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Numerical and experimental investigation on the post-buckling behavior of steel plate girders subjected to shear

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Abstract
The required moment of inertia for steel plate girders is commonly provided by using relatively slender webs to achieve high strength-to-weight ratio. A large part of the shear capacity of such plate girders depends on the post-buckling reserve strength of the web panels due to the diagonal tension field mechanism. The effectiveness of the tension field significantly depends on geometrical and mechanical properties of the panel as well as on the boundary conditions. This research study includes two experimental tests which have been conducted considering both rigid and non-rigid end posts. The acquired data in addition to the previous experimental studies were then used to verify the numerical models. Furthermore, the ultimate post buckling capacity of the plate girders are compared with the current specification codes. Findings show a good agreement between codes and numerical analyses for the web slenderness between 120 and 220. However, there is a rather large discrepancy in the results for the web slenderness outside this range.

1. Introduction
Steel plate girders with slender webs are commonly used in a variety of structural engineering applications because of their high strength-to-weight ratio and post-buckling reserve of strength and stiffness. For a given applied bending moment, the axial forces in the flanges of these girders decrease as the web depth (h) is increased; hence it is economical to make the webs as deep as possible [1]. In addition, in most practical ranges of span lengths –for which a plate girder is designed– the induced shearing stresses are relatively low as compared with the normal stresses in the flanges resulting from flexure. As a result, the thickness of the web plate is generally much smaller than that of the flanges. Consequently, the web panel can buckle at a relatively low value of the applied shear loading. Thus, such webs are often reinforced with vertical stiffeners to increase their buckling strength. The web design involves finding a combination of an optimum plate thickness and stiffener spacing that renders economy in terms of material and fabrication costs [2].

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The ultimate shear capacity of the girder consists of three contributions obtained from: i) the elastic critical load; ii) the post-critical membrane tension developed on an idealized tension band; and iii) The plastic moment capacity of the flanges [3].

The post-buckling capacity is taken into account in the ultimate limit state design methods in codes specifications and can be considerably larger than the elastic critical load of the web panel [4]. This study involves both experimental and numerical investigations. First the effect of web slenderness on the ultimate post-buckling capacity of the girders with both flexible and rigid end-posts is investigated. Then, the position of the plastic hinges in the flanges at the ultimate limit state are measured and compared with the result of code specifications and theoretical approaches.

2. State of the art

Wilson (1886) was the first who studied the post-buckling behavior of plate girder web panels. Wagner (1931) first presented a diagonal tension theory for aircraft structures with very thin web panels. A pure tension field theory proposed by Wagner is only suitable for structures where extremely thin plates are attached to very rigid boundary elements. This is the case, for instance, for an inner panel in a plate with rigid cross-beams and stiffeners.

For civil engineering purposes, the major break-through for analytical methods based on tension field action was achieved by Basler and Thurlimann (1961). At that time, shear stresses in the webs of the plate girders were limited to the elastic critical load. Basler developed design methods to specify the size of stiffeners needed for efficient girder design to achieve the required tension field action [5]. The American Institute of Steel Construction (AISC) first adopted post-buckling strength into its specifications in 1963, and in 1973 the American Association of State Highway and Transportation Officials (AASHTO) followed suit. Cooper et al. (1964) conducted four tests on two welded constructional alloy steel plate girders to investigate the applicability of the shear strength theory to the girders [6].

The tension field action for the plate girders was summarized in Structural Stability Research Council (SSRC), 2010. Since then, several researchers have developed alternative and refined methods which have been corroborated in extensive test programs. Although these classical failure theories assumed different yield zones, the following fundamental assumption was implicit in all the theories: “compressive stresses that develop in the direction perpendicular to the tension diagonal do not increase any further once elastic buckling has taken place [7]”. The difference between these methods is mainly due to various assumptions of how the tension band is attached to the edges of the web, and about the bending stiffness of the flanges. In the model by Basler and Thurlimann, the flanges are assumed to have small bending stiffness; hence, the tension field is attached only to the transverse stiffeners. The Cardiff method –described by Porter, Rockey and Evans (1975)– utilizes the moment capacity of the flanges and assumes the tension field to be attached both to the flanges and the transverse stiffeners [8].

