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Simplified method for determining the critical elastic distortional buckling load of thin-walled cold-formed steel sections

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Abstract
The Direct Strength Method (DSM) is a recently adopted design approach by the North American Specifications for Cold-Formed Steel Structural Members (AISI S100) for calculating the nominal strength of cold-formed steel (CFS) sections. The critical elastic buckling load shall be obtained in order to employ the DSM in computing the nominal strength of CFS members. The AISI S100 provides simplified methods for determining the critical elastic distortional buckling load. However it is found that the AISI S100 simplified methods are over conservative for the industrial standard C and Z sections in U.S. This paper presents revised simplified methods for calculating the critical elastic distortional buckling loads of typical CFS C and Z sections in bending and axial compression loading respectively. The new methods yield more accurate results but similar computation cost compared to the existing methods. The new methods can be added to the DSM methodologies for designing CFS members

1. Introduction

The Direct Strength Method (DSM) developed by Schafer and Peköz (1998) uses strength curves for the entire member to determine the nominal strength for cold-formed steel (CFS) members. It avoids the computation of the effective widths by requiring the knowledge of critical elastic buckling solutions for the entire member. The development of DSM is based on the same empirical assumption as the effective width method that the ultimate strength is a function of the critical elastic buckling load and the yield stress of the material. The strength curves for DSM are calibrated by a large amount of experimental data. One advantage of DSM is that the distortional buckling failure is explicitly addressed as a limit state in design. The DSM is first adopted by the North American Specifications for Cold-Formed Steel Structural Members in 2004 as an alternative method and included in the Appendix of AISI S100. The latest AISI S100 (2007) adds DSM based provisions for the distortional buckling strength and provides three methods to calculate the elastic distortional buckling stress. The 1st method (Section C3.1.4 (a) for flexural members and C4.2(a) for compression members) is an empirical simplified method with limited

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application to unrestrained C- and Z-sections with simple lip stiffeners. The 2nd method is a general analytical method for a broad range of cross-section shapes but it consists of complex and lengthy calculation procedures. The 3rd method specified in AISI S100 is the rational elastic buckling analysis using the finite element method or similar computational and analytical methods to determine the elastic buckling load with a high degree of accuracy.

The simplified method in AISI S100 offers simple steps with closed-formed equations, therefore is preferred by the design professions. However the simplified method was developed in a conservative way due to the large scatter of elastic buckling loads of sections within the specified geometric limits. The authors observe that the elastic buckling load calculated by the simplified method for industrial standard CFS sections is always less than the finite strip results which can be regarded as the accurate values. And the difference between the two methods can be as high as 58% for flexural member and 69% for compression member when the accurate result is used as the reference for comparison. Revised simplified method is therefore required to provide more accurate elastic distortional buckling solutions. The paper proposes two simplified methods for flexural and compression members respectively for determining the critical distortional buckling stress for industrial standard C and Z sections.

2. Simplified method for distortional buckling load of flexural members

The proposed simplified method for determining the distortional buckling moment of flexural members is based on the existing simplified method in AISI S100 and calibrated by comparing with the accurate results from the finite strip analysis. A large number of industrial standard CFS C and Z sections are used for developing the new method. The C sections are chosen from the technical catalog of the Steel Stud Manufacturer Association (SSMA 2010). The Z sections are standard members utilized by the Metal Building Manufacturer Association, and the geometries are obtained from the AISI Cold-Formed Steel Design Manual (AISI D100 2008). The CUFSM v2.6 software (CUFSM 2003) is used for the finite strip analyses.

The existing simplified method in AISI S100 (2007) is expressed as follows.

Elastic Critical Distortional Buckling Moment = M_{crd}

$$M_{crd} = S_f \; F_d \tag{1}$$

S_f = Elastic section modulus of full unreduced section relative to extreme compression fiber

F_d = Elastic distortional buckling stress

$$F_d = \beta k_{AISI} \frac{\pi^2 E}{12(1-\mu^2)} \left(\frac{t}{b_o}\right)^2 \tag{2}$$

where

$$k_{AISI} = 0.6\left(\frac{b_o D Sin\theta}{h_o t}\right)^{0.7} \tag{3}$$

h_o = Out-to-out web depth

t = Base steel thickness

b_o = Out-to-out flange width

D = Out-to-out lip dimension
θ = Lip angle
β = A value accounting for moment gradient
E = Modulus of elasticity
μ = Poisson ratio

The key equation in the simplified method is Eq. 3 which approximates the elastic buckling coefficient of members. The proposed new method adopts the same main equations (Eqs. 1 and 2) but employs a revised equation for the elastic buckling coefficient, *k*, as expressed below.

$$k_{new} = 0.95\left(\frac{b_o D Sin\theta}{h_o t}\right)^{0.8} \quad (4)$$

Tables 1 and 2 list the accurate buckling coefficients and the comparison for Z and C sections respectively. The accurate buckling coefficient, k_{FSA}, is determined with the assistance of the finite strip method using CUFSM (2003).

