SteelDay Eve 2013

The Evolution of Stability Provisions in the AISC Specification

Charles J. Carter[1]

Introduction

Buildings and the structural systems that support them have evolved, as have the requirements and practices upon which their design and construction are based. The treatment of stability effects has varied significantly through the history of (and before) the AISC Specification. Despite differences in the provisions in AISC Specifications of differing eras, it can be generalized that stability effects are addressed in all AISC Specifications in some combination of three features in the AISC Specification: the column buckling equation(s), the compression and flexure interaction equation(s), and the analysis requirements.

In this paper, the evolution of stability analysis and design provisions will be traced through observation of the changes made in these three areas up to the most current requirements: those included in the 2005 and 2010 versions of AISC 360 *Specification for Structural Steel Buildings* (AISC, 2005; AISC, 2010). These historical developments will be summarized by time period as follows:

1. Before the AISC Specification Existed
2. The First AISC Specification – 1923
3. Developments Leading to the 1963 AISC Specification
4. Refinements in Allowable Stress Design (1969 through 1989)
5. Load and Resistance Factor Design (1986 through 1999)
6. The 2005 and 2010 AISC Specifications

Additionally, the influence of changing characteristics of buildings and the structures supporting them will be summarized.

A frame of reference is needed to make more sense out of the historical developments as they are reviewed. The best frame of reference may be the current state-of-the-art, which can be found in Chapter C and Appendix 7 in the 2005 AISC Specification, and Chapter C and Appendices 7 and 8 in the 2010 AISC Specification. A detailed discussion is available in the corresponding Commentary sections, but for the purposes of this paper it is sufficient to review the list provided in Section C1.1 of factors that must be accounted for in stability analysis and design:

1. Flexural, shear, and axial deformations – these are the member deformations and all other component and connection deformations that contribute to displacements of the structure;

---

[1] Charles J. Carter, S.E., P.E., is Chief Structural Engineer at the American Institute of Steel Construction, Chicago, IL

2. Second-order effects – these are the increases that occur in forces and moments due to displacements of the structure induced by the loads, including both P-Δ effects (displacements of points of intersection of members) and P-δ effects (deformations of the members between points of intersection);
3. Geometric imperfections – these are the initial out-of-plumbness of the structure and the initial out-of-straightness of the members;
4. Stiffness reductions due to inelasticity – these are the effects of residual stresses; and,
5. Variability in component and system stiffness – these are the effects of variations in material and cross-sectional properties of members, as well as the other effects generally accounted for in the resistance factors (LRFD) and safety factors (ASD).

As will be explained in greater detail when the 2005 and 2010 AISC Specification are discussed, these factors combine to affect both the demand side (the analysis results) and the strength side (the member strength equations). It will also be highlighted throughout the review of historical developments which of these effects were considered and which were not.

Griffis and White (2013) provide an excellent compilation of current state-of-the-art for stability analysis and design. In preparing that publication, the history of column design, interaction, and stability was explored. That work is used extensively in this paper, and credited as Griffis and White (2008) because it was not included in the final manuscript of Griffis and White (2013)

Before the AISC Specification Existed

Griffis and White (2008) provide an excellent summary of stability-related developments in the pre-1923 era, including some of the very early developments:

> "While columns and frames have been in use for many centuries, it was not until 1729 that van Musschenbroek published the first paper concerning the strength of columns (Salmon, 1921). An empirical column curve was presented for a rectangular column taking the form,
>
> $$P = k\frac{bd^2}{L^2}$$
>
> where P is the column strength, b and d are the column width and depth, respectively, L is the column length, and k is an empirical factor. Interestingly, this equation has remarkable similarity to those still in use today.
>
> In 1759, Euler published his now famous treatise on the buckling of columns (Salmon, 1921). The original buckling load determined by Euler was for a column with one end fixed and the other free – a flagpole column. His equation took the form,

$$P = \frac{\pi^2 C}{4L^2}$$

where P is the buckling strength, L is the column length, and the constant C is the "absolute elasticity", which was defined as depending on the elastic properties of the material. Euler was the first to recognize that column strength could also be a problem of stability and not just a matter of crushing the material. Euler investigated the purely elastic phenomenon of buckling.

We know today that elastic instability of columns occurs only with very slender columns, and the theories that define inelastic column strength began to emerge over a century after the above developments in elasticity. Engesser published his tangent modulus theory in 1889, and followed this in 1895 with a revised theory called the reduced modulus theory.

The reduced modulus theory of inelastic buckling was accepted as the correct buckling theory until 1947 when Shanley published a paper giving the buckling load of a centrally loaded column as the tangent modulus load (Beedle, 1964). The critical buckling stress was given by the equation,

$$F_{cr} = \frac{\pi^2 E_t}{\left(\frac{L}{r}\right)^2}$$

E_t is the tangent modulus of the stress-strain relationship of the material at the critical stress. Indeed, in 1924 the forerunner to the Column Research Council declared this tangent modulus equation as the proper basis for establishing column load formulas (Beedle, 1964).

The first discussion on problems of the stability of members that were part of rectangular frames came in 1893 by Engesser (Timoshenko and Gere, 1961). Yet it would not be understood until the late 1940s that the key to the tangent modulus concept for steel column buckling was the inclusion of the effects of residual stresses that existed in the cross-section of the column even before the application of external load."

Regarding the five factors that influence stability, it is not surprising that few of them were explicitly addressed. Implicitly, however, there were compensating factors that explain why buildings rarely experienced problems with stability. Workmanship requirements for materials, fabrication, and erection, the customary use of heavy masonry infill details that added uncalculated strength and stiffness, and the factors of safety used probably served to manage P-Δ and P-δ effects, member, component, and connection deformations, and the effects of geometric imperfections in the erected structure. Analysis methods of this era were also conservative by nature, and although the term "skyscraper" had come into existence, it described buildings that were on the order of 10 stories tall.

It is interesting that the concept that would come to be known as effective length and consideration of the impact of residual stresses date as far back as these works. Nonetheless, the state of knowledge was focused on the column buckling equation. Recognition had not yet been made of the role of combined compression and flexure in column behavior; nor had the relationship between stability effects and analysis been realized.

The First AISC Specification – 1923

In 1923, AISC published the work upon which five eminent engineers collaborated – the *Specification of the American Institute of Steel Construction, Inc. for the Design Fabrication, and Erection of Structural Steel for Buildings* (AISC, 1923). Their deliberations resulted in nine pages of text in a document that was proposed for acceptance by the engineering community and steel construction industry. We know today that this document became the cornerstone upon which all else since has been built.

The allowable stress for column buckling was given as:

$$F_a = \frac{18{,}000}{1+\dfrac{l^2}{18{,}000r^2}} \leq 15{,}000$$

where l is the unsupported length of the column and r is the corresponding least radius of gyration of the section. Thus, the 15,000 psi allowable plateau applied for values of l/r up to 60 and the parabolic formulation controlled thereafter. It was also allowed for "short lengths or where lateral deflection is prevented" to use F_a = 18,000 psi.

Consideration of the combination of stresses was required, but the basis of the requirements was fairly crude: "Members subject to both direct and bending stresses shall be so proportioned that the greatest combined stresses shall not exceed the allowed limits." There was no mention of second-order effects, amplification factors, effective length factors, or overall frame behavior (Griffis and White, 2008). Thus, it remained in this period that the impact of stability on interaction and analysis had not been recognized.

Regarding the five factors that influence stability, it continued that few of them were explicitly addressed. However, for most buildings, the aforementioned implicit compensating factors continued to exist as well, albeit with one important exception. Such landmark structures as the Empire State Building, Manhattan Tower, and Chrysler Building were being designed to rise to heights approaching and exceeding 1,000 ft despite a continuing lack of clear understanding of system buckling, secondary effects in frames, and effective length factors (Griffis and White, 2013). Certainly the compensating factors were now beginning to be tested.

It is also interesting to note how performance-oriented some of the text was in those original nine pages:

1. The writers commented to the reader in their introduction that "The question of design is all-important. It necessarily presupposes that the design is good, made by and executed under the supervision of competent structural engineers; that proper provision is made for secondary stresses, excentric [sic] loads, unequal distribution of stresses on rivets, etc.; that the details are suitable and that the workmanship is high grade.".
2. Section 2 included general requirements that "To obtain a satisfactory structure, the following major requirements must be fulfilled. (a) The material used must be suitable, of uniform quality, and without defects affecting the strength or service of the structure. (b) Proper loads and conditions must be assumed in the design. (c) The unit stresses must be suitable for the material used. (d) The workmanship must be good, so that defects or injuries are not produced in the manufacture. (e) The computations and design must be properly made so that the unit stresses specified shall not be exceeded, and the structure and its details shall possess the requisite strength and rigidity.
3. Section 9 repeated that "Full provision shall be made for stresses caused by excentric [sic] loads."
4. Section 22(a) required that "The frame of all steel skeleton buildings shall be carried up true and plumb, …". The first AISC *Code of Standard Practice*, which was also a proposed standard when first published by AISC in 1924, established this as no greater than 1/500 for interior columns nor 1/1000 for exterior columns.

Some of these requirements may have been intended to mitigate the effects of stability, but the effectiveness in doing so probably related more to the use of traditional techniques than any meaning that might have been taken from these general statements.

Developments Leading to the 1963 AISC Specification

After the first AISC Specification was published in 1923, several revisions were issued up to and including the 1963 AISC Specification. This period was a time of growth in both knowledge and application, but treatment of stability evolved at a much slower pace. Griffis and White (2008) summarize this as follows:

"By the 1936 AISC Specification, column design was based upon the following equations:

For $l/r < 120$

$$F_a = 17{,}000 - 0.485\left(\frac{l}{r}\right)^2$$

For $l/r \geq 120$

$$F_a = \frac{18{,}000}{1 + \dfrac{l^2}{18{,}000 r^2}}$$

Considerable research was undertaken in the late 1940s and 1950s studying the influence of residual stresses and other factors such as initial out-of-straightness, eccentricity of load, end fixity, transverse loads and the effect of local and lateral buckling on column strength. This work culminated with the Column Research Council (now Structural Stability Research Council - SSRC) publishing column strength curves that serve as the basis for many code provisions today (CRC, 1960).

The tangent modulus concept for steel column design gave way to direct consideration of the effects of residual stresses, which it was discovered existed in the cross-section of the column even before the application of external load. The effects of initial out-of-straightness, eccentricity of load, end fixity, transverse loads, and local buckling on column strength became known.

Column buckling provisions were based upon the Johnson parabola (CRC, 1960) in the inelastic range until it merged with the Euler curve for elastic strength.

Members subject to both axial and bending stresses were proportioned by an interaction equation reflecting a simple combination of stresses. There was no mention of second-order effects, amplification factors, effective length factors, or overall frame behavior.

