Proceedings of the
Annual Stability Conference
Structural Stability Research Council
St. Louis, Missouri, April 16-20, 2013

Mapping Web-Tapered Member to a Prismatic Member for Buckling Analysis of Sway Frames-Closed Form Equation

E.S. Salem[1]

Abstract
Web-tapered members are widely used among steel designers in industrial buildings for economic purposes. Nevertheless, buckling analysis of web-tapered members isn't as easy as prismatic ones. It takes up much time and effort since calculating the effective buckling length requires usage of charts other than the widely popular alignment chart for prismatic members. In addition, previous charts for tapered members have no mathematical expression, which do not lend themselves to computerized programming. In an effort to put an end to the above stumbling block, closed form equations are proposed in this study to cover most of the practical cases encountered by design engineers. Sway uninhibited frame buckles in a sway mode when it loses its lateral stiffness. Therefore, the lateral stiffness of sway uninhibited frame is the main controlling parameter in buckling analysis. To map a tapered member to a prismatic one, the two members have to contribute the same to lateral stiffness. Equating the contribution to lateral stiffness from the tapered member and its equivalent prismatic one results in an equivalent prismatic moment of inertia. The mapping between the two is through a simple closed form equation. This procedure has endless uses as it can be applied to a linearly web-tapered member with equal or unequal flanges, or tapered-constant-web member (i.e. part of the web member is tapered while the rest is prismatic) to cover all practical cases for sway frame structures encountered by steel designers.

1. Introduction
The critical buckling load of non-prismatic members attracts the attention of many researchers due to the wide spread of utilizing these members in many structural systems. Ermopoulos (1997, 1999) presented an analytical solution by solving the nonlinear equilibrium equations developed using slope deflection method. The result is presented in the form of charts or tables. Li (2000) Applied Bessel Function to solve the governing equation for critical buckling load. A power series approach for solving buckling differential equation with variable coefficients was proposed by Al-Sadder (2004). Simple and fast approximate procedures were proposed by Bazeos and Karabalis (2006) and Saffari et al. (2008) and presented in chart format. Numerical parametric study for finding a closed-form expression for buckling load of non-uniform members subjected to non-uniform axial load was proposed by Serna et al. (2011). Valipour and Bradford (2012) developed a systematic approach for deriving shape functions for tapered frame element

[1] Assistant Professor, Al-Azhar University, Cairo, Egypt <emadsalem4@yahoo.com>

using the principle of virtual force and concept of force interpolation. Most of the previous approaches are either cumbersome to be implemented by steel designers or incorporated in a computer design program.

In this research, a new method for converting tapered frame member to a prismatic one is proposed so that alignment chart method will be applicable to tapered frames. In order to even computerize the design of tapered steel frames, instead of using the alignment chart its nonlinear equation can be incorporated in the design program. For buckling analysis, it is crucial to determine the moment of inertia that should be used to calculate the critical buckling load. This calculation is always confusing, especially when the column is tapered. The new closed form equation will put an end to all the confusion as it accurately calculates the required equivalent moment of inertia in a simple direct formula. As a result, the constraint on using the alignment chart or its equation for web-tapered members in sway uninhibited frames is removed.

2. Proposed Procedures

Sway uninhibited frame buckles in a sway mode when it loses its lateral stiffness. Therefore, the lateral stiffness of sway uninhibited frame is the main controlling parameter in buckling analysis. To map a tapered member to a prismatic one, the two members have to contribute the same to lateral stiffness (and hence the lateral displacement due to a unit horizontal load). From this condition, an equivalent prismatic member can be produced. Only the flexural stiffness was considered in the calculations of the lateral displacement: the axial and shear stiffnesses were not considered. To perform the calculations of the lateral displacement, the moment of inertia variation along the tapered member must be known. The moment of inertia variation was selected based on that developed by SAP 2000 (2009) as shown in the verification manual as:

$$I(x)=\left[\sqrt{I_1}\left[1-\frac{x}{L}\right]+\sqrt{I_2}\left[\frac{x}{L}\right]\right]^2 \tag{1}$$

where I_1 and I_2 are the moments of inertia at small and large ends of the tapered member (Fig. 1), respectively, L is the member length, and $I(x)$ is the moment of inertia at distance x from the small end. Eq. 1 represents with sufficient accuracy the variation of moment of inertia along a tapered member with constant flange width and linearly varying depth. To simplify the calculation of the equivalent moment of inertia,

define: $a=\sqrt{I_2}$, $b=\sqrt{I_1}$, $c=\frac{b-a}{L}$

Therefore, Eq. 1 can be written as:

$$I(x)=[b-cx]^2 \tag{2}$$

The moment of inertia calculated using Eq. 2 is compared with the actual moment of inertia of the tapered member as shown in Fig. 3. The tapered member dimensions are given in Table 1, as shown in Fig. 3. It is clear that Eq. 2 represents, with acceptable accuracy, the moment of inertia variation along the tapered member.

2.1 Tapered Column of One Segment

To calculate the lateral stiffness of the frame shown in Fig. 2-a, a horizontal force, H, of a unit load is applied, as shown in Fig. 2-b, and the corresponding bending moment is shown in Fig. 2-c. To have the same contribution to lateral stiffness of both tapered column and its equivalent prismatic one, the lateral stiffness and hence the lateral displacement due to the unit load must be equal. As a result, Eq. 3 has to be satisfied for both columns shown in Fig. 3.

$$\frac{1}{EI_{eq}}\int_0^L [M(x)]^2\,dx = \frac{1}{E}\int_0^L \frac{[M(x)]^2}{I(x)}dx \tag{3}$$

where E is the modulus of elasticity, L is the column length, $M(x)$ is the bending moment at distance x from the small end, and I_{eq} is the moment of inertia of the equivalent prismatic column. From Fig. 3, the bending moment variation can be expressed as:

$$M(x) = \frac{M}{L}x \tag{4}$$

where M is the bending moment at the column top. Substituting Eqs. 2 and 4 into Eq. 3 and noting that M/L is constant give:

$$\frac{1}{I_{eq}}\int_0^L x^2 dx = \int_0^L \frac{x^2}{[b-cx]^2}dx \tag{5}$$

The integration and simplification of Eq. 5 result in:

$$I_{eq} = \frac{(b-a)^2}{3\left[1+\frac{b}{a}+\frac{2b}{(b-a)}Ln\left(\frac{a}{b}\right)\right]} \tag{6}$$

where Ln is the natural logarithm. Eq. 6 maps a tapered frame column of I-section (with constant width and varying depth) to a prismatic one with an equivalent moment of inertia, I_{eq}. Note that as constant a approaches b (for a prismatic member) in Eq. 6, I_{eq} becomes zero. However, by applying the limit to Eq. 6, it can be shown that I_{eq} approaches b^2.

