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Modal identification in nonlinear collapse analysis of thin-walled members

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Abstract

The objective of this paper is to provide a method for applying modal identification, (i.e. the separation of general deformations into fundamental buckling deformations classes: local, distortional, global, shear, and transverse extension) to collapse analysis of thin-walled members modeled using material and geometric nonlinear shell finite element analysis. The advantage of such a modal identification is the ability to categorize and reduce the complicated deformations that occur in a shell finite element model – and ultimately to (a) quantitatively associate failures with particular classes, e.g. state a model is a local failure, and (b) track the evolution of the classes, e.g., mixed local and distortional buckling leads to a distortional failure in a given model. Ultimately, this will aid Specification development, which must simplify complicated behavior down to strength predictions in isolated buckling classes. The modal identification method is enabled by creating a series of base functions within the fundamental buckling deformation classes that can then be compared to the general finite element displacements. The base functions are constructed using the constrained finite strip method (cFSM) for general end boundary conditions, previously developed by the authors. A fairly sizeable minimization problem is required for assigning the contributions to the fundamental buckling deformation classes. The procedure is illustrated with numerical examples of cold-formed steel members modeled to collapse with geometric and material nonlinearity provided. In future work it is planned to explore members with changes in geometry (e.g., holes and tapers) such that exact match between the *c*FSM base functions and the developed finite element models is impossible.

1. Introduction

The strength of thin-walled members is often governed by its buckling behavior due to high cross-sectional slenderness. As commonly acknowledged, thin-walled member instabilities can be generally categorized as: local (local-plate), distortional, and global (Euler) buckling. As currently reflected in design specifications, such as AISI-S100 [1], appropriate separation and identification of the buckling modes are necessary because of the different post-buckling strength and interactions between the modes.

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Given the many advances in computation, the Finite Element Method (FEM) employing shell finite elements has become more popular in analyzing thin-walled structures both for elastic (eigenmode) buckling and nonlinear collapse analyses. FEM's unique applicability to handle complex geometry and boundary conditions makes it a natural choice in many situations. However, FEM itself provides no means of modal identification, and instead requires a laborious and completely subjective procedure employing visual investigation.

For elastic buckling analysis, the newly developed constrained finite strip method (*c*FSM) [2-7] provides the ability to decompose buckling modes as well as categorize (identify) arbitrary buckling modes (or displacements) into the fundamental deformation classes of global (G), distortional (D), local (L), shear (S), and transverse extension (T). G, D, and L are generally the most relevant classes, and since S and T are commonly handled together they are referred to simply as ST in the following. Recently, the base functions (or vectors) in *c*FSM have been employed to quantitatively identify buckling modes of shell finite element models for elastic buckling analysis [8-9]. Specifically, the generalized base vectors in [9] provide a means to potentially handle arbitrary end boundary conditions.

However, to date, for shell element based nonlinear FEM collapse analysis, failure mode identification is still investigated based on subjective, and largely visual, engineering observations. In this paper, the *c*FSM based modal identification procedure is applied to the nonlinear collapse analysis of shell finite element models with both geometric and material nonlinearity. Note, generalized beam theory has been utilized in nonlinear elastic analysis for a similar purpose [10].

2. Modal identification using cFSM base functions

The constrained Finite Strip Method (*c*FSM) is originally derived from the semi-analytical finite strip method (FSM) by separating the original displacement field of FSM into the four categorized and reduced buckling mode classes: G, D, L, and ST. This separation is based on mechanical assumptions, as has been typical of the *c*FSM literature [2-7] Application of the mechanical assumptions for each mode class defines a subspace of the original full deformation space that is limited to each mode class. Namely, *c*FSM defines constraint matrices, R_M , where M is G, D, L, or ST. The columns of R_M equate to a set of base vectors for subspace M. Transformation inside each subspace is possible [4, 6-7]. R_G , R_D , R_L , and R_{ST} taken together, as constraint matrix R, span the entire original nodal FSM displacement and represent a transformation of the solution from the original nodal degrees of freedom to a basis where G, D, L, and ST deformations are separated. (Note, columns of R are referred to as 'base vectors'. These vectors are the nodal representations of special displacement functions, thus, they may equally be called 'base functions'.)

2.1 FEM eigenbuckling identification

In standard FEM notation, the eigenbuckling problem may be stated as:

$$(K_e - \lambda K_g) \varphi = 0 \tag{1}$$

where, K_e is the elastic stiffness, K_g is the geometric stiffness which depends on the applied loading, λ is the eigenvalue (load factors), and φ is the eigenmode (buckling mode) vector.