Although the concept of effective length was discussed in the Commentary that accompanied the 1961 AISC Specification, it was not until the 1963 AISC Specification that the effective length, *KL*, became explicit in the AISC Specification. The now-well-known alignment chart[2] was first published in a comprehensive paper (Kavanaugh, 1962) discussing column and frame buckling.

New strength formulas were introduced for columns based upon the basic column strength estimate suggested by the Column Research Council (CRC, 1960). An amplification factor was introduced in one of the two interaction equations to account for the fact that lateral displacement generates a secondary moment that must be accounted for in the member bending stress. For the first time, stability against sidesway of a frame was recognized in the interaction equations and design procedure.

[2] First introduced from unpublished notes in 1959 as the J & L Charts by Julian and Lawrence for incorporation into the Boston Building Code (Griffis and White, 2013).

The resulting equations for column buckling were:

For $KL/r < C_c$

$$F_a = \frac{\left(1 - \frac{\left(KL/r\right)^2}{2C_c^{\,2}}\right)F_y}{FS}$$

For $KL/r \geq C_c$

$$F_a = \frac{149{,}000{,}000}{\left(KL/r\right)^2}$$

where

$$FS = \frac{5}{3} + \frac{3\left(KL/r\right)}{8C_c} - \frac{\left(KL/r\right)^3}{8C_c^{\,3}}$$

$$C_c = \sqrt{\frac{2\pi^2 E}{F_y}}$$

The resulting interaction equations were:

For $f_a/F_a \leq 0.15$

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0$$

For $f_a/F_a > 0.15$

$$\frac{f_a}{F_a} + \frac{C_m F_b}{\left(1 - \frac{f_a}{F'_e}\right)F_b} \leq 1.0$$

The term $(1 - f_a / F_e')$ was the new amplification factor and C_m was a moment modifier term."

Regarding the five factors that influence stability, the substantial growth in knowledge in this era resulted in the explicit inclusion of many stability factors in design provisions and more explicit consideration of stability effects in design office practices. The treatment of stability that evolved in this era continued to be based largely on modification made on the design-side of the equations, however. The effects of stability on the analysis side remained to be developed.

Additionally, the introduction of *K* was not without compromise. The methods available to calculate *K* in all but the most simple of cases required a number of assumptions, few and often none of which were actually satisfied in real structures (Kavanaugh, 1962). Regardless, what would come to be known as the effective length method was accepted because it did something and that was better than doing nothing.

This era also marked the beginning of changes in the way buildings were designed and constructed, particularly at the end of it. Building systems were evolving and architecture was demanding changes in the way structures were configured. Computational techniques were on the verge of sophistication. Stability effects were soon to become a by-product of these changes in methods and technologies.

Refinements in Allowable Stress Design (1969 through 1989)
There were three major revisions of the AISC Specification made in this time period:

1. The 1969 AISC Specification
2. The 1978 AISC Specification
3. The 1989 AISC (ASD) Specification[3]

The Column Research Council was now known as the Structural Stability Research Council, and their work fed into the AISC Specification. None of these revisions made any substantive changes to the column buckling equations or the interaction equations. There were other developments, however, and Griffis and White (2008) summarize the 1969 and 1978 developments as follows:

> "The 1969 Specification for the first time explicitly mentioned the word stability in the provisions and required that "General stability shall be provided for the structure as a whole and for each compression element". More attention was given to overall frame behavior as opposed to member behavior. The Commentary to Section 1.8 Stability and Slenderness Ratios focused more on determination of the effective length factor *K* which was a subject of much attention among practitioners during this period. The Commentary referred to the SSRC Guide second edition (Johnston ,1966) which covered the subject in some detail.
>
> It was not until the 1978 Specification (AISC, 1978) that the subject of structural analysis and determination of secondary effects on frames was covered, albeit in the Commentary to Section 1.8 on Stability and Slenderness Ratios. For the first time the term "P-Δ" was used and its impact on frame behavior highlighted. Reference was made to research at Lehigh University on the load carrying capacity of rigid multistory frames subjected to gravity and lateral loads using second-order analysis methods. This discussion also referenced a fairly comprehensive treatment of this subject in the third edition of the SSRC Guide (Johnston, 1976)."

[3] All AISC Specifications that predated the 1989 ASD Specification were written using the allowable stress design method. The load and resistance factor design method was introduced in 1986 is omitted from this section and discussed in the next section.

The 1989 ASD Specification was little more than an editorial reorganization of the 1978 AISC Specification to align with the format and organization of the new, alternative 1986 LRFD Specification. There were some substantive changes, but changes in stability considerations were insignificant in spite of the significant advancements in stability analysis and design requirements that had been made in the 1986 LRFD Specification. This may have contributed to a mistaken perception in the engineering community that stability was a concern when the LRFD method was used, but not when the ASD method was used.

In spite of the advancements noted above, considerations of stability effects in design office practice largely remained constant – and usually were limited to whatever effects were considered by the terms that explicitly had been added to the equations in the AISC Specification. Changes in buildings literally soared in almost every respect. The heights of skyscrapers reached the pinnacle of the century, and every system used in building architecture, structure, and mechanics was being developed and innovated. The curtain wall and open floor plans with longer and longer spans dominated.

Even the marketplace was changing, with construction management firmly split as a discipline away from the role of the architect. CM influence would further drive the lightness and flexibility of structural systems as economic evaluations dictated that framing be economized by more exact designs and limited use of lateral framing.

The more advanced projects like the Sears Tower, John Hancock Tower, and Standard Oil Building in Chicago and World Trade Center Towers in New York City undoubtedly received significant attention to stability effects. Nonetheless, building systems were changing and this impact would manifest itself in many ways, including a heightened need for more advanced stability analysis and design requirements.

The John Hancock Tower in Boston is perhaps the most notable example of a building with design and construction aspects that outpaced the technology of the code in the area of stability. A project that ultimately was beset with a number of different structural issues, stability was cited as the reason that the majority of the panels in its all-glass façade popped out. Other examples exist, but none are as dramatic in their illustration of the need for further development of provisions to address stability in the design of buildings.

Load and Resistance Factor Design (1986 through 1999)

Concurrent with the development of the load and resistance factor design method as a replacement for allowable stress design, significant work was being done by AISC, the Structural Stability Research Council (SSRC), and others to advance the state-of-the-art of stability analysis and design. These efforts came together, and stability design requirements were advanced with the release of the first LRFD Specification in 1986.

A new column curve was also introduced into this new Specification (Griffis and White, 2008), based on column strength curve 2P of the 4th edition of the SSRC Guide (Galambos, 1988). The resulting equations for column buckling were:

For $\lambda_c \leq 1.5$

$$\phi F_{cr} = 0.85\left(0.658^{\lambda_c^2}\right)F_y$$

For $\lambda_c > 1.5$

$$\phi F_{cr} = 0.85\left[\frac{0.877}{\lambda_c^2}\right]F_y$$

where

$$\lambda_c = \frac{Kl}{r\pi}\sqrt{\frac{F_y}{E}}$$

Second-order effects were required to be included in the analysis results (that is, included in the calculation of P_u and M_u), which simplified the resulting interaction equations:

For $P_u/\phi P_n \geq 0.2$

$$\frac{P_u}{\phi P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi M_{nx}} + \frac{M_{uy}}{\phi M_{ny}}\right) \leq 1.0$$

For $P_u/\phi P_n < 0.2$

$$\frac{P_u}{2\phi P_n} + \left(\frac{M_{ux}}{\phi M_{nx}} + \frac{M_{uy}}{\phi M_{ny}}\right) \leq 1.0$$

Griffis and White (2008) summarize other significant developments as follows:

> "A more comprehensive treatment of stability of frames and second-order effects was introduced in the new LRFD Specification published in 1986 (AISC, 1986). Here, as part of a reorganization of the Specification provisions, an entire Chapter C was devoted to the subject of frame behavior. For the first time the Specification specifically required that "Second-order (P-Δ) effects shall be considered in the design of frames." Requirements were placed on the structural analysis to include axial deformations and the effects of frame instability under ultimate loads, a point not always realized under the allowable stress method. … In addition, continuing a trend toward specifying requirements for the structural analysis in order to properly

> address second-order effects, Chapter H on combined forces contained an approximate second-order analysis procedure, introducing the now commonly used B_1-B_2 method. The second-order amplification factor B_2 was permitted to be calculated using either the story buckling approach or the story stiffness approach. Frame stability was recognized as a system or story buckling phenomenon.
>
> The 1993 LRFD Specification in Chapter C on frames expanded the stability treatment of steel frame structures – both braced and unbraced. For the first time, the "destabilizing effects of gravity loaded columns…" were required to be considered in the moment frame analysis and design. This important fact was frequently overlooked in many building designs before this requirement was formally made a part of the Specification. Emphasis was placed in the Commentary to Chapter C on acceptable methods to calculate the effective length factor *K*, given the difficulty of this topic for practicing engineers. Consideration of the leaning column effect and its use in frames not meeting the requirements of the alignment chart commonly used in practice was introduced. The effect of leaning columns on different versions of *K* factor equations was discussed in detail. The $B_1 - B_2$ method was moved from Chapter H on combined forces to Chapter C on frames.
>
> In the 1999 Specification, Chapter C was expanded to include a discussion of stability bracing of frames as well as column and beam bracing. Specific stability bracing requirements emerged for braced frames where a minimum strength and stiffness requirement was placed on the story or panel of a steel building. Equations for the strength and stiffness requirements for braces in columns and beams were introduced."

These advancements were largely lost to the engineering community when debate about ASD and LRFD ensued. Indeed, this duality prevailed throughout this era with LRFD being developed further while ASD lay fallow – and without the advancements in stability analysis and design requirements that would have been made in it had LRFD never come into existence. It was unfortunate that this translated into the perception that stability was an LRFD issue, not an ASD issue.

These advancements also were made within the context of the effective length method. As a result, they merely pecked at the problems with that method that were mentioned previously and ignored many new problems.

The 2005 AISC Specification

As the 1990s gave way to the new century, AISC established a direction to resolve the debate between ASD and LRFD advocates. A unified specification was created and the previously separate ASD and LRFD methods were combined, using the best of both as the single approach for the future. At about the same time, the developmental work being done by AISC, SSRC, and others on stability analysis and design had achieved critical mass. This landmark work was incorporated into the 2005 AISC Specification.

The equations for column buckling were returned to more familiar terms. The LRFD alternative is shown below[4]:

For $KL/r \le 800/\sqrt{F_y}$

$$\phi F_{cr} = 0.9\left(0.658^{\frac{F_y}{F_e}}\right)F_y$$

For $\lambda_c > 1.5$

$$\phi F_{cr} = 0.9\left(0.877F_e\right)$$

where

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$

Second-order effects were required to be included in the analysis results, which simplified the resulting interaction equations:

For $P_r/P_c \ge 0.2$

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \le 1.0$$

For $P_r/P_c < 0.2$

$$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \le 1.0$$

In these equations, the numerator of each term represents the demand determined by analysis and the denominator of each term represents the corresponding available strength determined with the member strength provisions in the AISC Specification. All quantities are calculated using LRFD load combinations and resistance factors, or ASD load combinations and safety factors.

