



Stability of cylindrical steel panels under uniform axial compression.

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Abstract

The architects intervene more and more in the design of the steel bridges. By using curved elements, the stability verifications become more difficult due to a lack of information in the standards. The present study aims to develop a method for predicting the ultimate strength of a cylindrically unstiffened curved plate subjected to uniform axial compression. The methodology is based on the formal procedure adopted by Eurocode 3, for all type of stability verification. A series of numerical simulation is carried out in order to clarify and examine the fundamental buckling behavior of curved panels. On the basis of the numerical results, the formulae for elastic buckling and ultimate strength are derived.

Keyword: Stability, Cylindrical Curved Panels, Steel Bridge, GMNIA Analysis, ANsys.

1. Introduction

Curved plate members have been used in various applications including civil engineering construction. The typical examples are bridge with orthotropic deck (Fig. 1) or web of in-plane curved I-girder. At the present time, steel construction is based on minimum weight for economy. One of the main tasks for designers is to reduce as much as possible the thickness for lighter structure and economical reason. Stability problem can occur in such curved panels. Unfortunately this question is not dealt with in Eurocode 3 nowadays. Actually EN 1993-1-5 only gives specifications about the flat plate or slightly curved plate (with the condition $b^2/Rt < 1$) and EN 1993-1-6 deals only with the full revolution cylinder shell. The issues of curved panel in some architectural bridges are often situated in the middle of these two cases.

In that case, the use of finite element modeling (F.E.M) becomes necessary. But F.E.M requires very sophisticated models and time-consuming calculations. Moreover, the results supplied by a common program should be very carefully analyzed. Consequently, the daily practitioners need a reliable and relatively simple hand calculation method which is the aim of this work.

2. Literature review

The papers related to the buckling theory of curved panels are not so numerous in comparison to the literature devoted to plate buckling or cylindrical shell buckling. Initially, the behavior of

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curved plates is expected to be similar to the behavior of a revolution cylinder since a curved plate is a part of a full revolution cylinder. At the first step, the ultimate load of such curved plate is equal to the buckling critical load.



Figure 1: Renault Bridge - Seguin Island, France.

Several researchers developed formulae for buckling loads of curved plates in compression. The first theoretical expressions were developed by Redshaw (Redshaw1933) and Timoshenko (Timoshenko1936). Their work assumed that the expressions for the three displacements satisfy either the boundary conditions or the equilibrium equations, but not the two. By using this famous linearized Donnell's equation (Donnell1934), Batdorf (Batdorf1947) obtained the same expression from Timoshenko. Another similar Timoshenko's expression is also achieved by using the Schapitz's criterion (Spier1969).

Test data on curved plates were obtained by Cox and Clenshaw (Cox1941), Crate and Levin (Crate1943), Jackson and Hall (Jackson1947), Welter (Welter1946), and Schuette (Schuette1948). The ultimate loads coming from the tests are quite different from the theoretical values coming from the application of previous derived equations for the buckling load. These discrepancies were not explained until the works of Batdorf and al. (Batdorf1947), who attempted a synthesis of test data of Crate and Levin and of Cox and Clenshaw. This modified solution is presented as drawing curves, also known as Batdorf's curve, and has been adopted in the aeronautical fields till now (Bruhn1973).

In a second step, the researchers wondered if the critical buckling load really represented the failure load for each curved panel, whatever its curvature radius is. It may be expected that at least for small curved panel (close to single plate), that the ultimate load increases after buckling taking the post-critical strength reserve into account. On the other hand, for a panel with large curvature, the ultimate load is expected to decrease after buckling. This remark is verified by the experimental work of Wenzek (Wenzek1938). Some expressions for post-buckling behaviors of curved panels are first proposed by Pope (Pope1963), Sekine and Tamate (Sekine1969) and Gerard (Gerard1957). However the rigorous mathematical solution is not yet proposed because a hand-made resolution of the equation is very complicated to achieve because of the curvature.