Table 1: Dimensions and moment of inertia of the tapered column

Section	b_f (mm)	t_f (mm)	h_w (mm)	t_w (mm)	$I \times 10^4$ (mm^4)
1	250	10	490	8	189691
2	250	10	990	8	39098

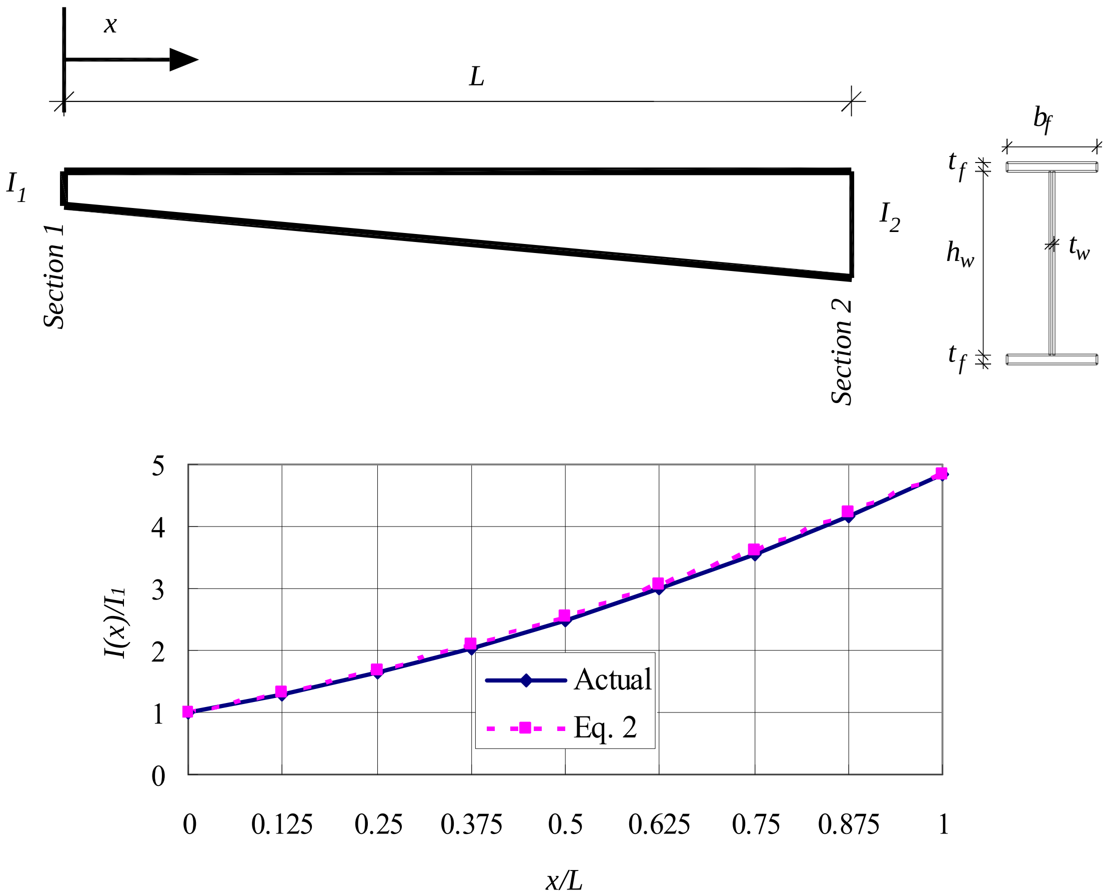


Figure 1: Comparison between actual moment of inertia and moment of inertia calculated using Eq.2

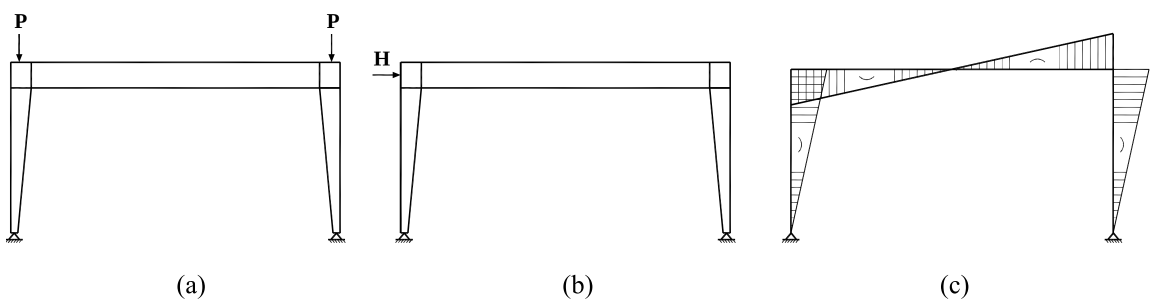
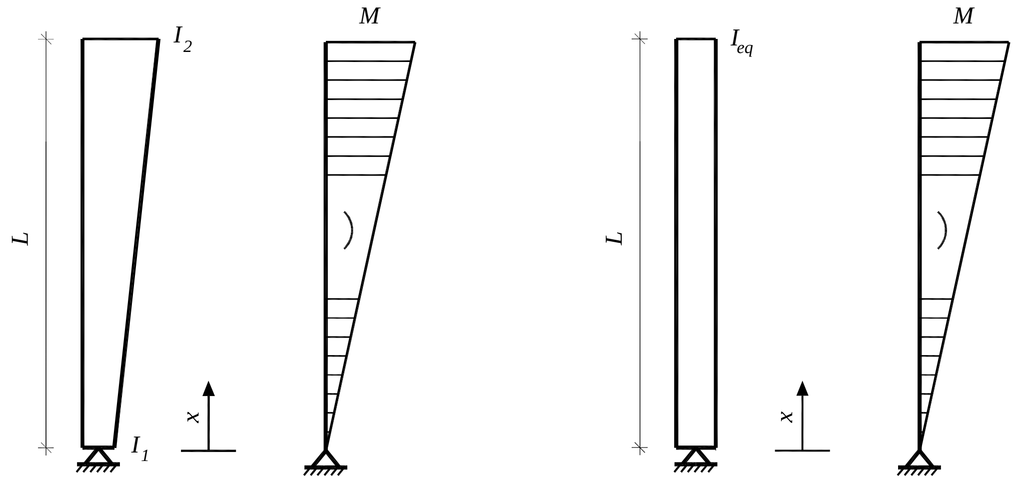


Figure 2: Tapered hinged base frame: (a) column load, (b) horizontal unit load, (c) corresponding bending moment

2.2 Tapered Column of Two Segments

For the case where the tapered column consists of two different segments (as shown in Fig. 4-a), each segment has different flanges and web thicknesses, the same procedure can be followed. Segment 1 of the column is mapped to a prismatic member using Eq. 6 with $a = \sqrt{I_{12}}$, and $b = \sqrt{I_{11}}$

where I_{11} and I_{12} are moments of inertia for segment 1 at small and large ends of the tapered segment, respectively.