Also, for the first time, the impact of stability was addressed properly on both the demand side (the analysis results) and the strength side (the member strength equations). The

[4] The ASD alternative is similar with $\Omega = 1.67$ used as a divisor instead of the $\phi = 0.9$ used as a multiplier.

2005 AISC Specification included three prescriptive approaches for stability analysis and design:

1. The Direct Analysis Method (Appendix 7)
2. The Effective Length Method (Section C2.2a)
3. The First-Order Analysis Method (Section C2.2b)

Each of these methods is further illustrated in the information provided in Attachment A (Carter and Geschwindner, 2008).

The Direct Analysis Method was new in 2005. The Effective Length Method remains similar to the traditional approach, but has been modified. The First-Order Analysis Method was new in 2005, and offers an approach that permits a first-order analysis but still satisfies the requirements of the AISC Specification. These methods also are addressed in the 2010 AISC Specification (AISC, 2010) with simplifications, refinements and clarifications based upon usage sine the 2005 AISC Specification was published.

Direct Analysis Method
From Carter and Geschwindner (2008):

> "The direct analysis method is permitted for any ratio of second-order drift, Δ_{2nd}, to first-order drift, Δ_{1st}, and required when this ratio exceeds 1.5. It requires the use of:
>
> 1. A direct second-order analysis or a first-order analysis with B_1-B_2 amplification.
> 2. The nominal frame geometry with an additional lateral load of $N_i = 0.002Y_i$, where Y_i is the total gravity load on level i from LRFD load combinations, or 1.6 times ASD load combinations.
> 3. The reduced stiffnesses $EA*$ and $EI*$ (including in B_1-B_2 amplification, if used).
> 4. LRFD load combinations, or ASD load combinations multiplied by 1.6. This multiplier ensures that the drift level is consistent for LRFD and ASD when determining second-order effects. The forces and moments obtained in this analysis are then divided by 1.6 for ASD member design.
>
> The following exceptions apply as alternatives in item 2 above:
>
> a. If the out-of-plumb geometry of the structures is used, the notional loads can be omitted.
> b. When the ratio of second-order drift to first-order drift is equal to or less than 1.5, the notional load can be applied as a minimum lateral load, not an additional lateral load. Note that the unreduced stiffnesses, *EA* and *EI*, are used in this comparison.
> c. When the actual out-of-plumbness is known, it is permitted to adjust the notional loads proportionally.
>
> For all frames designed with this method, $K = 1.0$."

Regarding the five factors that influence stability, it is not surprising that all of them are explicitly addressed in this method – the direct analysis method represents the current state-of-the-art:

1. Member, component, and connection deformations are addressed directly in the analysis.
2. Second-order effects (both P-Δ and P-δ effects) are addressed directly in the analysis, either by rigorous second-order analysis or a first-order analysis with B_1-B_2 amplification;
3. Structural out-of-plumbness is addressed with the use of notional loads (or direct modeling of the initial out-of-plumbness).
4. Member out-of-straightness is accounted for in the column design equations for its effect on member strength, and in the use of a reduced stiffness for its effect on the structure stiffness.
5. Residual stresses are accounted for in the column design equations for their effect on member strength, and in the use of a reduced stiffness for their effect on the structure stiffness.
6. Variability in component and system stiffness is accounted for in the resistance and safety factors for its effect on member strength, and in the use of a reduced stiffness for its effect on the structure stiffness.

Effective Length Method
From Carter and Geschwindner (2008):

> "[This] is essentially the traditional effective length method with an additional requirement for a minimum lateral load. It is permitted when the ratio of second-order drift, Δ_{2nd}, to first-order drift, Δ_{1st}, is equal to or less than 1.5, and requires the use of:
>
> 1. A direct second-order analysis or a first-order analysis with B_1-B_2 amplification.
> 2. The nominal frame geometry with a minimum lateral load (a "notional load") $N_i = 0.002Y_i$, where Y_i is the total gravity load on level i from LRFD load combinations (or 1.6 times ASD load combinations). This notional load is specified to capture the effects of initial out-of-plumbness up to the AISC *Code of Standard Practice* maximum value of 1:500. In this method, N_i is not applied when the actual lateral load is larger than the calculated notional load.
> 3. The nominal stiffnesses EA and EI.
> 4. LRFD load combinations, or ASD load combinations multiplied by 1.6. This multiplier on ASD load combinations ensures that the drift level is consistent for LRFD and ASD when determining second-order effects. The forces and moments obtained in this analysis are then divided by 1.6 for ASD member design.
>
> When the ratio of second-order drift to first-order drift, which is given by B_2, is equal to or less than 1.1, $K = 1.0$ can be used in the design of moment frames. Otherwise,

for moment frames, K is determined from a sidesway buckling analysis. Section C2.2a(4) indicates that for braced frames, $K = 1.0$."

Regarding the five factors that influence stability, it is slightly less obvious how the simple addition of a notional load to the traditional effective length method could result in all of them being explicitly addressed in this method. Nonetheless:

1. Member, component, and connection deformations are addressed directly in the analysis.
2. Second-order effects (both P-Δ and P-δ effects) are addressed directly in the analysis, either by rigorous second-order analysis or a first-order analysis with B_1-B_2 amplification;
3. Structural out-of-plumbness is addressed with the use of notional loads.
4. Member out-of-straightness is accounted for in the column design equations for its effect on member strength.
5. Residual stresses are accounted for in the column design equations for their effect on member strength.
6. Variability in component and system stiffness is accounted for in the resistance and safety factors for its effect on member strength.
7. Note that K must be calculated (exceptions above noted) and used to account for the effects on structure stiffness of member out-of-straightness, residual stresses, and variability in component and system stiffness.

The basis in the use of K in this method means that some of the aforementioned limitations and criticisms of the effective length method are still applicable.

First-Order Analysis Method
From Carter and Geschwindner (2008):

"The first-order analysis method is permitted when:

1. The ratio of second-order drift, Δ_{2nd}, to first-order drift, Δ_{1st}, is equal to or less than 1.5.
2. The column axial force $\alpha P_r \leq 0.5 P_y$, where $\alpha = 1.0$ for LRFD, 1.6 for ASD.

This method requires the use of:

1. A first-order analysis.
2. The nominal frame geometry with an additional lateral load $N_i = 2.1(\Delta/L)Y_i \geq 0.0042Y_i$, applied in all load cases.
3. The nominal stiffnesses EA and EI.
4. B_1 as a multiplier on the total moment in beam-columns.
5. LRFD load combinations, or ASD load combinations multiplied by 1.6. This multiplier on ASD load combinations ensures that the drift level is consistent for LRFD and ASD when determining the notional loads. The forces and

moments obtained in this analysis are then divided by 1.6 for ASD member design.

For all frames designed with this method, $K = 1.0$."

Regarding the five factors that influence stability, it is even less obvious how they are explicitly addressed in this method. This is for good reason – they are all satisfied implicitly because the method is a mathematical manipulation of the Direct Analysis Method based upon the characteristics of typical building frames. For this reason, this method should be used with care to ensure that it is appropriate.

A General Observation About the Progression of Provisions and Design Practices

As can be seen in the descriptions, the stability provisions and corresponding designs have always been related, with one advancing slightly and affecting the other in a stair-stepping fashion. Early in the time-span, stability design provisions were comparatively crude, but they were acceptable then because framing systems were very conservative and very redundant in the typical building. As provisions advanced, framing systems changed. Stability problems sometimes influenced changes in provisions, systems, or both.

Today's provisions would have been unnecessary in the early days of the portal-frame building with heavy masonry infill. Yesterday's provisions are inadequate for today's economized and open buildings. Tomorrow's buildings and provisions will undoubtedly follow a similar trend.

Conclusions

The preceding historical summary of developments in stability analysis and design provisions shows an extensive path of progress from the first basis of column design in elastic buckling to the current state-of-the-art. Additionally, the key developments, particularly those made in the AISC Specification, have been highlighted.

The most current methods – those presented in the 2005 and 2010 AISC Specifications – have been available for several years now, and are being used in design offices and implemented by companies that write software for use in design offices. The Direct Analysis Method, which first appeared in the 2005 AISC Specification, offers a clean approach that sheds the deficiencies that have been noted in the methods of the past.

References

American Institute of Steel Construction, Inc. (AISC) (1923), *Specification of the American Institute of Steel Construction, Inc. for the Design Fabrication, and Erection of Structural Steel for Buildings,* Chicago, IL

American Institute of Steel Construction, Inc. (AISC) (1936), *Specification for the Design, Fabrication and Erection of Structural Steel for Buildings,* Chicago, IL

American Institute of Steel Construction, Inc. (AISC) (1961), *Specification for the Design, Fabrication and Erection of Structural Steel for Buildings,* Chicago, IL

American Institute of Steel Construction, Inc. (AISC) (1963), *Specification for the Design, Fabrication and Erection of Structural Steel for Buildings,* Chicago, IL
American Institute of Steel Construction, Inc. (AISC) (1969), *Specification for the Design, Fabrication and Erection of Structural Steel for Buildings,* Chicago, IL
American Institute of Steel Construction, Inc. (AISC) (1978), *Specification for the Design, Fabrication and Erection of Structural Steel for Buildings,* Chicago, IL
American Institute of Steel Construction, Inc. (AISC) (1989), *Specification for Structural Steel Buildings,* Chicago, IL
American Institute of Steel Construction, Inc. (AISC) (1986), *Load and Resistance Factor Design Specification for Structural Steel Buildings,* Chicago, IL
American Institute of Steel Construction, Inc. (AISC) (1989), *Specification for Structural Steel Buildings – Allowable Stress Design and Plastic Design,* Chicago, IL
American Institute of Steel Construction, Inc. (AISC) (1999), *Load and Resistance Factor Design Specification for Structural Steel Buildings,* Chicago, IL
American Institute of Steel Construction, Inc. (AISC) (2005), *Specification for Structural Steel Buildings*, ANSI/AISC 360-05, Chicago, IL.
American Institute of Steel Construction, Inc. (AISC) (2010), *Specification for Structural Steel Buildings*, ANSI/AISC 360-10, Chicago, IL.
Beedle, L, et al, *Structural Steel Design,* Ronald Press Company, New York, 1964
Bleich, F., *Buckling Strength of Metal Structures,* McGraw Hill Book Company, New York, 1952
Carter, C. and Geschwindner, L., (2008) "A Comparison of Frame Stability Analysis Methods In ANSI/AISC 360-05", 3rd Qtr. AISC *Engineering Journal*, Volume 45, No. 3, pp. 159-170.
Column Research Council (CRC) (1952), *The Basic Column Formula,* CRC Technical Memorandum No. 1, May.
Column Research Council (CRC) (1960), *Guide to Design Criteria for Metal Compression Members,* Column Research Council.
Griffis, L., and White, D., (2013) *Stability of Structural Steel Buildings*, AISC Design Guide No. 28, to be published in 2013
Griffis, L., and White, D., (2008) *Unpublished history of column design, interaction, and stability*, originally prepared for AISC Design Guide No. 28.
Johnston, B.G. (1966), *Guide to Design Criteria for Metal Compression Members,* Edited by B. Johnston, Column Research Council, John Wiley and Sons, New York, 1976 (2nd ed).
Johnston, B.G. (1976), *Guide to Stability Design Criteria for Metal Structures,* Edited by B. Johnston, Structural Stability Research Council, John Wiley and Sons, New York, 1976 (3rd ed).
Kavanaugh, T. C. (1962) "Effective Length of Framed Columns," *Transactions of the American Society of Civil Engineers*, Vol. 127, pp81-101.
Salmon, E.H., Columns, *A Treatise on the Strength and Design of Compression Members,* Oxford Technical Publications, London, 1921
Timoshenko,S. and Gere J. (1961). *Theory of Elastic Stability,* McGraw Hill Book Company, New York.

