



New shear design criteria for plate girders

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Abstract

Shear design of steel plate girders has been one of the most controversial subjects in structural design. There have been a number of competing theories as to the postbuckling behavior of the plate girder web panels and how to assess the ultimate shear strength of plate girders. The shear design provisions differ from code to code. In this study, a review of existing shear failure theories is made. In addition, the shear design provisions specified in AASHTO LRFD and AISC specifications are examined. The AASHTO and AISC Specifications still contain remnants of unproven theoretical results. New shear design criteria are proposed for a more rational assessment of the design shear strength of steel plate girders and the design of the transverse stiffeners.

1. Introduction

The approach to assess the overall shear strength of plate girders differs from theory to theory. More than a dozen of competing failure theories and their derivatives have been proposed since Basler (1961) first presented the diagonal tension field theory for plate girders used in civil engineering structures (SSRC 2010). The shear design provisions in AISC and AASHTO Specifications have been fundamentally based on Basler (1961) since the first adoptions in 1963 and 1973, respectively. Among other modified theories, the Cardiff model (Porter et al. 1975) was adopted in British standard, BS 5400 (1982). Eurocode 3 (2006) is based on Höglund (1997). Recently Yoo and Lee (2006) and Lee et al. (2009a; 2009b) unveiled the true postbuckling mechanism developed in ordinary plate girders and expounded the inadequacies in the previous failure theories. This study presents a comprehensive shear design procedure for a rational determination of the design shear strength and the design of intermediate transverse stiffeners of plate girders.

2. Shear Strength of Web Panels

Unless the web panel is thick enough, elastic buckling will take place prior to shear yielding and then the postbuckling strength will develop due to the diagonal tension field action. The flanges

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also can contribute to the shear strength through the so-called frame action (Vierendeel truss analogy).

2.1 Elastic Shear Buckling Strength of Web Panels

The elastic buckling stress of the web panel subjected to shear is given by the following equation:

$$\tau_{cr} = k \frac{\pi^2 E}{12(1-\mu^2)(D/t_w)^2} \quad (1)$$

where k = shear buckling coefficient; D = web depth; t_w = web thickness; E = modulus of elasticity; μ = Poisson's ratio. The transverse stiffeners are designed to function as a simply supported boundary. On the other hand, the web panel is elastically restrained at the juncture between the web panel and flanges. The degree of the elastic restraint largely depends upon a relative bending stiffness of the flanges to the web.

The nature and degree of the elastic restraint at the flange-web juncture was first numerically investigated on broad ranges of various geometric parameters by Lee et al. (1996). Through an examination of over 300 plate girder models, they suggested the following simple formula to evaluate the shear buckling coefficient:

$$k = k_{ss} + \frac{4}{5}(k_{sf} - k_{ss}) \left[1 - \frac{2}{3} \left(2 - \frac{t_f}{t_w} \right) \right] \quad \text{for} \quad \frac{1}{2} < \frac{t_f}{t_w} < 2 \quad (2)$$

$$k = k_{ss} + \frac{4}{5}(k_{sf} - k_{ss}) \quad \text{for} \quad \frac{t_f}{t_w} \geq 2 \quad (3)$$

where t_f = flange thickness and k_{ss} , k_{sf} = the shear buckling coefficients of plates with simple-simple boundary and simple-fixed boundary, respectively. The validity of Eqs. 2 and 3 has been well acknowledged by many other researchers including Abu-Hamed (2010), Keerthan and Mahendran (2010, 2011), and White et al. (2001). It is of interest to note that for web bend-buckling resistance, AASHTO LRFD (2010) uses $k_b = 36.0$ for the bend-buckling coefficient following the same methodology given by the Eq. 3. The numerical value of 36 is approximately equal to $k_b = k_{ss} + 0.8(k_{sf} - k_{ss})$, where $k_{ss} = 23.9$ and $k_{sf} = 39.6$ are the bend-buckling coefficients for simple-simple and simple-fixed conditions, respectively. Prior to the 4th edition (2007), AASHTO Specifications used $k_b = 32.0$, which is approximately halfway between 23.9 and 39.6. It is believed that the equations suggested by Lee et al. (1996) can also be effectively used for a realistic evaluation of the elastic shear buckling strength as numerous researchers have ascertained the validity and accuracy of the equations.