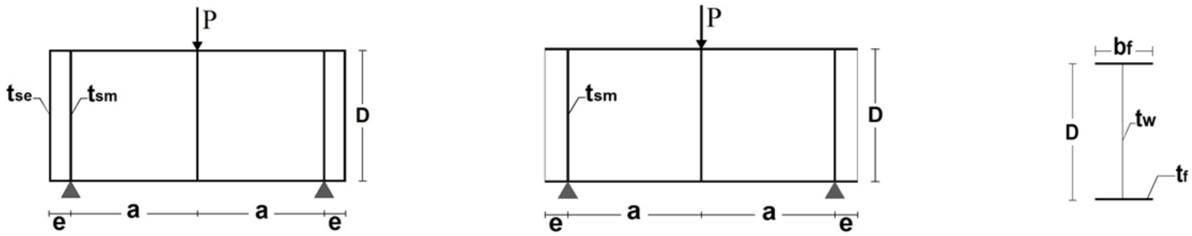
Unfortunately, the tension field action is limited to the interior panels of plate girders in AASHTO and AISC codes as it is shown in the work which is done by White and Barker (2008) and White et al. (2008) [9], [10]. Yoo and Lee (2006) demonstrated that this assumption is too conservative and pointed out that the tension field action is possible in the end panel of steel plate girders [11].

The ultimate shear resistance of steel plate girders has been studied extensively, both experimentally and theoretically, resulting in the development of the well-established Cardiff tension-field and Hoglund's rotating stress-field theories [12], [13]. Theoretical predictions of ultimate shear resistance of slender plate girders can be made using Cardiff tension field theory developed by Rockey. The theory is based on an assumed equilibrium tension field in the girder, which satisfies the approach for a lower-bound strength prediction provided the material possesses sufficient ductility for the stress field to develop. In this method, full account may be taken of the post-buckling reserve of strength. In Eurocode 3 [14], the tension field design method is based on this theory and it is applicable for transversely stiffened girders having web panel aspect ratio between 1.0 and 3.0. Höglund' s rotating stress field theory is based on a system of perpendicular bars in compression and tension, which are assumed to represent the web panel. The simple post-buckling design method is based on this theory and it is applicable to both stiffened and non-stiffened girders. Höglund (1973) reports on shear buckling capacity of steel and aluminum girders and states that the rotated stress field method –with some modifications– was found to give the best agreement with 273 tests on steel plate girders as well as 93 tests on aluminum alloy plate girders in shear. The method is simple to use and is applicable to un-stiffened, transversely, and longitudinally stiffened flat plate webs, as well as to the trapezoidal corrugated webs.

3. Experimental study

It is obvious that experimental tests are imperative to really understand and validate any theoretical or numerical results in research. The experimental investigation described in this paper sought to investigate the response of steel web plates taking into account the condition –rigid or non-rigid– of their end posts. The rigidity of the end posts is given by the thickness, number –normally one or two–, and the distance of vertical stiffeners at each ends of the girder and it is directly related to the rotation capacity of the tension band that is developed as a resistant mechanism after buckling occurs. The objective of the present experimental and numerical study is to investigate the following aspects: i) to study the behavior of girders with very slender web panels; ii) to investigate the effects of rigidity of the end posts on shear behavior of the girder.

The two laboratory tested girders in this study were conceived slender enough to ensure that shear buckling would occur during the tests, followed by complete development of the tension field in the post-buckling range. The chosen girders for this test program had similar geometrical and material properties but one girder was designed with rigid end posts *(RPG)* whereas the other exhibited non-rigid end posts *(NPG)*. This means that the effects of the end posts could be isolated in the tests and directly compared. Fig. 1 and Table 1 show a summary of material and geometrical properties of the two girders, where f_{yf} and f_{yw} are yield strength of the flanges and web, respectively. The geometry of the tested girders was directly related to the objective of the test program. Therefore, to minimize the flexural effects compared to shear ones, the girders were designed as short span and large depth elements which were tested as simply supported girders subjected to a concentrated load at mid-span.



