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Cross Section Optimization Using Simulated Annealing of Cold-Formed Steel Channel Columns

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Abstract
Cold-formed profiles has been used in large scale in Building industry, due to the easy way to product them and the wide range of sections feasible to accomplish with the project needs. The search for a maximum performance of structural elements with the low use of material is a challenge of today and future for engineering. This paper presents a numeric study to obtaining minimum weight of cold-formed channel columns, with and without lips, using the prescriptions of AISI 2007. Flexural, torsional and torsional–flexural buckling of columns was considered as constraints. The optimization was made through the method of Simulated Annealing. Several numeric simulations are presented and discussed to validate the proposal and an experimental example that qualifies the implementation. The relations between lips, web width and flange width are analyzed. Finally, the process shows excellent results to reduce the cross section area.

1. Introduction and Background

Cold-formed steel structures constitute an alternative for small and medium size steel structures. The great advantage of those sections consists in the possibility of adjusting the form to the needs of the member in the overall group of the structure, obtaining it with the minimum possible weight. In practice that advantage it is not totally explored, therefore the simple search of a section with its maximum relationship of strength versus weight is not a trivial activity of the engineer's day-by-day. In this sense, the present paper integrates a research line that aims to study the optimization of component elements of structures with cold-formed sections. First, is introduced the calculation process for channel sections with or without lips to determine the load capacity.

The Simulated Annealing (SA) technique was applied to formulate the optimization of the problem. The relationship between web and flanges width is evaluated for channel section without lips, as well the relations among web, flange and lips in channel section with lips. In the optimization process of the section to get the design load, the method of the effective widths was used. All the calculations were done according the American Standard AISI (2007). Second following that process, each one of the possible elastic loads of buckling (global flexure,

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torsional, local and its combinations) was obtained, and the smallest of them is the maximum load nominal. The distortional buckling mode of the section was not considered in this study.

The optimization of cold-formed steel members is still incipient in the literature. Seaburg and Salmon (1971) studied the minimum weight of hat sections using the direct methods and the gradient for the minimum of the function weight, given a single example due to numeric complexity of the problem.

Dinovitzer [1992] optimized the length of the lip of a channel section based on the Canadian standard of cold-formed steel. The procedure used for the optimization was not based on the resistance load of the member, but in the capacity to increase the resistant moment of the section while minimizing the cross section area.

Castelluci et al [1997] presented a channel section with lips optimized for bending, including two intermediate stiffeners on the web, those authors conclude that with only 5% of area increment was possible to get 15% more capacity of the section. The work, however, does not mention the optimization technique used.

Adeli and Karim (1997) developed a model of neural networks for non linear problems, and applied to simple supported beams with I and Z sections. For the verification of the sections they used as reference the AISI 1996 Allowable stress Design (ASD).

Al-Mosawi and Saka (2000) included in the optimization procedures the bending stress of cold-formed and get optimal sections of symmetric and asymmetric channel and Z sections subjected to uniform loads, however, they used only as constraints the normal stress and displacements.

Tian (2003) presented a theoretical and experimental study for obtaining the minimum weight of channel section subjected to compression prescribing a fixed length and a resistant axial load. For the load design calculations it uses as reference the British Standard BS 5959, and a non-linear method with restrictions (SQP- sequential quadratic programming). Include, also, a simplified procedure, in which the buckling stresses are equivalent. Tian conclusions mention an optimal relation among the web and flange width. The studied sections supplied an increase of 50% in the load design capacity to the axial compression without increasing the area.

The use of Simulated Annealing (SA) was choosen due to the success in other applications developed by the authors (2005, 2008). Also, a research developed by Degertekin (2007) shown that SA obtained lighter frames than Genetic Algorithm (GA); similar conclusions were obtained by Grigoletti (2008).

2. Methodology

2.1 Verification of channel section with and without lips subject to compression

The verification of the sections was made starting from the precepts of the American Standard AISI (2007), described in a brief way in the present item.

According to the section C4 of AISI [1], the nominal axial resistance P_n should be defined in the following way:

$$P_n = A_e F_n \quad (1)$$

Where A_e is the effective area at stress F_n area, Fn shall be calculated as follows:

If $\lambda c \leq 1{,}5$ then (2)

$$F_n = \left(0{,}658^{\lambda_c^2}\right) F_y$$

For $\lambda c > 1{,}5$ then (3)

$$F_n = \left[\frac{0{,}877}{\lambda_c^2}\right] F_y$$

Where (4)

$$\lambda = \sqrt{\frac{F_y}{F_e}}$$

In the previous relationship, Fe is the minimum value of elastic buckling stress, flexural, torsional and flexural-torsional, with the equations C4.1.1-1 to C.4.1.2-3 (2007).

