DESIGN EXAMPLES

For Use in First Semester
Structural Steel Design Classes

Version 14.0

AMERICAN INSTITUTE
OF
STEEL CONSTRUCTION

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Notes: Design Examples for use in first semester structural steel design course

The AISC *Manual* and *Specifications* committees prepare design examples to illustrate the application of the provisions in the AISC *Specification for Structural Steel Buildings*. The complete set of design examples includes 155 example problems totaling 884 pages, and is a free download that can be found at www.aisc.org/designexamples.

The AISC Partners in Education Committee has condensed the set of Design Examples to include 43 example problems that will be most likely to address topics that are studied in a first semester structural steel design course. These 43 example problems can be found on the following pages.

PREFACE

The primary objective of these design examples is to provide illustrations of the use of the 2010 AISC *Specification for Structural Steel Buildings* (ANSI/AISC 360-10) and the 14th Edition of the AISC *Steel Construction Manual*. The design examples provide coverage of all applicable limit states whether or not a particular limit state controls the design of the member or connection.

In addition to the examples which demonstrate the use of the *Manual* tables, design examples are provided for connection designs beyond the scope of the tables in the Manual. These design examples are intended to demonstrate an approach to the design, and are not intended to suggest that the approach presented is the only approach. The committee responsible for the development of these design examples recognizes that designers have alternate approaches that work best for them and their projects. Design approaches that differ from those presented in these examples are considered viable as long as the *Specification*, sound engineering, and project specific requirements are satisfied.

Part I of these examples is organized to correspond with the organization of the Specification. The Chapter titles match the corresponding chapters in the Specification.

Part II is devoted primarily to connection examples that draw on the tables from the Manual, recommended design procedures, and the breadth of the Specification. The chapters of Part II are labeled II-A, II-B, II-C, etc.

Part III addresses aspects of design that are linked to the performance of a building as a whole. This includes coverage of lateral stability and second order analysis, illustrated through a four-story braced-frame and moment-frame building.

The Design Examples are arranged with LRFD and ASD designs presented side by side, for consistency with the AISC *Manual*. Design with ASD and LRFD are based on the same nominal strength for each element so that the only differences between the approaches are which set of load combinations from ASCE/SEI 7-10 are used for design and whether the resistance factor for LRFD or the safety factor for ASD is used.

CONVENTIONS

The following conventions are used throughout these examples:

1. The 2010 AISC *Specification for Structural Steel Buildings* is referred to as the AISC *Specification* and the 14th Edition AISC *Steel Construction Manual,* is referred to as the AISC *Manual*.

2. The source of equations or tabulated values taken from the AISC *Specification* or AISC *Manual* is noted along the right-hand edge of the page.

3. When the design process differs between LRFD and ASD, the designs equations are presented side-by-side. This rarely occurs, except when the resistance factor, ϕ, and the safety factor, Ω, are applied.

4. The results of design equations are presented to three significant figures throughout these calculations.

ACKNOWLEDGMENTS

The AISC Committee on Manuals reviewed and approved V14.0 of the AISC *Design Examples*:

William A. Thornton, Chairman
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The AISC Committee on Manuals gratefully acknowledges the contributions of the following individuals who assisted in the development of this document: Leigh Arber, Eric Bolin, Janet Cummins, Thomas Dehlin, William Jacobs, Richard C. Kaehler, Margaret Matthew, Heath Mitchell, Thomas J. Schlafly, and Sriramulu Vinnakota.

TABLE OF CONTENTS

PART I. EXAMPLES BASED ON THE AISC SPECIFICATION

Chapter C
Design for Stability

C1. GENERAL STABILITY REQUIREMENTS

The AISC *Specification* requires that the designer account for both the stability of the structural system as a whole, and the stability of individual elements. Thus, the lateral analysis used to assess stability must include consideration of the combined effect of gravity and lateral loads, as well as member inelasticity, out-of-plumbness, out-of-straightness and the resulting second-order effects, P-Δ and P-δ. The effects of "leaning columns" must also be considered, as illustrated in the examples in this chapter and in the four-story building design example in Part III of AISC *Design Examples*.

P-Δ and P-δ effects are illustrated in AISC *Specification Commentary* Figure C-C2.1. Methods for addressing stability, including P-Δ and P-δ effects, are provided in AISC *Specification* Section C2 and Appendix 7.

C2. CALCULATION OF REQUIRED STRENGTHS

The calculation of required strengths is illustrated in the examples in this chapter and in the four-story building design example in Part III of AISC *Design Examples*.

C3. CALCULATION OF AVAILABLE STRENGTHS

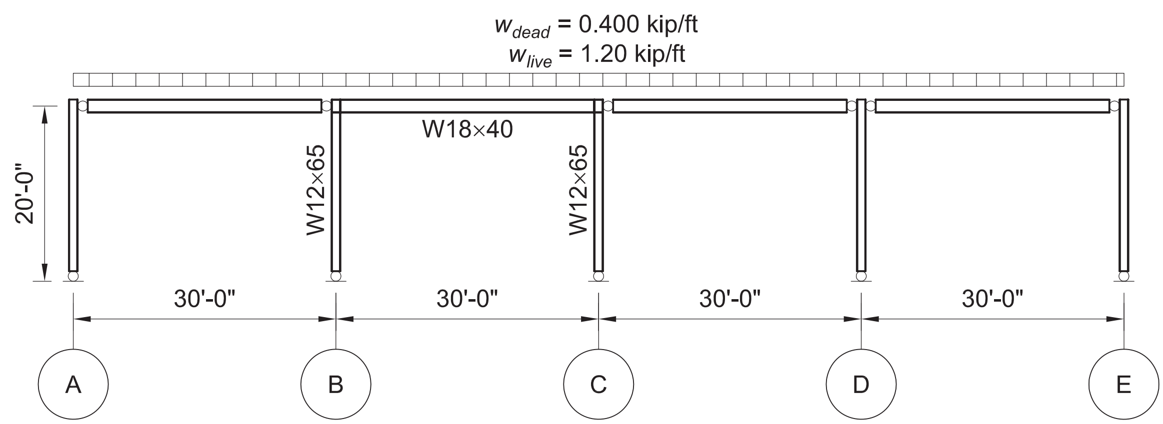
The calculation of available strengths is illustrated in the four-story building design example in Part III of AISC *Design Examples*.

EXAMPLE C.1A DESIGN OF A MOMENT FRAME BY THE DIRECT ANALYSIS METHOD

Given:

Determine the required strengths and effective length factors for the columns in the rigid frame shown below for the maximum gravity load combination, using LRFD and ASD. Use the direct analysis method. All members are ASTM A992 material.

Columns are unbraced between the footings and roof in the x- and y-axes and are assumed to have pinned bases.



Solution:

From *Manual* Table 1-1, the W12×65 has A = 19.1 in.^{2}

The beams from grid lines A to B, and C to E and the columns at A, D and E are pinned at both ends and do not contribute to the lateral stability of the frame. There are no P-Δ effects to consider in these members and they may be designed using K=1.0.

The moment frame between grid lines B and C is the source of lateral stability and therefore must be designed using the provisions of Chapter C of the AISC *Specification*. Although the columns at grid lines A, D and E do not contribute to lateral stability, the forces required to stabilize them must be considered in the analysis. For the analysis, the entire frame could be modeled or the model can be simplified as shown in the figure below, in which the stability loads from the three "leaning" columns are combined into a single column.

From Chapter 2 of ASCE/SEI 7, the maximum gravity load combinations are:

LRFD	ASD
$w_u = 1.2D + 1.6L$ $= 1.2(0.400\text{ kip/ft}) + 1.6(1.20\text{ kip/ft})$ $= 2.40\text{ kip/ft}$	$w_a = D + L$ $= 0.400\text{ kip/ft} + 1.20\text{ kip/ft}$ $= 1.60\text{ kip/ft}$

Per AISC *Specification* Section C2.1, for LRFD perform a second-order analysis and member strength checks using the LRFD load combinations. For ASD, perform a second-order analysis using 1.6 times the ASD load combinations and divide the analysis results by 1.6 for the ASD member strength checks.

Frame Analysis Gravity Loads

The uniform gravity loads to be considered in a second-order analysis on the beam from B to C are:

LRFD	ASD
$w_u{}' = 2.40$ kip/ft	$w_a{}' = 1.6(1.60 \text{ kip/ft})$ $= 2.56$ kip/ft

Concentrated gravity loads to be considered in a second-order analysis on the columns at B and C contributed by adjacent beams are:

LRFD	ASD
$P_u{}' = (15.0 \text{ ft})(2.40 \text{ kip/ft})$ $= 36.0$ kips	$P_a{}' = 1.6(15.0 \text{ ft})(1.60 \text{ kip/ft})$ $= 38.4$ kips

Concentrated Gravity Loads on the Pseudo "Leaning" Column

The load in this column accounts for all gravity loading that is stabilized by the moment frame, but is not directly applied to it.

LRFD	ASD
$P_{uL}{}' = (60.0 \text{ ft})(2.40 \text{ kip/ft})$ $= 144$ kips	$P_{aL}{}' = 1.6(60.0 \text{ ft})(1.60 \text{ kip/ft})$ $= 154$ kips

Frame Analysis Notional Loads

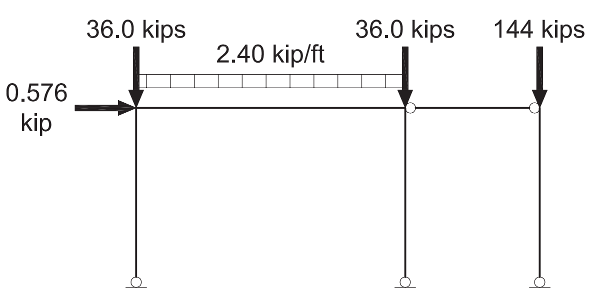
Per AISC *Specification* Section C2.2, frame out-of-plumbness must be accounted for either by explicit modeling of the assumed out-of-plumbness or by the application of notional loads. Use notional loads.

From AISC *Specification* Equation C2-1, the notional loads are:

LRFD	ASD
$\alpha = 1.0$ $Y_i = (120 \text{ ft})(2.40 \text{ kip/ft})$ $= 288$ kips $N_i = 0.002\alpha Y_i$ (*Spec*. Eq. C2-1) $= 0.002(1.0)(288 \text{ kips})$ $= 0.576$ kips	$\alpha = 1.6$ $Y_i = (120 \text{ ft})(1.60 \text{ kip/ft})$ $= 192$ kips $N_i = 0.002\alpha Y_i$ (*Spec*. Eq. C2-1) $= 0.002(1.6)(192 \text{ kips})$ $= 0.614$ kips

Summary of Applied Frame Loads

<table>
<tr><th>LRFD</th><th>ASD</th></tr>
<tr><td>



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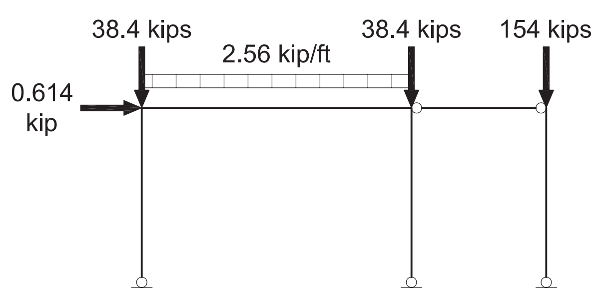


</td></tr>
</table>

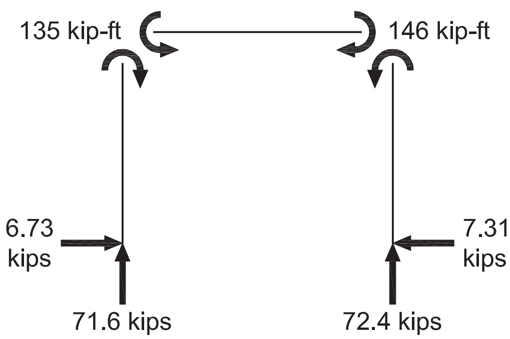
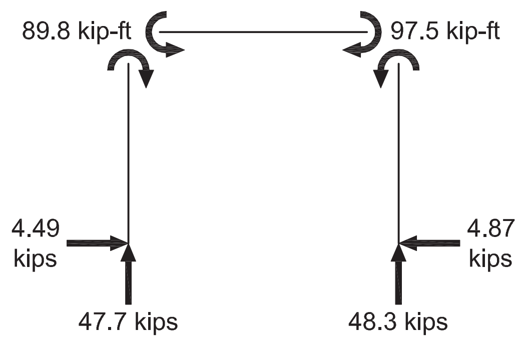
Return to Table of Contents

Per AISC *Specification* Section C2.3, conduct the analysis using 80% of the nominal stiffnesses to account for the effects of inelasticity. Assume, subject to verification, that $\alpha P_r / P_y$ is no greater than 0.5; therefore, no additional stiffness reduction is required.

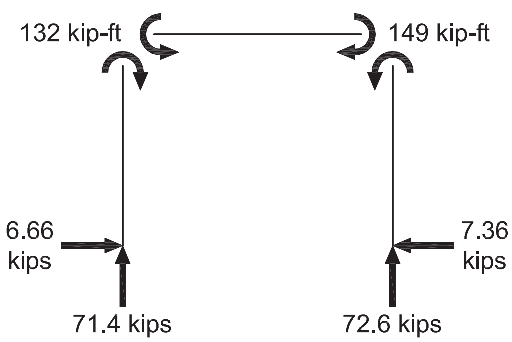
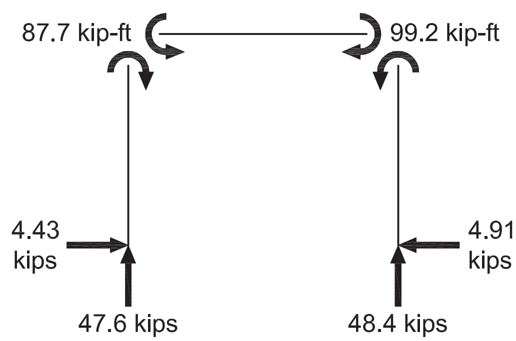
50% of the gravity load is carried by the columns of the moment resisting frame. Because the gravity load supported by the moment resisting frame columns exceeds one third of the total gravity load tributary to the frame, per AISC *Specification* Section C2.1, the effects of P-δ upon P-Δ must be included in the frame analysis. If the software used does not account for P-δ effects in the frame analysis, this may be accomplished by adding joints to the columns between the footing and beam.

Using analysis software that accounts for both P-Δ and P-δ effects, the following results are obtained:

First-order results

LRFD	ASD
$\Delta_{1st} = 0.149$ in.	$\Delta_{1st} = 0.159$ in. (prior to dividing by 1.6)
 	

Second-order results

LRFD	ASD
$\Delta_{2nd} = 0.217$ in. $\dfrac{\Delta_{2nd}}{\Delta_{1st}} = \dfrac{0.217\text{ in.}}{0.149\text{ in.}}$ $= 1.46$	$\Delta_{2nd} = 0.239$ in. (prior to dividing by 1.6) $\dfrac{\Delta_{2nd}}{\Delta_{1st}} = \dfrac{0.239\text{ in.}}{0.159\text{ in.}}$ $= 1.50$
 	

Check the assumption that $\alpha P_r / P_y \leq 0.5$ and therefore, $\tau_b = 1.0$:

$P_y = F_y A_g$
$= 50 \text{ ksi}(19.1 \text{ in.}^2)$
$= 955 \text{ kips}$

LRFD	ASD
$\dfrac{\alpha P_r}{P_y} = \dfrac{1.0(72.6 \text{ kips})}{955 \text{ kips}}$ $= 0.0760 \leq 0.5$ **o.k.**	$\dfrac{\alpha P_r}{P_y} = \dfrac{1.6(48.4 \text{ kips})}{955 \text{ kips}}$ $= 0.0811 \leq 0.5$ **o.k.**

The stiffness assumption used in the analysis, τ_b = 1.0, is verified.

Although the second-order sway multiplier is approximately 1.5, the change in bending moment is small because the only sway moments are those produced by the small notional loads. For load combinations with significant gravity and lateral loadings, the increase in bending moments is larger.

Verify the column strengths using the second-order forces shown above, using the following effective lengths (calculations not shown):

Columns:
Use KL_x = 20.0 ft
Use KL_y = 20.0 ft

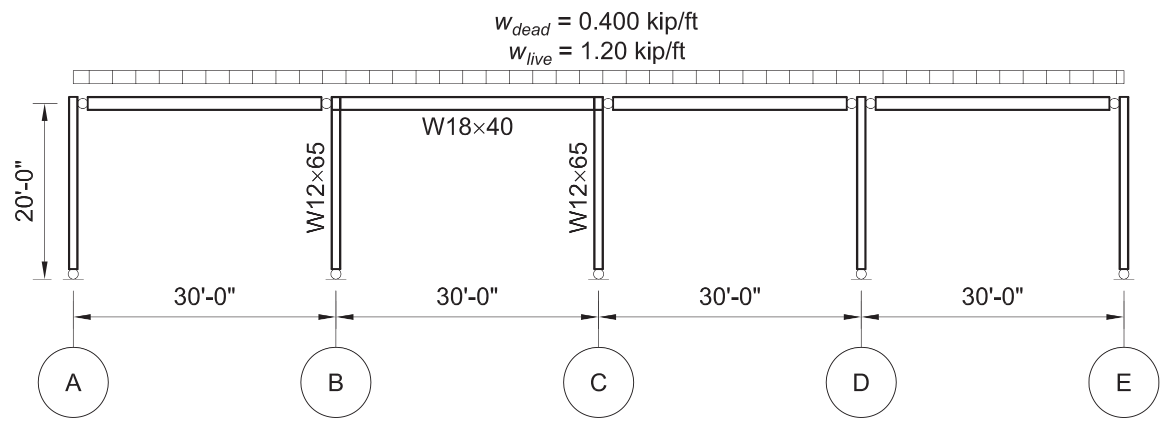
EXAMPLE C.1B DESIGN OF A MOMENT FRAME BY THE EFFECTIVE LENGTH METHOD

Repeat Example C.1A using the effective length method.

Given:

Determine the required strengths and effective length factors for the columns in the rigid frame shown below for the maximum gravity load combination, using LRFD and ASD. Use the effective length method.

Columns are unbraced between the footings and roof in the *x*- and *y*-axes and are assumed to have pinned bases.



Solution:

From *Manual* Table 1-1, the W12×65 has I_x = 533 in.^{4}

The beams from grid lines A to B, and C to E and the columns at A, D and E are pinned at both ends and do not contribute to the lateral stability of the frame. There are no P-Δ effects to consider in these members and they may be designed using K=1.0.

The moment frame between grid lines B and C is the source of lateral stability and therefore must be designed using the provisions of Appendix 7 of the AISC *Specification*. Although the columns at grid lines A, D and E do not contribute to lateral stability, the forces required to stabilize them must be considered in the analysis. For the analysis, the entire frame could be modeled or the model can be simplified as shown in the figure below, in which the stability loads from the three “leaning” columns are combined into a single column.

Check the limitations for the use of the effective length method given in Appendix 7, Section 7.2.1:

(1) The structure supports gravity loads through nominally vertical columns.
(2) The ratio of maximum second-order drift to the maximum first-order drift will be assumed to be no greater than 1.5, subject to verification following.

From Chapter 2 of ASCE/SEI 7, the maximum gravity load combinations are:

LRFD	ASD
$w_u = 1.2D + 1.6L$ $= 1.2(0.400\text{ kip/ft}) + 1.6(1.20\text{ kip/ft})$ $= 2.40\text{ kip/ft}$	$w_a = D + L$ $= 0.400\text{ kip/ft} + 1.20\text{ kip/ft}$ $= 1.60\text{ kip/ft}$

Per AISC *Specification* Appendix 7, Section 7.2.1, the analysis must conform to the requirements of AISC *Specification* Section C2.1, with the exception of the stiffness reduction required by the provisions of Section C2.3.

Per AISC *Specification* Section C2.1, for LRFD perform a second-order analysis and member strength checks using the LRFD load combinations. For ASD, perform a second-order analysis at 1.6 times the ASD load combinations and divide the analysis results by 1.6 for the ASD member strength checks.

Frame Analysis Gravity Loads

The uniform gravity loads to be considered in a second-order analysis on the beam from B to C are:

LRFD	ASD
$w_u' = 2.40$ kip/ft	$w_a' = 1.6(1.60 \text{ kip/ft})$ $= 2.56$ kip/ft

Concentrated gravity loads to be considered in a second-order analysis on the columns at B and C contributed by adjacent beams are:

LRFD	ASD
$P_u' = (15.0 \text{ ft})(2.40 \text{ kip/ft})$ $= 36.0$ kips	$P_a' = 1.6(15.0 \text{ ft})(1.60 \text{ kip/ft})$ $= 38.4$ kips

Concentrated Gravity Loads on the Pseudo "Leaning" Column

The load in this column accounts for all gravity loading that is stabilized by the moment frame, but is not directly applied to it.

LRFD	ASD
$P_{uL}' = (60.0 \text{ ft})(2.40 \text{ kip/ft})$ $= 144$ kips	$P_{aL}' = 1.6(60.0 \text{ ft})(1.60 \text{ kip/ft})$ $= 154$ kips

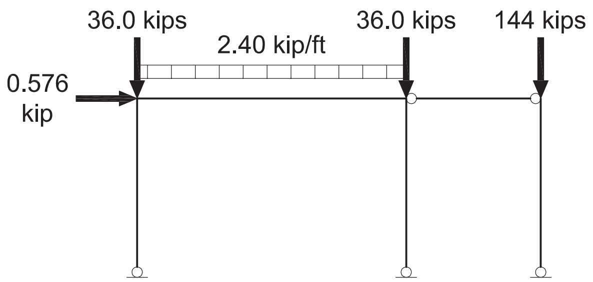
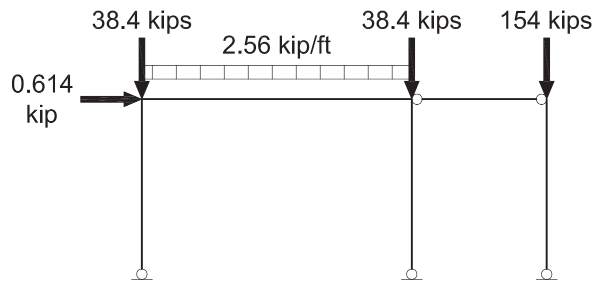
Frame Analysis Notional Loads

Per AISC *Specification* Appendix 7, Section 7.2.2, frame out-of-plumbness must be accounted for by the application of notional loads in accordance with AISC *Specification* Section C2.2b.

From AISC *Specification* Equation C2-1, the notional loads are:

LRFD	ASD
$\alpha = 1.0$ $Y_i = (120 \text{ ft})(2.40 \text{ kip/ft})$ $= 288$ kips $N_i = 0.002\alpha Y_i$ (*Spec.* Eq. C2-1) $= 0.002(1.0)(288 \text{ kips})$ $= 0.576$ kips	$\alpha = 1.6$ $Y_i = (120 \text{ ft})(1.60 \text{ kip/ft})$ $= 192$ kips $N_i = 0.002\alpha Y_i$ (*Spec.* Eq. C2-1) $= 0.002(1.6)(192 \text{ kips})$ $= 0.614$ kips

Summary of Applied Frame Loads

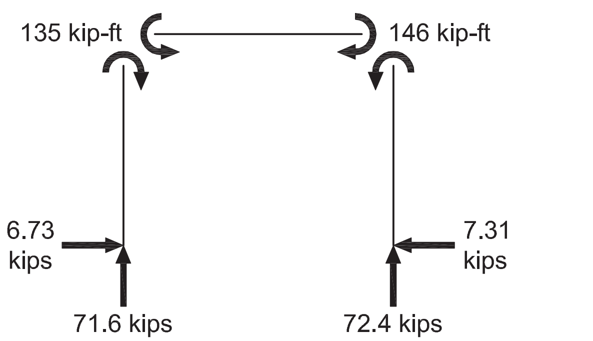
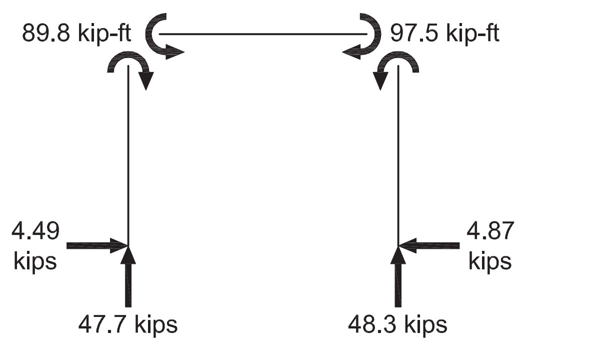
<table>
<tr><th>LRFD</th><th>ASD</th></tr>
<tr><td>
</td><td>
</td></tr>
</table>

Per AISC *Specification* Appendix 7, Section 7.2.2, conduct the analysis using the full nominal stiffnesses.

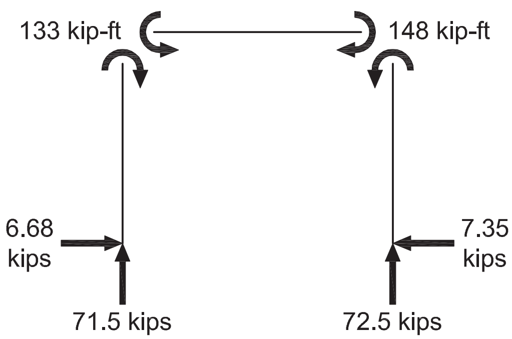
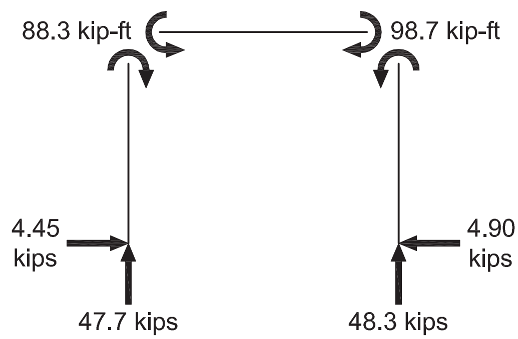
50% of the gravity load is carried by the columns of the moment resisting frame. Because the gravity load supported by the moment resisting frame columns exceeds one third of the total gravity load tributary to the frame, per AISC *Specification* Section C2.1, the effects of P-δ must be included in the frame analysis. If the software used does not account for P-δ effects in the frame analysis, this may be accomplished by adding joints to the columns between the footing and beam.

Using analysis software that accounts for both P-Δ and P-δ effects, the following results are obtained:

First-order results

<table>
<tr><th>LRFD</th><th>ASD</th></tr>
<tr><td>Δ_{1st} = 0.119 in.

</td><td>Δ_{1st} = 0.127 in. (prior to dividing by 1.6)

</td></tr>
</table>

Second-order results

LRFD	ASD
$\Delta_{2nd} = 0.159 \text{ in.}$ $\dfrac{\Delta_{2nd}}{\Delta_{1st}} = \dfrac{0.159 \text{ in.}}{0.119 \text{ in.}}$ $= 1.34$  	$\Delta_{2nd} = 0.174 \text{ in. (prior to dividing by 1.6)}$ $\dfrac{\Delta_{2nd}}{\Delta_{1st}} = \dfrac{0.174 \text{ in.}}{0.127 \text{ in.}}$ $= 1.37$ 

The assumption that the ratio of the maximum second-order drift to the maximum first-order drift is no greater than 1.5 is verified; therefore, the effective length method is permitted.

Although the second-order sway multiplier is approximately 1.35, the change in bending moment is small because the only sway moments for this load combination are those produced by the small notional loads. For load combinations with significant gravity and lateral loadings, the increase in bending moments is larger.

