Proceedings of the
Annual Stability Conference
Structural Stability Research Council
Grapevine, Texas, April 18-21, 2012

Curvature Limitations of Non-Composite Girder Bridges at Construction Stage

Imad Eldin Khalafalla[1], Khaled Sennah[2]

ABSTRACT

In bridges with light curvature, the curvature effects on bending, shear and torsional shear stresses may be ignored if they are within acceptable range. Treating horizontally curved bridges as straight ones with certain limitations is one of the methods to simplify the analysis and design procedure. Recently the Canadian Highway Bridge Design Code, CHBDC, the AASHTO Guide Specifications for Horizontally Curved Bridges, and AASHTO-LRFD Bridge Design Specifications have specified certain limitations to treat horizontally curved bridges as straight ones, but still more investigations are needed to examine these limitations for different bridges configurations (i.e. open I girders versus closed box girders). In this study, a series of non-composite horizontally curved braced steel I-girder bridges at construction stage were analyzed under dead load condition using the finite-element program, ABAQUS. Major internal forces developed in the members were determined, namely: flexural stresses, vertical deflections and vertical support reactions for different degrees of curvature. These values were then compared with those for straight bridges of similar span and cross-section configuration. Both single and two-span horizontally curved non-composite steel I-girder bridges were considered in this study. The design parameters considered in this study were degree of curvature, span length, number of lanes, number of girders, and span-to-depth ratio. The stipulations made in bridge codes for treating a curved bridge as straight one were then examined. Based on the data generated from the parametric study, sets of empirical expressions were developed to evaluate girder flexural stress, deflection and support reaction magnification factors in a curved bridge system as related to a straight bridge system. Then, these expressions were extended to establish more reliable expressions for curvature limitations to treat a curved bridge as straight one as opposed to the available curvature limitation equation available in CHDBC and AASHTO Specifications. It should be noted that the current study proved that the available CHBDC and AASHTO curvature limitations underestimates the structural response of few curved bridge configurations.

1. INTRODUCTION

The growing demands on bridges of curved alignment present real challenges to structural engineers, especially in the design of crowded urban areas where multi-level interchanges must be built within inflexible geometric restrictions. However, due to the existence of curvature, the design and

[1] PhD Student, Ryerson University, Toronto, ON, Canada <kimadeld@ryerson.ca>
[2] Professor, Ryerson University, Toronto, ON, Canada <ksennah@ryerson.ca >

construction of bridges become greatly more complicated than that of straight bridges, and their structural behavior still not well understood. Treating the horizontally curved bridges as straight ones with certain limitations on the central angle is one of the recommended methods to simplify the analysis and design procedure. CHBDC (CSA, 2006), AASHTO-LRFD Bridge Design Specifications (AASHTO, 2006) and the AASHTO Guide Specifications for Horizontally Curved Bridges, (Guide, 2003), have specified certain limitations to treat horizontally curved bridges as straight ones. The American Association of State Highway and Transportation Officials Guide Specifications for Horizontally Curved Highway Bridges states that for composite steel I-girder bridges, the effect of curvature may be ignored in the determination of vertical bending moment, when the following three conditions are met: (i) girders are concentric, (3) bearing lines are not skewed more than 10 degrees from radial, and (iii) the arc span divided by the girder radius, L/R, is less than 0.06 radians. It also specifies that the arc length, L, is the arc length of the girder in case of simple span bridges, 0.9 times the arc length of the girder for end spans of continuous bridges and 0.8 times the arc length of the girder for interior spans of continuous bridges. It should be noted that the superseded edition of such guide for curved bridges (AASHTO, 1993) specified limiting central angle of the steel girder to the values shown in Table 1, as a function of the available number of steel girders and span continuity. Since the AASHTO Guide for Curved bridges is pertained to steel bridges only, no limits were set forth for concrete bridges. The AASHTO-LRFD Bridge Design Specifications specified curvature limitation similar to those of the superseded version of ASSHTO Guide for Curved Bridges, listed in Table 1. On other hand, the Canadian Highway Bridge Design Code states that for bridges that are curved in plan and that are built with shored construction, the simplified method of analysis can be applied by treating the bridge as a straight, when the following two conditions are met: (i) there are at least two intermediate diaphragms per span; and (ii) $L^2/BR \leq 0.5$, where B is the width of the bridge, L is the centreline curved span length, and R is the radius of curvature. Given the discrepancies in such limitations in different bridge codes, there is a strong need for more investigations to examine these limitations for different bridges configurations, by considering design parameters such as longitudinal flexural stresses in the steel girders, support reactions, and vertical deflection. Also, none of these bridge codes provides curvature limitations for bridges with unshored construction at construction stage.