During the last few decades, the rapid development of computers and finite element codes allow the buckling and GMNIA analysis of complicated structures. The stability problem of curved panels came back in the research field by the use of numerical methods. Recognizing a need for updating the NASA¹ structural stability monographs, Domb and Leigh (Domb2001) proposed an update of the currently-used design curve by using home-made code MARC (F.E.M). Other

¹ National Aeronautics and Space Administration

studies are also proposed in the field of ship building where generally, curved plates are so thick that plastic buckling occurs instead of elastic (Maeno2004). This is the opposite case to the aeronautical field where the elastic buckling only occurs. However, for the ship industry, information on the elastic post-buckling behavior may be also useful when considering the reduced thickness due to corrosion. Yumura and al. (Yumura2005) investigated the buckling/plastic collapse behavior on the basis of the results of numerical calculation. Park and al. (Park2006; Park2008; Park2009) provided a simplified method to estimate the ultimate strength based from Faulkner's formulae for a single plate with newly defined slenderness parameter in order to consider curvature effects.

Although there have been numerous contribution studies to this stability question, further studies on the buckling/collapse behaviors of cylinder curved panels have to be produced especially in the civil engineering construction field.

The present study aims to propose a simple method, as a formal process in the new Eurocode 3, for all types of stability verification (Fig. 2).

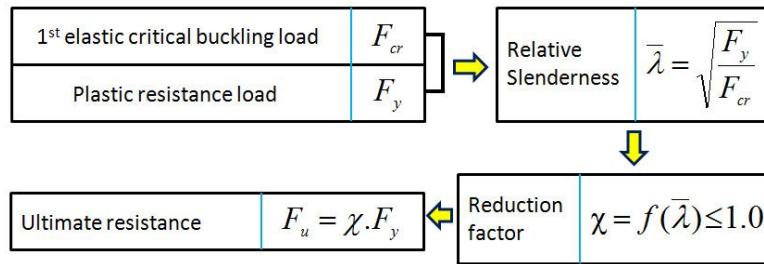


Figure 2: Manual calculation design process adopted in Eurocode 3.

This study is limited to cylindrical curved, unstiffened, and square panels subjected to a longitudinal uniform compression. Except for the stiffening condition, this configuration is one of the most common one for a bottom flange in the box girder bridge.

3. Finite element modeling

The cylindrical shell geometry is modeled with finite element method code ANSYS version 11 (Fig. 3) and its standard structural shell element, Shell 181 (Ansys User Manual 2001). This element is a quadrilateral 4-node element involving both bending and membrane capability along with six degrees of freedom at each node. This element is suitable from thin to moderately-thick shell structures.

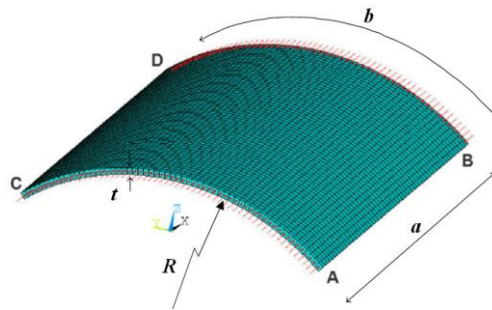


Figure 3: Cylindrical curved panel modeled by ANSYS.

The panel is assumed to be simply supported at all edges. The boundary conditions impose on the curved load edges (AC & BD) to remain straight during loading. The two other edges (AB & CD) are free to wave in-plane. These conditions are usually used for the modeling of compression plate coming from thin-walled steel cross sections. The external load is introduced by means of an external uniform compression.

The steel material is assumed to be elastic-perfectly plastic for the material non-linear analysis (MNA) and elastic-plastic with linear strain hardening as indicated in EN 1993-1-5 C.6 for the material non-linear second-order analysis with initial imperfections (GMNIA). The Young's modulus E and Poisson's ratio ν are taken equal respectively to 210 GPa and 0.3. The steel grade is S355 with maximum yield strength equal to $f_y = 355\text{MPa}$ for thicknesses up to 16 mm and reduced with the thickness increase according to EN 1993-1-1.

