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Analytical model for stiffened steel infill plates

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Abstract
Vertical and horizontal plate stiffeners are designed for the steel infill plate of the steel plate wall (SPW) systems to improve buckling stability and to prevent early elastic buckling of infill plates. Nevertheless, a lack of analytical model exists that are capable of replicating stiffened plates' load-displacement behavior. In this paper, an analytical method is proposed that is capable of modeling the load-displacement behavior of stiffened infill plates for steel plate wall (SPW) systems. The analytical model is an addendum to the previously proposed plate frame interaction (PFI) method used for determining the shear load-displacement of unstiffened steel infill plates of SPW systems. To evaluate the proposed model, the predictions of this model are compared to results obtained from tests previously conducted, as well as those gained from finite element (FE) analyses performed for this study. Considering the simplicity of the analytical mode, the result of the evaluation indicates that the model is providing the stiffness and strength of stiffened infill plates with an average error of -7% and 15% respectively.

1. Introduction

The steel shear wall (SPW) system has been used in a number of buildings in Japan and North America as part of the lateral force-resisting system, in the past there decades (Astaneh-Asl, 2001). In earlier days, SPWs were treated like vertically oriented plate girders and design procedures tended to be overly conservative. Web buckling was prevented through extensive stiffening or by selecting an appropriately thick web plate, until more information became available on the post-buckling characteristics of web plates.

In today's designs, the SPW system is designed to buckle elastically, develop a tension field and finally yield under extreme loading (Kharrazi, 2005). To increase the elastic buckling capacity, the common practice is to increase the web thickness, or to use horizontal or/and vertical plate stiffeners. To analyze and design SPW systems, mainly two well-known analytical methods exist, which are: a). the strip model (Driver et al., 1994; Timler and Kulak, 1983), and b). the plate frame interaction (PFI) model (Sabouri- Ghomi et al., 2005, and Kharrazi, 2005).
In the strip model, the infill panel is modeled using strips that are only carrying axial tension load. The strip model is limited when it comes to wall configurations with opening or stiffeners

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(Rezai, 1999).On the other hand, the PFI model is a general method capable of modeling a wide range of SPW configurations. These configurations include walls with and without opening, with and without plate stiffeners, as well as walls with thin or thick infill plates (Sabouri-Ghomi and Roberts, 1991; Sabouri- Ghomi, 2001; Sabouri- homiet al., 2005; Kharrazi, 2005). The PFI model can determine shear force and displacement values that corresponded to pre- and post-critical buckling state, post-yield state and ultimate capacity of an individual panel.

In this paper, an analytical model is proposed to predict the behavior of the stiffened steel infill plate under pure shear load, using the PFI model. This analytical rational is an addendum to the PFI model established by Sabouri-Ghomi and Roberts, (1991); Sabouri-Ghomi, (2001); Sabouri-Ghomi et al., (2005); and further developed by Kharrazi, 2005. The effectiveness of this rational is then valuated by comparing its results to those obtained from experimental studies, and finite element (FE) analysis.

2. Experimental study

Takahashi et al. (1973) conducted a series of experimental and FE studies on stiffened thin plated SPWs in the early 1970s (Takahashi et al., 1973). They conducted quasi-static cyclic tests of 12 one-storey infill plate and 2 two-storey specimens in two phases. For the first phase of testing, 12 one-storey infill plate specimens were tested with overall width and height dimensions of 2100mm and 900mm, respectively, and with steel plate thicknesses of 4.5, 3.2 and 2.5mm. The parameters investigated were the spacing and width of stiffeners on both sides, or on one side of the steel panels together with the strength, hysteresis curve and post-buckling behavior of steel panels. The specimens were considered as quarter-scaled SPWs in comparison to typical building dimensions. All specimens had vertical or vertical and horizontal stiffeners welded on one or both sides of the steel plate with the exception of one specimen. The boundary frames were extremely stiff members so that any mid-height bending deformation is small enough to be considered negligible. These highly stiff frame members were connected using pin-hinges at both ends. All specimens were cyclic loaded along their diagonals with the aim of creating a relatively pure shear in the steel panels. The stiffener configurations of the tested SPW specimens are shown in Figures 1 to 3. Outcomes of the first test phase showed that all specimens were able to withstand large deformations and exhibited very stable and ductile behavior. It was reported that in some specimens the steel panels underwent global buckling, because the width of the transverse stiffeners (as a result their moment of inertia) were small. In other tests, local buckling was reported in the sub-panel, due to the relatively large spacing of associated stiffeners (with high moment of inertia). In addition, elastic buckling of the panel, as well as, plastic buckling were observed. Specimens with double-sided stiffeners indicated stability improvements better than those with one-sided stiffeners. For infill plates designed with stiffeners that have a relative high moment of inertia, the hysteresis diagram was observed to take the shape of a spindle. Figure 4(a) displays the hysteresis curve of specimens constructed of 2.3 mm thick steel plates without stiffeners; (specimen P-2.3). Figure 4(b) shows the hysteresis curve of the specimen with the same plate thickness but with horizontal and vertical stiffeners; (specimen P-2.3-M2-60).