Table 1: Limiting central angle for neglecting curvature in determining primary bending moment (AASHTO, 1993, 2006)

Numbers of girders	Central angle for one span	Central angle for two or more spans
2	2º	3º
3 or 4	3º	4º
5 or more	4º	5º

2. DESCRIPTION OF BRIDGE PROTOTYPES CONSIDERED IN THIS STUDY

A Total of 126 bridge configurations were analyzed in this study. Half of these bridges were of single span, while the other half were continuous over two-equal spans. The arc lengths of the curved bridges or the girder lengths in case of straight bridges were taken 15, 25 and 35 m. The span-to-radius of curvature ratio, L/R, were taken as 0.0, 0.1, 0.2, 0.3, 0.4, 0.5 and 0.6 to cover all possible ranges of L/R obtained from the curvature limitations in bridge codes. The cross-section of the studied bridges was made of non-composite steel I-girders (i.e. braced steel girders with no composite action with wet concrete deck slab during construction stage. Figure 1 shows schematic diagram of such multi-girder cross-section. Depth-to-span ratios, d/L of the steel girders were taken 1/25 for the single-span bridges and 1/30 for the two-span bridges. Number of girders, N, was taken 4 for the 8-m bridge width, 6 for 12-m bridge width and 8 for 16-m bridge width, while maintaining the girder spacing as 2 m. Steel web thickness was considered 16 mm, while the top and bottom flange width and thickness were taken as 300 and 16 mm, respectively. Number of transverse lines of vertical cross-bracings with top and bottom chord members, shown in Figure 1, including the support lines, was taken as 3, 5 and 7 for the 15-, 25-, and 25-m centerline curved span lengths, respectively. Bracing members were spaced at equal intervals between the support lines and were made of L102x102x9.5 single steel angles. Bridge analysis considered herein was based on the following assumptions: (i) all materials were elastic and homogenous; (ii) the effect of road superelevation, and curbs were ignored; and (iii) bridges had constant radii of curvature between support lines. The modulus of elasticity and possion's ration for steel material were taken 200 GPa and 0.30, respectively. Table 2 presents a summary of straight bridge configurations considered in this study. The bridge designation given in Table 2 represents the parameters involved. For example, in the first bridge designation "L15W8N4S", L15 means that the bridge is of 15 m span, W8 means that the bridge total width is 8 m; N4 means that the bridge cross-section has 4 girders, and S means that the bridge is single span. If the last symbol is "C", this means that the bridge is continuous over two-equal spans.

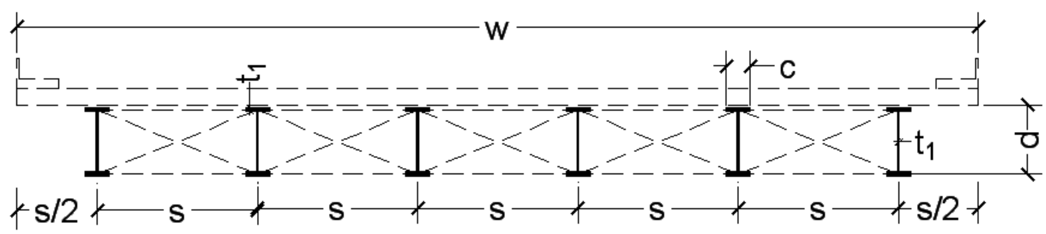


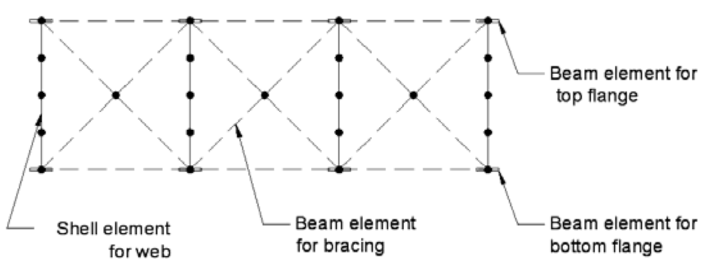
Fig. 1: Schematic diagram of the steel I-girder bridge cross-section

3. FINITE-ELEMENT MODELLING

In this study, three-dimensional finite-element (FEA) modelling was performed using ABAQUS software version 6.6 (Habbitt et al, 2009). In order to simplify the analysis, the structure was divided into a number of components such as steel webs, steel top flange, steel bottom flange, cross bracing, top and bottom chords. A four-node shell element, type S4R, with six degrees of freedom at each node was selected to model the steel webs. A three-dimensional two-node beam element, type B31H, was chosen to model the top and bottom steel flanges, cross-bracing, top and bottom chord, Figure 2 shows the finite-element representation and Abaqus model view of non-composite four steel I-girders. To model bearings along the support lines, bottom nodes of the web along the support line were restrained from moving vertically. In case of single span bridges, the bottom node of the girder close to, or at, bridge centreline, was restrained from moving transversally at one side of the bridge and restrained from moving both longitudinally and transversally at the other side of the bridge. This type of bearing modelling is illustrated in Fig. 3.a. In case of two-span continuous bridges, the hinge condition at the interior support was imposed at the lower node of the middle girder of the bridge cross-section by restraining movements in the three directions, U1, U2 and U3. The corresponding bottom node at the other ends of the girder web at ends of the bridge were restrained transversally only to provide temperature-free superstructure. Figure 3.b shows such boundary conditions for the continuous span bridges.