(a) tapered column (b) equivalent prismatic column

Figure 3: Tapered and equivalent prismatic column in sway frame with corresponding bending moments

For segment 2, since the bending moment at the small end of the segment is not zero, Eq. 6 can not be applied to this segment. To simplify the calculations of equivalent moment of inertia for segment 2, the origin for the coordinates is moved to the bottom of the segment as shown in Fig. 4-b. The bending moment applied to this segment is separated into uniform and linearly varying moment as shown in Fig. 4-c.

For segment 2 to yield the same contribution to the lateral stiffness of the frame, Eq. 7 must be satisfied.

$$\frac{1}{EI_{eq2}}\left(\int_0^{L_2}[M(x)]^2\,dx\right)=\frac{1}{E}\left(\int_0^{L_2}\frac{[M(x)]^2}{I(x)}\,dx\right) \tag{7}$$

where I_{eq2} is the moment of inertia of the equivalent prismatic section for segment 2. Substituting Eq. 2 into Eq. 7 and noting that M/L is constant result in:

$$\frac{1}{I_{eq2}}\left(\int_0^{L_2}L_1^{\,2}dx+\int_0^{L_2}x^2dx\right)=\int_0^{L_2}\frac{L_1^{\,2}}{[b-cx]^2}\,dx+\int_0^{L_2}\frac{x^2}{[b-cx]^2}\,dx \tag{8}$$

After integration, simplification, and definition of $a=\frac{L_1}{L_2}$, Eq. 8 becomes:

$$I_{eq2}=\frac{a^2+\frac{1}{3}}{\frac{a^2}{ab}+\frac{1}{(b-a)^2}\left[1+\frac{b}{a}+\frac{2b}{(b-a)}Ln\left(\frac{a}{b}\right)\right]} \tag{9}$$

Note that Eq. 9 is transformed into Eq. 6 for $a=0$, which means that Eq. 9 can be used as a general equation for mapping any part of tapered column to a prismatic one when the tapered section varies linearly along the column. For using Eq. 9 as a general equation, a is the distance from zero moment to the small section of the segment under consideration divided by the segment length.

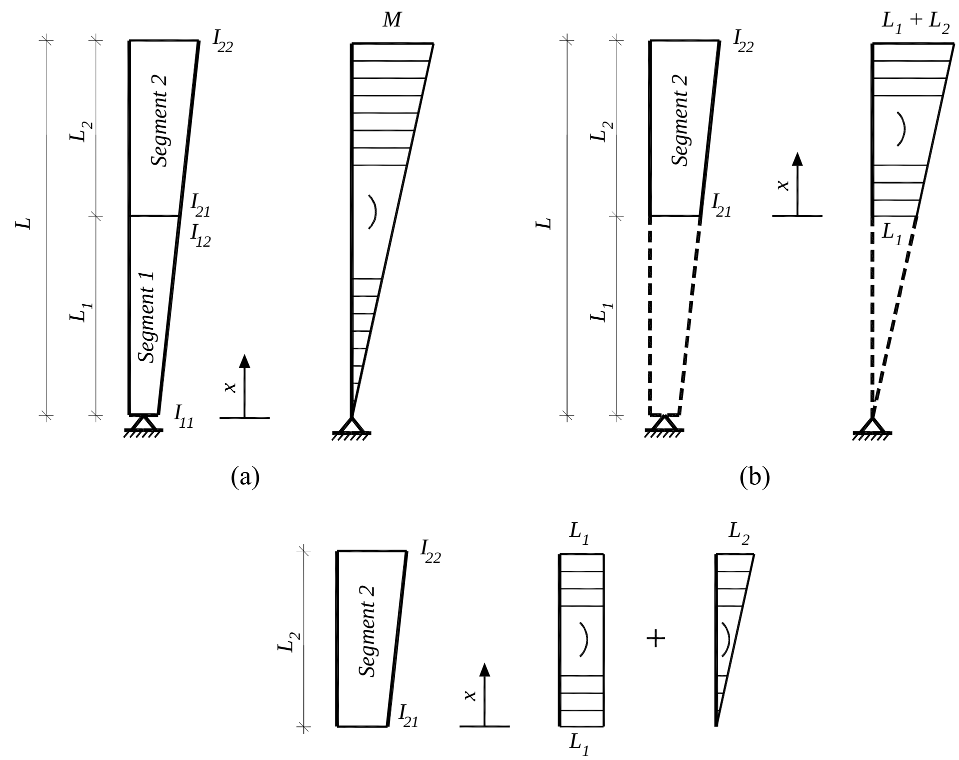


Figure 4: Tapered column of two segments

For the case where the tapered column consists of two different segments, each segment has different flanges and web thicknesses; Eq. 9 is applied for each segment separately to yield a prismatic section for each segment with equivalent moments of inertia I_{eq1} and I_{eq2} as shown in Fig. 5.

To convert the two prismatic sections with equivalent moments of inertia I_{eq1} and I_{eq2} to one prismatic section with an equivalent moment of inertia I_{eq}, follow the same procedures.

$$\frac{1}{EI_{eq}}\left(\int_0^L [M(x)]^2\,dx\right) = \frac{1}{EI_{eq_1}}\int_0^{L_1}[M(x)]^2\,dx + \frac{1}{EI_{eq_2}}\int_{L_1}^{L}[M(x)]^2\,dx \tag{10}$$

$$\frac{1}{I_{eq}}\int_0^L x^2\,dx = \frac{1}{EI_{eq_1}}\int_0^{L_1} x^2\,dx + \frac{1}{EI_{eq_2}}\int_{L_1}^{L} x^2\,dx \tag{11}$$

After integration, simplification, and definition of $h = \dfrac{I_{eq_1}}{I_{eq_2}}$, Eq. 11 gives:

$$I_{eq} = \frac{I_{eq_1}}{h + (1-h)\left(\frac{L_1}{L_1 + L_2}\right)^3} \quad (12)$$

Eq. 12 converts the two equivalent prismatic sections for segment 1 and 2 to one equivalent prismatic section.