Existing efforts towards FEM eigenbuckling identification have focused on categorizing the buckling mode φ into the four buckling mode classes, G, D, L and ST, in terms of participation percentages. This is completed, first by taking the base vectors in *R* of *c*FSM and interpolating them to all the nodal locations in the FEM displacement field. The new *c*FSM base functions written in the FEM degrees of freedom are referred as Φ . Then, a sizeable minimization problem is solved by approximating the eigenmode vector φ as a linear combination of the base function in Φ .

The base functions are formed based upon certain assumed boundary conditions in *c*FSM. The analytically attractive simply supported boundary conditions (S-S) may not be used without modification as these base functions cannot handle buckling problems where translational movements are allowed at the ends [8]. Generalized base functions are proposed in [9] which include the base vector of clamped-free (C-F) and free-clamped (F-C) boundary conditions. The longitudinal shape functions for S-S and C-F boundary conditions are expressed as follows:

S-S:
$$Y_{[m]} = \sin(m\pi y/a)$$
 C-F: $Y_{[m]} = 1 - \cos[(m-1/2)\pi y/a]$ (2)

For mode identification of members with general boundary conditions, the proposed generalized base functions are the combination of S-S functions with several longitudinal terms and the C-F and F-C base functions with one (m=1) longitudinal term. These base functions are applied in the examples shown subsequently.



Figure 1 longitudinal field diagram: (a) C-F&F-C m=1; (b) S-S m=1,2,...,10

2.2 Extension to FEM nonlinear collapse analysis

The nonlinear collapse analysis problem, in an FEM context, can be expressed as:

$$(K_e + K_g + K_p) d = F \tag{3}$$

where, K_e is the conventional elastic stiffness matrix, K_g is the geometric stiffness matrix and depends upon the current forces applied on the structure, K_p is the plastic reduction matrix to account for yielding, d is the displacement vector, and F is the consistent nodal forces applied on the structure. Note, different from the eigenbuckling problem above, K_e and K_g here are formulated upon the deformed shape in each increment step (or iteration) when analyzing the nonlinear problem by iterative methods, such as the Newton-Raphson or arc-length method.

For modal identification in the nonlinear collapse analysis the vector to be identified is d, the displacement vector, as opposed to φ , although, the same identification procedure can be performed on d to categorize the buckling modes of each step during the collapse analysis by using the generalized base functions. One note, the base functions from *c*FSM used to identify the deformation in the nonlinear collapse analysis are based on the perfect geometry. However, the deformation vectors are normally based on a model of the member with initial geometric

imperfections. Based on the author's studies, adding the imperfection field to the deformation vector provides a means to approximately address this geometric discrepancy. Consequently, the deformation vector to be identified is actually:

$$d_{id} = d + d_{imp} \tag{4}$$

where, d_{id} is the deformation vector to be identified in terms of G, D, L, and ST, d is the deformation vector from Eq. (3), and d_{imp} is the imperfection deviating from the perfect model.

3. FEM nonlinear collapse modeling

3.1 Modeling parameters

Shell finite element modeling of thin-walled structures for ultimate strength prediction and investigation of collapse behavior necessitates the inclusion of both geometric and material nonlinearity. The solutions of these models are highly sensitive to model inputs, such as geometric imperfections, residual stresses, plastic strain, yield criteria, material model, boundary conditions, and also the fundamental mechanics, particularly with regard to element selection and solution schemes [11]. All the analyses performed herein utilize the commercial finite element package ABAQUS [12]. In this study, residual stresses and plastic strains are not included in this model. Additional model assumptions are discussed in details as follows.

3.1.1Mesh and boundary conditions

Prediction of the peak strength and failure behavior of thin-walled structural members requires a fine mesh in nonlinear FEM models to provide reliable results. The shell element used here is the S4 shell element from the ABAQUS library of elements [12], the S4 is a 4-node linear element (fully integrated). In addition, the element aspect ratio is controlled between $\frac{1}{2}$ to 2 to avoid element distortion under large deformations. Specifically, for all the channel sections studied in this paper, the mesh of the cross section is provided in Section 3.2.

The end boundary conditions simulate local-plate simply supported conditions, which imply warping fixity at the member ends. Specifically, all translational degrees of freedom are fixed at one end of the model, at the opposite end the transverse translational degrees of freedom are fixed, while the longitudinal translational degrees of freedom are tied to a single reference node, where the end shortening (loading) is performed.

3.1.2 Material model

The material is assumed to be homogeneous and isotropic and modeled as elastic-perfectly plastic (von Mises yield criteria with isotropic hardening) with Young's modulus E=210,000 MPa, Poisson's ratio v=0.3, and a yield stress of 345 MPa.