Table 2: Geometry of the bridges considered in the parametric study

Bridge Type	Span (L), m	No. of girders (N)	Bridge width (W), m	No. of spans	Cross-section dimensions (mm)				
					W	S	c	d	t1
L15W8N4S	15	4	8	1	8000	2000	300	700	16
L15W12N6S	15	6	12	1	12000	2000	300	700	16
L15W16N8S	15	8	16	1	16000	2000	300	700	16
L15W8N4C	15	4	8	2	8000	2000	300	700	16
L15W12N6C	15	6	12	2	12000	2000	300	700	16
L15W16N8C	15	8	16	2	16000	2000	300	700	16
L25W8N4S	25	4	8	1	8000	2000	300	1000	16
L25W12N6S	25	6	12	1	12000	2000	300	1000	16
L25W16N8S	25	8	16	1	16000	2000	300	1000	16
L25W8N4C	25	4	8	2	8000	2000	300	900	16
L25W12N6C	25	6	12	2	12000	2000	300	900	16
L25W16N8C	25	8	16	2	16000	2000	300	900	16
L35W8N4S	35	4	8	1	8000	2000	300	1400	16
L35W12N6S	35	6	12	1	12000	2000	300	1400	16
L35W16N8S	35	8	16	1	16000	2000	300	1400	16
L35W8N4C	35	4	8	2	8000	2000	300	1200	16
L35W12N6C	35	6	12	2	12000	2000	300	1200	16
L35W16N8C	35	8	16	2	16000	2000	300	1200	16



(a) Finite-element representation of steel I-girder

(b) Abaqus model view of steel I-girder

Fig. 2: Steel I-girder bridge cross-section

4. PARAMETRIC STUDY

A practical-design-oriented parametric study was conducted on selected curved bridge systems to determine (i) longitudinal bending stress magnification factor (SMF); (ii) deflection magnification factor (DMF); and (iii) reaction magnification factor (RMF). These design parameters are explained as follows:

$$SMF = \sigma_c / \sigma_s \tag{1}$$

Where SMF is the stress magnification factor for a curved bridge; σ_c and σ_s are the maximum vertical bending stresses found from the FEA for a curved bridge and a straight bridge of similar geometry and material characteristics, respectively.

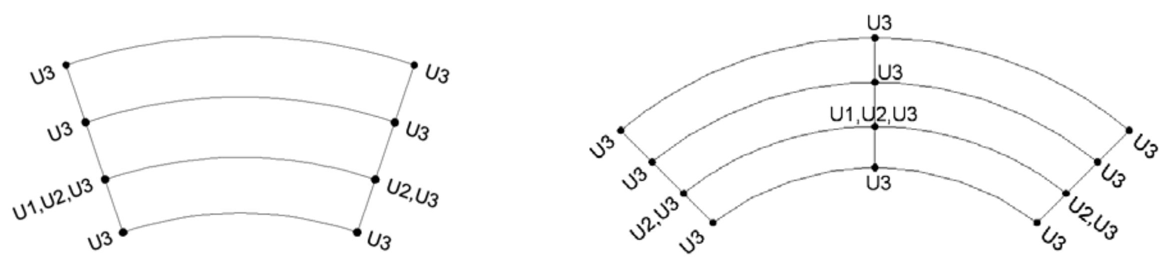
$$DMF = \Delta_c \ / \ \Delta_s \tag{2}$$

Where DMF is the deflection magnification factor for a curved bridge; Δ_c and Δ_s are the maximum vertical deflections found from the FEA for a curved bridge and a straight bridge of similar geometry and material characteristics, respectively.

$$RMF = R_c \ / \ R_s \tag{3}$$

Where RMF is the reaction magnification factor for a curved bridge; R_c and R_s are the maximum vertical reaction forces found from the FEA, for a curved bridge and a straight bridge of similar geometry and material characteristics, respectively.

In order to precisely determine the parameters affecting the above-mentioned magnification factors, a sensitivity study was first undertaken to determine the influence of the different parameters that may affect these magnification factors (Khalafalla, 2009). It was found that the key parameters that affect the structural response of a curved bridge system are: (i) span-to-radius of curvature ratio, L/R; (ii) bridge span length, L; (iii) bridge width, W; and (vi) bridge continuity. A database was generated from the parametric study to develop empirical expressions for evaluating the magnification factors for girder longitudinal flexural stress, deflection, and support reaction.