Table 1 Buckling coefficient for Z sections in bending

Section	k_{FSA}	k_{AISI}/k_{FSA}	k_{new}/k_{FSA}
12ZS3.25x105	1.71	0.561	0.950
12ZS3.25x070	2.26	0.540	0.946
12ZS2.75x085	1.61	0.602	1.020
12ZS2.25x070	1.34	0.704	1.189
10ZS3.25x105	1.91	0.571	0.984
10ZS2.75x085	1.90	0.579	1.000
10ZS2.75x070	2.18	0.566	0.993
10ZS2.25x070	1.68	0.638	1.097
10ZS2.25x059	1.9	0.626	1.093
9ZS2.25x105	1.36	0.667	1.121
9ZS2.25x085	1.61	0.640	1.094
9ZS2.25x065	1.97	0.612	1.071
8ZS3.25x105	2.17	0.587	1.036
8ZS3.25x085	2.55	0.567	1.018
8ZS2.75x070	2.55	0.565	1.015
8ZS2.25x065	2.13	0.615	1.088
8ZS2.25x059	2.32	0.599	1.070
7ZS2.25x085	1.93	0.636	1.116
7ZS2.25x065	2.32	0.620	1.112
7ZS2.25x059	2.55	0.599	1.084
6ZS2.25x085	2.13	0.642	1.144
6ZS2.25x070	2.43	0.631	1.142
6ZS2.25x059	2.84	0.599	1.101
4ZS2.25x065	3.26	0.653	1.238
3.5ZS1.5x065	2.11	0.668	1.194
3.5ZS1.5x059	2.27	0.657	1.185

Table 2 Buckling coefficient for C sections in bending

Section	k_{FSA}	k_{AISI}/k_{FSA}	k_{new}/k_{FSA}
1200S250-97	1.27	0.562	0.912
1200S250-54	1.92	0.560	0.964
1200S200-68	1.11	0.705	1.159
1200S162-54	0.67	1.015	1.636
1000S250-97	1.61	0.503	0.832
1000S250-43	3.11	0.460	0.825
1000S200-97	1.16	0.598	0.966
1000S200-54	1.75	0.597	1.023
1000S162-68	0.88	0.747	1.198
1000S162-43	1.17	0.774	1.300
800S250-97	1.95	0.486	0.821
800S200-68	2.07	0.502	0.860
800S200-33	3.82	0.451	0.831
800S162-97	1.04	0.576	0.912
800S162-54	1.53	0.590	0.991
800S137-43	1.55	0.497	0.816
600S250-97	2.31	0.502	0.873
600S200-33	5.05	0.417	0.791
600S162-97	1.47	0.499	0.813
600S162-54	2.19	0.505	0.872
600S137-68	1.32	0.518	0.836
600S137-43	1.79	0.526	0.889
550S162-54	2.37	0.496	0.864
550S162-33	3.7	0.448	0.820
400S162-68	2.56	0.488	0.857
400S162-43	3.6	0.478	0.880
400S137-54	2.13	0.501	0.862
400S137-33	3.2	0.471	0.850
362S200-68	3.52	0.514	0.953
362S162-54	3.12	0.504	0.917
362S162-33	4.88	0.455	0.869
362S137-43	2.77	0.484	0.861
350S162-54	3.19	0.505	0.921
350S162-33	4.98	0.457	0.875
250S162-68	3.27	0.531	0.978
250S162-43	4.56	0.524	1.011
250S137-54	2.73	0.543	0.979
250S137-33	4.19	0.500	0.946