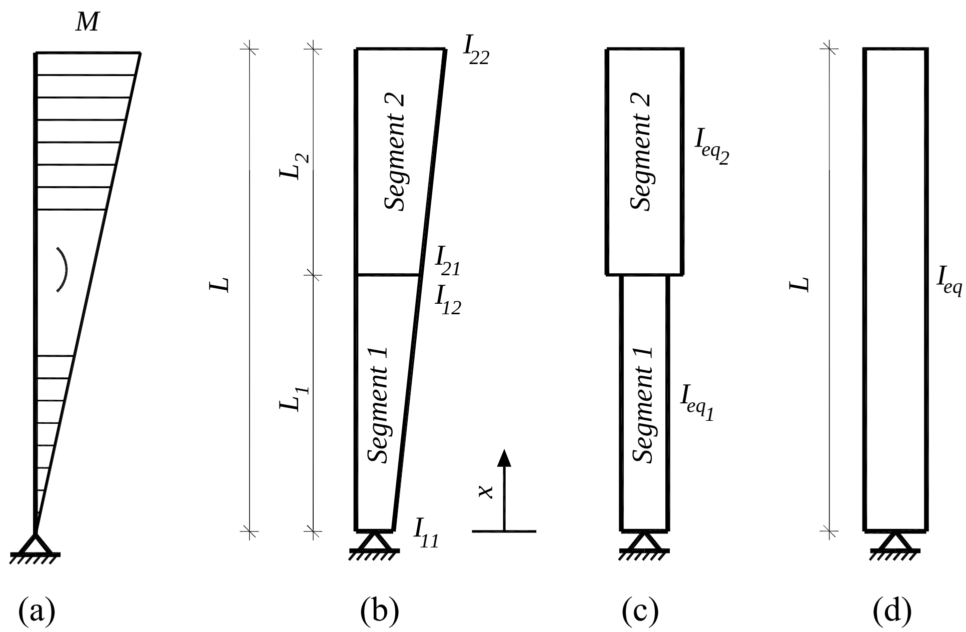


Figure 5: Converting a two segment tapered column to an equivalent prismatic column

For a column shown in Fig. 6 with two different segments where the first segment is prismatic and the second one is tapered, $I_{eq1} = I_1$ while I_{eq2} is calculated using Eq. 9. Applying Eq. 12 yields an equivalent moment of inertia, I_{eq}, for the column.

2.3 Tapered Column of Three Segments

For the case where the tapered column consists of three different segments (as shown in Fig. 7-a), the same procedures can be followed. Segments 1 and 2 of the column are mapped to prismatic members using Eq. 9 with $a_i = \sqrt{I_{i2}}$, $b_i = \sqrt{I_{i1}}$, where a_i is the distance from zero moment to the small section of segment i divided by segment length and I_{ij} = moment of inertia of segment i at end j, and j=1,2 represents small and large ends of the tapered segment, respectively.

After calculating an equivalent moment of inertia for each segment, Eq. 12 is applied for segments 1 and 2 to yield an equivalent moment of inertia, $I_{eq1\text{-}2}$, for segment 1-2. Eq. 12 is then applied again for segment 1-2 and segment 3 to yield an equivalent moment of inertia for the three segments, I_{eq}.

These procedures can be applied to a tapered member having any number of segments where the web height is linearly varying. For a tapered member with nonlinear variation in web height, the column can be divided into a number of segments, where each segment can be approximated by a linear variation in web height and hence the same procedures can be applied.

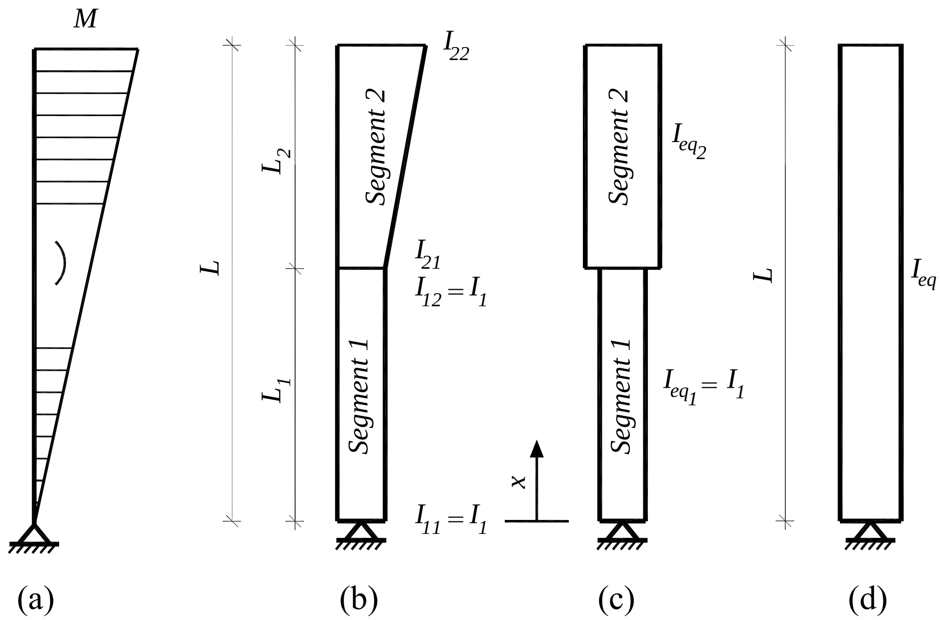


Figure 6: Converting a constant-tapered column to an equivalent prismatic column

3. Elastic Critical Buckling Load of Tapered Frame

Having converted the tapered member to a prismatic one for columns and rafters, the elastic critical buckling load, P_{cr}, can be determined using the effective length method (AISC 2010). With the procedures developed in this study, the constraint on using the alignment chart or its closed form equation in calculating the critical buckling load of sidesway uninhibited tapered frames is removed.

