Proceedings of the
Annual Stability Conference
Structural Stability Research Council
Grapevine, Texas, April 18-21, 2012

Moment-shear interaction in plate girders

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Abstract
AASHTO and AISC specifications had adopted Basler's interaction equation. AASHTO LRFD specifications, however, have completely neglected the interaction effect of bending on the shear strength since the 3rd edition in 2004. AISC LRFD specification followed suit from the 13th edition in 2005. In this study, revisited are the interaction equations specified in design codes that were developed for bare-steel sections. The interaction effect in composite sections was also investigated. It was found that neglecting the interaction could lead to unsafe designs when shear buckling is not associated web failures. The interaction should not be neglected when web failures are governed by yielding due to combined bending and shear regardless of whether sections are composite or noncomposite.

1. Introduction

The interaction equation derived by Basler (1961) had long been used in AASHTO LRFD Specifications until it was first completely discarded in the 3rd edition (2004). The commentary of AASHTO LRFD Specifications (2004) reads: "White et al. (2004) shows that the equations of these specifications sufficiently capture the resistance of a reasonably comprehensive body of experimental test results without the need to consider moment-shear interaction." The 3rd edition (2005) of AISC LRFD Specification followed suit.

However, it needs to be noted that most of the previous experimental tests investigated plate girders having slender web panels in which shear buckling takes place first prior to shear yielding under pure shear condition. Since the Basler's interaction was derived without considering shear buckling, it may overestimate the interaction effect when web failures are associated with shear buckling. However, the neglect of the interaction can lead to significantly unsafe designs when web panels fail due to yielding under combined bending and shear as in the Basler's model. Eurocode 3 (2006) still requires checking the bending and shear interaction for the design of steel plate girders. In this paper, the interaction equations specified in design codes, which were formulated for bare-steel sections, were revisited. Also, the moment-shear interaction in composite sections was theoretically investigated.

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2. Bare-Steel Sections

2.1 Interaction Equations Specified in Design Codes

2.1.1 Basler's Interaction Equation

Basler (1961) developed his interaction equation assuming a simple stress distribution shown in Fig. 1 for bending moments greater than the flange moment M_f that is the maximum bending moment that can be carried by the flanges. The flange moment M_f can be written as:

$$M_f = \frac{M_y}{1+\frac{a_r}{6}} \tag{1}$$

where $a_r =$ ratio of the web area to the flange area.

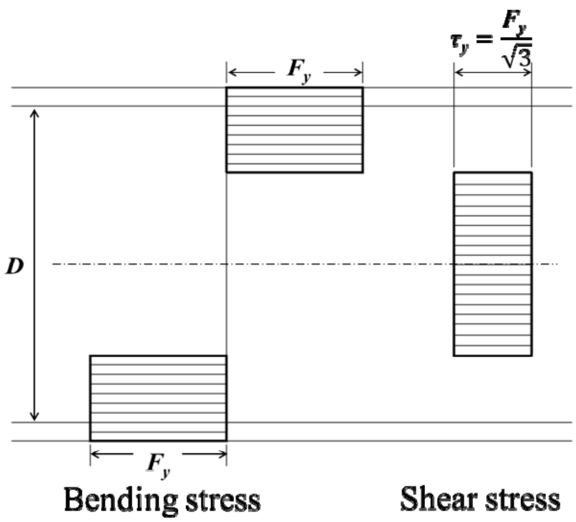

Fig. 1 Stress distribution assumed in Basler's interaction model

Basler disregarded local buckling of the web due to shear. Assuming $a_r = 2.0$ conservatively, the Basler's interaction equation becomes

$$\left(\frac{V}{V_p}\right)^2 = 2.2 - 1.6\frac{M}{M_y} \tag{2}$$

For simplicity, Eq. 2 may be rewritten as:

$$\frac{V}{V_p} = 2.2 - 1.6\frac{M}{M_y} \tag{3}$$

Eq. 3 became the basis for AASHTO LRFD and AISC specifications. It is valid when an applied bending moment M is in between M_f and M_y. The shear strength becomes $0.6V_p$ when the bending moment is equal to M_y.