3.1.3 Imperfections

Careful treatment of geometric imperfections is of significant importance in modeling coldformed steel members, because the ultimate strength and post-buckling mechanisms are both imperfection sensitive. Both the imperfection distribution and magnitude are important. In this study, for modeling convenience, the distribution of the imperfection are seeded from the local and/or distortional buckling mode shapes generated from a CUFSM analysis [7], and the magnitude is a function of the plate thickness. It is worth noting here that if one wants to simulate tests or provide strength predictions of cold-formed steel member in a more reliable and accurate manner, the distribution and magnitude of imperfections should be more closely tied to measured data [11, 13].

3.1.4 Solution control

Nonlinear collapse analysis is sensitive to the solution scheme. Methods, such as displacement control, arc-length method, and artificial damping, are typically potentially appropriate for collapse modeling of cold-formed steel members. The method employed in the study is the arc-length method (modified Riks method [14] in ABAQUS).

3.2 Cross-sections under study

Three cold-formed steel members, as shown in Figure 2, are selected to study the proposed modal identification method for nonlinear collapse analysis. In Figure 2, the nodes illustrate the mesh density within the cross-section. All the members are modeled as columns under the boundary conditions specified in Section 3.1.1. Member (a) with a length of 600 mm in Figure 2 is a local dominant model (first buckling mode is the local buckling mode in the eigenbuckling analysis). Member (b) with a length of 1200 mm in Figure 2 is a distortional dominant model. Member (c) with a length of 700 mm in Figure 2 is a local-distortional interacted model, which means the first FEM eigenbuckling mode is a mixed local and distortional buckling mode. The relatively short length for each section is intended to exclude global buckling from dominating.



4. Modal identification of FEM nonlinear collapse: perfect initial geometry (no initial imperfections)

Cold-formed steel members have geometric imperfections during manufacturing. Thus, perfect geometry (no imperfections) does not exist in reality. However, studying the perfect geometry can still provide useful insights about the behavior of cold-formed steel members, as is shown herein. For the cross-sections in Figure 2, the first eigenbuckling mode and full nonlinear collapse models are both analyzed with shell finite element analysis.

While eigenbuckling analysis is based on a linear perturbation, static analysis is based on incremental load-deflection response. For modal identification, determining which displacements to consider in the identification analysis requires some judgment. For example, in this study, the applied load (actually an applied displacement) of column end shortening is highly correlated with the fourth global buckling mode (G4) of the *c*FSM base functions used for modal identification. Similarly, in major-axis bending moment the applied deformations are also highly correlated with a global mode *c*FSM base function. In this study, modal identification results both with and without these correlated global modes are provided. For perfect initial geometry, modal identification results without the end shortening (G4) deformation (i.e., the *adjusted* participations) are summarized in Table 1 along with the elastic buckling load and peak load.

		Participation (%)			Load (kN)		
Model		G	D	L	ST	P _{cr}	P _n
Local dominant model	1 st eigenmode	0.4	4.8	94.6	0.1	20.8	
	at peak	0.7	12.8	85.4	1.1		72.9
	at post	0.4	55.9	41.0	2.8		
Distortional dominant model	1 st eigenmode	6.0	89.2	4.7	0.1	173.4	
	at peak	0.1	93.8	4.6	1.6		163.8
	at post	0.1	82.6	14.8	2.5		
Local-distortional interacted model	1 st eigenmode	2.9	38.5	58.1	0.4	283.8	
	at peak	0.2	50.5	47.1	2.2		296.4
	at post	18.4	52.0	27.5	2.1		

Table 1 Participations of eigenmode and nonlinear collapse models

Note: at peak refers to the peak of the load-displacement response in a nonlinear collapse analysis at post refers to collapse mechanisms at force levels less than 80% of peak deformation

4.1 Local dominant model

Modal identification results for the local dominant model (without imperfections) are summarized in Table 1 and shown throughout the nonlinear collapse analysis in Figure 3b. If G4 (the end shortening mode) is included in the determination of the participations then G modes dominate the response in the pre-buckling regime and have significant percentages in the buckling and post-peak regimes (see upper plot of Figure 3b). However, if G4 participations are removed, then the eigenmode and peak modal identification results are similar (Table 1) and only in the collapse/post-peak range does distortional buckling significantly participate.