The material specifications and dimensions of the tested samples are given in Table 1. In Table 1 each specimen is named based on their configuration and dimension. Each sample name consists of four parts: in the first part, P and PR stand for unstiffened and stiffened infill panel,

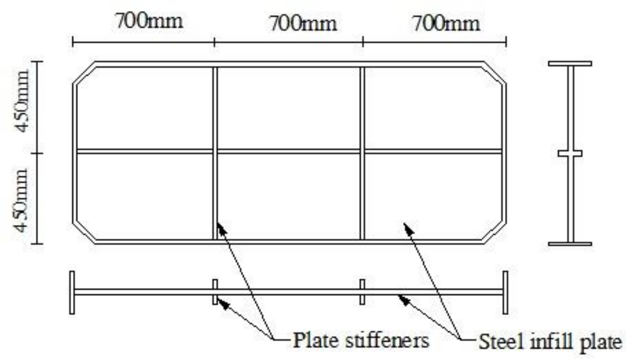


Figure 1: Specimen configuration M1

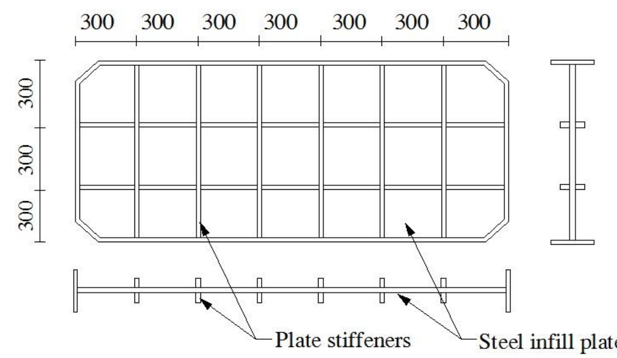


Figure2: Specimen configuration M2

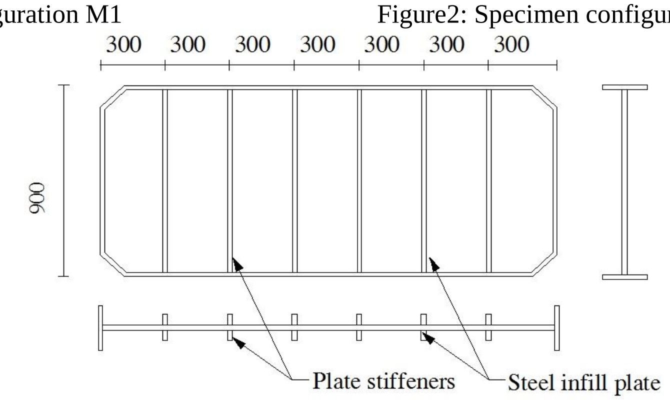


Figure 3: Specimen configuration G

respectively. The first set of numbers (second part of the naming convention) represents the thickness of the infill plate in millimeters, and the third part represents the type of configuration as shown in Figures 1 to 3. The last part indicates the width of the stiffeners. Only the first sample (P-2.3) is made of infill plate without stiffeners. From the experimental study, the P-2.3, PR-3.2-M2-15, PR-4.5-M1-15, and PR-4.5-G-10 specimens were observed to undergo elastic global buckling, which was related to the stiffeners being too slender (Takahashi et al., 1973). The sub-panels of the PR-2.3-M2-60 specimen were observed to undergo elastic local buckling and the resistance of the infill panel is found to be mostly because of the post-buckling - tension field action. The steel infill plates of the PR-4.5-M1-55, and PR-4.5-M1-35 specimens experience buckling as they reached their shear yield point. The rest of the specimens were observed to undergo plastic (local) buckling in their sub-panels. The backbone of the hysteresis obtained from the tests results are shown in Figure 5. These figures also contain results from analytical and numerical analyses done as part of this study, which will be explained next.

