Proceedings of the
Annual Stability Conference
Structural Stability Research Council
Pittsburgh, Pennsylvania, May 10-14, 2011

Buckling Strength of Tapered Bridge Girders under Shear and Bending

Metwally Abu-Hamd[1] and Iman Abu-Hamd[2]

Abstract
This paper presents the results of finite element analysis studies of the local buckling strength of web tapered plate girder used in bridges when subjected to shear and bending loads. A complete girder finite element model subjected to uniform bending stress without shear and to uniform shear stress without bending is used. A parametric study is performed to investigate the effect of major design parameters such as web and flange slenderness, tapering angle, tapered panel aspect ratio on the buckling strength. Comparisons with present prismatic members design code provisions are presented. Conclusions are made regarding the effect of design parameters on the behavior of web tapered girders.

1. Introduction
Web tapered girders are usually used in bridges to achieve economy by varying the web depth according to variation of the bending moments and shear forces resulting from applied loads. This variation leads to lighter design than conventional prismatic girders. Current design codes, e.g., AASHTO (2009), are based on theoretical and experimental research on prismatic girders. There are very few theoretical and experimental investigations into the structural behavior of web-tapered girders under shear and/or bending moments, Mirambell (2000). Consequently, there are no specific provisions in current design codes for the design of tapered girders.

Theoretical solutions of plate buckling problems are based on the simplifying assumptions of simply supported plate panels. These solutions do not consider the real boundary conditions at the web-flange and web-stiffener connections which are known from experimental investigations to be somewhere between simply supported and fixed depending on the relative slenderness of the flange and the stiffener. Finite Element Analysis has been used effectively to obtain the elastic buckling stress under a wider scope of design variables related to applied stresses and actual boundary conditions. The buckling stress is obtained by solving an eigen-value problem with the eigen-values representing the buckling load factors and the eigen-vectors representing the buckling mode shapes. The finite element models used may be a single isolated panel or a complete girder model. Numerical solutions obtained from isolated single panel models give conservative buckling strength values as compared to results obtained from complete girder models, Maiorana (2009).

[1] Professor, Structural Engineering Dept, Cairo University, Egypt, abuhamd@eng.cu.edu.eg
[2] Assistant Lecturer, Housing and Building National Research Center, Egypt.

Furthermore, it is well known that a slender plate element does not fail by buckling, but exhibits significant post-buckling strength. Initial geometric imperfections change the plate stability behavior from a bifurcation problem into a load-deflection problem. Based on previous studies, Chacon (2009), initial geometrical imperfections and residual stresses may reduce the buckling strength by up to 15 % for stocky girders and has negligible effects on slender girders. As the present aims at identify the effect of major design parameters on the buckling strength of tapered girders, initial geometrical imperfections and residual stresses are not included in the present study so the results give the elastic buckling strengths without any post-buckling effect.

2. Finite Element Analysis

2.1 Elastic Buckling Strength

The theoretical elastic buckling stress of a rectangular plate, σ_{cr}, is given by the widely known formula:

$$\sigma_{cr} = k_{\sigma} \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{d} \right)^2 \tag{1}$$

where E is the modulus of elasticity, ν is Poisson's ratio, t is the thickness of the plate, d is the width of the plate, and k_{σ} is the plate buckling factor, which depends on the type of stress distribution and the edge support conditions.

Finite Element Analysis, Earls (2007), Ziemian (2010), may be used effectively to obtain the elastic buckling stress under a wider scope of design variables related to applied stresses and actual boundary conditions. The buckling stress is obtained by solving the eigen-value problem:

$$\mathbf{K}_E = \lambda \, \mathbf{K}_G \tag{2}$$

Where $\mathbf{K}_E$ is the elastic stiffness matrix, $\mathbf{K}_G$ is the geometric stiffness matrix, and λ is the eigen-value which represents the buckling load factor. The corresponding eigen-vector represents the mode shapes of the buckled plate. The finite element models used may be a single isolated panel or a complete girder model. Numerical solutions obtained from isolated single panel models give conservative buckling strength values as compared to results obtained from complete girder models. In this paper, a finite element model representing a complete girder is used to investigate the elastic buckling strength of web tapered plate girder used in bridges when subjected to shear and bending loads.

