Basic Introduction to Nonlinear Analysis

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The function of a structural engineer is to design — not to analyze
Norris and Wilbur 1960

Analysis is a means to an end rather than the end itself.

Role of the analysis:
- forces, moments and deflections ⇒ design equations
- insight into the behavior of a structure ⇒ better the understanding, better the design

Limit States Design:
Prior to limit of resistance, significant nonlinear response, including
- geometrical effects (P-Δ, P-δ)
- material effects (yielding, cracking, crushing)
- combined effects

Impetus:
Limit States Design
- AISC Ch. C: P-Δ, P-δ (App. 7)
- App. 1: Inelastic Design
- Seismic: Pushover Analysis
- Other: Progressive Collapse

Nonlinear Analysis
- Available Software
- Education
- Research

Lecture Overview
- Brief Introduction (done!)
- Computer Structural Analysis (Review?)
- Basis for Material Nonlinear Models
- Incorporating Geometric Nonlinear Behavior
- Critical Load Analysis
- Overview of MASTAN2 software
- Summary and Concluding Remarks

Nonlinear Analysis
- Hand methods
  - Second-order effects
    - i.e. Moment Amplification Factors (B1 and B2 factors)
  - Material nonlinear effects
    - i.e. plastic analysis (upper and lower bound theories)
- Computer Methods (focus of today's lecture)
  - Lots of variations
    - all use some basic concepts (most important to today)
    - one approach will be presented (basis for MASTAN2)
- Please keep in mind
  - All methods are approximate
  - Not a substitute, but a complement to good engineering
# of Equil. Eqs. = # of Unknown Displacements

State-of-the-Art Crystal Ball?  Not quite.
- Translator "apples to oranges"
- Two Requirements:
  - Equilibrium (equations in terms of Fs and Ms, 1 per d.o.f.)
  - Compatibility (equations in terms of Δs and θs, 1 per d.o.f.)
- Constitutive Relationship (i.e. Hooke’s Law, σ = E ε)
- Generalized to Force-to-Displacement (i.e. F = kΔ)
- Re-write equilibrium eqs. in terms of unknown displacements

Translator: Forces → Displacements

How does the computer get these results?

Big Question:
Where do these known stiffness coefficients k’s come from?

Little Answer:
Function of member’s material and geometric properties, including its orientation.

Substituting into Equil. Eqs.

Member AD:

\[
\begin{align*}
40 &= (k_{AD} u_{AD} + k_{BD} v_{BD} + k_{CD} u_{CD} + k_{AD} u_D) \\
\end{align*}
\]

Member BD:

\[
\begin{align*}
40 &= (k_{BD} u_{BD} + k_{CD} v_{CD} + k_{CD} u_{CD} + k_{BD} u_D) \\
\end{align*}
\]

Member CD:

\[
\begin{align*}
40 &= (k_{CD} u_{CD} + k_{BD} v_{BD} + k_{CD} u_{CD} + k_{AD} u_D) \\
\end{align*}
\]

Substituting into Equil. Eqs. (cont.)

Member AD:

\[
\begin{align*}
0 &= (k_{AD} u_{AD} + k_{BD} v_{BD} + k_{CD} u_{CD} + k_{AD} u_D) \\
\end{align*}
\]

Member BD:

\[
\begin{align*}
0 &= (k_{BD} u_{BD} + k_{CD} v_{CD} + k_{CD} u_{CD} + k_{BD} u_D) \\
\end{align*}
\]

Member CD:

\[
\begin{align*}
0 &= (k_{CD} u_{CD} + k_{BD} v_{BD} + k_{CD} u_{CD} + k_{CD} u_D) \\
\end{align*}
\]
So, where are we at?