It is noticed that the initial imperfections have not been sufficiently investigated yet for curved panels (Featherston2003). However, the fabrication of a curved panel is very similar to a plate; it is thus proposed to use the equivalent initial imperfection of plate as given by EN1993-1-5. The imperfections are now defined according to the shape of the first critical buckling mode of the curved panel with a maximum deflection $w_o = \max(a/200; b/200)$.

The residual stress is also studied with its patterns extracted from the Swedish design code BSK for welded sections (BSK99). The obtained numerical results show that the influence of the residual stresses can be covered in a good way by the use of an equivalent geometrical imperfection as defined in EN 1993-1-5. The influence of the residual stress due to bending procedure is not considered in this study.

4. Elastic critical resistance

In order to investigate the elastic buckling resistance of the studied unstiffened curved panel, a series of elastic eigenvalue analysis is performed. Results of these simulations will also allow the verification of the accuracy of the theoretical formulae found in the literature and mentioned above. The investigated panels have a constant width $b = 3\text{m}$, a constant depth $a = 3\text{m}$ and a constant thickness $t = 20\text{mm}$. Only the radius R will vary in order to change the non-dimensional curvature parameter Z from 0 to 100. These dimensions have been chosen in order to be in the scope of steel bridges.

The curvature parameter adopted in this study is calculated as $Z = b^2/Rt$. It is a modification of Batdorf's parameter Z_B (Batdorf1947) in order not to take into account the effects of Poisson's ratio. It should be noted that in case of a curved panel the curvature parameter Z is highly significant. Indeed the maximum distance between an arc of circle (length b and radius R) and its chord is given by $b^2/8R$. Thus the parameter $Z = b^2/Rt$ is proportional to this maximum distance divided by the thickness. Z is thus a nondimensional measure of the deviation from the flatness and a representative parameter for a curved panel.

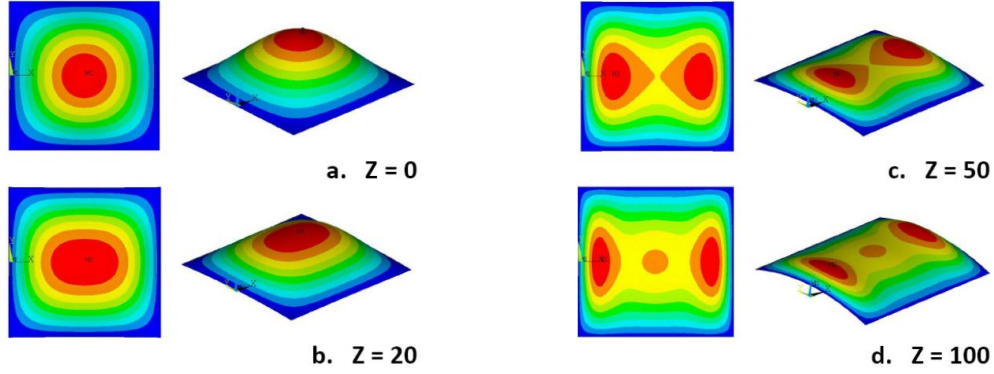


Figure 4: First elastic buckling mode for different curvature.

Some results are shown in Fig. 4 to illustrate the evolution of the first buckling mode shapes with the curvature parameter Z . Z is equal to zero for a plate (Fig. 4a). For small Z values, the curved panel buckles with a nearly square wave. However when Z increases, the buckling mode shape progressively evolves, the out-of-plane deformation decreases in the middle of the panel whereas the maximum deflection becomes localized along the two curved edges of the panel.