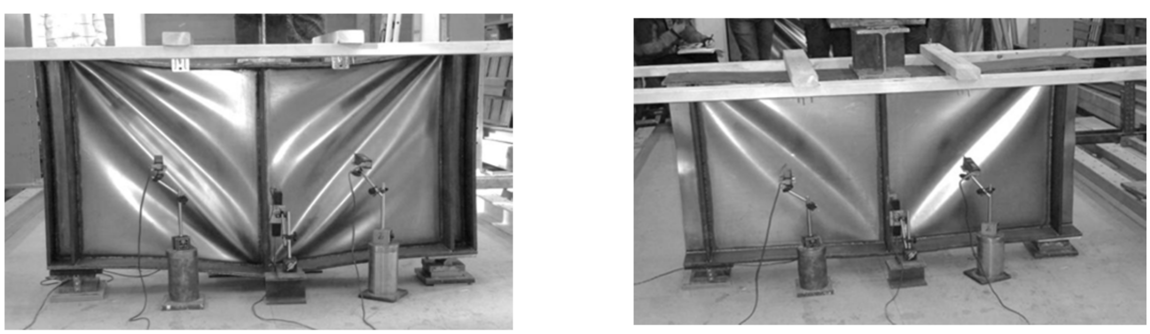
a) Rigid end post (RPG) b) Non-Rigid end post (NPG) c) Cross section

Figure 1: The geometry (described in Table.1) and the loading conditions of the tested girders

Table 1: Dimensional and material properties of the tested girders

Test	t_f *(mm)*	b_f *(mm)*	t_w *(mm)*	D *(mm)*	a *(mm)*	t_{sm} *(mm)*	t_{se} *(mm)*	e *(mm)*	f_{yf} *(MPa)*	f_{yw} *(MPa)*
RPG	15	250	2	800	750	15	15	100	235	210
NPG	15	250	2	800	750	15	-	-	235	210

The tests were performed at the laboratory of Structural Engineering, Lund University. The lateral and vertical displacements at key points were continuously measured during the laboratory test using linear displacement transducers. Both girder specimens were tested up to the complete development of the tension field under the action of an applied point load at mid-span, through a rigid specimen bolted to the jack. The tests were carried out under displacement control using a hydraulic actuator with a loading capacity of 1000kN. Displacement control permits more accurate reproduction of the ductile response of the beam after buckling takes place. There were two LVDTs on the center of each panel to measure the out of plane displacement and one LVDT at mid-span for the vertical displacements. In Fig. 2, the two specimens are shown after loading and the deformation of web plate girder caused by full forming tension field is clearly visible in both specimens. In these tests the position of plastic hinges in the flanges were also measured for both cases.



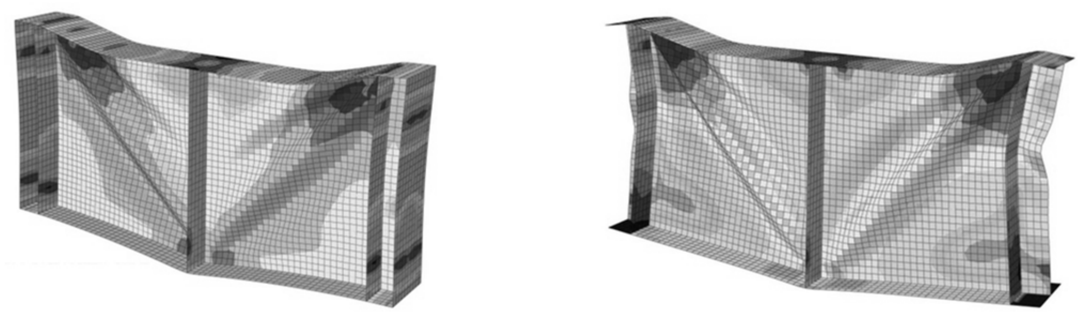
a) Rigid end post (RPG) b) Non-Rigid end post (NPG)

Figure 2: Post-buckling behavior of the tested plate girders

4. Numerical study

In the current study, the Finite Element program ABAQUS 6.11, and its predefined S4R element was implemented in all eigenvalue and incremental non-linear analyses. The S4R element is a four-node reduced integration quadrilateral shell element with three rotational and three translational degrees of freedom, per node. The element is capable of modeling elastic, plastic, and large-strain behaviors and is used to simulate both membrane and flexural behaviors. For the verification purposes, the results obtained via numerical analyses were compared to the theoretical calculations. For convergence, simple-detached panels were meshed into sufficient number of elements to allow the development of the shear buckling modes and displacements.