4. Finite element analysis

All specimens were modeled using FE method. Both, the specimens with double-sided stiffeners, as well as, the specimens with one-sided stiffeners were modeled. In the modeling of the specimens, SHELL S4R elements were used using the commercial available FE analysis software ABAQUS/Explicit version 5.7. SHELL S4R is suitable for analyzing thin to moderately-thick shell structures. It is a 4-node element with six degrees of freedom at each node: translations in the x, y, and z directions, and rotations about the x, y, and z-axes. SHELL S4R is well-suited for linear, large rotation, and/or large strain nonlinear applications. Change in shell thickness is accounted for in nonlinear analyses. To account for buckling deformation, the

Table 1: Specimen configurations, material specifications, test outcomes, as well as, numerical results after Takahashi et al. (1973)

Sample Name[1]	Plate thick t (mm)	Stiffener			Material Specification		Calculated Results				Maximum Measured Stresses (Test results) (N/mm^2)
		Area Cross Section (mm^2)	Configuration	Moment of Inertia (I) (mm^4)	Yield Stress (N/mm^2)	Ultimate Stress (N/mm^2)	Buckling Stress (N/mm^2)	Uni-Axial Yield Stress (N/mm^2)	Shear yield stress (N/mm^2)	Break (or Ultimate) Stress (N/mm^2)	
P-2.3	2.3	N/A	N/A	N/A	310	504	6	155	178	252	166
PR-2.3-M2-60	2.3	2.3 x 60	Double-sided M2	8.1e4	310	504	105	155	178	252	214
PR-3.2-M2-15	3.2	3..2 x 15	One-sided,M2	0.36e4	280	451	50	140	162	226	172
PR-3.2-M2-25	3.2	3.2 x 25	One-sided,M2	1.67e4	280	451	130	140	162	226	175
PR-3.2-M2-40	3.2	3..2 x 40	Double-sided M2	1.71e4	280	451	130	140	162	226	173
PR-3.2-M2-60	3.2	4..5 x 60	Double-sided M2	8.1e4	232	380	130	116	134	190	171
PR-4.5-M1-15	4.5	4..5 x 15	One-sided,M1	0.51e4	237	354	41	119	137	177	142
PR-4.5-M1-35	4.5	4..5 x 35	One-sided,M1	6.43e4	237	354	105	119	137	177	142
PR-4.5-M1-55	4.5	4..5 x 55	Double-sided M1	6.24e4	237	354	104	119	137	177	148
PR-4.5-G-10	4.5	4..5 x 10	One-sided,G	0.15e4	237	354	37	119	137	177	139
PR-4.5-G-30	4.5	4..5 x 30	One-sided,G	4.05e4	237	354	130	119	137	177	147
PR-4.5-G-50	4.5	4..5 x 50	Double-sided, G	4.67e4	237	354	130	119	137	177	151

[1] Each sample name consist of four parts: P and PR stands for un-stiffened, and stiffened infill panel, respectively, first set of number represent the thickness of the infill plate in mm, and third part represents the type of configuration as shown in Figure 4 and the last number indicates the width of the stiffeners.

solver including the large deformation was selected. Figure 6 shows a sample mesh of the modeled specimens. The results obtained from the finite element analysis are plotted in Figures 5 to 16 together with the backbone curve. In this figures, to simplify the backbone further as well as to make the comparison of these curvature easier, a bi-linear curve is plotted for each backbone curve. The stiffness and strength from the FE analysis are tabulated in Table 2.