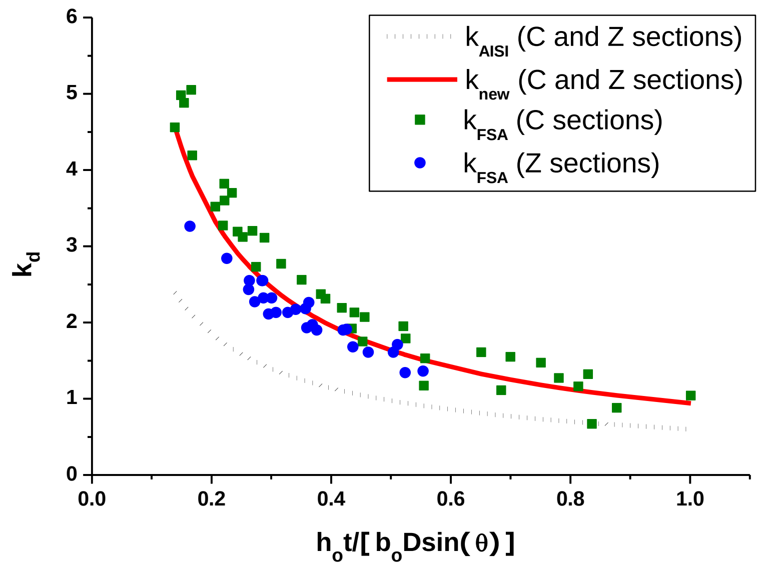

Figure 1: Comparison of buckling coefficient for Z and C sections in bending

Fig. 1 illustrates the comparison of the two simplified methods with the accurate solution. It can be found that the AISI S100 method is consistently over conservative for those analyzed C and Z sections. The average k_{AISI}/k_{FSA} ratio is 0.569 with a coefficient of variation of 0.167. The proposed method is calibrated according to the finite stripe results, and it yields an average ratio of 0.994 with a coefficient of variation of 0.150. Table 3 summarizes the statistical results. The proposed method gives better statistically better performance than the existing AISI S100 method. Particularly, the less coefficient of variation in the new method indicates better reliability than the existing method.

Table 3 Summary of the statistical results for flexural members

	k_{AISI}/k_{FSA}	k_{new}/k_{FSA}
Number of sections	64	64
Average	0.569	0.994
Standard deviation	0.095	0.149
Coefficient of variation	0.167	0.150

Since the new method is calibrated by US industrial standard C and Z sections, the following dimensional limits shall apply for the use of this new method.

(1) $35 \le h_o / t \le 230$

(2) $15 \le b_o / t \le 60$

(3) $5 \le D / t \le 18$

(4) $\theta = 90°$ for C-section, $\theta = 50°$ for Z-section

(5) $1.5 \le h_o / b_o \le 7.4$

3. Simplified method for distortional buckling load of compression members

The AISI S100 simplified method for distortional buckling load of concentrically loaded compression members uses essentially the same procedure for the flexural members except that a different equation for the buckling coefficient is adopted. The AISI S100 simplified method for compression members is expressed as follows.

Elastic critical distortional buckling load: P_{crd}

$$P_{crd} = A_g F_d$$

A_g is cross area of the cross-section,

F_d is elastic distortional buckling stress calculated. The distortional buckling stress

$$F_d = \alpha k_{AISI} \frac{\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b_o}\right)^2$$

where

$$k_{AISI} = 0.1\left(\frac{b_o D Sin\theta}{h_o t}\right)^{1.4} \tag{7}$$

α = A value that accounts for the benefit of an unbraced length. The definition of the rest variables is same as those defined in Section 2.

Based on the analyses of the same CFS section listed in Tables 1 and 2, a new expression (Eq. 8) of the buckling coefficient is developed for the compression members.

$$k_{new} = 0.16\left(\frac{b_o D Sin\theta}{h_o t}\right)^{1.5} \tag{8}$$

The same dimensional limits as defined in Section 2 are applicable for the proposed simplified method for compression members.

Tables 4 and 5 respectively list the finite strip results and the comparison for Z and C sections in compression.

Fig. 2 shows the comparison between the accurate solution and the two simplified methods. Compare to Fig. 1, the compression members yield greater scatter than the flexural members when the same parameters are used to relate the cross-section geometries to the elastic buckling load. Fig. 2 indicates that the AISI S100 simplified method captures the lower bound of the accurate results. The average k_{AISI}/k_{FSA} ratio is 0.55 with a coefficient of variation of 0.312. The proposed simplified method has a better approximate to the accurate results with an average k_{new}/k_{FSA} ratio of 0.978 with a coefficient of variation of 0.297. Table 6 summarizes the statistics of the comparison for compression members.

Table 4 Buckling coefficient for Z sections in compression

Section	k_{FSA}	k_{AISI}/k_{FSA}	K_{new}/k_{FSA}
12ZS3.25x105	0.59	0.434	0.743
12ZS3.25x070	0.76	0.546	0.967
12ZS2.75x085	0.44	0.588	1.008
12ZS2.25x070	0.29	0.851	1.453
10ZS3.25x105	0.85	0.388	0.676
10ZS2.75x085	0.68	0.495	0.863
10ZS2.75x070	0.77	0.551	0.977
10ZS2.25x070	0.46	0.689	1.198
10ZS2.25x059	0.52	0.755	1.332
9ZS2.25x105	0.46	0.501	0.850
9ZS2.25x085	0.53	0.561	0.969
9ZS2.25x065	0.62	0.648	1.145
8ZS3.25x105	1.21	0.372	0.663
8ZS3.25x085	1.41	0.413	0.749
8ZS2.75x070	1.18	0.488	0.886
8ZS2.25x065	0.80	0.596	1.066
8ZS2.25x059	0.87	0.615	1.109
7ZS2.25x085	0.86	0.489	0.867
7ZS2.25x065	1.04	0.555	1.005
7ZS2.25x059	1.12	0.579	1.059
6ZS2.25x085	1.07	0.484	0.870
6ZS2.25x070	1.24	0.526	0.962
6ZS2.25x059	1.42	0.564	1.048
4ZS2.25x065	1.96	0.641	1.230
3.5ZS1.5x065	1.15	0.479	0.866
3.5ZS1.5x059	1.24	0.500	0.911