Some results from the Finite Element modeling of two girders are showed in Fig. 3. Mild steel material properties, with the elastic modulus $E = 210GPa$, and the Poisson's ratio $\nu = 0.3$ were used throughout the study. The material behavior was assumed bilinear elastic-plastic with a strain-hardening. In the incremental nonlinear analysis, an initial imperfection shape corresponding to the lowest Eigen mode of the elastic shear buckling –as suggested by Bathe (1996)– was applied. The magnitude of imperfection according to *EC3* is the minimum of (D/200 and a/200), where D and a are shown in Fig.1.



a) Rigid end post (RPG) b) Non-Rigid end post (NPG)

Figure 3: Finite element modeling of the tested plate girders

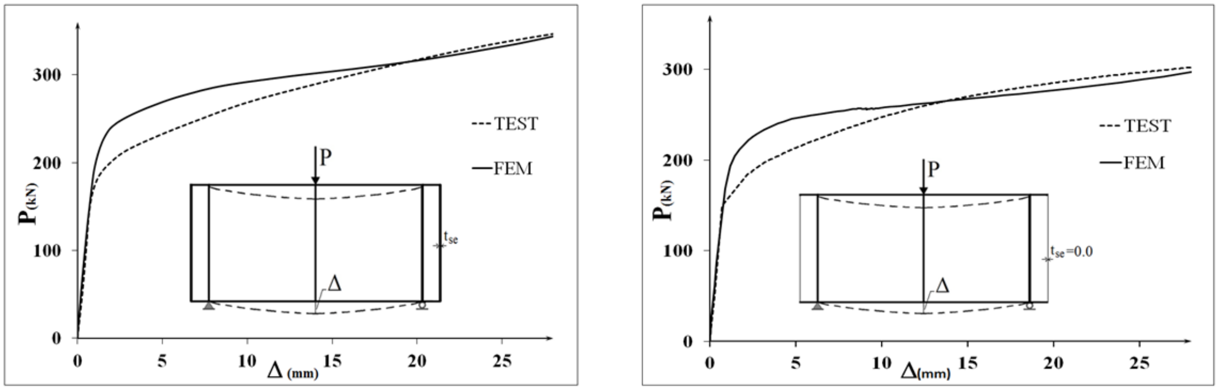
Moreover, it is important to outline that the laboratory test results were used as a means to validate the numerical model used in this research. Being validated, the model has been employed as a tool to develop further numerical analysis.

5. Result of analysis

5.1 The in-plane load-deflection curves

The in-plane load-deflection curves corresponding to the numerical and experimental studies are shown in Fig. 4 for both rigid (RPG), and non-rigid (NPG) end post cases. There is a relatively good agreement between the result of FE model and the tests both in terms of the initial stiffness and the ultimate strength.

The post-buckling behavior –depicted by the load-deflection curves– shows an increasing stiffness of the system subsequent to the onset of buckling as a stable post-buckling behavior. For systems with a "stable post-buckling" curve, the ultimate shear resistance depends on the steepness of post-buckling region as well as the ratio of critical-load to yielding-load. In addition, it is practically insensitive to initial imperfections.



a) Rigid end post (RPG) b) Non-Rigid end post (NPG)

Figure 4: Load-deflection curves of the tested plate girders

5.2 The effect of web Slenderness on ultimate shear strength

Web slenderness significantly influences the ultimate shear-buckling load of the plate girders. However, girders with very thin webs possess significant post-buckling strength which is far beyond the shear buckling load. The result of two laboratory tests performed at Lund University along with currently available similar test results conducted by Estrada(2007) [15], Konishi(1965) [13], Skaloud(1971) [13] were used to investigate the effect of web slenderness on ultimate shear strength of the girders. A summary of the dimensions and the material properties of the specimens are shown in Table 2.