It is now interested in the comparison between the first elastic buckling stress obtained by ANsys and by applying the theoretical formulae established by Redshaw, Timoshenko and Domb and Leight. Batdorf's formula will not be considered because his theoretical background is similar to Timoshenko's formula and a modified formula has been proposed in the work by Domb and Leight. To simplify the comparison, all these formulae have been written in order to show the buckling coefficient k_c^{curved} for a curved panel in uniform compression:

$$\sigma_{cr}^{curved} = k_c^{curved} \cdot \sigma_E \quad (1)$$

where σ_E is the classic Euler critical stress, which is given as:

$$\sigma_E = \frac{\pi^2 E}{12(1 - \nu^2)} \left(\frac{t}{b}\right)^2 \quad (2)$$

The studied formulae from the literature are now written as follows:

Redshaw:

$$k_c^{curved} = 2 \left(1 + \sqrt{1 + \frac{12(1-\nu^2)}{\pi^4} Z^2} \right) \quad (3)$$

Timoshenko:

$$k_c^{curved} = \begin{cases} 4 + \frac{3(1-\nu^2)}{\pi^4} Z^2 & \text{if } Z \leq \frac{2\pi^4}{\sqrt{3(1-\nu^2)}} \\ \frac{4\sqrt{3}}{\pi^2} & \text{if not} \end{cases} \quad (4)$$

Domb and Leight :

$$k_c^{\text{curved}} = \begin{cases} 10 \sum_{i=0}^3 c_i (\log Z_b)^i & 1 \leq Z \leq 23.15 \\ c(Z)^d & 23.15 < Z \end{cases} \quad (5)$$

with $Z_B = Z(1-\nu^2)$; $c_0 = 0.6021$; $c_1 = 0.005377$; $c_2 = 0.192495$; $c_3 = 0.00267$; $c = 0.4323$; and $d = 0.9748$.

Fig. 5 illustrates the comparison by plotting the buckling coefficient k_c^{curved} as a function of the curvature parameter Z . It should be noticed that Z varies as the square of the dimensions. Consequently a logarithmic plotting is needed.

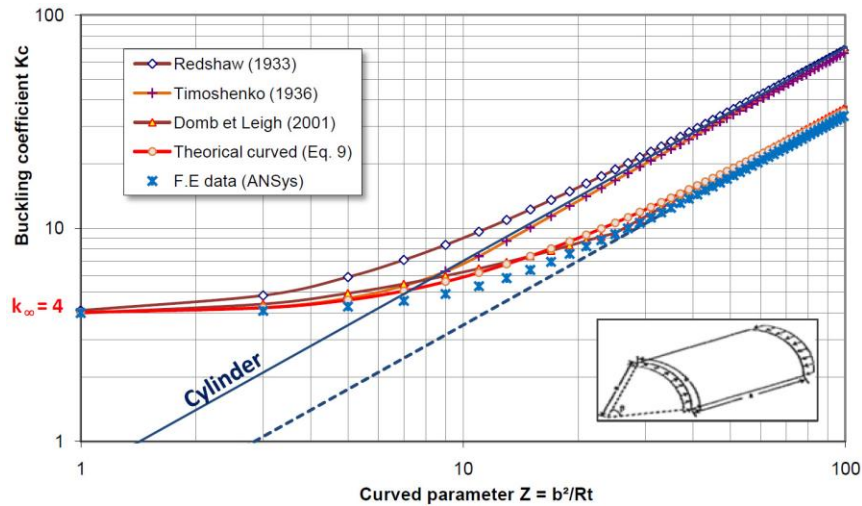


Figure 5: Comparison of different buckling coefficient expressions with F.E results.

First we notice that all formulae converge on the value of a plate ($k_{\text{plate}} = 4$) if Z tends towards 0. The buckling coefficient increases with Z because the higher the Z value is, the higher is the circumferential membrane strain which holds up the buckling deflection. Nevertheless these formulae were developed in different ways so that their infinite behaviors differ. The theoretical formulae of Redshaw and Timoshenko converge asymptotically towards the straight line whose slope is equal to the buckling coefficient of a revolution cylinder. However the formula of Domb and Leight and the F.E results only converge on the half of this value. This latter similar behavior is logical because Domb and Leight derived their proposal from the F.E numerical results.