- We have two equilibrium equations (1 per d.o.f.) in terms of a lot of displacements:
  \[ u_D = 40 = (k_{A3}^D u_A^{3D} + k_{B3}^D u_B^{3D} + k_{D3}^D u_D^{3D} + k_{CD}^D v_D^{3D}) + (k_{A4}^D u_A^{4D} + k_{B4}^D u_B^{4D} + k_{D4}^D u_D^{4D} + k_{CD}^D v_D^{4D}) + (k_{A5}^D u_A^{5D} + k_{B5}^D u_B^{5D} + k_{D5}^D u_D^{5D} + k_{CD}^D v_D^{5D}) \]
  \[ v_D = 0 = (k_{A3}^D u_A^{3D} + k_{C3}^D v_A^{3D} + k_{D3}^D u_D^{3D} + k_{CD}^D v_D^{3D}) + (k_{A4}^D u_A^{4D} + k_{C4}^D v_A^{4D} + k_{D4}^D u_D^{4D} + k_{CD}^D v_D^{4D}) + (k_{A5}^D u_A^{5D} + k_{C5}^D v_A^{5D} + k_{D5}^D u_D^{5D} + k_{CD}^D v_D^{5D}) \]

What card haven’t we played yet?

Time for some serious simplifying

- Applying Compatibility to Equil. Eqs.:
  \[ \text{All} = u_D \]
  \[ \text{All} = v_D \]

Which simplifies to...

With all displacements, solve for member forces...

Member AD:
\[ f_A = k_A^D u_A^{3D} + k_A^D u_A^{4D} + k_A^D u_A^{5D} + k_A^{CD} v_A^{3D} + k_A^{CD} v_A^{4D} + k_A^{CD} v_A^{5D} \]
\[ f_A = k_A^{3D} u_A^{3D} + k_A^{4D} u_A^{4D} + k_A^{5D} u_A^{5D} + k_A^{CD} v_A^{3D} + k_A^{CD} v_A^{4D} + k_A^{CD} v_A^{5D} \]
\[ f_A = k_A^{CD} u_A^{CD} + k_A^{CD} u_A^{CD} + k_A^{CD} u_A^{CD} + k_A^{CD} v_A^{CD} + k_A^{CD} v_A^{CD} + k_A^{CD} v_A^{CD} \]

Member BD:
\[ f_B = k_B^D u_B^{3D} + k_B^D u_B^{4D} + k_B^D u_B^{5D} + k_B^{CD} v_B^{3D} + k_B^{CD} v_B^{4D} + k_B^{CD} v_B^{5D} \]
\[ f_B = k_B^{3D} u_B^{3D} + k_B^{4D} u_B^{4D} + k_B^{5D} u_B^{5D} + k_B^{CD} v_B^{3D} + k_B^{CD} v_B^{4D} + k_B^{CD} v_B^{5D} \]
\[ f_B = k_B^{CD} u_B^{CD} + k_B^{CD} u_B^{CD} + k_B^{CD} u_B^{CD} + k_B^{CD} v_B^{CD} + k_B^{CD} v_B^{CD} + k_B^{CD} v_B^{CD} \]

Member CD:
\[ f_C = k_C^D u_C^{3D} + k_C^D u_C^{4D} + k_C^D u_C^{5D} + k_C^{CD} v_C^{3D} + k_C^{CD} v_C^{4D} + k_C^{CD} v_C^{5D} \]
\[ f_C = k_C^{3D} u_C^{3D} + k_C^{4D} u_C^{4D} + k_C^{5D} u_C^{5D} + k_C^{CD} v_C^{3D} + k_C^{CD} v_C^{4D} + k_C^{CD} v_C^{5D} \]
\[ f_C = k_C^{CD} u_C^{CD} + k_C^{CD} u_C^{CD} + k_C^{CD} u_C^{CD} + k_C^{CD} v_C^{CD} + k_C^{CD} v_C^{CD} + k_C^{CD} v_C^{CD} \]

Compatibility Eqs. (consistent deflections)

Member-to-Member

Member-to-Support
\[ u_D = 40 = u_D^{3D} = 0 \]
\[ v_D = 0 = v_D^{3D} = 0 \]
\[ u_B = u_B^{4D} = 0 \]
\[ v_B = v_B^{4D} = 0 \]
\[ u_D = u_D^{5D} = 0 \]
\[ v_D = v_D^{5D} = 0 \]