2.2 Description of Girder Model

Fig. 1a shows the geometric configuration and dimensions of the complete girder model used in the present study. It represents a bridge girder with five segments, two of which are tapered. The panel length of the prismatic segments is equal to 4 meters each. The tapered segment length is varied between 2 and 6 meters at 2 meters intervals to give different tapered panel aspect ratios of 1, 2, and 3. The deeper end web depth is taken equal to 2 meters while the smaller end depth is varied between 1 meter and 2 meters at 0.25 meter intervals to give different tapering angles of 0.125, 0.25, 0.375, and 0.5.

Two load cases are applied to the model as shown in Fig. 1b and 1c.The first load case comprise two concentrated moments (M_1 and M_2) at tapered panel end. The moment value is taken equal to the yield moment of the respective section so that the tapered panel is subjected to a uniform bending stress which equals the yield stress F_y. The resulting shear from this moment gradient is balanced by additionally applying a uniform distributed moment (m) to the tapered panel which results in a shear force on the tapered panel equals to the shear from moment gradient but opposite in direction. Consequently this load case produces pure uniform bending stress on the tapered panel. In the second load case, the model is subjected to two point loads (V_1 and V_2) at tapered panel ends. The load values are chosen to produce a shear stress of 0.58*F_y at each end. As these loads subjects the tapered panel to high bending stresses, additional uniform distributed moment (m) is applied to the tapered panel and another opposite point moment (M_e) is applied at the girder ends. The values of these additional moments are calculated to balance the bending moment resulting from the applied shear so that the tapered panel is subjected to uniform pure shear stress. The total number of models studied is 180 models for the bending load case and 180 models for the shear loading case.

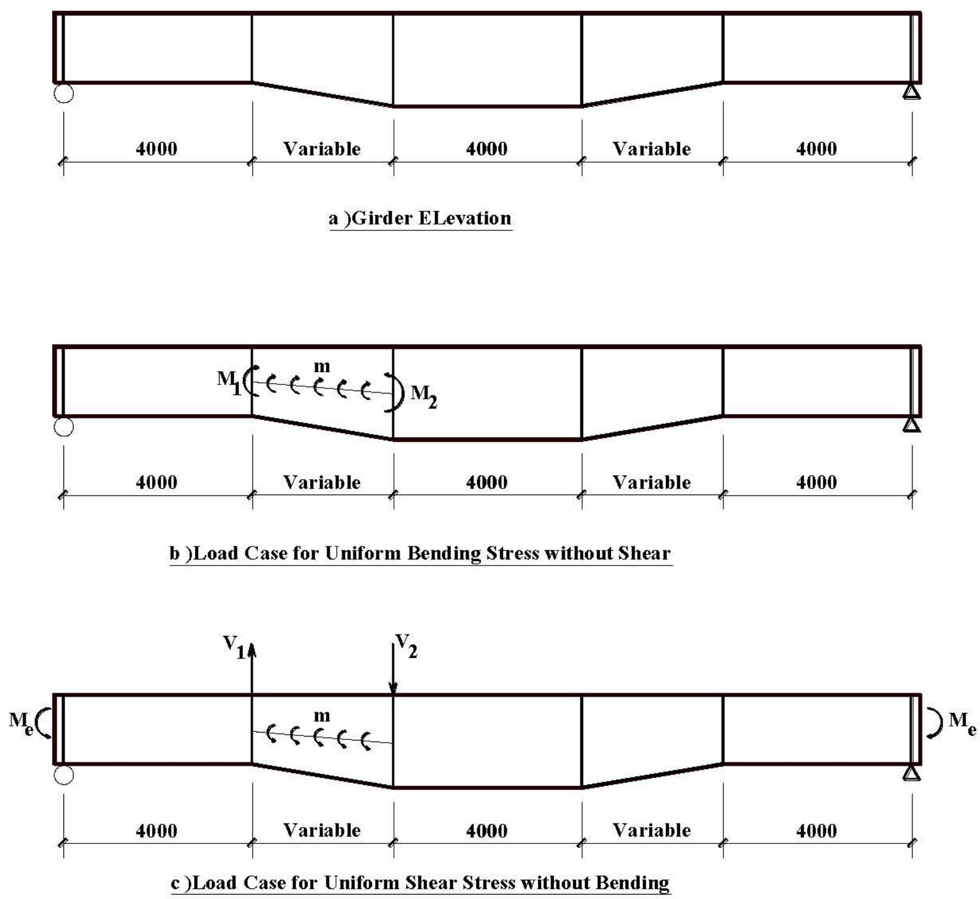
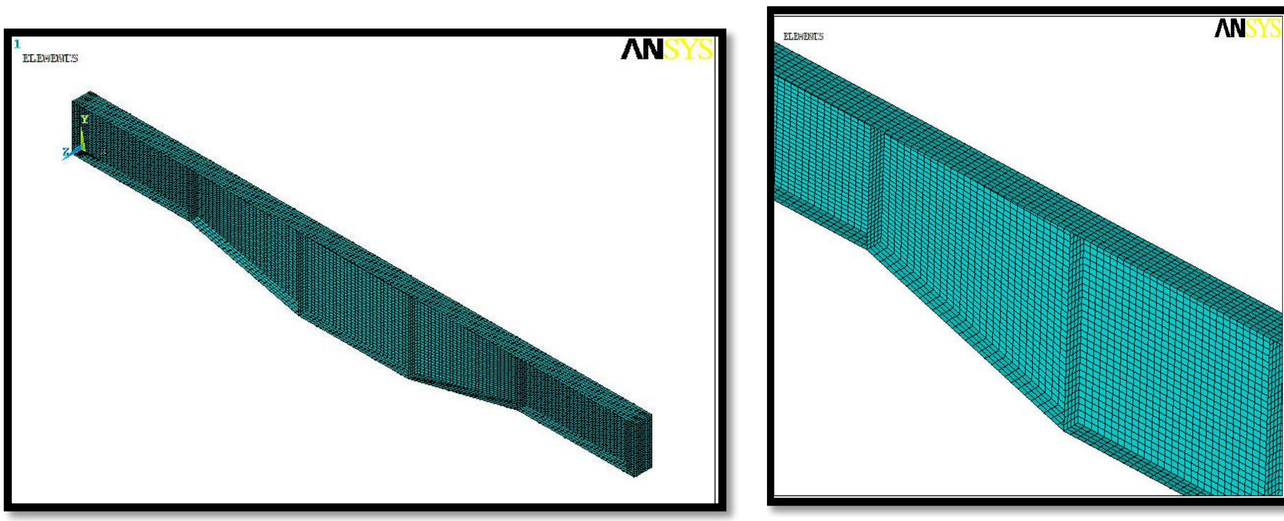