2.2 Flange Contribution to Shear Strength

The flanges can directly contribute to the overall shear strength of a plate girder by resisting shear acting on the flange cross sections. Therefore, as the flange size increases, the relative portion of the flange contribution becomes greater. White and Barker (2008) comprehensively

compares the shear strengths calculated from existing design equations and those observed in tests. White and Barker (2008) reads: “The Lee and Yoo models tend to under predict test strengths by a significant margin as the flange size increases.” It is absolutely true because the equation proposed by Lee and Yoo (1998) was formulated using the results of nonlinear finite element analysis of an isolated plate girder model, in which the flanges cannot directly resist the applied shear but can indirectly contribute to the elastic buckling strength by providing an elastic restraint to the web panel. Other theories consider the direct flange contribution either by an explicit manner (Cardiff model) or implicitly (Basler 1961).

The flanges are, of course, able to contribute to the shear strength unless the bending stress reaches the yield stress. As per von Mises yield criterion, once the flanges of compact or noncompact sections reach the yield stress, there is no room for shear in the flanges at all. In order to reflect the direct flange contribution in the design, the maximum flange stress must be kept below the flange yield stress, thereby reducing bending strength. However, reducing bending capacity may not only result in a less economical design but also could make the design too complicated for relatively small improvement. Bending moment is the primary force component in plate girders. It would make girder designs simple and more economical to leave the potential bending capacities (the plastic moment and the yield moment) intact and let the web panel take shear alone.

2. 3 Suggested Design Equations for Design Shear Strength

The major classical failure theories including Basler (1961) and Porter et al. (1975), and Höglund (1997) were formulated fundamentally based upon the assumption that the anchoring mechanism is the unique source of the tension field. That is, in order for the postbuckling strength to develop the diagonal tension field must be anchored by the flanges, adjacent panels, and/or rigid end stiffeners. However, Lee and Yoo (1998) first found that a laterally supported panel (a panel simply supported at all the edges) is able to develop postbuckling strength quantitatively close to those observed in tests of ordinary plate girders without recourse to any external anchors such as the flanges and adjacent panels. Then, they suggested a simple design formula to predict the postbuckling strength through nonlinear finite element analyses of an isolated plate girder model.

Thereafter, Yoo and Lee (2006) unveiled what is behind the postbuckling mechanism developed in the simply supported panel. They observed that contrary to the classical theories, the principal compressive stresses continuously increase near the edges of the simply supported panel after buckling. Development of a tension field is possible in the simply supported panel due to the increments of the compressive stresses without relying on the flanges and adjacent panels. This study dictates that there exist two independent sources of the tension field developed in the web panels: (1) the anchoring mechanism developed by means of the flanges and/or adjacent panels; and (2) the function of lateral supports. This study has been gaining currency (Keerthan and Mahendran 2010, 2011, Alina et al 2009, 2011).

Recently, Lee et al. (2009a, b) revisited the anchoring mechanism that is assumed to be the unique source of the web postbuckling strength in the classical tension field theories. They found that even with the flanges that are heavy enough to function as the rigid anchor, the anchoring mechanism cannot completely develop unless the flanges are supported by incompressible transverse stiffeners. The primary reason why the anchoring mechanism by the

flanges contributes little to the postbuckling strength in ordinary plate girders is that the transverse stiffeners are axially too flexible to be treated as incompressible. They also found that the plastic hinge-like failure mechanism is not due to bending caused by the anchoring action of flanges but due to direct shear. A similar finding was reported in Alina et al. (2009). Lee et al. (2009b) concluded that the utilization of the flange anchoring mechanism in practical designs, to any meaningful extent, is beyond the realms of possibility because it requires an unimaginably high axial stiffness of the transverse stiffeners. It was also found that the horizontal anchoring action by adjacent panels is not possible in normal plate girders.

Granted that existing design equations based upon either Basler (1961) or Höglund (1997) have served a useful purpose in the past, a more valid rational design equation proposed by Lee and Yoo (1998) needs to be reflected in new design codes for a more rational design. The Lee and Yoo equation is given by

$$V_n = V_p (0.6C + 0.4) \quad (4)$$

The ratios C of the elastic shear buckling strength V_{cr} to the shear yield strength (plastic shear strength) V_p are to be determined by the current AASHTO Specifications (2010). AISC Specification (2010) has slightly different constants for C .