Calculate the in-plane effective length factor, K_x, using the "story stiffness method" and Equation C-A-7-5 presented in Commentary Appendix 7, Section 7.2. Take $K_x = K_2$

$$K_x = K_2 = \sqrt{\frac{\Sigma P_r}{\left(0.85+0.15R_L\right)P_r}\left(\frac{\pi^2 EI}{L^2}\right)\left(\frac{\Delta_H}{\Sigma HL}\right)} \geq \sqrt{\frac{\pi^2 EI}{L^2}\left(\frac{\Delta_H}{1.7HL}\right)} \qquad \text{(}\textit{Spec.}\text{ Eq. C-A-7-5)}$$

Calculate the total load in all columns, ΣP_r

LRFD	ASD
$\Sigma P_r = 2.40 \text{ kip/ft}\left(120 \text{ ft}\right)$ $= 288 \text{ kips}$	$\Sigma P_r = 1.60 \text{ kip/ft}\left(120 \text{ ft}\right)$ $= 192 \text{ kips}$

Calculate the ratio of the leaning column loads to the total load, R_L

LRFD	ASD
$R_L = \dfrac{\Sigma P_r - \Sigma P_{r\ moment\ frame}}{\Sigma P_r}$ $= \dfrac{288 \text{ kips} - \left(71.5 \text{ kips} + 72.5 \text{ kips}\right)}{288 \text{ kips}}$ $= 0.500$	$R_L = \dfrac{\Sigma P_r - \Sigma P_{r\ moment\ frame}}{\Sigma P_r}$ $= \dfrac{192 \text{ kips} - \left(47.7 \text{ kips} + 48.3 \text{ kips}\right)}{192 \text{ kips}}$ $= 0.500$

Calculate the Euler buckling strength of an individual column.

$$\frac{\pi^2 EI}{L^2} = \frac{\pi^2\left(29{,}000\text{ ksi}\right)\left(533\text{ in.}^4\right)}{\left(240\text{ in.}\right)^2}$$
$$= 2{,}650\text{ kips}$$

Calculate the drift ratio using the first-order notional loading results.

LRFD	ASD
$\dfrac{\Delta_H}{L} = \dfrac{0.119\text{ in.}}{240\text{ in.}}$ $= 0.000496\text{ in./in.}$	$\dfrac{\Delta_H}{L} = \dfrac{0.127\text{ in.}}{240\text{ in.}}$ $= 0.000529\text{ in./in.}$

For the column at line C:

LRFD	ASD
$K_x = \sqrt{\dfrac{288\text{ kips}}{\left[0.85+0.15\left(0.500\right)\right]\left(72.4\text{ kips}\right)} \times\left(2{,}650\text{ kips}\right)\left(\dfrac{0.000496\text{ in./in.}}{0.576\text{ kips}}\right)}$ $\geq \sqrt{2{,}650\text{ kips}\left(\dfrac{0.000496\text{ in./in.}}{1.7\left(7.35\text{ kips}\right)}\right)}$ $= 3.13 \geq 0.324$ Use $K_x = 3.13$	$K_x = \sqrt{\dfrac{1.6\left(192\text{ kips}\right)}{\left[0.85+0.15\left(0.500\right)\right]\left(1.6\right)\left(48.3\text{ kips}\right)} \times\left(2{,}650\text{ kips}\right)\left(\dfrac{0.000529\text{ in./in.}}{0.614\text{ kips}}\right)}$ $\geq \sqrt{2{,}650\text{ kips}\left(\dfrac{0.000529\text{ in./in.}}{1.7\left(1.6\right)\left(4.90\text{ kips}\right)}\right)}$ $= 3.13 \geq 0.324$ Use $K_x = 3.13$

Note that it is necessary to multiply the column loads by 1.6 for ASD in the expression above.

Verify the column strengths using the second-order forces shown above, using the following effective lengths (calculations not shown):

Columns:

Use $K_xL_x = 3.13\left(20.0\text{ ft}\right) = 62.6\text{ ft}$

Use $K_yL_y = 20.0$ ft

Chapter D
Design of Members for Tension

D1. SLENDERNESS LIMITATIONS

Section D1 does not establish a slenderness limit for tension members, but recommends limiting L/r to a maximum of 300. This is not an absolute requirement. Rods and hangers are specifically excluded from this recommendation.

D2. TENSILE STRENGTH

Both tensile yielding strength and tensile rupture strength must be considered for the design of tension members. It is not unusual for tensile rupture strength to govern the design of a tension member, particularly for small members with holes or heavier sections with multiple rows of holes.

For preliminary design, tables are provided in Part 5 of the AISC *Manual* for W-shapes, L-shapes, WT-shapes, rectangular HSS, square HSS, round HSS, Pipe and 2L-shapes. The calculations in these tables for available tensile rupture strength assume an effective area, A_e, of $0.75A_g$. If the actual effective area is greater than $0.75A_g$, the tabulated values will be conservative and calculations can be performed to obtain higher available strengths. If the actual effective area is less than $0.75A_g$, the tabulated values will be unconservative and calculations are necessary to determine the available strength.

D3. EFFECTIVE NET AREA

The gross area, A_g, is the total cross-sectional area of the member.

In computing net area, A_n, AISC *Specification* Section B4.3 requires that an extra $1/16$ in. be added to the bolt hole diameter.

A computation of the effective area for a chain of holes is presented in Example D.9.

Unless all elements of the cross section are connected, $A_e = A_nU$, where U is a reduction factor to account for shear lag. The appropriate values of U can be obtained from Table D3.1 of the AISC *Specification*.

D4. BUILT-UP MEMBERS

The limitations for connections of built-up members are discussed in Section D4 of the AISC *Specification*.

D5. PIN-CONNECTED MEMBERS

An example of a pin-connected member is given in Example D.7.

D6. EYEBARS

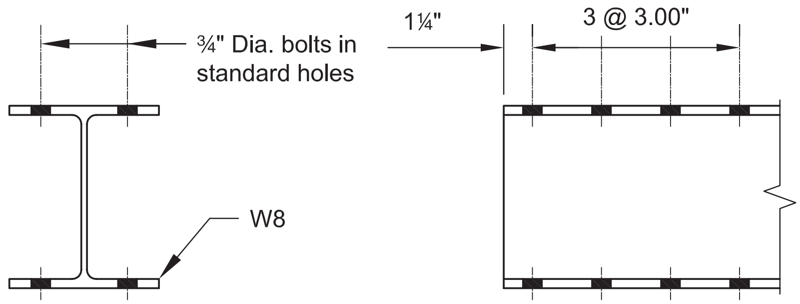
An example of an eyebar is given in Example D.8. The strength of an eyebar meeting the dimensional requirements of AISC *Specification* Section D6 is governed by tensile yielding of the body.

Return to Table of Contents

EXAMPLE D.1 W-SHAPE TENSION MEMBER

Given:

Select an 8-in. W-shape, ASTM A992, to carry a dead load of 30 kips and a live load of 90 kips in tension. The member is 25 ft long. Verify the member strength by both LRFD and ASD with the bolted end connection shown. Verify that the member satisfies the recommended slenderness limit. Assume that connection limit states do not govern.



Solution:

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
P_u = 1.2(30 kips) + 1.6(90 kips) = 180 kips	P_a = 30 kips + 90 kips = 120 kips

From AISC *Manual* Table 5-1, try a W8×21.

From AISC *Manual* Table 2-4, the material properties are as follows:

W8×21
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Tables 1-1 and 1-8, the geometric properties are as follows:

W8×21
A_g = 6.16 in.^{2}
b_f = 5.27 in.
t_f = 0.400 in.
d = 8.28 in.
r_y = 1.26 in.

WT4×10.5
$\bar{y}$ = 0.831 in.

Tensile Yielding

From AISC *Manual* Table 5-1, the tensile yielding strength is:

LRFD	ASD
277 kips > 180 kips **o.k.**	184 kips > 120 kips **o.k.**

Tensile Rupture

Verify the table assumption that $A_e/A_g \geq 0.75$ for this connection.

Calculate the shear lag factor, U, as the larger of the values from AISC *Specification* Section D3, Table D3.1 case 2 and case 7.

From AISC *Specification* Section D3, for open cross sections, U need not be less than the ratio of the gross area of the connected element(s) to the member gross area.

$$
\begin{aligned}
U &= \frac{2b_f t_f}{A_g} \\
&= \frac{2(5.27\text{ in.})(0.400\text{ in.})}{6.16\text{ in.}^2} \\
&= 0.684
\end{aligned}
$$

Case 2: Check as two WT-shapes per AISC *Specification* Commentary Figure C-D3.1, with $\bar{x} = \bar{y} = 0.831$ in.

$$
\begin{aligned}
U &= 1 - \frac{\bar{x}}{l} \\
&= 1 - \frac{0.831\text{ in.}}{9.00\text{ in.}} \\
&= 0.908
\end{aligned}
$$

Case 7:

$b_f = 5.27$ in.
$d = 8.28$ in.
$b_f < \frac{2}{3}d$
$U = 0.85$

Use $U = 0.908$.

Calculate A_n using AISC *Specification* Section B4.3.

$$
\begin{aligned}
A_n &= A_g - 4(d_h + \tfrac{1}{16}\text{ in.})t_f \\
&= 6.16\text{ in.}^2 - 4(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(0.400\text{ in.}) \\
&= 4.76\text{ in.}^2
\end{aligned}
$$

Calculate A_e using AISC *Specification* Section D3.

$$
\begin{aligned}
A_e &= A_n U \qquad (Spec.\text{ Eq. D3-1}) \\
&= 4.76\text{ in.}^2(0.908) \\
&= 4.32\text{ in.}^2
\end{aligned}
$$

$$
\begin{aligned}
\frac{A_e}{A_g} &= \frac{4.32\text{ in.}^2}{6.16\text{ in.}^2} \\
&= 0.701 < 0.75\text{; therefore, table values for rupture are not valid.}
\end{aligned}
$$

The available tensile rupture strength is,

$$P_n = F_u A_e = 65\text{ ksi}(4.32\text{ in.}^2) = 281\text{ kips}$$ (*Spec*. Eq. D2-2)

From AISC *Specification* Section D2, the available tensile rupture strength is:

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(281\text{ kips})$ $= 211\text{ kips}$ 211 kips > 180 kips **o.k.**	$\Omega_t = 2.00$ $\dfrac{P_n}{\Omega_t} = \dfrac{281\text{ kips}}{2.00}$ $= 141\text{ kips}$ 141 kips > 120 kips **o.k.**

Check Recommended Slenderness Limit

$$\frac{L}{r} = \left(\frac{25.0\text{ ft}}{1.26\text{ in.}}\right)\left(\frac{12.0\text{ in.}}{\text{ft}}\right)$$

$= 238 < 300$ from AISC *Specification* Section D1 **o.k.**

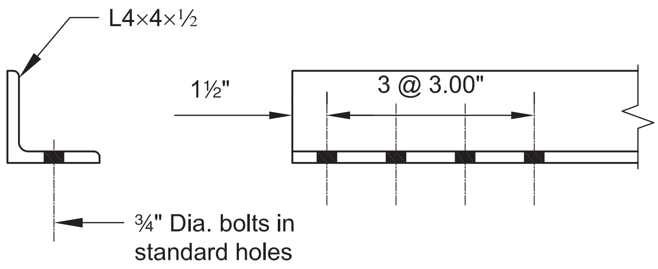
The W8×21 available tensile strength is governed by the tensile rupture limit state at the end connection.

See Chapter J for illustrations of connection limit state checks.

EXAMPLE D.2 SINGLE ANGLE TENSION MEMBER

Given:

Verify, by both ASD and LRFD, the tensile strength of an L4×4×½, ASTM A36, with one line of (4) ¾-in.-diameter bolts in standard holes. The member carries a dead load of 20 kips and a live load of 60 kips in tension. Calculate at what length this tension member would cease to satisfy the recommended slenderness limit. Assume that connection limit states do not govern.



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

L4×4×½
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-7, the geometric properties are as follows:

L4×4×½
A_g = 3.75 in.^{2}
r_z = 0.776 in.
$\bar{y}$ = 1.18 in. = $\bar{x}$

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
P_u = 1.2(20 kips) + 1.6(60 kips) = 120 kips	P_a = 20 kips + 60 kips = 80.0 kips

Tensile Yielding

$$P_n = F_y A_g \qquad (Spec.\ \text{Eq. D2-1})$$
$$= 36 \text{ ksi}(3.75 \text{ in.}^2)$$
$$= 135 \text{ kips}$$

From AISC *Specification* Section D2, the available tensile yielding strength is:

LRFD	ASD
ϕ_t = 0.90 $\phi_t P_n$ = 0.90(135 kips) = 122 kips	Ω_t = 1.67 $\dfrac{P_n}{\Omega_t} = \dfrac{135 \text{ kips}}{1.67}$ = 80.8 kips

Tensile Rupture

Calculate U as the larger of the values from AISC *Specification* Section D3, Table D3.1 Case 2 and Case 8.

From AISC *Specification* Section D3, for open cross sections, U need not be less than the ratio of the gross area of the connected element(s) to the member gross area, therefore,

$$U = 0.500$$

Case 2:

$$\begin{aligned} U &= 1 - \frac{\bar{x}}{l} \\ &= 1 - \frac{1.18\text{ in.}}{9.00\text{ in.}} \\ &= 0.869 \end{aligned}$$

Case 8, with 4 or more fasteners per line in the direction of loading:

$$U = 0.80$$

Use $U = 0.869$.

Calculate A_n using AISC *Specification* Section B4.3.

$$\begin{aligned} A_n &= A_g - (d_h + \tfrac{1}{16})t \\ &= 3.75\text{ in.}^2 - (\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(\tfrac{1}{2}\text{ in.}) \\ &= 3.31\text{ in.}^2 \end{aligned}$$

Calculate A_e using AISC *Specification* Section D3.

$$\begin{aligned} A_e &= A_nU \\ &= 3.31\text{ in.}^2(0.869) \\ &= 2.88\text{ in.}^2 \end{aligned}$$ (*Spec*. Eq. D3-1)

$$\begin{aligned} P_n &= F_uA_e \\ &= 58\text{ ksi}(2.88\text{ in.}^2) \\ &= 167\text{ kips} \end{aligned}$$ (*Spec*. Eq. D2-2)

From AISC *Specification* Section D2, the available tensile rupture strength is:

LRFD	ASD
$\phi_t = 0.75$ $\phi_tP_n = 0.75(167\text{ kips})$ $= 125\text{ kips}$	$\Omega_t = 2.00$ $\dfrac{P_n}{\Omega_t} = \dfrac{167\text{ kips}}{2.00}$ $= 83.5\text{ kips}$

The L4×4×½ available tensile strength is governed by the tensile yielding limit state.

LRFD	ASD
$\phi_tP_n = 122\text{ kips}$ 122 kips > 120 kips **o.k.**	$\dfrac{P_n}{\Omega_t} = 80.8\text{ kips}$ 80.8 kips > 80.0 kips **o.k.**

Recommended L_{max}

Using AISC *Specification* Section D1:

$$
\begin{aligned}
L_{max} &= 300r_z \\
&= (300)(0.776\text{ in.})\left(\frac{\text{ft}}{12.0\text{ in.}}\right) \\
&= 19.4\text{ ft}
\end{aligned}
$$

Note: The L/r limit is a recommendation, not a requirement.

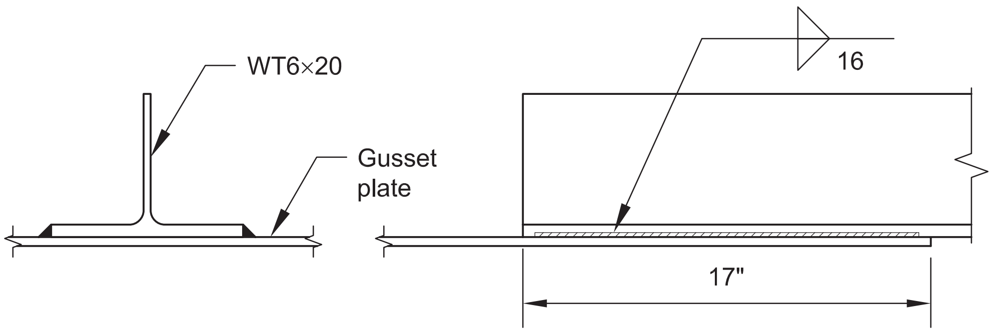
See Chapter J for illustrations of connection limit state checks.

Return to Table of Contents

EXAMPLE D.3 WT-SHAPE TENSION MEMBER

Given:

A WT6×20, ASTM A992 member has a length of 30 ft and carries a dead load of 40 kips and a live load of 120 kips in tension. The end connection is fillet welded on each side for 16 in. Verify the member tensile strength by both LRFD and ASD. Assume that the gusset plate and the weld are satisfactory.



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

WT6×20
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-8, the geometric properties are as follows:

WT6×20
A_g = 5.84 in.^{2}
b_f = 8.01 in.
t_f = 0.515 in.
r_x = 1.57 in.
$\bar{y}$ = 1.09 in. = $\bar{x}$ (in equation for U)

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
P_u = 1.2(40 kips) + 1.6(120 kips) = 240 kips	P_a = 40 kips + 120 kips = 160 kips

Tensile Yielding

Check tensile yielding limit state using AISC *Manual* Table 5-3.

LRFD	ASD
$\phi_t P_n$ = 263 kips > 240 kips **o.k.**	$\dfrac{P_n}{\Omega_t}$ = 175 kips > 160 kips **o.k.**

Tensile Rupture

Check tensile rupture limit state using AISC *Manual* Table 5-3.

LRFD	ASD
$\phi_t P_n = 214 \text{ kips} < 240 \text{ kips}$ **n.g.**	$\dfrac{P_n}{\Omega_t} = 142 \text{ kips} < 160 \text{ kips}$ **n.g.**

The tabulated available rupture strengths may be conservative for this case; therefore, calculate the exact solution.

Calculate U as the larger of the values from AISC *Specification* Section D3 and Table D3.1 case 2.

From AISC *Specification* Section D3, for open cross-sections, U need not be less than the ratio of the gross area of the connected element(s) to the member gross area.

$$
\begin{aligned}
U &= \frac{b_f t_f}{A_g} \\
&= \frac{8.01 \text{ in.}(0.515 \text{ in.})}{5.84 \text{ in.}^2} \\
&= 0.706
\end{aligned}
$$

Case 2:

$$
\begin{aligned}
U &= 1 - \frac{\bar{x}}{l} \\
&= 1 - \frac{1.09 \text{ in.}}{16.0 \text{ in.}} \\
&= 0.932
\end{aligned}
$$

Use $U = 0.932$.

Calculate A_n using AISC *Specification* Section B4.3.

$$
\begin{aligned}
A_n &= A_g \text{ (because there are no reductions due to holes or notches)} \\
&= 5.84 \text{ in.}^2
\end{aligned}
$$

Calculate A_e using AISC *Specification* Section D3.

$$
\begin{aligned}
A_e &= A_n U \\
&= 5.84 \text{ in.}^2(0.932) \\
&= 5.44 \text{ in.}^2
\end{aligned}
$$

(*Spec*. Eq. D3-1)

Calculate P_n.

$$
\begin{aligned}
P_n &= F_u A_e \\
&= 65 \text{ ksi}(5.44 \text{ in.}^2) \\
&= 354 \text{ kips}
\end{aligned}
$$

(*Spec*. Eq. D2-2)

From AISC *Specification* Section D2, the available tensile rupture strength is:

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(354 \text{ kips})$ $= 266 \text{ kips}$ 266 kips > 240 kips **o.k.**	$\Omega_t = 2.00$ $\dfrac{P_n}{\Omega_t} = \dfrac{354 \text{ kips}}{2.00}$ $= 177 \text{ kips}$ 177 kips > 160 kips **o.k.**

Alternately, the available tensile rupture strengths can be determined by modifying the tabulated values. The available tensile rupture strengths published in the tension member selection tables are based on the assumption that $A_e = 0.75A_g$. The actual available strengths can be determined by adjusting the values from AISC *Manual* Table 5-3 as follows:

LRFD	ASD
$\phi_t P_n = 214 \text{ kips}\left(\dfrac{A_e}{0.75A_g}\right)$ $= 214 \text{ kips}\left(\dfrac{5.44 \text{ in.}^2}{0.75\left(5.84 \text{ in.}^2\right)}\right)$ $= 266 \text{ kips}$	$\dfrac{P_n}{\Omega_t} = 142 \text{ kips}\left(\dfrac{A_e}{0.75A_g}\right)$ $= 142 \text{ kips}\left(\dfrac{5.44 \text{ in.}^2}{0.75\left(5.84 \text{ in.}^2\right)}\right)$ $= 176 \text{ kips}$

The WT6×20 available tensile strength is governed by the tensile yielding limit state.

LRFD	ASD
$\phi_t P_n = 263 \text{ kips}$ 263 kips > 240 kips **o.k.**	$\dfrac{P_n}{\Omega_t} = 175 \text{ kips}$ 175 kips > 160 kips **o.k.**

Recommended Slenderness Limit

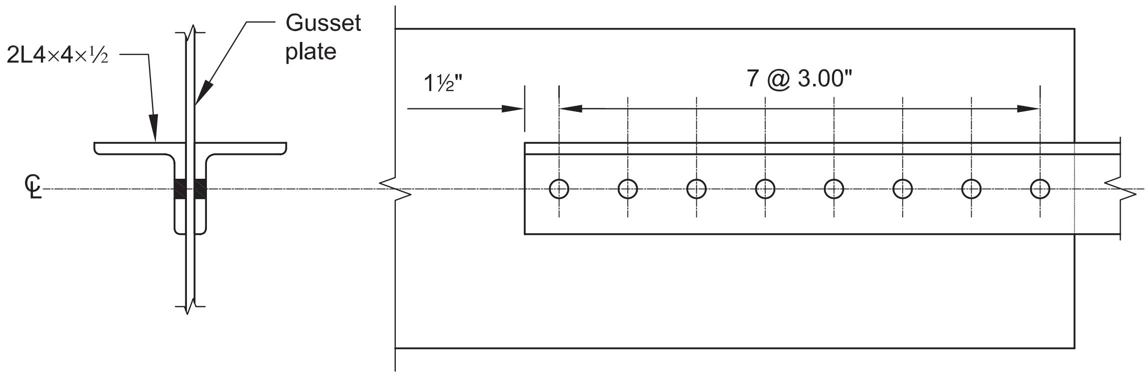
$$\frac{L}{r} = \left(\frac{30.0 \text{ ft}}{1.57 \text{ in.}}\right)\left(\frac{12.0 \text{ in.}}{\text{ft}}\right)$$
$$= 229 < 300 \text{ from AISC } \textit{Specification} \text{ Section D1} \quad \textbf{o.k.}$$

See Chapter J for illustrations of connection limit state checks.

EXAMPLE D.6 DOUBLE ANGLE TENSION MEMBER

Given:

A 2L4×4×½ (⅜-in. separation), ASTM A36, has one line of (8) ¾-in.-diameter bolts in standard holes and is 25 ft in length. The double angle is carrying a dead load of 40 kips and a live load of 120 kips in tension. Verify the member tensile strength. Assume that the gusset plate and bolts are satisfactory.



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-7 and 1-15, the geometric properties are as follows:

L4×4×½
A_g = 3.75 in.^{2}
$\bar{x}$ = 1.18 in.

2L4×4×½ (s = ⅜ in.)
r_y = 1.83 in.
r_x = 1.21 in.

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
P_n = 1.2(40 kips) + 1.6(120 kips) = 240 kips	P_n = 40 kips + 120 kips = 160 kips

Tensile Yielding

$P_n = F_yA_g$ (*Spec*. Eq. D2-1)
= 36 ksi(2)(3.75 in.^{2})
= 270 kips

From AISC *Specification* Section D2, the available tensile yielding strength is:

LRFD	ASD
$\phi_t = 0.90$ $\phi_t P_n = 0.90(270 \text{ kips})$ $= 243 \text{ kips}$	$\Omega_t = 1.67$ $\dfrac{P_n}{\Omega_t} = \dfrac{270 \text{ kips}}{1.67}$ $= 162 \text{ kips}$

Tensile Rupture

Calculate U as the larger of the values from AISC *Specification* Section D3, Table D3.1 case 2 and case 8.

From AISC *Specification* Section D3, for open cross-sections, U need not be less than the ratio of the gross area of the connected element(s) to the member gross area.

$$U = 0.500$$

Case 2:

$$\begin{aligned} U &= 1 - \frac{\bar{x}}{l} \\ &= 1 - \frac{1.18 \text{ in.}}{21.0 \text{ in.}} \\ &= 0.944 \end{aligned}$$

Case 8, with 4 or more fasteners per line in the direction of loading:

$$U = 0.80$$

Use $U = 0.944$.

Calculate A_n using AISC *Specification* Section B4.3.

$$\begin{aligned} A_n &= A_g - 2(d_h + \tfrac{1}{16} \text{ in.})t \\ &= 2(3.75 \text{ in.}^2) - 2(\tfrac{13}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(\tfrac{1}{2} \text{ in.}) \\ &= 6.63 \text{ in.}^2 \end{aligned}$$

Calculate A_e using AISC *Specification* Section D3.

$$\begin{aligned} A_e &= A_n U \\ &= 6.63 \text{ in.}^2(0.944) \\ &= 6.26 \text{ in.}^2 \end{aligned}$$ (*Spec*. Eq. D3-1)

Calculate P_n.

$$\begin{aligned} P_n &= F_u A_e \\ &= 58 \text{ ksi}(6.26 \text{ in.}^2) \\ &= 363 \text{ kips} \end{aligned}$$ (*Spec*. Eq. D2-2)

From AISC *Specification* Section D2, the available tensile rupture strength is:

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(363 \text{ kips})$ $= 272 \text{ kips}$	$\Omega_t = 2.00$ $\dfrac{P_n}{\Omega_t} = \dfrac{363 \text{ kips}}{2.00}$ $= 182 \text{ kips}$

The double angle available tensile strength is governed by the tensile yielding limit state.

LRFD	ASD
243 kips > 240 kips **o.k.**	162 kips > 160 kips **o.k.**

Recommended Slenderness Limit

$$\frac{L}{r_x} = \left(\frac{25.0 \text{ ft}}{1.21 \text{ in.}}\right)\left(\frac{12.0 \text{ in.}}{\text{ft}}\right)$$
$= 248 < 300$ from AISC *Specification* Section D1 **o.k.**

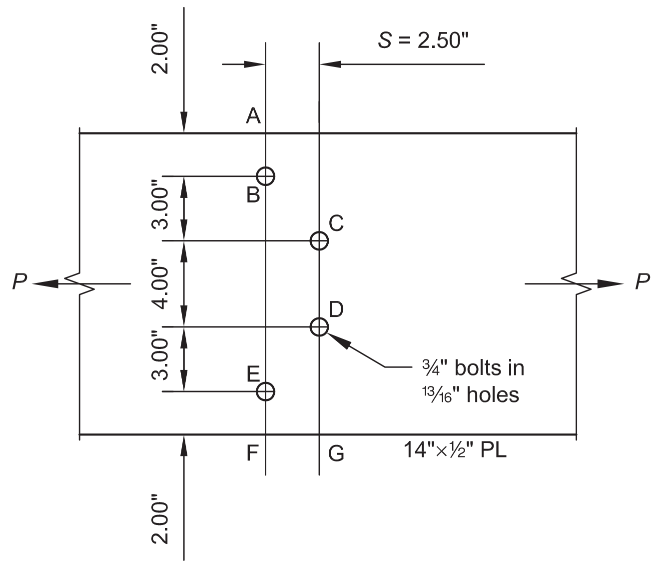
Note: From AISC *Specification* Section D4, the longitudinal spacing of connectors between components of built-up members should preferably limit the slenderness ratio in any component between the connectors to a maximum of 300.

See Chapter J for illustrations of connection limit state checks.

EXAMPLE D.9 PLATE WITH STAGGERED BOLTS

Given:

Compute A_n and A_e for a 14-in.-wide and ½-in.-thick plate subject to tensile loading with staggered holes as shown.



Solution:

Calculate net hole diameter using AISC *Specification* Section B4.3.

$$
\begin{aligned}
d_{net} &= d_h + \tfrac{1}{16} \text{ in.} \\
&= \tfrac{13}{16} \text{ in.} + \tfrac{1}{16} \text{ in.} \\
&= 0.875 \text{ in.}
\end{aligned}
$$

Compute the net width for all possible paths across the plate. Because of symmetry, many of the net widths are identical and need not be calculated.