Figure 1: Tapered Girder Model and Loading

2.3 Finite Element Model

Fig. 2 shows the finite element model representing the model girder. All plate elements were modeled with an iso-parametric finite strain shell element designated as "Shell 181" in ANSYS element library. Shell 181 is a four-noded shell element with six degrees of freedom per node and has geometric and material nonlinearities capabilities. It is well suited for linear, large rotation, and /or large strain nonlinear applications. In the construction of the finite element model, convergence was achieved by using 12 elements through the flange width and 50 elements through both the web depth and each girder segment length. The displacement boundary conditions at girder ends were specified to give a hinged support at one end and a roller support at the other end. Lateral torsional buckling was prevented by restraining the movement in the out-of-plane direction of all nodes along the web-to-flange connection. The material properties used are Elastic modulus E=210 GPa, yield stress Fy=350 MPa, and Poisson's ratio ν=0.3.



b) Tapered Panel

a) Complete Girder

Figure 2: Finite Element Model

3. Bending Buckling Results

Fig. 3 shows the variation of the elastic bend buckling stress (F_{cr}/F_y) against the web slenderness ratio λ_w for different values of the flange slenderness ratio λ_w, the aspect ratio α and the tapering angle φ. The bend buckling strength for a prismatic member having the same section as the deeper end according to AASHTO (2009) is also plotted on the same Figure. This value corresponds to a buckling stress factor k_σ of 36, regardless of the flange slenderness. Based on the results of the studied girder models, the following may be concluded:

1- As expected, the effect of the web and flange slenderness ratios follows the same trend as prismatic members so that the bend buckling strength decreases inversely with the increase of web and flange slenderness ratios. The bend buckling strength is affected by the flange slenderness so that using a constant value of 36 for k_σ gives un-conservative results for slender flanges, Abu-Hamd (2010).

2- The combined effect of web and flange slenderness is clearly shown in Figure (4) by plotting the buckling stress factor k_σ , computed from the finite element results, against the parameter (λ_w / λ_f). The results show that k_σ increases almost linearly with (λ_w / λ_f) up to a value of ~15 then k_σ keeps an almost constant value afterwards.
3- The bend buckling strength increases with the increase of the tapering angle φ as shown in Fig. 4.
4- Based on the results of the studied girder models, the bend buckling strength of tapered girders may be approximately calculated using the prismatic girder equations with the girder depth substituted by an equivalent depth H_e as shown in Fig. 5.