AISC (2010) and AASHTO LRFD (2010) treat web panels with the aspect ratio d_o / D greater than 3 as unstiffened panels so that the use of postbuckling strength is not permitted in both. The notion behind this limit is that the enhanced resistance due to the tension field action could be reduced when the panel aspect ratio becomes large because the angle of the inclined diagonal tension lines becomes too shallow. However, in fact, there are few experimental data available giving credence to this restriction. Lee et al. (2008) investigated web panels having the aspect ratios from 3 to 6, and found that Eq. (4) can also be used for such long web panels. It has been reported that the placement of intermediate transverse stiffeners greatly facilitates the shipping and handling of slender girders. Unless this is the reason, there is no justification in limiting the aspect ratio of the web panel to 3.

As per AISC (2010) and AASHTO (2010), the use of postbuckling strength is not permitted for end panels in that they are not fully anchored by adjacent panels unlike interior panels. However, this restriction should be retracted since the postbuckling strength developed in normal plate girders is not due to either vertical anchoring mechanism by the flanges or horizontal anchoring mechanism by adjacent panels (Yoo and Lee 2006, Lee et al. 2009a, b). Given that end panels are mostly subjected to higher shears than interior panels, the removal of this restriction would allow a more economical design.

Lee and Yoo (1998) reported that out-of-plane bending actions triggered by initial imperfections could substantially lessen the postbuckling strength for thicker web panels. They proposed the strength reduction factor R reflecting the detrimental effect of a large initial imperfection ($\Delta_{in} = D/120$) that is the maximum initial out-of-flatness allowed in Bridge Welding Code (2010). As White and Barker (2008) pointed out, the reduction as presented in Lee and Yoo (1998) might be too severe for stocky web panels so that the reduction factor for thicker webs was linearly adjusted by Lee et al. (2011). The reduction factor is given by:

$$R = 0.8 + 0.2 \frac{1.10 - (D/t_w) \sqrt{F_{yw}/kE}}{1.10} \quad \text{for } \frac{D}{t_w} \leq 1.10 \sqrt{kE/F_{yw}} \quad (5)$$

$$R = 0.8 + 0.2 \frac{(D/t_w) \sqrt{F_{yw}/kE} - 1.10}{1.10} \quad \text{for } 1.10 \sqrt{kE/F_{yw}} \leq \frac{D}{t_w} \leq 2.20 \sqrt{kE/F_{yw}} \quad (6)$$

$$R = 1.0 \quad \text{for } \frac{D}{t_w} \geq 2.20 \sqrt{kE/F_{yw}} \quad (7)$$

Then, the final form of the design equation for the ultimate shear strength is written as:

$$V_n = R \lambda V_p (0.6C + 0.4) \quad (8)$$

Hassanein (2010) later confirmed the similar effect of large initial imperfections.

3. Design of Transverse Stiffeners

The function of intermediate transverse stiffeners attached on the plate girder web panel is to provide nodal lines or simple support conditions during local buckling due to shear, thereby increasing the shear strength. AASHTO (2007) made a major revision regarding the design of transverse stiffeners based on Kim et al. (2004). Similar provisions were newly specified in AISC (2010). In this study, a new methodology is proposed for a simple and systematic design of the transverse stiffeners. Following the new approach, a set of design equations are obtained through linear buckling and nonlinear finite element analyses. The results of new design equations are compared with those of the current AASHTO and AISC Specifications.

3.1 New Design Methodology for Transverse Stiffeners

Approaches towards the design of the transverse stiffeners are directly related to the design shear strength of the web panel. The transverse stiffeners are to be designed to have a proper flexural rigidity that ensures that the web panel is able to develop its design shear strength. Fig. 1 shows the AASHTO shear strength curve for panels, which is divided into the three zones: yield zone; inelastic buckling zone; elastic buckling zone.