(a) Single-span bridge (b) Two-span bridge

Fig. 3: Boundary conditions for single- and two-span bridge models

5. RESULTS FROM THE PARAMTERIC STUDY

The database generated from the parametric study was used to investigate the effect of curvature on stress, deflection, and reaction magnification factors. The effect of curvature on the stress magnification factor is presented in Figs 4 through 7. It can be observed that as the span-to-radius ratio, L/R increases, the stress magnification factor increases. For example, in Fig. 4, as L/R ratio increases from 0.1 to 0.6 for a simply-supported bridge of 15 m span and 8 m bridge width, the SMF increased from 1.95 to 8.53. One may also observe that the change of bridge width from 8 m to 12 and 16 m has insignificant effect of the stress magnification factor. Similar observations are depicted in Figs. 6 and 7 for the two-span bridges of 25 and 35 m span, respectively. Figs. 8 to 11 depict the effect of span-to-radius of curvature, L/R, on deflection magnification factor, DMF. It can be observed that the DMF increases with increase in L/R. It can also be observed that the DMF decreases with increase in bridge span. The effect of degree of curvature on reaction magnification was examined in this parametric study. Figs. 12 to 15 show the reaction magnification factor, RMF, for curved bridges of width 8, 12 and 16 m (i.e. W8, W12 and W16 as shown in the Figures). It is clear that the RMF increases with the increase in L/R ratio, as compared to that for a straight bridge of similar configurations. For example, in Fig. 13, as the L/R ratio increased from 0.1 to 0.6 for simply-supported steel I-girder bridge with 35 m span length, and 8 m width , the RMF increased from 1.16 to 2.08.

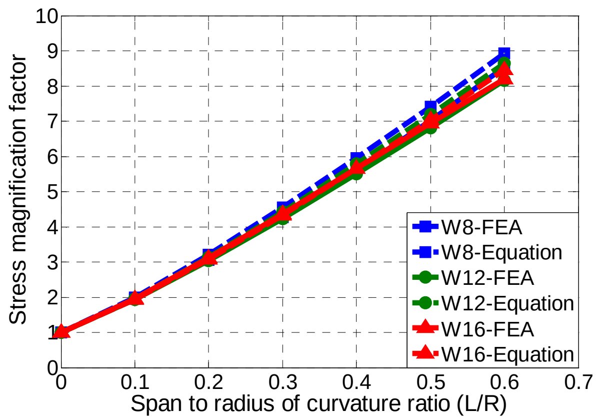


Fig. 4: Effect of curvature on the SMF for simply-supported bridges of 15 m span and different width

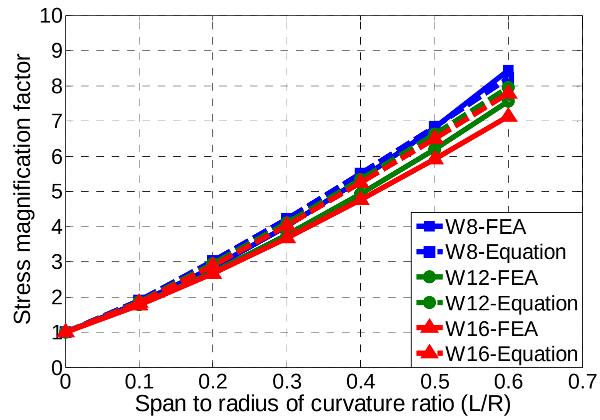


Fig. 5: Effect of curvature on SMF for simply-supported bridges of 25 m span and different bridge width

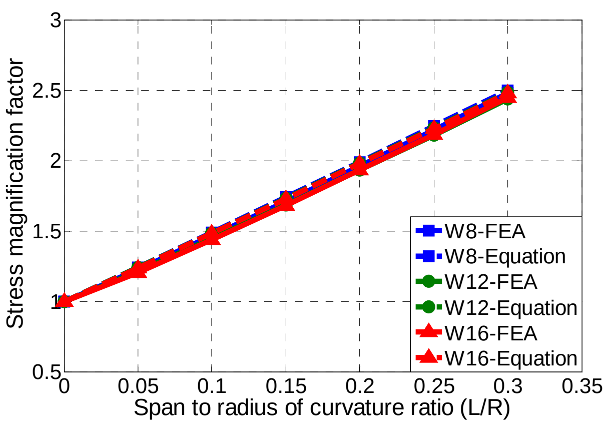


Fig. 6: Effect of Curvature on the SMF for two-Span bridges with 25 m span and different bridge width