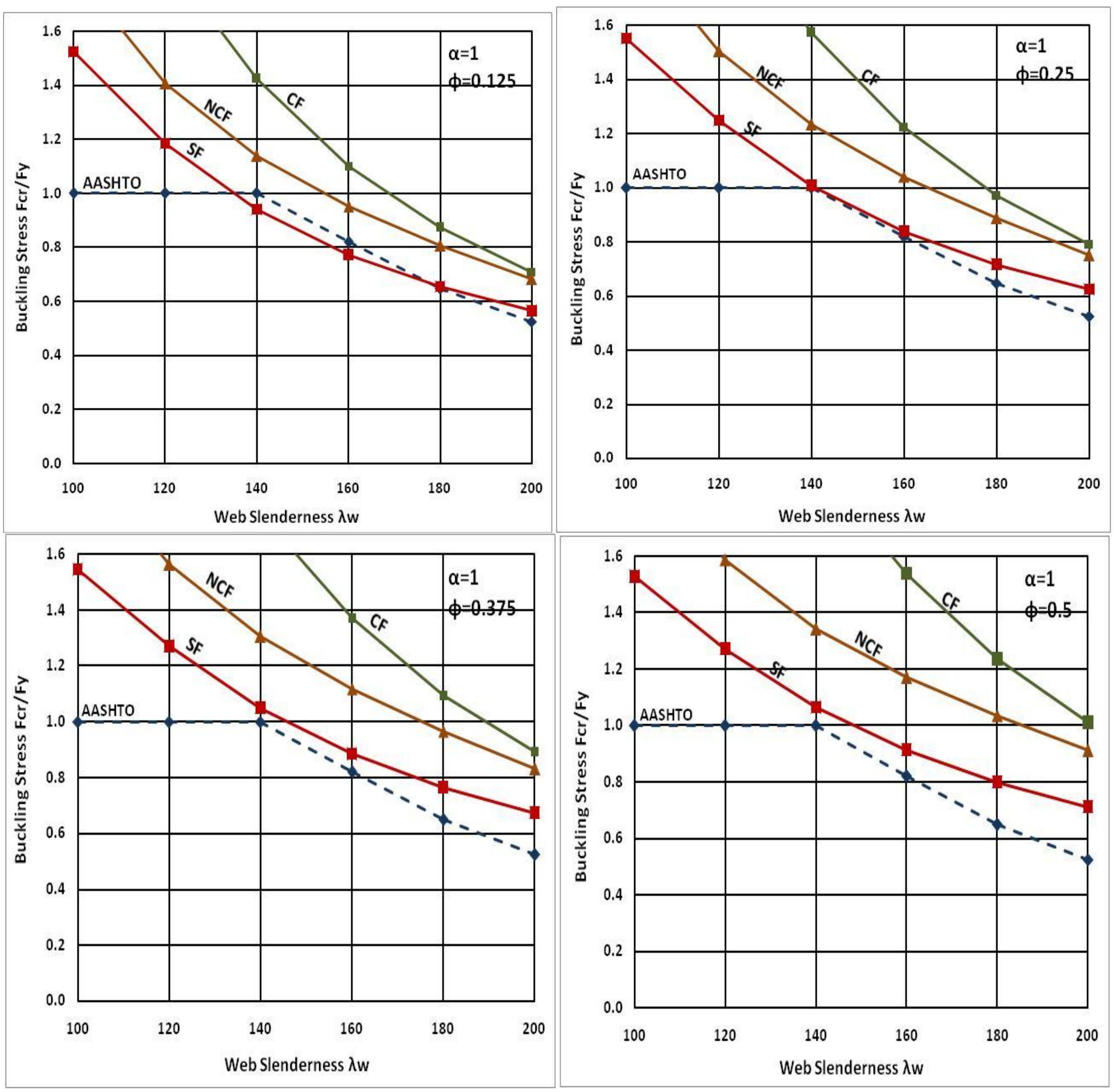


Figure 3: Variation of Bend Buckling Stress with Web Slenderness for different values of α and φ