To calculate P_{cr} using the proposed procedures, a tapered member is mapped to a prismatic one using Eq. 9. For cases where the tapered member consists of two segments, Eq. 9 is applied for each segment and then Eq. 12 is applied to yield an equivalent prismatic section for the two segments. For cases where columns or rafters have tapered segments and prismatic segments, Eq. 9 is applied only for tapered segments. The effective buckling length factor, K, is determined from the alignment chart base equation for sidesway uninhibited frames, AISC (2010):

$$\frac{G_A G_B (p/K)^2 - 36}{6(G_A + G_B)} - \frac{(p/K)}{tan(p/K)} = 0 \tag{13}$$

where

$$G = \frac{\sum \left(E_c I_{eq_c} / L_c\right)}{\sum \left(E_g I_{eq_g} / L_g\right)} \tag{14}$$

where E_c is the elastic modulus of the column, I_{eq_c} is the equivalent moment of inertia of the column, L_c is the unsupported length of the column, E_g is the elastic modulus of the girder, I_{eq_g}

is the equivalent moment of inertia of the girder, and L_g is the unsupported length of the girder. The elastic critical buckling load is then calculated using

$$P_{cr} = \frac{\pi^2 E I_{eq_c}}{(K L_c)^2} \tag{15}$$

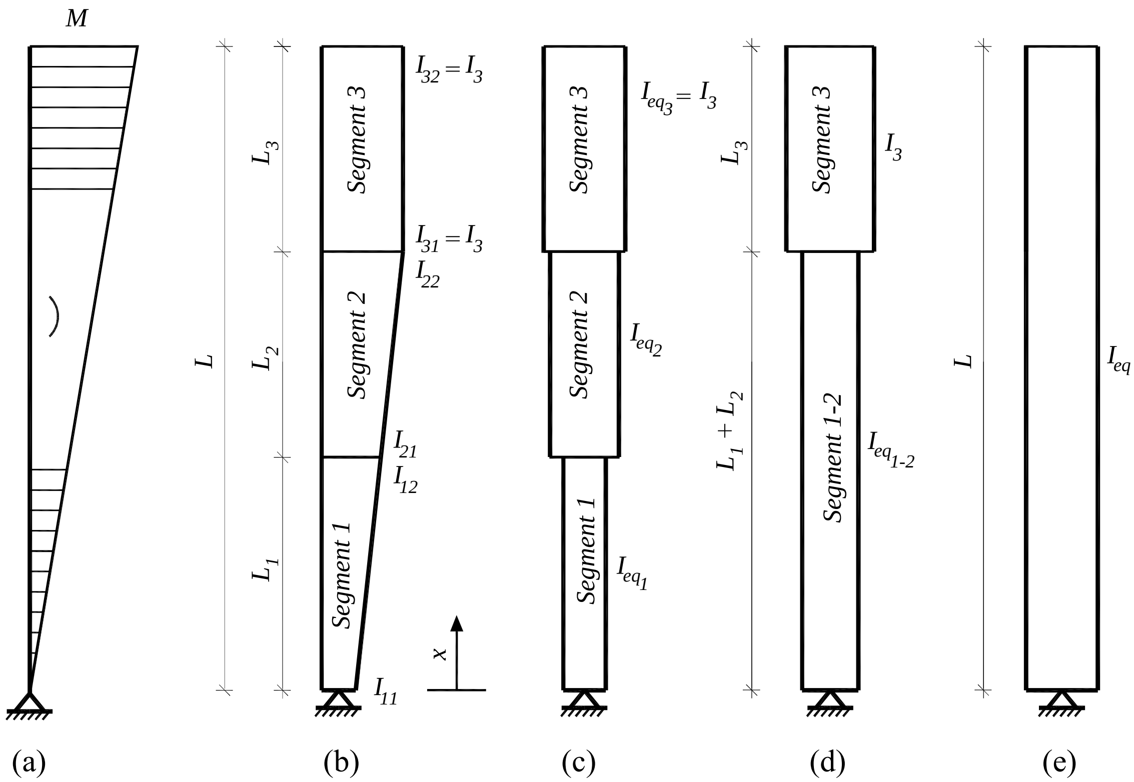


Figure 7: Converting a tapered-constant column to an equivalent prismatic column

3.1 Verifications

To verify the accuracy of the proposed procedures in calculating the critical buckling load of a tapered frame using effective length method, two general purpose finite element analysis programs were used: ABAQUS (2010) and SAP 2000 (2009). Only bending stiffness was considered in the analyses: the axial and shear stiffnesses were not considered. The results from the finite element analyses were compared with the results from the present study.

3.1.1 Example 1

A hinged base frame with tapered columns and prismatic rafter with a main span of 24m and column height of 8m is shown in Fig. 8. The tapered column has a linearly varying web height while each flange has a constant width and thickness. The geometry of different sections shown in Fig. 8 is given in Table 2. The steel modulus of elasticity is taken E=200GPa. The tapered column equivalent moment of inertia obtained from the present study is I_{eq}=1569 x 10^6 mm^4. This value is calculated using Eq. 6, or Eq. 9 with $\alpha = 0$, from the data given in Table 3. The effective buckling length factor, K, is calculated using Eq. 13. In Eq. 13, G_B is taken equal to infinity to represent a true hinge. G_A is calculated using I_{eq_c} =1569 x 10^6 mm^4, L_c=8 m, I_{eq_g} =2203 x 10^6 mm^4, and L_g=24 m. Solving Eq. 13 yields K=2.676. The critical buckling load, P_{cr}=6768 kN, is calculated using Eq. 15

For the analysis using SAP 2000, tapered element was used with parabolic variation for moment of inertia. Since tapered element is not available in ABAQUS element library, the tapered column was divided into 8 segments and the moment of inertia for each segment was calculated using the section at the middle of the segment. The critical buckling load, P_{cr}, calculated using ABAQUS and SAP 2000 is compared with that calculated using the present study in Table 4. The unconservative error in calculating P_{cr} using the present study is 2.39%.

3.1.2 Example 2

The same frame as in example 1 except using a small prismatic rafter section as given in Table 5. The steel modulus of elasticity is taken E=200 GPa. The tapered column has the same equivalent moment of inertia as example1. The effective buckling length factor, K, is calculated using Eq. 13. In Eq. 13, G_B is taken equal to infinity to represent a true hinge. G_A is calculated using I_{eq_c} =1569 x 10^6 mm^4, L_c=8 m, I_{eq_g} =4766 x 10^5 mm^4, and L_g=24 m. Solving Eq. 13 yields K=4.434. The critical buckling load, P_{cr}=2464 kN, is calculated using Eq. 15

The critical buckling load, P_{cr}, calculated using ABAQUS and SAP 2000 is compared with that calculated using the present study in Table 6. The unconservative error in calculating P_{cr} using the present study is 0.29%.