APPENDIX

(Carter and Geschwindner paper from 2008 AISC *Engineering Journal*)

A Comparison of Frame Stability Analysis Methods in ANSI/AISC 360-05

CHARLES J. CARTER and LOUIS F. GESCHWINDNER

ANSI/AISC 360-05 *Specification for Structural Steel Buildings* (AISC, 2005a), hereafter referred to as the AISC *Specification*, includes three prescriptive approaches for stability analysis and design. Table 2-1 in the 13th Edition AISC *Steel Construction Manual* (AISC, 2005b), hereafter referred to as the AISC *Manual*, provides a comparison of the methods and design options associated with each. A fourth approach, referred to as the Simplified Method, is also presented in the AISC *Manual* (see page 2-12) and on the AISC *Basic Design Values* cards. These four methods are illustrated in this paper in order to give the reader a general understanding of the differences between them:

1. The Second-Order Analysis Method (Section C2.2a)

2. The First-Order Analysis Method (Section C2.2b)

3. The Direct Analysis Method (Appendix 7)

4. The Simplified Method (Manual page 2-12; AISC *Basic Design Values* cards)

Two simple unbraced frames are used in this paper. The one-bay frame shown in Figure 1 has a rigid roof element spanning between a flagpole column (Column A) and leaning column (Column B). Drift is not limited for this frame, which results in a higher ratio of second-order drift to first-order drift, and allows illustration of the detailed requirements in each method for the calculation of K-factors, notional loads, and required and available strengths. The three-bay frame shown in Figure 2 has rigid roof elements spanning between two flagpole columns (Columns D and E) and two leaning columns (Columns C and F). This frame is used with a drift limit of $L/400$ to illustrate the simplifying effect a drift limit can have on the analysis requirements in each method.

Although these example frames are not realistic frames, the results obtained are representative of the impact of second-order elastic and inelastic effects on strength requirements in real frames, particularly when the number of moment connections is reduced. The loads shown in Figures 1 and 2 are from the controlling load and resistance factor design (LRFD) load combination and the corresponding designs are performed using LRFD. The process is essentially identical for allowable strength design (ASD), where ASD load combinations are used with α = 1.6 as a multiplier, when required in each method, to account for the second-order effects at the ultimate load level.

When it is required to include second-order effects, the B_1-B_2 amplification is used with a first-order analysis throughout this paper. A direct second-order analysis is straightforward and could have been used instead of the B_1-B_2 amplification.

THE ONE-BAY FRAME

A trial shape is selected using a first-order analysis without consideration of drift limits or second-order effects. Thereafter, that trial shape is used as the basis for comparison of the four methods discussed earlier.

Charles J. Carter is vice president and chief structural engineer, American Institute of Steel Construction, Chicago, IL.

Louis F. Geschwindner is vice president of special projects, American Institute of Steel Construction, and professor emeritus of architectural engineering at Pennsylvania State University, University Park, PA.

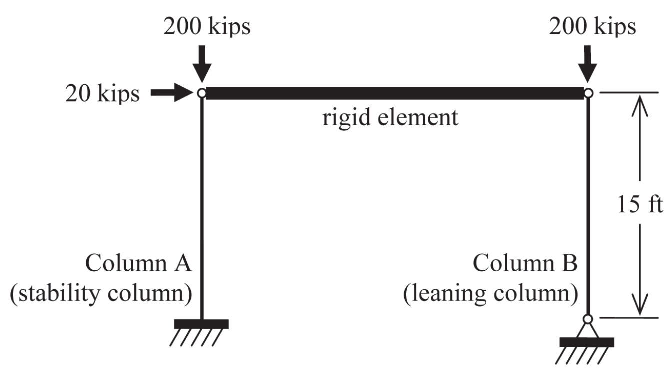

Fig. 1. One-bay unbraced frame used in examples.

Selection of Trial Shape Based Upon Strength Consideration Only

Based upon the loading shown in Figure 1, the first-order axial force, strong-axis moment, and design parameters for Column A are:

P_u = 200 kips M_{ux} = (20 kips) (15 ft)
K_x = 2.0 = 300 kip-ft
K_y = 1.0 C_b = 1.67
L_x = L_y = 15 ft L_b = 15 ft

Note that K_x = 2.0, the theoretical value for a column with a fixed base and top that is free to rotate and translate, is used rather than the value of 2.1 recommended for design in the AISC *Specification* Commentary Table C-C2.2. The value of 2.0 is used because it is consistent with the formulation of the lateral stiffness calculation below. Note also that the impact of the leaning column on K_x is ignored in selecting the trial size, although it will be considered in subsequent sections when K_x cannot be taken equal to 1 for Column A. Out of the plane of the frame, K_y is taken as 1.0.

A simple rule of thumb for trial beam-column selection is to use an equivalent axial force equal to P_u plus 24/d times M_u, where d is the nominal depth of the column (Geschwindner, Disque and Bjorhovde, 1994). Using d = 14 in. for a W14, the equivalent axial force is 714 kips and an ASTM A992 W14×90 is selected as the trial shape.

The lateral stiffness of the frame depends on Column A only and is:

$$k = 3EI/L^3$$
$$= 3(29{,}000 \text{ ksi})(999 \text{ in.}^4)/(15 \text{ ft} \times 12 \text{ in./ft})^3$$
$$= 14.9 \text{ kips/in.}$$

The corresponding first-order drift of the frame is:

$$\Delta_{1st} = (20 \text{ kips})/(14.9 \text{ kips/in.})$$
$$= 1.34 \text{ in.}$$

Note that this is a very flexible frame with Δ_{1st}/L = 1.34/(15 ft × 12 in./ft) = 1/134.

Design by Second-Order Analysis (Section C2.2a)

Design by second-order analysis is essentially the traditional effective length method with an additional requirement for a minimum lateral load. It is permitted when the ratio of second-order drift, Δ_{2nd}, to first-order drift, Δ_{1st}, is equal to or less than 1.5, and requires the use of:

1. A direct second-order analysis or a first-order analysis with B_1-B_2 amplification.
2. The nominal frame geometry with a minimum lateral load (a "notional load") $N_i = 0.002Y_i$, where Y_i is the total gravity load on level i from LRFD load combinations (or 1.6 times ASD load combinations). This notional load is specified to capture the effects of initial out-of-plumbness up to the AISC *Code of Standard Practice* maximum value of 1:500. In this method, N_i is not applied when the actual lateral load is larger than the calculated notional load.
3. The nominal stiffnesses EA and EI.
4. LRFD load combinations, or ASD load combinations multiplied by 1.6. This multiplier on ASD load combinations ensures that the drift level is consistent for LRFD and ASD when determining second-order effects. The forces and moments obtained in this analysis are then divided by 1.6 for ASD member design.

When the ratio of second-order drift to first-order drift, which is given by B_2, is equal to or less than 1.1, K = 1.0 can be used in the design of moment frames. Otherwise, for moment frames, K is determined from a sidesway buckling analysis. Section C2.2a(4) indicates that for braced frames, K = 1.0.

For the example frame given in Figure 1, the minimum lateral load based upon the total gravity load, Y_i, is:

$$Y_i = 200 \text{ kips} + 200 \text{ kips}$$
$$= 400 \text{ kips}$$

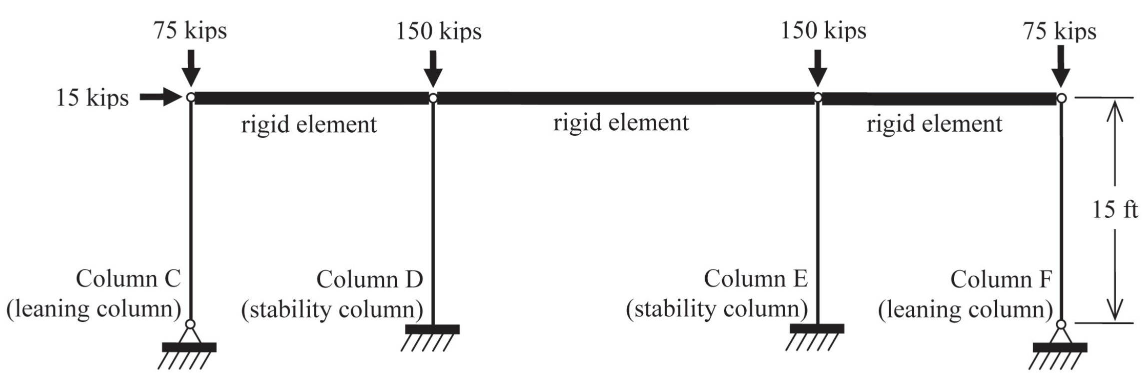

Fig. 2. Three-bay unbraced frame used in examples.

$$N_i = 0.002\,Y_i$$
$$= 0.002\,(400 \text{ kips})$$
$$= 0.8 \text{ kips}$$

Because this notional load is less than the actual lateral load, it need not be applied. For a load combination that did not include a lateral load, the notional load would need to be included in the analysis.