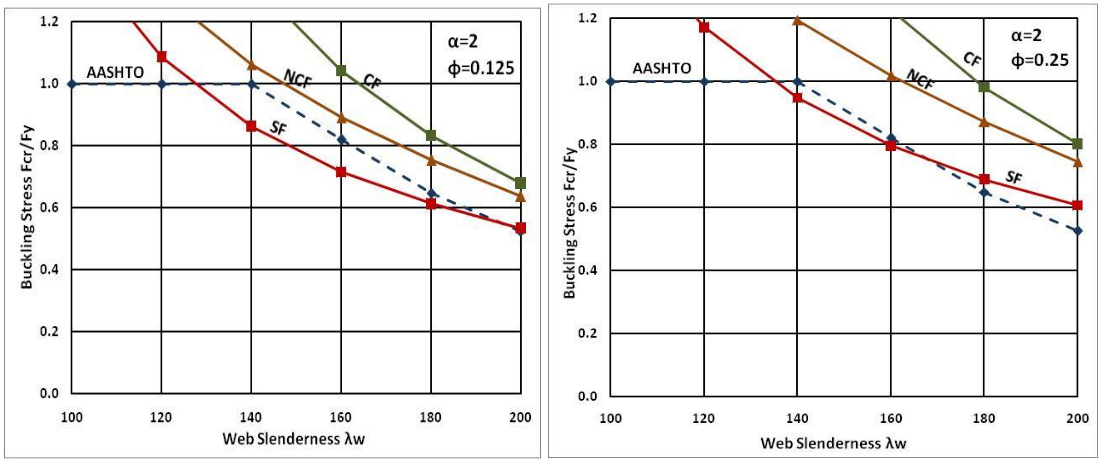


Figure 3: Variation of Bend Buckling Stress with Web Slenderness for different values of α and φ (contd.)

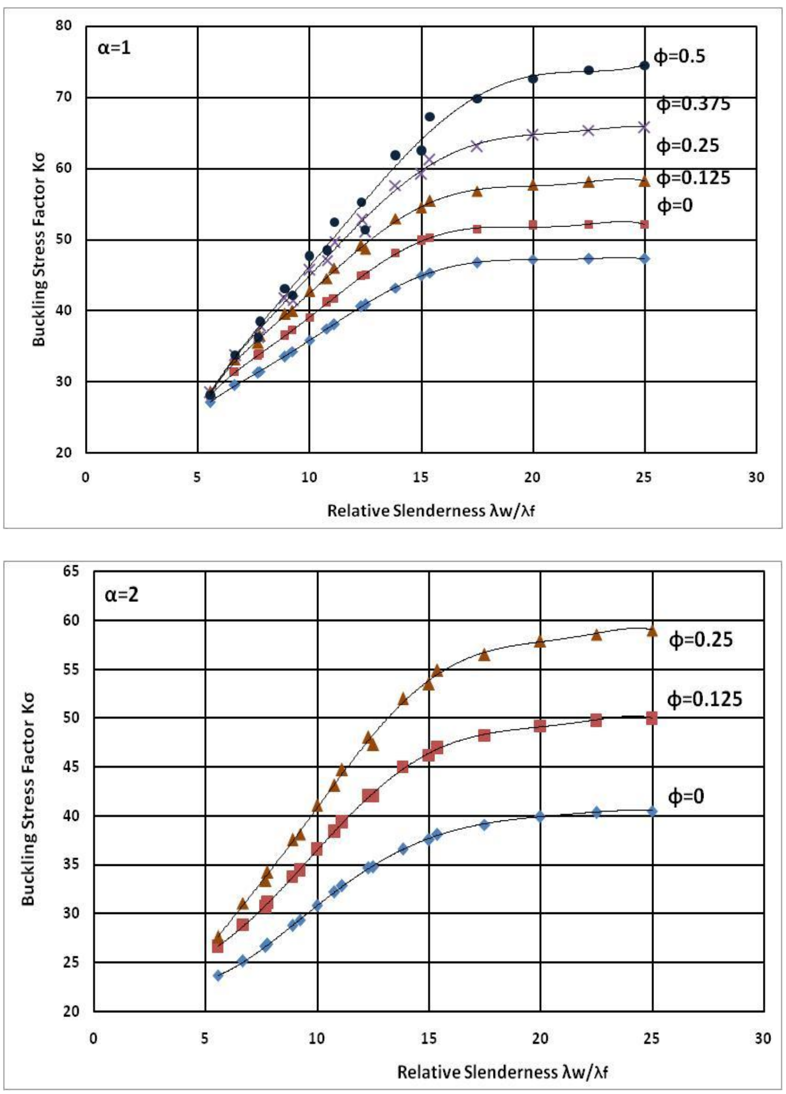


Figure 4: Variation of Bend Buckling Stress Factor with Relative Slenderness for different values of α and φ