3.1.3 Example 3

A hinged base frame with a main span of 36m and a column height of 10m is shown in Fig. 9. The column consists of two tapered segments while the rafter consists of a prismatic segment and a tapered one. Each tapered segment, for the columns and rafters, has a linearly varying web height while each flange has a constant width and thickness. The geometry of different sections shown in Fig. 9 is given in Table 7. The steel modulus of elasticity is taken E=200GPa. Each tapered segment of the column and rafter is mapped to an equivalent prismatic section using Eq. 9 and the resulting equivalent moment of inertia is given in Table 8. The two prismatic sections of the column and rafter are then mapped to one prismatic section using Eq. 12. The resulting equivalent moments of inertia for column and rafter are 3849 x 10^6 mm^4 and 2117 x 10^6 mm^4, respectively, as given in Table 9. The effective buckling length factor, K, is calculated using Eq. 13. In Eq. 13, G_B is taken equal to infinity to represent a true hinge. G_A is calculated using I_{eq_c} =3849 x 10^6 mm^4, L_c=10 m, I_{eq_g} =2117 x 10^6 mm^4, and L_g=2 x (12.06+6.03)=36.18 m. Solving Eq. 13 yields K=3.78. The critical buckling load, P_{cr}=5323 kN, is calculated using Eq. 15. The critical buckling load, P_{cr}, calculated using SAP 2000 is compared with that calculated using the present study in Table 10. The unconservative error in calculating P_{cr} using the present study is 2.9%.

3.1.4 Example 4

A hinged base frame with a main span of 36m and a column height of 12m is shown in Fig. 10. The column consists of two tapered segments and one prismatic segment. The rafter consists of two tapered segments. Each tapered segment, for the columns and rafters, has a linearly varying web height while each flange has a constant width. The first tapered segment of the column has an equal flange thickness while the remaining two segments have unequal flange thicknesses. The two rafter segments have unequal flange thicknesses. The geometry of different sections shown in Fig. 10 is given in Table 11. The steel modulus of elasticity is taken E=200GPa. Each tapered segment of the column and rafter is mapped to an equivalent prismatic section using Eq. 9 and the resulting equivalent moment of inertia is given in Table 12. Since the column has three different segments, the first two segments 1C and 2C are mapped to one equivalent prismatic section 1C-2C using Eq. 12. Segments 1C-2C and C3 are then mapped to one equivalent prismatic section (1C-2C)-C3 using Eq. 12 to yield an equivalent prismatic column section. The two equivalent prismatic sections of rafter are mapped to one equivalent prismatic section using Eq. 12. The resulting equivalent moments of inertia for column and rafter are 2351 x 10^6 mm^4 and 1423 x 10^6 mm^4, respectively, as given in Table 13. The effective buckling length factor, K, is calculated using Eq. 13. In Eq. 13, G_B is taken equal to infinity to represent a true hinge. G_A is calculated using I_{eq_c} =2351 x 10^6 mm^4, L_c=12 m, I_{eq_g} =1423 x 10^6 mm^4, and L_g=2 x (12.06+6.03)=36.18 m. Solving Eq. 13 yields K=3.42. The critical buckling load, P_{cr}=2757 kN, is calculated using Eq. 15. The critical buckling load, P_{cr}, calculated using SAP 2000 is compared with the present study in Table 14. The unconservative error in calculating P_{cr} using the present study is 2.7%.

4. Inelastic Buckling Load of Tapered Column

The inelastic buckling load of a tapered column in a sway frame can be calculated following the same concept developed by Kaehler et al. (2011) in the AISC design guide for frame design using web-tapered members. The ratio of the elastic critical buckling load, P_{cr}, (calculated using the present study) to the required axial stress of the column, P_r, defines the scalar ratio, g_e.

$$g_e = \frac{P_{cr}}{P_r} \tag{16}$$

For a tapered column where the in-plane flexural buckling is the governing mode, the critical buckling strength of the tapered column can be calculated using the equations developed by Kaehler et al. (2011):

When $\frac{QF_y}{g_e f_r} \leq 2.25$

$$F_{cr} = \left[0.658^{\frac{QF_y}{g_e f_r}} \right] QF_y \tag{17}$$

When $\frac{QF_y}{g_e f_r} > 2.25$

$$F_{cr} = 0.877 g_e f_r \tag{18}$$

where F_{cr} is the nominal buckling strength, Q is the slenderness reduction factor, and f_r is the required axial stress calculated at the location of the smallest area along the column, and F_y is the yield strength.

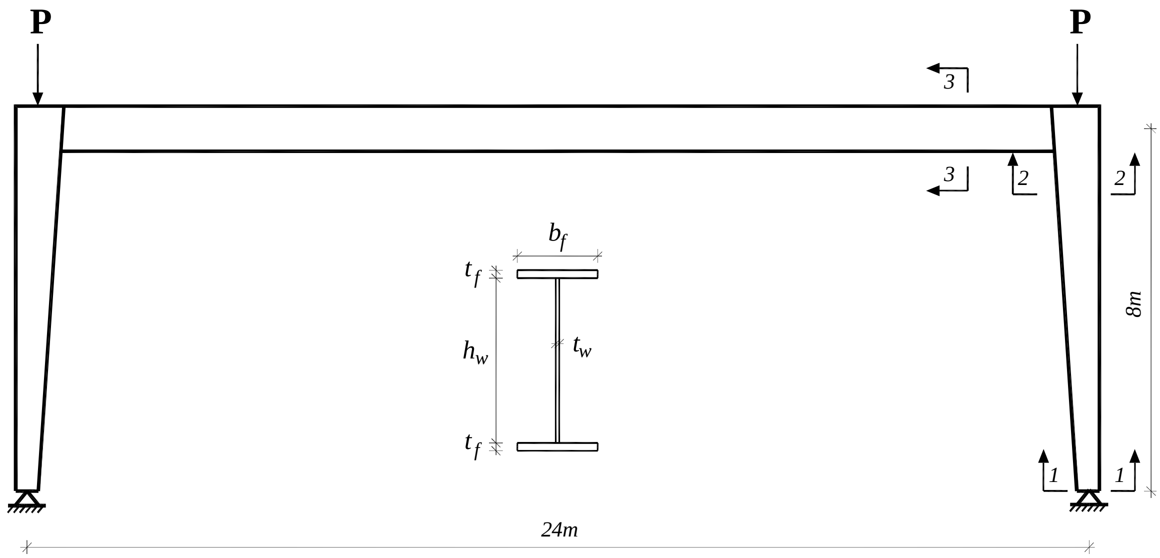


Figure 8: Frame geometry and loads (Examples 1 and 2)