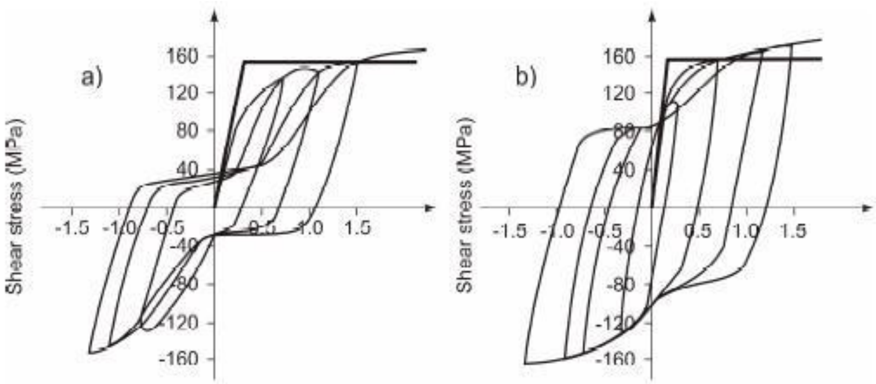
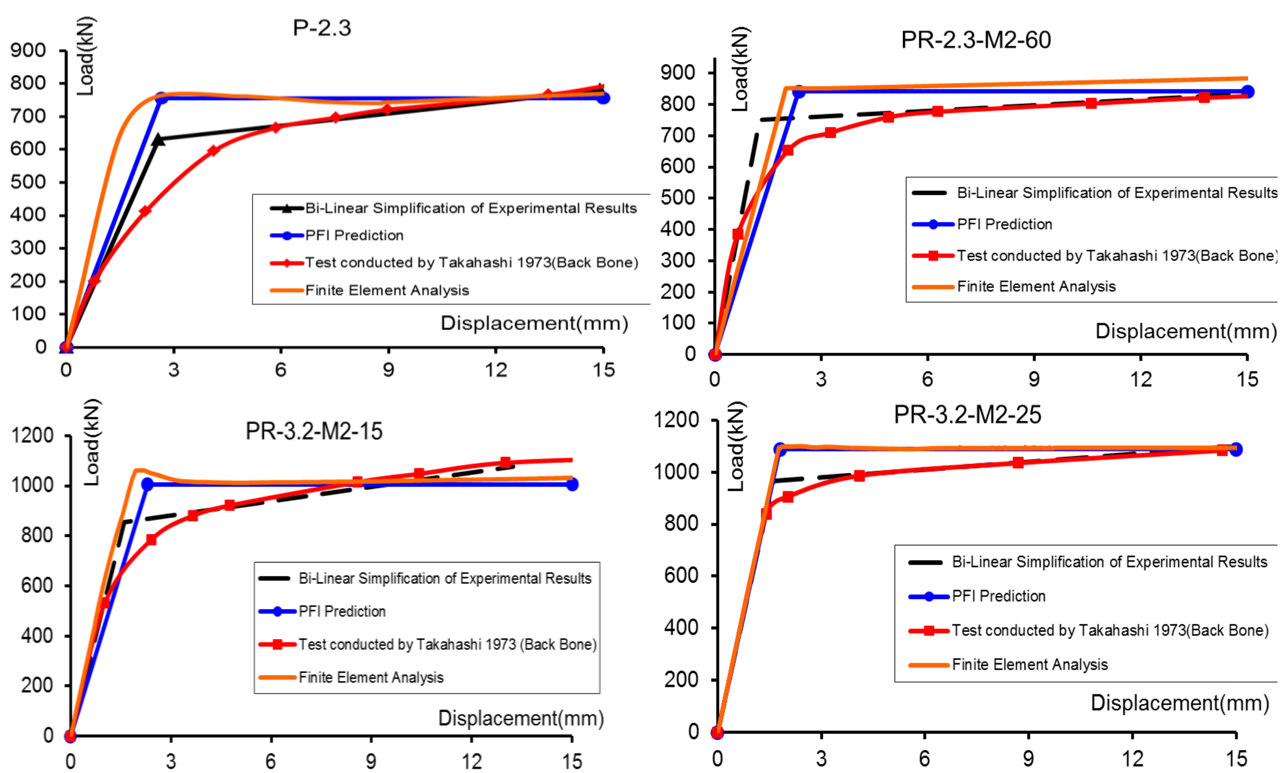


Figure 4: Hysteresis behavior of a) an unstiffened (P-2.3) and b) a heavily stiffened steel plate wall (PR-2.3-M2-60) (Takahashi et al., 1973)



