5})$$

where

$$\begin{aligned} A_{gv} &= (2 \text{ angles})(l - l_{ev})t \\ &= (2 \text{ angles})(14\frac{1}{2} \text{ in.} - 1\frac{1}{4} \text{ in.})(\frac{5}{8} \text{ in.}) \\ &= 16.6 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{nv} &= A_{gv} - (2 \text{ angles})(n - 0.5)(d_h + \frac{1}{16} \text{ in.})t \\ &= 16.6 \text{ in.}^2 - (2 \text{ angles})(5 - 0.5)(\frac{15}{16} \text{ in.} + \frac{1}{16} \text{ in.})(\frac{5}{8} \text{ in.}) \\ &= 11.0 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{nt} &= (2 \text{ angles})[l_{eh} - 0.5(d_h + \frac{1}{16} \text{ in.})]t \\ &= (2 \text{ angles})[1\frac{1}{4} \text{ in.} - 0.5(\frac{15}{16} \text{ in.} + \frac{1}{16} \text{ in.})](\frac{5}{8} \text{ in.}) \\ &= 0.938 \text{ in.}^2 \end{aligned}$$

$$U_{bs} = 1.0$$

and

$$\begin{aligned} R_{bsv} &= 0.60(65 \text{ ksi})(11.0 \text{ in.}^2) + 1.0(65 \text{ ksi})(0.938 \text{ in.}^2) \leq 0.60(50 \text{ ksi})(16.6 \text{ in.}^2) + 1.0(65 \text{ ksi})(0.938 \text{ in.}^2) \\ &= 490 \text{ kips} < 559 \text{ kips} \end{aligned}$$

Therefore:

$$R_{bsv} = 490 \text{ kips}$$

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture of the angles is:

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi R_{bsv} = 0.75(490 \text{ kips})$ $= 368 \text{ kips} > 75 \text{ kips} \quad \mathbf{o.k.}$	$\frac{R_{bsv}}{\Omega} = \frac{490 \text{ kips}}{2.00}$ $= 245 \text{ kips} > 50 \text{ kips} \quad \mathbf{o.k.}$

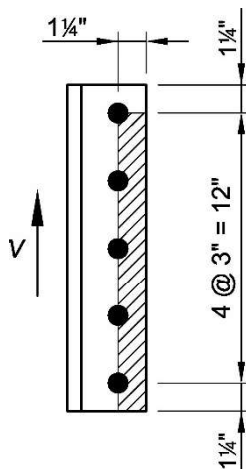


Fig. II.A-1B-4. Block shear rupture of angles for shear load only.

The block shear rupture failure path due to axial load only could occur as an L- or U-shape. Assuming an L-shaped tearout relative to the axial load on the angles, the nominal block shear rupture strength of the angles is determined as follows. The tearout pattern is shown in Figure II.A-1B-5.

$$R_{bsn} = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \leq 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \quad (\text{from Spec. Eq. J4-5})$$

where

$$\begin{aligned} A_{gv} &= (2 \text{ angles}) l_{eh} t \\ &= (2 \text{ angles}) (1 \frac{1}{4} \text{ in.}) (\frac{5}{8} \text{ in.}) \\ &= 1.56 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{nv} &= A_{gv} - (2 \text{ angles}) (0.5) (d_h + \frac{1}{16} \text{ in.}) t \\ &= 1.56 \text{ in.}^2 - (2 \text{ angles}) (0.5) (\frac{15}{16} \text{ in.} + \frac{1}{16} \text{ in.}) (\frac{5}{8} \text{ in.}) \\ &= 0.935 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{nt} &= (2 \text{ angles}) [(l - l_{ev}) - (n - 0.5) (d_h + \frac{1}{16} \text{ in.})] t \\ &= (2 \text{ angles}) [(14 \frac{1}{2} \text{ in.} - 1 \frac{1}{4} \text{ in.}) - (5 - 0.5) (\frac{15}{16} \text{ in.} + \frac{1}{16} \text{ in.})] (\frac{5}{8} \text{ in.}) \\ &= 10.9 \text{ in.}^2 \end{aligned}$$

$$U_{bs} = 1.0$$

and

$$\begin{aligned} R_{bsn} &= 0.60(65 \text{ ksi}) (0.935 \text{ in.}^2) + 1.0(65 \text{ ksi}) (10.9 \text{ in.}^2) \leq 0.60(50 \text{ ksi}) (1.56 \text{ in.}^2) + 1.0(65 \text{ ksi}) (10.9 \text{ in.}^2) \\ &= 745 \text{ kips} < 755 \text{ kips} \end{aligned}$$

Therefore:

$$R_{bsn} = 745 \text{ kips}$$

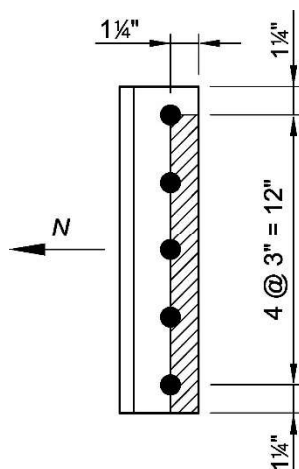


Fig. II.A-1B-5. Block shear rupture of angles for axial load only—L-shape.

From AISC Specification Section J4.3, the available strength for the limit state of block shear rupture of the angles is:

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi R_{bsn} = 0.75(745 \text{ kips})$ $= 559 \text{ kips} > 60 \text{ kips} \quad \text{o.k.}$	$\frac{R_{bsn}}{\Omega} = \frac{745 \text{ kips}}{2.00}$ $= 373 \text{ kips} > 40 \text{ kips} \quad \text{o.k.}$

The nominal strength for the limit state of block shear rupture assuming a U-shaped tearout relative to the axial load on the angles is determined as follows. The tearout pattern is shown in Figure II.A-1B-6.

$$R_{bsn} = 0.60F_u A_{nv} + U_{bs}F_u A_{nt} \leq 0.60F_y A_{gv} + U_{bs}F_u A_{nt} \quad (\text{from Spec. Eq. J4-5})$$

where

$$\begin{aligned} A_{gv} &= (2 \text{ angles})(2 \text{ planes})l_{eh}t \\ &= (2 \text{ angles})(2 \text{ planes})(1 \frac{1}{4} \text{ in.})(\frac{5}{8} \text{ in.}) \\ &= 3.13 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{nv} &= (2 \text{ angles})(2 \text{ planes})[l_{eh} - 0.5(d_h + \frac{1}{16} \text{ in.})]t \\ &= (2 \text{ angles})(2 \text{ planes})[1 \frac{1}{4} \text{ in.} - 0.5(\frac{15}{16} \text{ in.} + \frac{1}{16} \text{ in.})](\frac{5}{8} \text{ in.}) \\ &= 1.88 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{nt} &= (2 \text{ angles})[l - (n-1)(d_h + \frac{1}{16} \text{ in.})]t \\ &= (2 \text{ angles})[12.0 \text{ in.} - (5-1)(\frac{15}{16} \text{ in.} + \frac{1}{16} \text{ in.})](\frac{5}{8} \text{ in.}) \\ &= 10.0 \text{ in.}^2 \end{aligned}$$

$$U_{bs} = 1.0$$

and

$$\begin{aligned} R_{bsn} &= 0.60(65 \text{ ksi})(1.88 \text{ in.}^2) + 1.0(65 \text{ ksi})(10.0 \text{ in.}^2) \leq 0.60(50 \text{ ksi})(3.13 \text{ in.}^2) + 1.0(65 \text{ ksi})(10.0 \text{ in.}^2) \\ &= 723 \text{ kips} < 744 \text{ kips} \end{aligned}$$

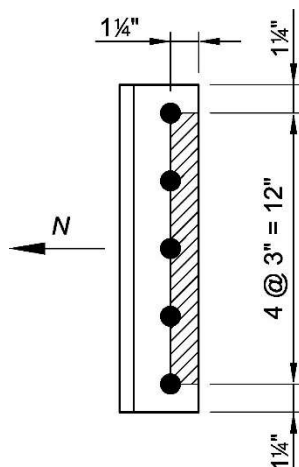


Fig. II.A-1B-6. Block shear rupture of angles for axial load only—U-shape.

Therefore:

$$R_{bsn} = 723 \text{ kips}$$

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture of the angles is:

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi R_{bsn} = 0.75(723 \text{ kips})$ $= 542 \text{ kips} > 60 \text{ kips} \quad \text{o.k.}$	$\frac{R_{bsn}}{\Omega} = \frac{723 \text{ kips}}{2.00}$ $= 362 \text{ kips} > 40 \text{ kips} \quad \text{o.k.}$

Considering the interaction of shear and axial loads, apply a formulation that is similar to AISC *Manual* Equation 12-1:

LRFD	ASD
$\left(\frac{V_r}{\phi R_{bsv}} \right)^2 + \left(\frac{N_r}{\phi R_{bsn}} \right)^2 \leq 1$	$\left(\frac{V_r}{R_{bsv}/\Omega} \right)^2 + \left(\frac{N_r}{R_{bsn}/\Omega} \right)^2 \leq 1$
$\left(\frac{75 \text{ kips}}{368 \text{ kips}} \right)^2 + \left(\frac{60 \text{ kips}}{559 \text{ kips}} \right)^2 = 0.0531 \leq 1 \quad \text{o.k.}$	$\left(\frac{50 \text{ kips}}{245 \text{ kips}} \right)^2 + \left(\frac{40 \text{ kips}}{373 \text{ kips}} \right)^2 = 0.0531 \leq 1 \quad \text{o.k.}$

Block Shear Rupture of Angles—Outstanding Legs

The nominal strength for the limit state of block shear rupture relative to the shear load on the angles is determined as follows. The tearout pattern is shown in Figure II.A-1B-7.

$$R_n = 0.60F_u A_{nv} + U_{bs}F_u A_{nt} \leq 0.60F_y A_{gv} + U_{bs}F_u A_{nt} \quad (\text{Spec. Eq. J4-5})$$

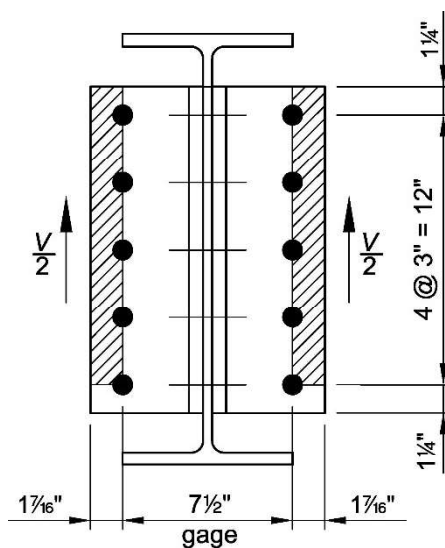


Fig. II.A-1B-7. Block shear rupture of outstanding legs of angles.