The different boundary conditions explain this discrepancy for high Z values. Indeed the curved cylindrical panels have four free cut edges whereas the revolution cylinders have two, so that the circumferential membrane strain develops in different ways. The F.E results indicate that the buckling coefficient of a curved panel (constant width b) will converge on the half value of the buckling coefficient of the corresponding revolution cylinder ($b = 2\pi R$). To clarify this observation, we carried out further F.E simulations by studying a cylindrical curved panel which is assumed to have a constant length, radius and thickness ($a = 9$ m; $R = 3$ m; $t = 15$ mm), and a variable width b from 0 to $2\pi R$. The ratio ($\rho = \sigma_{\text{curved}}/\sigma_{\text{cylinder}}$) between the critical buckling stress

of the curved panel and of the revolution cylinder is plotted against the curvature parameter Z (Fig. 6).

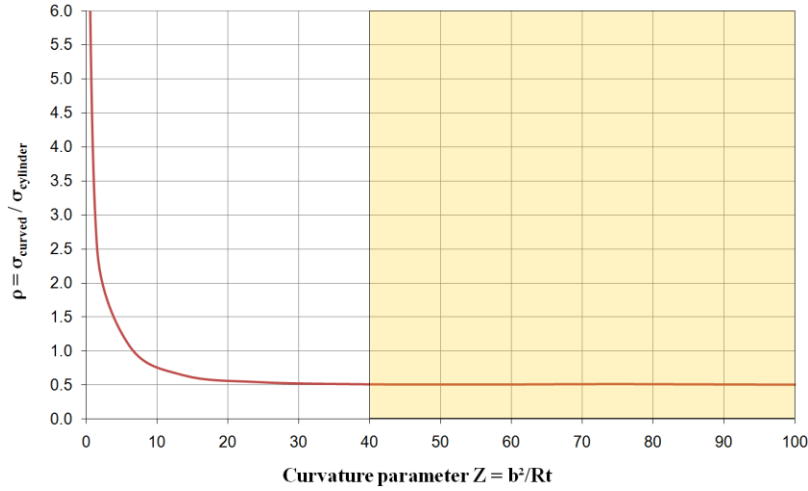


Figure 6: Study of convergence for cylindrical curved panel.

The ratio ρ decreases with the Z increase, from the infinity and the vertical asymptote for $Z = 0$ (i.e, $b = 0$) to the 0.5 value for $Z = 4\pi^2 R/t$. The reached structure is a revolution cylinder with a straight cut along its length a . It should be also noticed that for the example the 0.5 value is already almost reached for $Z \geq 40$ i.e. $b = 1:16\text{m}$ which is far from the value $2\pi R = 18.8 \text{ m}$.

Based on the previous analysis, we proposed the following new formula for the buckling coefficient of a curved panel under uniform axial compression. It has been developed by assuming the following mathematical basis:

$$k_c^{\text{curved}} = A \left(1 + \sqrt{1 + B \cdot Z^2} \right) \quad (6)$$

where A and B can be interpreted with respect to the previous identified limit behaviors.

- Behavior as a flat panel for $Z = 0$

$$k_c^{\text{curved}} \Big|_{Z=0} = k_c^{\text{plate}} \rightarrow A = \frac{k_c^{\text{plate}}}{2} \quad (7)$$

- Behavior as a revolution cut cylinder for $Z \rightarrow \infty$

$$\lim_{Z \rightarrow \infty} k_c^{\text{curved}} = \frac{1}{2} k_c^{\text{plate}} \rightarrow B = \frac{48(1-\nu^2)}{\pi^4 (k_c^{\text{plate}})^2} \quad (8)$$

The proposal formula for the buckling coefficient is now rewritten as:

$$k_c^{\text{curved}} = \frac{k_c^{\text{plate}}}{2} \left(1 + \sqrt{1 + \frac{48(1-\nu^2)}{\pi^4 (k_c^{\text{plate}})^2} \cdot Z^2} \right) \quad (9)$$

One of the main interests of the proposed equation (Eq. 9) is the explicit reference to the limit behaviors by including the plate buckling coefficient k_c^{plate} . This proposal is also more accurate

and simpler than Domb's and Leight's formula (Fig. 5) which refers to several calibrated numerical coefficients in order to minimize the error. Moreover all these coefficients do not have the same measurement unit. It should be noted that the proposed formula shows a slight discrepancy with the F.E. results especially for the transition zone from the plate to the cylinder. However this critical buckling load should be considered as a parameter in the hand-calculation procedure that we are developing for the determination of the ultimate strength. The discrepancy between Eq. 9 and the F.E. results is partly corrected during the calibration process of the reduction factor χ (cf. below).