The calculation of area is in agreement with the method of the effective widths, according to the section B2 of AISI (2007).

2.2 Optimization formulation for cold-formed steel columns

Optimum design formulation of cold-formed steel columns is based on AISI specification (2007) as exposed in the section 2.1. The objective function is the area of the cross section (see Figure 1) with four design variables: web length (A), flange length (B), lip length (C) and thickness (t). Equation (5) represents the objective function, and Equations (6) to (11) express the calculations for the area gross section properties.

$$f(A, B, C, t) = A_g \quad (5)$$

Where:

$$A_g = t[a + 2b + 2u + \alpha(2c + 2u)] \quad (6)$$

$$a = A - (2r + t) \quad (7)$$

$$b = B - \left[r + \frac{t}{2} + \alpha(r + \frac{t}{2})\right] \quad (8)$$

$$c = \alpha\left[c - \left(r + \frac{t}{2}\right)\right] \quad (9)$$

$$u = \frac{\pi r}{2} \quad (10)$$

$$\alpha\begin{cases}1 \rightarrow C \neq 0 \\ 0 \rightarrow C = 0\end{cases} \tag{11}$$

To simplify the problem, the corner radius is used with the same value of the thickness. In the case of channel columns, section without lips, the number of variables in the objective function is reduced to three.

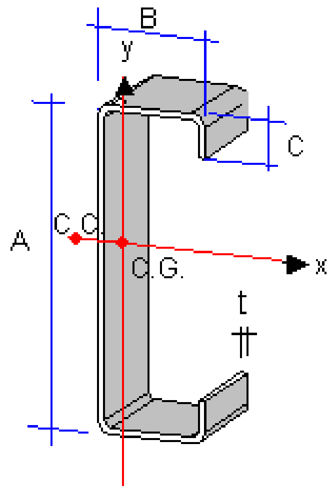

Fig. 1– Channel section

The dimensions were considered as continuous variables, with lower limits and upper limits for each dimension defined in function of the inherent limitations to the production process.

Besides the limits imposed to the dimensions, the strength resistance is also a restriction, P_d be equal or greater than the load P_s. A section that attends all those limits, with the restrictions, it as feasible solution for the problem, being constituted as good choose.

The problem of minimize the section have the following restrictions:

$$P_s \leq P_n \tag{12}$$

$$50\text{mm} \leq \text{A} \leq 1000\text{mm} \tag{13}$$

$$30\text{mm} \leq \text{B} \leq 1000\text{mm} \tag{14}$$

$$1\text{mm} \leq \text{t} \leq 25\text{mm} \tag{15}$$

The presented formulation was implemented using the Simulated Annealing method, which consists of a heuristic method for global optimization developed by Kirkpatrick et al. (1997). Simulated Annealing utilizes a non-descending strategy, unlike the optimization approaches normally employed, trying to avoid convergence to a local minimum accepting also, according to a specific criterion, solutions that increase the value of the function. The method was developed in analogy upon the process of annealing of a solid, when a state of minimum energy is being searched. The denomination annealing is given to the process of heating a solid to its point of fusion, followed by a slow cooling. In this process, slow cooling is essential to maintain a thermal equilibrium in which the atoms are able to reorganize themselves in a structure with minimum energy. If the solid is cooled abruptly, its atoms will form an irregular and weak structure, with high energy in consequence of the internal effort spent. In computational therms, the annealing can be seen as a stochastic procedure of determination of the organization of the atoms with minimum energy. To high temperatures the atoms move freely being able to, with big probability, achieve positions that will increase the energy of the system. When the temperature is reduced, the atoms can move gradually to form a regular structure, and the energy increase probability is reduced.

In the optimization technique, the objective function corresponds to the energy of the solid. Similarly to the annealing in thermodynamics, the process initiates with a high value of T, for which a new solution is generated. This new solution will be automatically accepted if it generates a decrease in the value of the function. Being the new value of the function greater than the previous one, the acceptation will be given according to a probabilistic criterion, being the acceptation function

$$p = \exp\left(\frac{-\Delta f}{T}\right) \tag{16}$$

and the new solution accepted if p is bigger than a number between zero and one, generated randomly. While T is high, the majority of the solutions are accepted, being T reduced gradually to each trial series, in the neighborhoods of the current solution.