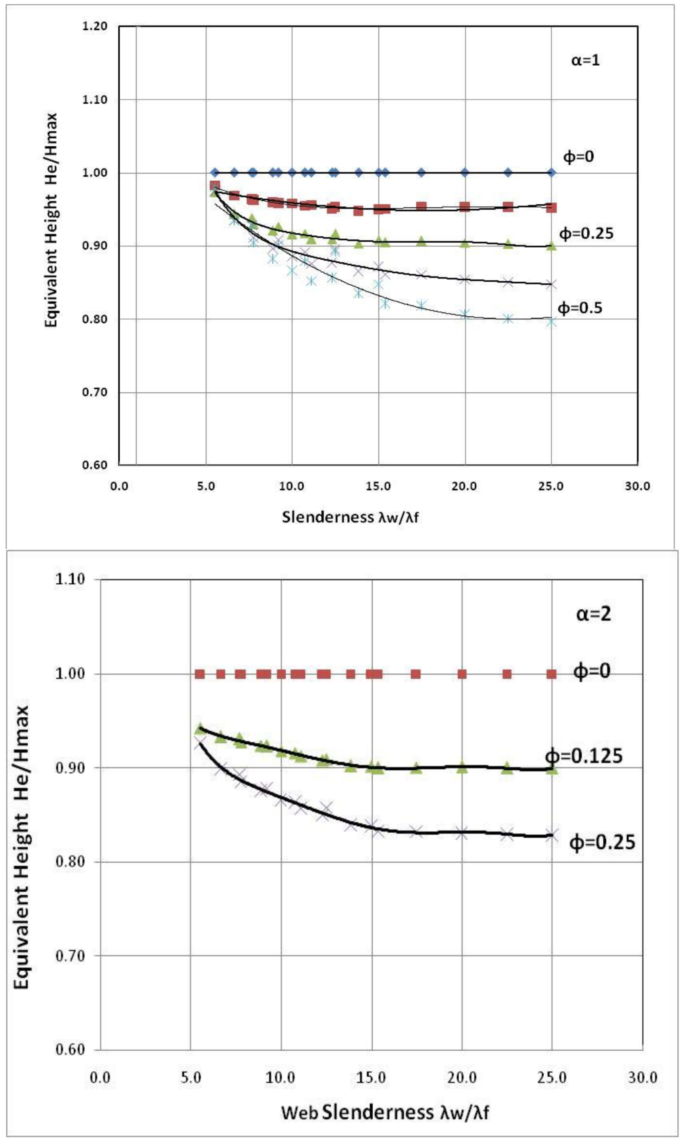


Figure 5: Height of Equivalent Prismatic Girder

4. Shear Buckling Results

Fig. 6 shows the variation of the elastic shear buckling stress ($q_{cr}/0.58*F_y$) against the web slenderness ratio λ_w for different values of the flange slenderness ratio λ_w, the aspect ratio α and the tapering angle φ. The shear buckling strength for a prismatic member having the same section as the deeper end according to AASHTO (2009) is also plotted on the same Figure. Based on the results of the studied girder models, the following may be concluded:

1- As expected, the effect of the web and flange slenderness ratios follows the same trend as prismatic members so that the shear buckling strength decreases inversely with the increase of web and flange slenderness ratios.

2- The combined effect of web and flange slenderness is clearly shown in Fig. 7 by plotting the shear buckling stress factor k_q , computed from the finite element results, against the parameter (λ_w / λ_f). The results show that k_q increases almost linearly with (λ_w / λ_f) up to a value of ~15 then k_σ keeps an almost constant value afterwards.

3- The shear buckling strength increases with the increase of the tapering angle φ as shown in Figure (7).