For Column A, using first-order analysis and B_1-B_2 amplification:

$P_{nt} = 200$ kips, $P_{lt} = 0$ kips
$M_{nt} = 0$ kip-ft, $M_{lt} = 300$ kip-ft

For P-δ amplification, since there are no moments associated with the no-translation case, there is no need to calculate B_1. For P-Δ amplification, the first-order drift ratio is determined from the calculated drift of 1.34 in. Thus,

$$\Delta_{1st}/L = (1.34 \text{ in.})/(15 \text{ ft} \times 12 \text{ in./ft})$$
$$= 0.00744$$

For moment frames, $R_m = 0.85$ and from Equation C2-6b with $\Delta_H = \Delta_{1st}$ and $\Sigma H = 20$ kips,

$$\Sigma P_{e2} = R_m \Sigma H/(\Delta_{1st}/L)$$
$$= 0.85\,(20 \text{ kips})/(0.00744)$$
$$= 2{,}280 \text{ kips}$$

For design by LRFD, $\alpha = 1.0$ and ΣP_{nt} is the sum of the gravity loads. Thus,

$$\alpha \Sigma P_{nt}/\Sigma P_{e2} = 1.0\,(200 \text{ kips} + 200 \text{ kips})/2{,}280 \text{ kips}$$
$$= 0.175$$

From Equation C2-3, the amplification is:

$$B_2 = \frac{1}{1 - \dfrac{\alpha \Sigma P_{nt}}{\Sigma P_{e2}}} \geq 1$$
$$= \frac{1}{1 - 0.175} \geq 1$$
$$= 1.21 \geq 1.0$$
$$= 1.21$$

Because $B_2 = 1.21$, the second-order drift is less than 1.5 times the first-order drift. Thus, the use of this method is permitted. Because $B_2 > 1.1$, K cannot be taken as 1.0 for column design in the moment frame with this method. Thus, K must be calculated, including the leaning-column effect. Several approaches are available in the AISC *Specification* Commentary to include this effect. A simple approach that uses the ratio of the load on the leaning columns to the load on the stabilizing columns had been provided in previous Commentaries and is used here (Lim and McNamara, 1972):

$$\Sigma P_{leaning}/\Sigma P_{stability} = (200 \text{ kips})/(200 \text{ kips})$$
$$= 1$$

$$K_x^* = K_x(1 + \Sigma P_{leaning} / \Sigma P_{stability})^{1/2}$$
$$= 2.0(1 + 1)^{1/2}$$
$$= 2.83$$

The amplified axial force (Equation C2-1b) and associated design parameters for this method are:

$$P_r = P_{nt} + B_2 P_{lt}$$
$$= 200 \text{ kips} + 1.21(0 \text{ kips})$$
$$= 200 \text{ kips}$$

$$K_x^* = 2.83,\ K_y = 1.0$$

$$L_x = L_y = 15 \text{ ft}$$

The amplified moment (Equation C2-1a) and associated design parameters for this method are:

$$M_{rx} = B_1 M_{nt} + B_2 M_{lt}$$
$$= (0 \text{ kip-ft}) + 1.21\,(300 \text{ kip-ft})$$
$$= 363 \text{ kip-ft}$$

$$C_b = 1.67$$

$$L_b = 15 \text{ ft}$$

Based upon these design parameters, the axial and strong-axis available flexural strengths of the ASTM A992 W14×90 are:

$$P_c = \phi_c P_n$$
$$= 721 \text{ kips}$$

$$M_{cx} = \phi_b M_{nx}$$
$$= 573 \text{ kip-ft}$$

To determine which interaction equation is applicable, the ratio of the required axial compressive strength to available axial compressive strength must be determined.

$$\frac{P_r}{P_c} = \frac{200 \text{ kips}}{721 \text{ kips}}$$
$$= 0.277$$

Thus, because $P_r/P_c \geq 0.2$, Equation H1-1a is applicable.

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}}\right) = 0.277 + \frac{8}{9}\left(\frac{363 \text{ kip-ft}}{573 \text{ kip-ft}}\right)$$
$$= 0.840$$

The W14×90 is adequate because $0.840 \leq 1.0$.

Design by First-Order Analysis (Section C2.2b)

The first-order analysis method is permitted when:

1. The ratio of second-order drift, Δ_{2nd}, to first-order drift, Δ_{1st}, is equal to or less than 1.5.

2. The column axial force $\alpha P_r \leq 0.5 P_y$, where $\alpha = 1.0$ for LRFD, 1.6 for ASD.

This method requires the use of:

1. A first-order analysis.

2. The nominal frame geometry with an additional lateral load $N_i = 2.1(\Delta/L)Y_i \geq 0.0042Y_i$, applied in all load cases.

3. The nominal stiffnesses EA and EI.

4. B_1 as a multiplier on the total moment in beam-columns.

5. LRFD load combinations, or ASD load combinations multiplied by 1.6. This multiplier on ASD load combinations ensures that the drift level is consistent for LRFD and ASD when determining the notional loads. The forces and moments obtained in this analysis are then divided by 1.6 for ASD member design.

For all frames designed with this method, $K = 1.0$.

For the example frame given in Figure 1, the additional lateral load is based on the first-order drift ratio, Δ/L, and the total gravity load, Y_i. Thus, with $\Delta = \Delta_{1st}$,

$$\begin{aligned}\Delta_{1st}/L &= (1.34 \text{ in.})/(15 \text{ ft} \times 12 \text{ in./ft})\\ &= 0.00744\end{aligned}$$

$$\begin{aligned}Y_i &= 200 \text{ kips} + 200 \text{ kips}\\ &= 400 \text{ kips}\end{aligned}$$

$$\begin{aligned}N_i &= 2.1(\Delta_{1st}/L)Y_i \geq 0.0042Y_i\\ &= 2.1(0.00744)(400 \text{ kips}) \geq 0.0042(400 \text{ kips})\\ &= 6.25 \text{ kips} \geq 1.68 \text{ kips}\\ &= 6.25 \text{ kips}\end{aligned}$$

It was previously determined in the illustration of design by second-order analysis example that the second-order drift is less than 1.5 times the first-order drift. Additionally,

$$\begin{aligned}\alpha P_r &= 1.0(200 \text{ kips})\\ &= 200 \text{ kips}\end{aligned}$$

And for a W14×90,

$$\begin{aligned}0.5P_y &= 0.5F_yA_g\\ &= 0.5(50 \text{ ksi})(26.5 \text{ in.}^2)\\ &= 663 \text{ kips}\end{aligned}$$

Because $\Delta_{2nd} < 1.5\Delta_{1st}$ and $\alpha P_r < 0.5P_y$, the use of this method is permitted.

The loading for this method is the same as that shown in Figure 1, except for the addition of a notional load of 6.25 kips coincident with the lateral load of 20 kips shown, resulting in a column moment, M_u, of 394 kip-ft.

This moment must be amplified by B_1 as determined from Equation C2-2. The Euler buckling load is calculated with $K_1 = 1.0$. Thus,

$$\begin{aligned}P_{e1} &= \pi^2EI/(K_1L)^2\\ &= \pi^2(29{,}000 \text{ ksi})(999 \text{ in.}^4)/(1.0 \times 15 \text{ ft} \times 12 \text{ in./ft})^2\\ &= 8{,}830 \text{ kips}\end{aligned}$$

The moment on one end of the column is zero, so the moment gradient term is:

$$\begin{aligned}C_m &= 0.6 - 0.4(M_1/M_2)\\ &= 0.6 - 0.4(0/394 \text{ kip-ft})\\ &= 0.6\end{aligned}$$

From Equation C2-2,

$$\begin{aligned}\alpha P_r/P_{e1} &= 1.0(200 \text{ kips})/(8{,}830 \text{ kips})\\ &= 0.0227\end{aligned}$$

$$\begin{aligned}B_1 &= \frac{C_m}{1-\dfrac{\alpha P_r}{P_{e1}}} \geq 1\\ &= \frac{0.6}{1-0.0227} \geq 1.0\\ &= 0.614 \geq 1.0\\ &= 1.0\end{aligned}$$

The axial force and associated design parameters for this method are:

$$\begin{aligned}P_r &= 200 \text{ kips}\\ K_x &= K_y = 1.0\\ L_x &= L_y = 15 \text{ ft}\end{aligned}$$

The amplified moment and associated design parameters for this method are:

$$\begin{aligned}M_{rx} &= B_1M_u\\ &= 1.0\,(394 \text{ kip-ft})\\ &= 394 \text{ kip-ft}\end{aligned}$$

$$C_b = 1.67$$

$$L_b = 15 \text{ ft}$$

Based on these design parameters, the axial and strong-axis available flexural strengths of the ASTM A992 W14×90 are:

$$\begin{aligned}P_c &= \phi_cP_n = 1{,}000 \text{ kips}\\ M_{cx} &= \phi_bM_{nx} = 573 \text{ kip-ft}\end{aligned}$$

To determine which interaction equation is applicable, the ratio of the required axial compressive strength to available axial compressive strength must be determined.

$$\begin{aligned}\frac{P_r}{P_c} &= \frac{200 \text{ kips}}{1{,}000 \text{ kips}}\\ &= 0.200\end{aligned}$$

Thus, because $P_r/P_c \geq 0.2$, Equation H1-1a is applicable.

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}}\right) = 0.200 + \frac{8}{9}\left(\frac{394 \text{ kip-ft}}{573 \text{ kip-ft}}\right)$$
$$= 0.811$$

The W14×90 is adequate since 0.811 ≤ 1.0.

Design by Direct Analysis (Appendix 7)

The Direct Analysis Method is permitted for any ratio of second-order drift, Δ_{2nd}, to first-order drift, Δ_{1st}, and required when this ratio exceeds 1.5. It requires the use of:

1. A direct second-order analysis or a first-order analysis with B_1-B_2 amplification.

2. The nominal frame geometry with an additional lateral load of $N_i = 0.002Y_i$, where Y_i is the total gravity load on level i from LRFD load combinations, or 1.6 times ASD load combinations.

3. The reduced stiffnesses EA^* and EI^* (including in B_1-B_2 amplification, if used).

4. LRFD load combinations, or ASD load combinations multiplied by 1.6. This multiplier ensures that the drift level is consistent for LRFD and ASD when determining second-order effects. The forces and moments obtained in this analysis are then divided by 1.6 for ASD member design.

The following exceptions apply as alternatives in item 2:

a. If the out-of-plumb geometry of the structures is used, the notional loads can be omitted.

b. When the ratio of second-order drift to first-order drift is equal to or less than 1.5, the notional load can be applied as a minimum lateral load, not an additional lateral load. Note that the unreduced stiffnesses, *EA* and *EI*, are used in this comparison.

c. When the actual out-of-plumbness is known, it is permitted to adjust the notional loads proportionally.

For all frames designed with this method, $K = 1.0$.

It was previously determined in the illustration of design by second-order analysis example that the second-order drift is less than 1.5 times the first-order drift (note that this check is properly made using the unreduced stiffnesses, *EA* and *EI*).

Thus, the notional load can be applied as a minimum lateral load, and that minimum is:

$$Y_i = 200 \text{ kips} + 200 \text{ kips}$$
$$= 400 \text{ kips}$$

$$N_i = 0.002Y_i$$
$$= 0.002(400 \text{ kips})$$
$$= 0.8 \text{ kips}$$

Because this notional load is less than the actual lateral load, it need not be applied. For a load combination that does not include a lateral load, the notional load would need to be included in the analysis.