After simplifying...
\[ u_D = 40 = (k_{A3}^D + k_{B3}^D + k_{D3}^D) u_D^{3D} + (k_{A4}^D + k_{B4}^D + k_{D4}^D) u_D^{4D} + (k_{A5}^D + k_{B5}^D + k_{D5}^D) u_D^{5D} \]
\[ v_D = 0 = (k_{A3}^D + k_{C3}^D + k_{D3}^D) v_D^{3D} + (k_{A4}^D + k_{C4}^D + k_{D4}^D) v_D^{4D} + (k_{A5}^D + k_{C5}^D + k_{D5}^D) v_D^{5D} \]

Since k’s are known, we have 2 Equations and 2 Unknowns

\[ u_D = \# \quad \text{and} \quad v_D = \## \]

Solve for Unknown Displacements

Summary of Computer Approach

- For each d.o.f., write an equilibrium equation:
  \[ F_{\text{external}} = \sum f_{\text{member}} \]

- Re-write (translate) each member force in terms of its end displacements (Stiffness Eqs.)
  \[ f_{\text{member}} = \sum k_{\text{member}} \Delta_{\text{member end}} \]

- Substitute Stiffness Eqs. into above Equil. Eqs.

- Simplify Equil. Eqs. by applying member-to-member and member-to-support compatibility conditions

- Solve n Equil. Eqs. for the n unknown displacements

- Use Stiffness Eqs. to calculate member forces

- Apply Equil. Eqs. to solve for reactions
Lot’s of Questions

- So, this is how most commercial programs such as SAP2000, RISA, STAAD, etc. get the answer?
  - Yes! Known as "Direct Stiffness Method"
- So, all such programs will give the same answer?
  - Yes, as long as it is a static 1st-order elastic analysis.
- Wait a minute...Is this the basic analysis procedure for the "finite element method"?
  - Yes! Bit more tricky to get k’s, σ’s, and ε’s

Two Big Questions

- Where do those stiffness coefficients come from?
  - You mean the ones that relate member end forces to member end displacements?
  - Yeah, those k’s! <More to come on this>
- What happens when we go static nonlinear or even dynamic?
  - Same basic procedure, but apply loads in increments and perform a series of analyses. Then, sum incremental results. <Much more to come on this!>

Important Points

- The only opportunity for most computer analysis software to model the actual behavior of the structure is through the member stiffness terms.
- So, to include
  - first-order effects
  - second-order effects
  - material nonlinear behavior
  - Must modify member stiffness!!!
- Let’s review member stiffness

Stiffness Coefficients, k’s

- Let’s start with high school physics
  - Extension Spring Experiment

Before:
- Force
- Displacement
After:
- \( F = k \Delta \)

Stiffness Coefficients, k’s (cont.)

- More “advanced” high school physics lab
  - Rotational Spring Experiment

Before:
- \( M \)
- \( \theta \)
After:
- \( M = k \theta \)

How about real structural members?

- Axial force member

Before:
- \( F \)
- \( \Delta \)
After:
- \( F = k(A,L,E) \Delta \)

- Stiffness k function of:
  - Geometry: Area and Length (A↑, k↑ & L↑, k↓)
  - Material: Elastic Modulus (E↑, k↑)
How about real members? (cont.)

Flexural members
- Before:
  - After:

Stiffness $k$ function of:
- Geometry: Moment of Inertia & Length ($I$, $k$ & $L$, $k$)
- Material: Elastic Modulus ($E$, $k$)

Before:

After:

$F = k(I, L, E)$

$M = k(I, L, E)$

$\theta$

Other factor impacting stiffness

Orientation of member
- consider axial force member:
  - Vertical Member
  - Horizontal Member

$Fv = 0$

$Fh = E/A/L$

$kv = E/A/L$

$kh = (\sin^2 \phi) E/A/L$

Vertical Member

Horizontal Member

$\phi$

Summary: Three Perspectives

 Reality: What you see...