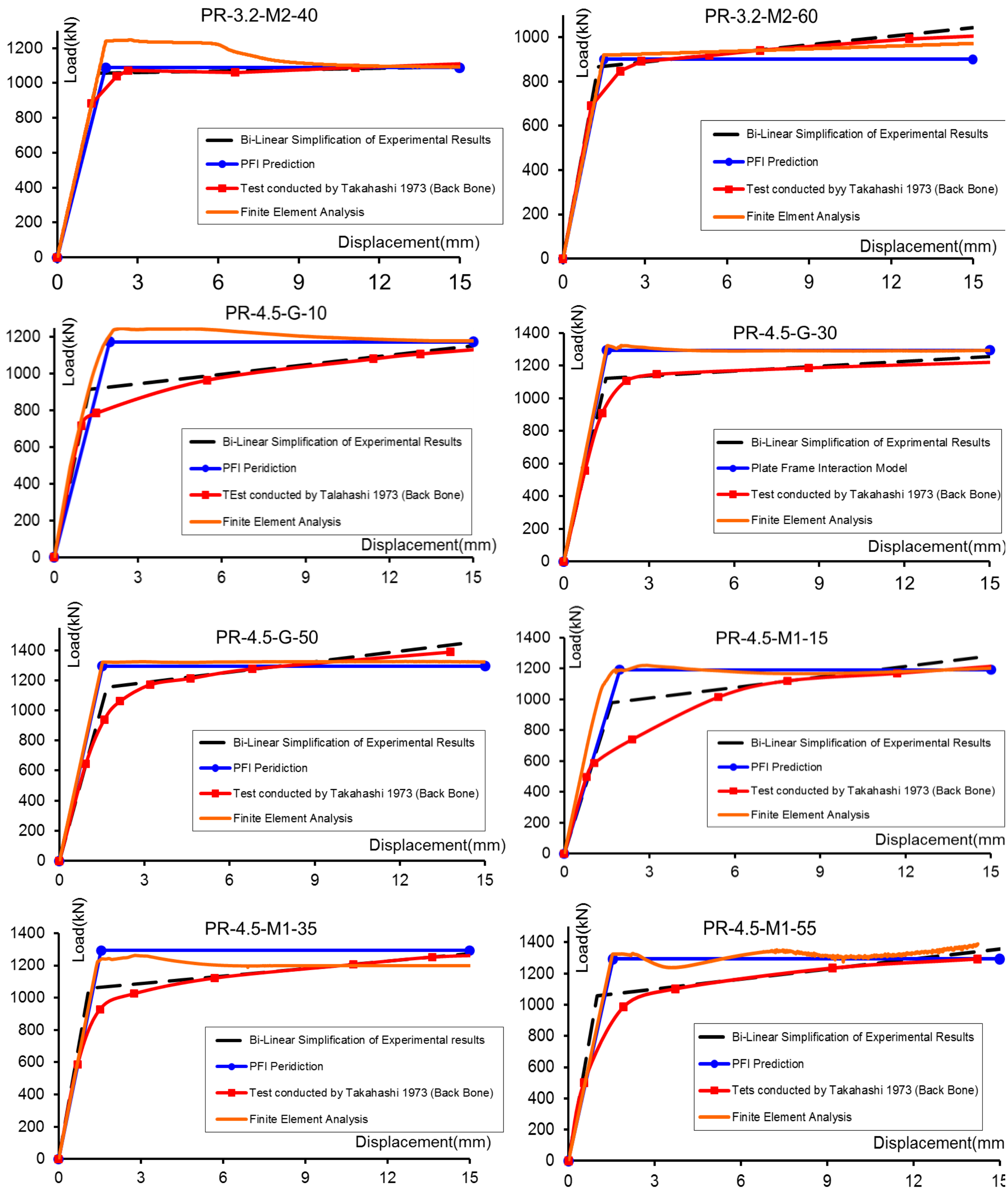


Figure 5: Shear Load Displacement for all specimens

Table 2: Stiffness and yield strength of the plates

Sample Name	Yield Strength (kN)			Yield Displacement (mm)			Stiffness (kN/mm)		
	Test	PFI	FEA (Totally Yielded)	Test	PFI	FEA (Start Point)	Test	PFI	FEA (Before any yield)
P-2.3	631	755.8	761.2	2.58	2.66	2.07	244.6	284.4	415.5
PR-2.3-M2-60	751	842.1	795.5	1.25	2.36	1.97	600.8	356.3	447.5
PR-3.2-M2-15	855	1005.5	1019	1.61	2.3	2.1	531.6	436.5	596.3
PR-3.2-M2-25	965	1086.3	1010	1.6	1.8	1.8	603.8	603.1	614.2
PR-3.2-M2-40	1069	1086.3	1012	1.53	1.8	1.74	698.7	603.1	641.5
PR-3.2-M2-60	866	900.1	847	1.27	1.49	1.51	681.9	603.1	636.8
PR-4.5-M1-15	980	1191.9	1162	1.49	1.96	1.67	657.7	609.1	835.8
PR-4.5-M1-35	1068	1292.9	1200	1.25	1.55	1.5	854.4	834.7	991.5
PR-4.5-M1-55	1116	1292.9	1180	1	1.55	1.5	1116	834.7	978.0
PR-4.5-G-10	916	1173.3	1178	1.25	1.98	1.4	732.8	592.2	819.1
PR-4.5-G-30	1124	1293.1	1195	1.48	1.52	1.58	759.5	848.1	817.9
PR-4.5-G-50	1156	1293.1	1202	1.67	1.52	1.61	692.2	848.1	830.4