Table 2. Geometrical details and material properties of specimens

Name	Test ID	a (mm)	D (mm)	t_w (mm)	t_f (mm)	b_f (mm)	D/t_w	f_{yf} (MPa)	f_{yw} (MPa)
This investigation	RPG & NPG	750	800	2	15	250	400	235	210
Konishi	B	1200	1200	4.5	12	240	267	500	500
Skaloud	TG1	1000	1000	2.5	5.17	160	400	286	203.7
	TG2	1000	1000	2.5	10.1	200	400	286	203.7
	TG3	1000	1000	2.5	16.5	200	400	286	203.7
	TG4	1000	1000	2.5	20.2	200	400	286	203.7
	TG5	1000	1000	2.5	29.7	250	400	286	203.7
Estrada	r700ad15	1050	700	4	20	200	175	266.8	301.4
	r600ad2	1200	600	4	20	200	150	266.8	301.4
	r500ad25	1250	500	4	20	200	125	266.8	301.4
	r400ad325	1300	400	4	20	200	100	266.8	301.4

For the mentioned specimens, the ultimate shear strengths were determined by means of: i) numerical analysis; theoretical approaches such as ii) Basler and iii) Cardiff methods; and the current code specifications such as iv) AISC [16], v) AASHTO [17], and vi) Eurocode3 [14]. The post-buckling capacities obtained by different methods were normalized to the maximum load achieved in the corresponding test. The results are shown in Table 3.

Table 3. The ratio of ultimate shear resistance to the corresponding test results

Name	Test ID	FEM	Basler	Cardiff	AASHTO	AISC	EC3
This investigation	RPG	1.07	0.68	0.97	0.65	0.68	0.65
	NPG	1.09	0.81	1.17	0.78	0.81	0.67
Konishi	B	0.94	1.35	1.16	1.3	1.35	0.75
Skaloud	TG1	0.88	1.26	0.97	1.20	1.25	0.68
	TG2	1.01	1.20	1.18	1.14	1.18	0.75
	TG3	1.05	1.01	1.25	0.96	0.99	0.8
	TG4	0.94	0.84	1.17	0.80	0.83	0.77
	TG5	0.92	0.62	1.19	0.59	0.61	0.87
Estrada	r700ad15	1.09	0.99	1.19	0.92	0.94	0.97
	r600ad2	1.1	1.02	1.25	0.91	0.94	1.08
	r500ad25	0.99	1.01	1.27	0.88	0.9	1.11
	r400ad325	0.98	1.07	1.33	0.91	0.92	1.09

The results of table 3 show that:

1. There is a very good agreement between the numerical and the tests results. The discrepancy for most cases is in the range of 5-10%.
2. The ultimate shear strengths obtained by AISC are relatively close to those obtained by Basler's theory [18]. This is due to the fact that AISC’s approach for calculation of the post-buckling shear strength of girders is based on the Basler theory [18].
3. The predicted values by Basler theory, AISC and AASHTO show a decreasing trend of the shear strength with increase of the flange thickness (see e.g. the data related to Skaloud’s specimens). This is due to the fact that the contribution of the flanges to the shear resistance is ignored in these methods.
4. Cardiff’s model gives more conservative results than the other models.
5. Eurocode3 gives acceptable results with maximum 11% discrepancy to the tests results for the specimens with web slenderness between 100 and 175. However, Eurocode3 significantly underestimates the shear strength for the specimens with web slenderness larger than 250, e.g. 267 and 400.

The effect of web slenderness on ultimate shear strength of the specimens that were tested at Lund University was also numerically studied by changing the thickness of the web. The results of numerical analysis and Eurocode3 predictions are shown in Fig. 5. The results show that Eurocode3 is slightly conservative for web-slenderness smaller than 180. However, Eurocode3 is significantly on the safe side for web-slenderness greater than 180. It is also shown in Fig. 6 that the discrepancies between the result of Eurocode3 and the numerical study is less than 15% for the web slenderness ratios between 120 and 220 which represent the majority of practical cases. However, the discrepancies between Eurocode3 and the numerical results are higher, outside this range.

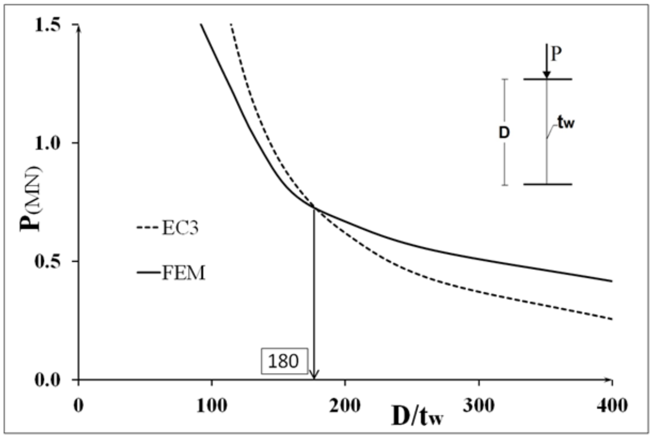


Figure 5: Ultimate shear resistance versus web-slenderness according to EC3 and FEM