2.1.2 Interaction Equations in Eurocode 3

Using the simple stress diagrams shown in Fig. 1 as in Basler's model, the following equation can be obtained for compact sections where local shear buckling of the web does not take place prior to shear yielding:

$$\frac{M}{M_p}+\left(1-\frac{M_f}{M_p}\right)\left(\frac{V}{V_p}\right)^2=1 \text{ if } M>M_f \tag{4}$$

Eq. 4 gives zero shear strength when the bending moment reaches the plastic moment M_p. EN 1993-1-1 (2003) of Eurocode 3 uses an empirically-modified version of Eq. 4 based upon experimental test results: The web panels are able to carry a considerable shear even when sections reach the plastic moment. Omitting the partial safety factor, the interaction given in EN 1993-1-1 (2003) can be rewritten as:

$$\frac{M}{M_p}+\left(1-\frac{M_f}{M_p}\right)\left(\frac{2V}{V_p}-1\right)^2=1 \text{ if } V>0.5V_p \tag{5}$$

Eq. 5 for a section having the area ratio $a_r=1.11$ ($M_f/M_p=0.783$) is plotted in Fig. 2.

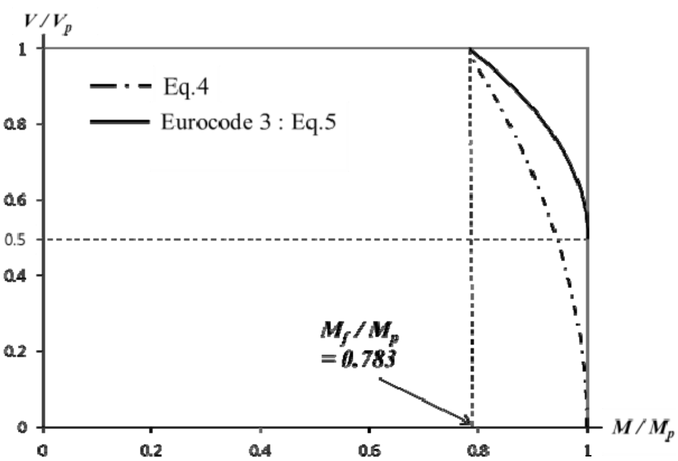

Fig. 2 Interaction Equation (Eq. 5) in Eurocode 3: $M_f/M_p=0.783$

The shear strength at plastic moment M_p becomes $0.5V_p$. Eq. 5 is only for cross sections in class 1 and 2. EN 1993-1-5 (2006) of Eurocode 3 modified Eq. 5 by simply replacing V_p with $V_{bw.R}$ in order to cover the whole classes (1, 2, 3, and 4) of cross sections as:

$$\frac{M}{M_p}+\left(1-\frac{M_f}{M_p}\right)\left(\frac{2V}{V_{bw.R}}-1\right)^2=1 \text{ if } V>0.5V_{bw.R} \tag{6}$$

$V_{bw.R}$ is the shear strength of the web panel under pure shear.

2.2 Investigation of the AASHTO and Eurocode 3 Equations

2.2.1 Basler's Interaction Equation

The interaction equation (Basler 1961) was developed for noncompact sections, in which the maximum bending capacity is M_y. Yet, the application of the Basler's interaction equation was extended to compact sections in AASHTO LRFD and AISC specifications with little adjustments. Omitting the resistance factors, the interaction equation in AASHTO (1998) turns into the original form of Eq. 3 as:

$$\frac{V}{V_n} = 2.2 - 1.6\frac{M}{M_n} \leq 1 \qquad (7)$$

The only difference is that V_p and M_y were replaced with V_n and M_n, respectively. For compact sections without shear buckling, Eq. 7 becomes

$$\frac{V}{V_p} = 2.2 - 1.6\frac{M}{M_p} \leq 1 \qquad (8)$$

Eq. 8 dictates that the shear strength V at M_p is $0.6V_p$, which is the same as that obtained at M_y from Eq. 3. This is contradictory to the original Basler's model. This means that Eq. 8 is not applicable to compact sections.