where

$$\begin{aligned} A_{gv} &= (2 \text{ angles})(l - l_{ev})t \\ &= (2 \text{ angles})(14\frac{1}{2} \text{ in.} - 1\frac{1}{4} \text{ in.})(\frac{5}{8} \text{ in.}) \\ &= 16.6 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{nv} &= A_{gv} - (2 \text{ angles})(n - 0.5)(d_h + \frac{1}{16} \text{ in.})t \\ &= 16.6 \text{ in.}^2 - (2 \text{ angles})(5 - 0.5)(\frac{15}{16} \text{ in.} + \frac{1}{16} \text{ in.})(\frac{5}{8} \text{ in.}) \\ &= 11.0 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{nt} &= (2 \text{ angles})[l_{eh} - 0.5(d_h + \frac{1}{16} \text{ in.})]t \\ &= (2 \text{ angles})[1\frac{7}{16} \text{ in.} - 0.5(\frac{15}{16} \text{ in.} + \frac{1}{16} \text{ in.})](\frac{5}{8} \text{ in.}) \\ &= 1.17 \text{ in.}^2 \end{aligned}$$

$$U_{bs} = 1.0$$

and

$$\begin{aligned} R_n &= 0.60(65 \text{ ksi})(11.0 \text{ in.}^2) + 1.0(65 \text{ ksi})(1.17 \text{ in.}^2) \leq 0.60(50 \text{ ksi})(16.6 \text{ in.}^2) + 1.0(65 \text{ ksi})(1.17 \text{ in.}^2) \\ &= 505 \text{ kips} < 574 \text{ kips} \end{aligned}$$

Therefore:

$$R_n = 505 \text{ kips}$$

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture of the angles is:

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$

$\phi R_n = 0.75(505 \text{ kips})$ $= 379 \text{ kips} > 75 \text{ kips} \quad \mathbf{o.k.}$	$\frac{R_n}{\Omega} = \frac{505 \text{ kips}}{2.00}$ $= 253 \text{ kips} > 50 \text{ kips} \quad \mathbf{o.k.}$
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Shear Strength of Beam Web

From AISC *Specification* Section J4.2(a), the available shear yield strength of the beam web is determined as follows:

$$\begin{aligned}
 A_{gv} &= dt_w \\
 &= (18.0 \text{ in.})(0.355 \text{ in.}) \\
 &= 6.39 \text{ in.}^2
 \end{aligned}$$

$$\begin{aligned}
 R_n &= 0.60F_y A_{gv} && (\text{Spec. Eq. J4-3}) \\
 &= 0.60(50 \text{ ksi})(6.39 \text{ in.}^2) \\
 &= 192 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(192 \text{ kips})$ $= 192 \text{ kips} > 75 \text{ kips} \quad \mathbf{o.k.}$	$\Omega = 1.50$ $\frac{R_n}{\Omega} = \frac{192 \text{ kips}}{1.50}$ $= 128 \text{ kips} > 50 \text{ kips} \quad \mathbf{o.k.}$

The limit state of shear rupture of the beam web does not apply in this example because the beam is uncoped.

Tensile Strength of Beam

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the beam is determined as follows:

$$\begin{aligned}
 R_n &= F_y A_g && (\text{Spec. Eq. J4-1}) \\
 &= (50 \text{ ksi})(14.7 \text{ in.}^2) \\
 &= 735 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(735 \text{ kips})$ $= 662 \text{ kips} > 60 \text{ kips} \quad \mathbf{o.k.}$	$\Omega = 1.67$ $\frac{R_n}{\Omega} = \frac{735 \text{ kips}}{1.67}$ $= 440 \text{ kips} > 40 \text{ kips} \quad \mathbf{o.k.}$

From AISC *Specification* Section J4.1(b), determine the available tensile rupture strength of the beam. The effective net area is $A_e = A_n U$. No cases in AISC *Specification* Table D3.1 apply to this configuration; therefore, U is determined from AISC *Specification* Section D3.

$$\begin{aligned}
 A_n &= A_g - n(d_h + 1/16 \text{ in.})(t_w) \\
 &= 14.7 \text{ in.}^2 - 5(15/16 \text{ in.} + 1/16 \text{ in.})(0.355 \text{ in.}) \\
 &= 12.9 \text{ in.}^2
 \end{aligned}$$

As stated in AISC *Specification* Section D3, the value of U can be determined as the ratio of the gross area of the connected element (beam web) to the member gross area.

$$\begin{aligned}
 U &= \frac{(d - 2t_f)(t_w)}{A_g} \\
 &= \frac{[18.0 \text{ in.} - 2(0.570 \text{ in.})](0.355 \text{ in.})}{14.7 \text{ in.}^2} \\
 &= 0.407
 \end{aligned}$$

$$\begin{aligned}
 A_e &= A_n U & (\text{Spec. Eq. D3-1}) \\
 &= (12.9 \text{ in.}^2)(0.407) \\
 &= 5.25 \text{ in.}^2
 \end{aligned}$$

$$\begin{aligned}
 R_n &= F_u A_e & (\text{Spec. Eq. J4-2}) \\
 &= (65 \text{ ksi})(5.25 \text{ in.}^2) \\
 &= 341 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi R_n = 0.75(341 \text{ kips})$ $= 256 \text{ kips} > 60 \text{ kips} \quad \mathbf{o.k.}$	$\frac{R_n}{\Omega} = \frac{341 \text{ kips}}{2.00}$ $= 171 \text{ kips} > 40 \text{ kips} \quad \mathbf{o.k.}$

Block Shear Rupture Strength of Beam Web

Block shear rupture is only applicable in the direction of the axial load because the beam is uncoped and the limit state is not applicable for an uncoped beam subject to vertical shear. Assuming a U-shaped tearout relative to the axial load, and assuming a horizontal edge distance of $l_{eh} = 1\frac{3}{4} \text{ in.} - \frac{1}{4} \text{ in.} = 1\frac{1}{2} \text{ in.}$ to account for a possible beam underrun of $\frac{1}{4} \text{ in.}$, the block shear rupture strength is:

$$R_n = 0.60F_u A_{nv} + U_{bs}F_u A_{nt} \leq 0.60F_y A_{gv} + U_{bs}F_u A_{nt} \quad (\text{Spec. Eq. J4-5})$$

where

$$\begin{aligned}
 A_{gv} &= (2)l_{eh}t_w \\
 &= (2)(1\frac{1}{2} \text{ in.})(0.355 \text{ in.}) \\
 &= 1.07 \text{ in.}^2
 \end{aligned}$$

$$\begin{aligned}
 A_{nv} &= A_{gv} - (2)(0.5)(d_h + 1/16 \text{ in.})t_w \\
 &= 1.07 \text{ in.}^2 - (2)(0.5)(15/16 \text{ in.} + 1/16 \text{ in.})(0.355 \text{ in.}) \\
 &= 0.715 \text{ in.}^2
 \end{aligned}$$

$$\begin{aligned}
 A_{nt} &= [l - (n-1)(d_h + \frac{1}{16} \text{ in.})] t_w \\
 &= [12.0 \text{ in.} - (5-1)(\frac{15}{16} \text{ in.} + \frac{1}{16} \text{ in.})] (0.355 \text{ in.}) \\
 &= 2.84 \text{ in.}^2
 \end{aligned}$$

$$U_{bs} = 1.0$$

and

$$\begin{aligned}
 R_n &= 0.60(65 \text{ ksi})(0.715 \text{ in.}^2) + 1.0(65 \text{ ksi})(2.84 \text{ in.}^2) \leq 0.60(50 \text{ ksi})(1.07 \text{ in.}^2) + 1.0(65 \text{ ksi})(2.84 \text{ in.}^2) \\
 &= 212 \text{ kips} < 217 \text{ kips}
 \end{aligned}$$

Therefore:

$$R_n = 212 \text{ kips}$$

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture of the beam web is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(212 \text{ kips})$ $= 159 \text{ kips} > 60 \text{ kips} \quad \mathbf{o.k.}$	$\Omega = 2.00$ $\frac{R_n}{\Omega} = \frac{212 \text{ kips}}{2.00}$ $= 106 \text{ kips} > 40 \text{ kips} \quad \mathbf{o.k.}$

Conclusion

The connection is found to be adequate as given for the applied loads.

EXAMPLE IIA-2A BOLTED/WELDED DOUBLE-ANGLE CONNECTION**Given:**

Verify the available strength of a double-angle shear connection with welds on the support legs (welds B) and bolts in the supported-beam-web legs, as shown in Figure II.A-2A-1. The ASTM A992/A992M W36×231 beam is attached to an ASTM A992/A992M W14×90 column flange supporting the following beam end reactions:

$$R_D = 37.5 \text{ kips}$$

$$R_L = 113 \text{ kips}$$

Use ASTM A572/A572M Grade 50 angles and 70-ksi weld electrodes.

This example is repeated using the following two procedures:

Part A: Determine the available connection strength using the tables in *Manual* Part 10.

Part B: Determine the available connection strength by checking individual limit states.

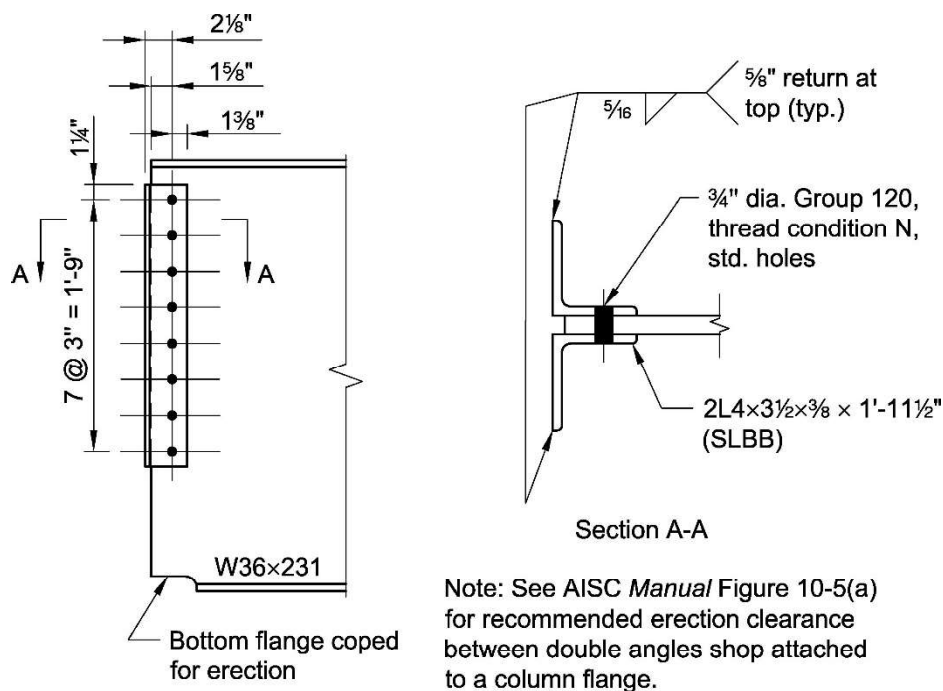


Fig. II.A-2A-1. Connection geometry for Example II.A-2A.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam and column
ASTM A992/A992M

$$F_y = 50 \text{ ksi}$$

$$F_u = 65 \text{ ksi}$$

Angles

ASTM A572/A572M Grade 50

$$F_y = 50 \text{ ksi}$$

$$F_u = 65 \text{ ksi}$$

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam

W36×231

$t_w = 0.760$ in.