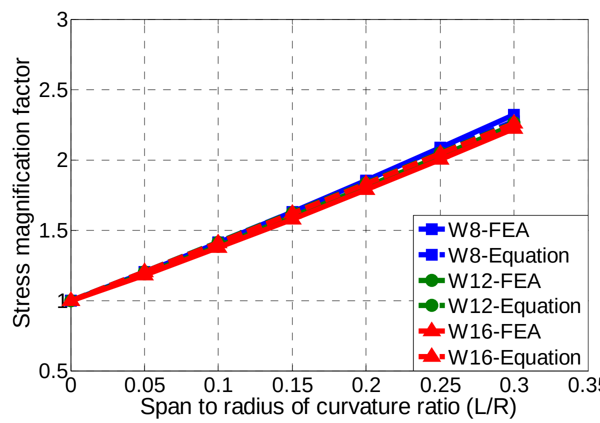


Fig. 7: Effect of curvature of the SMF for two-span bridges with 35 m span and different bridge width

6. CORRELATION OF FEA RESULTS WITH THE CURRENT CURVATURE LIMITATIONS IN BRIDGE CODES

As stated earlier, the aim of this study is to examine the available equations for curvature limitations in bridge codes so that the curved bridge can be treated as a straight one in structural design. To achieve this task, the data generated from the parametric study was used to develop expressions for the magnification factors for longitudinal flexural stress, support reaction and vertical deflection of the curved bridge. Using a statistical package for curve fit, two sets of empirical equations were developed for single-span and two-span bridges considered in this study, respectively. The general empirical equation for the magnification factors took the following form:

$$MF = a\left(\frac{L}{R}\right)^{b} \times (L)^{c} \times (B)^{d} + e \tag{4}$$

Where MF is the magnification factor for structural quantities; L is the bridge span length in meters; R is the radius of curvature in meters; B is the bridge width in meters; a, b, c, d, and e are equation variables.

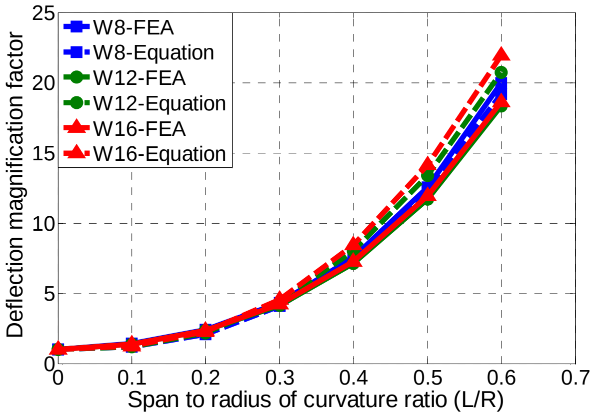


Fig. 8: Effect of curvature on the DMF for simply-supported bridges of 25 m span and different width

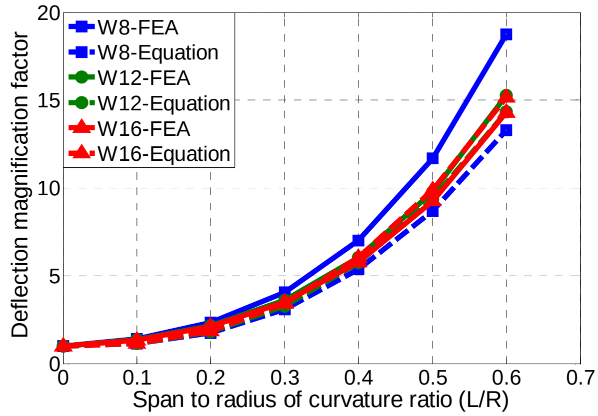


Fig. 9: Effect of curvature on DMF for simply-supported bridges of 35 m span and different bridge width

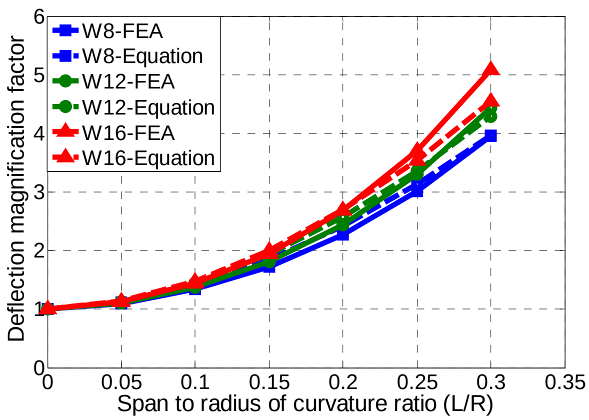


Fig. 10: Effect of Curvature on the DMF for two-Span bridges with 15 m span and different bridge width