Also, it needs to be noted that the Basler's equation was developed for web panels where shear buckling is not associated with the failure. Whether it is also applicable to web panels in which web failures are associated with shear buckling will be examined later.

2.2.2 Interaction Equation specified in Eurocode (3)

Eq. 5 specified in Eurocode 3 (2003) was also originally intended for web panels in which web failures are not associated with shear buckling. There have been no theoretical studies reported investigating the interaction behavior in plate girders having web panels in the shear buckling zone presumably due to complexities involved. For this reason, Eurocode 3 (2006) transformed Eq. 5 into Eq. 6 for a broad use with just minor adjustments. Herein, the applicability of Eq. 6 to web panels in the shear buckling zone is examined.

Consider a compact plate girder having the following geometric and material properties: D/t_w = 90; a_r = 1.11; and F_y = 345 MPa, where D = web depth; t_w = web thickness. For d_o/D = 2.0, the web panel falls into the elastic buckling zone according to AASHTO specifications as can be seen from Fig. 3, where d_o is transverse stiffener spacing. Its shear strength V_n under pure shear is $0.804V_p$ and V_n is equivalent to $V_{bw.R}$ in Eurocode 3 (2006). Although the values of V_n and $V_{bw.R}$ are not the same due to the difference in the associated postbuckling theories, there should be no problem in using V_n instead of $V_{bw.R}$ as long as Eq. 6 is theoretically valid.

Assuming the bending moment M is $0.9M_p$, Eq. 6 gives the nominal shear strength equal to $0.84V_n(=0.675V_p)$ for d_o/D = 2.0. When d_o/D = 0.5, the web panel falls into the shear yield zone as can be seen from Fig. 4 but Eq. 6 still gives the same reduction factor 0.84 at the same bending moment. The web panel with d_o/D = 0.5 will, therefore, fail by yielding due combined bending and shear when $M = 0.9M_p$ and $V = 0.84V_p$. It means that as long as an applied shear does not exceed $0.84V_p$ for the given bending moment $M = 0.9M_p$, yielding will never take place. The shear strength $V_n = 0.804V_p$ of the web panel with d_o/D = 2.0 under pure shear is less than $0.84V_p$. Although shear reaches $0.804V_p$, yielding under combined bending and shear will never take place. This implies that shear failure governs the design for d_o/D = 2.0. The reduction factor 0.84, which was originally intended to consider yielding under combined bending and shear, is meaningless for the web with d_o/D = 2.0. The shear strength $V_n = 0.804V_p$ under pure shear, therefore, should be taken as the nominal shear strength for the web with d_o/D = 2.0. This finding also means that the Basler's interaction equation should not directly applied to noncompact sections when shear buckling is associated with web failures.