Column

W14×90

$t_f = 0.710$ in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(37.5 \text{ kips}) + 1.6(113 \text{ kips})$ $= 226 \text{ kips}$	$R_a = 37.5 \text{ kips} + 113 \text{ kips}$ $= 151 \text{ kips}$

Part A—Determine the Available Connection Strength Using the Tables in Manual Part 10

Weld Design

Use AISC *Manual* Table 10-2 (welds B) with $n = 8$. Try $\frac{5}{16}$ in. weld size, $l = 23\frac{1}{2}$ in. From AISC *Manual* Table 10-2, the minimum support thickness is:

$$t_{min} = 0.238 \text{ in.} < 0.710 \text{ in.} \quad \text{o.k.}$$

LRFD	ASD
$\phi R_n = 318 \text{ kips} > 226 \text{ kips} \quad \text{o.k.}$	$\frac{R_n}{\Omega} = 212 \text{ kips} > 151 \text{ kips} \quad \text{o.k.}$

Angle Thickness

From AISC *Specification* Section J2.2b, the minimum angle thickness for a $\frac{5}{16}$ in. fillet weld is:

$$\begin{aligned} t &= w + \frac{1}{16} \text{ in.} \\ &= \frac{5}{16} \text{ in.} + \frac{1}{16} \text{ in.} \\ &= \frac{3}{8} \text{ in.} \end{aligned}$$

Try 2L4×3½×¾ (SLBB).

Angle Design

AISC *Manual* Table 10-1a includes checks for shear rupture and block shear rupture of the angles.

Check 8 rows of ¾-in.-diameter bolts in standard holes and ¾ in. angle thickness. From AISC *Manual* Table 10-1a:

LRFD	ASD
$\phi R_n = 362 \text{ kips} > 226 \text{ kips} \quad \text{o.k.}$	$\frac{R_n}{\Omega} = 241 \text{ kips} > 151 \text{ kips} \quad \text{o.k.}$

Available Shear Transfer Strength at Bolt Holes

The available shear transfer strength at bolt holes is the sum of the effective strength of individual bolts per the User Note in AISC *Specification* Section J3.7, which is the least of (1) the available bolt shear strength determined in

accordance with AISC *Specification* Section J3.7, (2) the available bearing or tearout strength of the angles at the bolt hole determined in accordance with AISC *Specification* Section J3.11a, and (3) the available bearing or tearout strength of the beam web or support element at the bolt hole determined in accordance with AISC *Specification* Section J3.11a.

From AISC *Manual* Table 10-1b, the available bolt shear strength per bolt for $\frac{3}{4}$ -in.-diameter, Group 120 bolts with threads not excluded from the shear plane (thread condition N) in double shear is:

LRFD	ASD
$\phi r_n = 17.9 \text{ kips (2)}$ $= 35.8 \text{ kips}$	$\frac{r_n}{\Omega} = 11.9 \text{ kips (2)}$ $= 23.8 \text{ kips}$

From AISC *Manual* Table 10-1b, the available bearing and tearout strength of the angle per bolt for $\frac{3}{4}$ -in.-diameter bolts in standard holes is:

LRFD	ASD
For the edge bolt ($l_{ev} = 1\frac{1}{4} \text{ in.}$): $\phi r_n = (49.4 \text{ kip/in.})(\frac{3}{8} \text{ in.})(2 \text{ angles})$ $= 37.1 \text{ kips}$	For the edge bolt ($l_{ev} = 1\frac{1}{4} \text{ in.}$): $\frac{r_n}{\Omega} = (32.9 \text{ kip/in.})(\frac{3}{8} \text{ in.})(2 \text{ angles})$ $= 24.7 \text{ kips}$
For the non-edge bolts ($s = 3 \text{ in.}$): $\phi r_n = (87.8 \text{ kip/in.})(\frac{3}{8} \text{ in.})(2 \text{ angles})$ $= 65.9 \text{ kips}$	For the non-edge bolts ($s = 3 \text{ in.}$): $\frac{r_n}{\Omega} = (58.5 \text{ kip/in.})(\frac{3}{8} \text{ in.})(2 \text{ angles})$ $= 43.9 \text{ kips}$

From AISC *Manual* Table 10-1b, the available bearing and tearout strength of the beam web per bolt for $\frac{3}{4}$ -in.-diameter bolts in standard holes is:

LRFD	ASD
For the non-edge bolts ($s = 3 \text{ in.}$): $\phi r_n = (87.8 \text{ kip/in.})(0.760 \text{ in.})$ $= 66.7 \text{ kips}$	For the non-edge bolts ($s = 3 \text{ in.}$): $\frac{r_n}{\Omega} = (58.5 \text{ kip/in.})(0.760 \text{ in.})$ $= 44.5 \text{ kips}$

At the top connection bolt, the available shear transfer strength is the minimum of the available bolt shear strength, the available bearing and tearout strength of the angle for a non-edge bolt, and available bearing and tearout strength of the beam web for a non-edge bolt:

LRFD	ASD
$\phi r_{n,top} = \min \begin{Bmatrix} 35.8 \text{ kips,} \\ 65.9 \text{ kips,} \\ 66.7 \text{ kips} \end{Bmatrix}$ $= 35.8 \text{ kips}$	$\frac{r_{n,top}}{\Omega} = \min \begin{Bmatrix} 23.8 \text{ kips,} \\ 43.9 \text{ kips,} \\ 44.5 \text{ kips} \end{Bmatrix}$ $= 23.8 \text{ kips}$

At the middle connection bolts, the available shear transfer strength is the minimum of the available bolt shear strength, the available bearing and tearout strength of the angle for a non-edge bolt, and available bearing and tearout strength of the beam web for a non-edge bolt:

LRFD	ASD
$\phi r_{n,mid} = \min \begin{cases} 35.8 \text{ kips,} \\ 65.9 \text{ kips,} \\ 66.7 \text{ kips} \end{cases}$ $= 35.8 \text{ kips}$	$\frac{r_{n,mid}}{\Omega} = \min \begin{cases} 23.8 \text{ kips,} \\ 43.9 \text{ kips,} \\ 44.5 \text{ kips} \end{cases}$ $= 23.8 \text{ kips}$

At the bottom connection bolt, the available shear transfer strength is the minimum of the available bolt shear strength, the available bearing and tearout strength of the angles for an edge bolt, and available bearing and tearout strength of the beam web for a non-edge bolt:

LRFD	ASD
$\phi r_{n,bot} = \min \begin{cases} 35.8 \text{ kips,} \\ 37.1 \text{ kips,} \\ 66.7 \text{ kips} \end{cases}$ $= 35.8 \text{ kips}$	$\frac{r_{n,bot}}{\Omega} = \min \begin{cases} 23.8 \text{ kips,} \\ 24.7 \text{ kips,} \\ 44.5 \text{ kips} \end{cases}$ $= 23.8 \text{ kips}$

The available shear transfer strength at the bolt holes is:

LRFD	ASD
$\phi R_n = \phi r_{n,top} + \phi r_{n,mid} (n-2) + \phi r_{n,bot}$ $= 35.8 \text{ kips} + (35.8 \text{ kips})(8-2) + 35.8 \text{ kips}$ $= 286 \text{ kips} > 226 \text{ kips} \quad \mathbf{o.k.}$	$\frac{R_n}{\Omega} = \frac{r_{n,top}}{\Omega} + \frac{r_{n,mid}}{\Omega} (n-2) + \frac{r_{n,bot}}{\Omega}$ $= 23.8 \text{ kips} + (23.8 \text{ kips})(8-2) + 23.8 \text{ kips}$ $= 190 \text{ kips} > 151 \text{ kips} \quad \mathbf{o.k.}$

Available Beam Web Strength

The available beam web strength is the lesser of the limit states of block shear rupture, shear yielding, and shear rupture. In this example, because of the small size of the cope relative to the overall beam size, the coped section can be found to not control using AISC *Manual* Part 9.

Available Column Flange Strength

Because the thickness of the column flange, $t_f = 0.710$ in., is greater than the thickness of the angles, $t = \frac{3}{8}$ in., the available shear rupture strength of the angles is less than that of the column flange. The column flange is adequate for the required loading.

Summary

The available shear strength of the connection is controlled by the available shear transfer strength at the bolt holes.

LRFD	ASD
$\phi R_n = 286 \text{ kips} > 226 \text{ kips} \quad \mathbf{o.k.}$	$\frac{R_n}{\Omega} = 190 \text{ kips} > 151 \text{ kips} \quad \mathbf{o.k.}$

The connection is found to be adequate as given for the applied loads.

Part B—Determine the Available Connection Strength by Checking Individual Limit States

Available Shear Strength of Angles

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the angles is determined as follows:

$$\begin{aligned} A_{gv} &= (2 \text{ angles})lt \\ &= (2 \text{ angles})(23\frac{1}{2} \text{ in.})(\frac{3}{8} \text{ in.}) \\ &= 17.6 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} R_n &= 0.60F_y A_{gv} && (\text{Spec. Eq. J4-3}) \\ &= 0.60(50 \text{ ksi})(17.6 \text{ in.}^2) \\ &= 528 \text{ kips} \end{aligned}$$

LRFD	ASD
$\phi = 1.00$	$\Omega = 1.50$
$\phi R_n = 1.00(528 \text{ kips})$ $= 528 \text{ kips} > 226 \text{ kips} \quad \mathbf{o.k.}$	$\frac{R_n}{\Omega} = \frac{528 \text{ kips}}{1.50}$ $= 352 \text{ kips} > 151 \text{ kips} \quad \mathbf{o.k.}$

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the angle is determined using the net area determined in accordance with AISC *Specification* Section B4.3b.

$$\begin{aligned} A_{nv} &= (2 \text{ angles})\left[l - n\left(d_h + \frac{1}{16} \text{ in.}\right)\right]t \\ &= (2 \text{ angles})\left[23\frac{1}{2} \text{ in.} - 8\left(\frac{13}{16} \text{ in.} + \frac{1}{16} \text{ in.}\right)\right]\left(\frac{3}{8} \text{ in.}\right) \\ &= 12.4 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} R_n &= 0.60F_u A_{nv} && (\text{Spec. Eq. J4-4}) \\ &= 0.60(65 \text{ ksi})(12.4 \text{ in.}^2) \\ &= 484 \text{ kips} \end{aligned}$$