$w = 14.0 - \Sigma d_{net} + \Sigma \dfrac{s^2}{4g}$ from AISC *Specification* Section B4.3.

Line A-B-E-F: $w = 14.0 \text{ in.} - 2(0.875 \text{ in.})$
$= 12.3 \text{ in.}$

Line A-B-C-D-E-F: $w = 14.0 \text{ in.} - 4(0.875 \text{ in.}) + \dfrac{(2.50 \text{ in.})^2}{4(3.00 \text{ in.})} + \dfrac{(2.50 \text{ in.})^2}{4(3.00 \text{ in.})}$

$= 11.5 \text{ in.}$ **controls**

Line A-B-C-D-G: $w = 14.0 \text{ in.} - 3(0.875 \text{ in.}) + \dfrac{(2.50 \text{ in.})^2}{4(3.00 \text{ in.})}$

$= 11.9 \text{ in.}$

Line A-B-D-E-F: $w = 14.0 \text{ in.} - 3(0.875 \text{ in.}) + \dfrac{(2.50 \text{ in.})^2}{4(7.00 \text{ in.})} + \dfrac{(2.50 \text{ in.})^2}{4(3.00 \text{ in.})}$

$= 12.1 \text{ in.}$

Therefore, $A_n = 11.5 \text{ in.}(0.500 \text{ in.})$

$= 5.75 \text{ in.}^2$

Calculate U.

From AISC *Specification* Table D3.1 case 1, because tension load is transmitted to all elements by the fasteners,

$U = 1.0$

$$\begin{aligned} A_e &= A_n U \\ &= 5.75 \text{ in.}^2(1.0) \\ &= 5.75 \text{ in.}^2 \end{aligned} \qquad (\textit{Spec}. \text{ Eq. D3-1})$$

Chapter E
Design of Members for Compression

This chapter covers the design of compression members, the most common of which are columns. The AISC *Manual* includes design tables for the following compression member types in their most commonly available grades.

- wide-flange column shapes
- HSS
- double angles
- single angles

LRFD and ASD information is presented side-by-side for quick selection, design or verification. All of the tables account for the reduced strength of sections with slender elements.

The design and selection method for both LRFD and ASD designs is similar to that of previous AISC *Specifications*, and will provide similar designs. In this AISC *Specification*, ASD and LRFD will provide identical designs when the live load is approximately three times the dead load.

The design of built-up shapes with slender elements can be tedious and time consuming, and it is recommended that standard rolled shapes be used, when possible.

E1. GENERAL PROVISIONS

The design compressive strength, $\phi_c P_n$, and the allowable compressive strength, P_n/Ω_c, are determined as follows:

P_n = nominal compressive strength based on the controlling buckling mode

$\phi_c = 0.90$ (LRFD) $\qquad \Omega_c = 1.67$ (ASD)

Because F_{cr} is used extensively in calculations for compression members, it has been tabulated in AISC *Manual* Table 4-22 for all of the common steel yield strengths.

E2. EFFECTIVE LENGTH

In the AISC *Specification*, there is no limit on slenderness, KL/r. Per the AISC *Specification* Commentary, it is recommended that KL/r not exceed 200, as a practical limit based on professional judgment and construction economics.

Although there is no restriction on the unbraced length of columns, the tables of the AISC *Manual* are stopped at common or practical lengths for ordinary usage. For example, a double L3×3×¼, with a ⅜-in. separation has an r_y of 1.38 in. At a KL/r of 200, this strut would be 23'-0" long. This is thought to be a reasonable limit based on fabrication and handling requirements.

Throughout the AISC *Manual*, shapes that contain slender elements when supplied in their most common material grade are footnoted with the letter "c". For example, see a W14×22[c].

E3. FLEXURAL BUCKLING OF MEMBERS WITHOUT SLENDER ELEMENTS

Return to Table of Contents

Nonslender sections, including nonslender built-up I-shaped columns and nonslender HSS columns, are governed by these provisions. The general design curve for critical stress versus KL/r is shown in Figure E-1.

The term L is used throughout this chapter to describe the length between points that are braced against lateral and/or rotational displacement.

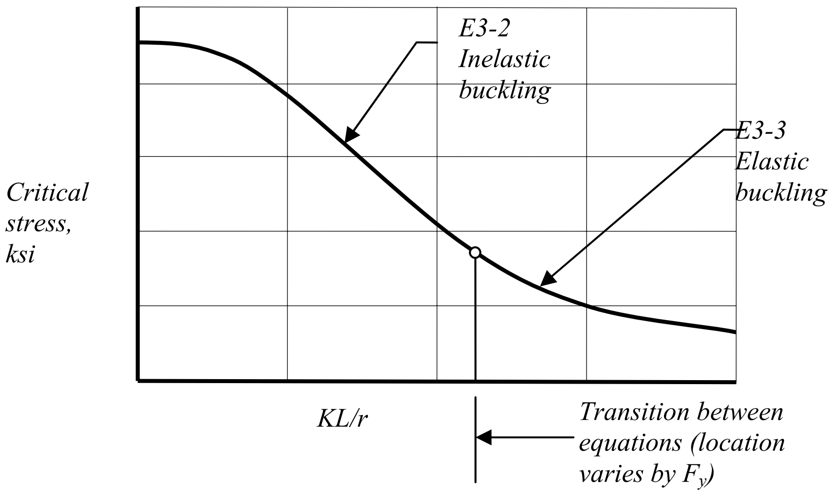


Fig. E-1. Standard column curve.

TRANSITION POINT LIMITING VALUES OF KL/r		
F_y, ksi	Limiting KL/r	$0.44F_y$, ksi
36	134	15.8
50	113	22.0
60	104	26.4
70	96	30.8

E4. TORSIONAL AND FLEXURAL-TORSIONAL BUCKLING OF MEMBERS WITHOUT SLENDER ELEMENTS

This section is most commonly applicable to double angles and WT sections, which are singly-symmetric shapes subject to torsional and flexural-torsional buckling. The available strengths in axial compression of these shapes are tabulated in Part 4 of the AISC *Manual* and examples on the use of these tables have been included in this chapter for the shapes with $KL_z = KL_y$.

E5. SINGLE ANGLE COMPRESSION MEMBERS

The available strength of single angle compression members is tabulated in Part 4 of the AISC *Manual*.

E6. BUILT-UP MEMBERS

There are no tables for built-up shapes in the AISC *Manual*, due to the number of possible geometries. This section suggests the selection of built-up members without slender elements, thereby making the analysis relatively straightforward.

E7. MEMBERS WITH SLENDER ELEMENTS

The design of these members is similar to members without slender elements except that the formulas are modified by a reduction factor for slender elements, Q. Note the similarity of Equation E7-2 with Equation E3-2, and the similarity of Equation E7-3 with Equation E3-3.

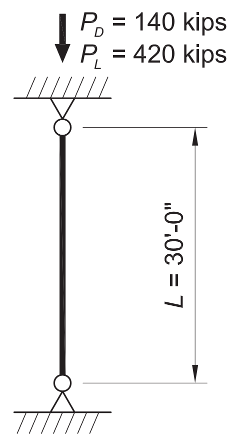
The tables of Part 4 of the AISC *Manual* incorporate the appropriate reductions in available strength to account for slender elements.

Design examples have been included in this Chapter for built-up I-shaped members with slender webs and slender flanges**.** Examples have also been included for a double angle, WT and an HSS shape with slender elements.

EXAMPLE E.1A W-SHAPE COLUMN DESIGN WITH PINNED ENDS

Given:

Select an ASTM A992 (F_y = 50 ksi) W-shape column to carry an axial dead load of 140 kips and live load of 420 kips. The column is 30 ft long and is pinned top and bottom in both axes. Limit the column size to a nominal 14-in. shape.



Solution:

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

LRFD	ASD
P_u = 1.2(140 kips) + 1.6(420 kips) = 840 kips	P_a = 140 kips + 420 kips = 560 kips

Column Selection

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0.

Because the unbraced length is the same in both the x-x and y-y directions and r_x exceeds r_y for all W-shapes, y-y axis bucking will govern.

Enter the table with an effective length, KL_y, of 30 ft, and proceed across the table until reaching the least weight shape with an available strength that equals or exceeds the required strength. Select a W14×132.

Return to Table of Contents

F_y = 50 ksi

Table 4-1 (continued)
Available Strength in Axial Compression, kips
W-Shapes

W14

Shape	W14×											
lb/ft	145		132		120		109		99		90	
Design	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
0	1280	1920	1160	1750	1060	1590	958	1440	871	1310	793	1190
6	1250	1880	1130	1700	1030	1550	932	1400	848	1270	772	1160
7	1240	1860	1120	1680	1020	1530	923	1390	839	1260	764	1150
8	1230	1840	1110	1660	1010	1510	913	1370	830	1250	755	1140
9	1210	1820	1090	1640	994	1490	901	1350	819	1230	745	1120
10	1200	1800	1080	1620	980	1470	888	1340	807	1210	735	1100
11	1180	1770	1060	1600	965	1450	874	1310	794	1190	723	1090
12	1160	1750	1040	1570	948	1430	859	1290	780	1170	710	1070
13	1140	1720	1020	1540	931	1400	843	1270	766	1150	697	1050
14	1120	1690	1000	1510	912	1370	826	1240	750	1130	682	1030
15	1100	1650	982	1480	892	1340	808	1210	733	1100	667	1000
16	1080	1620	960	1440	872	1310	789	1190	716	1080	652	979
17	1060	1590	937	1410	850	1280	770	1160	698	1050	635	955
18	1030	1550	913	1370	828	1240	750	1130	680	1020	618	929
19	1010	1510	888	1330	805	1210	729	1100	661	994	601	903
20	980	1470	862	1300	782	1180	708	1060	642	964	583	877
22	927	1390	810	1220	734	1100	664	998	602	904	547	822
24	872	1310	756	1140	685	1030	620	931	561	843	509	766
26	816	1230	702	1060	635	955	574	863	519	781	472	709
28	759	1140	648	974	586	880	529	796	478	719	434	653
30	703	1060	594	893	537	807	485	729	438	658	397	597

length, *KL* (ft), with respect to least radius of gyration, r_y

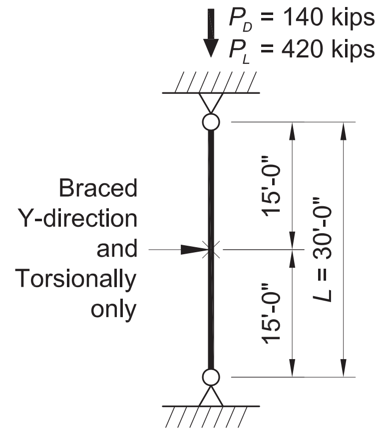
From AISC *Manual* Table 4-1, the available strength for a *y-y* axis effective length of 30 ft is:

LRFD	ASD
$\phi_c P_n$ = 893 kips > 840 kips **o.k.**	$\dfrac{P_n}{\Omega_c}$ = 594 kips > 560 kips **o.k.**

EXAMPLE E.1B W-SHAPE COLUMN DESIGN WITH INTERMEDIATE BRACING

Given:

Redesign the column from Example E.1A assuming the column is laterally braced about the *y-y* axis and torsionally braced at the midpoint.



Solution:

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

LRFD	ASD
P_u = 1.2(140 kips) + 1.6(420 kips) = 840 kips	P_a = 140 kips + 420 kips = 560 kips

Column Selection

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0.

Because the unbraced lengths differ in the two axes, select the member using the *y-y* axis then verify the strength in the *x-x* axis.

Enter AISC *Manual* Table 4-1 with a *y-y* axis effective length, KL_y, of 15 ft and proceed across the table until reaching a shape with an available strength that equals or exceeds the required strength. Try a W14×90. A 15 ft long W14×90 provides an available strength in the *y-y* direction of:

LRFD	ASD
$\phi_c P_n$ = 1,000 kips	$\dfrac{P_n}{\Omega_c}$ = 667 kips

The r_x/r_y ratio for this column, shown at the bottom of AISC *Manual* Table 4-1, is 1.66. The equivalent *y-y* axis effective length for strong axis buckling is computed as:

$$KL = \frac{30.0\text{ ft}}{1.66}$$
$$= 18.1\text{ ft}$$

Return to Table of Contents

F_y = 50 ksi

Table 4-1 (continued)
Available Strength in Axial Compression, kips
W-Shapes

W14

Shape		W14×											
lb/ft		145		132		120		109		99		90	
Design		P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
length, *KL* (ft), with respect to least radius of gyration, r_y	0	1280	1920	1160	1750	1060	1590	958	1440	871	1310	793	1190
	6	1250	1880	1130	1700	1030	1550	932	1400	848	1270	772	1160
	7	1240	1860	1120	1680	1020	1530	923	1390	839	1260	764	1150
	8	1230	1840	1110	1660	1010	1510	913	1370	830	1250	755	1140
	9	1210	1820	1090	1640	994	1490	901	1350	819	1230	745	1120
	10	1200	1800	1080	1620	980	1470	888	1340	807	1210	735	1100
	11	1180	1770	1060	1600	965	1450	874	1310	794	1190	723	1090
	12	1160	1750	1040	1570	948	1430	859	1290	780	1170	710	1070
	13	1140	1720	1020	1540	931	1400	843	1270	766	1150	697	1050
	14	1120	1690	1000	1510	912	1370	826	1240	750	1130	682	1030
	15	1100	1650	982	1480	892	1340	808	1210	733	1100	667	1000
	16	1080	1620	960	1440	872	1310	789	1190	716	1080	652	979
	17	1060	1590	937	1410	850	1280	770	1160	698	1050	635	955
	18	1030	1550	913	1370	828	1240	750	1130	680	1020	618	929
	19	1010	1510	888	1330	805	1210	729	1100	661	994	601	903
	20	980	1470	862	1300	782	1180	708	1060	642	964	583	877
	22	927	1390	810	1220	734	1100	664	998	602	904	547	822
	24	872	1310	756	1140	685	1030	620	931	561	843	509	766
	26	816	1230	702	1060	635	955	574	863	519	781	472	709
	28	759	1140	648	974	586	880	529	796	478	719	434	653
	30	703	1060	594	893	537	807	485	729	438	658	397	597

From AISC *Manual* Table 4-1, the available strength of a W14×90 with an effective length of 18 ft is:

LRFD	ASD
$\phi_c P_n = 929 \text{ kips} > 840 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 618 \text{ kips} > 560 \text{ kips}$ **o.k.**

The available compressive strength is governed by the *x-x* axis flexural buckling limit state.

The available strengths of the columns described in Examples E.1A and E.1B are easily selected directly from the AISC *Manual* Tables. The available strengths can also be verified by hand calculations, as shown in the following Examples E.1C and E.1D.

EXAMPLE E.1C W-SHAPE AVAILABLE STRENGTH CALCULATION

Given:

Calculate the available strength of a W14×132 column with unbraced lengths of 30 ft in both axes. The material properties and loads are as given in Example E.1A.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W14×132
A_g = 38.8 in.^{2}
r_x = 6.28 in.
r_y = 3.76 in.

Slenderness Check

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0.

Because the unbraced length is the same for both axes, the y-y axis will govern.

$$\frac{K_y L_y}{r_y} = \left(\frac{1.0(30.0 \text{ ft})}{3.76 \text{ in.}}\right)\left(\frac{12.0 \text{ in}}{\text{ft}}\right)$$
$$= 95.7$$

For F_y = 50 ksi, the available critical stresses, $\phi_c F_{cr}$ and F_{cr}/Ω_c for KL/r = 95.7 are interpolated from AISC *Manual* Table 4-22 as follows:

LRFD	ASD
$\phi_c F_{cr} = 23.0 \text{ ksi}$ $\phi_c P_n = 38.8 \text{ in.}^2 (23.0 \text{ ksi})$ $= 892 \text{ kips} > 840 \text{ kips}$ **o.k.**	$\dfrac{F_{cr}}{\Omega_c} = 15.4 \text{ ksi}$ $\dfrac{P_n}{\Omega_c} = 38.8 \text{ in.}^2 (15.4 \text{ ksi})$ $= 598 \text{ kips} > 560 \text{ kips}$ **o.k.**

Note that the calculated values are approximately equal to the tabulated values.

EXAMPLE E.1D W-SHAPE AVAILABLE STRENGTH CALCULATION

Given:

Calculate the available strength of a W14×90 with a strong axis unbraced length of 30.0 ft and weak axis and torsional unbraced lengths of 15.0 ft. The material properties and loads are as given in Example E.1A.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W14×90
A_g = 26.5 in.^{2}
r_x = 6.14 in.
r_y = 3.70 in.

Slenderness Check

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0.

$$\frac{KL_x}{r_x} = \frac{1.0(30.0\text{ ft})}{6.14\text{ in.}}\left(\frac{12\text{ in.}}{\text{ft}}\right)$$
$$= 58.6 \qquad \textbf{governs}$$
$$\frac{KL_y}{r_y} = \frac{1.0(15.0\text{ ft})}{3.70\text{ in.}}\left(\frac{12\text{ in.}}{\text{ft}}\right)$$
$$= 48.6$$

Critical Stresses

The available critical stresses may be interpolated from AISC *Manual* Table 4-22 or calculated directly as follows:

Calculate the elastic critical buckling stress, F_e.

$$F_e = \frac{\pi^2 E}{\left(\dfrac{KL}{r}\right)^2} \qquad \text{(}\textit{Spec}\text{. Eq. E3-4)}$$
$$= \frac{\pi^2(29{,}000\text{ ksi})}{(58.6)^2}$$
$$= 83.3\text{ ksi}$$

Calculate the flexural buckling stress, F_{cr}.

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}}$$
$$= 113$$

Because $\dfrac{KL}{r} = 58.6 \le 113,$

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right]F_y \qquad \text{(}\textit{Spec}\text{. Eq. E3-2)}$$
$$= \left[0.658^{\frac{50.0\text{ ksi}}{83.3\text{ ksi}}}\right]50.0\text{ ksi}$$
$$= 38.9\text{ ksi}$$

Nominal Compressive Strength

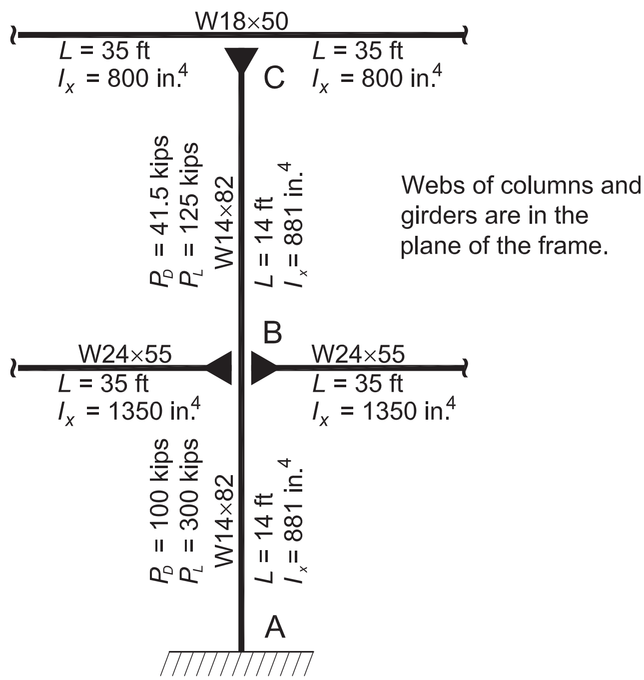
$$P_n = F_{cr}A_g \qquad \text{(}\textit{Spec}\text{. Eq. E3-1)}$$
$$= 38.9\text{ ksi}\left(26.5\text{ in.}^2\right)$$
$$= 1{,}030\text{ kips}$$

From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(1{,}030\text{ kips})$ $= 927\text{ kips} > 840\text{ kips}$ **o.k.**	$\Omega_c = 1.67$ $\dfrac{P_n}{\Omega_c} = \dfrac{1{,}030\text{ kips}}{1.67}$ $= 617\text{ kips} > 560\text{ kips}$ **o.k.**

EXAMPLE E.4A W-SHAPE COMPRESSION MEMBER (MOMENT FRAME)

This example is primarily intended to illustrate the use of the alignment chart for sidesway uninhibited columns in conjunction with the effective length method.



Given:

The member sizes shown for the moment frame illustrated here (sidesway uninhibited in the plane of the frame) have been determined to be adequate for lateral loads. The material for both the column and the girders is ASTM A992. The loads shown at each level are the accumulated dead loads and live loads at that story. The column is fixed at the base about the *x-x* axis of the column.

Determine if the column is adequate to support the gravity loads shown. Assume the column is continuously supported in the transverse direction (the *y-y* axis of the column).

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W18×50
I_x = 800 in.^{4}

W24×55
I_x = 1,350 in.^{4}

W14×82
A_g = 24.0 in.^{2}
I_x = 881 in.^{4}

Column B-C

From Chapter 2 of ASCE/SEI 7, the required compressive strength for the column between the roof and floor is:

LRFD	ASD
$P_u = 1.2(41.5 \text{ kips}) + 1.6(125 \text{ kips})$ $= 250 \text{ kips}$	$P_a = 41.5 + 125$ $= 167 \text{ kips}$

Effective Length Factor

Calculate the stiffness reduction parameter, τ_b, using AISC *Manual* Table 4-21.

LRFD	ASD
$\frac{P_u}{A_g} = \frac{250 \text{ kips}}{24.0 \text{ in.}^2}$ $= 10.4 \text{ ksi}$ $\tau_b = 1.00$	$\frac{P_a}{A_g} = \frac{167 \text{ kips}}{24.0 \text{ in.}^2}$ $= 6.96 \text{ ksi}$ $\tau_b = 1.00$

Therefore, no reduction in stiffness for inelastic buckling will be required.

Determine G_{top} and G_{bottom}.

$$G_{top} = \tau \frac{\sum(E_c I_c / L_c)}{\sum(E_g I_g / L_g)} \quad \text{(from \textit{Spec.} Comm. Eq. C-A-7-3)}$$

$$= (1.00)\frac{29{,}000 \text{ ksi}\left(\dfrac{881 \text{ in.}^4}{14.0 \text{ ft}}\right)}{2(29{,}000 \text{ ksi})\left(\dfrac{800 \text{ in.}^4}{35.0 \text{ ft}}\right)}$$

$$= 1.38$$

$$G_{bottom} = \tau \frac{\sum(E_c I_c / L_c)}{\sum(E_g I_g / L_g)} \quad \text{(from \textit{Spec.} Comm. Eq. C-A-7-3)}$$

$$= (1.00)\frac{2(29{,}000 \text{ ksi})\left(\dfrac{881 \text{ in.}^4}{14.0 \text{ ft}}\right)}{2(29{,}000 \text{ ksi})\left(\dfrac{1{,}350 \text{ in.}^4}{35.0 \text{ ft}}\right)}$$

$$= 1.63$$

From the alignment chart, AISC *Specification* Commentary Figure C-A-7.2, K is slightly less than 1.5; therefore use K = 1.5. Because the column available strength tables are based on the KL about the y-y axis, the equivalent effective column length of the upper segment for use in the table is:

$$KL = \frac{(KL)_x}{\left(\dfrac{r_x}{r_y}\right)}$$

$$= \frac{1.5(14.0 \text{ ft})}{2.44}$$

$$= 8.61 \text{ ft}$$

Take the available strength of the W14×82 from AISC *Manual* Table 4-1.

At KL = 9 ft, the available strength in axial compression is:

Return to Table of Contents

LRFD	ASD
$\phi_c P_n = 940 \text{ kips} > 250 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 626 \text{ kips} > 167 \text{ kips}$ **o.k.**

Column A-B

From Chapter 2 of ASCE/SEI 7, the required compressive strength for the column between the floor and the foundation is:

LRFD	ASD
$P_u = 1.2(100 \text{ kips}) + 1.6(300 \text{ kips})$ $= 600 \text{ kips}$	$P_a = 100 \text{ kips} + 300 \text{ kips}$ $= 400 \text{ kips}$

Effective Length Factor

Calculate the stiffness reduction parameter, τ_b, using AISC *Manual* Table 4-21.

LRFD	ASD
$\dfrac{P_u}{A_g} = \dfrac{600 \text{ kips}}{24.0 \text{ in.}^2}$ $= 25.0 \text{ ksi}$ $\tau_b = 1.00$	$\dfrac{P_a}{A_g} = \dfrac{400 \text{ kips}}{24.0 \text{ in.}^2}$ $= 16.7 \text{ ksi}$ $\tau_b = 0.994$

Determine G_{top} and G_{bottom} accounting for column inelasticity by replacing E_cI_c with $\tau_b(E_cI_c)$. Use $\tau_b = 0.994$.

$$G_{top} = \tau \frac{\Sigma(E_c I_c / L_c)}{\Sigma(E_g I_g / L_g)} \qquad \text{(from } \textit{Spec.} \text{ Comm. Eq. C-A-7-3)}$$

$$= (0.994)\frac{2\left(\dfrac{29{,}000 \text{ ksi}(881 \text{ in.}^4)}{14.0 \text{ ft}}\right)}{2\left(\dfrac{29{,}000 \text{ ksi}(1{,}350 \text{ in.}^4)}{35.0 \text{ ft}}\right)}$$

$$= 1.62$$

$G_{bottom} = 1.0$ (fixed) from AISC *Specification* Commentary Appendix 7, Section 7.2

From the alignment chart, AISC *Specification* Commentary Figure C-A-7.2, K is approximately 1.40. Because the column available strength tables are based on the KL about the y-y axis, the effective column length of the lower segment for use in the table is:

$$KL = \frac{(KL)_x}{\left(\dfrac{r_x}{r_y}\right)}$$

$$= \frac{1.40(14.0 \text{ ft})}{2.44}$$

$$= 8.03 \text{ ft}$$

Take the available strength of the W14×82 from AISC *Manual* Table 4-1.

At KL = 9 ft, (conservative) the available strength in axial compression is:

LRFD	ASD
$\phi_c P_n = 940 \text{ kips} > 600 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 626 \text{ kips} > 400 \text{ kips}$ **o.k.**

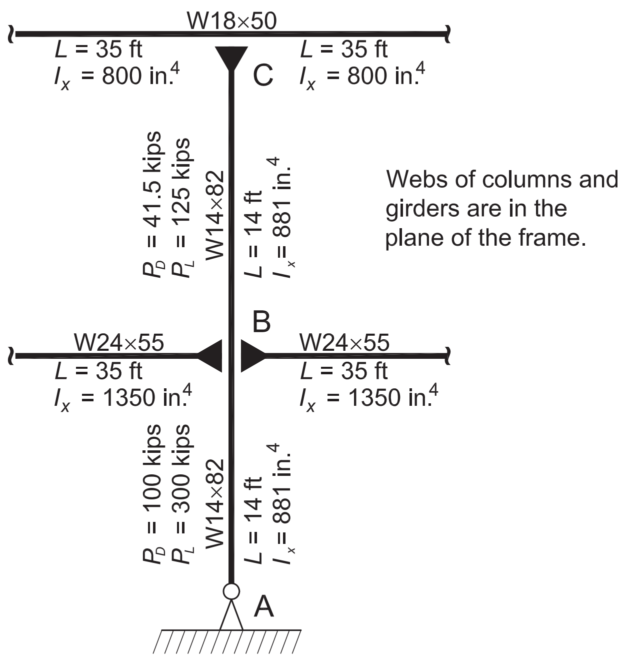
A more accurate strength could be determined by interpolation from AISC *Manual* Table 4-1.

Return to Table of Contents

EXAMPLE E.4B W-SHAPE COMPRESSION MEMBER (MOMENT FRAME)

Given:

Using the effective length method, determine the available strength of the column shown subject to the same gravity loads shown in Example E.4A with the column pinned at the base about the *x-x* axis. All other assumptions remain the same.



Solution:

As determined in Example E.4A, for the column segment B-C between the roof and the floor, the column strength is adequate.

As determined in Example E.4A, for the column segment A-B between the floor and the foundation,
$G_{top} = 1.62$

At the base,
$G_{bottom} = 10$ (pinned) from AISC *Specification* Commentary Appendix 7, Section 7.2

Note: this is the only change in the analysis.