The proposed equation (Eq. 9) will thus be used in the following to calculate the critical buckling load of the studied curved panels under uniform axial compression.

5. Ultimate strength

For stocky curved panels under uniform compression (i.e. small values of the slenderness parameter $\bar{\lambda}$), no shell buckling occurs and the ultimate strength is governed by the yielding of the whole cross-section. The yield resistance can also be calculated by the MNA analysis (material non-linear analysis with an elastic - perfectly plastic stress-strain behavior).

For more slender curved panels, the slenderness parameter $\bar{\lambda}$ increases, the failure mode becomes a shell buckling and the ultimate strength could be written as a fraction of the yield resistance defined by the reduction factor $\chi \leq 1$: $F_u = \chi \cdot F_y$.

The ultimate strength F_u can be calculated by F.E. using the non-linear analysis including the geometrical and material imperfections of the panel (GMNIA analysis) as precisely as possible. The maximum value of the load-deflection curve corresponds to F_u . Note that for such non-linear F.E. analysis, the arc-length method should be preferred for the plotting of a complex path into the buckling/post buckling regimes. The reduction factor χ is then numerically defined as the ratio between the maximum load F_u obtained by the GMNIA analysis and the yield resistance F_y .

5.1. Finite element simulation

A series of simulations (GMNIA analysis) was carried out for several curved panel configurations with a curvature parameter Z varying from 1 to 100. These geometrical configurations have been chosen in order to cover the scope of the steel bridge construction. The numerical results are presented in Fig. 7 where the reduction factor χ is plotted against the slenderness parameter $\bar{\lambda}$. The reduction factors coming from EN 1993-1-5 (top curve) for plate buckling and from EN 1993-1-6 (bottom curve) for revolution shell buckling are also plotted.

The numerical results show a high influence of the curvature parameter Z on the ultimate strength F_u . For a given relative slenderness $\bar{\lambda}$, the highest the panel curvature is the lowest the ultimate strength. It should also be noticed that all the marks are located between the two χ reduction factors coming from the European Standards. In other words, for small Z values the curved panel behaves as a plate (EN 1993-1-5), and if Z becomes higher, the behavior turns to the one of a cylinder (EN 1993-1-6).

In Fig. 7 it can be seen that different curved panels having the same curvature parameter Z numerically behave in a similar way. This gives an additional justification to our previous choice

of the Z parameter for representing the variability of this problem. The observed distribution of the marks for $Z = 1$ follows more or less the χ -curve from EN 1993-1-5 (plate buckling under uniform compression). It is also observed that out of its scope (for $Z > 1$ according to its clause 1.1(2)), the abused use of EN 1993-1-5 rules leads to an overestimation of the ultimate strength of the curved panel.