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi R_n = 0.75(484 \text{ kips})$ $= 363 \text{ kips} > 226 \text{ kips} \quad \mathbf{o.k.}$	$\frac{R_n}{\Omega} = \frac{484 \text{ kips}}{2.00}$ $= 242 \text{ kips} > 151 \text{ kips} \quad \mathbf{o.k.}$

Available Block Shear Rupture of Angles

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture of the angles is determined as follows:

$$R_{bsv} = 0.60F_u A_{nv} + U_{bs}F_u A_{nt} \leq 0.60F_y A_{gv} + U_{bs}F_u A_{nt} \quad (\text{from Spec. Eq. J4-5})$$

where

$$\begin{aligned} A_{gv} &= (2 \text{ angles})(l - l_{ev})t \\ &= (2 \text{ angles})(23\frac{1}{2} \text{ in.} - 1\frac{1}{4} \text{ in.})(\frac{3}{8} \text{ in.}) \\ &= 16.7 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned}
 A_{nv} &= A_{gv} - (2 \text{ angles})(n - 0.5)(d_h + 1/16 \text{ in.})t \\
 &= 16.7 \text{ in.}^2 - (2 \text{ angles})(8 - 0.5)(13/16 \text{ in.} + 1/16 \text{ in.})(3/8 \text{ in.}) \\
 &= 11.8 \text{ in.}^2
 \end{aligned}$$

$$\begin{aligned}
 A_{nt} &= (2 \text{ angles})[l_{eh} - 0.5(d_h + 1/16 \text{ in.})]t \\
 &= (2 \text{ angles})[13/8 \text{ in.} - 0.5(13/16 \text{ in.} + 1/16 \text{ in.})](3/8 \text{ in.}) \\
 &= 0.703 \text{ in.}^2
 \end{aligned}$$

$$U_{bs} = 1.0$$

and

$$\begin{aligned}
 R_{bsv} &= 0.60(65 \text{ ksi})(11.8 \text{ in.}^2) + 1.0(65 \text{ ksi})(0.703 \text{ in.}^2) \leq 0.60(50 \text{ ksi})(16.7 \text{ in.}^2) + 1.0(65 \text{ ksi})(0.703 \text{ in.}^2) \\
 &= 506 \text{ kips} < 547 \text{ kips}
 \end{aligned}$$

Therefore:

$$R_{bsv} = 506 \text{ kips}$$

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture of the angles is:

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi R_{bsv} = 0.75(506 \text{ kips})$ $= 380 \text{ kips} > 226 \text{ kips} \quad \text{o.k.}$	$\frac{R_{bsv}}{\Omega} = \frac{506 \text{ kips}}{2.00}$ $= 253 \text{ kips} > 151 \text{ kips} \quad \text{o.k.}$

Available Shear Transfer Strength at Bolt Holes

The available shear transfer strength at bolt holes is the sum of the effective strength of individual bolts per the User Note in AISC *Specification* Section J3.7, which is the least of (1) the available bolt shear strength determined in accordance with AISC *Specification* Section J3.7, (2) the available bearing or tearout strength of the angles at the bolt hole determined in accordance with AISC *Specification* Section J3.11a, and (3) the available bearing or tearout strength of the beam web at the bolt hole determined in accordance with AISC *Specification* Section J3.11a.

Bolt shear

From AISC *Manual* Table 7-1, the available shear strength for 3/4-in.-diameter, Group 120 bolts with threads not excluded from the shear plane (thread condition N) is:

LRFD	ASD
$\phi r_n = 17.9 \text{ kips}$	$\frac{r_n}{\Omega} = 11.9 \text{ kips}$

The available bearing strength of the angles is determined from AISC *Specification* Section J3.11, assuming deformation at service load is a design consideration:

$$\begin{aligned}
 r_n &= 2.4dtF_u && \text{(from Spec. Eq. J3-6a)} \\
 &= (2.4)(3/4 \text{ in.})(3/8 \text{ in.})(65 \text{ ksi}) \\
 &= 43.9 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi r_n = 0.75(43.9 \text{ kips})$ $= 32.9 \text{ kips}$	$\frac{r_n}{\Omega} = \frac{43.9 \text{ kips}}{2.00}$ $= 22.0 \text{ kips}$

The available tearout strength of the angles is determined from AISC *Specification* Section J3.11, assuming deformation at service load is a design consideration.

For edge bolt tearout, the clear distance along the line of action of the force, between the edge of the hole and the edge of the angle is:

$$\begin{aligned}
 l_c &= l_{ev} - 0.5d_h \\
 &= 1\frac{1}{4} \text{ in.} - 0.5(1\frac{3}{16} \text{ in.}) \\
 &= 0.844 \text{ in.}
 \end{aligned}$$

The available tearout strength of the angles at the edge bolt is:

$$\begin{aligned}
 r_n &= 1.2l_c t F_u && \text{(from Spec. Eq. J3-6c)} \\
 &= (1.2)(0.844 \text{ in.})(\frac{3}{8} \text{ in.})(65 \text{ ksi}) \\
 &= 24.7 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi r_n = 0.75(24.7 \text{ kips})$ $= 18.5 \text{ kips}$	$\frac{r_n}{\Omega} = \frac{24.7 \text{ kips}}{2.00}$ $= 12.4 \text{ kips}$

For non-edge bolt tearout in the angles, the clear distance is between bolt holes:

$$\begin{aligned}
 l_c &= s - d_h \\
 &= 3 \text{ in.} - 1\frac{3}{16} \text{ in.} \\
 &= 2.19 \text{ in.}
 \end{aligned}$$

The available tearout strength of the angles at non-edge bolts is:

$$\begin{aligned}
 r_n &= 1.2l_c t F_u && \text{(from Spec. Eq. J3-6c)} \\
 &= (1.2)(2.19 \text{ in.})(\frac{3}{8} \text{ in.})(65 \text{ ksi}) \\
 &= 64.1 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi r_n = 0.75(64.1 \text{ kips})$ $= 48.1 \text{ kips}$	$\frac{r_n}{\Omega} = \frac{64.1 \text{ kips}}{2.00}$ $= 32.1 \text{ kips}$

The available bearing strength of the beam web is determined from AISC *Specification* Section J3.11, assuming deformation at service load is a design consideration:

$$\begin{aligned}
 r_n &= 2.4dtF_u && \text{(from Spec. Eq. J3-6a)} \\
 &= (2.4)(\frac{3}{4} \text{ in.})(0.760 \text{ in.})(65 \text{ ksi}) \\
 &= 88.9 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi r_n = 0.75(88.9 \text{ kips})$ $= 66.7 \text{ kips}$	$\frac{r_n}{\Omega} = \frac{88.9 \text{ kips/bolt}}{2.00}$ $= 44.5 \text{ kips/bolt}$

The available tearout strength of the beam web is determined from AISC *Specification* Section J3.11, assuming deformation at service load is a design consideration.

There is no edge bolt tearout for the beam web. For non-edge bolt tearout in the beam web, the clear distance is between bolt holes:

$$\begin{aligned}
 l_c &= s - d_h \\
 &= 3 \text{ in.} - \frac{13}{16} \text{ in.} \\
 &= 2.19 \text{ in.}
 \end{aligned}$$

The available tearout strength of the beam web at non-edge bolts is:

$$\begin{aligned}
 r_n &= 1.2l_c t F_u && \text{(from Spec. Eq. J3-6c)} \\
 &= (1.2)(2.19 \text{ in.})(0.760 \text{ in.})(65 \text{ ksi}) \\
 &= 130 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi r_n = 0.75(130 \text{ kips})$ $= 97.5 \text{ kips}$	$\frac{r_n}{\Omega} = \frac{130 \text{ kips}}{2.00}$ $= 65.0 \text{ kips}$

At the top connection bolt, the available shear transfer strength is the minimum of the available bolt shear strength (multiplied by 2 because the bolts are in double shear), the available bearing and tearout strength of the angles for a non-edge bolt (multiplied by 2 because there are two angles), and the available bearing and tearout strength of the beam web for a non-edge bolt:

LRFD	ASD
$\phi r_{n,top} = \min \left\{ \begin{array}{l} 17.9 \text{ kips}(2) = 35.8 \text{ kips,} \\ 32.9 \text{ kips}(2) = 65.8 \text{ kips,} \\ 48.1 \text{ kips}(2) = 96.2 \text{ kips,} \\ 66.7 \text{ kips,} \\ 97.5 \text{ kips} \end{array} \right\}$ $= 35.8 \text{ kips}$	$\frac{r_{n,top}}{\Omega} = \min \left\{ \begin{array}{l} 11.9 \text{ kips}(2) = 23.8 \text{ kips,} \\ 22.0 \text{ kips}(2) = 44.0 \text{ kips,} \\ 32.1 \text{ kips}(2) = 64.2 \text{ kips,} \\ 44.5 \text{ kips,} \\ 65.0 \text{ kips} \end{array} \right\}$ $= 23.8 \text{ kips}$

At the middle connection bolts, the available shear transfer strength is the minimum of the available bolt shear strength (multiplied by 2 because the bolts are in double shear), the available bearing and tearout strength of the angles for a non-edge bolt (multiplied by 2 because there are two angles), and the available bearing and tearout strength of the beam web for a non-edge bolt:

LRFD	ASD
$\phi r_{n,mid} = \min \left\{ \begin{array}{l} 17.9 \text{ kips}(2) = 35.8 \text{ kips,} \\ 32.9 \text{ kips}(2) = 65.8 \text{ kips,} \\ 48.1 \text{ kips}(2) = 96.2 \text{ kips,} \\ 66.7 \text{ kips,} \\ 97.5 \text{ kips} \end{array} \right\}$ $= 35.8 \text{ kips}$	$\frac{r_{n,mid}}{\Omega} = \min \left\{ \begin{array}{l} 11.9 \text{ kips}(2) = 23.8 \text{ kips,} \\ 22.0 \text{ kips}(2) = 44.0 \text{ kips,} \\ 32.1 \text{ kips}(2) = 64.2 \text{ kips,} \\ 44.5 \text{ kips,} \\ 65.0 \text{ kips} \end{array} \right\}$ $= 23.8 \text{ kips}$

At the bottom connection bolt, the available shear transfer strength is the minimum of the available bolt shear strength (multiplied by 2 because the bolts are in double shear), the available bearing and tearout strength of the angles for an edge bolt (multiplied by 2 because there are two angles), and the available bearing and tearout strength of the beam web for a non-edge bolt:

LRFD	ASD
$\phi r_{n,bot} = \min \left\{ \begin{array}{l} 17.9 \text{ kips}(2) = 35.8 \text{ kips,} \\ 32.9 \text{ kips}(2) = 65.8 \text{ kips,} \\ 18.5 \text{ kips}(2) = 37.0 \text{ kips,} \\ 66.7 \text{ kips,} \\ 97.5 \text{ kips} \end{array} \right\}$ $= 35.8 \text{ kips}$	$\frac{r_{n,bot}}{\Omega} = \min \left\{ \begin{array}{l} 11.9 \text{ kips}(2) = 23.8 \text{ kips,} \\ 22.0 \text{ kips}(2) = 44.0 \text{ kips,} \\ 12.4 \text{ kips}(2) = 24.8 \text{ kips,} \\ 44.5 \text{ kips,} \\ 65.0 \text{ kips} \end{array} \right\}$ $= 23.8 \text{ kips}$

The available shear transfer strength at the bolt holes is:

LRFD	ASD
$\phi R_n = \phi r_{n,top} + \phi r_{n,mid} (n-2) + \phi r_{n,bot}$ $= 35.8 \text{ kips} + (35.8 \text{ kips})(8-2) + 35.8 \text{ kips}$ $= 286 \text{ kips} > 226 \text{ kips} \quad \text{o.k.}$	$\frac{R_n}{\Omega} = \frac{r_{n,top}}{\Omega} + \frac{r_{n,mid}}{\Omega} (n-2) + \frac{r_{n,bot}}{\Omega}$ $= 23.8 \text{ kips} + (23.8 \text{ kips})(8-2) + 23.8 \text{ kips}$ $= 190 \text{ kips} > 151 \text{ kips} \quad \text{o.k.}$

Available Beam Web Strength

Because the beam is not coped at the top flange, limit states of block shear rupture and shear rupture of the beam are not applicable. The beam web is adequate for the required loading.

Available Weld Strength

The available weld strength is determined using AISC *Manual* Table 8-4, with Angle = 0°.

$$k = 0$$

$$e_x = al$$

$$= 4 \text{ in.}$$

$$a = \frac{4 \text{ in.}}{23.5 \text{ in.}}$$

$$= 0.170$$

Interpolating from AISC *Manual* Table 8-4:

$$C = 3.61$$

LRFD	ASD
$\phi R_n = \phi CC_1 D l$ $= 0.75(3.61)(1.0)(5)(23.5 \text{ in.})$ $= 318 \text{ kips} > 226 \text{ kips} \quad \mathbf{o.k.}$	$\frac{R_n}{\Omega} = \frac{CC_1 D l (2 \text{ welds})}{\Omega}$ $= \frac{(3.61)(1.0)(5)(23.5 \text{ in.})}{2.00}$ $= 212 \text{ kips} > 151 \text{ kips} \quad \mathbf{o.k.}$

Minimum Support Thickness

The minimum support thickness at the welds is determined as follows:

$$\begin{aligned}
 t_{min} &= \frac{3.09D}{F_u} && (\text{Manual Eq. 9-6}) \\
 &= \frac{3.09(5)}{65 \text{ ksi}} \\
 &= 0.237 \text{ in.} < 0.710 \text{ in.} \quad \mathbf{o.k.}
 \end{aligned}$$

Summary

The available shear strength of the connection is controlled by the available shear transfer strength at the bolt holes.

LRFD	ASD
$\phi R_n = 286 \text{ kips} > 226 \text{ kips} \quad \mathbf{o.k.}$	$\frac{R_n}{\Omega} = 190 \text{ kips} > 151 \text{ kips} \quad \mathbf{o.k.}$

The connection is found to be adequate as given for the applied loads.

EXAMPLE II.A-2B BOLTED/WELDED DOUBLE-ANGLE CONNECTION SUBJECT TO AXIAL AND SHEAR LOADING

Given:

Verify the available strength of a double-angle connection with welds in the supported-beam-web legs and bolts in the outstanding legs for an ASTM A992/A992M W18×50 beam, as shown in Figure II.A-2B-1, to support the following beam end reactions:

LRFD	ASD
Shear, $V_u = 75$ kips Axial tension, $N_u = 60$ kips	Shear, $V_a = 50$ kips Axial tension, $N_a = 40$ kips

Use ASTM A572/A572M Grade 50 angles and 70-ksi electrodes.

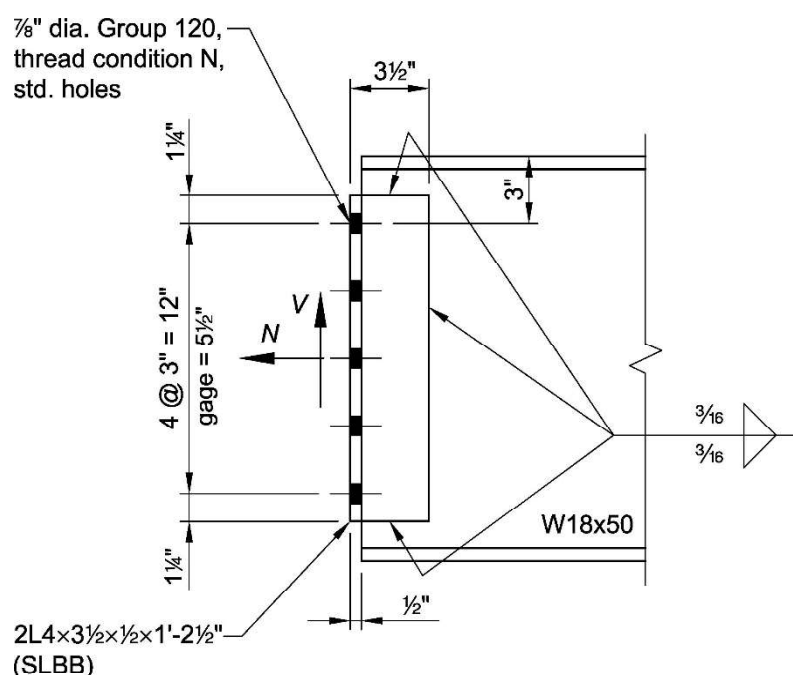


Fig. II.A-2B-1. Connection geometry for Example II.A-2B.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992/A992M
 $F_y = 50$ ksi
 $F_u = 65$ ksi

Angles
ASTM A572/A572M Grade 50
 $F_y = 50$ ksi
 $F_u = 65$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam

W18×50

$$A_g = 14.7 \text{ in.}^2$$

$$d = 18.0 \text{ in.}$$

$$t_w = 0.355 \text{ in.}$$

$$b_f = 7.50 \text{ in.}$$

$$t_f = 0.570 \text{ in.}$$

From AISC *Specification* Table J3.3, the hole diameter for 7/8-in.-diameter bolts with standard holes is:

$$d_h = 15/16 \text{ in.}$$

The resultant load is:

LRFD	ASD
$R_u = \sqrt{V_u^2 + N_u^2}$ $= \sqrt{(75 \text{ kips})^2 + (60 \text{ kips})^2}$ $= 96.0 \text{ kips}$	$R_a = \sqrt{V_a^2 + N_a^2}$ $= \sqrt{(50 \text{ kips})^2 + (40 \text{ kips})^2}$ $= 64.0 \text{ kips}$

The following bolt shear, bearing, and tearout calculations are for a pair of bolts.

Bolt Shear

From AISC *Manual* Table 7-1, the available shear strength for 7/8-in.-diameter, Group 120 bolts with threads not excluded from the shear plane (thread condition N) in double shear (or pair of bolts):

LRFD	ASD
$\phi r_n = 48.7 \text{ kips (for pair of bolts)}$	$\frac{r_n}{\Omega} = 32.5 \text{ kips (for pair of bolts)}$

Bolt Bearing on Angles

The available bearing strength of the double angle is determined from AISC *Specification* Section J3.11a, assuming deformation at the bolt hole is a design consideration:

$$\begin{aligned}
 r_n &= (2 \text{ bolts}) 2.4 d t F_u && \text{(from Spec. Eq. J3-6a)} \\
 &= (2 \text{ bolts}) (2.4) \left(\frac{7}{8} \text{ in.}\right) \left(\frac{1}{2} \text{ in.}\right) (65 \text{ ksi}) \\
 &= 137 \text{ kips (for pair of bolts)}
 \end{aligned}$$

The available bearing strength for a pair of bolts is:

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi r_n = 0.75 (137 \text{ kips})$ $= 103 \text{ kips (for pair of bolts)}$	$\frac{r_n}{\Omega} = \frac{137 \text{ kips}}{2.00}$ $= 68.5 \text{ kips (for pair of bolts)}$

The bolt shear strength controls over bearing in the angles.

Bolt Tearout on Angles

The available tearout strength of the angle is determined from AISC *Specification* Section J3.11a, assuming deformation at the bolt hole is a design consideration:

For the edge bolt:

$$\begin{aligned} l_c &= l_e - 0.5d_h \\ &= 1\frac{1}{4} \text{ in.} - 0.5\left(1\frac{5}{16} \text{ in.}\right) \\ &= 0.781 \text{ in.} \end{aligned}$$

$$\begin{aligned} r_n &= (2 \text{ bolts})1.2l_c t F_u && \text{(from Spec. Eq. J3-6c)} \\ &= (2 \text{ bolts})(1.2)(0.781 \text{ in.})(\frac{1}{2} \text{ in.})(65 \text{ ksi}) \\ &= 60.9 \text{ kips (for pair of bolts)} \end{aligned}$$

The available tearout strength of the angles for a pair of edge bolts is:

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi r_n = 0.75(60.9 \text{ kips})$ $= 45.7 \text{ kips (for pair of bolts)}$	$\frac{r_n}{\Omega} = \frac{60.9 \text{ kips}}{2.00}$ $= 30.5 \text{ kips (for pair of bolts)}$

The tearout strength controls over bolt shear and bearing for the edge bolts in the angles.

For the other bolts:

$$\begin{aligned} l_c &= s - d_h \\ &= 3 \text{ in.} - 1\frac{5}{16} \text{ in.} \\ &= 2.06 \text{ in.} \end{aligned}$$

$$\begin{aligned} r_n &= (2 \text{ bolts})1.2l_c t F_u && \text{(from Spec. Eq. J3-6c)} \\ &= (2 \text{ bolts})(1.2)(2.06 \text{ in.})(\frac{1}{2} \text{ in.})(65 \text{ ksi}) \\ &= 161 \text{ kips (for pair of bolts)} \end{aligned}$$

The available tearout strength for a pair of other bolts is:

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi r_n = 0.75(161 \text{ kips})$ $= 121 \text{ kips (for pair of bolts)}$	$\frac{r_n}{\Omega} = \frac{161 \text{ kips}}{2.00}$ $= 80.5 \text{ kips (for pair of bolts)}$

Bolt shear strength controls over tearout and bearing strength for the other bolts in the angles.

Strength of Bolted Connection

The effective strength for the bolted connection at the angles is determined by summing the effective strength for each bolt using the minimum available strength calculated for bolt shear, bearing on the angles, and tearout on the angles.