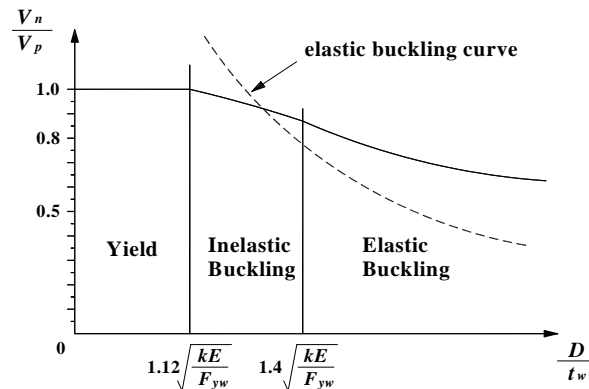


Fig. 1: AASHTO Shear Strength Curve

For web panels falling into the elastic buckling zone, in which the postbuckling strength is utilized, a special attention should be given to the design of the transverse stiffeners. For the

transverse stiffener to maintain the nodal lines straight and to ensure the web panel to be able to develop its potential postbuckling strength, a much greater flexural rigidity is necessary than for elastic buckling.

When the web panels fall into the yield zone, the elastic buckling strength is greater than the shear yield strength and therefore, the transverse stiffeners do not have to provide the nodal lines for elastic buckling. It is enough for the transverse stiffeners to ensure that the web panel is able to develop the shear yield strength.

The inelastic buckling zone may need to be divided into three subzones as shown in Fig. 2: zone I; zone II; and zone III. The design of the transverse stiffeners, however, can be greatly simplified by assuming zone I as the yield zone, and zone II and zone III as the elastic buckling zone. This assumption will also lead to slightly conservative designs; hence, it is adopted in this study.

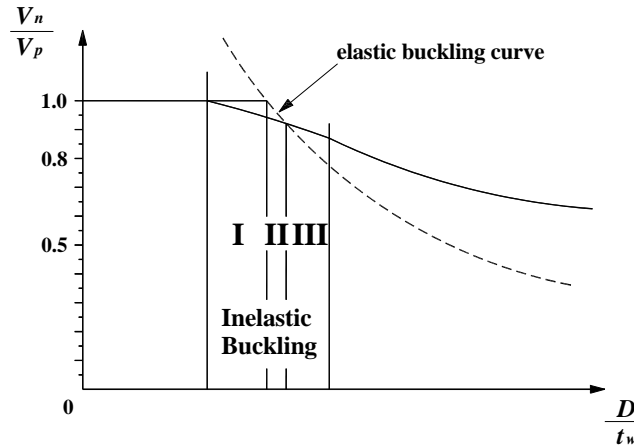


Fig. 2: Subdivision of Inelastic Buckling Zone

3.2 Flexural Rigidity Required for Elastic Buckling

The moment of inertia required to provide the web with the nodal line, specified in AASHTO LRFD Specifications prior to AASHTO (2007), is given by:

$$I_t \geq j d_o t_w^3 \quad (9)$$

where

$$j = \frac{2.5}{\left(\frac{d_o}{D}\right)^2} - 2 \geq 0.5 \quad (10)$$

Eq. 10 is a simplified version of the formula that Bleich (1952) developed using the results obtained by Stein and Fralich (1949). The minimum value of j is limited to 0.5 because Stein and Fralich (1949) did not cover aspect ratios greater than 1.0. In this study, a rigorous buckling analysis was carried out by the finite element analysis (ADINA) to investigate the required

moment of inertia of the transverse stiffener for a broader range of $d_o/D = 0.5 \sim 6.0$ and to examine the validity of Eq. 10. The results are comparatively summarized in Table 1. It is apparent that Eq. 10 is much too conservative. A new design equation is formulated for j from the FEA results and conservatively limited to 0.05 as:

$$j = \frac{2.12}{(d_o/D)^4} - \frac{5.98}{(d_o/D)^3} + \frac{5.29}{(d_o/D)^2} - \frac{1.10}{(d_o/D)} \geq 0.05 \quad (11)$$

Table 1 Comparison of j values: FEA vs. Eq. 10

d_o/D	0.5	0.75	1.0	2.0	3.0
j_{FEM}	4.94	0.43	0.33	0.12	0.05
$j_{Eq. 10}$	8.0	2.44	0.5	0.5	0.5