Important Point: Less vertical a member, the less stiffness to resist vertical loads.

Three Perspectives (cont.)

What you see on your computer screen:

Collection of elements connected by sharing common nodes

Three Perspectives

What your computer actually sees:

Assemblage of equivalent springs $\{F\} = [K]\{\Delta\}$
Analysis Review: Key Points

- Reviewed the "Direct Stiffness Method"
  - Equilibrium $\Rightarrow$ Translator $F(\Delta) \Rightarrow$ Compatibility
- Response of structure controlled by stiffness of members (a.k.a. springs)
- First-order elastic stiffness of member function of:
  - Material Property ($E$)
  - Geometric Properties ($A$, $I$, $L$, and orientation)
- Time to go nonlinear... let's begin with material nonlinear

Material Nonlinear (Inelastic)

- Best place to start is with a tensile test

Normal Stress: Structural Members

- For typical structural steel members ($L/d\approx10$), elastic/inelastic behavior controlled by normal stresses $\sigma$'s acting along the length axis of the member.
- Normal stress produced by:
  - Axial force ($P/A$)
  - Major and/or minor axis flexure ($Mc/I$)
  - Combination of above effects (i.e. $P/A + Mc/I$)
  - Warping (not today!)
- We will assume elastic-perfectly-plastic material (often done for steel)

Inelastic Behavior: Axial Force

- Originally: $\sigma = P/A < \sigma_y$
- Elastic: $\sigma = P/A < \sigma_y$
- Yield: $\sigma = \sigma_y$
- Post-Yield: $\sigma = \sigma_y$

Inelastic Behavior: Flexure

- $M_p = Z\sigma_y$
- $M_y = S\sigma_y$
- $\theta = 4EI/L$

Inelastic Behavior: Flexure (cont.)

- $M_p = Z\sigma_y$
- $M_y = S\sigma_y$
- $\theta = 4EI/L$
Inelastic Behavior: Flexure (cont.)

\[ M_p = Z \sigma_y \]
\[ M_y = S \sigma_y \]

where:

\[ k = \frac{4EI}{L} \]

\[ \theta = \frac{M}{M_p} \]

Section A-A

\[ E = 0 \]

EI ≪ EI_{elastic}

\[ M_y < M < M_p \]

\[ \sigma_y \]

\[ \sigma_f \]

\[ M = Z \sigma_y \]

\[ M_y = M_p \]

\[ M_p = M_y = M_f \]

\[ EI = 0 \rightarrow k = 0 \]

\[ EI > 0 \rightarrow k > 0 \]

\[ M = Z \sigma_y \]

\[ M_p = M_y = M_f \]

\[ EI = 0 \rightarrow k = 0 \]

\[ EI > 0 \rightarrow k > 0 \]

\[ M = Z \sigma_y \]

\[ M_p = M_y = M_f \]

Types of inelastic models

- We will employ a plastic hinge model
  - A.K.A. “Concentrated Plasticity”
  - Section is fully elastic or fully yielded
  - Plastic hinges only at element ends
- Distributed plasticity (still line elements)
  - A.K.A. “Plastic Zone”
  - Captures gradual yielding through depth and along length
  - More accurate, but computationally more $$$
- Finite element with continuum elements ($$$$)

Simple Example:

- E = 29,000 ksi
- \[ \sigma_y = 50 \text{ ksi} \]
- W12×65
- 100 kips
- 12' - 0"

Plastic Hinge Model - Assume section as fully elastic or fully plastic (neglect partial yielding)

Plastic Hinge at \( M = M_p \) or when \( M/M_p = 1.0 \)
Inelastic Behavior: Combination P & M

\[ M < M_p \]

\[ \sigma = \frac{Mc}{I} + \frac{P}{A} \]

