Proceedings of the
Annual Stability Conference
Structural Stability Research Council
St. Louis, Missouri, April 16-20, 2013

Practical Design of Complex Stability Bracing Configurations

C.D. Bishop[1], D.W. White[2]

Abstract
The analysis and design of bracing systems for complex frame geometries can prove to be an arduous task given current methods. AISC's Appendix 6 from the 2010 Specification for Structural Steel Buildings affords engineers a means for determining brace strength and stiffness requirements, but only for the most basic cases. This paper aims to shed some light on common aspects of certain bracing systems that lie well outside the scope of Appendix 6 as well as explain why the corresponding structures can be unduly penalized by Specification equations that were never derived for such use. Subsequently, a practical computational tool is proposed that can be used to accurately assess bracing demands while removing the interpretations needed by design engineers to "fit" their frames into the limited scope of AISC Appendix 6. The software tool is described in detail and several benchmark cases are presented to provide validation. Individual beams and complete framing systems are evaluated via the proposed software and compared to refined test simulations. Finally, recommendations are articulated governing the use of this new software as a means to accurately and safely assess bracing demands in complex bracing configurations.

1. Introduction

Metal buildings are structures that utilize extreme weight efficiency to provide large, open floor space at a relatively inexpensive price. Metal buildings are designed to the limits of applicable codes and standards in order to optimize their economy while still meeting safety standards and client objectives. Thus, locations in the standards where conservatism is abundant can unduly influence the steel system costs. Fig. 1 shows a rendering of a one-bay, two-frame segment of a typical metal building.

The American Institute of Steel Construction's *Specification for Structural Steel Buildings* (2010b) Appendix 6 – "Stability Bracing for Columns and Beams" provides simplified techniques for designing lateral and torsional braces required for stability. However, there are a number of specific attributes of metal building systems that are outside of the scope of Appendix 6. A number of these attributes are:

[1] Associate, Exponent, Inc., <cbishop@exponent.com>
[2] Professor, Georgia Institute of Technology, <don.white@ce.gatech.edu>

- Metal building frames make extensive use of web-tapered members. AISC's Appendix 6 only considers prismatic members.
- The bracing stiffnesses provided are assumed to be equal at each brace per Appendix 6. This is often not achieved in practice due to variations in girt or purlin size and in bracing diagonal lengths and angles of inclination.
- Appendix 6 assumes uniform spacing of braces. However, metal building frames generally have unequal spacing of the girts/purlins as well as diagonal braces.
- Knee joints and other joints may not provide rigid restraint against twisting and lateral movement at the rafter and column ends, yet the AISC equations are based on the assumption of rigid bracing at the member ends.
- Warping and lateral bending restraint from joints is not considered, and continuity with more lightly-loaded adjacent member segments addressed only to a limited extent.
- The combined action of diaphragms and discrete flange braces may contribute significantly to the stability of critical segments. AISC does not offer any guidance for assessing the stiffness provided to the system by multiple bracing types.
- AISC's Appendix 6 targets the design of the braces for a single upper-bound estimate of the stiffness and strength requirements. However, some economy may be gained by recognizing that the bracing stiffness and strength demands often reduce very sharply as one moves away from a critical bracing location.

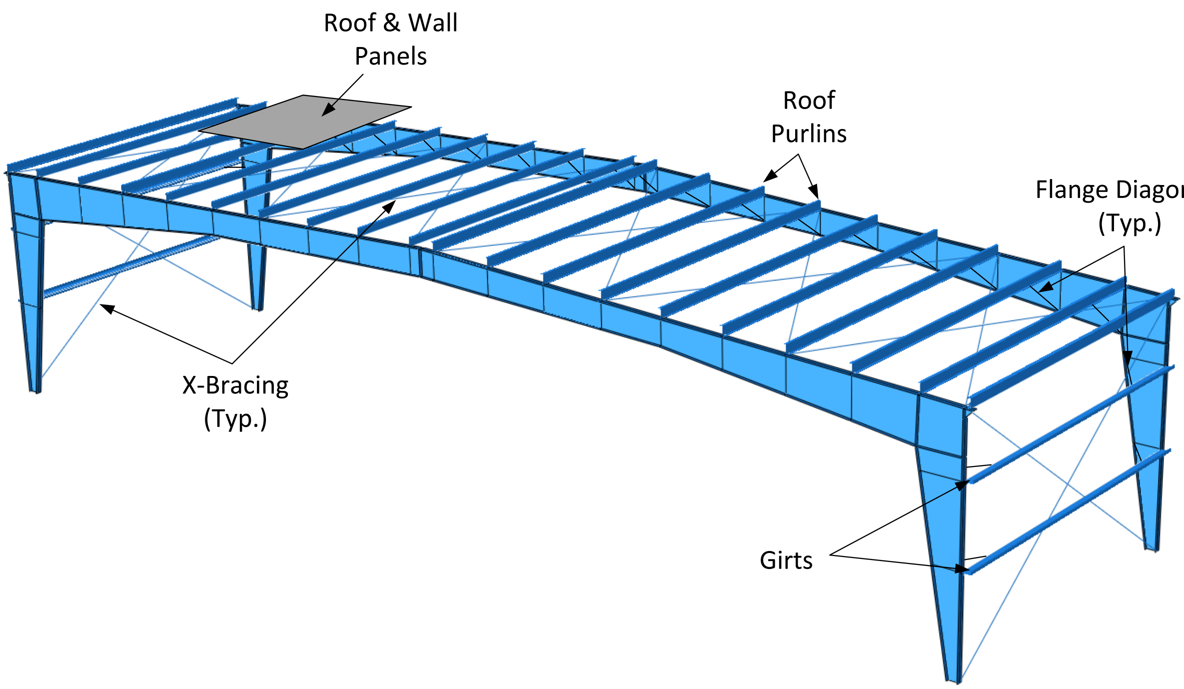

Figure 1: Two representative clear-span metal building frames

Although it is believed that many of the above attributes result in conservative designs, engineers are left to interpret and adapt the current AISC codified equations well beyond the intent of the Specification to design bracing for their buildings. This could lead to inadequate designs and, in extreme situations, to adverse effects on life-safety.