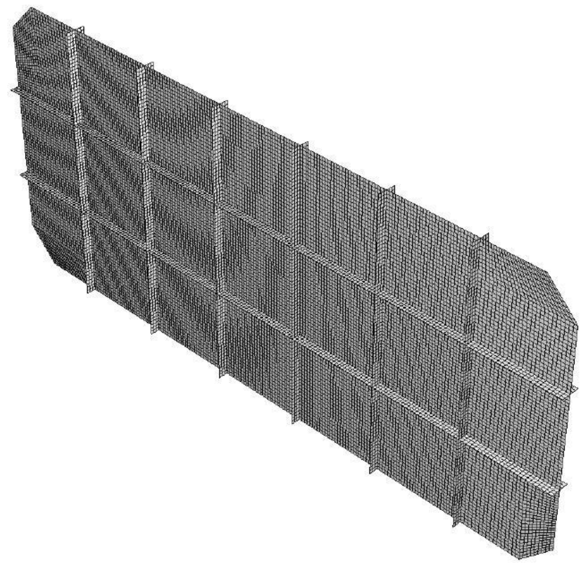


Figure 6: Snap shot of the meshed model (PR-2.3-M2-60)

5. Stiffened plate behavior prediction using PFI method

The stiffened steel plate infill panel under shear load could generally buckle in two modes: (1) global buckling mode, and (2) local buckling mode (Figure 7). Stiffeners are designed to force the buckling of the infill plate from a global buckling mode to a localized buckling in the sub-panels (i.e. local buckling mode). To achieve a local buckling mode, steel panels are typically stiffened using vertical and horizontal stiffeners. Global buckling occurs when stiffeners are too slender (or have low moment of inertia). As a result, the entire steel infill plate is going to globally buckle including the plate stiffeners (Figure 18). To predict the infill panels behavior, including the overall load-displacement, and the sub-panels buckling mode, an analytical model is proposed in here using the PFI method and the classical buckling theory (Sabouri-Ghomi et. al, 2007).

5.1 Global Buckling Mode

The critical shear buckling stress for the global buckling mode is obtained assuming the steel panel with orthotropic stiffness. That means the stiffened steel plate was considered with two different stiffness values in each global direction. The critical buckling shear stress for the global mode, τ_{crg}, is obtained from (Timoshenko, 1961, and Sayed-Ahmed, 2001):

$$\tau_{crg} = \left(D_x^{0.75} D_y^{0.25}\right) \frac{K_g \pi^2}{td^2} \leq \tau_{sy} \tag{1}$$

$$D_x = \frac{EI_x}{s_x} + \frac{Et^3}{12(1-\nu^2)} \tag{2}$$

$$D_y = \frac{EI_y}{s_y} + \frac{Et^3}{12(1-\nu^2)} \tag{3}$$

t is the thickness of the steel plate, as well as, I_x and I_y are the stiffeners moment of inertia around the X and Y axes, respectively. b and d are width and height of the infill plate, respectively. E is the module of elasticity, ν is the Poisson ratio, and s_x, s_y are the stiffeners spacing in X and Y directions, respectively (Figure 3). τ_{sy} stands for shear yield stress of the steel plate, which is $\tau_{sy} = \tau_y / \sqrt{3}$, and τ_y is the yield stress of the steel plate. K_g is the global buckling factor, (which is a function of Dx , Dy , b , d , as well as the steel plates boundary condition. The minimum value of K_g for plate to frame connection with pinned and rigid characteristics is 3.64 and 6.9, respectively (Sabouri-Ghomi et. al, 2007).

5.2 Local Buckling Mode

The elastic-critical shear buckling stress, τ_{crl} for local buckling of sub-panels (surrounded by the stiffeners) is obtained from the classical stability equation (Timoshenko, 1961):

$$\tau_{crl} = \frac{K_l \pi^2 E}{12(1-\nu^2)}\left(\frac{t}{s_x}\right)^2 \leq \tau_{sy} \tag{4}$$

where K_l is the local buckling factor obtained from:

$$K_l = 5.35 + 4\left(\frac{s_x}{s_y}\right)^2 \quad \text{for} \quad \frac{s_y}{s_x} \geq 1 \tag{5}$$

$$K_l = 5.35\left(\frac{s_x}{s_y}\right)^2 + 4 \quad \text{for} \quad \frac{s_y}{s_x} \leq 1 \tag{6}$$

In Equation 4, the stiffener-to-plate connections are considered pinned connections (i.e. hinged boundary condition).