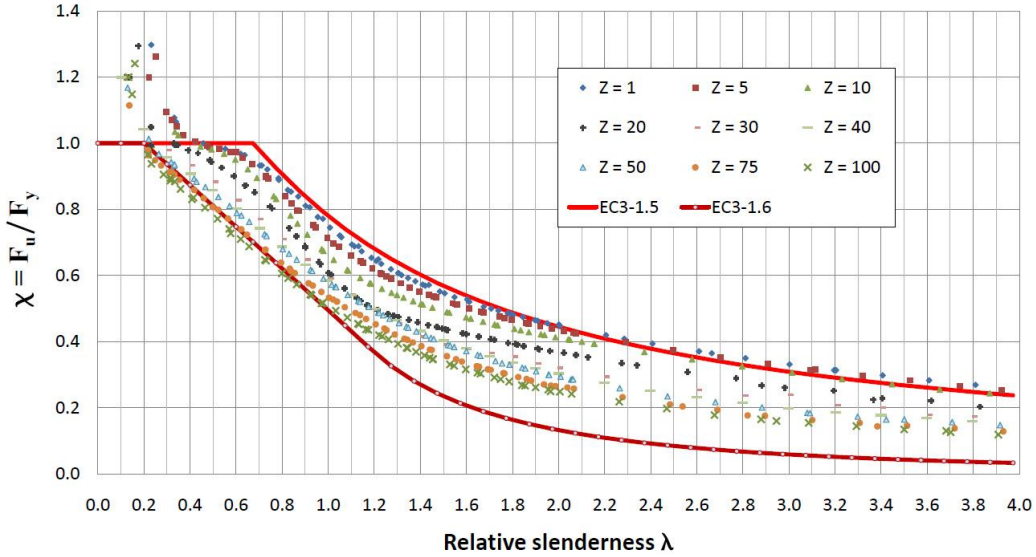


Figure 7: Ultimate strength of curved panels as a function of a curvature parameter Z .

For high values of Z ($Z \geq 40$) and for the slenderness $\bar{\lambda} \leq 1.0$, the numerical results follow more or less the χ -curve from EN 1993-1-6. Nevertheless a non-negligible discrepancy should be noticed when the slenderness becomes higher and EN 1993-1-6 becomes too conservative. This gap is mainly due to the post-critical strength reserve of curved panels which leads to a critical buckling load smaller than the ultimate load. This strength reserve is also related to the different boundary conditions of the curved panel edges and of the revolution cylinder.

5.2. New capacity curve approaches

The previous observations lead us to look for a new reduction factor χ allowing filling the gap between EN 1993-1-5 and EN 1993-1-6. This new coefficient will obviously depend on the curvature parameter Z . The generally elastic-plastic stability curve from EN 1993-1-6 has been adopted in this work in order to preserve a consistent framework for all kind of stability problem.

The ultimate strength of the cylindrical curved panel, for axial compression may be written as follow:

$$\chi = \begin{cases} 1 & \text{when } \bar{\lambda} \leq \bar{\lambda}_0 \\ 1 - \beta \left(\frac{\bar{\lambda} - \bar{\lambda}_0}{\bar{\lambda}_p - \bar{\lambda}_0} \right)^\eta & \text{when } \bar{\lambda}_0 < \bar{\lambda} \leq \bar{\lambda}_p \\ \frac{A \cdot \bar{\lambda} + B}{\bar{\lambda}^2} & \text{when } \bar{\lambda}_p < \bar{\lambda} \end{cases} \quad (10)$$

Where $\bar{\lambda}_0$ is the squash limit relative slenderness; $\bar{\lambda}_p$ is the plastic limit relative slenderness; β is the plastic range factor, η is the interaction exponent; A and B are the two parameters characterizing the elastic buckling. The formulae of six parameters are all derived from the F.E results, and written as follow:

$$\bar{\lambda}_0 = 0.2 + 0.473 \left(e^{-Z/20} \right) \quad (11)$$

$$\bar{\lambda}_p = 0.673 + \frac{Z}{10 + 1.8 \cdot Z} \quad (12)$$

$$\beta = \frac{Z}{8 + 1.7 Z} \quad (13)$$

$$\eta = 1 \quad (14)$$

$$A = \frac{1}{1 + 0.01 Z} \quad (15)$$

$$B = (1 - \beta) \bar{\lambda}_p^2 + A \bar{\lambda}_p \quad (16)$$

The accuracy of the new proposed formula (Eq. 10) has been checked by a comparison with the F.E. results; Fig. 8 shows that the proposed formula fits well with the numerical database. The mean value is approximatively equal to 1 and the maximum standard deviation is equal $\sigma = 0.051$ for $Z = 50$.