From the alignment chart, AISC *Specification* Commentary Figure C-A-7.2, K is approximately equal to 2.00. Because the column available strength tables are based on the effective length, KL, about the *y-y* axis, the effective column length of the segment A-B for use in the table is:

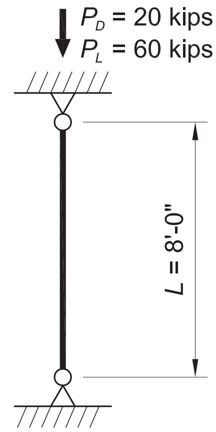
$$
\begin{aligned}
KL &= \frac{(KL)_x}{\left(\dfrac{r_x}{r_y}\right)} \\
&= \frac{2.00(14.0 \text{ ft})}{2.44} \\
&= 11.5 \text{ ft}
\end{aligned}
$$

Interpolate the available strength of the W14×82 from AISC *Manual* Table 4-1.

LRFD	ASD
$\phi_c P_n = 861 \text{ kips} > 600 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 573 \text{ kips} > 400 \text{ kips}$ **o.k.**

EXAMPLE E.5 DOUBLE ANGLE COMPRESSION MEMBER WITHOUT SLENDER ELEMENTS

Given:



Verify the strength of a 2L4×3½×⅜ LLBB (¾-in. separation) strut, ASTM A36, with a length of 8 ft and pinned ends carrying an axial dead load of 20 kips and live load of 60 kips. Also, calculate the required number of pretensioned bolted or welded intermediate connectors required.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-7 and 1-15, the geometric properties are as follows:

L4×3½×⅜ LLBB
r_z = 0.719 in.

2L4×3½×⅜ LLBB
r_x = 1.25 in.
r_y = 1.55 in. for ⅜-in. separation
r_y = 1.69 in. for ¾-in. separation

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

LRFD	ASD
$P_u = 1.2(20\text{ kips}) + 1.6(60\text{ kips})$ $= 120\text{ kips}$	$P_a = 20\text{ kips} + 60\text{ kips}$ $= 80.0\text{ kips}$

Table Solution

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0.

For $(KL)_x$ = 8 ft, the available strength in axial compression is taken from the upper (X-X) portion of AISC *Manual* Table 4-9 as:

LRFD	ASD
$\phi_c P_n = 127\text{ kips} > 120\text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 84.7\text{ kips} > 80.0\text{ kips}$ **o.k.**

For buckling about the y-y axis, the values are tabulated for a separation of ⅜ in.

To adjust to a spacing of ¾ in., $(KL)_y$ is multiplied by the ratio of the r_y for a ⅜-in. separation to the r_y for a ¾-in. separation. Thus,

$$(KL)_y = 1.0(8.00\text{ ft})\left(\frac{1.55\text{ in.}}{1.69\text{ in.}}\right)$$

$= 7.34 \text{ ft}$

The calculation of the equivalent $(KL)_y$ in the preceding text is a simplified approximation of AISC *Specification* Section E6.1. To ensure a conservative adjustment for a ¾-in. separation, take $(KL)_y$ = 8 ft.

The available strength in axial compression is taken from the lower (*Y*-*Y*) portion of AISC *Manual* Table 4-9 as:

LRFD	ASD
$\phi_c P_n = 130 \text{ kips} > 120 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 86.3 \text{ kips} > 80.0 \text{ kips}$ **o.k.**

Therefore, *x*-*x* axis flexural buckling governs.

Intermediate Connectors

From AISC *Manual* Table 4-9, at least two welded or pretensioned bolted intermediate connectors are required. This can be verified as follows:

$$a = \text{distance between connectors}$$
$$= \frac{8.00 \text{ ft}\left(12 \text{ in./ft}\right)}{3 \text{ spaces}}$$
$$= 32.0 \text{ in.}$$

From AISC *Specification* Section E6.2, the effective slenderness ratio of the individual components of the built-up member based upon the distance between intermediate connectors, *a*, must not exceed three-fourths of the governing slenderness ratio of the built-up member.

Therefore, $\dfrac{Ka}{r_i} \leq \dfrac{3}{4}\left(\dfrac{Kl}{r}\right)_{max}$

Solving for *a* gives, $a \leq \dfrac{3r_i\left(\dfrac{KL}{r}\right)_{max}}{4K}$

$$\frac{KL}{r_x} = \frac{1.0\left(8.00 \text{ ft}\right)\left(12 \text{ in./ft}\right)}{1.25 \text{ in.}}$$
$$= 76.8 \quad \textbf{controls}$$

$$\frac{KL}{r_y} = \frac{1.0\left(8.00 \text{ ft}\right)\left(12 \text{ in./ft}\right)}{1.69 \text{ in.}}$$
$$= 56.8$$

Thus, $a \le \dfrac{3r_z\left(\dfrac{KL}{r}\right)_{max}}{4K}$

$= \dfrac{3(0.719\text{ in.})(76.8)}{4(1.0)}$

$= 41.4\text{ in.} > 32.0\text{ in.}$ **o.k.**

Note that one connector would not be adequate as 48.0 in. > 41.4 in. The available strength can be easily determined by using the tables of the AISC *Manual*. Available strength values can be verified by hand calculations, as follows:

Calculation Solution

From AISC *Manual* Tables 1-7 and 1-15, the geometric properties are as follows:

L4×3½×⅜
$J = 0.132\text{ in.}^4$
$r_y = 1.05\text{ in.}$
$\bar{x} = 0.947\text{ in.}$

2L4×3½×⅜ LLBB (¾ in. separation)
$A_g = 5.36\text{ in.}^2$
$r_y = 1.69\text{ in.}$
$\bar{r}_o = 2.33\text{ in.}$
$H = 0.813$

Slenderness Check

$$\lambda = \frac{b}{t}$$
$$= \frac{4.00\text{ in.}}{3/8\text{ in.}}$$
$$= 10.7$$

Determine the limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a Case 3

$$\lambda_r = 0.45\sqrt{E/F_y}$$
$$= 0.45\sqrt{29{,}000\text{ ksi}/36\text{ ksi}}$$
$$= 12.8$$

$\lambda < \lambda_r$; therefore, there are no slender elements.

For compression members without slender elements, AISC *Specification* Sections E3 and E4 apply.

The nominal compressive strength, P_n, shall be determined based on the limit states of flexural, torsional and flexural-torsional buckling.

Flexural Buckling about the *x-x* Axis

$$\frac{KL}{r_x} = \frac{1.0(8.00\text{ ft})(12\text{ in./ft})}{1.25\text{ in.}}$$
$$= 76.8$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \qquad \text{(}\mathit{Spec}\text{. Eq. E3-4)}$$
$$= \frac{\pi^2(29{,}000\text{ ksi})}{(76.8)^2}$$
$$= 48.5\text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000\text{ ksi}}{36\text{ ksi}}}$$
$$= 134 > 76.8\text{, therefore}$$

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right]F_y \qquad \text{(}\mathit{Spec}\text{. Eq. E3-2)}$$
$$= \left[0.658^{\frac{36\text{ ksi}}{48.5\text{ ksi}}}\right](36\text{ ksi})$$
$$= 26.4\text{ ksi} \qquad \textbf{controls}$$

Torsional and Flexural-Torsional Buckling

For nonslender double angle compression members, AISC *Specification* Equation E4-2 applies.

F_{cry} is taken as F_{cr}, for flexural buckling about the y-y axis from AISC *Specification* Equation E3-2 or E3-3 as applicable.

Using AISC *Specification* Section E6, compute the modified KL/r_y for built up members with pretensioned bolted or welded connectors. Assume two connectors are required.

$$a = 96.0\text{ in.}/3$$
$$= 32.0\text{ in.}$$

$$r_i = r_z\text{ (single angle)}$$
$$= 0.719\text{ in.}$$

$$\frac{a}{r_i} = \frac{32\text{ in.}}{0.719\text{ in.}}$$
$$= 44.5 > 40\text{, therefore}$$

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + \left(\frac{K_i a}{r_i}\right)^2} \text{ where } K_i = 0.50 \text{ for angles back-to-back} \qquad \text{(}\mathit{Spec}\text{. Eq. E6-2b)}$$
$$= \sqrt{(56.8)^2 + \left(\frac{0.50(32.0\text{ in.})}{0.719\text{ in.}}\right)^2}$$

$= 61.0 \le 134$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \qquad \text{(}\textit{Spec}\text{. Eq. E3-4)}$$

$$= \frac{\pi^2 (29{,}000 \text{ ksi})}{(61.0)^2}$$

$$= 76.9 \text{ ksi}$$

$$F_{cry} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y \qquad \text{(}\textit{Spec}\text{. Eq. E3-2)}$$

$$= \left[0.658^{\frac{36 \text{ ksi}}{76.9 \text{ ksi}}}\right](36 \text{ ksi})$$

$$= 29.6 \text{ ksi}$$

$$F_{crz} = \frac{GJ}{A_g \bar{r}_o^2} \qquad \text{(}\textit{Spec}\text{. Eq. E4-3)}$$

$$= \frac{(11{,}200 \text{ ksi})(2 \text{ angles})(0.132 \text{ in.}^4)}{(5.36 \text{ in.}^2)(2.33 \text{ in.})^2}$$

$$= 102 \text{ ksi}$$

$$F_{cr} = \left(\frac{F_{cry} + F_{crz}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{cry}F_{crz}H}{(F_{cry} + F_{crz})^2}}\right] \qquad \text{(}\textit{Spec}\text{. Eq. E4-2)}$$

$$= \left(\frac{29.6 \text{ ksi} + 102 \text{ ksi}}{2(0.813)}\right)\left[1 - \sqrt{1 - \frac{4(29.6 \text{ ksi})(102 \text{ ksi})(0.813)}{(29.6 \text{ ksi} + 102 \text{ ksi})^2}}\right]$$

$= 27.7$ ksi **does not control**

Nominal Compressive Strength

$$P_n = F_{cr} A_g \qquad \text{(}\textit{Spec}\text{. Eq. E4-1)}$$

$$= 26.4 \text{ ksi}(5.36 \text{ in.}^2)$$

$$= 142 \text{ kips}$$

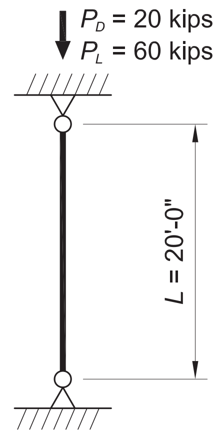
From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(142 \text{ kips})$ $= 128 \text{ kips} > 120 \text{ kips}$ **o.k.**	$\Omega_c = 1.67$ $\frac{P_n}{\Omega_c} = \frac{142 \text{ kips}}{1.67}$ $= 85.0 \text{ kips} > 80.0 \text{ kips}$ **o.k.**

Return to Table of Contents

EXAMPLE E.7 WT COMPRESSION MEMBER WITHOUT SLENDER ELEMENTS

Given:



Select an ASTM A992 nonslender WT-shape compression member with a length of 20 ft to support a dead load of 20 kips and live load of 60 kips in axial compression. The ends are pinned.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

LRFD	ASD
$P_u = 1.2(20 \text{ kips}) + 1.6(60 \text{ kips})$ $= 120 \text{ kips}$	$P_a = 20 \text{ kips} + 60 \text{ kips}$ $= 80.0 \text{ kips}$

Table Solution

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0.

Therefore, $(KL)_x = (KL)_y$ = 20.0 ft.

Select the lightest nonslender member from AISC *Manual* Table 4-7 with sufficient available strength about both the x-x axis (upper portion of the table) and the y-y axis (lower portion of the table) to support the required strength.

Try a WT7×34.

The available strength in compression is:

LRFD	ASD
$\phi_c P_{nx} = 128 \text{ kips} > 120 \text{ kips}$ **controls** **o.k.**	$\dfrac{P_{nx}}{\Omega_c} = 85.5 \text{ kips} > 80.0 \text{ kips}$ **controls** **o.k.**
$\phi_c P_{ny} = 221 \text{ kips} > 120 \text{ kips}$ **o.k.**	$\dfrac{P_{ny}}{\Omega_c} = 147 \text{ kips} > 80.0 \text{ kips}$ **o.k.**

The available strength can be easily determined by using the tables of the AISC *Manual*. Available strength values can be verified by hand calculations, as follows.

Calculation Solution

From AISC *Manual* Table 1-8, the geometric properties are as follows.

WT7×34

$A_g = 10.0 \text{ in.}^2$
$r_x = 1.81 \text{ in.}$
$r_y = 2.46 \text{ in.}$
$J = 1.50 \text{ in.}^4$
$\overline{y} = 1.29 \text{ in.}$
$I_x = 32.6 \text{ in.}^4$
$I_y = 60.7 \text{ in.}^4$
$d = 7.02 \text{ in.}$
$t_w = 0.415 \text{ in.}$
$b_f = 10.0 \text{ in.}$
$t_f = 0.720 \text{ in.}$

Stem Slenderness Check

$$\lambda = \frac{d}{t_w}$$
$$= \frac{7.02 \text{ in.}}{0.415 \text{ in.}}$$
$$= 16.9$$

Determine the stem limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a Case 4

$$\lambda_r = 0.75\sqrt{\frac{E}{F_y}}$$
$$= 0.75\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 18.1$$

$\lambda < \lambda_r$; therefore, the stem is not slender

Flange Slenderness Check

$$\lambda = \frac{b_f}{2t_f}$$
$$= \frac{10 \text{ in.}}{2(0.720 \text{ in.})}$$
$$= 6.94$$

Determine the flange limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a Case 1

$$\lambda_r = 0.56\sqrt{\frac{E}{F_y}}$$
$$= 0.56\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 13.5$$

$\lambda < \lambda_r$; therefore, the flange is not slender

There are no slender elements.

For compression members without slender elements, AISC *Specification* Sections E3 and E4 apply. The nominal compressive strength, P_n, shall be determined based on the limit states of flexural, torsional and flexural-torsional buckling.

Flexural Buckling About the *x-x* Axis

$$\frac{KL}{r_x} = \frac{1.0(20.0\text{ ft})(12\text{ in./ft})}{1.81\text{ in.}}$$
$$= 133$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}}$$
$$= 113 < 133\text{, therefore, AISC } \textit{Specification} \text{ Equation E3-3 applies}$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \qquad \text{(}\textit{Spec}\text{. Eq. E3-4)}$$
$$= \frac{\pi^2(29{,}000\text{ ksi})}{(133)^2}$$
$$= 16.2\text{ ksi}$$

$$F_{cr} = 0.877F_e \qquad \text{(}\textit{Spec}\text{. Eq. E3-3)}$$
$$= 0.877(16.2\text{ ksi})$$
$$= 14.2\text{ ksi} \qquad \textbf{controls}$$

Torsional and Flexural-Torsional Buckling

Because the WT7×34 section does not have any slender elements, AISC *Specification* Section E4 will be applicable for torsional and flexural-torsional buckling. F_{cr} will be calculated using AISC *Specification* Equation E4-2.

Calculate F_{cry}.

F_{cry} is taken as F_{cr} from AISC *Specification* Section E3, where $KL/r = KL/r_y$.

$$\frac{KL}{r_y} = \frac{1.0(20.0\text{ ft})(12\text{ in./ft})}{2.46\text{ in.}}$$
$$= 97.6 \le 113\text{, therefore, AISC } \textit{Specification} \text{ Equation E3-2 applies}$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \qquad \text{(}\textit{Spec}\text{. Eq. E3-4)}$$
$$= \frac{\pi^2(29{,}000\text{ ksi})}{(97.6)^2}$$
$$= 30.0\text{ ksi}$$

$$F_{cry} = F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y$$ (*Spec*. Eq. E3-2)

$$= \left[0.658^{\frac{50.0\,\text{ksi}}{30.0\,\text{ksi}}}\right] 50.0\,\text{ksi}$$

$$= 24.9 \text{ ksi}$$

The shear center for a T-shaped section is located on the axis of symmetry at the mid-depth of the flange.

$$x_o = 0.0 \text{ in.}$$

$$y_o = \bar{y} - \frac{t_f}{2}$$

$$= 1.29 \text{ in.} - \frac{0.720 \text{ in.}}{2}$$

$$= 0.930 \text{ in.}$$

$$\bar{r}_o^2 = x_o^2 + y_o^2 + \frac{I_x + I_y}{A_g}$$ (*Spec*. Eq. E4-11)

$$= (0.0 \text{ in.})^2 + (0.930 \text{ in.})^2 + \frac{32.6 \text{ in.}^4 + 60.7 \text{ in.}^4}{10.0 \text{ in.}^2}$$

$$= 10.2 \text{ in.}^2$$

$$\bar{r}_o = \sqrt{\bar{r}_o^2}$$

$$= \sqrt{10.2 \text{ in.}^2}$$

$$= 3.19 \text{ in.}$$

$$H = 1 - \frac{x_o^2 + y_o^2}{\bar{r}_o^2}$$ (*Spec*. Eq. E4-10)

$$= 1 - \frac{(0.0 \text{ in.})^2 + (0.930 \text{ in.})^2}{10.2 \text{ in.}^2}$$

$$= 0.915$$

$$F_{crz} = \frac{GJ}{A_g \bar{r}_o^2}$$ (*Spec*. Eq. E4-3)

$$= \frac{(11{,}200 \text{ ksi})(1.50 \text{ in.}^4)}{(10.0 \text{ in.}^2)(10.2 \text{ in.}^2)}$$

$$= 165 \text{ ksi}$$

$$F_{cr} = \left(\frac{F_{cry} + F_{crz}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{cry}F_{crz}H}{(F_{cry} + F_{crz})^2}}\right]$$ (*Spec*. Eq. E4-2)

$$= \left(\frac{24.9\text{ ksi} + 165\text{ ksi}}{2(0.915)}\right)\left[1 - \sqrt{1 - \frac{4(24.9\text{ ksi})(165\text{ ksi})(0.915)}{(24.9\text{ ksi} + 165\text{ ksi})^2}}\right]$$

$= 24.5$ ksi **does not control**

x-x axis flexural buckling governs, therefore,

$$P_n = F_{cr} A_g \qquad (Spec.\ \text{Eq. E3-1})$$

$$= 14.2\text{ ksi}\left(10.0\text{ in.}^2\right)$$

$$= 142\text{ kips}$$

From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c P_n = 0.90(142\text{ kips})$ $= 128\text{ kips} > 120\text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = \dfrac{142\text{ kips}}{1.67}$ $= 85.0\text{ kips} > 80.0\text{ kips}$ **o.k.**

Chapter F
Design of Members for Flexure

INTRODUCTION

This *Specification* chapter contains provisions for calculating the flexural strength of members subject to simple bending about one principal axis. Included are specific provisions for I-shaped members, channels, HSS, tees, double angles, single angles, rectangular bars, rounds and unsymmetrical shapes. Also included is a section with proportioning requirements for beams and girders.

There are selection tables in the AISC *Manual* for standard beams in the commonly available yield strengths. The section property tables for most cross sections provide information that can be used to conveniently identify noncompact and slender element sections. LRFD and ASD information is presented side-by-side.

Most of the formulas from this chapter are illustrated by the following examples. The design and selection techniques illustrated in the examples for both LRFD and ASD will result in similar designs.

F1. GENERAL PROVISIONS

Selection and evaluation of all members is based on deflection requirements and strength, which is determined as the design flexural strength, $\phi_b M_n$, or the allowable flexural strength, M_n/Ω_b,
where

M_n = the lowest nominal flexural strength based on the limit states of yielding, lateral torsional-buckling, and local buckling, where applicable

$\phi_b = 0.90$ (LRFD) $\qquad \Omega_b = 1.67$ (ASD)

This design approach is followed in all examples.

The term L_b is used throughout this chapter to describe the length between points which are either braced against lateral displacement of the compression flange or braced against twist of the cross section. Requirements for bracing systems and the required strength and stiffness at brace points are given in AISC *Specification* Appendix 6.

The use of C_b is illustrated in several of the following examples. AISC *Manual* Table 3-1 provides tabulated C_b values for some common situations.

F2. DOUBLY SYMMETRIC COMPACT I-SHAPED MEMBERS AND CHANNELS BENT ABOUT THEIR MAJOR AXIS

AISC *Specification* Section F2 applies to the design of compact beams and channels. As indicated in the User Note in Section F2 of the AISC *Specification*, the vast majority of rolled I-shaped beams and channels fall into this category. The curve presented as a solid line in Figure F-1 is a generic plot of the nominal flexural strength, M_n, as a function of the unbraced length, L_b. The horizontal segment of the curve at the far left, between $L_b = 0$ ft and L_p, is the range where the strength is limited by flexural yielding. In this region, the nominal strength is taken as the full plastic moment strength of the section as given by AISC *Specification* Equation F2-1. In the range of the curve at the far right, starting at L_r, the strength is limited by elastic buckling. The strength in this region is given by AISC *Specification* Equation F2-3. Between these regions, within the linear region of the curve between $M_n = M_p$ at L_p on the left, and $M_n = 0.7M_y = 0.7F_yS_x$ at L_r on the right, the strength is limited by inelastic buckling. The strength in this region is provided in AISC *Specification* Equation F2-2.

The curve plotted as a heavy solid line represents the case where C_b = 1.0, while the heavy dashed line represents the case where C_b exceeds 1.0. The nominal strengths calculated in both AISC *Specification* Equations F2-2 and F2-3 are linearly proportional to C_b, but are limited to M_p as shown in the figure.

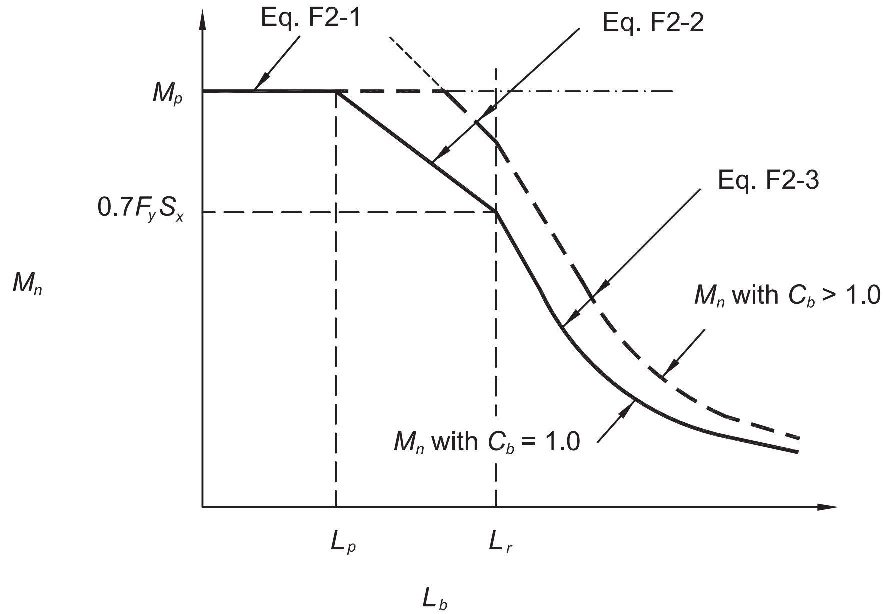


Fig. F-1. Beam strength versus unbraced length.

$$M_n = M_p = F_yZ_x \quad \text{(}\textit{Spec}\text{. Eq. F2-1)}$$

$$M_n = C_b\left[M_p - \left(M_p - 0.7F_yS_x\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \quad \text{(}\textit{Spec}\text{. Eq. F2-2)}$$

$$M_n = F_{cr}S_x \le M_p \quad \text{(}\textit{Spec}\text{. Eq. F2-3)}$$

where

$$F_{cr} = \frac{C_b\pi^2E}{\left(\dfrac{L_b}{r_{ts}}\right)^2}\sqrt{1 + 0.078\frac{Jc}{S_xh_o}\left(\frac{L_b}{r_{ts}}\right)^2} \quad \text{(}\textit{Spec}\text{. Eq. F2-4)}$$

The provisions of this section are illustrated in Example F.1(W-shape beam) and Example F.2 (channel).

Plastic design provisions are given in AISC *Specification* Appendix 1. L_{pd}, the maximum unbraced length for prismatic member segments containing plastic hinges is less than L_p.

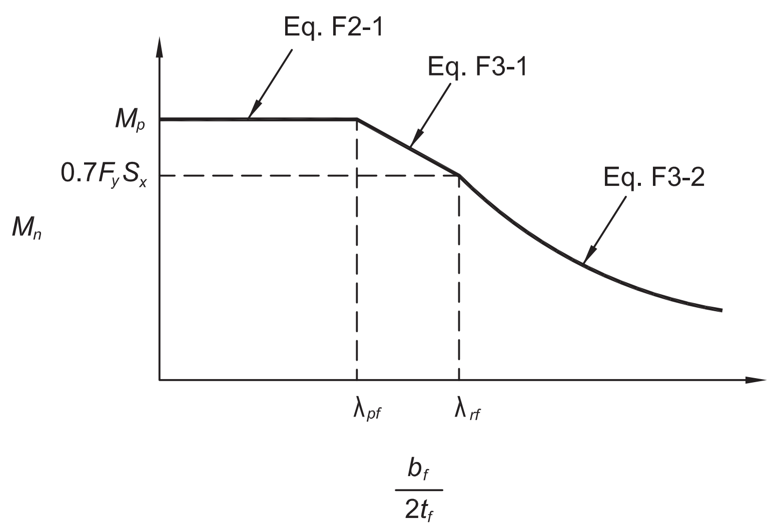
Return to Table of Contents

F3. DOUBLY SYMMETRIC I-SHAPED MEMBERS WITH COMPACT WEBS AND NONCOMPACT OR SLENDER FLANGES BENT ABOUT THEIR MAJOR AXIS

The strength of shapes designed according to this section is limited by local buckling of the compression flange. Only a few standard wide flange shapes have noncompact flanges. For these sections, the strength reduction for F_y = 50 ksi steel varies. The approximate percentages of M_p about the strong axis that can be developed by noncompact members when braced such that $L_b \le L_p$ are shown as follows:

W21×48 = 99%	W14×99 = 99%	W14×90 = 97%	W12×65 = 98%
W10×12 = 99%	W8×31 = 99%	W8×10 = 99%	W6×15 = 94%
W6×8.5 = 97%			

The strength curve for the flange local buckling limit state, shown in Figure F-2, is similar in nature to that of the lateral-torsional buckling curve. The horizontal axis parameter is $\lambda = b_f/2t_f$. The flat portion of the curve to the left of λ_{pf} is the plastic yielding strength, M_p. The curved portion to the right of λ_{rf} is the strength limited by elastic buckling of the flange. The linear transition between these two regions is the strength limited by inelastic flange buckling.



Fig, F-2. Flange local buckling strength.

$$M_n = M_p = F_y Z_x \qquad (Spec.\ \text{Eq. F2-1})$$

$$M_n = \left[M_p - \left(M_p - 0.7F_yS_x\right)\left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right)\right] \qquad (Spec.\ \text{Eq. F3-1})$$

$$M_n = \frac{0.9Ek_cS_x}{\lambda^2} \qquad (Spec.\ \text{Eq. F3-2})$$

where

$k_c = \dfrac{4}{\sqrt{h/t_w}}$ from AISC *Specification* Table B4.1b footnote [a], where $0.35 \le k_c \le 0.76$

The strength reductions due to flange local buckling of the few standard rolled shapes with noncompact flanges are incorporated into the design tables in Chapter 3 of the AISC *Manual*.

There are no standard I-shaped members with slender flanges. The noncompact flange provisions of this section are illustrated in Example F.3.

F4. OTHER I-SHAPED MEMBERS WITH COMPACT OR NONCOMPACT WEBS BENT ABOUT THEIR MAJOR AXIS

This section of the AISC *Specification* applies to doubly symmetric I-shaped members with noncompact webs and singly symmetric I-shaped members (those having different flanges) with compact or noncompact webs.

F5. DOUBLY SYMMETRIC AND SINGLY SYMMETRIC I-SHAPED MEMBERS WITH SLENDER WEBS BENT ABOUT THEIR MAJOR AXIS

This section applies to I-shaped members with slender webs, formerly designated as "plate girders".