5.3 Shear Load-Displacement

Once the overall static buckling behavior of the infill plate is determined, the post buckling shear load displacement can be calculated. To draw the shear load-displacement diagram, the limiting elastic shear displacement U_{we} , and the shear strength of the web plate, F_{wu} is needed to be determined. These values are obtained from the PFI method (Sabouri-Ghomi et al., 2005, and Kharrazi, 2005):

$$U_{we} = d\left(\frac{\tau_{cr}}{G} + \frac{2\sigma_{ty}}{E \sin 2\theta}\right) \tag{7}$$

$$F_{wu} = bt\left(\tau_{cr} + \frac{\sigma_{ty} \sin 2\theta}{2}\right) \tag{8}$$

where τ_{cr} is the critical buckling shear and is determined using the minimum value resulted from Equations 1 and 4, θ is the angle of inclination for the tension field (which is in this paper for simplicity considered 45 degrees), and the shear yield values, σ_{ty} is obtained from:

$$3\tau_{cr}^2 + 3\tau_{cr}\sigma_{ty}\sin 2\theta + \sigma_{ty}^2 - \sigma_y^2 = 0 \quad (9)$$

and the elastic module of shear, G , from:

$$G = \frac{E}{2(1+\nu)} \quad (10)$$

where σ_y is the yield stress of the steel plate.

Using these values the limiting elastic shear displacement, we U and the shear strength of the web plate, F_{wu} , are obtained for all the tested specimens and are tabulated in Table 3. These values are obtained using the module of elasticity, E , equal to 210 GPa and the Poisson ratio, ν , equal to 0.3. The tabulated results are plotted in Figures 5 to 16. In these figures, the hysteresis curve is represented by its backbone curve. Each figure includes not only the result from the PFI method but also the FE results.

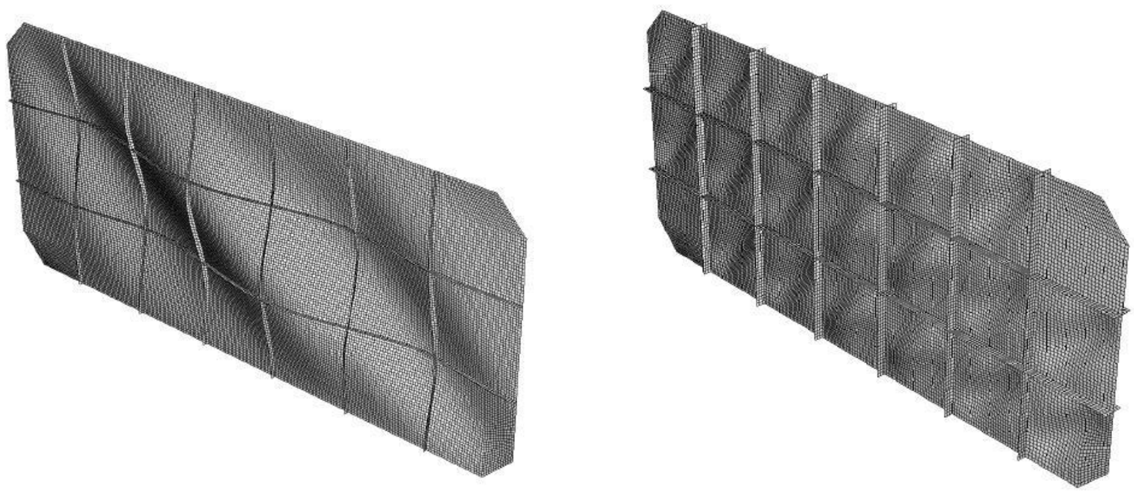


Figure 7: Buckling deformation of stiffened plate: global buckling mode (left), and local buckling mode (right)

6. PFI Evaluation using Experimental Study

To evaluate the proposed model, the results of this model are compared to those obtained from the test and the FE method. Primary aspects of this model such as stiffness, and yielding strength of the steel plate were the criteria used to assess the effectiveness of the proposed rational.

The error values for the stiffness, and for the yield strength of the infill panel is given in Table 4. This table compares FE result with test results (i.e. the bi-linear simplification), as well as, the results of the proposed analytical model with the test results (i.e. the bi-linear simplification). It also includes the comparison of FE outcomes with the results of the proposed analytical model. The model is capable to predict the stiffness with an average error of -7%, and the overall strength of the infill panel with an error of 15%. A negative percentage means that the value is under predicted. The FE method is capable to predict the stiffness with an average error of 9%, and the yield strength of the infill panel with an error of 10%. Comparing the outcomes of the analytical model and the FE model, an average error of -14% was found for predicting the

stiffness and an average error of 5% was determined for predicting the overall strength of the infill panel.
Given the simplicity of the rational method, it could be considered as an alternative approach, (when approximations needed quickly), to the existing numerical modeling techniques, and resulting in outcomes that could be used for design of stiffened SPW systems.