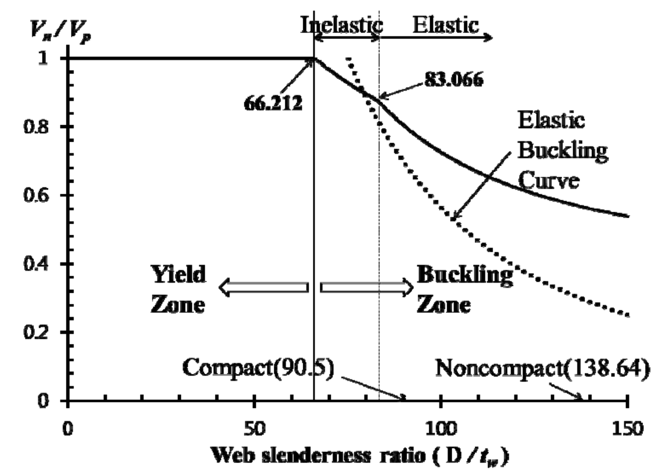

Fig. 3 AASHTO shear strength curve for $F_y = 345$ MPa and $d_o/D = 2.0$

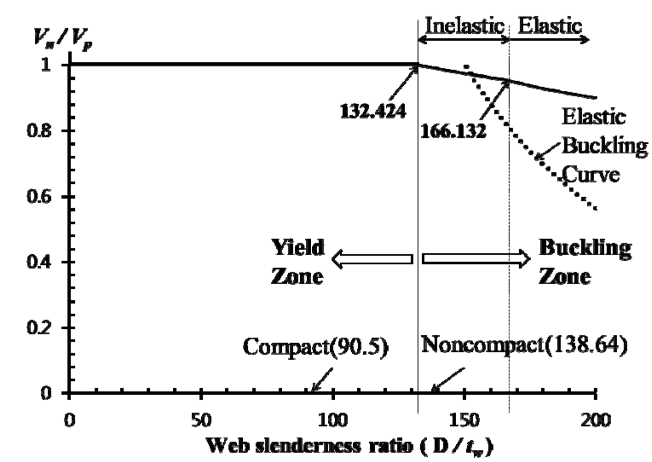

Fig. 4 AASHTO shear strength curve for $F_y = 345$ MPa and $d_o/D = 0.5$

2.3 New Methodology to Determine Nominal Shear Strength for Webs in Shear Buckling Zone
It has been shown that the direct application of Eq. 6 to web panels falling into the shear buckling zone could result in a considerable underestimation of the nominal shear strength. Unlike the case of sections having web panels falling into the shear yield zone, developing a generally-applicable interaction equation is virtually not feasible when web failures are associated with local buckling. This is not only because the methods to assess the shear strength V_n under pure shear including postbuckling strength differ from theory to theory and from code to code but also because of complexities involved.

As demonstrated above, the shear strength V_n determined under pure shear condition can be taken as the nominal shear strength if it is less than the reduced shear strength V determined without considering local shear buckling. That is to say, there is no need to consider the interaction if web panels fail by shear. When V_n is greater than the reduced shear strength V determined without considering local shear buckling, yielding due to combined bending and shear takes place first prior to shear failure and therefore, the reduced shear strength V should be taken as the nominal shear strength.

3. Composite Sections
Composite sections are often used in bridge constructions in order to maximize the contribution of concrete decks to bending resistance. Basler's interaction equation (Basler 1961), which was derived for noncomposite sections, had long been applied to composite sections in AASHTO LRFD and AISC specifications without any theoretical background until the interaction effect was totally neglected. Due to lack of studies regarding composite sections, it has yet to be testified whether the interaction effect could be safely neglected in composite sections or not. In this study, a theoretical investigation was carried out to assess the interaction effect on the shear strength of composite girders. The shear-carrying capacity of concrete is not considered. Composite sections are categorized into two types according to whether they are under positive bending or negative bending.

3.1 Composite Sections in Positive Bending

3.1.1 Interaction Equation
As in the case of bare-steel sections, the flange plastic moment M_f including the concrete moment, first of all, needs to be determined in order to assess the interaction effect for a given bending moment. If an applied bending moment is less than or equal to the flange plastic moment, there is no interaction as in bare-steel sections. When the maximum compressive force in the effective concrete area is greater than the sum of the forces in the top and bottom flanges at yield, the neutral axis at M_f will be within the concrete. Otherwise, the neutral axis at M_f will be within the top flange.

As the bending moment increases beyond the flange plastic moment M_f, the web will begin to participate in carrying the bending moment approaching the plastic moment M_p. Fig. 5 (a) depicts shear stresses τ_{xy} for an applied shear force V in the web prior to applying bending moment. When the whole web reaches the yield condition as per the von Mises yield criterion, the bending stress σ_x will be uniform as shown in Fig. 5 (b). The stress distribution shown in Fig. 5 is identical to another distribution given in Basler (1961) known to be more accurate (Salmon and Johnson 1996).