Table 2: Cross section dimensions and properties (Example 1)

Section	b_f (mm)	t_f (mm)	h_w (mm)	t_w (mm)	$I \times 10^4$ (mm^4)
1	250	12	500	8	47660
2	250	12	1000	8	220300
3	250	12	1000	8	220300

Table 3: Equivalent moment of inertia (Example 1)

Segment	$I_1 \times 10^4$ (mm^4)	$I_2 \times 10^4$ (mm^4)	$a \times 10^2$ (mm^2)	$b \times 10^2$ (mm^2)	$I_{eq} \times 10^4$ (mm^4)	Comments
1	47660	220300	469.36	218.32	156900	Use Eq. 6

Table 4: Critical buckling load comparison (Example 1)

	SAP	ABAQUS	Present Study
Buckling load P_{cr} (kN)	6610	6547	6768
Difference %	0.00	-0.95%	+2.39%

Table 5: Cross section dimensions and properties (Example 2)

Section	b_f (mm)	t_f (mm)	h_w (mm)	t_w (mm)	$I \times 10^4$ (mm^4)
1	250	12	500	8	47660
2	250	12	1000	8	220300
3	250	12	500	8	47660

Table 6: Critical buckling load comparison (Example 2)

	SAP	ABAQUS	Present study
Buckling load P_{cr} (kN)	2457	2447	2464
Difference %	0.00	-0.41%	+0.29%

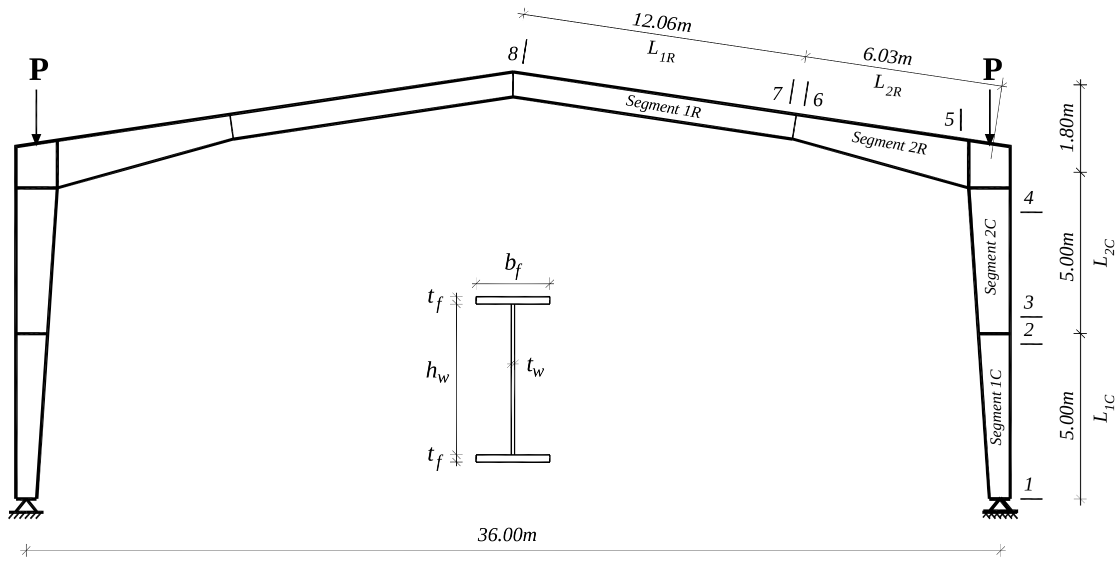


Figure 9: Frame geometry and loads (Example 3)

Table 7: Cross section dimensions and properties (Example 3)

Section	b_f (mm)	t_f (mm)	h_w (mm)	t_w (mm)	$I \times 10^4$ (mm^4)
1	330	16	400	10	51040
2	330	16	804	10	220800
3	330	20	796	12	270200
4	330	20	1200	12	664000
5	330	16	1168	12	529500
6	330	16	768	12	207600
7	200	12	776	8	105700
8	200	12	776	8	105700

Table 8: Equivalent moment of inertia for each segment (Example 3)

Segment	$I_1 \times 10^4$ (mm^4)	$I_2 \times 10^4$ (mm^4)	$a \times 10^2$ (mm^2)	$b \times 10^2$ (mm^2)	L_1 (mm)	L_2 (mm)	$I_{eq} \times 10^4$ (mm^4)	Comments
1C	51040	220800	469.94	225.93	0	5000	159400	Use Eq. 9
2C	270200	664000	814.87	519.82	5000	5000	482400	Use Eq. 9
1R	105700	105700		(Prismatic section)			105700	
2R	207600	529500	727.64	455.62	12060	6030	366300	Use Eq. 9

Table 9: Equivalent moment of inertia for columns and rafters (Example 3)

Segment	$I_{eq1} \times 10^4$ (mm^4)	$I_{eq2} \times 10^4$ (mm^4)	L_1 (mm)	L_2 (mm)	$I_{eq} \times 10^4$ (mm^4)	Comments
1C-2C	159400	482400	5000	5000	384900	Use Eq. 12
1R-2R	105700	366300	12060	6030	211700	Use Eq. 12

Table 10: Critical buckling load comparison (Example 3)