3 Numerical results

Some results obtained from a program developed by the authors using the formulation proposed in this paper. In all of the numeric simulations the following data were considered: Buckling length coefficients $K_x = K_y = 1$ and $K_t = 0,7$, module of longitudinal elasticity E = 200 GPa, coefficient of Poisson $\upsilon = 0,3$ and yield stress of $f_y = 350$ MPa.

The first analyses were developed in a section with the following characteristics: height A = 88.5 mm, width B = 37.5 mm, height of the lip C = 7.65 mm, thickness t = 1.5 mm and total length L = 2400 mm (generating the effective lengths $K_{lx} = K_{ly} = 2400$ mm and $K_{lt} = 1680$ mm). For the described section it was obtained the resistant load $P_n = 10.72$ kN.

In the sequence several analyses were made with the computational code developed, at least ten runs searching by the smallest cross section Ag with the load originally proposed. The table 1 presents the results obtained from two simulations, which are, sections without and with lips (designated, respectively, for the initials PSE and PCE), comparing them with the original section (designated by P0). In this table, the last column indicates the percentage of area reduction (and, in consequence, the weight).

The data on the table 1, shows a great reduction in the section area obtained from the optimization process. As expected, the lips presence increases in a considerable way its efficiency, generating an additional reduction in the total amount of material (approximately 15%,). It was observed although the resistance of the initial section P0 is limited by the flexure buckling stress in relation to the vertical axis y, while in bigger sections the dominant buckling mode is bending and compression acting simultaneous.

Table 1–Initial reference section and optimized section with and without lips

Section	A (mm)	B (mm)	C (mm)	t (mm)	F =AG (mm^2)	Reduction (%)
P0	88.50	37.50	7.65	1.50	253.41	-
PSE	80.86	48.62	-	1.15	200.84	20.70
PCE	69.33	40.07	13.52	1.00	169.96	32.90

It is important to emphasize that such economy on the section area (weight reduction) is obtained preserving safety conditions. The values of the load capacity for the three column channel analyzed are the same.

With the goal of validating the employed procedure for the optimization of the section, were made several analyses with different initial values for each variable on the column channel section. For all the analysis made was noted that the final result does not have dependency of the initial values adopted.

A second series of analyses was made for the same data employees in the previous analyses, however for increasing loads. Considering load Ps with the values of 10, 20, 30, 40 and 50 kN, was searched optimal dimensions for sections with and without lips. The results of those analyses are exposed in figures 2 to 6.

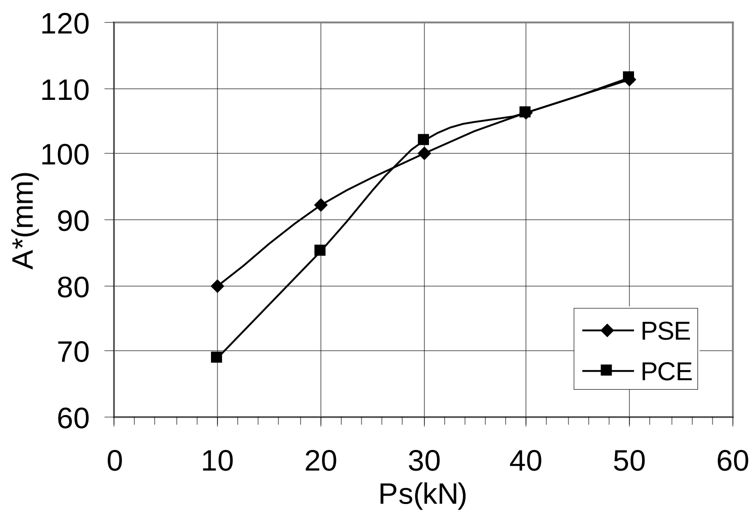

Fig. 2–Relation Load (Ps) versus web length (A*)

Based on those figures presented, a linear relation is observed in the dimensions of the sections with the increase of the load. This relationship, however, it is not verified in relation to the lip length (Fig. 4). Nevertheless, it is evident the significant contribution of the lips for the reduction of the section area. In the cases here analyzed, this reduction varied from 14.2 to 25.3%.