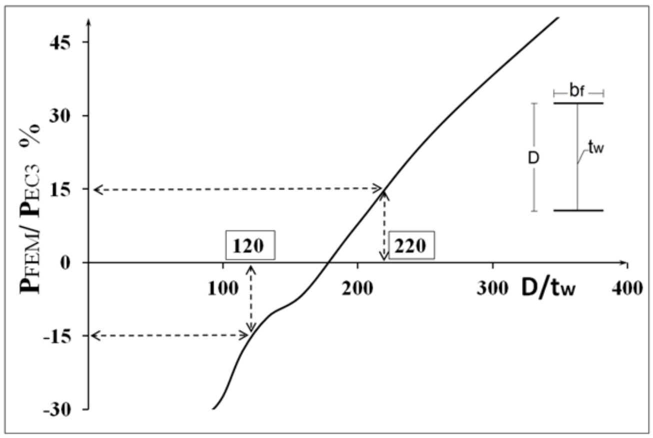


Figure 6: The effect of web slenderness on the discrepancy between EC3 and FEM

Furthermore, the accuracy of the post-buckling strength value that is given by Eurocode3 approach was also studied for the other tested specimens, shown in Table 3. The results show –as expected– that Eurocode3 slightly overestimates the shear strength for the web slenderness ratios less than 180 and it significantly underestimates the shear strength for the web slenderness ratios larger than 180.

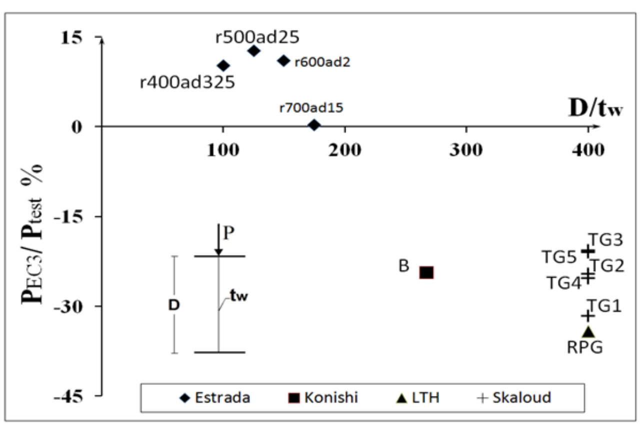
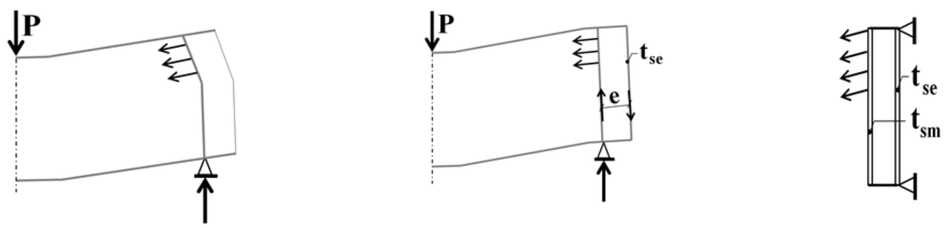


Figure 7: The variation of web slenderness via differences ultimate shear resistance in EC3 to the test results

5.3 The effect of end posts

The post-buckling resistance of steel-plate girders significantly depends on the tension field that develops in the web panels. It behaves as diagonal members of a Pratt truss, where the flanges of the girder are the chords, and the posts are its transverse stiffeners. The effect of the rigidity of the end-posts on the ultimate shear capacity of the girders was investigated by varying the thickness of end-stiffener, t_{se} (see Fig. 8b). The rigid end-post behaves similarly to an I-shaped beam attached at the ends of the girder. Such a short beam takes the induced horizontal stresses of the tension field in the web panel by its bending rigidity, (see Fig. 8c).



a) Non-rigid end-post b) Rigid end-post c)

Figure 8: Tension field and the effect of end-post rigidity

The in-plane load-deflection curves of the two experimental tests –the rigid-end post case where $t_{se} = 15mm$, and the non-rigid end-post case where $t_{se} = 0.0$– are shown in Fig. 9. The curves were developed by more numerical analyses for different thicknesses of the end transverse stiffeners. The results show that providing a rigid end-post has no effect on the initial stiffness of system and only increases the ultimate shear strength of the girder. It also shows that the shear resistance of the girder considerably increases by providing relatively thin transversal stiffeners at the end of girder. However, increasing the thickness of stiffener over a given value does not increase significantly the shear resistance of the girder.