3.3 Flexural Rigidity for Web Panels in Yield Zone

For web panels in the yield zone, the shear yield strength becomes the shear strength. In the yield zone, Eq. 9 is too conservative because the transverse stiffeners do not have to provide nodal lines until it reaches the theoretical buckling stress, which is greater than the shear yield stress. Transverse stiffeners are not needed at all for webs whose shear yield stress is less than or equal to the buckling stress of an infinitely long panel. This condition is given by:

$$k \frac{\pi^2 E}{12(1-\mu^2)} \frac{1}{(D/t_w)^2} \geq \frac{F_{yw}}{\sqrt{3}} \quad (12)$$

where F_{yw} is the yield stress of the web. Using the shear buckling coefficient $k = 5.34$ for the infinitely long simply-supported panel and Poisson's ratio $\mu = 0.3$, Eq. 12 can be rewritten as:

$$D/t_w \leq 2.90 \sqrt{\frac{E}{F_{yw}}} \quad (13)$$

Eq. 13 represents zone A shown in Fig. 3, where the transverse stiffeners are not necessary. In AISC Specification (2010), this limit is given by:

$$D/t_w \leq 2.46 \sqrt{\frac{E}{F_{yw}}} \quad (14)$$

When the shear yield strength is equal to the elastic buckling strength of the web panel, the required moment of inertia of the transverse stiffener is the same as that required for elastic buckling calculated from Eq. 1. This condition is given by:

$$k \frac{\pi^2 E}{12(1-\mu^2)} \frac{1}{(D/t_w)^2} = \frac{F_{yw}}{\sqrt{3}} \quad (15)$$

or

$$D/t_w = 1.25 \sqrt{kE/F_{yw}} \quad (16)$$

When $2.90\sqrt{E/F_{yw}} \leq D/t_w \leq 1.25\sqrt{kE/F_{yw}}$ (zone B in Fig. 3), the transverse stiffener is necessary and its moment of inertia can be determined by reducing the j value for elastic buckling calculated from Eq. 11.

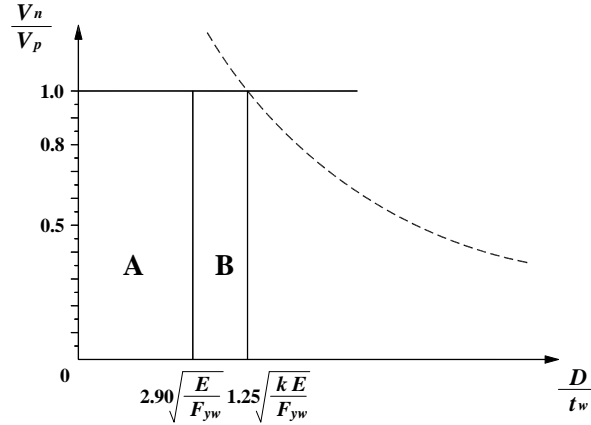


Fig. 3: Subdivision of Yield Zone for Stiffener Design

In zone B, the required moment of inertia is the largest at $D/t_w = 1.25\sqrt{kE/F_{yw}}$ and it is gradually reduced to zero as D/t_w decreases. For the sake of design simplicity and conservativeness as well, the moment of inertia required for $D/t_w = 1.25\sqrt{kE/F_{yw}}$ can be used in zone B because zone B is mostly very narrow as can be seen from Figs. 4.

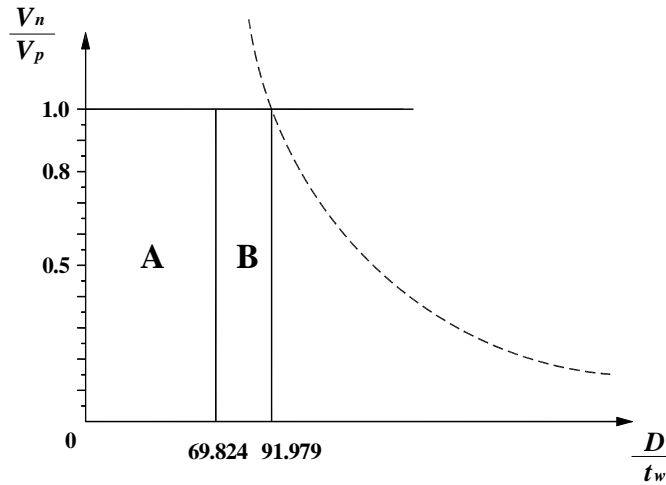


Fig. 4: Zone B ($d_o/D = 1.0$, $k = 9.34$, $F_{yw} = 345$ MPa)