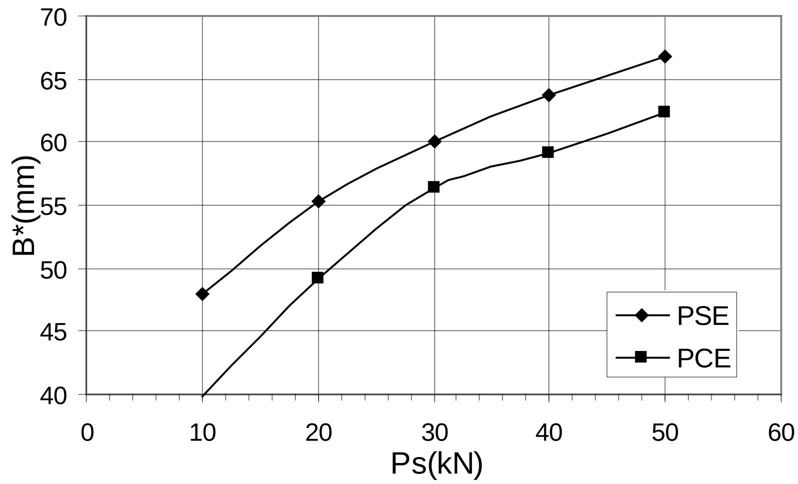

Fig. 3– Load (Ps) x flange length (B *)

In spite of the variation of the dimensions of the section with the increase of the load, the relationship between the height and the base of the profile without lips is a constant with the value of 1.66. For sections with lips, this relationship varies from 1.73 to 1.81.

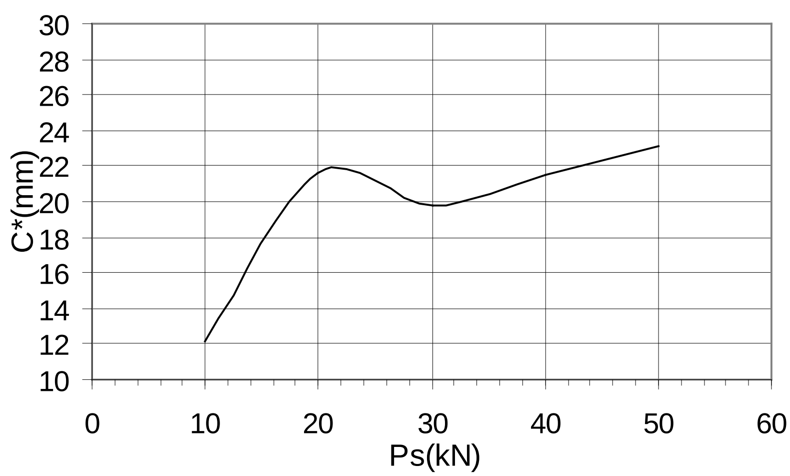

Fig. 4–Load (Ps) versus x lip length (C *)

We highlight that the relationships limits width to thickness, defined by AISI (2007), do not been imposed as restrictions in the present work, therefore, all the sections optimized was below those limits (200 for the web, and 60 for flanges).

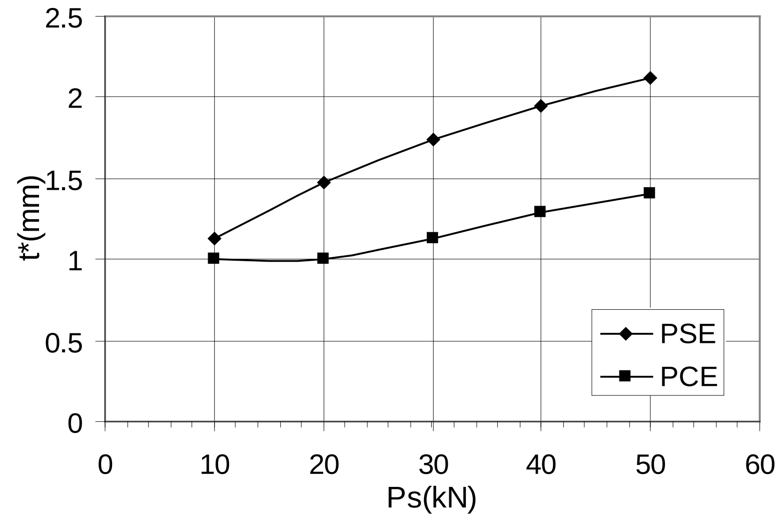

Fig. 5–Load (Ps) versus thickness (t *)