F6. I-SHAPED MEMBERS AND CHANNELS BENT ABOUT THEIR MINOR AXIS

I-shaped members and channels bent about their minor axis are not subject to lateral-torsional buckling. Rolled or built-up shapes with noncompact or slender flanges, as determined by AISC *Specification* Tables B4.1a and B4.1b, must be checked for strength based on the limit state of flange local buckling using Equations F6-2 or F6-3 as applicable.

The vast majority of W, M, C and MC shapes have compact flanges, and can therefore develop the full plastic moment, M_p, about the minor axis. The provisions of this section are illustrated in Example F.5.

F7. SQUARE AND RECTANGULAR HSS AND BOX-SHAPED MEMBERS

Square and rectangular HSS need only be checked for the limit states of yielding and local buckling. Although lateral-torsional buckling is theoretically possible for very long rectangular HSS bent about the strong axis, deflection will control the design as a practical matter.

The design and section property tables in the AISC *Manual* were calculated using a design wall thickness of 93% of the nominal wall thickness. Strength reductions due to local buckling have been accounted for in the AISC *Manual* design tables. The selection of rectangular or square HSS with compact flanges is illustrated in Example F.6. The provisions for rectangular or square HSS with noncompact flanges are illustrated in Example F.7. The provisions for HSS with slender flanges are illustrated in Example F.8. Available flexural strengths of rectangular and square HSS are listed in Tables 3-12 and 3-13, respectively.

F8. ROUND HSS

The definition of HSS encompasses both tube and pipe products. The lateral-torsional buckling limit state does not apply, but round HSS are subject to strength reductions from local buckling. Available strengths of round HSS and Pipes are listed in AISC *Manual* Tables 3-14 and 3-15, respectively. The tabulated properties and available flexural strengths of these shapes in the AISC *Manual* are calculated using a design wall thickness of 93% of the nominal wall thickness. The design of a Pipe is illustrated in Example F.9.

F9. TEES AND DOUBLE ANGLES LOADED IN THE PLANE OF SYMMETRY

The AISC *Specification* provides a check for flange local buckling, which applies only when the flange is in compression due to flexure. This limit state will seldom govern. A check for local buckling of the web was added in the 2010 edition of the *Specification*. Attention should be given to end conditions of tees to avoid inadvertent fixed end moments which induce compression in the web unless this limit state is checked. The design of a WT-shape in bending is illustrated in Example F.10.

F10. SINGLE ANGLES

Section F10 permits the flexural design of single angles using either the principal axes or geometric axes (*x-x* and *y-y* axes). When designing single angles without continuous bracing using the geometric axis design provisions, M_y must be multiplied by 0.80 for use in Equations F10-1, F10-2 and F10-3. The design of a single angle in bending is illustrated in Example F.11.

F11. RECTANGULAR BARS AND ROUNDS

There are no design tables in the AISC *Manual* for these shapes. The local buckling limit state does not apply to any bars. With the exception of rectangular bars bent about the strong axis, solid square, rectangular and round bars are not subject to lateral-torsional buckling and are governed by the yielding limit state only. Rectangular bars bent about the strong axis are subject to lateral-torsional buckling and are checked for this limit state with Equations F11-2 and F11-3, as applicable.

These provisions can be used to check plates and webs of tees in connections. A design example of a rectangular bar in bending is illustrated in Example F.12. A design example of a round bar in bending is illustrated in Example F.13.

F12. UNSYMMETRICAL SHAPES

Due to the wide range of possible unsymmetrical cross sections, specific lateral-torsional and local buckling provisions are not provided in this *Specification* section. A general template is provided, but appropriate literature investigation and engineering judgment are required for the application of this section. A Z-shaped section is designed in Example F.14.

F13. PROPORTIONS OF BEAMS AND GIRDERS

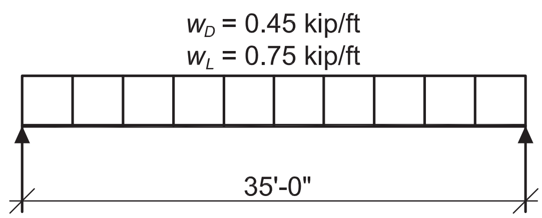
This section of the *Specification* includes a limit state check for tensile rupture due to holes in the tension flange of beams, proportioning limits for I-shaped members, detail requirements for cover plates and connection requirements for built-up beams connected side-to-side. Also included are unbraced length requirements for beams designed using the moment redistribution provisions of AISC *Specification* Section B3.7.

Return to Table of Contents

EXAMPLE F.1-1A W-SHAPE FLEXURAL MEMBER DESIGN IN STRONG-AXIS BENDING, CONTINUOUSLY BRACED

Given:

Select an ASTM A992 W-shape beam with a simple span of 35 ft. Limit the member to a maximum nominal depth of 18 in. Limit the live load deflection to L/360. The nominal loads are a uniform dead load of 0.45 kip/ft and a uniform live load of 0.75 kip/ft. Assume the beam is continuously braced.



Beam Loading & Bracing Diagram
(*full lateral support*)

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
w_u = 1.2(0.45 kip/ft) +1.6 (0.75 kip/ft) = 1.74 kip/ft $M_u = \dfrac{1.74\text{ kip/ft}\left(35.0\text{ ft}\right)^2}{8}$ = 266 kip-ft	w_a = 0.45 kip/ft + 0.75 kip/ft = 1.20 kip/ft $M_a = \dfrac{1.20\text{ kip/ft}\left(35.0\text{ ft}\right)^2}{8}$ = 184 kip-ft

Required Moment of Inertia for Live-Load Deflection Criterion of L/360

$$\Delta_{max} = \frac{L}{360}$$
$$= \frac{35.0\text{ ft}(12\text{ in./ft})}{360}$$
$$= 1.17\text{ in.}$$

$$I_{x(reqd)} = \frac{5w_L l^4}{384E\Delta_{max}} \text{ from AISC } \textit{Manual} \text{ Table 3-23 Case 1}$$
$$= \frac{5(0.750\text{ kip/ft})(35.0\text{ ft})^4(12\text{ in./ft})^3}{384\,(29{,}000\text{ ksi})(1.17\text{ in.})}$$
$$= 746\text{ in.}^4$$

Beam Selection

Select a W18×50 from AISC *Manual* Table 3-2.

Per the User Note in AISC *Specification* Section F2, the section is compact. Because the beam is continuously braced and compact, only the yielding limit state applies.

From AISC *Manual* Table 3-2, the available flexural strength is:

LRFD	ASD
$\phi_b M_n = \phi_b M_{px}$ $= 379\text{ kip-ft} > 266\text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = \dfrac{M_{px}}{\Omega_b}$ $= 252\text{ kip-ft} > 184\text{ kip-ft}$ **o.k.**

From AISC *Manual* Table 3-2, I_x = 800 in.^{4} > 746 in.^{4} **o.k.**

Return to Table of Contents

EXAMPLE F.1-1B W-SHAPE FLEXURAL MEMBER DESIGN IN STRONG-AXIS BENDING, CONTINUOUSLY BRACED

Given:

Verify the available flexural strength of the W18×50, ASTM A992 beam selected in Example F.1-1A by applying the requirements of the AISC *Specification* directly.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

W18×50
ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W18×50
$Z_x = 101$ in.^{3}

The required flexural strength from Example F.1-1A is:

LRFD	ASD
$M_u = 266$ kip-ft	$M_a = 184$ kip-ft

Nominal Flexural Strength, M_n

Per the User Note in AISC *Specification* Section F2, the section is compact. Because the beam is continuously braced and compact, only the yielding limit state applies.

$$\begin{aligned} M_n &= M_p \\ &= F_y Z_x \qquad (\textit{Spec}.\text{ Eq. F2-1}) \\ &= 50\text{ ksi}(101\text{ in.}^3) \\ &= 5{,}050\text{ kip-in. or }421\text{ kip-ft} \end{aligned}$$

From AISC *Specification* Section F1, the available flexural strength is:

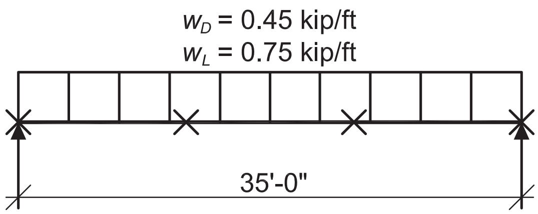
LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(421\text{ kip-ft})$ $= 379\text{ kip-ft} > 266\text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{421\text{ kip-ft}}{1.67}$ $= 252\text{ kip-ft} > 184\text{ kip-ft}$ **o.k.**

Return to Table of Contents

EXAMPLE F.1-2A W-SHAPE FLEXURAL MEMBER DESIGN IN STRONG-AXIS BENDING, BRACED AT THIRD POINTS

Given:

Verify the available flexural strength of the W18×50, ASTM A992 beam selected in Example F.1-1A with the beam braced at the ends and third points. Use the AISC *Manual* tables.



Beam Loading & Bracing Diagram
(*bracing at ends and third points*)

Solution:

The required flexural strength at midspan from Example F.1-1A is:

LRFD	ASD
M_u = 266 kip-ft	M_a = 184 kip-ft

Unbraced Length

$$L_b = \frac{35.0 \text{ ft}}{3}$$
$$= 11.7 \text{ ft}$$

By inspection, the middle segment will govern. From AISC *Manual* Table 3-1, for a uniformly loaded beam braced at the ends and third points, C_b = 1.01 in the middle segment. Conservatively neglect this small adjustment in this case.

Available Strength

Enter AISC *Manual* Table 3-10 and find the intersection of the curve for the W18×50 with an unbraced length of 11.7 ft. Obtain the available strength from the appropriate vertical scale to the left.

From AISC *Manual* Table 3-10, the available flexural strength is:

LRFD	ASD
$\phi_b M_n \approx 302$ kip-ft > 266 kip-ft **o.k.**	$\dfrac{M_n}{\Omega_b} \approx 201$ kip-ft > 184 kip-ft **o.k.**

EXAMPLE F.1-2B W-SHAPE FLEXURAL MEMBER DESIGN IN STRONG-AXIS BENDING, BRACED AT THIRD POINTS

Given:

Verify the available flexural strength of the W18×50, ASTM A992 beam selected in Example F.1-1A with the beam braced at the ends and third points. Apply the requirements of the AISC *Specification* directly.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W18×50
S_x = 88.9 in.^{3}

The required flexural strength from Example F.1-1A is:

LRFD	ASD
M_u = 266 kip-ft	M_a = 184 kip-ft

Nominal Flexural Strength, M_n

Calculate C_b.

For the lateral-torsional buckling limit state, the nonuniform moment modification factor can be calculated using AISC *Specification* Equation F1-1.

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} \quad \text{(Spec. Eq. F1-1)}$$

For the center segment of the beam, the required moments for AISC *Specification* Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: M_{max} = 1.00, M_A = 0.972, M_B = 1.00, and M_C = 0.972.

$$C_b = \frac{12.5(1.00)}{2.5(1.00)+3(0.972)+4(1.00)+3(0.972)} = 1.01$$

For the end-span beam segments, the required moments for AISC *Specification* Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: M_{max} = 0.889, M_A = 0.306, M_B = 0.556, and M_C = 0.750.

$$C_b = \frac{12.5(0.889)}{2.5(0.889)+3(0.306)+4(0.556)+3(0.750)} = 1.46$$

Thus, the center span, with the higher required strength and lower C_b, will govern.

From AISC *Manual* Table 3-2:

$L_p = 5.83$ ft
$L_r = 16.9$ ft

For a compact beam with an unbraced length of $L_p < L_b \leq L_r$, the lesser of either the flexural yielding limit state or the inelastic lateral-torsional buckling limit state controls the nominal strength.

$M_p = 5{,}050$ kip-in. (from Example F.1-1B)

$$M_n = C_b \left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_p \qquad (\textit{Spec}.\ \text{Eq. F2-2})$$

$$= 1.01\left\{ 5{,}050 \text{ kip-in.} - \left[5{,}050 \text{ kip-in.} - 0.7(50 \text{ ksi})(88.9 \text{ in.}^3) \right]\left(\frac{11.7 \text{ ft} - 5.83 \text{ ft}}{16.9 \text{ ft} - 5.83 \text{ ft}} \right) \right\} \leq 5{,}050 \text{kip-in.}$$

$$= 4{,}060 \text{ kip-in. or } 339 \text{ kip-ft}$$

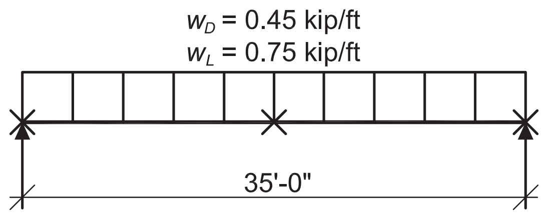
From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(339 \text{ kip-ft})$ $= 305 \text{ kip-ft} > 266 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{339 \text{ kip-ft}}{1.67}$ $= 203 \text{ kip-ft} > 184 \text{ kip-ft}$ **o.k.**

EXAMPLE F.1-3A W-SHAPE FLEXURAL MEMBER DESIGN IN STRONG-AXIS BENDING, BRACED AT MIDSPAN

Given:

Verify the available flexural strength of the W18×50, ASTM A992 beam selected in Example F.1-1A with the beam braced at the ends and center point. Use the AISC *Manual* tables.



Beam Loading & Bracing Diagram
(*bracing at ends & midpoint*)

Solution:

The required flexural strength at midspan from Example F.1-1A is:

LRFD	ASD
M_u = 266 kip-ft	M_a = 184 kip-ft

Unbraced Length

$$L_b = \frac{35.0\text{ ft}}{2}$$
$$= 17.5\text{ ft}$$

From AISC *Manual* Table 3-1, for a uniformly loaded beam braced at the ends and at the center point, C_b = 1.30. There are several ways to make adjustments to AISC *Manual* Table 3-10 to account for C_b greater than 1.0.

Procedure A

Available moments from the sloped and curved portions of the plots from AISC *Manual* Table 3-10 may be multiplied by C_b, but may not exceed the value of the horizontal portion (ϕM_p for LRFD, M_p/Ω for ASD).

Obtain the available strength of a W18×50 with an unbraced length of 17.5 ft from AISC *Manual* Table 3-10.

Enter AISC *Manual* Table 3-10 and find the intersection of the curve for the W18×50 with an unbraced length of 17.5 ft. Obtain the available strength from the appropriate vertical scale to the left.

LRFD	ASD
$\phi_b M_n \approx 222$ kip-ft From *Manual* Table 3-2, $\phi_b M_p = 379$ kip-ft (upper limit on $C_b M_n$)	$\dfrac{M_n}{\Omega_b} \approx 148$ kip-ft From *Manual* Table 3-2, $\dfrac{M_p}{\Omega_b} = 252$ kip-ft (upper limit on $C_b M_n$)

LRFD	ASD
Adjust for C_b. 1.30(222 kip-ft) = 289 kip-ft Check Limit. 289 kip-ft ≤ $\phi_b M_p$ = 379 kip-ft **o.k.** Check available versus required strength. 289 kip-ft > 266 kip-ft **o.k.**	Adjust for $C_{b.}$ 1.30(147 kip-ft) = 191 kip-ft Check Limit. 191 kip-ft ≤ $\dfrac{M_p}{\Omega_b}$ = 252 kip-ft **o.k.** Check available versus required strength. 191 kip-ft > 184 kip-ft **o.k.**

Procedure B

For preliminary selection, the required strength can be divided by C_b and directly compared to the strengths in AISC *Manual* Table 3-10. Members selected in this way must be checked to ensure that the required strength does not exceed the available plastic moment strength of the section.

Calculate the adjusted required strength.

LRFD	ASD
M_u' = 266 kip-ft/1.30 = 205 kip-ft	M_a' = 184 kip-ft/1.30 = 142 kip-ft

Obtain the available strength for a W18×50 with an unbraced length of 17.5 ft from AISC *Manual* Table 3-10.

LRFD	ASD
$\phi_b M_n$ ≈ 222 kip-ft > 205 kip-ft **o.k.**	$\dfrac{M_n}{\Omega_b}$ ≈ 148 kip-ft > 142 kip-ft **o.k.**
$\phi_b M_p$ = 379 kip-ft > 266 kip-ft **o.k.**	$\dfrac{M_p}{\Omega_b}$ = 252 kip-ft > 184 kip-ft **o.k.**

EXAMPLE F.1-3B W-SHAPE FLEXURAL MEMBER DESIGN IN STRONG-AXIS BENDING, BRACED AT MIDSPAN

Given:

Verify the available flexural strength of the W18×50, ASTM A992 beam selected in Example F.1-1A with the beam braced at the ends and center point. Apply the requirements of the AISC *Specification* directly.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W18×50
r_{ts} = 1.98 in.
S_x = 88.9 in.^{3}
J = 1.24 in.^{4}
h_o = 17.4 in.

The required flexural strength from Example F.1-1A is:

LRFD	ASD
M_u = 266 kip-ft	M_a = 184 kip-ft

Nominal Flexural Strength, M_n

Calculate C_b.

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} \qquad \text{(}Spec\text{. Eq. F1-1)}$$

The required moments for AISC *Specification* Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: M_{max}= 1.00, M_A = 0.438, M_B = 0.751, and M_C = 0.938.

$$\begin{aligned} C_b &= \frac{12.5(1.00)}{2.5(1.00)+3(0.438)+4(0.751)+3(0.938)} \\ &= 1.30 \end{aligned}$$

From AISC *Manual* Table 3-2:

L_p = 5.83 ft
L_r = 16.9 ft

For a compact beam with an unbraced length $L_b > L_r$, the limit state of elastic lateral-torsional buckling applies.

Calculate F_{cr} with L_b = 17.5 ft.

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2}\sqrt{1+0.0078\frac{Jc}{S_x h_o}\left(\frac{L_b}{r_{ts}}\right)^2} \quad \text{where } c = 1.0 \text{ for doubly symmetric I-shapes}$$ (*Spec*. Eq. F2-4)

$$= \frac{1.30\pi^2(29{,}000 \text{ ksi})}{\left(\frac{17.5 \text{ ft}(12 \text{ in./ft})}{1.98 \text{ in.}}\right)^2}\sqrt{1+0.078\frac{1.24 \text{ in.}^4(1.0)}{(88.9 \text{ in.}^3)(17.4 \text{ in.})}\left(\frac{17.5 \text{ ft}(12 \text{ in./ft})}{1.98 \text{ in.}}\right)^2}$$

$$= 43.2 \text{ ksi}$$

$M_n = F_{cr}S_x \le M_p$ (*Spec*. Eq. F2-3)

$= 43.2 \text{ ksi}(88.9 \text{ in.}^3)$

$= 3{,}840$ kip-in. $\le 5{,}050$ kip-in. (from Example F.1-1B)

M_n = 3,840 kip-in or 320 kip-ft

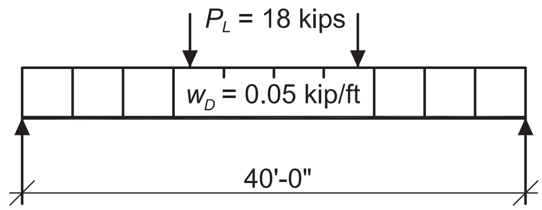
From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(320 \text{ kip-ft})$ $= 288 \text{ kip-ft}$ 288 kip-ft > 266 kip-ft **o.k.**	$\Omega_b = 1.67$ $\frac{M_n}{\Omega_b} = \frac{320 \text{ kip-ft}}{1.67}$ $= 192 \text{ kip-ft}$ 192 kip-ft > 184 kip-ft **o.k.**

EXAMPLE F.3A W-SHAPE FLEXURAL MEMBER WITH NONCOMPACT FLANGES IN STRONG-AXIS BENDING

Given:

Select an ASTM A992 W-shape beam with a simple span of 40 ft. The nominal loads are a uniform dead load of 0.05 kip/ft and two equal 18 kip concentrated live loads acting at the third points of the beam. The beam is continuously braced. Also calculate the deflection.



Beam Loading & Bracing Diagram
(*continuous bracing*)

Note: A beam with noncompact flanges will be selected to demonstrate that the tabulated values of the AISC *Manual* account for flange compactness.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength at midspan is:

LRFD	ASD
$w_u = 1.2(0.05\text{ kip/ft})$ $= 0.0600\text{ kip/ft}$	$w_a = 0.05\text{ kip/ft}$
$P_u = 1.6(18\text{ kips})$ $= 28.8\text{ kips}$	$P_a = 18\text{ kips}$
$M_u = \dfrac{(0.0600\text{ kip/ft})(40.0\text{ ft})^2}{8} + (28.8\text{ kips})\dfrac{40.0\text{ ft}}{3}$ $= 396\text{ kip-ft}$	$M_a = \dfrac{(0.0500\text{ kip/ft})(40.0\text{ ft})^2}{8} + (18.0\text{ kips})\dfrac{40.0\text{ ft}}{3}$ $= 250\text{ kip-ft}$

Beam Selection

For a continuously braced W-shape, the available flexural strength equals the available plastic flexural strength.

Select the lightest section providing the required strength from the bold entries in AISC *Manual* Table 3-2.

Try a W21×48.

This beam has a noncompact compression flange at F_y = 50 ksi as indicated by footnote “f” in AISC *Manual* Table 3-2. This shape is also footnoted in AISC *Manual* Table 1-1.

From AISC *Manual* Table 3-2, the available flexural strength is:

LRFD	ASD
$\phi_b M_n = \phi_b M_{px}$ $= 398 \text{ kip-ft} > 396 \text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = \dfrac{M_{px}}{\Omega_b}$ $= 265 \text{ kip-ft} > 250 \text{ kip-ft}$ **o.k.**

Note: The value M_{px} in AISC *Manual* Table 3-2 includes the strength reductions due to the noncompact nature of the shape.

Deflection

$I_x = 959 \text{ in.}^4$ from AISC *Manual* Table 1-1

The maximum deflection occurs at the center of the beam.

$$\Delta_{max} = \frac{5w_D l^4}{384EI} + \frac{P_L l^3}{28EI} \text{ from AISC } \textit{Manual} \text{ Table 3-23 cases 1 and 9}$$

$$= \frac{5(0.0500 \text{ kip/ft})(40.0 \text{ ft})^4(12 \text{ in./ft})^3}{384\,(29{,}000 \text{ ksi})(959 \text{ in.}^4)} + \frac{18.0 \text{ kips}(40.0 \text{ ft})^3(12 \text{ in./ft})^3}{28(29{,}000 \text{ ksi})(959 \text{ in}^4)}$$

$$= 2.66 \text{ in.}$$

This deflection can be compared with the appropriate deflection limit for the application. Deflection will often be more critical than strength in beam design.

EXAMPLE F.3B W-SHAPE FLEXURAL MEMBER WITH NONCOMPACT FLANGES IN STRONG-AXIS BENDING

Given:

Verify the results from Example F.3A by calculation using the provisions of the AISC *Specification*.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W21×48
$S_x = 93.0$ in.^{3}
$Z_x = 107$ in.^{3}
$\dfrac{b_f}{2t_f} = 9.47$

The required flexural strength from Example F.3A is:

LRFD	ASD
$M_u = 396$ kip-ft	$M_a = 250$ kip-ft

Flange Slenderness

$$\lambda = \frac{b_f}{2t_f} = 9.47$$

The limiting width-to-thickness ratios for the compression flange are:

$$\lambda_{pf} = 0.38\sqrt{\frac{E}{F_y}} \quad \text{from AISC } \textit{Specification} \text{ Table B4.1b Case 10}$$
$$= 0.38\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 9.15$$

$$\lambda_{rf} = 1.0\sqrt{\frac{E}{F_y}} \quad \text{from AISC } \textit{Specification} \text{ Table B4.1b Case 10}$$
$$= 1.0\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 24.1$$

$\lambda_{rf} > \lambda > \lambda_{pf}$, therefore, the compression flange is noncompact. This could also be determined from the footnote "f" in AISC *Manual* Table 1-1.

Nominal Flexural Strength, M_n

Because the beam is continuously braced, and therefore not subject to lateral-torsional buckling, the available strength is governed by AISC *Specification* Section F3.2, Compression Flange Local Buckling.

$$\begin{aligned} M_p &= F_y Z_x \\ &= 50 \text{ ksi}(107 \text{ in.}^3) \\ &= 5{,}350 \text{ kip-in. or } 446 \text{ kip-ft} \end{aligned}$$

$$M_n = \left[M_p - \left(M_p - 0.7 F_y S_x \right) \left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}} \right) \right] \qquad \text{(}\textit{Spec}\text{. Eq. F3-1)}$$

$$\begin{aligned} &= \left\{ 5{,}350 \text{ kip-in.} - \left[5{,}350 \text{ kip-in} - 0.7\left(50 \text{ ksi}\right)\left(93.0 \text{ in.}^3\right) \right] \left(\frac{9.47 - 9.15}{24.1 - 9.15} \right) \right\} \\ &= 5{,}310 \text{ kip-in. or } 442 \text{ kip-ft} \end{aligned}$$

From AISC *Specification* Section F1, the available flexural strength is:

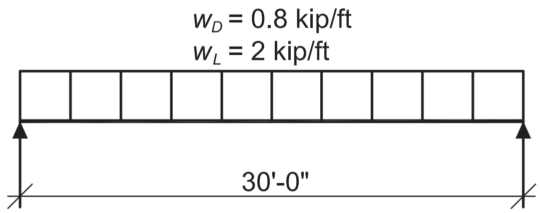
LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90\left(442 \text{ kip-ft}\right)$ $= 398 \text{ kip-ft} > 396 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{442 \text{ kip-ft}}{1.67}$ $= 265 \text{ kip-ft} > 250 \text{ kip-ft}$ **o.k.**

Note that these available strengths are identical to the tabulated values in AISC *Manual* Table 3-2, which account for the noncompact flange.

EXAMPLE F.4 W-SHAPE FLEXURAL MEMBER, SELECTION BY MOMENT OF INERTIA FOR STRONG-AXIS BENDING

Given:

Select an ASTM A992 W-shape flexural member by the moment of inertia, to limit the live load deflection to 1 in. The span length is 30 ft. The loads are a uniform dead load of 0.80 kip/ft and a uniform live load of 2 kip/ft. The beam is continuously braced.



Beam Loading & Bracing Diagram
(*full lateral support*)

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.800 \text{ kip/ft}) + 1.6(2 \text{ kip/ft})$ $= 4.16 \text{ kip/ft}$ $M_u = \dfrac{4.16 \text{ kip/ft}(30.0 \text{ ft})^2}{8}$ $= 468 \text{ kip-ft}$	$w_a = 0.80 \text{ kip/ft} + 2 \text{ kip/ft}$ $= 2.80 \text{ kip/ft}$ $M_a = \dfrac{2.80 \text{ kip/ft}(30.0 \text{ ft})^2}{8}$ $= 315 \text{ kip-ft}$

Minimum Required Moment of Inertia

The maximum live load deflection, Δ_{max}, occurs at midspan and is calculated as:

$$\Delta_{max} = \frac{5w_L l^4}{384EI} \quad \text{from AISC } \textit{Manual} \text{ Table 3-23 case 1}$$

Rearranging and substituting Δ_{max} = 1.00 in.,

$$\begin{aligned} I_{min} &= \frac{5(2 \text{ kips/ft})(30.0 \text{ ft})^4 (12 \text{ in./ft})^3}{384(29{,}000 \text{ ksi})(1.00 \text{ in.})} \\ &= 1{,}260 \text{ in.}^4 \end{aligned}$$

Beam Selection

Select the lightest section with the required moment of inertia from the bold entries in AISC *Manual* Table 3-3.

Try a W24×55.