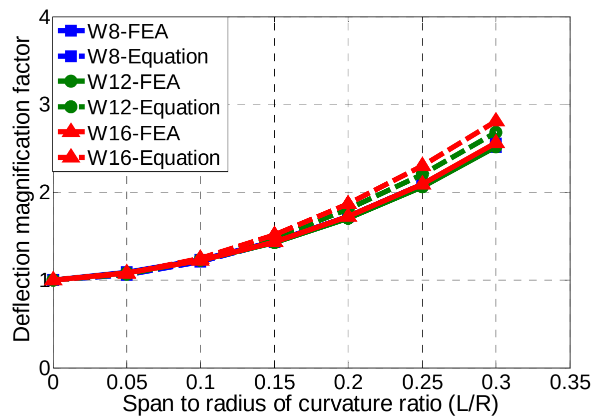


Fig. 11: Effect of curvature of the DMF for two-span bridges with 35 m span and different bridge width

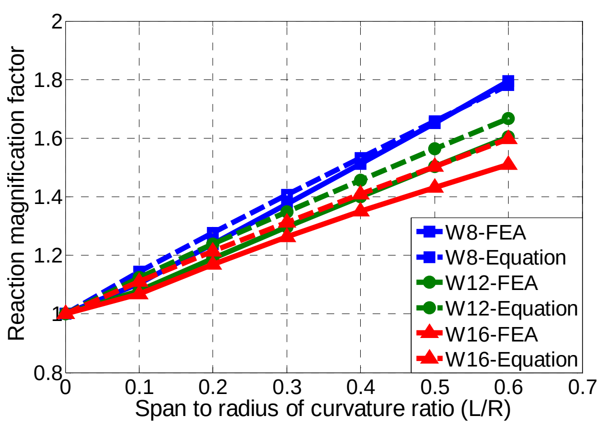


Fig. 12. Effect of curvature on the RMF for simply-supported bridges of 25 m span and different width

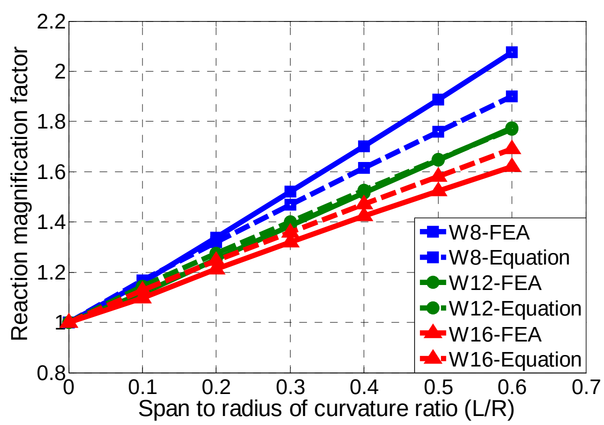


Fig. 13. Effect of curvature on RMF for simply-supported bridges of 35 m span and different bridge width

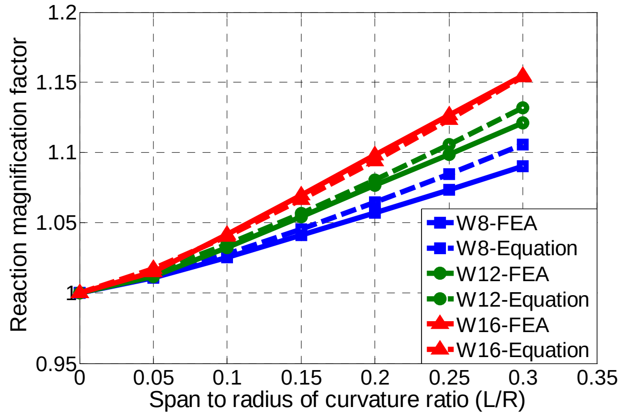


Fig. 14: Effect of Curvature on the RMF for two-Span bridges with 15 m span and different bridge width

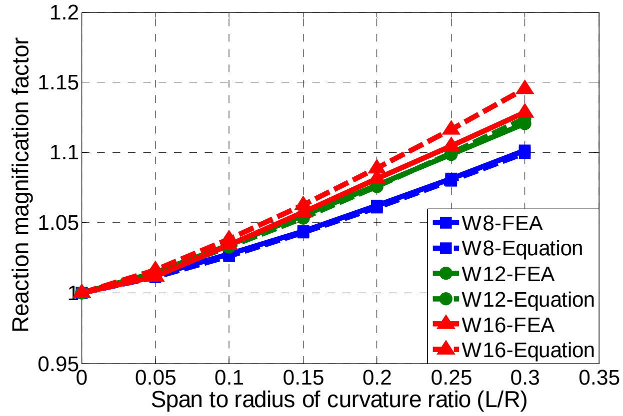


Fig. 15: Effect of curvature of the RMF for two-span bridges with 25 m span and different bridge width

Table 3 presents the developed magnification factor expressions for non-composite steel I-girder bridges. To establish confidence on the proposed expressions, Figs. 4 to 15 show plots of the values obtained from these expressions and those from the FEA analysis. Good correlation is observed. The developed empirical expressions were then extended to establish expressions for curvature limitations to treat a horizontally curved bridge as a straight one by setting the magnification factors to 5% and 10% tolerance (i.e. magnification factors were set equal to 1.05 and 1.10 to produce curvature limitations). It should be noted that the 10% tolerance was used as the basis to develop the curvature limitations specified in Table 1 for AASHTO Guide for Curved Bridges (AASHTO 1993). However, the authors believe that 5% tolerance (i.e. underestimation) in design parameters is acceptable. This means that the increase in straining actions for a curved system is equal or less than those for straight bridge of similar configuration.