Plastic Hinge Criterion:

\[ \frac{P}{P_y} = 1.0 \]
\[ \frac{M}{M_p} = 1.0 \]
\[ \sigma = \sigma_y \]
\[ P = 0 \quad M = M_p \]
\[ P > P_y \quad M > M_p \]

Material Nonlinear Analysis

- Employ "Direct Stiffness Method" applying loads in increments: \[ (K)(\Delta) = (F) \]
- During the load increment, check to see if plastic hinge(s) form. If so, scale back load increment accordingly.
- Reduce stiffness of yielded members and continue load increments
  \[ k = k_{\text{elastic}} + k_{\text{plastic}} \]
  with \( k_{\text{plastic}} = \) plastic reduction
- Continue to accumulate results of load increments until all of load is applied or a plastic mechanism forms.

Simple Example (with axial force):

- \[ P = 400 \text{ kips} \]
- \[ E = 29,000 \text{ ksi} \]
- \[ \sigma_y = 50 \text{ ksi} \]
Second-Order Effects

- A.K.A. "Geometric Nonlinear Behavior"
- Equilibrium Equations
  - Reality: Should be formulated on deformed shape
  - Difficulty: Deformed shape (deformations) is a function of the member forces, which are in turn a function of the deformations (Chicken 'n Egg)
  - Remedy: Perform a series of analyses with loads applied in small increments and update geometry after each load increment.

Equilibrium Equations

- Formulated on Undeformed Shape
- Formulated on Deformed Shape

Focus on Lateral Stiffness

- Formulated on Undeformed Shape: Linear Response
- Formulated on Deformed Shape

Different reactions and member forces.
Focus on Lateral Stiffness (cont.)

- Formulated on Deformed Shape: Nonlinear Response
  - Before:  
  - After: Effective lateral stiffness is reduced

Before:
\[ k_{\text{spring}} \Delta P \Delta H \]

After:
\[ \Delta k_{\text{lateral}} < k_{\text{spring}} \]

Effective lateral stiffness is reduced

Focus on Lateral Stiffness (cont.)

- Equilibrium Formulated on Deformed Shape
  - Let's start by assuming \( L' \approx L \),
    \[ \Sigma M_o = 0 \]
    \[ R = H + P \Delta/L \]
    \[ k_{\text{spring}} \Delta = H + P \Delta/L \]
    \[ H = k_{\text{spring}} \Delta - P \Delta/L \]
    \[ H = (k_{\text{spring}} - P/L) \Delta \]

Lateral Stiffness (slope of response curve)
\[ H = k_{\text{lateral}} \Delta \] with \( k_{\text{lateral}} = k_{\text{spring}} - P/L \)

Some thoughts here...
- This simple analysis becomes less "accurate" as \( \Delta/L \) becomes large (i.e. \( \Delta/L \gg 1/5 \))
  - Remedy: Perform an incremental analysis and update geometry after each load increment...hence, limit \( \Delta/L \) in each step to some small amount
  - Keep in mind serviceability limits are often something like \( \Delta/L < 1/400 \)
- Most importantly, \( k_{\text{lateral}} = k_{\text{spring}} - P/L \) takes on the form:
  \[ k_{2\text{nd-Order El.}} = k_{1\text{st-Order El.}} + k_g \] Geometric Stiffness

Geometric Stiffness
- Effective lateral stiffness of a member:
  - decreases as a member is compressed
    - \( k_g \) is negative for compressive \( P \)
      - backpacker example
  - increases when subjected to tension
    - \( k_g \) is positive for tensile \( P \)
      - guitar string example
- Employing geometric stiffness approach
  - Other methods exist (i.e. stability functions)

How about real members? (recall...)
- Flexural members subjected to axial force
  \[ F = k(I,L,E,P) \Delta \] with \( k = 4EI/L - 2PL/15 \)
  \[ M = k(I,L,E,P) \theta \]