7. Conclusions

In this paper, an analytical model was proposed for stiffened infill panels using the PFI method. The model is capable to predict the buckling mode that is weather the infill panel has buckled in local buckling mode or global buckling mode. To derive this analytical model, the classical buckling relations were used. To evaluate the proposed model, the outcomes of FE analyses conducted for this study and previous experimental studies were utilized.
In this paper, the proposed analytical model, which is also used in the PFI method, was found to predict the SPW systems behavior with acceptable error. The model is capable to predict the stiffness with an average error of -7%, and the overall strength of the infill panel with an error of 15%. Given the simplicity, the analytical model is a very simple approach for analysis and design of this type of infill plates.

Table 3: Shear load-displacement calculation using the proposed method

Specimen	t (mm)	K_g	K_l	I_x (mm^4)	I_y (mm^4)	s_x (mm)	s_y (mm)	D_x $(N.mm)$	D_y $(N.mm)$	τ_{sy} (N/mm^2)	τ_{crg} (N/mm^2)	τ_{crl} (N/mm^2)	τ_{cr} (N/mm^2)	σ_{ty} (N/mm^2)	U_{we} (mm)	F_{wu} (kN)
P-2.3	2.3	5.06	N/A	N/A	N/A	N/A	N/A	N/A	N/A	179	6	N/A	5	302	2.64	753.4
PR-2.3-M2-60	2.3	5.06	9.35	8.1e4	8.1	300	300	570e5	570e5	179	178	104	104	140	2.36	842.1
PR-3.2-M2-15	3.2	5.06	9.35	0.36e4	0.36	300	300	31.5e5	31.5e5	162	59	161	44	212	2.26	1005.5
PR-3.2-M2-25	3.2	5.06	9.35	1.67e4	1.67	300	300	128.9e5	128.9e5	162	161	161	162	0	1.80	1086.3
PR-3.2-M2-40	3.2	5.06	9.35	1.71e4	1.71	300	300	131.7e5	131.7e5	162	161	161	162	0	1.80	1086.3
PR-3.2-M2-60	3.2	5.06	9.35	8.1e4	8.1	300	300	570e5	570e5	134	134	134	134	0	1.49	900.1
PR-4.5-M1-15	4.5	5.06	16.94	0.51e4	0.51	700	450	31e5	39.7e5	137	44	132	34	184	1.96	1191.9
PR-4.5-M1-35	4.5	5.06	16.94	6.43e4	6.43	700	450	211e5	318e5	137	132	137	133	8	1.55	1292.9
PR-4.5-M1-55	4.5	5.06	16.94	6.24e4	6.24	700	450	307e5	205e5	137	132	137	133	8	1.55	1292.9
PR-4.5-G-10	4.5	5.06	5.79	0.15e4	0.15	300	N/A	28e5	17.5e5	137	38	137	25	199	1.98	1173.3
PR-4.5-G-30	4.5	5.06	5.79	4.05e4	4.05	300	N/A	301e5	17.5e5	137	137	137	137	0	1.53	1293.1
PR-4.5-G-50	4.5	5.06	5.79	4.67e4	4.67	300	N/A	360e5	17.5e5	137	137	137	137	0	1.53	1293.1

Table 4: Error calculation for stiffness and strength of the infill plates

Specimen	Yield Strength			Yield Displacement			Stiffness		
	PFI | TEST	PFI | FEA	FEA | TEST	PFI | TEST	PFI | FEA	FEA | TEST	PFI | TEST	PFI | FEA	FEA | TEST
P-2.3	20%	-1%	21%	3%	28%	-20%	16%	-32%	70%
PR-2.3-M2-60	12%	6%	6%	89%	20%	58%	-41%	-20%	-26%
PR-3.2-M2-15	17%	-1%	19%	43%	10%	30%	-18%	-27%	12%
PR-3.2-M2-25	13%	8%	5%	13%	0%	13%	0%	-2%	2%
PR-3.2-M2-40	2%	7%	-5%	18%	4%	14%	-14%	-6%	-8%
PR-3.2-M2-60	4%	6%	-2%	18%	-1%	19%	-12%	-5%	-7%
PR-4.5-M1-15	22%	3%	19%	31%	17%	12%	-7%	-27%	27%
PR-4.5-M1-35	21%	8%	12%	24%	3%	20%	-2%	-16%	16%
PR-4.5-M1-55	16%	10%	6%	55%	3%	50%	-25%	-15%	-12%
PR-4.5-G-10	28%	0%	29%	58%	42%	12%	-19%	-28%	12%
PR-4.5-G-30	15%	8%	6%	3%	-3%	6%	12%	4%	8%
PR-4.5-G-50	12%	8%	4%	-9%	-5%	-4%	23%	2%	20%
Average (%)	15%	5%	10%	29%	10%	18%	-7%	-14%	9%
Standard Deviation (%)	7%	4%	10%	27%	14%	20%	17%	12%	23%

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