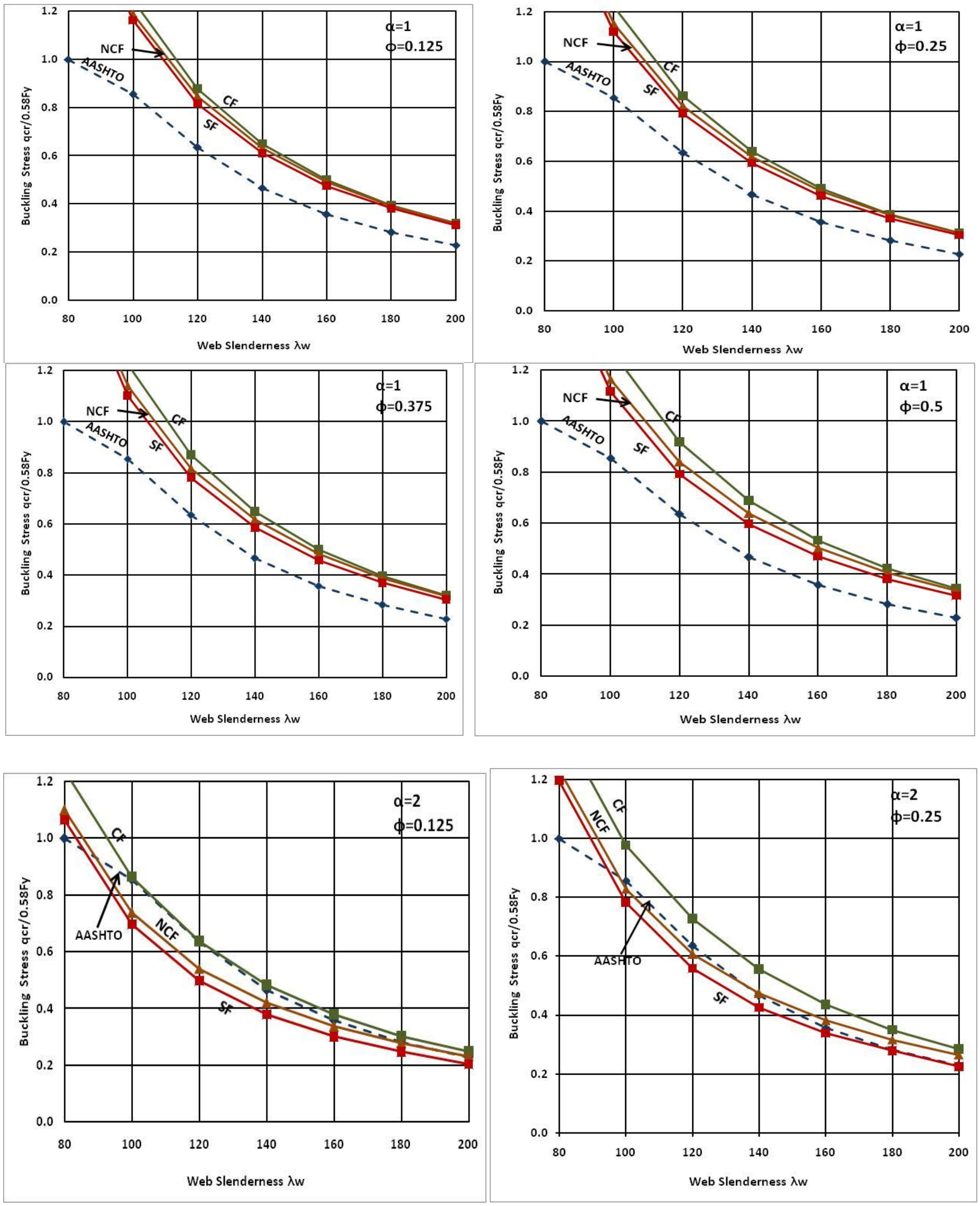


Figure 6: Variation of Shear Buckling Stress with Web Slenderness for different values of α and φ