$I_x = 1{,}350 \text{ in.}^4 > 1{,}260 \text{ in.}^4$ **o.k.**

Because the W24×55 is continuously braced and compact, its strength is governed by the yielding limit state and AISC *Specification* Section F2.1

From AISC *Manual* Table 3-2, the available flexural strength is:

LRFD	ASD
$\phi_b M_n = \phi_b M_{px}$ $= 503$ kip-ft 503 kip-ft > 468 kip-ft **o.k.**	$\dfrac{M_n}{\Omega_b} = \dfrac{M_{px}}{\Omega_b}$ $= 334$ kip-ft 334 kip-ft > 315 kip-ft **o.k.**

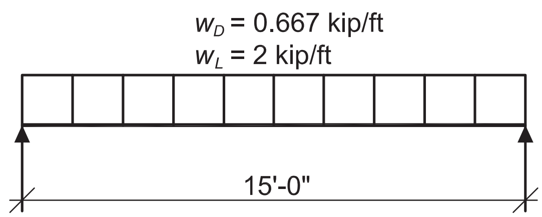
Return to Table of Contents

EXAMPLE F.5 I-SHAPED FLEXURAL MEMBER IN MINOR-AXIS BENDING

Given:

Select an ASTM A992 W-shape beam loaded in its minor axis with a simple span of 15 ft. The loads are a total uniform dead load of 0.667 kip/ft and a uniform live load of 2 kip/ft. Limit the live load deflection to L/240. The beam is braced at the ends only.

Note: Although not a common design case, this example is being used to illustrate AISC *Specification* Section F6 (I-shaped members and channels bent about their minor axis).



Beam Loading & Bracing Diagram
(*braced at ends only*)

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.667 \text{ kip/ft}) + 1.6(2 \text{ kip/ft})$ $= 4.00 \text{ kip/ft}$ $M_u = \dfrac{4.00\text{kip/ft}(15.0 \text{ ft})^2}{8}$ $= 113 \text{ kip-ft}$	$w_a = 0.667 \text{ kip/ft} + 2 \text{ kip/ft}$ $= 2.67 \text{ kip/ft}$ $M_a = \dfrac{2.67\text{kip/ft}(15.0 \text{ ft})^2}{8}$ $= 75.1 \text{ kip-ft}$

Minimum Required Moment of Inertia

The maximum live load deflection permitted is:

$$\begin{aligned}\Delta_{max} &= \frac{L}{240}\\ &= \frac{15.0 \text{ ft}(12 \text{ in./ft})}{240}\\ &= 0.750 \text{ in.}\end{aligned}$$

$$I_{req} = \frac{5w_L l^4}{384E\Delta_{max}} \quad \text{from AISC } \textit{Manual} \text{ Table 3-23 case 1}$$

$$= \frac{5(2.00 \text{ kip/ft})(15.0 \text{ ft})^4(12 \text{ in./ft})^3}{384(29{,}000 \text{ ksi})(0.750 \text{ in.})}$$

$= 105 \text{ in.}^4$

Beam Selection

Select the lightest section from the bold entries in AISC *Manual* Table 3-5, due to the likelihood that deflection will govern this design.

Try a W12×58.

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W12×58
$S_y = 21.4 \text{ in.}^3$
$Z_y = 32.5 \text{ in.}^3$
$I_y = 107 \text{ in.}^4 > 105 \text{ in.}^4$ **o.k.**

AISC *Specification* Section F6 applies. Because the W12×58 has compact flanges per the User Note in this Section, the yielding limit state governs the design.

$$M_n = M_p = F_y Z_y \leq 1.6 F_y S_y \qquad \text{(\textit{Spec}. Eq. F6-1)}$$
$$= 50 \text{ ksi}(32.5 \text{ in.}^3) \leq 1.6(50 \text{ ksi})(21.4 \text{ in.}^3)$$
$$= 1{,}630 \text{ kip-in.} \leq 1{,}710 \text{ kip-in.} \quad \textbf{o.k.}$$

$M_n = 1{,}630$ kip-in. or 136 kip-ft

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(136 \text{ kip-ft})$ $= 122 \text{ kip-ft}$ 122 kip-ft > 113 kip-ft **o.k.**	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{136 \text{ kip-ft}}{1.67}$ $= 81.4 \text{ kip-ft}$ 81.4 kip-ft > 75.1 kip-ft **o.k.**

Return to Table of Contents

Chapter G
Design of Members for Shear

INTRODUCTION

This chapter covers webs of singly or doubly symmetric members subject to shear in the plane of the web, single angles, HSS sections, and shear in the weak direction of singly or doubly symmetric shapes.

Most of the equations from this chapter are illustrated by example. Tables for all standard ASTM A992 W-shapes and ASTM A36 channels are included in the AISC *Manual*. In the tables, where applicable, LRFD and ASD shear information is presented side-by-side for quick selection, design and verification.

LRFD and ASD will produce identical designs for the case where the live load effect is approximately three times the dead load effect.

G1. GENERAL PROVISIONS

The design shear strength, $\phi_v V_n$, and the allowable shear strength, V_n/Ω_v, are determined as follows:

V_n = nominal shear strength based on shear yielding or shear buckling

$$V_n = 0.6F_y A_w C_v \qquad \text{(\textit{Spec}. Eq. G2-1)}$$

$\phi_v = 0.90$ (LRFD) $\qquad\qquad \Omega_v = 1.67$ (ASD)

Exception: For all current ASTM A6, W, S and HP shapes except W44×230, W40×149, W36×135, W33×118, W30×90, W24×55, W16×26 and W12×14 for F_y = 50 ksi:

$\phi_v = 1.00$ (LRFD) $\qquad\qquad \Omega_v = 1.50$ (ASD)

AISC *Specification* Section G2 does not utilize tension field action. AISC *Specification* Section G3 specifically addresses the use of tension field action.

Strong axis shear values are tabulated for W-shapes in AISC *Manual* Tables 3-2 and 3-6, for S-shapes in AISC *Manual* Table 3-7, for C-shapes in AISC *Manual* Table 3-8, and for MC-shapes in AISC *Manual* Table 3-9. Weak axis shear values for W-shapes, S-shapes, C-shapes and MC-shapes, and shear values for angles, rectangular HSS and box members, and round HSS are not tabulated.

G2. MEMBERS WITH UNSTIFFENED OR STIFFENED WEBS

As indicated in the User Note of this section, virtually all W, S and HP shapes are not subject to shear buckling and are also eligible for the more liberal safety and resistance factors, ϕ_v = 1.00 (LRFD) and Ω_v = 1.50 (ASD). This is presented in Example G.1 for a W-shape. A channel shear strength design is presented in Example G.2.

G3. TENSION FIELD ACTION

A built-up girder with a thin web and transverse stiffeners is presented in Example G.8.

G4. SINGLE ANGLES

Rolled angles are typically made from ASTM A36 steel. A single angle example is illustrated in Example G.3.

G5. RECTANGULAR HSS AND BOX-SHAPED MEMBERS

The shear height, h, is taken as the clear distance between the flanges less the inside corner radius on each side. If the corner radii are unknown, h shall be taken as the corresponding outside dimension minus 3 times the thickness. A rectangular HSS example is provided in Example G.4.

G6. ROUND HSS

For all round HSS and pipes of ordinary length listed in the AISC *Manual*, F_{cr} can be taken as $0.6F_y$ in AISC *Specification* Equation G6-1. A round HSS example is illustrated in Example G.5.

G7. WEAK AXIS SHEAR IN DOUBLY SYMMETRIC AND SINGLY SYMMETRIC SHAPES

For examples of weak axis shear, see Example G.6 and Example G.7.

G8. BEAMS AND GIRDERS WITH WEB OPENINGS

For a beam and girder with web openings example, see AISC Design Guide 2, *Steel and Composite Beams with Web Openings* (Darwin, 1990).

EXAMPLE G.1A W-SHAPE IN STRONG AXIS SHEAR

Given:

Determine the available shear strength and adequacy of a W24×62 ASTM A992 beam using the AISC *Manual* with end shears of 48 kips from dead load and 145 kips from live load.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

LRFD	ASD
V_u = 1.2(48.0 kips) + 1.6(145 kips) = 290 kips	V_a = 48.0 kips + 145 kips = 193 kips

From AISC *Manual* Table 3-2, the available shear strength is:

LRFD	ASD
$\phi_v V_n$ = 306 kips 306 kips > 290 kips **o.k.**	$\dfrac{V_n}{\Omega_v}$ = 204 kips 204 kips > 193 kips **o.k.**

Return to Table of Contents

EXAMPLE G.1B W-SHAPE IN STRONG AXIS SHEAR

Given:

The available shear strength, which can be easily determined by the tabulated values of the AISC *Manual*, can be verified by directly applying the provisions of the AISC *Specification*. Determine the available shear strength for the W-shape in Example G.1A by applying the provisions of the AISC *Specification*.

Solution:

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W24×62
d = 23.7 in.
t_w = 0.430 in.

Except for very few sections, which are listed in the User Note, AISC *Specification* Section G2.1(a) is applicable to the I-shaped beams published in the AISC *Manual* for F_y = 50 ksi.

$C_v = 1.0$ (*Spec*. Eq. G2-2)

Calculate A_w.

$A_w = dt_w$ from AISC *Specification* Section G2.1b
$= 23.7 \text{ in.}(0.430 \text{ in.})$
$= 10.2 \text{ in.}^2$

Calculate V_n.

$V_n = 0.6F_yA_wC_v$ (*Spec*. Eq. G2-1)
$= 0.6(50 \text{ ksi})(10.2 \text{ in.}^2)(1.0)$
$= 306 \text{ kips}$

From AISC *Specification* Section G2.1a, the available shear strength is:

LRFD	ASD
$\phi_v = 1.00$ $\phi_vV_n = 1.00(306 \text{ kips})$ $= 306 \text{ kips}$	$\Omega_v = 1.50$ $\dfrac{V_n}{\Omega_v} = \dfrac{306 \text{ kips}}{1.50}$ $= 204 \text{ kips}$

EXAMPLE G.2A C-SHAPE IN STRONG AXIS SHEAR

Given:

Verify the available shear strength and adequacy of a C15×33.9 ASTM A36 channel with end shears of 17.5 kips from dead load and 52.5 kips from live load.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

LRFD	ASD
V_u = 1.2(17.5 kips) + 1.6(52.5 kips) = 105 kips	V_a = 17.5 kips + 52.5 kips = 70.0 kips

From AISC *Manual* Table 3-8, the available shear strength is:

LRFD	ASD
$\phi_v V_n$ = 117 kips 117 kips > 105 kips **o.k.**	$\dfrac{V_n}{\Omega_v}$ = 77.6 kips 77.6 kips > 70.0 kips **o.k.**

EXAMPLE G.2B C-SHAPE IN STRONG AXIS SHEAR

Given:

The available shear strength, which can be easily determined by the tabulated values of the AISC *Manual*, can be verified by directly applying the provisions of the AISC *Specification*. Determine the available shear strength for the channel in Example G.2A.

Solution:

From AISC *Manual* Table 1-5, the geometric properties are as follows:

C15×33.9
d = 15.0 in.
t_w = 0.400 in.

AISC *Specification* Equation G2-1 is applicable. All ASTM A36 channels listed in the AISC *Manual* have $h/t_w \leq 1.10\sqrt{k_v E / F_y}$; therefore,

$$C_v = 1.0 \qquad \text{(\textit{Spec}. Eq. G2-3)}$$

Calculate A_w.

$$\begin{aligned} A_w &= dt_w \text{ from AISC \textit{Specification} Section G2.1b} \\ &= 15.0 \text{ in.}(0.400 \text{ in.}) \\ &= 6.00 \text{ in.}^2 \end{aligned}$$

Calculate V_n.

$$\begin{aligned} V_n &= 0.6F_yA_wC_v \qquad \text{(\textit{Spec}. Eq. G2-1)} \\ &= 0.6(36 \text{ ksi})(6.00 \text{ in.}^2)(1.0) \\ &= 130 \text{ kips} \end{aligned}$$

Available Shear Strength

The values of ϕ_v = 1.00 (LRFD) and Ω_v = 1.50 (ASD) do not apply to channels. The general values ϕ_v = 0.90 (LRFD) and Ω_v = 1.67 (ASD) must be used.

LRFD	ASD
$\phi_v V_n = 0.90(130 \text{ kips}) = 117 \text{ kips}$	$\dfrac{V_n}{\Omega_v} = \dfrac{130 \text{ kips}}{1.67} = 77.8 \text{ kips}$

EXAMPLE G.6 DOUBLY SYMMETRIC SHAPE IN WEAK AXIS SHEAR

Given:

Verify the available shear strength and adequacy of a W21×48 ASTM A992 beam with end shears of 20.0 kips from dead load and 60.0 kips from live load in the weak direction.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W21×48
$b_f = 8.14$ in.
$t_f = 0.430$ in.

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

LRFD	ASD
$V_u = 1.2(20.0 \text{ kips}) + 1.6(60.0 \text{ kips})$ $= 120$ kips	$V_a = 20.0 \text{ kips} + 60.0 \text{ kips}$ $= 80.0$ kips

From AISC *Specification* Section G7, for weak axis shear, use AISC *Specification* Equation G2-1 and AISC *Specification* Section G2.1(b) with $A_w = b_f t_f$ for each flange, $h/t_w = b/t_f$, $b = b_f/2$ and $k_v = 1.2$.

Calculate A_w. (Multiply by 2 for both shear resisting elements.)

$$\begin{aligned} A_w &= 2b_f t_f \\ &= 2(8.14 \text{ in.})(0.430 \text{ in.}) \\ &= 7.00 \text{ in.}^2 \end{aligned}$$

Calculate C_v.

$$\begin{aligned} h/t_w &= b/t_f \\ &= \frac{(8.14 \text{ in.})/2}{0.430 \text{ in.}} \\ &= 9.47 \end{aligned}$$

$$\begin{aligned} 1.10\sqrt{k_v E/F_y} &= 1.10\sqrt{1.2(29{,}000 \text{ ksi}/50 \text{ ksi})} \\ &= 29.0 \geq 9.47\text{, therefore, } C_v = 1.0 \end{aligned}$$ (*Spec*. Eq. G2-3)

Note: For all ASTM A6 W-, S-, M- and HP-shapes when $F_y \leq 50$ ksi, $C_v = 1.0$, except some M-shapes noted in the User Note at the end of AISC *Specification* Section G2.1.

Calculate V_n.

$$\begin{aligned} V_n &= 0.6F_y A_w C_v \\ &= 0.6(50 \text{ ksi})(7.00 \text{ in.}^2)(1.0) \\ &= 210 \text{ kips} \end{aligned}$$ (*Spec*. Eq. G2-1)

From AISC *Specification* Section G1, the available shear strength is:

LRFD	ASD
$\phi_v = 0.90$ $\phi_v V_n = 0.90(210\text{ kips})$ $= 189\text{ kips}$ 189 kips > 120 kips **o.k.**	$\Omega_v = 1.67$ $\dfrac{V_n}{\Omega_v} = \dfrac{210\text{ kips}}{1.67}$ $= 126\text{ kips}$ 126 kips > 80.0 kips **o.k.**

Chapter H
Design of Members for Combined Forces and Torsion

For all interaction equations in AISC *Specification* Chapter H, the required forces and moments must include second-order effects, as required by Chapter C of the AISC *Specification*. ASD users of the 1989 AISC *Specification* are accustomed to using an interaction equation that includes a partial second-order amplification. Second order effects are now calculated in the analysis and are not included in these interaction equations.

Return to Table of Contents

EXAMPLE H.1A W-SHAPE SUBJECT TO COMBINED COMPRESSION AND BENDING ABOUT BOTH AXES (BRACED FRAME)

Given:

Using AISC *Manual* Table 6-1, determine if an ASTM A992 W14×99 has sufficient available strength to support the axial forces and moments listed as follows, obtained from a second-order analysis that includes P-δ effects. The unbraced length is 14 ft and the member has pinned ends. $KL_x = KL_y = L_b = 14.0$ ft.

LRFD	ASD
P_u = 400 kips M_{ux} = 250 kip-ft M_{uy} = 80.0 kip-ft	P_a = 267 kips M_{ax} = 167 kip-ft M_{ay} = 53.3 kip-ft

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

The combined strength parameters from AISC *Manual* Table 6-1 are:

LRFD	ASD
$p = \dfrac{0.887}{10^3 \text{ kips}}$ at 14.0 ft $b_x = \dfrac{1.38}{10^3 \text{ kip-ft}}$ at 14.0 ft $b_y = \dfrac{2.85}{10^3 \text{ kip-ft}}$ Check limit for AISC *Specification* Equation H1-1a. From AISC *Manual* Part 6, $\dfrac{P_u}{\phi_c P_n} = pP_u$ $= \left(\dfrac{0.887}{10^3 \text{ kips}}\right)(400 \text{ kips})$ $= 0.355$ Because $pP_u \geq 0.2$, $pP_u + b_xM_{ux} + b_yM_{uy} \leq 1.0$ (*Manual* Eq. 6-1)	$p = \dfrac{1.33}{10^3 \text{ kips}}$ at 14.0 ft $b_x = \dfrac{2.08}{10^3 \text{ kip-ft}}$ at 14.0 ft $b_y = \dfrac{4.29}{10^3 \text{ kip-ft}}$ Check limit for AISC *Specification* Equation H1-1a. From AISC *Manual* Part 6, $\dfrac{P_a}{P_n / \Omega_c} = pP_a$ $= \left(\dfrac{1.33}{10^3 \text{ kips}}\right)(267 \text{ kips})$ $= 0.355$ Because $pP_a \geq 0.2$, $pP_a + b_xM_{ax} + b_yM_{ay} \leq 1.0$ (*Manual* Eq. 6-1)

LRFD	ASD
$0.355+\left(\dfrac{1.38}{10^3\text{ kip-ft}}\right)(250\text{ kip-ft})$ $+\left(\dfrac{2.85}{10^3\text{ kip-ft}}\right)(80.0\text{ kip-ft})\leq 1.0$ $= 0.355 + 0.345 + 0.228$ $= 0.928 \leq 1.0$ **o.k.**	$0.355+\left(\dfrac{2.08}{10^3\text{ kip-ft}}\right)(167\text{ kip-ft})$ $+\left(\dfrac{4.29}{10^3\text{ kip-ft}}\right)(53.3\text{ kip-ft})\leq 1.0$ $= 0.355 + 0.347 + 0.229$ $= 0.931 \leq 1.0$ **o.k.**

AISC *Manual* Table 6-1 simplifies the calculation of AISC *Specification* Equations H1-1a and H1-1b. A direct application of these equations is shown in Example H.1B.

Return to Table of Contents

EXAMPLE H.1B W-SHAPE SUBJECT TO COMBINED COMPRESSION AND BENDING MOMENT ABOUT BOTH AXES (BRACED FRAME)

Given:

Using AISC *Manual* tables to determine the available compressive and flexural strengths, determine if an ASTM A992 W14×99 has sufficient available strength to support the axial forces and moments listed as follows, obtained from a second-order analysis that includes P-δ effects. The unbraced length is 14 ft and the member has pinned ends. $KL_x = KL_y = L_b = 14.0$ ft.

LRFD	ASD
$P_u = 400$ kips $M_{ux} = 250$ kip-ft $M_{uy} = 80.0$ kip-ft	$P_a = 267$ kips $M_{ax} = 167$ kip-ft $M_{ay} = 53.3$ kip-ft

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

The available axial and flexural strengths from AISC *Manual* Tables 4-1, 3-10 and 3-4 are:

LRFD	ASD
at $KL_y = 14.0$ ft, $P_c = \phi_c P_n = 1{,}130$ kips	at $KL_y = 14.0$ ft, $P_c = \dfrac{P_n}{\Omega_c} = 750$ kips
at $L_b = 14.0$ ft, $M_{cx} = \phi M_{nx} = 642$ kip-ft	at $L_b = 14.0$ ft, $M_{cx} = M_{nx}/\Omega = 428$ kip-ft
$M_{cy} = \phi M_{ny} = 311$ kip-ft	$M_{cy} = \dfrac{M_{ny}}{\Omega} = 207$ kip-ft
$\dfrac{P_u}{\phi_c P_n} = \dfrac{400\text{ kips}}{1{,}130\text{ kips}}$ $= 0.354$	$\dfrac{P_a}{P_n/\Omega_c} = \dfrac{267\text{ kips}}{750\text{ kips}}$ $= 0.356$
Because $\dfrac{P_u}{\phi_c P_n} \geq 0.2$, use AISC *Specification* Equation H1-1a.	Because $\dfrac{P_a}{P_n/\Omega_c} \geq 0.2$, use AISC *Specification* Equation H1-1a.
$\dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \leq 1.0$ $\dfrac{400\text{ kips}}{1{,}130\text{ kips}} + \dfrac{8}{9}\left(\dfrac{250\text{ kip-ft}}{642\text{ kip-ft}} + \dfrac{80.0\text{ kip-ft}}{311\text{ kip-ft}}\right)$ $= 0.354 + \dfrac{8}{9}(0.389 + 0.257)$ $= 0.928 < 1.0$ **o.k.**	$\dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \leq 1.0$ $\dfrac{267\text{ kips}}{750\text{ kips}} + \dfrac{8}{9}\left(\dfrac{167\text{ kip-ft}}{428\text{ kip-ft}} + \dfrac{53.3\text{ kip-ft}}{207\text{ kip-ft}}\right)$ $= 0.356 + \dfrac{8}{9}(0.390 + 0.257)$ $= 0.931 < 1.0$ **o.k.**

EXAMPLE H.2 W-SHAPE SUBJECT TO COMBINED COMPRESSION AND BENDING MOMENT ABOUT BOTH AXES (BY AISC *SPECIFICATION* SECTION H2)

Given:

Using AISC *Specification* Section H2, determine if an ASTM A992 W14×99 has sufficient available strength to support the axial forces and moments listed as follows, obtained from a second-order analysis that includes P-δ effects. The unbraced length is 14 ft and the member has pinned ends. $KL_x = KL_y = L_b = 14.0$ ft. This example is included primarily to illustrate the use of AISC *Specification* Section H2.

LRFD	ASD
P_u = 360 kips M_{ux} = 250 kip-ft M_{uy} = 80.0 kip-ft	P_a = 240 kips M_{ax} = 167 kip-ft M_{ay} = 53.3 kip-ft

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W14×99
A = 29.1 in.^{2}
S_x = 157 in.^{3}
S_y = 55.2 in.^{3}

The required flexural and axial stresses are:

LRFD	ASD
$f_{ra} = \dfrac{P_u}{A}$ $= \dfrac{360\text{ kips}}{29.1\text{ in.}^2}$ $= 12.4\text{ ksi}$ $f_{rbx} = \dfrac{M_{ux}}{S_x}$ $= \dfrac{250\text{ kip-ft}}{157\text{ in.}^3}\left(\dfrac{12\text{ in.}}{\text{ft}}\right)$ $= 19.1\text{ ksi}$	$f_{ra} = \dfrac{P_a}{A}$ $= \dfrac{240\text{ kips}}{29.1\text{ in.}^2}$ $= 8.25\text{ ksi}$ $f_{rbx} = \dfrac{M_{ax}}{S_x}$ $= \dfrac{167\text{ kip-ft}}{157\text{ in.}^3}\left(\dfrac{12\text{ in.}}{\text{ft}}\right)$ $= 12.8\text{ ksi}$

LRFD	ASD
$f_{rby} = \dfrac{M_{uy}}{S_y}$ $= \dfrac{80.0\text{ kip-ft}}{55.2\text{ in.}^3}\left(\dfrac{12\text{ in.}}{\text{ft}}\right)$ $= 17.4\text{ ksi}$	$f_{rby} = \dfrac{M_{ay}}{S_y}$ $= \dfrac{53.3\text{ kip-ft}}{55.2\text{ in.}^3}\left(\dfrac{12\text{ in.}}{\text{ft}}\right)$ $= 11.6\text{ ksi}$

Calculate the available flexural and axial stresses from the available strengths in Example H.1B.

LRFD	ASD
$F_{ca} = \phi_c F_{cr}$ $= \dfrac{\phi_c P_n}{A}$ $= \dfrac{1{,}130\text{ kips}}{29.1\text{ in.}^2}$ $= 38.8\text{ ksi}$ $F_{cbx} = \dfrac{\phi_b M_{nx}}{S_x}$ $= \dfrac{642\text{ kip-ft}}{157\text{ in.}^3}\left(\dfrac{12\text{ in.}}{\text{ft}}\right)$ $= 49.1\text{ ksi}$ $F_{cby} = \dfrac{\phi_b M_{ny}}{S_y}$ $= \dfrac{311\text{ kip-ft}}{55.2\text{ in.}^3}\left(\dfrac{12\text{ in.}}{\text{ft}}\right)$ $= 67.6\text{ ksi}$	$F_{ca} = \dfrac{F_{cr}}{\Omega_c}$ $= \dfrac{P_n}{\Omega_c A}$ $= \dfrac{750\text{ kips}}{29.1\text{ in.}^2}$ $= 25.8\text{ ksi}$ $F_{cbx} = \dfrac{M_{nx}}{\Omega_b S_x}$ $= \dfrac{428\text{ kip-ft}}{157\text{ in.}^3}\left(\dfrac{12\text{ in.}}{\text{ft}}\right)$ $= 32.7\text{ ksi}$ $F_{cby} = \dfrac{M_{ny}}{\Omega_b S_y}$ $= \dfrac{207\text{ kip-ft}}{55.2\text{ in.}^3}\left(\dfrac{12\text{ in.}}{\text{ft}}\right)$ $= 45.0\text{ ksi}$

As shown in the LRFD calculation of F_{cby} in the preceding text, the available flexural stresses can exceed the yield stress in cases where the available strength is governed by yielding and the yielding strength is calculated using the plastic section modulus.

Combined Stress Ratio

From AISC *Specification* Section H2, check the combined stress ratios as follows:

LRFD	ASD
$\left\lvert \dfrac{f_{ra}}{F_{ca}} + \dfrac{f_{rbx}}{F_{cbx}} + \dfrac{f_{rby}}{F_{cby}} \right\rvert \le 1.0$ (from *Spec.* Eq. H2-1) $\left\lvert \dfrac{12.4\text{ ksi}}{38.8\text{ ksi}} + \dfrac{19.1\text{ ksi}}{49.1\text{ ksi}} + \dfrac{17.4\text{ ksi}}{67.6\text{ ksi}} \right\rvert = 0.966 \le 1.0$ **o.k.**	$\left\lvert \dfrac{f_{ra}}{F_{ca}} + \dfrac{f_{rbx}}{F_{cbx}} + \dfrac{f_{rby}}{F_{cby}} \right\rvert \le 1.0$ (from *Spec.* Eq. H2-1) $\left\lvert \dfrac{8.25\text{ ksi}}{25.8\text{ ksi}} + \dfrac{12.8\text{ ksi}}{32.7\text{ ksi}} + \dfrac{11.6\text{ ksi}}{45.0\text{ ksi}} \right\rvert = 0.969 \le 1.0$ **o.k.**

A comparison of these results with those from Example H.1B shows that AISC *Specification* Equation H1-1a will produce less conservative results than AISC *Specification* Equation H2-1 when its use is permitted.

Note: This check is made at a point on the cross-section (extreme fiber, in this example). The designer must therefore determine which point on the cross-section is critical, or check multiple points if the critical point cannot be readily determined.

EXAMPLE H.3 W-SHAPE SUBJECT TO COMBINED AXIAL TENSION AND FLEXURE

Given:

Select an ASTM A992 W-shape with a 14-in. nominal depth to carry forces of 29.0 kips from dead load and 87.0 kips from live load in axial tension, as well as the following moments due to uniformly distributed loads:

$M_{xD} = 32.0$ kip-ft
$M_{xL} = 96.0$ kip-ft

$M_{yD} = 11.3$ kip-ft
$M_{yL} = 33.8$ kip-ft

The unbraced length is 30.0 ft and the ends are pinned. Assume the connections are made with no holes.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
$P_u = 1.2(29.0 \text{ kips}) + 1.6(87.0 \text{ kips})$ $= 174$ kips $M_{ux} = 1.2(32.0 \text{ kip-ft}) + 1.6(96.0 \text{ kip-ft})$ $= 192$ kip-ft $M_{uy} = 1.2(11.3 \text{ kip-ft}) + 1.6(33.8 \text{ kip-ft})$ $= 67.6$ kip-ft	$P_a = 29.0 \text{ kips} + 87.0 \text{ kips}$ $= 116$ kips $M_{ax} = 32.0 \text{ kip-ft} + 96 \text{ kip-ft}$ $= 128$ kip-ft $M_{ay} = 11.3 \text{ kip-ft} + 33.8 \text{ kip-ft}$ $= 45.1$ kip-ft

Try a W14×82.