LRFD	ASD
$\phi R_n = (1 \text{ bolt})(45.7 \text{ kips})$ $+ (4 \text{ bolts})(48.7 \text{ kips})$ $= 241 \text{ kips} > 75 \text{ kips} \quad \mathbf{o.k.}$	$\frac{R_n}{\Omega} = (1 \text{ bolt})(30.5 \text{ kips})$ $+ (4 \text{ bolts})(32.5 \text{ kips})$ $= 161 \text{ kips} > 50 \text{ kips} \quad \mathbf{o.k.}$

Shear and Tension Interaction in Bolts

The required shear stress for each bolt is determined as follows:

$$f_{rv} = \frac{V_r}{nA_b}$$

where

$$A_b = 0.601 \text{ in.}^2 \text{ (from AISC Manual Table 7-1)}$$

$$n = 10 \text{ bolts}$$

LRFD	ASD
$f_{rv} = \frac{75 \text{ kips}}{(10 \text{ bolts})(0.601 \text{ in.}^2)}$ $= 12.5 \text{ ksi}$	$f_{rv} = \frac{50 \text{ kips}}{(10 \text{ bolts})(0.601 \text{ in.}^2)}$ $= 8.32 \text{ ksi}$

The nominal tensile stress modified to include the effects of shear stress is determined from AISC *Specification* Section J3.8 as follows. From AISC *Specification* Table J3.2:

$$F_{nt} = 90 \text{ ksi}$$

$$F_{nv} = 54 \text{ ksi}$$

LRFD	ASD
$\phi = 0.75$ $F'_t = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_{rv} \leq F_{nt} \quad (\text{Spec. Eq. J3-3a})$ $= 1.3(90 \text{ ksi}) - \frac{90 \text{ ksi}}{0.75(54 \text{ ksi})}(12.5 \text{ ksi}) \leq 90 \text{ ksi}$ $= 89.2 \text{ ksi} < 90 \text{ ksi} \quad \mathbf{o.k.}$	$\Omega = 2.00$ $F'_t = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_{rv} \leq F_{nt} \quad (\text{Spec. Eq. J3-3b})$ $= 1.3(90 \text{ ksi}) - \frac{2.00(90 \text{ ksi})}{54 \text{ ksi}}(8.32 \text{ ksi}) \leq 90 \text{ ksi}$ $= 89.3 \text{ ksi} < 90 \text{ ksi} \quad \mathbf{o.k.}$

Using the value of $F'_t = 89.2 \text{ ksi}$ determined for LRFD, the nominal tensile strength of one bolt is:

$$\begin{aligned}
 r_n &= F'_t A_b && (\text{from Spec. Eq. J3-2}) \\
 &= (89.2 \text{ ksi})(0.601 \text{ in.}^2) \\
 &= 53.6 \text{ kips}
 \end{aligned}$$

The available tensile strength due to combined tension and shear is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = n\phi r_n$ $= (10 \text{ bolts})(0.75)(53.6 \text{ kips})$ $= 402 \text{ kips} > 60 \text{ kips} \quad \mathbf{o.k.}$	$\Omega = 2.00$ $\frac{R_n}{\Omega} = n \frac{r_n}{\Omega}$ $= (10 \text{ bolts}) \left(\frac{53.6 \text{ kips}}{2.00} \right)$ $= 268 \text{ kips} > 40 \text{ kips} \quad \mathbf{o.k.}$

Prying Action on Bolts

From AISC *Manual* Part 9, the available tensile strength of the bolts in the outstanding angle legs taking prying action into account is determined as follows:

$$\begin{aligned}
 b &= \frac{\text{gage} - t_w - t}{2} \\
 &= \frac{5\frac{1}{2} \text{ in.} - 0.355 \text{ in.} - \frac{1}{2} \text{ in.}}{2} \\
 &= 2.32 \text{ in.} \\
 a &= \frac{\text{angle leg}(2) + t_w - \text{gage}}{2} \leq 1.25b \\
 &= \frac{(4.00 \text{ in.})(2) + 0.355 \text{ in.} - 5\frac{1}{2} \text{ in.}}{2} \leq 1.25(2.32 \text{ in.}) \\
 &= 1.43 \text{ in.} \leq 2.90 \text{ in.} \\
 &= 1.43 \text{ in.}
 \end{aligned}$$

Note: Although it is not shown in this example, if the distance from the bolt centerline to the edge of the supporting element is smaller than $a = 1.43 \text{ in.}$, use the smaller a in the following calculation.

$$\begin{aligned}
 a' &= a + \frac{d_b}{2} && (\text{Manual Eq. 9-23}) \\
 &= 1.43 \text{ in.} + \frac{\frac{7}{8} \text{ in.}}{2} \\
 &= 1.87 \text{ in.}
 \end{aligned}$$

$$\begin{aligned}
 b' &= b - \frac{d_b}{2} && (\text{Manual Eq. 9-24}) \\
 &= 2.32 \text{ in.} - \frac{\frac{7}{8} \text{ in.}}{2} \\
 &= 1.88 \text{ in.}
 \end{aligned}$$

$$\begin{aligned}
 \rho &= \frac{b'}{a'} && (\text{Manual Eq. 9-29}) \\
 &= \frac{1.88 \text{ in.}}{1.87 \text{ in.}} \\
 &= 1.01
 \end{aligned}$$

Note that end distances of $1\frac{1}{4} \text{ in.}$ are used on the angles, so p is the average pitch of the bolts:

$$\begin{aligned}
 p &= \frac{l}{n} \\
 &= \frac{14\frac{1}{2} \text{ in.}}{5} \\
 &= 2.90 \text{ in.}
 \end{aligned}$$

Check that $p \leq s$:

$$\begin{aligned}
 p &\leq s \\
 2.90 \text{ in.} &< 3 \text{ in.} \quad \mathbf{o.k.}
 \end{aligned}$$

$$\begin{aligned}
 d' &= d_h \\
 &= 1\frac{5}{16} \text{ in.}
 \end{aligned}$$

$$\begin{aligned}
 \delta &= 1 - \frac{d'}{p} && (\text{Manual Eq. 9-28}) \\
 &= 1 - \frac{1\frac{5}{16} \text{ in.}}{2.90 \text{ in.}} \\
 &= 0.677
 \end{aligned}$$

The angle thickness required to develop the available strength of the bolt with no prying action is determined as follows:

LRFD	ASD
$T_c = 40.2 \text{ kips/bolt}$ (from previous calculations)	$T_c = 26.8 \text{ kips/bolt}$ (from previous calculations)
$\phi_b = 0.90$	$\Omega_b = 1.67$
$t_c = \sqrt{\frac{4T_c b'}{\phi_b p F_u}} \quad (\text{Manual Eq. 9-30a})$	$t_c = \sqrt{\frac{\Omega_b 4T_c b'}{p F_u}} \quad (\text{Manual Eq. 9-30b})$
$= \sqrt{\frac{4(40.2 \text{ kips/bolt})(1.88 \text{ in.})}{0.90(2.90 \text{ in.})(65 \text{ ksi})}}$	$= \sqrt{\frac{1.67(4)(26.8 \text{ kips/bolt})(1.88 \text{ in.})}{(2.90 \text{ in.})(65 \text{ ksi})}}$
$= 1.33 \text{ in.}$	$= 1.34 \text{ in.}$

$$\begin{aligned}
 \alpha' &= \frac{1}{\delta(1+\rho)} \left[\left(\frac{t_c}{t} \right)^2 - 1 \right] && (\text{Manual Eq. 9-38}) \\
 &= \frac{1}{0.677(1+1.01)} \left[\left(\frac{1.34 \text{ in.}}{\frac{1}{2} \text{ in.}} \right)^2 - 1 \right] \\
 &= 4.54
 \end{aligned}$$

Because $\alpha' > 1$, the angles have insufficient strength to develop the bolt strength, therefore:

$$\begin{aligned}
 Q &= \left(\frac{t}{t_c} \right)^2 (1 + \delta) && (\text{Manual Eq. 9-39c}) \\
 &= \left(\frac{\frac{1}{2} \text{ in.}}{1.34 \text{ in.}} \right)^2 (1 + 0.677) \\
 &= 0.233
 \end{aligned}$$

The available tensile strength of the bolts taking prying action into account is determined from AISC *Manual* Equation 9-40 as follows:

LRFD	ASD
$\phi R_n = nT_{c, adj}$ $= nQT_c$ $= (10 \text{ bolts})(0.233)(40.2 \text{ kips/bolt})$ $= 93.7 \text{ kips} > 60 \text{ kips} \quad \mathbf{o.k.}$	$\frac{R_n}{\Omega} = nT_{c, adj}$ $= nQT_c$ $= (10 \text{ bolts})(0.233)(26.8 \text{ kips/bolt})$ $= 62.4 \text{ kips} > 40 \text{ kips} \quad \mathbf{o.k.}$

Weld Design

The resultant load angle on the weld is:

LRFD	ASD
$\theta = \tan^{-1} \left(\frac{N_u}{V_u} \right)$ $= \tan^{-1} \left(\frac{60 \text{ kips}}{75 \text{ kips}} \right)$ $= 38.7^\circ$	$\theta = \tan^{-1} \left(\frac{N_a}{V_a} \right)$ $= \tan^{-1} \left(\frac{40 \text{ kips}}{50 \text{ kips}} \right)$ $= 38.7^\circ$

From AISC *Manual* Table 8-8 for Angle = 30° (which will lead to a conservative result), using a total beam setback of ½ in. + ¼ in. = ¾ in. (the ¼ in. is included to account for mill underrun):

$$l = 14\frac{1}{2} \text{ in.}$$

$$kl = 3\frac{1}{2} \text{ in.} - \frac{3}{4} \text{ in.}$$

$$= 2.75 \text{ in.}$$

$$k = \frac{kl}{l}$$

$$= \frac{2.75 \text{ in.}}{14\frac{1}{2} \text{ in.}}$$

$$= 0.190$$

$$x = 0.0269 \text{ by interpolation}$$

$$al = 3\frac{1}{2} \text{ in.} - xl$$

$$= 3\frac{1}{2} \text{ in.} - 0.0269(14\frac{1}{2} \text{ in.})$$

$$= 3.11 \text{ in.}$$

$$a = \frac{al}{l}$$

$$= \frac{3.11 \text{ in.}}{14\frac{1}{2} \text{ in.}}$$

$$= 0.214$$

$$C = 2.69 \text{ by interpolation}$$

The required weld size is determined using AISC *Manual* Equation 8-30, as follows:

LRFD	ASD
$D_{min} = \frac{R_u}{\phi C C_1 l}$ $= \frac{96.0 \text{ kips}}{0.75(2.69)(1)(14\frac{1}{2} \text{ in.})(2 \text{ sides})}$ $= 1.64 \text{ sixteenths}$	$D_{min} = \frac{\Omega R_a}{C C_1 l}$ $= \frac{2.00(64.0 \text{ kips})}{2.69(1)(14\frac{1}{2} \text{ in.})(2 \text{ sides})}$ $= 1.64 \text{ sixteenths}$

Use a $\frac{3}{16}$ in. fillet weld (minimum size from AISC *Specification* Table J2.4).