For Column A, using first-order analysis and B_1-B_2 amplification:

$P_{nt} = 200$ kips, $\quad P_{lt} = 0$ kips
$M_{nt} = 0$ kip-ft, $\quad M_{lt} = 300$ kip-ft

To determine the second-order amplification, the reduced stiffness, EI^*, must be calculated.

$$\alpha P_r = 1.0(200 \text{ kips})$$
$$= 200 \text{ kips}$$

and

$$0.5P_y = 0.5F_yA_g$$
$$= 0.5(50 \text{ ksi})(26.5 \text{ in.}^2)$$
$$= 663 \text{ kips}$$

Thus, because $\alpha P_r < 0.5P_y$, $\tau_b = 1.0$ and

$$EI^* = 0.8\tau_b EI$$
$$= 0.8EI$$

For P-δ amplification, since there are no moments associated with the no-translation case, there is no need to calculate B_1. For P-Δ amplification, the reduced stiffness EI^* must be used to determine the first-order drift. Because $EI^* = 0.8EI$, the first-order drift based upon EI^* is 25% larger than that calculated previously. Thus,

$$\Delta_{1st} = 1.25(1.34 \text{ in.})$$
$$= 1.68 \text{ in.}$$

The first-order drift ratio is determined from the amplified drift of 1.68 in.

$$\Delta_{1st}/L = (1.68 \text{ in.})/(15 \text{ ft} \times 12 \text{ in./ft})$$
$$= 0.00933$$

For moment frames, $R_M = 0.85$ and from Equation C2-6b with $\Delta_H = \Delta_{1st}$ and $\Sigma H = 20$ kips,

$$\Sigma P_{e2} = R_M \frac{\Sigma H}{\left(\Delta_{1st}/L\right)}$$
$$= 0.85\frac{20 \text{ kips}}{(0.00933)}$$
$$= 1{,}820 \text{ kips}$$

For design by LRFD, α = 1.0 and ΣP_{nt} is the sum of the gravity loads. Thus,

$$\alpha\Sigma P_{nt}/\Sigma P_{e2} = 1.0(200 \text{ kips} + 200 \text{ kips})/1{,}820 \text{ kips} = 0.220$$

From Equation C2-3, the amplification is:

$$B_2 = \frac{1}{1-\dfrac{\alpha\,\Sigma P_{nt}}{\Sigma P_{e2}}} \geq 1$$

$$= \frac{1}{(1-0.220)} \geq 1.0$$

$$= 1.28 \geq 1.0$$

$$= 1.28$$

It is worth noting that use of the reduced axial stiffness, $EA^* = 0.8EA$, in members that contribute to lateral stability is also required in this method. However, due to the characteristics of the structures chosen for this example, there are no axial deformations that impact the stability of the structure.

The amplified axial force (Equation C2-1b) and associated design parameters for this method are:

$$P_r = P_{nt} + B_2P_{lt}$$

$$= 200 \text{ kips} + 1.28(0 \text{ kips})$$

$$= 200 \text{ kips}$$

$$K_x = K_y = 1.0$$

$$L_x = L_y = 15 \text{ ft}$$

The amplified moment (Equation C2-1a) and associated design parameters for this method are:

$$M_{rx} = B_1M_{nt} + B_2M_{lt}$$

$$= (0 \text{ kip-ft}) + 1.28(300 \text{ kip-ft})$$

$$= 384 \text{ kip-ft}$$

$$C_b = 1.67$$

$$L_b = 15 \text{ ft}$$

Based upon these design parameters, the axial and strong-axis available flexural strengths of the ASTM A992 W14×90 are:

$$P_c = \phi_cP_n = 1{,}000 \text{ kips}$$

$$M_{cx} = \phi_bM_{nx} = 573 \text{ kip-ft}$$

To determine which interaction equation is applicable, the ratio of the required axial compressive strength to available axial compressive strength must be determined.

$$\frac{P_r}{P_c} = \frac{200 \text{ kips}}{1{,}000 \text{ kips}}$$

$$= 0.200$$

Thus, because $P_r/P_c \geq 0.2$, Equation H1-1a is applicable.

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}}\right) = 0.200 + \frac{8}{9}\left(\frac{384 \text{ kip-ft}}{573 \text{ kip-ft}}\right)$$

$$= 0.796$$

The W14×90 is adequate since 0.796 ≤ 1.0.

The Simplified Method

This method is provided in the AISC *Basic Design Values* Cards and the 13th Edition *Steel Construction Manual* (AISC, 2005b), and excerpted as shown in Figure 3. This simplified method is derived from the effective length method (Design by Second-Order Analysis; Section C2.2a) using B_1-B_2 amplification with B_1 taken equal to B_2. Note that the user note in Section C2.1b says that B_1 may be taken equal to B_2 as long as B_1 is less than 1.05. However, it is also conservative to take B_1 equal to B_2 any time B_1 is less than B_2. Although it cannot universally be stated that B_1 is always equal to or less than B_2, this is the case for typical framing. It is left to engineering judgment to confirm that this criterion is satisfied when applying the simplified method.

This method is permitted when the ratio of second-order drift, Δ_{2nd}, to first-order drift, Δ_{1st}, is equal to or less than 1.5 as with the Design by Second-Order Analysis method. It allows the use of a first-order analysis based on nominal stiffnesses, EA and EI, with a minimum lateral load $N_i = 0.002Y_i$, where Y_i is the total gravity load on level i from LRFD load combinations or ASD load combinations. The 1.6 multiplier on ASD load combinations is not used at this point but its effect is included in the determination of the amplification multiplier upon entering the table.

The ratio of total story gravity load (times 1.0 in LRFD, 1.6 in ASD) to the story lateral load is used to enter the table in Figure 3. The second-order amplification multiplier is determined from the value in the table corresponding to the calculated load ratio and design story drift limit. While linear interpolation between tabular values is permitted, it is important to note that the tabular values have, in essence, only two significant digits. Accordingly, the value determined should not be calculated to more than one decimal place. The tabular value is used to amplify all forces and moments in the analysis.

When the ratio of second-order drift to first-order drift is equal to or less than 1.1, K = 1.0 can be used in the design of moment frames. Otherwise, for moment frames, K is determined from a sidesway buckling analysis. For braced frames, K = 1.0.

For the example frame given in Figure 1, the minimum lateral load is:

$$Y_i = 200 \text{ kips} + 200 \text{ kips}$$
$$= 400 \text{ kips}$$

$$N_i = 0.002Y_i$$
$$= 0.002(400 \text{ kips})$$
$$= 0.8 \text{ kips}$$

Because this notional load is less than the actual lateral load, it need not be applied. For a load combination that does not include a lateral load, the notional load would need to be included in the analysis.

The actual first-order drift of the trial frame corresponds to a drift ratio of L/134 and the load ratio is:

$$1.0 \times (200 \text{ kips} + 200 \text{ kips})/(20 \text{ kips}) = 20$$

Entering the table in the column for a load ratio of 20, the corresponding multiplier for a drift ratio of H/134 is 1.3 (determined by interpolation to one decimal place). This multiplier is less than 1.5; thus, $\Delta_{2nd} < 1.5\Delta_{1st}$ and the use of this method is permitted. However, because the multiplier is greater than 1.1, K cannot be taken as 1.0 for column design in the moment frame with this method. Thus, K must be calculated, including the leaning column effect. Using the same approach as previously discussed (Lim and McNamara, 1972):

$$\Sigma P_{leaning}/\Sigma P_{stability} = (200 \text{ kips})/(200 \text{ kips})$$
$$= 1$$

$$K_x^* = K_x(1 + \Sigma P_{leaning}/\Sigma P_{stability})^{1/2}$$
$$= 2.0(1 + 1)^{1/2}$$
$$= 2.83$$

The amplified axial force (with the full axial force amplified by B_2) and associated design parameters for this method are:

$$P_r = 1.3P_u$$
$$= 1.3 \text{ (200 kips)}$$
$$= 260 \text{ kips}$$

$$K_x^* = 2.83,\ K_y = 1.0$$

$$L_x = L_y = 15 \text{ ft}$$

The amplified moment (with the full moment amplified by B_2) and associated design parameters for this method are:

$$M_{rx} = 1.3M_u$$
$$= 1.3(300 \text{ kip-ft})$$
$$= 390 \text{ kip-ft}$$

$$C_b = 1.67$$

$$L_b = 15 \text{ ft}$$

Based on these design parameters, the available axial compressive strength and strong-axis available flexural strength of the ASTM A992 W14×90 are:

$$P_c = \phi_c P_n = 721 \text{ kips}$$
$$M_{cx} = \phi_b M_{nx} = 573 \text{ kip-ft}$$

To determine which interaction equation is applicable, the ratio of the required axial compressive strength to available axial compressive strength must be determined.

$$\frac{P_r}{P_c} = \frac{260 \text{ kips}}{721 \text{ kips}}$$
$$= 0.361$$

Simplified Method

Step 1. Perform first-order analysis. Use 0.2% of total story gravity load as minimum lateral load in all load combinations.
Step 2. Establish the design story drift limit and determine the lateral load required to produce it.
Step 3. Determine the ratio of the total story gravity load to the lateral load determined in Step 2. For ASD, multiply by 1.6.
Step 4. Multiply first-order results by the tabular value. K=1, except for moment frames when the tabular value is greater than 1.1.

Design Story Drift Limit	**Ratio from Step 3 (times 1.6 for ASD, 1.0 for LRFD)**										
	0	**5**	**10**	**20**	**30**	**40**	**50**	**60**	**80**	**100**	**120**
***H*/100**	1	1.1	1.1	1.3	1.4				When ratio exceeds 1.5, simplified method requires a stiffer structure.		
***H*/200**	1	1	1.1	1.1	1.2	1.3	1.3	1.4			
***H*/300**	1	1	1	1.1	1.1	1.2	1.2	1.3	1.4	1.5	
***H*/400**	1	1	1	1.1	1.1	1.1	1.2	1.2	1.3	1.3	1.4
***H*/500**	1	1	1	1	1.1	1.1	1.1	1.2	1.2	1.3	1.3

K=1

Fig. 3. Simplified method from AISC basic design values cards.

Thus, because $P_r/P_c \geq 0.2$, Equation H1-1a is applicable.

$$\frac{P_r}{P_c}+\frac{8}{9}\left(\frac{M_{rx}}{M_{cx}}\right)=0.361+\frac{8}{9}\left(\frac{390 \text{ kip-ft}}{573 \text{ kip-ft}}\right)$$
$$= 0.966$$

The W14×90 is adequate since $0.966 \leq 1.0$.

Summary for the One-Bay Frame

All methods illustrated in the foregoing sections produce similar designs. The results are tabulated here for comparison, where the result of the beam-column interaction equation is given for each method. A lower interaction equation result for the same column shape signifies a prediction of higher strength.

Method	Interaction Equation
Second-Order	0.840
First-Order	0.811
Direct Analysis	0.796
Simplified	0.966

In this example, the direct analysis method predicts the highest strength, while the simplified method predicts the lowest strength. This would be expected because the Direct Analysis Method was developed as the most accurate approach while the simplified method was developed to produce a quick yet conservative solution.