	SAP	Present study
Buckling load P_{cr} (kN)	5176	5323
Difference %	0.00	+2.9%

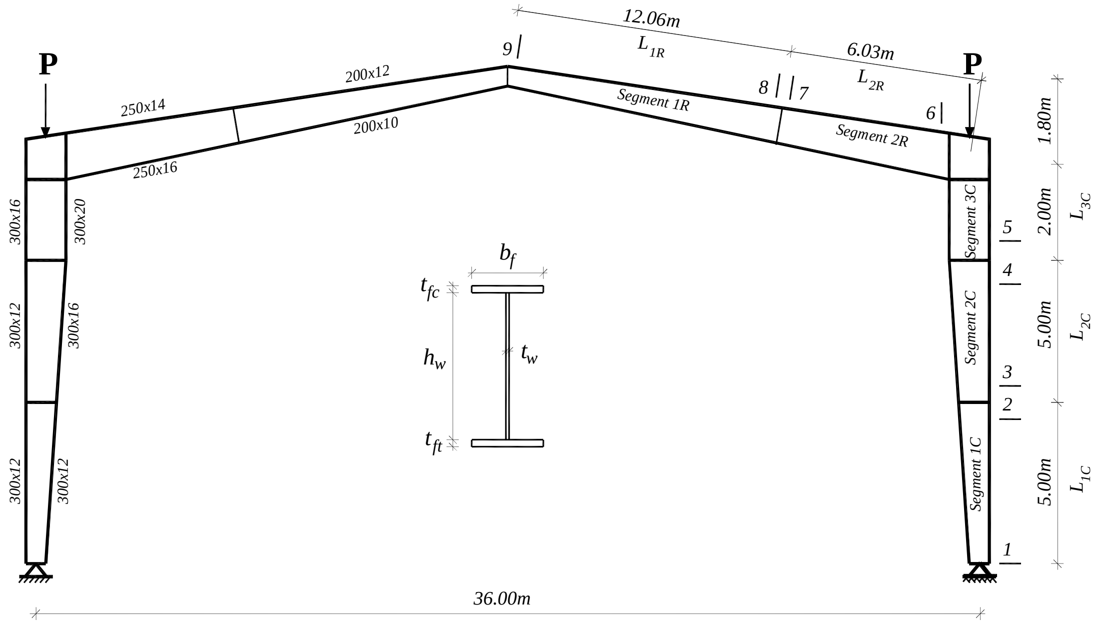


Figure 10: Frame geometry and loads (Example 4)

Table 11: Cross section dimensions and properties (Example 4)

Section	b_f (mm)	t_{fc} (mm)	t_{ft} (mm)	h_w (mm)	t_w (mm)	$I \times 10^4$ (mm^4)
1	300	12	12	400	8	34830
2	300	12	12	700	8	114100
3	300	16	12	700	10	134500
4	300	16	12	1000	10	297300
5	300	20	16	1000	10	361400
6	250	16	14	1000	10	276200
7	250	16	14	800	10	167000
8	200	12	10	800	8	106300
9	200	12	10	400	8	22770

Table 12: Equivalent moment of inertia for each segment (Example 4)

Segment	$I_1 \times 10^4$ (mm^4)	$I_2 \times 10^4$ (mm^4)	$a \times 10^2$ (mm^2)	$b \times 10^2$ (mm^2)	L_1 (mm)	L_2 (mm)	$I_{eq} \times 10^4$ (mm^4)	Comments
1C	34830	114100	337.82	186.63	0	5000	87080	Use Eq. 9
2C	134500	297300	545.27	366.75	5000	5000	224300	Use Eq. 9
3C	361400	361400					361400	
1R	22770	106300	325.97	150.89	0	12060	75550	Use Eq. 9
2R	167000	276200	525.51	408.62	12060	6030	226600	Use Eq. 9

Table 13: Equivalent moment of inertia for columns and rafters (Example 4)

Segment	$I_{eq1} \times 10^4$ (mm^4)	$I_{eq2} \times 10^4$ (mm^4)	L_1 (mm)	L_2 (mm)	L_3 (mm)	$I_{eq} \times 10^4$ (mm^4)	Comments
1C-2C	87080	224300	5000	5000	-	187400	Use Eq. 12
(1C-2C)-3C	187400	361400	$L_1 + L_2$=10000		2000	235100	Use Eq. 12
1R-2R	75550	226600	12060	6030	-	142300	Use Eq. 12

Table 14: Critical buckling load comparison (Example 4)

	SAP	Present study
Buckling load P_{cr} (kN)	2686	2757
Difference %	0.00	+2.7%

5. Conclusions

The new closed form equations developed in this study for mapping web tapered member to a prismatic one simplify the buckling load calculations for tapered sway frames. Sway uninhibited frame buckles in a sway mode when it loses its lateral stiffness. Therefore, the lateral stiffness of sway uninhibited frame is the main controlling parameter in buckling analysis. To map a tapered member to a prismatic one, the two members have to contribute the same to lateral stiffness. From this condition, an equivalent prismatic member can be produced. Most of practical configurations of tapered frames encountered by steel designers are covered. The developed procedures eliminate the need for using charts other than the widely accepted alignment chart which has a closed form equation. As a result, calculation of buckling load for tapered sway frames can be easily incorporated into design programs. The error in calculating the critical buckling load using the closed form equations developed in this study is less than 3%, which is acceptable for practical design.

References

ABAQUS, V.6.10. (2010). Dassalut Systems/Simulia, Providence, RI, USA.

AISC (2010). Specifications for structural steel buildings, ANSI/AISC 360-10, American Institute of Steel Construction, Chicago, IL.

Al-Sadder, Z.S. (2004). “Exact expressions for stability functions of a general non-prismatic beam-column member.” *J. Constr. Steel Res.*, 60 (11) 1561-84.

Bazeos, N., Karabalis, D.L. (2006). “Efficient computation of buckling loads for plane steel frames with tapered members.” *Engrg. Struct.*, 28 (5) 771-5.

Ermopoulos, J.Ch. (1997). “Equivalent buckling length of non-uniform members.” *J. Constr. Steel Res.*, 42 (2) 141-58.

Ermopoulos, J.Ch. (1999). “Buckling length of non-uniform members under stepped axial loads.” *Comput. Struct.,* 73 (4) 573-82.

Kaehler, R.C., White, D.W., Kim, Y.D., (2011). Frame design using web-tapered members, American Institute of Steel Construction, Chicago, IL.

Li, Q.S., (2000). “Buckling of elastically restrained non-uniform columns.” *Engrg. Struct.*, 22 (10) 1231-43.

Saffari, H., Rahgozar, R., Jahanshahi, R. (2008). “An efficient method for computation of effective length factor of columns in steel gabled frame with tapered members.” *J. Constr. Steel Res.*, 64 (4) 400-6.

SAP 2000, V.14. (2009). Computers and Structures, Inc., Berkeley, CA, USA.

Serna, M.A., Ibanez, J.R., Lopez, A. (2011). “Elastic flexural buckling of non-uniform members: closed-form expression and equivalent load approach.” *J. Constr. Steel Res.*, 67 (9) 1078-85.

Valipour, H.R., Bradford, M.A. (2012). “A new shape function for tapered three-dimensional beams with flexible connections.” *J. Constr. Steel Res.*, 70 (1) 43-50.