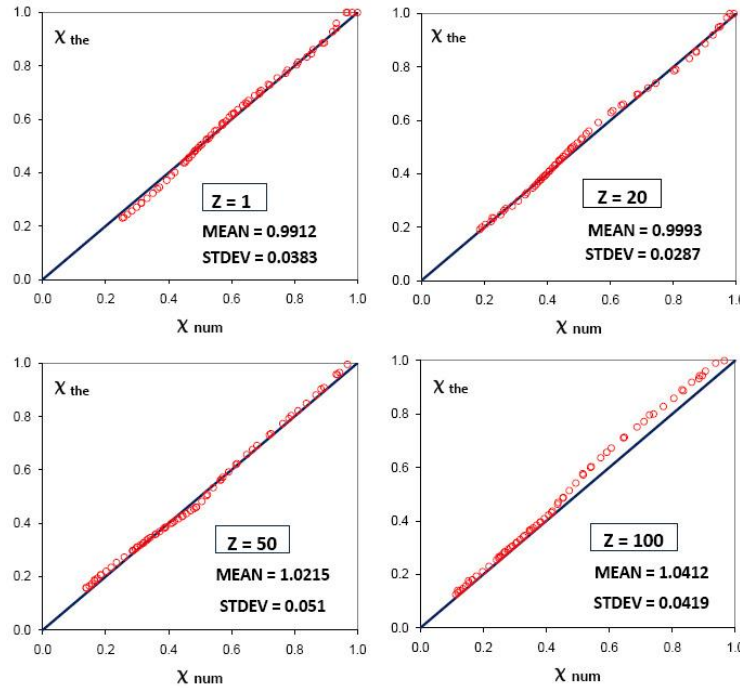


Figure 8: Correlation of the proposed formula with the ultimate strength obtained by F.E.M.

6. Conclusion

In this paper, the critical buckling behavior and the ultimate strength of a cylindrical curved panel have been investigated by means of linear and non-linear buckling F.E. analysis. The behavior of a cylindrical curved panel usually depends on different factors, mainly the relative slenderness ratio $\bar{\lambda}$, the imperfection factors and the curvature parameter Z . The results can be summarized as follows:

- The available literature about the buckling theory of curved panels is rather limited in comparison to the plate or revolution cylinder buckling; A thorough research about the collapse behavior of the cylindrical curved panels has not yet been performed, especially for predicting the ultimate strength.
- The elastic critical buckling load of a cylindrical curved panel increases with the curvature parameter Z . The available formulae give very different results when compared to the numerical F.E. results (buckling analysis as well as non-linear analysis). A semi-empirical formula for the buckling coefficient has thus been proposed as a function of the curvature parameter Z and by including the influence of the boundary conditions.
- The ultimate strength is very sensitive to the panel imperfections. Due to a lack of available imperfection measurements for the cylindrical curved panels, the initial imperfections have been assumed to be based on the first critical buckling mode shape with a maximum deflection $w_0 = \max(a/200; b/200)$ as given in EN1993-1-5 for plate buckling.
- The residual stresses have also been studied but their influence could be covered by the chosen equivalent geometrical imperfection.
- To our knowledge there is a lack of experimental tests about these cylindrical curved panels. Although the F.E method is widely recognized to be a powerful tool for predicting the ultimate strength of a plate or cylinder buckling, the accuracy of the results is sensitive to the initial geometrical and material imperfections which can be better known through new experimental measurements.
- A reduction factor χ has been developed for predicting the ultimate strength. This χ -expression is based on those indicated in EN 1993-1-6, in order to preserve the consistent framework. The included parameters have been calibrated using the database of the numerical F.E. results especially built to cover the steel bridge scope. The provided formula for χ is valid for $Z < 100$.

This study has been carried out in the frame of a running PhD thesis essentially devoted to the civil engineering construction field (steel bridge). In a next step, numerous other researches will be carried out, by adding longitudinal stiffeners to the curved panels or by using various load cases (transverse compression and shear).

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