From AISC *Manual* Tables 1-1 and 3-2, the geometric properties are as follows:

W14×82
$A = 24.0 \text{ in.}^2$
$S_x = 123 \text{ in.}^3$
$Z_x = 139 \text{ in.}^3$
$S_y = 29.3 \text{ in.}^3$
$Z_y = 44.8 \text{ in.}^3$
$I_y = 148 \text{ in.}^4$
$L_p = 8.76$ ft
$L_r = 33.2$ ft

Nominal Tensile Strength

From AISC *Specification* Section D2(a), the nominal tensile strength due to tensile yielding on the gross section is:

$$P_n = F_y A_g \qquad (\textit{Spec}.\ \text{Eq. D2-1})$$
$$= 50 \text{ ksi}(24.0 \text{ in.}^2)$$
$$= 1{,}200 \text{ kips}$$

Note that for a member with holes, the rupture strength of the member would also have to be computed using AISC *Specification* Equation D2-2.

Nominal Flexural Strength for Bending About the X-X Axis

Yielding

From AISC *Specification* Section F2.1, the nominal flexural strength due to yielding (plastic moment) is:

$$
\begin{aligned}
M_{nx} &= M_p \\
&= F_y Z_x \qquad \text{(}Spec\text{. Eq. F2-1)} \\
&= 50 \text{ ksi}(139 \text{ in.}^3) \\
&= 6{,}950 \text{ kip-in.}
\end{aligned}
$$

Lateral-Torsional Buckling

From AISC *Specification* Section F2.2, the nominal flexural strength due to lateral-torsional buckling is determined as follows:

Because $L_p < L_b \le L_r$, i.e., 8.76 ft < 30.0 ft < 33.2 ft, AISC *Specification* Equation F2-2 applies.

Lateral-Torsional Buckling Modification Factor, C_b

From AISC *Manual* Table 3-1, C_b = 1.14, without considering the beneficial effects of the tension force. However, per AISC *Specification* Section H1.2, C_b may be increased because the column is in axial tension.

$$
\begin{aligned}
P_{ey} &= \frac{\pi^2 E I_y}{L_b^2} \\
&= \frac{\pi^2 (29{,}000 \text{ ksi})(148 \text{ in.}^4)}{\left[30.0 \text{ ft}(12.0 \text{ in./ft})\right]^2} \\
&= 327 \text{ kips}
\end{aligned}
$$

LRFD	ASD
$\sqrt{1+\frac{\alpha P_u}{P_{ey}}} = \sqrt{1+\frac{1.0(174 \text{ kips})}{327 \text{ kips}}}$ $= 1.24$	$\sqrt{1+\frac{\alpha P_a}{P_{ey}}} = \sqrt{1+\frac{1.6(116 \text{ kips})}{327 \text{ kips}}}$ $= 1.25$

$$
\begin{aligned}
C_b &= 1.24(1.14) \\
&= 1.41
\end{aligned}
$$

$$
M_n = C_b\left[M_p - \left(M_p - 0.7F_y S_x\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(}Spec\text{. Eq. F2-2)}
$$

$$
\begin{aligned}
&= 1.41\left\{6{,}950 \text{ kip-in.} - \left[6{,}950 \text{ kip-in.} - 0.7(50 \text{ ksi})(123 \text{ in.}^3)\right]\left(\frac{30.0 \text{ ft} - 8.76 \text{ ft}}{33.2 \text{ ft} - 8.76 \text{ ft}}\right)\right\} \\
&= 6{,}560 \text{ kip-in.} \le M_p
\end{aligned}
$$

Therefore, use M_n = 6,560 kip-in. or 547 kip-ft **controls**

Local Buckling

Per AISC *Manual* Table 1-1, the cross section is compact at F_y = 50 ksi; therefore, the local buckling limit state does not apply.

Nominal Flexural Strength for Bending About the Y-Y Axis and the Interaction of Flexure and Tension

Because a W14×82 has compact flanges, only the limit state of yielding applies for bending about the *y-y* axis.

$$M_{ny} = M_p = F_y Z_y \le 1.6 F_y S_y \qquad (Spec.\ \text{Eq. F6-1})$$
$$= 50\text{ ksi}(44.8\text{ in.}^3) \le 1.6(50\text{ ksi})(29.3\text{ in.}^3)$$
$$= 2{,}240\text{ kip-in.} \le 2{,}340\text{ kip-in.}$$

Therefore, use M_{ny} = 2,240 kip-in. or 187 kip-ft

Available Strength

From AISC *Specification* Sections D2 and F1, the available strength is:

LRFD	ASD
$\phi_b = \phi_t = 0.90$	$\Omega_b = \Omega_t = 1.67$
$P_c = \phi_t P_n$ $= 0.90(1{,}200\text{ kips})$ $= 1{,}080\text{ kips}$	$P_c = \dfrac{P_n}{\Omega_t}$ $= \dfrac{1{,}200\text{ kips}}{1.67}$ $= 719\text{ kips}$
$M_{cx} = \phi_b M_{nx}$ $= 0.90(547\text{ kip-ft})$ $= 492\text{ kip-ft}$	$M_{cx} = \dfrac{M_{nx}}{\Omega_b}$ $= \dfrac{547\text{ kip-ft}}{1.67}$ $= 328\text{ kip-ft}$
$M_{cy} = \phi_b M_{ny}$ $= 0.90(187\text{ kip-ft})$ $= 168\text{ kip-ft}$	$M_{cy} = \dfrac{M_{ny}}{\Omega_b}$ $= \dfrac{187\text{ kip-ft}}{1.67}$ $= 112\text{ kip-ft}$

Interaction of Tension and Flexure

Check limit for AISC *Specification* Equation H1-1a.

LRFD	ASD
$\dfrac{P_r}{\phi_t P_n} = \dfrac{P_u}{\phi_t P_n}$ $= \dfrac{174\text{ kips}}{1{,}080\text{ kips}}$ $= 0.161 < 0.2$	$\dfrac{P_r}{P_n/\Omega_t} = \dfrac{P_a}{P_n/\Omega_t}$ $= \dfrac{116\text{ kips}}{719\text{ kips}}$ $= 0.161 < 0.2$

Therefore, AISC *Specification* Equation H1-1b applies.

$$\frac{P_r}{2P_c}+\left(\frac{M_{rx}}{M_{cx}}+\frac{M_{ry}}{M_{cy}}\right)\le 1.0 \qquad \text{(\textit{Spec}. Eq. H1-1b)}$$

LRFD	ASD
$\frac{174\text{ kips}}{2\left(1{,}080\text{ kips}\right)}+\frac{192\text{ kip-ft}}{492\text{ kip-ft}}+\frac{67.6\text{ kip-ft}}{168\text{ kip-ft}}\le 1.0$	$\frac{116\text{ kips}}{2\left(719\text{ kips}\right)}+\frac{128\text{ kip-ft}}{328\text{ kip-ft}}+\frac{45.1\text{ kip-ft}}{112\text{ kip-ft}}\le 1.0$
$0.873 \le 1.0$ **o.k.**	$0.874 \le 1.0$ **o.k.**

Return to Table of Contents

EXAMPLE H.4 W-SHAPE SUBJECT TO COMBINED AXIAL COMPRESSION AND FLEXURE

Given:

Select an ASTM A992 W-shape with a 10-in. nominal depth to carry axial compression forces of 5.00 kips from dead load and 15.0 kips from live load. The unbraced length is 14.0 ft and the ends are pinned. The member also has the following required moment strengths due to uniformly distributed loads, not including second-order effects:

M_{xD} = 15 kip-ft
M_{xL} = 45 kip-ft

M_{yD} = 2 kip-ft
M_{yL} = 6 kip-ft

The member is not subject to sidesway (no lateral translation).

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From Chapter 2 of ASCE/SEI 7, the required strength (not considering second-order effects) is:

LRFD	ASD
P_u = 1.2(5.00 kips) + 1.6(15.0 kips) = 30.0 kips	P_a = 5.00 kips + 15.0 kips = 20.0 kips
M_{ux} = 1.2(15.0 kip-ft) + 1.6(45.0 kip-ft) = 90.0 kip-ft	M_{ax} = 15.0 kip-ft + 45.0 kip-ft = 60.0 kip-ft
M_{uy} = 1.2(2.00 kip-ft) + 1.6(6.00 kip-ft) = 12.0 kip-ft	M_{ay} = 2.00 kip-ft + 6.00 kip-ft = 8.00 kip-ft

Try a W10×33.

From AISC *Manual* Tables 1-1 and 3-2, the geometric properties are as follows:

W10×33
A = 9.71 in.^{2}
S_x = 35.0 in.^{3}
Z_x = 38.8 in.^{3}
I_x = 171 in.^{4}
r_x = 4.19 in.
S_y = 9.20 in.^{3}
Z_y = 14.0 in.^{3}
I_y = 36.6 in.^{4}
r_y = 1.94 in.
L_p = 6.85 ft
L_r = 21.8 ft

Available Axial Strength

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0.

Because $KL_x = KL_y$ = 14.0 ft and $r_x > r_y$, the y-y axis will govern.

From AISC *Manual* Table 4-1, the available axial strength is:

LRFD	ASD
$P_c = \phi_c P_n$ $= 253$ kips	$P_c = \dfrac{P_n}{\Omega_c}$ $= 168$ kips

Required Flexural Strength (including second-order amplification)

Use the approximate method of second-order analysis procedure from AISC *Specification* Appendix 8. Because the member is not subject to sidesway, only P-δ amplifiers need to be added.

$$B_1 = \frac{C_m}{1 - \alpha P_r / P_{e1}} \geq 1 \qquad \text{(\textit{Spec}. Eq. A-8-3)}$$

$C_m = 1.0$

The x-x axis flexural magnifier is,

$$P_{e1} = \frac{\pi^2 EI_x}{(K_1 L_x)^2} \qquad \text{(from \textit{Spec}. Eq. A-8-5)}$$

$$= \frac{\pi^2 (29{,}000\text{ ksi})(171\text{ in.}^4)}{\left[(1.0)(14.0\text{ ft})(12\text{ in./ft})\right]^2}$$

$$= 1{,}730 \text{ kips}$$

LRFD	ASD
$\alpha = 1.0$ $B_1 = \dfrac{1.0}{1 - 1.0(30.0\text{ kips}/1{,}730\text{ kips})}$ $= 1.02$ $M_{ux} = 1.02(90.0 \text{ kip-ft})$ $= 91.8$ kip-ft	$\alpha = 1.6$ $B_1 = \dfrac{1.0}{1 - 1.6(20.0\text{ kips}/1{,}730\text{ kips})}$ $= 1.02$ $M_{ax} = 1.02(60.0 \text{ kip-ft})$ $= 61.2$ kip-ft

The y-y axis flexural magnifier is,

$$P_{e1} = \frac{\pi^2 EI_y}{(K_1 L_y)^2} \qquad \text{(from \textit{Spec}. Eq. A-8-5)}$$

$$= \frac{\pi^2 (29{,}000\text{ ksi})(36.6\text{ in.}^4)}{\left[(1.0)(14.0\text{ ft})(12\text{ in./ft})\right]^2}$$

$$= 371 \text{ kips}$$

LRFD	ASD
$\alpha = 1.0$ $B_1 = \dfrac{1.0}{1-1.0\left(30.0\text{ kips}/371\text{ kips}\right)}$ $= 1.09$ $M_{uy} = 1.09(12.0\text{ kip-ft})$ $= 13.1\text{ kip-ft}$	$\alpha = 1.6$ $B_1 = \dfrac{1.0}{1-1.6\left(20.0\text{ kips}/371\text{ kips}\right)}$ $= 1.09$ $M_{ay} = 1.09(8.00\text{ kip-ft})$ $= 8.72\text{ kip-ft}$

Nominal Flexural Strength about the X-X Axis

Yielding

$$\begin{aligned} M_{nx} = M_p &= F_yZ_x \\ &= 50\text{ ksi}(38.8\text{ in.}^3) \\ &= 1{,}940\text{ kip-in} \end{aligned}$$ (*Spec*. Eq. F2-1)

Lateral-Torsional Buckling

Because $L_p < L_b \le L_r$, i.e., 6.85 ft < 14.0 ft < 21.8 ft, AISC *Specification* Equation F2-2 applies.

From AISC *Manual* Table 3-1, $C_b = 1.14$

$$M_{nx} = C_b\left[M_p - \left(M_p - 0.7F_yS_x\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$$ (*Spec*. Eq. F2-2)

$$= 1.14\left\{1{,}940\text{ kip-in.} - \left[1{,}940\text{ kip-in.} - 0.7\left(50\text{ ksi}\right)\left(35.0\text{ in.}^3\right)\right]\left(\frac{14.0\text{ ft} - 6.85\text{ ft}}{21.8\text{ ft} - 6.85\text{ ft}}\right)\right\}$$

$$= 1{,}820\text{ kip-in.} \le 1{,}940\text{ kip-in.}$$

Therefore, use M_{nx} = 1,820 kip-in. or 152 kip-ft **controls**

Local Buckling

Per AISC *Manual* Table 1-1, the member is compact for F_y = 50 ksi, so the local buckling limit state does not apply.

Nominal Flexural Strength about the Y-Y Axis

Determine the nominal flexural strength for bending about the *y-y* axis from AISC *Specification* Section F6. Because a W10×33 has compact flanges, only the yielding limit state applies.

From AISC *Specification* Section F6.2,

$$\begin{aligned} M_{ny} = M_p &= F_yZ_y \le 1.6F_yS_y \\ &= 50\text{ ksi}(14.0\text{ in.}^3) \le 1.6(50\text{ ksi})(9.20\text{ in.}^3) \\ &= 700\text{ kip-in} \le 736\text{ kip-in.} \end{aligned}$$ (*Spec*. Eq. F6-1)

Therefore, use M_{ny} = 700 kip-in. or 58.3 kip-ft

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $M_{cx} = \phi_b M_{nx}$ $= 0.90(152 \text{ kip-ft})$ $= 137 \text{ kip-ft}$ $M_{cy} = \phi_b M_{ny}$ $= 0.90(58.3 \text{ kip-ft})$ $= 52.5 \text{ kip-ft}$	$\Omega_b = 1.67$ $M_{cx} = \dfrac{M_{nx}}{\Omega_b}$ $= \dfrac{152 \text{ kip-ft}}{1.67}$ $= 91.0 \text{ kip-ft}$ $M_{cy} = \dfrac{M_{ny}}{\Omega_b}$ $= \dfrac{58.3 \text{ kip-ft}}{1.67}$ $= 34.9 \text{ kip-ft}$

Check limit for AISC *Specification* Equations H1-1a and H1-1b.

LRFD	ASD
$\dfrac{P_r}{P_c} = \dfrac{P_u}{\phi_c P_n}$ $= \dfrac{30.0 \text{ kips}}{253 \text{ kips}}$ $= 0.119 < 0.2$, therefore, use AISC *Specification* Equation H1-1b $\dfrac{P_r}{2P_c} + \left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \leq 1.0$ (*Spec*. Eq. H1-1b) $\dfrac{30.0 \text{ kips}}{2(253 \text{ kips})} + \left(\dfrac{91.8 \text{ kip-ft}}{137 \text{ kip-ft}} + \dfrac{13.1 \text{ kip-ft}}{52.5 \text{ kip-ft}}\right)$ $0.0593 + 0.920 = 0.979 \leq 1.0$ **o.k.**	$\dfrac{P_r}{P_c} = \dfrac{P_a}{P_n / \Omega_c}$ $= \dfrac{20.0 \text{ kips}}{168 \text{ kips}}$ $= 0.119 < 0.2$, therefore, use AISC *Specification* Equation H1-1b $\dfrac{P_r}{2P_c} + \left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \leq 1.0$ (*Spec*. Eq. H1-1b) $\dfrac{20.0 \text{ kips}}{2(168 \text{ kips})} + \left(\dfrac{61.2 \text{ kip-ft}}{91.0 \text{ kip-ft}} + \dfrac{8.72 \text{ kip-ft}}{34.9 \text{ kip-ft}}\right)$ $0.0595 + 0.922 = 0.982 \leq 1.0$ **o.k.**

Chapter J
Design of Connections

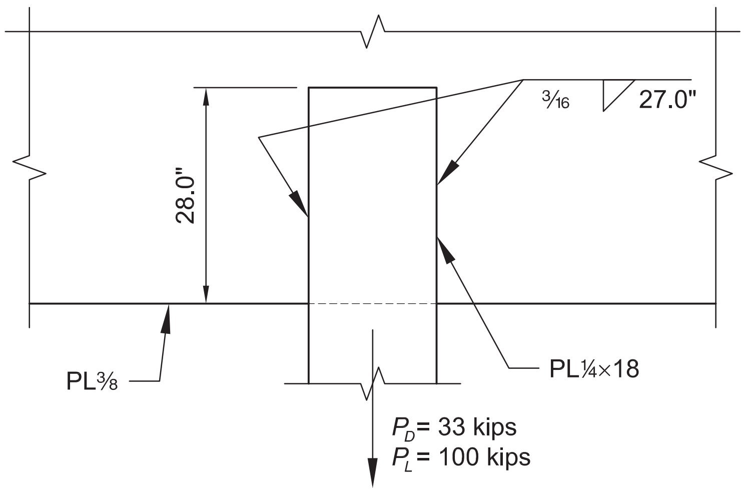
Chapter J of the AISC *Specification* addresses the design and review of connections. The chapter's primary focus is the design of welded and bolted connections. Design requirements for fillers, splices, column bases, concentrated forces, anchors rods and other threaded parts are also covered. Special requirements for connections subject to fatigue are not covered in this chapter.

EXAMPLE J.1 FILLET WELD IN LONGITUDINAL SHEAR

Given:

A ¼-in. × 18-in. wide plate is fillet welded to a ⅜-in. plate. The plates are ASTM A572 Grade 50 and have been properly sized. Use 70-ksi electrodes. Note that the plates would normally be specified as ASTM A36, but F_y = 50 ksi plate has been used here to demonstrate the requirements for long welds.

Verify the welds for the loads shown.



Solution:

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
P_u = 1.2(33.0 kips) + 1.6(100 kips) = 200 kips	P_a = 33.0 kips + 100 kips = 133 kips

Maximum and Minimum Weld Size

Because the thickness of the overlapping plate is ¼ in., the maximum fillet weld size that can be used without special notation per AISC *Specification* Section J2.2b, is a 3⁄16-in. fillet weld. A 3⁄16-in. fillet weld can be deposited in the flat or horizontal position in a single pass (true up to 5⁄16-in.).

From AISC *Specification* Table J2.4, the minimum size of fillet weld, based on a material thickness of ¼ in. is ⅛ in.

Length of Weld Required

The nominal weld strength per inch of 3⁄16-in. weld, determined from AISC *Specification* Section J2.4(a) is:

$$
\begin{aligned}
R_n &= F_{nw}A_{we} \qquad \text{(}\textit{Spec}\text{. Eq. J2-4)}\\
&= (0.60\,F_{EXX})(A_{we})\\
&= 0.60(70\text{ ksi})\left(\tfrac{3}{16}\text{ in.}/\sqrt{2}\right)\\
&= 5.57\text{ kips/in.}
\end{aligned}
$$

LRFD	ASD
$\dfrac{P_u}{\phi R_n} = \dfrac{200 \text{ kips}}{0.75(5.57 \text{ kips/in.})}$ $= 47.9$ in. or 24 in. of weld on each side	$\dfrac{P_a \Omega}{R_n} = \dfrac{133 \text{ kips}(2.00)}{5.57 \text{ kips/in.}}$ $= 47.8$ in. or 24 in. of weld on each side

From AISC *Specification* Section J2.2b, for longitudinal fillet welds used alone in end connections of flat-bar tension members, the length of each fillet weld shall be not less than the perpendicular distance between them.

$24 \text{ in.} \geq 18 \text{ in.}$ **o.k.**

From AISC *Specification* Section J2.2b, check the weld length to weld size ratio, because this is an end loaded fillet weld.

$$\frac{L}{w} = \frac{24 \text{ in.}}{3/16 \text{ in.}}$$

$= 128 > 100$. therefore, AISC *Specification* Equation J2-1 must be applied, and the length of weld increased, because the resulting β will reduce the available strength below the required strength.

Try a weld length of 27 in.

The new length to weld size ratio is:

$$\frac{27.0 \text{ in.}}{3/16 \text{ in.}} = 144$$

For this ratio:

$$\beta = 1.2 - 0.002(l/w) \leq 1.0 \qquad (\textit{Spec}. \text{ Eq. J2-1})$$
$$= 1.2 - 0.002(144)$$
$$= 0.912$$

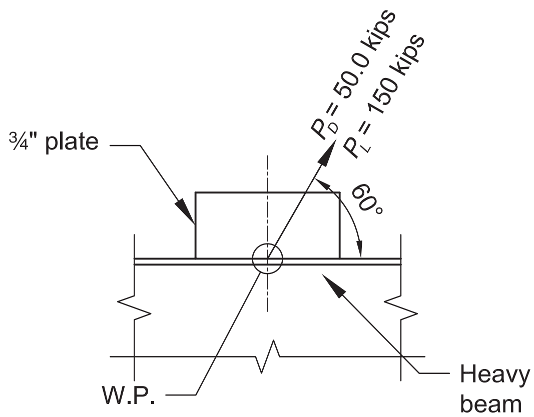
Recheck the weld at its reduced strength.

LRFD	ASD
$\phi R_n = (0.912)(0.75)(5.57 \text{ kips/in.})(54.0 \text{ in.})$ $= 206 \text{ kips} > P_u = 200 \text{ kips}$ **o.k.** Therefore, use 27 in. of weld on each side.	$\dfrac{R_n}{\Omega} = \dfrac{(0.912)(5.57 \text{ kips/in.})(54.0 \text{ in.})}{2.00}$ $= 137 \text{ kips} > P_a = 133 \text{ kips}$ **o.k.** Therefore, use 27 in. of weld on each side.

EXAMPLE J.2 FILLET WELD LOADED AT AN ANGLE

Given:

Design a fillet weld at the edge of a gusset plate to carry a force of 50.0 kips due to dead load and 150 kips due to live load, at an angle of 60° relative to the weld. Assume the beam and the gusset plate thickness and length have been properly sized. Use a 70-ksi electrode.



Solution:

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
$P_u = 1.2(50.0 \text{ kips}) + 1.6(150 \text{ kips})$ $= 300 \text{ kips}$	$P_a = 50.0 \text{ kips} + 150 \text{ kips}$ $= 200 \text{ kips}$

Assume a 5⁄16-in. fillet weld is used on each side of the plate.

Note that from AISC *Specification* Table J2.4, the minimum size of fillet weld, based on a material thickness of ¾ in. is ¼ in.

Available Shear Strength of the Fillet Weld Per Inch of Length

From AISC *Specification* Section J2.4(a), the nominal strength of the fillet weld is determined as follows:

$$A_{we} = \frac{5/16 \text{ in.}}{\sqrt{2}}$$
$$= 0.221 \text{ in.}$$

$$F_{nw} = 0.60F_{EXX}\left(1.0 + 0.5\sin^{1.5}\theta\right) \qquad (\textit{Spec}.\ \text{Eq. J2-5})$$
$$= 0.60(70 \text{ ksi})\left(1.0 + 0.5\sin^{1.5}60^\circ\right)$$
$$= 58.9 \text{ ksi}$$

$$R_n = F_{nw}A_{we} \qquad (\textit{Spec}.\ \text{Eq. J2-4})$$
$$= 58.9 \text{ ksi}(0.221 \text{ in.})$$
$$= 13.0 \text{ kip/in.}$$

From AISC *Specification* Section J2.4(a), the available shear strength per inch of weld length is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(13.0 \text{ kip/in.})$ $= 9.75 \text{ kip/in.}$ For 2 sides: $\phi R_n = 2(0.75)(13.0 \text{ kip/in.})$ $= 19.5 \text{ kip/in.}$	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{13.0 \text{ kip/in.}}{2.00}$ $= 6.50 \text{ kip/in.}$ For 2 sides: $\dfrac{R_n}{\Omega} = \dfrac{2(13.0 \text{ kip/in.})}{2.00}$ $= 13.0 \text{ kip/in.}$

Required Length of Weld

LRFD	ASD
$l = \dfrac{300 \text{ kips}}{19.5 \text{ kip/in.}}$ $= 15.4 \text{ in.}$ Use 16 in. on each side of the plate.	$l = \dfrac{200 \text{ kips}}{13.0 \text{ kip/in.}}$ $= 15.4 \text{ in.}$ Use 16 in. on each side of the plate.

EXAMPLE J.3 COMBINED TENSION AND SHEAR IN BEARING TYPE CONNECTIONS

Given:

A ¾-in.-diameter ASTM A325-N bolt is subjected to a tension force of 3.5 kips due to dead load and 12 kips due to live load, and a shear force of 1.33 kips due to dead load and 4 kips due to live load. Check the combined stresses according to AISC *Specification* Equations J3-3a and J3-3b.

Solution:

From Chapter 2 of ASCE/SEI 7, the required tensile and shear strengths are:

LRFD	ASD
Tension: $T_u = 1.2(3.50 \text{ kips}) + 1.6(12.0 \text{ kips})$ $= 23.4 \text{ kips}$ Shear: $V_u = 1.2(1.33 \text{ kips}) + 1.6(4.00 \text{ kips})$ $= 8.00 \text{ kips}$	Tension: $T_a = 3.50 \text{ kips} + 12.0 \text{ kips}$ $= 15.5 \text{ kips}$ Shear: $V_a = 1.33 \text{ kips} + 4.00 \text{ kips}$ $= 5.33 \text{ kips}$

Available Tensile Strength

When a bolt is subject to combined tension and shear, the available tensile strength is determined according to the limit states of tension and shear rupture, from AISC *Specification* Section J3.7 as follows.

From AISC *Specification* Table J3.2,

$F_{nt} = 90 \text{ ksi}$, $F_{nv} = 54 \text{ ksi}$

From AISC *Manual* Table 7-1, for a ¾-in.-diameter bolt,

$A_b = 0.442 \text{ in.}^2$

The available shear stress is determined as follows and must equal or exceed the required shear stress.

LRFD	ASD
$\phi = 0.75$ $\phi F_{nv} = 0.75(54 \text{ ksi})$ $= 40.5 \text{ ksi}$ $f_{rv} = \dfrac{V_u}{A_b}$ $= \dfrac{8.00 \text{ kips}}{0.442 \text{ in.}^2}$ $= 18.1 \text{ ksi} \leq 40.5 \text{ ksi}$	$\Omega = 2.00$ $\dfrac{F_{nv}}{\Omega} = \dfrac{54 \text{ ksi}}{2.00}$ $= 27.0$ $f_{rv} = \dfrac{V_a}{A_b}$ $= \dfrac{5.33 \text{ kips}}{0.442 \text{ in.}^2}$ $= 12.1 \text{ ksi} \leq 27.0 \text{ ksi}$ **o.k.**

The available tensile strength of a bolt subject to combined tension and shear is as follows:

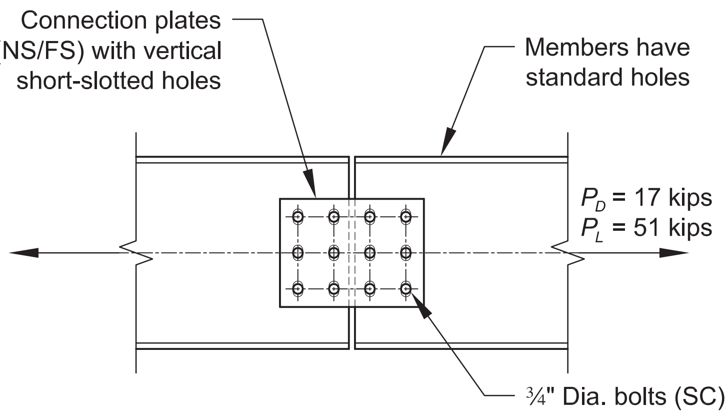
LRFD	ASD
$F'_{nt} = 1.3F_{nt} - \dfrac{F_{nt}}{\phi F_{nv}} f_{rv} \le F_{nt}$ (*Spec*. Eq. J3.3a) $= 1.3(90\text{ ksi}) - \dfrac{90\text{ ksi}}{0.75(54\text{ ksi})}(18.1\text{ ksi})$ $= 76.8\text{ ksi} \le 90\text{ ksi}$ $R_n = F'_{nt} A_b$ (*Spec*. Eq. J3-2) $= 76.8\text{ ksi}(0.442\text{ in.}^2)$ $= 33.9\text{ kips}$ For combined tension and shear, ϕ = 0.75 from AISC *Specification* Section J3.7 Design tensile strength: $\phi R_n = 0.75(33.9\text{ kips})$ $= 25.4\text{ kips} > 23.4\text{ kips}$ **o.k.**	$F'_{nt} = 1.3F_{nt} - \dfrac{\Omega F_{nt}}{F_{nv}} f_{rv} \le F_{nt}$ (*Spec*. Eq. J3.3b) $= 1.3(90\text{ ksi}) - \dfrac{2.00(90\text{ ksi})}{54\text{ ksi}}(12.1\text{ ksi})$ $= 76.7\text{ ksi} \le 90\text{ ksi}$ $R_n = F'_{nt} A_b$ (*Spec*. Eq. J3-2) $= 76.7\text{ ksi}(0.442\text{ in.}^2)$ $= 33.9\text{ kips}$ For combined tension and shear, Ω = 2.00 from AISC *Specification* Section J3.7 Allowable tensile strength: $\dfrac{R_n}{\Omega} = \dfrac{33.9\text{ kips}}{2.00}$ $= 17.0\text{ kips} > 15.5\text{ kips}$ **o.k.**

EXAMPLE J.4A SLIP-CRITICAL CONNECTION WITH SHORT-SLOTTED HOLES

Slip-critical connections shall be designed to prevent slip and for the limit states of bearing-type connections.