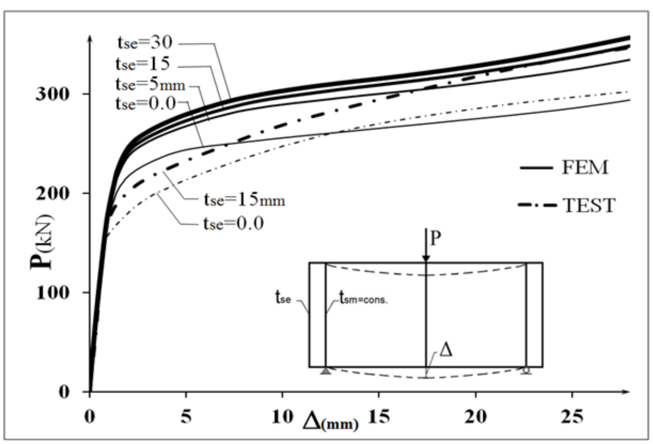


Figure 9: Force-Vertical displacement of plate girder with variation of thickness of end post

Moreover, the rigidity condition of end-post directly influences the position of plastic hinges on both flanges. The distance between the location of the plastic hinge and a vertical line passing through the closest support depends on the yield strength of material and the geometry of the web panel and the flanges as well as the rigidity of the end-posts; which can be expressed as:

$$C = \left(0.25 + \frac{1.6 b_f t_f^2 f_{yf}}{t_w h_w^2 f_{yw}}\right). a \tag{1}$$

where t_f, b_f, f_{yf} and t_w, h_w, f_{yw} are the dimensions and the yielding stress of the flanges and the web of the girder, respectively, and a is the longitudinal size of web panel between two adjacent transverse stiffeners. The position of plastic hinges versus the thickness of the end-post stiffener was numerically investigated for the rigid end-post specimen at Lund University. The results show that the distance between plastic hinges increases by increasing the thickness of the stiffener and consequently increasing the rigidity of the end-posts, see Fig. 10. This behavior can be approximated as a bilinear curve which gives constant value for the position of plastic hinges after a specific thickness of the stiffener.

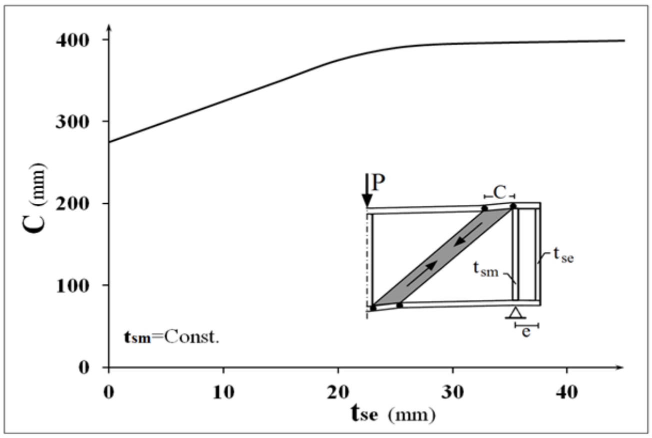


Figure 10: Plastic hinge location “C” as a function of the thickness of the end post “t_{se}”

5. Conclusion

In this paper, two laboratory tests on post-buckling shear strength of steel plate girders performed at Lund University along with some relevant experimental test results were analyzed and discussed with the results of nonlinear numerical analysis. The main conclusions are:

- The results show that Eurocode3 slightly overestimates the shear strength for web slenderness ratios less than 180 and it significantly underestimates the shear strength for the web slenderness ratios larger than 180.
- The ultimate shear strength can be significantly improved by using a relatively thin transverse end-post stiffener. However, increasing the thickness of that stiffener has negligible effect on increasing the ultimate shear strength of the girder. Similarly, increasing the thickness of end stiffener, linearly increases the distance of plastic hinges on the flanges and has not significant effect after a specific value.

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