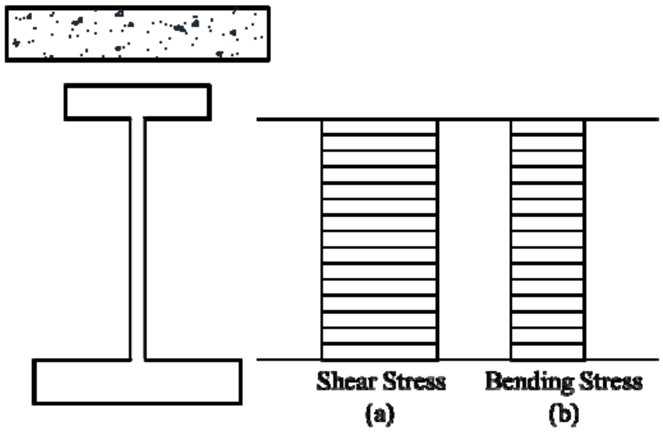

Fig. 5 Idealized stress distributions: (a) shear stress; (b) bending stress

As per the von Mises yield criterion, the bending stress in the web σ_x at yield is given by:

$$\sigma_x = \sqrt{F_y^2 - 3\tau_{xy}^2} \tag{9}$$

Letting $\eta = \dfrac{\sigma_x}{F_y}$, η becomes

$$\eta = \sqrt{1 - \left(\frac{\tau_{xy}}{F_y/\sqrt{3}}\right)^2} = \sqrt{1 - \left(\frac{V}{V_p}\right)^2} \tag{10}$$

where F_y is the yield stress of the web.

For a given bending moment M, η can be determined by considering force and moment equilibrium in the composite section. Once η is determined, the reduced shear strength V can be computed using Eq. 10 as:

$$\frac{V}{V_p} = \sqrt{1 - \eta^2} \tag{11}$$

3.1.2 Adjustment of Interaction Equation (Eq. 11)

The shear strength approaches zero as η approaches 1.0. At the plastic moment, η becomes 1.0. As done in Eq. 5 for bare-steel section, the adjustment may be necessary for loadings of high moment. In this study, the adjustment starts from $\eta = 0.5$, which corresponds to the yield moment of bare-steel sections. Eq. 11 gives $V/V_p = 0.8666$ at $\eta = 0.5$. Setting the reduced shear strength at M_p, i.e. when $\eta = 1.0$, equal to $0.5V_p$, the following equation can be obtained:

$$\frac{V}{V_p} = 0.8666 - 0.7332(\eta - 0.5) \cong 0.87 - 0.74(\eta - 0.5) \tag{12}$$

When η exceeds 0.5, Eq. 12 is to be used instead of Eq. 11. Eqs. 11 and 12 are plotted in Fig. 6.

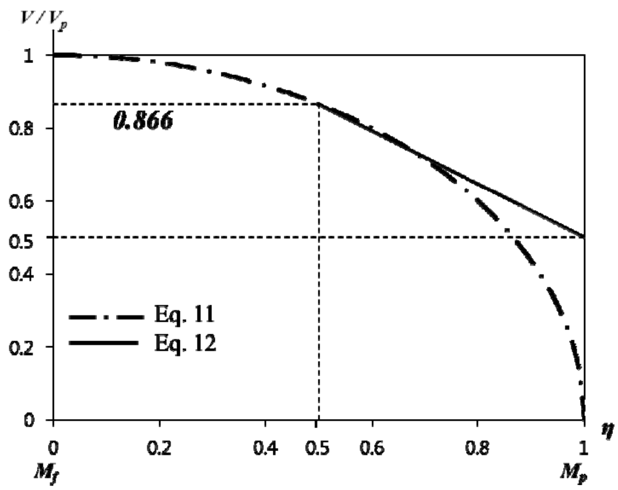

Fig. 6 Interaction curves: Eq. 11 and Eq. 12

3.1.3 Determination of Nominal Shear Strength

It needs to be noted that Eq. 11 and Eq. 12 were also formulated considering yielding due to the combined action of bending and shear. Therefore, they should not be directly applied to the web panels in which failures are associated with shear buckling.