From Eq. 16

$$t_w = \frac{D}{1.25\sqrt{kE/F_{yw}}} \quad (17)$$

Substituting Eq. 17 into Eq. 9 yields

$$I_t \geq \frac{jd_o D^3}{1.25^3 \left(\frac{kE}{F_{yw}}\right)^{1.5}} = \frac{jd_o D^3}{1.95 \left(\frac{kE}{F_{yw}}\right)^{1.5}} \quad (18)$$

3.4 Flexural Rigidity for Postbuckling of Web Panels

For the web panels in which the postbuckling strength is utilized, the transverse stiffeners should be able to maintain the nodal line straight not only for elastic buckling but also in the postbuckling stage, where severe out-of-plane deformations of the web panels could be developed. The optimum bending rigidity of the transverse stiffener sought here is such that it can keep the nodal line straight during postbuckling, thereby letting the model (Fig. 6) develop the same shear strength as that of the model shown in Fig. 7, which was used in Lee and Yoo (1998).

From the results of nonlinear FEA, a new design equation was formulated as:

$$I_t \geq njd_o t_w^3 \quad (19)$$

where j is calculated from Eq. 11 and n is a multiplication factor given by:

$$n = 10.67 \left(\frac{D}{t_w}\right) \sqrt{\frac{F_{yw}}{kE}} - 3.66 \left(\frac{d_o}{D}\right) - 8.58 \geq 1.0 \quad (20)$$

Fig. 8 compares the multiplication factors n obtained from the FEA and Eq. 19.

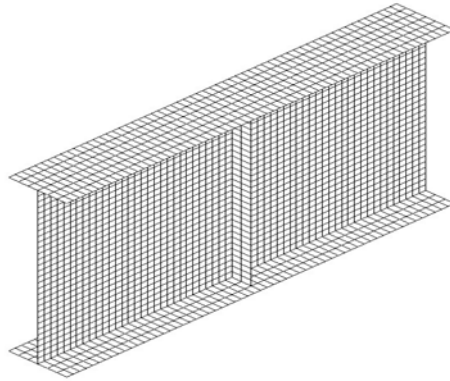


Fig. 6: Isolated plate girder model with transverse stiffener

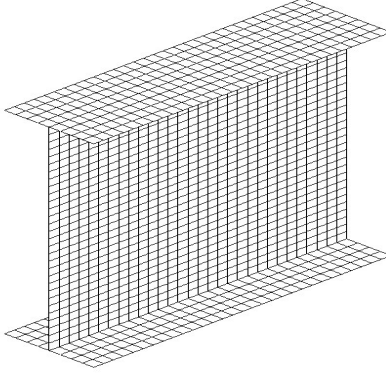


Fig. 7: Isolated plate girder model

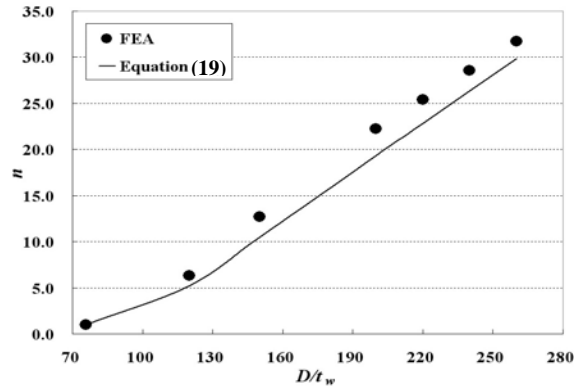


Fig. 8: Comparison of n for $d_o / D = 1.0$ and $F_{yw} = 345$ MPa: FEA vs. Eq. 19

3.5 Discussions of AASHTO LRFD and AISC Specifications

AASHTO (2007) changed for the first time the provisions for the design of the transverse stiffeners that had long been specified since AASHTO (1973) based on Kim et al. (2004), and AISC (2010) followed suit.

- When Web Panels Do Not Support Shear Forces Greater Than Shear Buckling Resistance

AASHTO (2007) reads: For transverse stiffeners adjacent to web panels in which neither panel supports shear forces larger than the shear buckling resistance, the moment of inertia shall be the smaller of Eq. 21 and Eq. 22:

$$I_t \geq bt_w^3 j \quad (21)$$

$$I_t \geq \frac{D^4 \rho_t^{1.3}}{40} \left(\frac{F_{yw}}{E} \right)^{1.5} \quad (22)$$

where ρ_t is the larger of F_{yw} or F_{crs} that is the buckling stress of the stiffener. Eq. 21 is a modified version of Eq. 9 (Kim et al. 2004) but j is to be calculated using Eq. 10 so that it still results in too conservative designs as shown in Fig. 9.