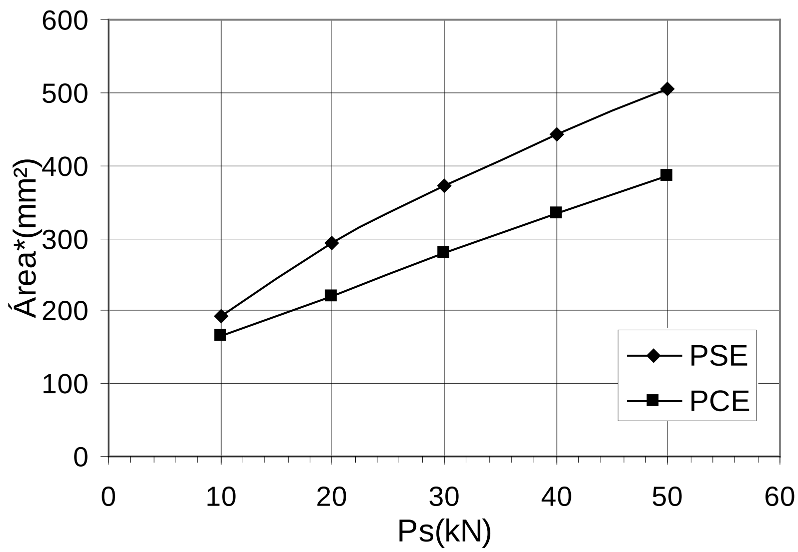

Fig. 6– Load (Ps) versus Gross Section (Ag *)

3 Experimental Analysis

To verify the optimization procedure here proposed, a section of dimensions 110x55x1.2mm was built, for a length of 1200mm and with the value of the load capacity, was subjected to the process of optimization, given a section with dimensions of 65x35x1.5mm. The thicknesses were limited by the commercial thickness available.

The yield stress was measured in four samples (see right side of Fig. 7), two of 1.2mm and two of 1.5mm of thickness Results are exposed on Table 2, and the load vs. displacement curve on Fig. 7

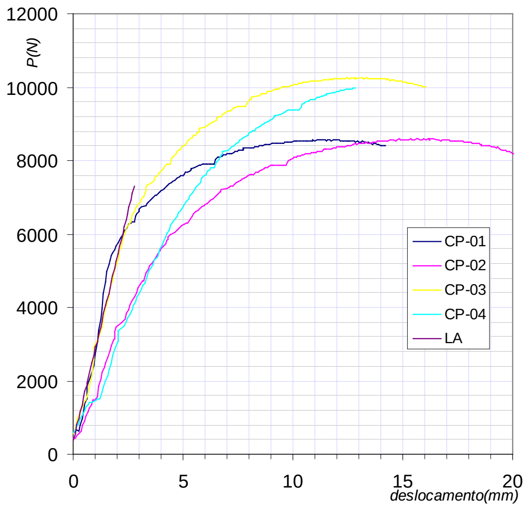

Fig 7– Load vs. displacement curve or tension stress test (left figure) and the samples tested (right)

Table 2–Results of the sample for tension stress thickness of 1.2 and 1.5mm

Sample	fy (MPa)	fu (MPa)	t (mm)
CP-01	221.9	352.8	1.2
CP-02	231.7	353.3	1.2
CP-03	201.4	337.8	1.5
CP-04	213.7	335.0	1.5
Medium	217.2	344.7	
Standard deviation	12.8	9.7	
CV	5.9	2.8	

Two specimens of each section were built (U110x55x1.2 and U65x35x1.5) for the compression to be tested (see Fig. 8). A general view of the test assembly is presented in the Fig. 9. The sections already tested are showed in Fig. 10.

Fig. 8–View of four channel column used in experimental tests (U110x55x1.2 and U65x35x1.5)

Fig. 9–Assembly of columns tests

Fig. 10–Section already tested showing deformation for local buckling

Table 3– Values of the sections tested experimentally

Section	Area (mm²)	P experimental (kN)	Reduction area (%)	Variation of load collapse (%)
110x55x1.2	259.27	16.70	-	-
65x35x1.5	197.00	16.08	24%	3.71

The comparison of the experimental results (see Tab. 3), shows that the optimized section reduces 24% in weight with relation to the original. A small variation was observed in the collapse load (3.7%) among both sections tested, what does not invalidate the results here exposed.

The continuity of this work will be carried out by more numerical and experimental tests, which may confirm the excellent applicability of optimization method here applied.

4 Conclusions

In this study, the shape optimization of cold-formed steel channel and lipped-channel columns under axial compression was presented by using Simulated Annealing, the design was prepared on the basis of the 2007 edition of the North American Specification for the Design of Cold-Formed Steel Structural Members.

From the numeric results, an important weight reduction was found on the members analyzed. With the experimental evaluation of the optimum process, and considering the available commercial limitations of thickness, a section was proposed and optimized with a defined compression resistance, the original section was an U110x55x1.2 and the optimum section obtained with the was an U65x35x1.5. The experimental results show a promising economy of 24% in weight.

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