If the reduced shear strength V obtained from Eq. 11 or Eq. 12 is less than the shear strength V_n that is determined under pure shear, then yielding due to the combined action of bending and shear will take place first before the web panel develops the shear strength V_n. In this case, the reduced shear strength V should be taken as the nominal shear strength as in bare-steel sections. If V is greater than V_n, shear failure will take place prior to yielding due to combined bending and shear and therefore, V_n becomes the nominal shear strength.

3.1.4 Design Example

The interaction behavior of a composite section under positive bending, which was used as design examples in AISI (1995), was investigated. Fig. 7 shows a composite section with the plastic neutral axis within the top flange: f_c' = 30 MPa; F_y = 345 MPa; D/t_w = 126.79; M_p = 29.05 x 10^9 N-m. The neutral axis at M_f is within the concrete: $d_{f.M}$ = 188 mm; and M_f = 20.97 x10^9 N-mm (= 0.72M_p).

When the bending moment M, for example, is 25.0 x 10^9 N-mm (86.06 % of M_p), η is 0.483. The reduced shear strength V determined from Eq. 12 is 0.8756V_p, which V falls below the AASHTO V_n curve for d_o / D = 0.5 shown in Fig. 4. This means yielding due to combined bending and shear governs the web failure so that the reduced shear strength V should be taken as the nominal shear strength. But, when $d_o / D = 2.0$, the reduced shear strength V falls above the curve as shown in Fig. 3 and therefore, there is no need to consider the interaction.

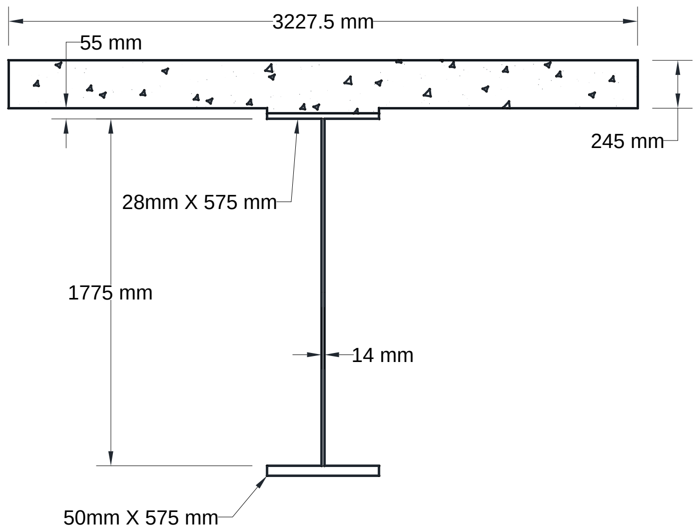

Fig. 7 Composite section in design example

3.2 Composite Sections in Negative Bending

As per AASHTO LRFD (2010), composite sections in negative bending are to be treated as noncompact sections. Neglecting the concrete in tension, a composite section subjected to negative bending is composed of the reinforcing bars and the steel girder. In practice, composite sections subjected to negative bending are usually designed such that the sum of areas of the reinforcing steels and top flanges is close to the area of the bottom flange. In turn, the interaction effect of bending on shear can be assessed following the procedure proposed for noncompact bare-steel sections in the companion paper.

4. Summary and Concluding Remarks

In this study, a theoretical investigation was carried out in order to evaluate the effect of bending moment on the shear strength of bare-steel and composite steel plate girders. The results are summarized as follow:

1. Basler's interaction equation should not be directly used for compact sections. Also, it could result in too conservative designs for web panels falling into shear buckling zone.
2. The interaction equation (Eq. 6) specified in Eurocode 3 (2006) cannot be directly applied to web panels falling into shear buckling zone.
3. For web panels in the shear buckling zone, the smaller one between the shear strength under pure shear and the reduced shear strength determined without considering shear buckling could be taken as the nominal shear strength.
4. The current provisions in AASHTO and AISC Specifications completely neglect the interaction effect regardless of web slenderness ratios. When web panels fall into the shear yield zone, neglecting the interaction effect, however, could lead to unsafe designs (up to 50% as can be seen from Fig. 2).

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