Given:

Select the number of ¾-in.-diameter ASTM A325 slip-critical bolts with a Class A faying surface that are required to support the loads shown when the connection plates have short slots transverse to the load and no fillers are provided. Select the number of bolts required for slip resistance only.



Solution:

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
$P_u = 1.2(17.0 \text{ kips}) + 1.6(51.0 \text{ kips})$ $= 102 \text{ kips}$	$P_a = 17.0 \text{ kips} + 51.0 \text{ kips}$ $= 68.0 \text{ kips}$

From AISC *Specification* Section J3.8(a), the available slip resistance for the limit state of slip for standard size and short-slotted holes perpendicular to the direction of the load is determined as follows:

$\phi = 1.00$ $\quad\Omega = 1.50$

μ = 0.30 for Class A surface
D_u = 1.13
h_f = 1.0, factor for fillers, assuming no more than one filler
T_b = 28 kips, from AISC *Specification* Table J3.1
n_s = 2, number of slip planes

$$R_n = \mu D_u h_f T_b n_s \qquad (Spec. \text{ Eq. J3-4})$$
$$= 0.30(1.13)(1.0)(28 \text{ kips})(2)$$
$$= 19.0 \text{ kips/bolt}$$

The available slip resistance is:

LRFD	ASD
$\phi R_n = 1.00(19.0 \text{ kips/bolt})$ $= 19.0 \text{ kips/bolt}$	$\dfrac{R_n}{\Omega} = \dfrac{19.0 \text{ kips/bolt}}{1.50}$ $= 12.7 \text{ kips/bolt}$

Required Number of Bolts

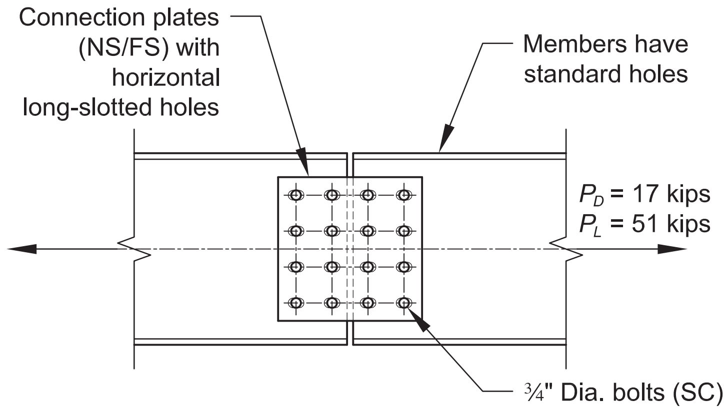
LRFD	ASD
$n_b = \dfrac{P_u}{\phi R_n}$ $= \dfrac{102 \text{ kips}}{19.0 \text{ kips/bolt}}$ $= 5.37 \text{ bolts}$ Use 6 bolts	$n_b = \dfrac{P_a}{\left(\dfrac{R_n}{\Omega}\right)}$ $= \dfrac{68.0 \text{ kips}}{(12.7 \text{ kips/bolt})}$ $= 5.37 \text{ bolts}$ Use 6 bolts

Note: To complete the design of this connection, the limit states of bolt shear, bolt bearing, tensile yielding, tensile rupture, and block shear rupture must be determined.

EXAMPLE J.4B SLIP-CRITICAL CONNECTION WITH LONG-SLOTTED HOLES

Given:

Repeat Example J.4A with the same loads, but assuming that the connected pieces have long-slotted holes in the direction of the load.



Solution:

The required strength from Example J.4A is:

LRFD	ASD
P_u = 102 kips	P_a = 68.0 kips

From AISC *Specification* Section J3.8(c), the available slip resistance for the limit state of slip for long-slotted holes is determined as follows:

$\phi = 0.70 \qquad \Omega = 2.14$

μ = 0.30 for Class A surface
D_u = 1.13
h_f = 1.0, factor for fillers, assuming no more than one filler
T_b = 28 kips, from AISC *Specification* Table J3.1
n_s = 2, number of slip planes

$$R_n = \mu D_u h_f T_b n_s \qquad \text{(Spec. Eq. J3-4)}$$
$$= 0.30(1.13)(1.0)(28\text{ kips})(2)$$
$$= 19.0\text{ kips/bolt}$$

The available slip resistance is:

LRFD	ASD
$\phi R_n = 0.70(19.0\text{ kips/bolt})$ $= 13.3\text{ kips/bolt}$	$\dfrac{R_n}{\Omega} = \dfrac{19.0\text{ kips/bolt}}{2.14}$ $= 8.88\text{ kips/bolt}$

Required Number of Bolts

LRFD	ASD
$n_b = \dfrac{P_u}{\phi R_n}$ $= \dfrac{102 \text{ kips}}{13.3 \text{ kips/bolt}}$ $= 7.67 \text{ bolts}$ Use 8 bolts	$n_b = \dfrac{P_a}{\left(\dfrac{R_n}{\Omega}\right)}$ $= \dfrac{68.0 \text{ kips}}{8.88 \text{ kips/bolt}}$ $= 7.66 \text{ bolts}$ Use 8 bolts

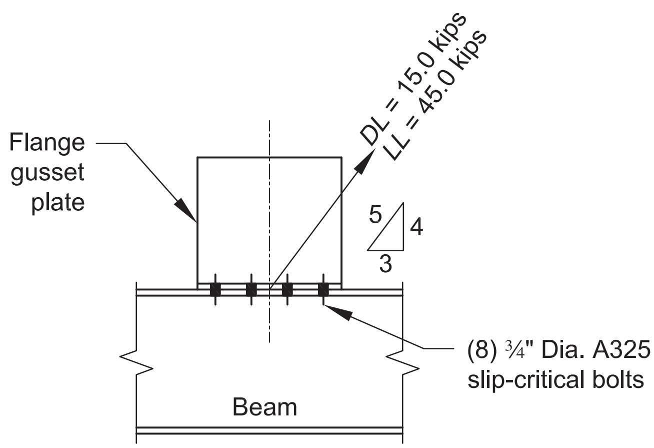
Note: To complete the design of this connection, the limit states of bolt shear, bolt bearing, tensile yielding, tensile rupture, and block shear rupture must be determined.

EXAMPLE J.5 COMBINED TENSION AND SHEAR IN A SLIP-CRITICAL CONNECTION

Because the pretension of a bolt in a slip-critical connection is used to create the clamping force that produces the shear strength of the connection, the available shear strength must be reduced for any load that produces tension in the connection.

Given:

The slip-critical bolt group shown as follows is subjected to tension and shear. Use ¾-in.-diameter ASTM A325 slip-critical Class A bolts in standard holes. This example shows the design for bolt slip resistance only, and assumes that the beams and plates are adequate to transmit the loads. Determine if the bolts are adequate.



Solution:

μ = 0.30 for Class A surface
D_u = 1.13
n_b = 8, number of bolts carrying the applied tension
h_f = 1.0, factor for fillers, assuming no more than one filler
T_b = 28 kips, from AISC *Specification* Table J3.1
n_s = 1, number of slip planes

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
$P_u = 1.2(15.0 \text{ kips})+1.6(45.0 \text{ kips})$ $= 90.0 \text{ kips}$ By geometry, $T_u = \frac{4}{5}(90.0 \text{ kips}) = 72.0 \text{ kips}$ $V_u = \frac{3}{5}(90.0 \text{ kips}) = 54.0 \text{ kips}$	$P_a = 15.0 \text{ kips} + 45.0 \text{ kips}$ $= 60.0 \text{ kips}$ By geometry, $T_a = \frac{4}{5}(60.0 \text{ kips}) = 48.0 \text{ kips}$ $V_a = \frac{3}{5}(60.0 \text{ kips}) = 36.0 \text{ kips}$

Available Bolt Tensile Strength

The available tensile strength is determined from AISC *Specification* Section J3.6.

From AISC *Specification* Table J3.2 for Group A bolts, the nominal tensile strength in ksi is, $F_{nt} = 90\text{ ksi}$. From AISC *Manual* Table 7-1, $A_b = 0.442\text{ in.}^2$

$$\begin{aligned} A_b &= \frac{\pi\left(\tfrac{3}{4}\text{ in.}\right)^2}{4} \\ &= 0.442\text{ in.}^2 \end{aligned}$$

The nominal tensile strength in kips is,

$$\begin{aligned} R_n &= F_{nt}A_b \qquad \text{(from } \textit{Spec}\text{. Eq. J3-1)} \\ &= 90\text{ ksi}\left(0.442\text{ in.}^2\right) \\ &= 39.8\text{ kips} \end{aligned}$$

The available tensile strength is,

LRFD	ASD
$\phi R_n = 0.75\left(\dfrac{39.8\text{ kips}}{\text{bolt}}\right) > \dfrac{72.0\text{ kips}}{8\text{ bolts}}$ $= 29.9\text{ kips/bolt} > 9.00\text{ kips/bolt}$ **o.k.**	$\dfrac{R_n}{\Omega} = \left(\dfrac{39.8\text{ kips/bolt}}{2.00}\right) > \dfrac{48.0\text{ kips}}{8\text{ bolts}}$ $= 19.9\text{ kips/bolt} > 6.00\text{ kips/bolt}$ **o.k.**

Available Slip Resistance Per Bolt

The available slip resistance of one bolt is determined using AISC *Specification* Equation J3-4 and Section J3.8.

LRFD	ASD
Determine the available slip resistance (T_u = 0) of a bolt. $\phi = 1.00$ $\phi R_n = \phi\mu D_u h_f T_b n_s$ $= 1.00(0.30)(1.13)(1.0)(28\text{ kips})(1)$ $= 9.49\text{ kips/bolt}$	Determine the available slip resistance (T_a = 0) of a bolt. $\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{\mu D_u h_f T_b n_s}{\Omega}$ $= \dfrac{0.30(1.13)(1.0)(28\text{ kips})(1)}{1.50}$ $= 6.33\text{ kips/bolt}$

Available Slip Resistance of the Connection

Because the clip-critical connection is subject to combined tension and shear, the available slip resistance is multiplied by a reduction factor provided in AISC *Specification* Section J3.9.

LRFD	ASD
Slip-critical combined tension and shear coefficient: $k_{sc} = 1 - \dfrac{T_u}{D_u T_b n_b}$ (*Spec*. Eq. J3-5a) $= 1 - \dfrac{72.0\text{ kips}}{1.13(28\text{ kips})(8)}$ $= 0.716$ $\phi = 1.00$	Slip-critical combined tension and shear coefficient: $k_{sc} = 1 - \dfrac{1.5T_a}{D_u T_b n_b}$ (*Spec*. Eq. J3-5b) $= 1 - \dfrac{1.5(48.0\text{ kips})}{1.13(28\text{ kips})(8)}$ $= 0.716$ $\Omega = 1.50$

LRFD	ASD
$\phi R_n = \phi R_n k_s n_b$ $= 9.49 \text{ kips/bolt}(0.716)(8 \text{ bolts})$ $= 54.4 \text{ kips} > 54.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{R_n}{\Omega} k_s n_b$ $= 6.33 \text{ kips/bolt}(0.716)(8 \text{ bolts})$ $= 36.3 \text{ kips} > 36.0 \text{ kips}$ **o.k.**

Note: The bolt group must still be checked for all applicable strength limit states for a bearing-type connection.

Chapter IIA
Simple Shear Connections

The design of simple shear connections is covered in Part 10 of the AISC *Steel Construction Manual*.

Return to Table of Contents

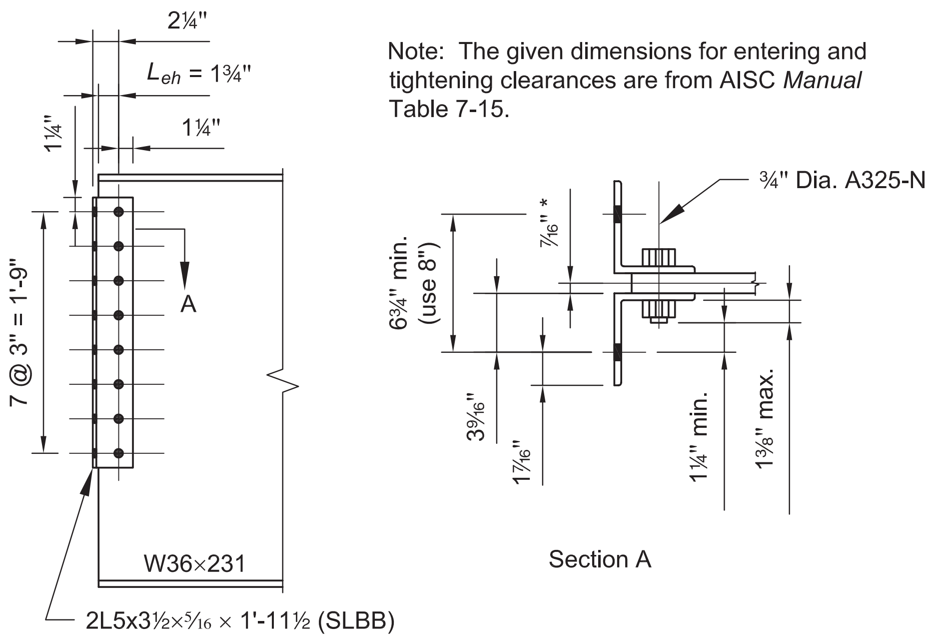
EXAMPLE II.A-1 ALL-BOLTED DOUBLE-ANGLE CONNECTION

Given:

Select an all-bolted double-angle connection between an ASTM A992 W36×231 beam and an ASTM A992 W14×90 column flange to support the following beam end reactions:

$R_D = 37.5$ kips
$R_L = 113$ kips

Use ¾-in.-diameter ASTM A325-N or F1852-N bolts in standard holes and ASTM A36 angles.



* This dimension (see sketch, Section A) is determined as one-half of the decimal web thickness rounded to the next higher 1⁄16 in. Example: 0.760"/2 = 0.380"; use 7⁄16 in. This will produce spacing of holes in the supporting beam slightly larger than detailed in the angles to permit spreading of angles (angles can be spread but not closed) at time of erection to supporting member. Alternatively, consider using horizontal short slots in the support legs of the angles.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

Column
ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

Angles

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W36×231
t_w = 0.760 in.

Column
W14×90
t_f = 0.710 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
R_u = 1.2(37.5 kips) + 1.6(113 kips) = 226 kips	R_a = 37.5 kips + 113 kips = 151 kips

Connection Design

AISC *Manual* Table 10-1 includes checks for the limit states of bearing, shear yielding, shear rupture, and block shear rupture on the angles, and shear on the bolts.

Try 8 rows of bolts and 2L5×3½×5⁄16 (SLBB).

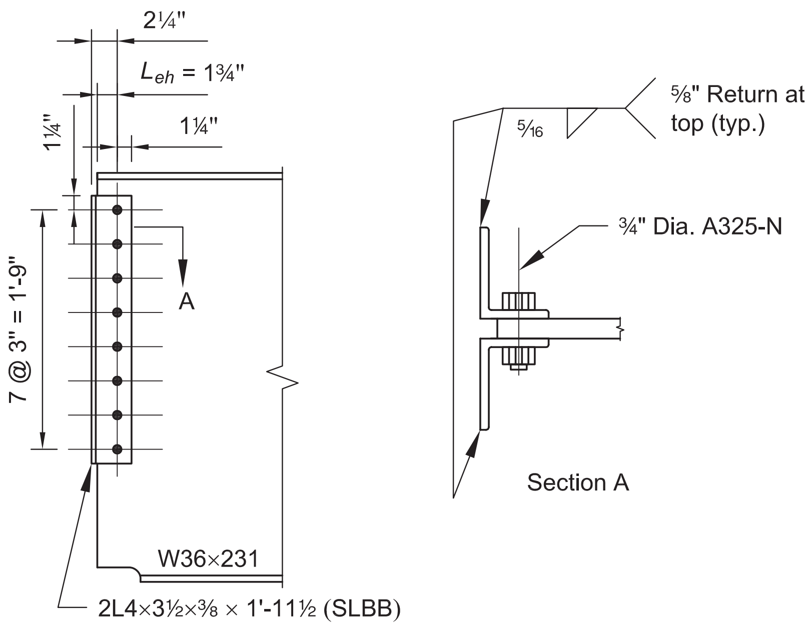
LRFD	ASD
ϕR_n = 247 kips > 226 kips **o.k.**	$\dfrac{R_n}{\Omega}$ = 165 kips > 151 kips **o.k.**
Beam web strength from AISC *Manual* Table 10-1:	Beam web strength from AISC *Manual* Table 10-1:
Uncoped, L_{eh} = 1¾ in.	Uncoped, L_{eh} = 1¾ in.
ϕR_n = 702 kips/in.(0.760 in.) = 534 kips > 226 kips **o.k.**	$\dfrac{R_n}{\Omega}$ = 468 kips/in.(0.760 in.) = 356 kips > 151 kips **o.k.**
Bolt bearing on column flange from AISC *Manual* Table 10-1:	Bolt bearing on column flange from AISC *Manual* Table 10-1:
ϕR_n = 1,400 kips/in.(0.710 in.) = 994 kips > 226 kips **o.k.**	$\dfrac{R_n}{\Omega}$ = 936 kips/in.(0.710 in.) = 665 kips > 151 kips **o.k.**

Return to Table of Contents

EXAMPLE II.A-2 BOLTED/WELDED DOUBLE-ANGLE CONNECTION

Given:

Repeat Example II.A-1 using AISC *Manual* Table 10-2 to substitute welds for bolts in the support legs of the double-angle connection (welds B). Use 70-ksi electrodes.



Note: Bottom flange coped for erection.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W36×231
t_w = 0.760 in.

Column
W14×90
$t_f = 0.710$ in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
$R_u = 1.2(37.5 \text{ kips}) + 1.6(113 \text{ kips})$ $= 226 \text{ kips}$	$R_a = 37.5 \text{ kips} + 113 \text{ kips}$ $= 151 \text{ kips}$

Weld Design using AISC Manual Table 10-2 (welds B)

Try 5⁄16-in. weld size, L = 23 ½ in.

$t_{f\,min} = 0.238 \text{ in.} < 0.710 \text{ in.}$ **o.k.**

LRFD	ASD
$\phi R_n = 279 \text{ kips} > 226 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 186 \text{ kips} > 151 \text{ kips}$ **o.k.**

Angle Thickness

The minimum angle thickness for a fillet weld from AISC *Specification* Section J2.2b is:

$$t_{min} = w + 1/16 \text{ in.}$$
$$= 5/16 \text{ in.} + 1/16 \text{ in.}$$
$$= 3/8 \text{ in.}$$

Try 2L4×3½×⅜ (SLBB).

Angle and Bolt Design

AISC *Manual* Table 10-1 includes checks for the limit states of bearing, shear yielding, shear rupture, and block shear rupture on the angles, and shear on the bolts.

Check 8 rows of bolts and ⅜-in. angle thickness.

LRFD	ASD
$\phi R_n = 286 \text{ kips} > 226 \text{ kips}$ **o.k.** Beam web strength: Uncoped, L_{eh} = 1¾ in. $\phi R_n = 702 \text{ kips/in.}(0.760 \text{ in.})$ $= 534 \text{ kips} > 226 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 191 \text{kips} > 151 \text{ kips}$ **o.k.** Beam web strength: Uncoped, L_{eh} = 1¾ in. $\dfrac{R_n}{\Omega} = 468 \text{ kips/in.}(0.760 \text{ in.})$ $= 356 \text{ kips} > 151 \text{ kips}$ **o.k.**

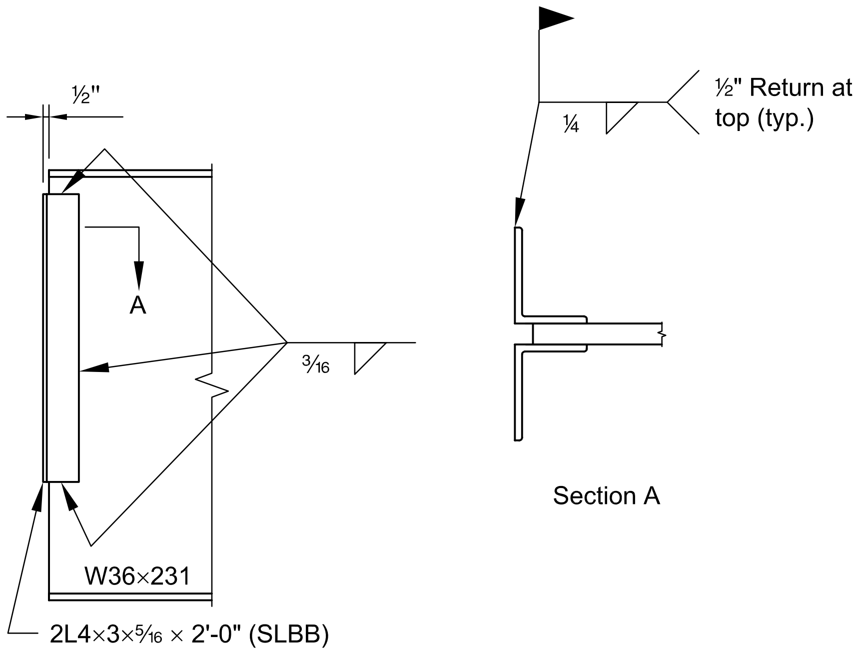
Note: In this example, because of the relative size of the cope to the overall beam size, the coped section does not control. When this cannot be determined by inspection, see AISC *Manual* Part 9 for the design of the coped section.

Return to Table of Contents

EXAMPLE II.A-3 ALL-WELDED DOUBLE-ANGLE CONNECTION

Given:

Repeat Example II.A-1 using AISC *Manual* Table 10-3 to design an all-welded double-angle connection between an ASTM A992 W36×231 beam and an ASTM A992 W14×90 column flange. Use 70-ksi electrodes and ASTM A36 angles.



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W36×231
t_w = 0.760 in.

Column
W14×90
t_f = 0.710 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
R_u = 1.2(37.5 kips) + 1.6(113 kips) = 226 kips	R_a = 37.5 kips + 113 kips = 151 kips

Design of Weld Between Beam Web and Angle (welds A)

Try 3/16-in. weld size, L = 24 in.

$t_{w\,min}$ = 0.286 in. < 0.760 in. **o.k.**

From AISC *Manual* Table 10-3:

LRFD	ASD
ϕR_n = 257 kips > 226 kips **o.k.**	$\dfrac{R_n}{\Omega}$ = 171 kips > 151 kips **o.k.**

Design of Weld Between Column Flange and Angle (welds B)

Try ¼-in. weld size, L = 24 in.

$t_{f\,min}$ = 0.190 in. < 0.710 in. **o.k.**

From AISC *Manual* Table 10-3:

LRFD	ASD
ϕR_n = 229 kips > 226 kips **o.k.**	$\dfrac{R_n}{\Omega}$ = 153 kips > 151 kips **o.k.**

Angle Thickness

Minimum angle thickness for weld from AISC *Specification* Section J2.2b:

$$t_{min} = w + \tfrac{1}{16}\text{ in.}$$
$$= \tfrac{1}{4}\text{ in.} + \tfrac{1}{16}\text{ in.}$$
$$= \tfrac{5}{16}\text{ in.}$$

Try 2L4×3×5/16 (SLBB).

Shear Yielding of Angles (AISC Specification Section J4.2)

$$A_{gv} = 2(24.0\text{ in.})(\tfrac{5}{16}\text{ in.})$$
$$= 15.0\text{ in.}^2$$

$$R_n = 0.60F_yA_{gv} \qquad (\textit{Spec}.\text{ Eq. J4-3})$$
$$= 0.60(36\text{ ksi})(15.0\text{ in.}^2)$$
$$= 324\text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(324 \text{ kips})$ $= 324 \text{ kips} > 226 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{324 \text{ kips}}{1.50}$ $= 216 \text{ kips} > 151 \text{ kips}$ **o.k.**

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W8

Shape	W8× 15				W8× 13				W8× 10[c,f]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	5.79	3.85	20.2	13.4	6.69	4.45	24.0	16.0	9.33	6.21	32.4	21.6
1	5.89	3.92	20.2	13.4	6.82	4.54	24.0	16.0	9.47	6.30	32.4	21.6
2	6.21	4.13	20.2	13.4	7.23	4.81	24.0	16.0	9.93	6.61	32.4	21.6
3	6.79	4.52	20.6	13.7	7.96	5.29	24.8	16.5	10.8	7.15	32.4	21.6
4	7.70	5.12	22.1	14.7	9.11	6.06	26.8	17.9	12.1	8.03	34.9	23.2
5	9.04	6.01	23.9	15.9	10.8	7.20	29.3	19.5	14.1	9.39	38.4	25.6
6	11.0	7.32	26.1	17.4	13.4	8.91	32.3	21.5	17.4	11.6	42.8	28.5
7	13.9	9.23	28.7	19.1	17.2	11.4	35.9	23.9	22.4	14.9	48.3	32.2
8	18.0	12.0	31.8	21.2	22.5	14.9	41.0	27.3	29.3	19.5	58.8	39.1
9	22.8	15.2	36.9	24.5	28.4	18.9	49.1	32.7	37.1	24.7	71.3	47.4
10	28.1	18.7	42.9	28.5	35.1	23.4	57.4	38.2	45.8	30.4	84.3	56.1
11	34.0	22.6	48.9	32.5	42.5	28.3	65.8	43.8	55.4	36.8	97.6	64.9
12	40.5	26.9	54.9	36.5	50.6	33.6	74.3	49.4	65.9	43.8	111	73.9
13	47.5	31.6	60.9	40.5	59.3	39.5	82.7	55.0	77.3	51.5	125	83.0
14	55.1	36.7	66.9	44.5	68.8	45.8	91.2	60.7	89.7	59.7	139	92.2

Other Constants and Properties

	W8×15 ASD	W8×15 LRFD	W8×13 ASD	W8×13 LRFD	W8×10 ASD	W8×10 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	103	68.3	127	84.8	177	118
$t_y \times 10^3$ (kips)$^{-1}$	5.79	3.85	6.69	4.45	8.68	5.78
$t_r \times 10^3$ (kips)$^{-1}$	7.51	5.00	8.68	5.79	11.3	7.50
r_x/r_y	3.76		3.81		3.83	
r_y, in.	0.876		0.843		0.841	

[c] Shape is slender for compression with F_y = 65 ksi.

[f] Shape does not meet compact limit for flexure with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y greater than or equal to 200.