The designs compared here are based on strength with no consideration of drift limitation, except to the extent that the actual drift impacts the magnitude of the second-order effects. The usual drift limits of approximately $L/400$ will necessitate framing members and configurations with more lateral stiffness than this frame provides. Hence, the designer may find that a frame configured for drift first will often require no increase in member size for strength, including second-order effects. This will be explored further with the three-bay frame.

THE THREE-BAY FRAME

For the frame shown in Figure 2, a trial shape is selected using a first-order drift limit of $L/600$ under a service level lateral load of 10 kips. Thereafter, that trial shape is used as the basis for comparison of the four methods used previously for the one-bay frame.

Selection of Trial Shape Based on the Drift Limit Only

For the dimensions shown in Figure 2:

$$L/600 = (15 \text{ ft} \times 12 \text{ in./ft})/600$$
$$= 0.300 \text{ in.}$$

The lateral stiffness of the frame depends on Columns D and E only, and based on a classical stiffness derivation with the given end conditions, it is calculated as follows:

$$k = 2 \times 3EI/L^3$$
$$= 2 \times 3(29{,}000 \text{ ksi})(I)/(15 \text{ ft} \times 12 \text{ in./ft})^3$$
$$= 0.0298(I)$$

With the service level lateral load on the frame of 10 kips:

$$0.0298(I) \geq (10 \text{ kips})/(0.300 \text{ in.})$$

Thus, I_{req} = 1,120 in.^{4} and an ASTM A992 W14×109 is selected as the trial shape with I_x = 1,240 in.^{4}

The actual lateral stiffness of the frame is:

$$k = 2 \times 3EI/L^3$$
$$= 2 \times 3(29{,}000 \text{ ksi})(1{,}240 \text{ in.}^4)/(15 \text{ ft} \times 12 \text{ in./ft})^3$$
$$= 37.0 \text{ kips/in.}$$

The corresponding first-order drift of the frame under the LRFD lateral load of 15 kips is:

$$\Delta_{1st} = (15 \text{ kips})/(37.0 \text{ kips/in.})$$
$$= 0.405 \text{ in.}$$

The first-order axial force, strong-axis moment, and design parameters for Columns D and E are:

P_u = 150 kips $\qquad M_{ux}$ = (15 kips)(15 ft)/2
K_x = 2.0 $\qquad$ = 113 kip-ft
K_y = 1.0 $\qquad C_b$ = 1.67
L_x = L_y = 15 ft $\qquad L_b$ = 15 ft

Note that K_x = 2.0, the theoretical value for a column with a fixed base and pinned top, is used rather than the value of 2.1 recommended for design in the AISC *Specification* Commentary Table C-C2.2. The value of 2.0 is used because it is consistent with the formulation of the lateral stiffness calculation that follows. Note also that the impact of the leaning column on K_x is ignored in selecting the trial size, although it will be considered in subsequent sections when K_x cannot be taken equal to 1.0 for Column A. Out of the plane of the frame, K_y is taken as 1.0.

Design by Second-Order Analysis (Section C2.2a)

For the example frame given in Figure 2, the minimum lateral load is:

$$Y_i = 75 \text{ kips} + 150 \text{ kips} + 150 \text{ kips} + 75 \text{ kips}$$
$$= 450 \text{ kips}$$

$$N_i = 0.002\, Y_i$$
$$= 0.002(450 \text{ kips})$$
$$= 0.90 \text{ kips}$$

Because this notional load is less than the actual lateral load, it need not be applied.

For Columns D and E, using first-order analysis and B_1-B_2 amplification:

P_{nt} = 150 kips, P_{lt} = 0 kips
M_{nt} = 0 kip-ft, M_{lt} = 113 kip-ft

For P-δ amplification, because there are no moments associated with the no-translation case, there is no need to calculate B_1. For P-Δ amplification, the first-order drift ratio is determined from the calculated drift of 0.405 in. Thus,

$$\Delta_{1st}/L = (0.405 \text{ in.})/(15 \text{ ft} \times 12 \text{ in./ft}) = 0.00225$$

For moment frames, R_m = 0.85 and from Equation C2-6b with $\Delta_H = \Delta_{1st}$ and ΣH = 15 kips,

$$\Sigma P_{e2} = R_M \frac{\Sigma H}{\left(\Delta_{1st}/L\right)} = 0.85 \frac{15 \text{ kips}}{(0.00225)} = 5{,}670 \text{ kips}$$

For design by LRFD, α = 1.0 and ΣP_{nt} is the sum of the gravity loads. Thus,

$$\alpha\Sigma P_{nt}/\Sigma P_{e2} = 1.0(75 \text{ kips} + 150 \text{ kips} + 150 \text{ kips} + 75 \text{ kips})/5{,}670 \text{ kips} = 0.0794$$

From Equation C2-3, the amplification is:

$$B_2 = \frac{1}{\left(1 - \dfrac{\alpha\Sigma P_{nt}}{\Sigma P_{e2}}\right)} \geq 1 = \frac{1}{(1 - 0.0794)} \geq 1.0 = 1.09 \geq 1.0 = 1.09$$

Because B_2 = 1.09, the second-order drift is less than 1.5 times the first-order drift. Thus, the use of this method is permitted. Because B_2 < 1.1, K can be taken as 1.0 for column design in the moment frame with this method.

The amplified axial force (Equation C2-1b) and associated design parameters for this method are:

$$P_r = P_{nt} + B_2 P_{lt} = 150 \text{ kips} + 1.09(0 \text{ kips}) = 150 \text{ kips}$$

$K_x = K_y = 1.0$

$L_x = L_y$ = 15 ft

The amplified moment (Equation C2-1a) and associated design parameters for this method are:

$$M_{rx} = B_1 M_{nt} + B_2 M_{lt} = (0 \text{ kip-ft}) + 1.09\,(113 \text{ kip-ft}) = 123 \text{ kip-ft}$$

C_b = 1.67

L_b = 15 ft

Based on these design parameters, the available axial compressive strength and strong-axis available flexural strength of the ASTM A992 W14×109 are:

$P_c = \phi_c P_n$ = 1,220 kips
$M_{cx} = \phi_b M_{nx}$ = 720 kip-ft

To determine which interaction equation is applicable, the ratio of the required axial compressive strength to available axial compressive strength must be determined.

$$\frac{P_r}{P_c} = \frac{150 \text{ kips}}{1{,}220 \text{ kips}} = 0.123$$

Thus, because P_r/P_c < 0.2, Equation H1-1b is applicable.

$$\frac{P_r}{2P_c} + \frac{M_{rx}}{M_{cx}} = \frac{0.123}{2} + \frac{123 \text{ kip-ft}}{720 \text{ kip-ft}} = 0.232$$

The W14×109 is adequate because 0.232 ≤ 1.0.

Design by First-Order Analysis (Section C2.2b)

For the example frame given in Figure 2, the additional lateral load (with $\Delta = \Delta_{1st}$) is:

$$\Delta_{1st}/L = (0.405 \text{ in.})/(15 \text{ ft} \times 12 \text{ in./ft}) = 0.00225$$

$$Y_i = 75 \text{ kips} + 150 \text{ kips} + 150 \text{ kips} + 75 \text{ kips} = 450 \text{ kips}$$

$$N_i = 2.1(\Delta_{1st}/L)Y_i \geq 0.0042Y_i = 2.1(0.00225)(450 \text{ kips}) \geq 0.0042(450 \text{ kips}) = 2.13 \text{ kips} \geq 1.89 \text{ kips} = 2.13 \text{ kips}$$

It was previously determined in the illustration of design by second-order analysis example that the second-order drift is less than 1.5 times the first-order drift. Additionally,

αP_r = 1.0(150 kips) = 150 kips

and for the ASTM A992 W14×109,

$$0.5P_y = 0.5F_y A_g = 0.5(50 \text{ ksi})(32.0 \text{ in.}^2) = 800 \text{ kips}$$

Because $\Delta_{2nd} < 1.5\Delta_{1st}$ and $\alpha P_r < 0.5P_y$, the use of this method is permitted.

The loading for this method is the same as shown in Figure 2, except for the addition of a notional load of 2.13 kips coincident with the lateral load of 15 kips shown, resulting in a moment M_u of 128 kip-ft in each column.

This moment must be amplified by B_1 as determined from Equation C2-2. The Euler buckling load is calculated with K_1 = 1.0. Thus,

$$\begin{aligned} P_{e1} &= \pi^2 EI / (K_1 L)^2 \\ &= \pi^2(29{,}000 \text{ ksi})(1{,}240 \text{ in.}^4)/(1.0 \times 15 \text{ ft} \times 12 \text{ in./ft})^2 \\ &= 11{,}000 \text{ kips} \end{aligned}$$

Because the moment on one end of the column is zero, the moment gradient term is:

$$\begin{aligned} C_m &= 0.6 - 0.4(M_1/M_2) \\ &= 0.6 - 0.4(0/128) \\ &= 0.6 \end{aligned}$$

From Equation C2-2,

$$\begin{aligned} \alpha P_r/P_{e1} &= 1.0(150 \text{ kips})/(11{,}000 \text{ kips}) \\ &= 0.0136 \end{aligned}$$

$$\begin{aligned} B_1 &= \frac{C_m}{1 - \dfrac{\alpha P_r}{P_{e1}}} \geq 1 \\ &= \frac{0.6}{1 - 0.0136} \geq 1.0 \\ &= 0.608 \geq 1.0 \\ &= 1.0 \end{aligned}$$

The axial force and associated design parameters for this method are:

$$\begin{aligned} P_r &= 150 \text{ kips} \\ K_x &= K_y = 1.0 \\ L_x &= L_y = 15 \text{ ft} \end{aligned}$$

The amplified moment and associated design parameters for this method are:

$$\begin{aligned} M_{rx} &= B_1 M_u \\ &= 1.0(128 \text{ kip-ft}) \\ &= 128 \text{ kip-ft} \\ C_b &= 1.67 \\ L_b &= 15 \text{ ft} \end{aligned}$$

Based on these design parameters, the available axial compressive strength and strong-axis available flexural strengths of the ASTM A992 W14×109 are:

$$\begin{aligned} P_c &= \phi_c P_n = 1{,}220 \text{ kips} \\ M_{cx} &= \phi_b M_{nx} = 720 \text{ kip-ft} \end{aligned}$$

To determine which interaction equation is applicable, the ratio of the required axial compressive strength to available axial compressive strength must be determined.

$$\begin{aligned} \frac{P_r}{P_c} &= \frac{150 \text{ kips}}{1{,}220 \text{ kips}} \\ &= 0.123 \end{aligned}$$

Thus, because $P_r/P_c < 0.2$, Equation H1-1b is applicable.

$$\begin{aligned} \frac{P_r}{2P_c} + \frac{M_{rx}}{M_{cx}} &= \frac{0.123}{2} + \frac{128 \text{ kip-ft}}{720 \text{ kip-ft}} \\ &= 0.239 \end{aligned}$$

The W14×109 is adequate because 0.239 ≤ 1.0.