Stiffness \( k \) function of:
- Geometry: Moment of Inertia & Length \((I, k_I & L, k_I)\)
- Material: Elastic Modulus \((E, k_E)\)
- Axial Force: Compressive \((P_f, k_f)\)

Closer look at stiffness terms...
- Flexural members subjected to axial force
  \[ F = k(I,L,E,P) \Delta \] with \( k = 12EI/L^3 - 6P/5L \)
  \[ M = k(I,L,E,P) \theta \]

Again, basic form:
\[ k_{2\text{nd-Order El.}} = k_{1\text{st-Order El.}} + k_g \]
**Geometric Nonlinear Analysis**

- Employ "Direct Stiffness Method" applying loads in increments: Solve Equil. Eqs. \( (dF) = [K](d\Delta) \)
- At start of increment, modify member stiffness to account for presence of member forces (such as axial force):
  - \( k = k_{\text{elastic}} + k_g \) with \( k_g = \) geometric stiffness
- At end of increment, update model of structural geometry to include displacements
- Continue to accumulate results of load increments \( (\Delta_i = \Delta_{i-1} + d\Delta \text{ and } f_i = f_{i-1} + df) \) until all of load is applied or elastic instability is detected.

**Comparison: 1st- and 2nd-Order Analysis Results**

- **2nd-Order Inelastic Analysis**
  - Employ "Direct Stiffness Method" applying loads in increments: Solve Equil. Eqs. \( (dF) = [K](d\Delta) \)
  - At start of increment, modify member stiffness to account for presence of member forces and any yielding:
    - \( k = k_{\text{elastic}} + k_{\text{geometric}} + k_{\text{plastic}} \)
  - At end of increment, update model of structural geometry to include displacements
  - Continue to accumulate results of load increments \( (\Delta_i = \Delta_{i-1} + d\Delta \text{ and } f_i = f_{i-1} + df) \) until all of load is applied or inelastic instability is detected.

**Critical Load Analysis (Basics)**

- Definition: Critical or buckling load is the load at which equilibrium may be satisfied by more than one deformed shape.

Big Q: How does computer software calculate this?
Critical Load Analysis (Background)

- Elastic stiffness of a member \( k = k_{el} + k_g \)
  - \( k_{el} \) is \( f(A \text{ or } I, L, \text{ and } E) \)
  - \( k_g \) is \( f(P, L) \), also note directly proportional to \( P \)
- Elastic stiffness of structure \([K] = \Sigma k\)
  - \([K_g] \) directly proportional to applied force
    - i.e. Double applied forces, hence, double internal force distribution and double \([K_g]\)
- To the computer, "buckling" will occur when our equilibrium equations \( (F) = [K] \Delta \) permit non-unique solutions, e.g. \( \text{det}[K] = 0 \).

Example

Demonstrate computational method for calculating the elastic critical load (buckling load) for the structural system shown.

Example: Key Stiffness Terms

Vertical Stiffness:
\[ P = k_{vertical} \Delta_{vert} \]

Lateral Stiffness:
\[ H = k_{lateral} \Delta_{lat} \]
\[ k_{lateral} = \frac{12EI}{L^3} - 6 \frac{P}{5L} \]

Example: Solution

1. Apply reference load, and use 1st-order elastic analysis to obtain internal force distribution.
2. Determine load factor \( \lambda \) at which system stiffness degrades to permit buckling.
\[ k_{lateral} = \frac{12EI}{L^3} - 6 \frac{P}{5L} \]
\[ k_{lateral} = 0 \text{ when } \lambda P = 10EI/L^2 \]
\[ P_{cr} = \lambda P = 10EI/L^2 \quad (P_{theory}=9.87EI/L^2) \]