This model demonstrates that a local dominant model (as determined by eigenanalysis and the fact that elastic local buckling is at 20.8 kN, but ultimate collapse at 72.9 kN), if properly considered, is indeed local dominant up to the peak strength, but even in a local dominant model the final collapse mechanisms may have significant contributions from other modes (e.g., distortional). This is consistent with experimental observations in [15]. Finally, it is also worth noting the role of the shear and transverse extension (ST) modes. Since no imperfection exists in this model the initial deformations (primarily associated with localized deformations due to the end boundary conditions) have a noticeable ST contribution. However, once the buckling fully initiates the ST participations decrease to near zero until the post-peak region.

4.2 Distortional dominant model

Modal identification results for the distortional dominant model (without imperfections) are summarized in Table 1 and shown throughout the nonlinear collapse analysis in Figure 4b. For the adjusted participations (i.e., G4 removed, as shown in lower plot of Figure 4b) the distortional mode participation at peak strength is similar to the distortional buckling mode participation in eigenbuckling analysis (see Table 1). However, while the local dominant model has significant post-buckling, the selected distortional dominant model is in the inelastic buckling regime (i.e., with strength of 163.8 kN and elastic buckling of 173.4 kN). Interestingly, in the collapse regime, local buckling participation grows slightly (presumably due to short wavelength localizations in the plastic mechanism) although distortional buckling remains dominant with participations near 80%.

4.3 Local-distortional interacted model

Modal identification results for the local-distortional interacted model (without imperfections) are summarized in Table 1 and shown throughout the nonlinear collapse analysis in Figure 5b. While the elastic buckling analysis suggests greater participation for the local mode, the collapse analysis reveals dominance of the distortional mode (with G4 removed). This is consistent with current understanding that distortional buckling has lower post-buckling reserve than local buckling and will dominate response even when at the same buckling load. The presence of a global mode in the collapse regimes is a surprise and warrants further study.

5. Modal identification of FEM nonlinear collapse: models with initial imperfections

Imperfections have a significant influence on nonlinear collapse analysis. A series of different imperfection distributions and magnitudes are explored for each of the three models and the results summarized here.

5.1 Local dominant model

For the local dominant model, five imperfection schemes are considered. The distributions are based directly on the local or distortional buckling mode shapes from CUFSM analysis [7] and listed along with the magnitude of the geometric imperfections in Table 2.

		1				
Case	Waves along member	Distribution	Magnitude	Peak load (kN)		
Ι	n/a	n/a	0	72.9		
II	5	Local	0.1t	72.1		
III	1	Outward distortional	0.1t	73.1		
IV	1	Inward distortional	0.1t	72.6		
V	1	Outward distortional	0.94t	68.3		

Note: (1) t is the thickness of the cross-section.

(2) Wave numbers are determined from the characteristic half-wavelengths of local and distortional buckling from CUFSM analysis.

Load-displacement responses for the local dominant models are provided in Figure 3a, and indicate generally mild imperfections sensitivity in the studied cases. The (modal) participations are provided in Figure 3c-f for the cases with imperfections II-V, respectively. When the local imperfection is presented (case II), slightly lower peak load is observed compared to the perfect geometry (case I), and participations (e.g., between local and distortional) remain essentially the

same. For this local dominant model, the strength of the member is not greatly influenced by a small amount of distortional imperfection (case II and IV, 0.1t), but is significantly reduced when the magnitude is bigger (case V, 0.94t). Illustrating the point that not all imperfections are detrimental, at small magnitudes the outward distortional imperfection (case III) actually increases the strength above the no imperfection results (case I).



The participation results of Figure 3c-f provide further insights. For a local dominant model with a local imperfection (case II) the results are similar to the no imperfection results (case I) and local participations dominate except in the post-peak range where distortional participation increases significantly.

In the local dominant model with small distortional imperfections (cases III and IV) the initial participations are largely distortional, consistent with the imperfection, but under increasing load the deformations move towards local modes and local participations increase and dominate prior to reaching the peak strength. Again, in the collapse regime, distortional buckling deformations start to grow rapidly and finally take over again.

However, even in the local dominant model, if the magnitude of the distortional imperfection is large enough (case V) then both local and distortional participations are observed throughout the loading regime. Consequently, this leads to a local-distortional interacted failure at the peak strength. In the collapse regime, distortional deformations once again dominate. For all the cases studied, if end shortening (G4) is excluded from the global buckling mode, then there are negligible global contributions during collapse.