Direct Analysis Method (Appendix 7)

It was previously determined in the illustration of design by second-order analysis example that the second-order drift is less than 1.5 times the first-order drift (note that this check is properly made using the unreduced stiffness *EI*). Thus, the notional load can be applied as minimum lateral load, and that minimum is:

$$\begin{aligned} Y_i &= 75 \text{ kips} + 150 \text{ kips} + 150 \text{ kips} + 75 \text{ kips} \\ &= 450 \text{ kips} \end{aligned}$$

$$\begin{aligned} N_i &= 0.002Y_i \\ &= 0.002(450 \text{ kips}) \\ &= 0.9 \text{ kip} \end{aligned}$$

Because this notional load is less than the actual lateral load, it need not be applied.

For Columns D and E, using first-order analysis and B_1-B_2 amplification:

$$\begin{aligned} P_{nt} &= 150 \text{ kips}, P_{lt} = 0 \text{ kips} \\ M_{nt} &= 0 \text{ kip-ft}, M_{lt} = 113 \text{ kip-ft} \end{aligned}$$

To determine the second-order amplification, the reduced stiffness, EI^*, must be calculated.

$$\begin{aligned} \alpha P_r &= 1.0(150 \text{ kips}) \\ &= 150 \text{ kips} \end{aligned}$$

and for the ASTM A992 W14×109,

$$\begin{aligned} 0.5P_y &= 0.5F_y A_g \\ &= 0.5(50 \text{ ksi})(32.0 \text{ in.}^2) \\ &= 800 \text{ kips} \end{aligned}$$

Thus, because $\alpha P_r < 0.5P_y$, τ_b = 1.0 and

$$\begin{aligned} EI^* &= 0.8\tau_b EI \\ &= 0.8EI \end{aligned}$$

For P-δ amplification, because there are no moments associated with the no-translation case, there is no need to calculate B_1. For P-Δ amplification, the reduced stiffness EI^* must be used to determine the first-order drift. Because

$EI^* = 0.8EI$, the first-order drift based on EI^* is 25% larger than that calculated previously. Thus,

$$\Delta_{1st} = 1.25(0.405 \text{ in.}) = 0.506 \text{ in.}$$

The first-order drift ratio is determined from the amplified drift of 0.506 in.

$$\Delta_{1st}/L = (0.506 \text{ in.})/(15 \text{ ft} \times 12 \text{ in./ft}) = 0.00281$$

For moment frames, $R_M = 0.85$ and from Equation C2-6b with $\Delta_H = \Delta_{1st}$ and $\Sigma H = 15$ kips,

$$\begin{aligned} \Sigma P_{e2} &= R_M \frac{\Sigma H}{\left(\Delta_{1st}/L\right)} \\ &= 0.85 \frac{15 \text{ kips}}{(0.00281)} \\ &= 4{,}540 \text{ kips} \end{aligned}$$

For design by LRFD, $\alpha = 1.0$ and ΣP_{nt} is the sum of the gravity loads. Thus,

$$\begin{aligned} \alpha\Sigma P_{nt}/\Sigma P_{e2} &= 1.0(75 \text{ kips} + 150 \text{ kips} + 150 \text{ kips} + 75 \text{ kips})/4{,}540 \text{ kips} \\ &= 0.0991 \end{aligned}$$

From Equation C2-3, the amplification is:

$$\begin{aligned} B_2 &= \frac{1}{\left(1 - \frac{\alpha \Sigma P_{nt}}{\Sigma P_{e2}}\right)} \geq 1 \\ &= \frac{1}{(1 - 0.0991)} \geq 1.0 \\ &= 1.11 \geq 1.0 \\ &= 1.11 \end{aligned}$$

It is worth noting that use of the reduced axial stiffness, $EA^* = 0.8EA$, in members that contribute to lateral stability is also required in this method. However, due to the characteristics of the structures chosen for this example, there are no axial deformations that impact the stability of the structure.

The amplified axial force (Equation C2-1b) and associated design parameters for this method are:

$$\begin{aligned} P_r &= P_{nt} + B_2 P_{lt} \\ &= 150 \text{ kips} + 1.11(0 \text{ kips}) \\ &= 150 \text{ kips} \end{aligned}$$

$$K_x = K_y = 1.0$$

$$L_x = L_y = 15 \text{ ft}$$

The amplified moment (Equation C2-1a) and associated design parameters for this method are:

$$\begin{aligned} M_{rx} &= B_1 M_{nt} + B_2 M_{lt} \\ &= (0 \text{ kip-ft}) + 1.11(113 \text{ kip-ft}) \\ &= 125 \text{ kip-ft} \end{aligned}$$

$$C_b = 1.67$$
$$L_b = 15 \text{ ft}$$

Based on these design parameters, the available axial compressive strength and strong-axis available flexural strengths of the ASTM A992 W14×109 are:

$$P_c = \phi_c P_n = 1{,}220 \text{ kips}$$
$$M_{cx} = \phi_b M_{nx} = 720 \text{ kip-ft}$$

To determine which interaction equation is applicable, the ratio of the required axial compressive strength to available axial compressive strength must be determined.

$$\frac{P_r}{P_c} = \frac{150 \text{ kips}}{1{,}220 \text{ kips}} = 0.123$$

Thus, because $P_r/P_c < 0.2$, Equation H1-1b is applicable.

$$\frac{P_r}{2P_c} + \frac{M_{rx}}{M_{cx}} = \frac{0.123}{2} + \frac{125 \text{ kip-ft}}{720 \text{ kip-ft}} = 0.235$$

The W14×109 is adequate because $0.235 \leq 1.0$.

The Simplified Method

For the example frame given in Figure 2, the minimum lateral load is:

$$\begin{aligned} Y_i &= 75 \text{ kips} + 150 \text{ kips} + 150 \text{ kips} + 75 \text{ kips} \\ &= 450 \text{ kips} \\ N_i &= 0.002 Y_i \\ &= 0.002(450 \text{ kips}) \\ &= 0.9 \text{ kips} \end{aligned}$$

Because this notional load is less than the actual lateral load, it need not be applied.

The 15-kip lateral load produces slightly less drift than that corresponding to the design story drift limit because the W14×109 has $I = 1{,}240$ in.^{4} (versus the 1,120 in.^{4} required to limit drift to L/400). The lateral load required to produce the design story drift limit is:

$$15 \text{ kips} \times (1{,}240 \text{ in.}^4)/(1{,}120 \text{ in.}^4) = 16.6 \text{ kips}$$

The load ratio is then:

$$1.0 \times (75 \text{ kips} + 150 \text{ kips} + 150 \text{ kips} + 75 \text{ kips})/(16.6 \text{ kips}) = 27.1$$

Entering the table in the row for H/400, the corresponding multiplier for a load ratio of 27.1 is 1.1 (determined by interpolation to one decimal place). Because this multiplier is less than 1.5, $\Delta_{2nd} < 1.5\Delta_{1st}$ and the use of this method is permitted. Additionally, because the multiplier is equal to 1.1, K can be taken as 1.0 for column design in the moment frame with this method.

The amplified axial force (with the full axial force amplified by B_2) and associated design parameters for this method are:

$$P_r = 1.1P_u = 1.1(150 \text{ kips}) = 165 \text{ kips}$$

$$K_x = K_y = 1.0$$

$$L_x = L_y = 15 \text{ ft}$$

The amplified moment (with the full moment amplified by B_2) and associated design parameters for this method are:

$$M_{rx} = 1.1M_u = 1.1(113 \text{ kip-ft}) = 124 \text{ kip-ft}$$

$$C_b = 1.67$$

$$L_b = 15 \text{ ft}$$

Based on these design parameters, the axial and strong-axis flexural available strengths of the ASTM A992 W14×109 are:

$$P_c = \phi_c P_n = 1{,}220 \text{ kips}$$
$$M_{cx} = \phi_b M_{nx} = 720 \text{ kip-ft}$$

To determine which interaction equation is applicable, the ratio of the required axial compressive strength to available axial compressive strength must be determined.

$$\frac{P_r}{P_c} = \frac{165 \text{ kips}}{1{,}220 \text{ kips}} = 0.135$$

Thus, because $P_r/P_c < 0.2$, Equation H1-1b is applicable.

$$\frac{P_r}{2P_c} + \frac{M_{rx}}{M_{cx}} = \frac{0.135}{2} + \frac{124 \text{ kip-ft}}{720 \text{ kip-ft}} = 0.240$$

The W14×109 is adequate because $0.240 \leq 1.0$.

Summary for the Three-Bay Frame

As before, all methods produce similar designs. The result of the beam-column interaction equation for each method is:

Method	Interaction Equation
Second-Order	0.232
First-Order	0.239
Direct Analysis	0.235
Simplified	0.240

In this example, the interaction equations predict values that are so close to each other that there is no practical difference in the results.

CONCLUSIONS

The following conclusions can be drawn from the foregoing examples:

1. If conservative assumptions are acceptable, the easiest method to apply is the Simplified Method, particularly when the drift limit is such that K can be taken equal to 1.

2. None of the analysis methods in the AISC *Specification* are particularly difficult to use. The First-Order Analysis Method and Direct Analysis Method both eliminate the need to calculate K, which can be a tedious process based upon assumptions that are rarely satisfied in real structures. Nonetheless, those who prefer to continue to use the approach of past specifications, the Effective Length Method, can do so, provided they incorporate the new requirement of a minimum lateral load in all load combinations.

3. Second-order effects and leaning columns have a significant impact on strength requirements, but usual drift limits such as $L/400$ sometimes can result in framing that requires no increase in member size for strength. For frames with little or no lateral load and/or heavy floor loading, it is more likely that stability will control, regardless of the drift limits. This should not be taken as a blanket indication that the use of a drift limit eliminates the need to consider stability effects. Rather, it simply means that drift-controlled designs may be less sensitive to second-order effects because the framing is naturally stiffer and provides reserve strength. Drift limits also result in significant simplification of the analysis requirements when the increased framing stiffness allows more frequent use of the simplifications allowed in the various methods, such as the use of $K = 1$.

REFERENCES

AISC (2005a), *Specification for Structural Steel Buildings*, ANSI/AISC 360-05, American Institute of Steel Construction, Chicago, IL.

AISC (2005b), *Steel Construction Manual*, 13th ed., American Institute of Steel Construction, Chicago, IL.

Geschwindner, L.F., Disque, R.O. and Bjorhovde, R. (1994), *Load and Resistance Factor Design of Steel Structures*, Prentice Hall, Englewood Cliffs, NJ.

Lim, L.C. and McNamara, R.J. (1972), "Stability of Novel Building System," Structural Design of Tall Buildings, Vol. II-16, *Proceedings*, ASCE-IABSE International Conference on the Planning and Design of Tall Buildings, Bethlehem, PA, pp. 499–524.