Table 3: Proposed magnification factor equations for steel I-girder bridges

Magnification factor type	Single-span	Two-span
Flexural stress	$SMF = 28.577(L/R)^{1.162} \times L^{-0.186} \times B^{-0.089} + 1$	$SMF = 24.884(L/R)^{1.02} \times L^{-0.478} \times B^{-0.02} + 1$
Deflection	$DMF = 1865.7(L/R)^{2.555} \times L^{-1.166} \times B^{0.204} + 1$	$DMF = 135(L/R)^{1.821} \times L^{-0.795} \times B^{0.254} + 1$
Support reaction	$RMF = 0.712(L/R)^{0.941} \times L^{0.425} \times B^{-0.383} + 1$	$RMF = 0.199(L/R)^{1.213} \times L^{-0.112} \times B^{0.543} + 1$

Note: L = arc length for single-span and arc length of one span in a two-span bridge.

Tables 4 and 5 summarize the developed expressions for the curvature limitations for non-composite steel I-girder bridges at construction stage based on 5% and 10% underestimation in design parameters. The developed equations are a function of the span-to-radius of curvature ratio, L/R, span length, L, and bridge width, B. It should be noted that for a given bridge span and width, the governing L/R to treat a curved bridge as a straight one in the structural design resulted from that for flexural stress limitations rather than from deflection, or support reaction limitations. Tables 6 and 7 present the correlation between the proposed L/R limitations for selected bridge spans and widths as obtained from the proposed equations listed in Tables 4 and 5. The correlations revealed that the CHBDC curvature limitation for slab-on-steel girder bridges considerably underestimates the structural response of the curved bridge system when treated as a straight one. This observation is less severe in case of curvature limitations specified in AASHTO code. However, the latter still underestimates the structural response of a curved bridge when treated as a straight one in design.

4: Proposed curvature limitations based on 5% underestimation in design parameters

Magnification factor type	Proposed equations with 5% underestimation	
	Single-span	Two-span
Flexural stress	$(L/R)^{1.162} \times L^{-0.186} \times B^{-0.089} \leq 0.002$	$(L/R)^{1.02} \times L^{-0.478} \times B^{-0.02} \leq 0.002$
Deflection	$(L/R)^{2.555} \times L^{-1.166} \times B^{0.204} \leq 2.68 \times 10^{-5}$	$(L/R)^{1.821} \times L^{-0.795} \times B^{0.254} \leq 0.0004$
Support reaction	$(L/R)^{0.941} \times L^{0.425} \times B^{-0.383} \leq 0.07$	$(L/R)^{1.213} \times L^{-0.112} \times B^{0.543} \leq 0.251$

Note: L = arc length for single-span and arc length of one span in a two-span bridge.

Table 5: Proposed curvature limitations based on 10% underestimation in design parameters

Magnification factor type	Proposed equations with 10% underestimation	
	Single-span	Two-span
Flexural stress	$(L/R)^{1.162} \times L^{-0.186} \times B^{-0.089} \leq 0.0035$	$(L/R)^{1.02} \times L^{-0.478} \times B^{-0.02} \leq 0.004$
Deflection	$(L/R)^{2.555} \times L^{-1.166} \times B^{0.204} \leq 5.36 \times 10^{-5}$	$(L/R)^{1.821} \times L^{-0.795} \times B^{0.254} \leq 0.0007$
Support reaction	$(L/R)^{0.941} \times L^{0.425} \times B^{-0.383} \leq 0.14$	$(L/R)^{1.213} \times L^{-0.112} \times B^{0.543} \leq 0.503$

Note: L = arc length for single-span and arc length of one span in a two-span bridge

Table 6: Correlation of the proposed L/R limitations and those obtained from bridge codes for single-span non-composite steel I-girder bridges