Beam Web Strength at Fillet Weld

The minimum beam web thickness required to match the shear rupture strength of a weld on both sides to that of the base metal is:

$$t_{min} = \frac{6.19 D_{min}}{F_u} \quad (\text{from Manual Eq. 9-7})$$

$$= \frac{6.19(1.64)}{65 \text{ ksi}}$$

$$= 0.156 \text{ in.} < 0.355 \text{ in.} \quad \text{o.k.}$$

Shear Strength of Angles

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the angles is determined as follows:

$$A_{gv} = (2 \text{ angles}) l t$$

$$= (2 \text{ angles})(14\frac{1}{2} \text{ in.})(\frac{1}{2} \text{ in.})$$

$$= 14.5 \text{ in.}^2$$

$$R_n = 0.60 F_y A_{gv} \quad (\text{Spec. Eq. J4-3})$$

$$= 0.60(50 \text{ ksi})(14.5 \text{ in.}^2)$$

$$= 435 \text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(435 \text{ kips})$ $= 435 \text{ kips} > 96.0 \text{ kips} \quad \text{o.k.}$	$\Omega = 1.50$ $\frac{R_n}{\Omega} = \frac{435 \text{ kips}}{1.50}$ $= 290 \text{ kips} > 64.0 \text{ kips} \quad \text{o.k.}$

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the angle is determined as follows. The effective net area is determined in accordance with AISC *Specification* Section B4.3b.

$$A_{nv} = (2 \text{ angles}) \left[l - n \left(d_h + \frac{1}{16} \text{ in.} \right) \right] t$$

$$= (2 \text{ angles}) \left[14\frac{1}{2} \text{ in.} - 5 \left(\frac{15}{16} \text{ in.} + \frac{1}{16} \text{ in.} \right) \right] \left(\frac{1}{2} \text{ in.} \right)$$

$$= 9.50 \text{ in.}^2$$

$$\begin{aligned}
 R_n &= 0.60F_u A_{nv} \\
 &= 0.60(65 \text{ ksi})(9.50 \text{ in.}^2) \\
 &= 371 \text{ kips}
 \end{aligned}
 \tag{Spec. Eq. J4-4}$$

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi R_n = 0.75(371 \text{ kips})$ $= 278 \text{ kips} > 96.0 \text{ kips} \quad \mathbf{o.k.}$	$\frac{R_n}{\Omega} = \frac{371 \text{ kips}}{2.00}$ $= 186 \text{ kips} > 64.0 \text{ kips} \quad \mathbf{o.k.}$

Tensile Strength of Angles—Beam Web Side

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the angles is determined as follows:

$$\begin{aligned}
 A_g &= (2 \text{ angles})lt \\
 &= (2 \text{ angles})(14\frac{1}{2} \text{ in.})(\frac{1}{2} \text{ in.}) \\
 &= 14.5 \text{ in.}^2
 \end{aligned}$$

$$\begin{aligned}
 R_n &= F_y A_g \\
 &= (50 \text{ ksi})(14.5 \text{ in.}^2) \\
 &= 725 \text{ kips}
 \end{aligned}
 \tag{Spec. Eq. J4-1}$$

LRFD	ASD
$\phi = 0.90$	$\Omega = 1.67$
$\phi R_n = 0.90(725 \text{ kips})$ $= 653 \text{ kips} > 60 \text{ kips} \quad \mathbf{o.k.}$	$\frac{R_n}{\Omega} = \frac{725 \text{ kips}}{1.67}$ $= 434 \text{ kips} > 40 \text{ kips} \quad \mathbf{o.k.}$

From AISC *Specification* Sections J4.1(b), the available tensile rupture strength of the angles is determined as follows:

$$R_n = F_u A_e \tag{Spec. Eq. J4-2}$$

Because the angle legs are welded to the beam web there is no bolt hole reduction and $A_e = A_g$; therefore, tensile rupture will not control.

Block Shear Rupture Strength of Angles—Outstanding Legs

The nominal strength for the limit state of block shear rupture of the angles assuming an L-shaped tearout relative to shear load, is determined as follows. The tearout pattern is shown in Figure II.A-2B-2.

$$R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \leq 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \tag{Spec. Eq. J4-5}$$

where

$$\begin{aligned}
 l_{eh} &= \frac{2(\text{angle leg}) + t_w - \text{gage}}{2} \\
 &= \frac{2(4 \text{ in.}) + 0.355 \text{ in.} - 5\frac{1}{2} \text{ in.}}{2} \\
 &= 1.43 \text{ in.}
 \end{aligned}$$

$$\begin{aligned}
 A_{nt} &= (2 \text{ angles}) [l_{eh} - 0.5(d_h + \frac{1}{16} \text{ in.})](t) \\
 &= (2 \text{ angles}) [1.43 \text{ in.} - 0.5(\frac{15}{16} \text{ in.} + \frac{1}{16} \text{ in.})](\frac{1}{2} \text{ in.}) \\
 &= 0.930 \text{ in.}^2
 \end{aligned}$$

$$\begin{aligned}
 A_{gv} &= (2 \text{ angles}) [l_{ev} + (n-1)s](t) \\
 &= (2 \text{ angles}) [1\frac{1}{4} \text{ in.} + (5-1)(3 \text{ in.})](\frac{1}{2} \text{ in.}) \\
 &= 13.3 \text{ in.}^2
 \end{aligned}$$

$$\begin{aligned}
 A_{nv} &= A_{gv} - (2 \text{ angles})(n-0.5)(d_h + \frac{1}{16} \text{ in.})(t) \\
 &= 13.3 \text{ in.}^2 - (2 \text{ angles})(5-0.5)(\frac{15}{16} \text{ in.} + \frac{1}{16} \text{ in.})(\frac{1}{2} \text{ in.}) \\
 &= 8.80 \text{ in.}^2
 \end{aligned}$$

$$U_{bs} = 1.0$$

and

$$\begin{aligned}
 R_n &= 0.60(65 \text{ ksi})(8.80 \text{ in.}^2) + 1.0(65 \text{ ksi})(0.930 \text{ in.}^2) \leq 0.60(50 \text{ ksi})(13.3 \text{ in.}^2) + 1.0(65 \text{ ksi})(0.930 \text{ in.}^2) \\
 &= 404 \text{ kips} < 459 \text{ kips}
 \end{aligned}$$

Therefore:

$$R_n = 404 \text{ kips}$$

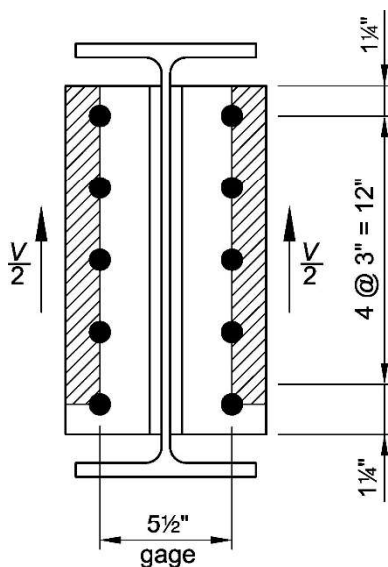


Fig. II.A-2B-2. Block shear rupture of outstanding legs of angles.

The available block shear rupture strength of the angles is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(404 \text{ kips})$ $= 303 \text{ kips} > 75 \text{ kips} \quad \mathbf{o.k.}$	$\Omega = 2.00$ $\frac{R_n}{\Omega} = \frac{404 \text{ kips}}{2.00}$ $= 202 \text{ kips} > 50 \text{ kips} \quad \mathbf{o.k.}$

Shear Strength of Beam

From AISC *Specification* Section J4.2(a), the available shear yield strength of the beam web is determined as follows:

$$\begin{aligned}
 A_{gv} &= dt_w \\
 &= (18.0 \text{ in.})(0.355 \text{ in.}) \\
 &= 6.39 \text{ in.}^2
 \end{aligned}$$

$$\begin{aligned}
 R_n &= 0.60F_y A_{gv} && (\text{Spec. Eq. J4-3}) \\
 &= 0.60(50 \text{ ksi})(6.39 \text{ in.}^2) \\
 &= 192 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(192 \text{ kips})$ $= 192 \text{ kips} > 75 \text{ kips} \quad \mathbf{o.k.}$	$\Omega = 1.50$ $\frac{R_n}{\Omega} = \frac{192 \text{ kips}}{1.50}$ $= 128 \text{ kips} > 50 \text{ kips} \quad \mathbf{o.k.}$

The limit state of shear rupture of the beam web does not apply in this example because the beam is uncoped.

Block Shear Rupture Strength of Beam Web

Assuming a U-shaped tearout along the weld relative to the axial load, and a total beam setback of $\frac{3}{4}$ in. (includes $\frac{1}{4}$ in. tolerance to account for possible mill underrun), the nominal block shear rupture strength is determined as follows.

$$R_n = 0.60F_u A_{nv} + U_{bs}F_u A_{nt} \leq 0.60F_y A_{gv} + U_{bs}F_u A_{nt} \quad (\text{Spec. Eq. J4-5})$$

where

$$\begin{aligned}
 A_{nt} &= lt_w \\
 &= (14\frac{1}{2} \text{ in.})(0.355 \text{ in.}) \\
 &= 5.15 \text{ in.}^2
 \end{aligned}$$

$$\begin{aligned}
 A_{gv} &= (2)(\text{leg} - \text{setback})t_w \\
 &= (2)(3\frac{1}{2} \text{ in.} - \frac{3}{4} \text{ in.})(0.355 \text{ in.}) \\
 &= 1.95 \text{ in.}^2
 \end{aligned}$$

Because the angles are welded and there is no reduction for bolt holes:

$$A_{nv} = A_{gv}$$

$$= 1.95 \text{ in.}^2$$

$$U_{bs} = 1$$

and

$$R_n = 0.60(65 \text{ ksi})(1.95 \text{ in.}^2) + 1.0(65 \text{ ksi})(5.15 \text{ in.}^2) \leq 0.60(50 \text{ ksi})(1.95 \text{ in.}^2) + 1.0(65 \text{ ksi})(5.15 \text{ in.}^2)$$

$$= 411 \text{ kips} > 393 \text{ kips}$$

Therefore:

$$R_n = 393 \text{ kips}$$

The available block shear rupture strength of the web is:

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi R_n = 0.75(393 \text{ kips})$ $= 295 \text{ kips} > 60 \text{ kips} \quad \mathbf{o.k.}$	$\frac{R_n}{\Omega} = \frac{393 \text{ kips}}{2.00}$ $= 197 \text{ kips} > 40 \text{ kips} \quad \mathbf{o.k.}$