Thoughts on Critical Load Analysis

- Computer analysis for a large system:
  - First, apply reference and perform analysis
    - Solve equilibrium eqs. \( (F_{ref}) = [K] \Delta \)
    - With displacements solve for member forces
  - Second, assemble \([K_{el}]\) and \([K_g]\) based on \( (F_{ref})\)
  - Finally, determine load factor \( \lambda \) causing instability; computationally this means find load factor \( \lambda \) at which \([K] = [K_{el}] + \lambda [K_g]\) becomes singular
    - Determine \( \lambda \) at which \( \text{det}([K_{el}] + \lambda [K_g]) = 0 \)
    - "Eigenvector" problem: Eigenvalues = Critical Load Factors, \( \lambda \)s
    - Eigenvectors = Buckling modes
  - Accuracy increases with more elements per compression members (2 often adequate)
Basic Introduction Complete

• Where do I go from here? (Learning to drive)
  - Review the slides (Read the driver’s manual)
  - Acquire nonlinear software (Borrow a friend’s car)
  - Work lots of examples (Go for a drive, scary at first…)
  - Apply nonlinear analysis in design (NASCAR? not quite)

Acquire nonlinear analysis software
  • Commercial programs
  • Educational software (i.e. MASTAN2)

Levels of Analysis:

1st-Order Elastic: $[K_e](\Delta) = \{F\}$
2nd-Order Elastic: $[K_e + K_g](d\Delta) = \{dF\}$
1st-Order Inelastic: $[K_e + K_p](d\Delta) = \{dF\}$
2nd-Order Inelastic: $[K_e + K_g + K_p](d\Delta) = \{dF\}$

Critical Load: $[K_e + \lambda K_g](d\Delta) = \{0\}$

Yield Surface:
Function of $P$, $M_{\text{major}}$, and $M_{\text{minor}}$

Planar Frame:

Mechanism

First hinge
Second hinge

Load Factor

Summary and Conclusions

Provided an introduction to nonlinear analysis
- Review of direct stiffness method
- Material nonlinear analysis (Inelastic hinge)
- Geometric nonlinear analysis (2nd-Order)
- 2nd-Order inelastic analysis (combine above)
- Critical load analysis (“eigenvalue analysis”)

Nonlinear... think modifying member stiffness!

Overview and availability of MASTAN2

Now, it’s your turn to take it for a spin…

MASTAN2:

- Educational software
- GUI ⇔ commercial programs
- Limited # of pre- and post-processing options to reduce learning curve
- Suite of linear and nonlinear 2D and 3D analysis routines
- Available with textbook or online at no cost

www.mastan2.com or www.aisc.org [Steel Tools]
Appendix

- Several examples to try out
- Solutions by MASTAN2
- Need a reference text with many examples? see Matrix Structural Analysis, 2nd Ed., by McGuire, Gallagher, and Ziemian (Wiley, 2000)
- See tutorial that comes with MASTAN2
- OK, time to jump in and start driving...
  <See Final Exam...>
Truss (Hoff et al.):

All members:

- \( A = 1.346 \times 10^{-2} \text{ in}^2 \)
- \( I = 6.954 \times 10^{-4} \text{ in}^4 \)
- \( E = 29,000 \text{ ksi} \)

Demonstrate:

- Elastic Critical Load
- 2nd-Order Elastic
- Experimental (\( P_{\text{limit}} = 220 \text{ lbs} \))

Elastic Critical Load: \( P_{\text{cr}} = 210.7 \text{ lbs} \)

2nd-Order Elastic: \( P_{\text{lim}} = 210 \text{ lbs} \)
Response Curves:

- Elastic Critical Load
- 2nd-Order Elastic
- 1st-Order Elastic

Beam-Column:

- W24×76
- E = 29,000 ksi
- L = 24'
- P = 210 lbs
- M = αPL

Demonstrate:
- Elastic Critical Load Analysis
  1. Flexural Buckling (α = 0.0)
  2. Torsional Flexural Buckling (α = 0.04)

MASTAN2 Model:

Elastic Critical Load Analysis (α = 0.0)

Suspension System:

- A = 5.40 in²
- σ_y = 150 ksi
- A = 50 in²
- I = 20,000 in⁴
- Z = 1,000 in³
- σ_y = 50 ksi

- Hinge Formation