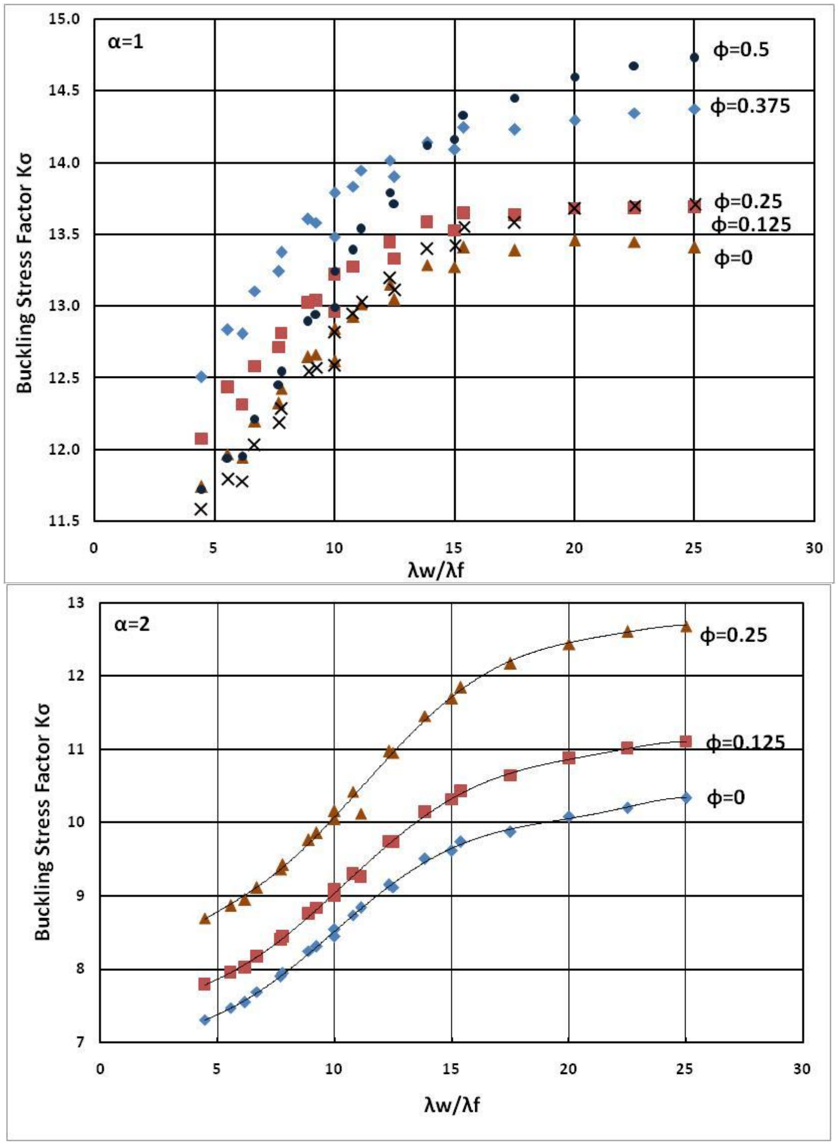


Figure 7: Variation of Shear Buckling Stress Factor with Relative Slenderness for different values of α and φ

5. Conclusions

This paper presents the results of finite element analysis studies of the local buckling strength of web tapered plate girder used in bridges when subjected to shear and bending loads. A complete girder finite element model subjected to uniform bending stress without shear and to uniform shear stress without bending is used. A parametric study is performed to investigate the effect of major design parameters such as web and flange slenderness, tapering angle, tapered panel aspect ratio on the buckling strength. The results of the parametric study showed the variation of the elastic buckling stress of web tapered girders with the major design parameters. More numerical and experimental investigation are still needed to arrive at clear design rules governing the buckling design of web tapered girders. This includes the effect of combined shear and bending and the post buckling behavior.

References

AASHTO (2009): LRFD Bridge Design Specifications, Washington, D.C.

Abu-Hamd (2010), "Effect of Local Buckling on the Design of Steel Plate Girders", *Proceedings of the Structural Stability Research Council, Annual Stability Conference,* pp 63-82.

ANSYS (2009) , Theory Manual, Swansea Company.

Chacon, R., et. al. (2009). “Influence of designer-assumed initial conditions on the numerical modeling of steel plate girders subjected to patch loading.” *Thin-Walled Structures,* Vol 47, pp 391-402.

Earls, C. J. (2007) "Observation on Eigenvalue buckling analysis within a finite element context." *Proceedings of the Structural Stability research Council, Annual Stability Conference.*

Maiorana, E., et. al. (2009). “Imperfections in steel girder webs with and without perforations under patch loading.” *Journal of Constructional Steel Research,* Vol 65, pp 1121-1129.

Mirambell, E., et. al. (2000). “Web buckling of tapered plate girders." *Proceedings Inst Civil Eng Struct* , Vol 140, pp 51-60.

Real, E., et. al. (2010). “Numerical and experimental research in tapered steel plate girders subjected to shear .” *SDSS’Rio 2010 Stability and Ductility of Steel Structures*, Batista, E., et al (Eds), pp 747-754.

Ziemian, R. (Ed.) (2010), Guide to stability design Criteria for metal structures," John Wiley & Sons.