Table 5 Buckling coefficient for C sections in compression

Section	k_{FSA}	k_{AISI}/k_{FSA}	k_{new}/k_{FSA}
1200S250-97	0.27	0.518	0.849
1200S250-54	0.40	0.798	1.388
1200S200-68	0.19	0.879	1.461
1200S162-54	0.11	1.148	1.870
1000S250-97	0.43	0.425	0.709
1000S250-43	0.82	0.698	1.264
1000S200-97	0.25	0.537	0.877
1000S200-54	0.38	0.803	1.390
1000S162-68	0.16	0.759	1.230
1000S162-43	0.21	1.099	1.865
800S250-97	0.74	0.338	0.577
800S200-68	0.56	0.538	0.931
800S200-33	1.02	0.807	1.501
800S162-97	0.23	0.434	0.694
800S162-54	0.33	0.693	1.175
800S137-43	0.20	0.828	1.373
600S250-97	1.22	0.306	0.538
600S200-33	1.98	0.622	1.192
600S162-97	0.45	0.332	0.547
600S162-54	0.68	0.498	0.870
600S137-68	0.32	0.405	0.660

Table 5 Buckling coefficient for C sections in compression (continued)

Section	k_{FSA}	k_{AISI}/k_{FSA}	k_{new}/k_{FSA}
600S137-43	0.43	0.578	0.987
550S162-54	0.85	0.453	0.798
550S162-33	1.27	0.602	1.113
400S162-68	1.26	0.344	0.611
400S162-43	1.82	0.452	0.841
400S137-54	0.92	0.344	0.597
400S137-33	1.37	0.462	0.842
362S200-68	2.09	0.434	0.813
362S162-54	1.71	0.401	0.736
362S162-33	2.61	0.524	1.011
362S137-43	1.31	0.382	0.685
350S162-54	1.79	0.404	0.744
350S162-33	2.74	0.525	1.016
250S162-68	1.98	0.422	0.786
250S162-43	2.88	0.552	1.076
250S137-54	1.72	0.355	0.646
250S137-33	2.60	0.468	0.895

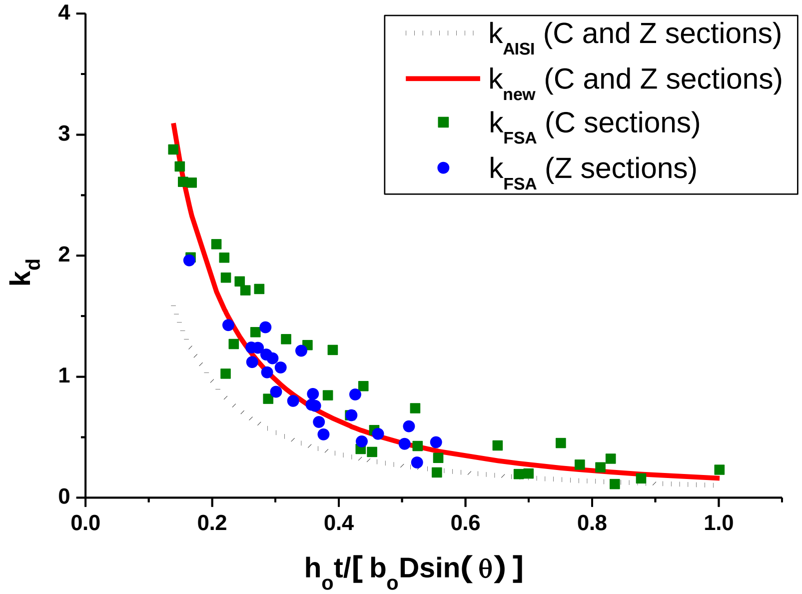

Figure 2: Comparison of buckling coefficient for Z and C sections in compression

Table 6 Summary of the statistical results for compression members

	k_{AISI}/k_{FSA}	k_{new}/k_{FSA}
Number of sections	64	64
Average	0.554	0.978
Standard deviation	0.173	0.290
Coefficient of variation	0.312	0.297

4. Conclusions

The AISI existing simplified methods for determining the elastic distortional buckling loads of CFS C and Z sections are found over conservative for industrial standard sections used in the US market. Based on the typical CFS sections, new simplified methods are developed to provide more accurate and reliable predictions for the elastic buckling load. The new methods can be integrated into the DSM method to obtain the distortional buckling strength of C and Z sections subjected to bending or axial compression.

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