Eq. 22 is a simplified version of the following equation developed in Kim et al. (2004):

$$I_t \geq \frac{bD^3 j}{1.4(Ek/F_{yw})^{1.5} \rho_t^{0.75}} \quad (23)$$

Eq. 23 was developed for web panels falling into the shear yield zone, which corresponds to Eq. 18. This provision is somewhat confusing because it requires checking both Eqs. 21 and 22. For web panels in the elastic buckling zone, it is not necessary to check Eq. 22 because it was developed for web panels in the yield zone, and vice versa. It appears more rational to clearly categorize web panels into two types (web panels in the yield zone; web panels in the elastic buckling zone) as suggested in the present study rather than checking both Eq. 21 and 22.

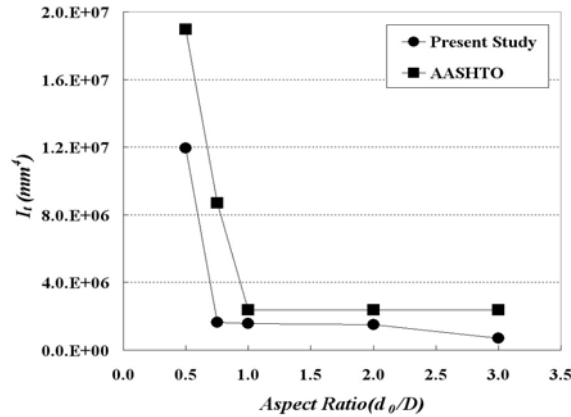


Fig. 9 Comparison of moments of inertia ($D = 2000$ mm, $D/t_w = 150$): Present study (Eq. 9 using Eq. 11) vs. AASHTO (Eq. 21)

- When Web Panels Support Shear Forces Greater Than Shear Buckling Resistance

AASHTO (2007) reads: “For transverse stiffeners adjacent to web panels in which the shear force is greater than the shear buckling resistance and thus the web postbuckling or tension-field resistance is required one or both panels, the moment of inertia of the transverse stiffeners shall satisfy Eq. 22.” As explained above, Eq. 22 was originally intended for web panels falling into the yield zone. The application of Eq. 22, therefore, will result in too conservative designs as can be seen from Fig. 10.

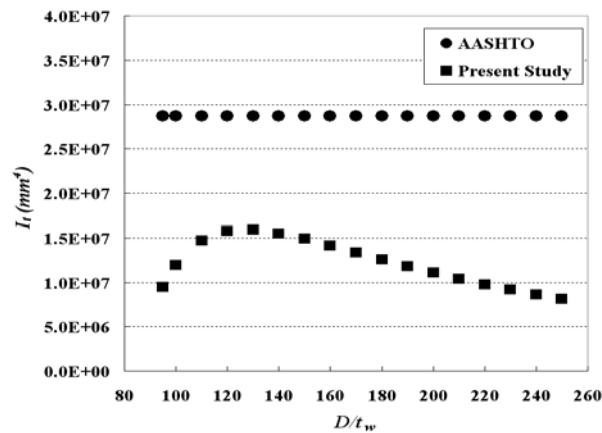


Fig. 10: Comparison of I_t ($D = 2000$ mm; $d_o/D = 1.0$, and $F_{yw} = F_{ys} = 345$ MPa): Present study (Eq.19) vs. AASHTO (Eq. 22)

4. Conclusion

A new procedure to determine the design shear strength of plate girder web panels is presented based on a new theory explaining the true mechanics behind the tension field action. It would seem logical to formulate national design specifications based on the sound theoretical basis explaining the complex nonlinear behavior behind the tension field action.

A systematic and simple methodology is proposed for a more rational and economical design of the transverse stiffeners for plate girders. A set of design equations for the new approach have been developed through synthesizing and characterizing linear buckling and nonlinear finite element analyses encompassing a wide range of parameters affecting the shear strength of plate girder web panels. It was found that the provisions specified in AASHTO LRFD Specifications (2010) and AISC Specification (2010) overall require considerably larger transverse stiffeners regardless of types of web panels. It is believed that the design equations developed in this study can be effectively used for a rational and economical design of the transverse stiffeners.

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