Bridge Parameters		Values of (L/R) according to the proposed equations with 5% tolerance			Values of (L/R) according to the proposed equations with 10% tolerance			Values of (L/R) according to CHBDC	Values of (L/R) according to AASHTO Guide specification 1993			Values of (L/R) according to AASHTO Guide specification 2003
L, m	B, m	S	D	R	S	D	R	$L^2/BR \leq 0.5$	2 girders	3 or 4 girders	5 or more girders	
15	8	0.01	0.05	0.04	0.01	0.06	0.08	0.27	0.03	0.05	0.07	0.06
15	12	0.01	0.05	0.05	0.01	0.06	0.10	0.40	0.03	0.05	0.07	0.06
15	16	0.01	0.04	0.05	0.01	0.06	0.11	0.53	0.03	0.05	0.07	0.06
25	8	0.01	0.06	0.03	0.02	0.08	0.07	0.16	0.03	0.05	0.07	0.06
25	12	0.01	0.06	0.04	0.02	0.08	0.08	0.24	0.03	0.05	0.07	0.06
25	16	0.01	0.06	0.04	0.02	0.07	0.09	0.32	0.03	0.05	0.07	0.06
35	8	0.01	0.07	0.03	0.02	0.09	0.06	0.11	0.03	0.05	0.07	0.06
35	12	0.01	0.07	0.03	0.02	0.09	0.07	0.17	0.03	0.05	0.07	0.06
35	16	0.01	0.07	0.04	0.02	0.09	0.08	0.23	0.03	0.05	0.07	0.06

Notes: S, stress; D, deflection; R, reaction

Table 7: Correlation of the proposed L/R limitations and those obtained from bridge codes for two-span non-composite steel I-girder bridges

Bridge Parameters		Values of (L/R) according to the proposed equations with 5% tolerance			Values of (L/R) according to the proposed equations with 10% tolerance			Values of (L/R) according to CHBDC	Values of (L/R) according to AASHTO Guide specification 1993			Values of (L/R) according to AASHTO Guide specification 2003
L, m	B, m	S	D	R	S	D	R	$L^2/BR \leq 0.5$	2 girders	3-4 girders	5 or more girders	
15	8	0.01	0.03	0.16	0.02	0.05	0.29	0.27	0.05	0.07	0.09	0.067
15	12	0.01	0.03	0.14	0.02	0.04	0.24	0.40	0.05	0.07	0.09	0.067
15	16	0.01	0.03	0.12	0.02	0.04	0.21	0.53	0.05	0.07	0.09	0.067
25	8	0.01	0.04	0.17	0.02	0.06	0.30	0.16	0.05	0.07	0.09	0.067
25	12	0.01	0.04	0.14	0.02	0.05	0.25	0.24	0.05	0.07	0.09	0.067
25	16	0.01	0.04	0.12	0.02	0.05	0.22	0.32	0.05	0.07	0.09	0.067
35	8	0.01	0.05	0.18	0.02	0.07	0.31	0.11	0.05	0.07	0.09	0.067
35	12	0.01	0.05	0.15	0.02	0.06	0.26	0.17	0.05	0.07	0.09	0.067
35	16	0.01	0.04	0.13	0.02	0.06	0.23	0.23	0.05	0.07	0.09	0.067

Notes: S, stress; D, deflection; R, reaction

7. CONCLUSIONS

Based on the results from the parametric study on non-composite steel I-girder bridges at construction stage, the following conclusions are drawn:

1. Curvature is the most important parameter affecting the structural behavior of horizontally curved bridges. A slight increase in the degree of curvature leads to a significant increase in the girder maximum flexural stresses, vertical deflection, and support reaction.
2. Curvature limitation specified in CHBDC significantly underestimates the structural response of the curved bridge superstructure when treated as a straight one in design at construction stage.
3. The AASHTO Guide Specifications for Horizontally Curved Bridges underestimate the structural response of the curved bridge superstructure when treated as a straight one in design at construction stage.
4. Two sets of empirical expressions for curvature limitations were developed for both single and two-span curved steel I-girder bridges considering 5% and 10% underestimation is design, respectively. It will be up to the bridge designers or code writers to decide on which percentage tolerance is acceptable. However, the authors believe that 5% tolerance in design is more acceptable in practice.

ACKNOWLEDGMENTS

The support of NSERC Discovery Grant to this research is very much appreciated.

REFERENCES

AASHTO. 1993. Guide Specification for Horizontally Curved Highway Bridges. American Association of State Highway and Transportation Officials Washington, D.C.

AASHTO. 2003. Guide Specification for Horizontally Curved Highway Bridges. American Association of State Highway and Transportation Officials Washington, D.C.

AASHTO. 2006. AASHTO-LRFD Bridge Design Specifications. American Association of State Highway and Transportation Officials, Washington, D.C.

CSA. 2006. Canadian Highway Bridge Design Code, CAN/CSA-06-06. Canadian Standards Association, Toronto, Ontario, Canada.

Hibbitt, D., Karlsson, I., and Sorenson, J. 2006. ABAQUS User's Manual version 6.6, Hibbitt, Karlsson and Sorenson Inc., Providence, RI.

Khalafalla, I., 2009, "Curvature Limitations in Bridge Codes', M.A.Sc. Thesis, Department of Civil Engineering, Ryerson University, Toronto, Ontario, Canada.