Tensile Strength of Beam

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the beam is determined from AISC *Specification* Equation J4-1:

$$R_n = F_y A_g \quad (\text{Spec. Eq. J4-1})$$

$$= (50 \text{ ksi})(14.7 \text{ in.}^2)$$

$$= 735 \text{ kips}$$

The available tensile yielding strength of the beam is:

LRFD	ASD
$\phi = 0.90$	$\Omega = 1.67$
$\phi R_n = 0.90(735 \text{ kips})$ $= 662 \text{ kips} > 60 \text{ kips} \quad \mathbf{o.k.}$	$\frac{R_n}{\Omega} = \frac{735 \text{ kips}}{1.67}$ $= 440 \text{ kips} > 40 \text{ kips} \quad \mathbf{o.k.}$

From AISC *Specification* Section J4.1(b), determine the available tensile rupture strength of the beam. The effective net area is $A_e = A_n U$, where U is determined from AISC *Specification* Table D3.1, Case 2. The value of $\bar{x}$ is determined by treating the W-shape as two channels back-to-back and finding the horizontal distance to the center of gravity of one of the channels from the centerline of the beam. (Note that the fillets are ignored.)

$$\begin{aligned}
\bar{x} &= \frac{\Sigma(A\bar{x})}{\Sigma A} \\
&= \frac{\left(\frac{t_w}{2}\right)(d - 2t_f)\left(\frac{t_w/2}{2}\right) + 2t_f\left(\frac{b_f}{2}\right)\left(\frac{b_f/2}{2}\right)}{\left(\frac{A_g}{2}\right)} \\
&= \frac{\left(\frac{0.355 \text{ in.}}{2}\right)[18.0 \text{ in.} - 2(0.570 \text{ in.})]\left(\frac{0.355 \text{ in.}/2}{2}\right) + 2(0.570 \text{ in.})\left(\frac{7.50 \text{ in.}}{2}\right)\left(\frac{7.50 \text{ in.}/2}{2}\right)}{\left(\frac{14.7 \text{ in.}^2}{2}\right)} \\
&= 1.13 \text{ in.}
\end{aligned}$$

The connection length, l , used in the determination of U will be reduced by $\frac{1}{4}$ in. to account for possible mill underrun. The shear lag factor, U , is:

$$\begin{aligned}
U &= 1 - \frac{\bar{x}}{l} \\
&= 1 - \frac{1.13 \text{ in.}}{(3 \text{ in.} - \frac{1}{4} \text{ in.})} \\
&= 0.589
\end{aligned}$$

The minimum value of U can be determined from AISC *Specification* Section D3, where U is the ratio of the gross area of the connected element to the member gross area.

$$\begin{aligned}
U &= \frac{A_{nt}}{A_g} \\
&= \frac{(d - 2t_f)t_w}{A_g} \\
&= \frac{[18.0 \text{ in.} - 2(0.570 \text{ in.})](0.355 \text{ in.})}{14.7 \text{ in.}^2} \\
&= 0.407
\end{aligned}$$

AISC *Specification* Table D3.1, Case 2, controls; use $U = 0.589$. Because the angles are welded and there is no reduction for bolt holes:

$$\begin{aligned}
A_n &= A_g \\
&= 14.7 \text{ in.}^2
\end{aligned}$$

$$\begin{aligned}
A_e &= A_n U && (\text{Spec. Eq. D3-1}) \\
&= (14.7 \text{ in.}^2)(0.589) \\
&= 8.66 \text{ in.}^2
\end{aligned}$$

$$\begin{aligned}
R_n &= F_u A_e && (\text{Spec. Eq. J4-2}) \\
&= (65 \text{ ksi})(8.66 \text{ in.}^2) \\
&= 563 \text{ kips}
\end{aligned}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(563 \text{ kips})$ $= 422 \text{ kips} > 60 \text{ kips} \quad \mathbf{o.k.}$	$\Omega = 2.00$ $\frac{R_n}{\Omega} = \frac{563 \text{ kips}}{2.00}$ $= 282 \text{ kips} > 40 \text{ kips} \quad \mathbf{o.k.}$

Conclusion

The connection is found to be adequate as given for the applied loads.

EXAMPLE II.A-3 ALL-WELDED DOUBLE-ANGLE CONNECTION**Given:**

Repeat Example II.A-1A using AISC *Manual* Table 10-3 and applicable provisions from the AISC *Specification* to verify the strength of an all-welded double-angle connection between an ASTM A992/A992M W36×231 beam and an ASTM A992/A992M W14×90 column flange, as shown in Figure II.A-3-1. Use 70-ksi electrodes and ASTM A572/A572M Grade 50 angles.

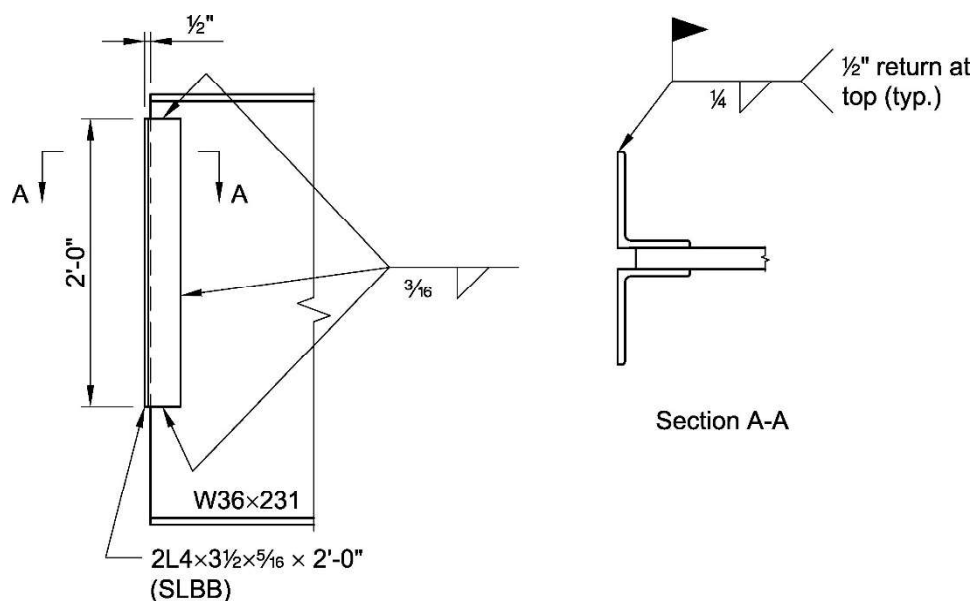


Fig. II.A-3-1. Connection geometry for Example II.A-3.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam and column
 ASTM A992/A992M
 $F_y = 50$ ksi
 $F_u = 65$ ksi

Angles
 ASTM A572/A572M Grade 50
 $F_y = 50$ ksi
 $F_u = 65$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
 W36×231
 $t_w = 0.760$ in.

Column
 W14×90
 $t_f = 0.710$ in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(37.5 \text{ kips}) + 1.6(113 \text{ kips})$ $= 226 \text{ kips}$	$R_a = 37.5 \text{ kips} + 113 \text{ kips}$ $= 151 \text{ kips}$

Design of Weld between Beam Web and Angles

Use AISC *Manual* Table 10-3 (Welds A). Try $\frac{3}{16}$ in. weld size, $l = 24$ in.

LRFD	ASD
$\phi R_n = 257 \text{ kips} > 226 \text{ kips} \quad \text{o.k.}$	$\frac{R_n}{\Omega} = 171 \text{ kips} > 151 \text{ kips} \quad \text{o.k.}$

From AISC *Manual* Table 10-3, the minimum beam web thickness is:

$$t_{w \min} = 0.286 \text{ in.} < 0.760 \text{ in.} \quad \text{o.k.}$$

Design of Weld between Column Flange and Angles

Use AISC *Manual* Table 10-3 (Welds B). Try $\frac{1}{4}$ in. weld size, $l = 24$ in.

LRFD	ASD
$\phi R_n = 260 \text{ kips} > 226 \text{ kips} \quad \text{o.k.}$	$\frac{R_n}{\Omega} = 173 \text{ kips} > 151 \text{ kips} \quad \text{o.k.}$

From AISC *Manual* Table 10-3, the minimum column flange thickness is:

$$t_{f \min} = 0.190 \text{ in.} < 0.710 \text{ in.} \quad \text{o.k.}$$

Angle Thickness

Minimum angle thickness for weld from AISC *Specification* Section J2.2b:

$$\begin{aligned}
 t_{\min} &= w + \frac{1}{16} \text{ in.} \\
 &= \frac{1}{4} \text{ in.} + \frac{1}{16} \text{ in.} \\
 &= \frac{5}{16} \text{ in.}
 \end{aligned}$$

Try 2L4 $\times$ 3 $\frac{1}{2}$ $\times$ $\frac{5}{16}$ (SLBB).

Shear Strength of Angles

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the angles is determined as follows:

$$\begin{aligned}
 A_{gv} &= (2 \text{ angles})lt \\
 &= (2 \text{ angles})(24 \text{ in.})(\frac{5}{16} \text{ in.}) \\
 &= 15.0 \text{ in.}^2
 \end{aligned}$$

$$\begin{aligned}
 R_n &= 0.60F_y A_{gv} && (\text{Spec. Eq. J4-3}) \\
 &= 0.60(50 \text{ ksi})(15.0 \text{ in.}^2) \\
 &= 450 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(450 \text{ kips})$ $= 450 \text{ kips} > 226 \text{ kips} \quad \mathbf{o.k.}$	$\Omega = 1.50$ $\frac{R_n}{\Omega} = \frac{450 \text{ kips}}{1.50}$ $= 300 \text{ kips} > 151 \text{ kips} \quad \mathbf{o.k.}$

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the angles is determined as follows:

$$\begin{aligned}
 A_{nv} &= (2 \text{ angles})lt \\
 &= (2 \text{ angles})(24 \text{ in.})(\frac{5}{16} \text{ in.}) \\
 &= 15.0 \text{ in.}^2
 \end{aligned}$$

$$\begin{aligned}
 R_n &= 0.60F_u A_{nv} && (\text{Spec. Eq. J4-4}) \\
 &= 0.60(65 \text{ ksi})(15.0 \text{ in.}^2) \\
 &= 585 \text{ kips}
 \end{aligned}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(585 \text{ kips})$ $= 439 \text{ kips} > 226 \text{ kips} \quad \mathbf{o.k.}$	$\Omega = 2.00$ $\frac{R_n}{\Omega} = \frac{585 \text{ kips}}{2.00}$ $= 293 \text{ kips} > 151 \text{ kips} \quad \mathbf{o.k.}$

Conclusion

The connection is found to be adequate as given for the applied loads.