5.2 Distortional dominant model

For the distortional dominant model, three imperfection schemes are studied: no imperfection (case I); local imperfections with 8 half-waves and a magnitude of 0.1t (case II); and distortional imperfections with 2 half-waves and a magnitude of 0.1t (case III). Load-displacement response and modal identification results shown throughout the nonlinear collapse analysis are provided in Figure 4. The distortional dominant model is far more imperfection sensitive than the local dominant model. Introduction of even a 0.1t imperfection results in peak strength decreases from the no imperfection (case I) model of 9% (below 149 kN) for the local imperfection (case II) and 14% loss for the distortional imperfection is more influential even in the collapse regime – as evidenced by the strongly local-distortional interacted failure that results from the local (case II) imperfection. If the imperfection matches the collapse, as in Case III, then the participation of the given mode, distortional in this case, will persist throughout the analysis. Such a "sympathetic" imperfection is likely to consistently be the most detrimental with to strength.

5.3 Local-distortional interacted model

For the local-distortional interacted model, similar to the distortional dominant model, three imperfection schemes are studied: no imperfection (case I); local imperfections with 5 half-waves and a magnitude of 0.1t (case II); and distortional imperfections with 2 half-waves and a magnitude of 0.1t (case III). Load-displacement response and modal identification results shown throughout the nonlinear collapse analysis are provided in Figure 5.



In this interacted model when the imperfection matches a particular mode (e.g. local), that mode starts out as dominant (under no load), then decreases its participation under deformation as the other mode (e.g., distortional) increases. However, past peak, the distortional mode dominates and the local mode decreases its participation while a global mode (exclusive of G4) also increases. The magnitude of the nonlinear collapse participations indicate the failure is an interaction between the two modes; however where the eigenbuckling analysis (Table 1) suggests local buckling will largely control, the collapse analysis suggests distortional buckling will dominate.

6. Discussion

The examples provided herein provide proof-of-concept for the extension of modal identification, driven by *c*FSM base vectors, to nonlinear collapse analysis. A key issue that arises in the analysis that requires further study is how to handle the primary displacement associated with a given loading. In the simple loading case provided here the primary displacement was essentially coincident with the G4 global mode, thus it was possible to consider participations both with and without this mode considered. However, for more complicated loading the separation is less clear. It is proposed that the entire linear elastic deformations associated with the applied loading on the perfect structure should be removed from the identification, but this idea needs further study.

In modal identification problems for eigenbuckling modes the necessity of shear and transverse extension (ST) is typically not obvious. However, consistent with observations in GBT [16] in the examples provided here ST has growing importance in the collapse regime, typically greater than 2% of the participation.

Though not specifically provided herein it is also worth noting that the error in the identification process is normally larger in the collapse regime due to the localization of the deformations. Potentially, this may be reduced by including base vectors with a larger number of longitudinal terms. However, the additional computational effort would be significant.

From a behavioral standpoint the dominance of distortional buckling in the post-peak regime of lipped channels, independent of the pre-peak deformations is an interesting result. Local failures, whereby local buckling dominates the response up to the peak strength are observed, but the collapse mechanisms that are triggered are typically better capture by the distortional base vectors.

Further, it is worth recalling that although the *c*FSM base vectors span a sufficient part of the FEM deformation space (and hence identification with only small errors is possible) the actual separations into the G, D, L, and ST spaces are based on linear elastic response. Specifically when it is observed that distortional buckling dominates in the post-peak regime, what this implies is that the largely plastic deformations associated with the collapse are best described by the elastic displacements associated with distortional buckling. This extension of elastic base vectors to describe plastic deformation fields is practical, and appears potentially useful in an engineering sense, but further work is needed before such an identification could be considered rigorous.

Finally, a long term goal of this work is to connect collapse mechanisms and their related energy dissipation to the G, D, L, and ST deformation spaces. Currently, no simple means exists for predicting energy dissipation in members, and modal identification and categorization provides new information to make these connections. Future work in this direction is anticipated.

7. Conclusions

A set of generalized base vectors developed from the constrained finite strip method are employed to identify the deformations of the nonlinear collapse analysis of a thin-walled member in terms of global, distortional, local, shear, or transverse extension deformations. The applicability of the method is demonstrated with numerical examples of geometrically perfect and imperfect cold-formed steel lipped channel members under uniform end shortening. The results indicate how the participation of a given deformation space evolves under load. For example, in the case of a member dominated by local buckling, the local mode has the only significant participation prior to collapse (peak strength); however, in the collapse regime distortional deformations have a large participation as they better describe the localized collapse mechanism that forms. The analysis results quantitatively demonstrate the interplay between local, distortional, and global buckling during collapse and even demonstrate the importance of shear and transverse extension (particularly during collapse). This paper provides proof-of-concept for the extension of *c*FSM base vectors to modal identification of nonlinear collapse models and significant discussion on future work that remains in this area.

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