Lastly, the AISC Specification Appendix 6 provisions estimate the maximum brace strength and stiffness demands throughout a given member generally assuming constant brace spacing and constant brace stiffness. However, this method is not practical for members with a large number of brace points along their length. When considering the sample frames shown in Fig. 1, several questions may come to mind:

1. Are the bracing requirements at the knee influenced significantly by the loading, cross-section geometry or arrangement of bracing at the ridge?
2. What constitutes a support point? That is, at what locations is the out-of-plane movement of the frame sufficiently restrained? Do any locations of attachment of the panel or rod bracing provide this restraint?
3. How do the rafters interact with the columns and vice versa with respect to the bracing demands?

In general, all the members and their bracing components work together as a system in structural frames such as that shown in Fig. 1. Therefore, the central question addressed in this paper is: What are the overall physical bracing strength and stiffness demands required to develop the required strength of the structure?

2. Toward a Comprehensive Bracing Tool

Due to the complex and interrelated nature of the attributes discussed above, this paper focuses on the development of a comprehensive computational bracing analysis tool for the direct assessment of stability bracing requirements. Any such tool would need to be robust enough to include consideration of all of the above items, yet remain simple enough for use in practice. The following is a summary of why such an analysis tool is required for bracing design in metal building structures.

The current AISC Specification equations for stability bracing are derived from solutions of the elastic eigenvalue buckling of members and their bracing systems. However, the nominal flexural strength of columns and rafters of metal building frames is usually controlled by *inelastic* lateral-torsional buckling, once the frames are in their final constructed configuration. The Appendix 6 equations address this (for LRFD of beam bracing) by a three-staged approximation:

1) Replacing the elastic critical moment M_{cr} by the design strength ϕM_n, thus implicitly estimating the bracing demands, as the elastic or inelastic strength limit ϕM_n is approached, by the normalized behavior of the system as the elastic buckling load is approached, then
2) Replacing ϕM_n by the required strength M_u, which is generally smaller than ϕM_n for a properly designed beam. This is based on the implicit approximation that the partial bracing stiffness demands can be estimated conservatively by this manipulation. Furthermore, where applicable (i.e., only for nodal lateral beam bracing), the Specification allows the use of L_q in place of L_b, where L_q is taken as the unbraced length that reduces ϕM_n to M_u. This completes the approximations in the AISC bracing equations for the partial bracing stiffness demands.

3) Lastly, for cases involving partial bracing, the AISC equations assume that the strength requirement for partial bracing is estimated sufficiently simply by using M_u in the equations for the strength requirement (along with the use of L_q). This appears to be an acceptable approximation for practical partial bracing cases approaching full bracing, but it can break down for weak partial bracing, where the amplification of the initial imperfection displacements may become substantial.

The various factors listed above may render the bracing system design over-conservative by as much as 10x that required based on inelastic load-deflection solutions when applied to typical metal building frames (Sharma 2010). A portion of the conservatism observed by Sharma (2010) may be due to the approximations detailed above; however, a larger portion is likely due to the fact that many of the metal building system attributes are not addressed explicitly by the AISC equations.

Tran (2009) showed that the AISC procedure for calculating the strength for columns works very well using the inelastic column stiffness reduction factor, τ_a. He performed "exact" inelastic eigenvalue buckling analyses for the column shown in Fig. 3 (exact in the sense that the solution is based on τ_a in a fashion such that the eigenvalue for a given bracing stiffness gives the column inelastic, or elastic, design strength ϕP_n) and having the following attributes:

- Prismatic, nodal-laterally braced W14x90 column
- F_y = 50 ksi
- Five equal unbraced lengths, $L_b = L_{by}$ = 15 ft
- Equal brace stiffness, β = 20 kips/in.
- Constant axial load, P

A comparison of the inelastic eigenvalue buckling analysis results with the AISC Direct Analysis Method solution for the column maximum strength as well as distributed plasticity simulation analysis results is shown in Fig. 4, where *DM*, *InE*, and *DP* represent the Direct Analysis Method result, the inelastic eigenvalue buckling solution, and the distributed plasticity simulation, respectively.

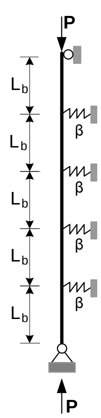

Figure 3 – Sample column

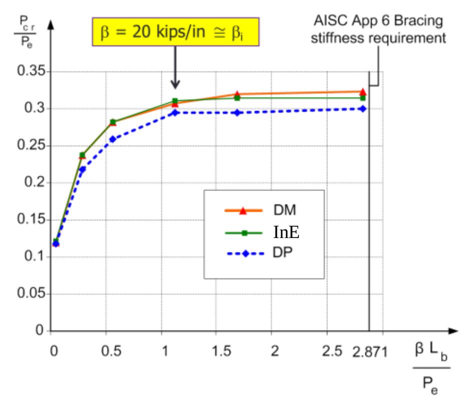

Figure 4 – Comparison of analysis results

One can observe from Fig. 4 that all of these solutions produce similar results, and all suggest that a bracing stiffness significantly less than the AISC Appendix 6 requirement is sufficient to develop the full-bracing resistance for this column example. Tran (2009) showed that for this problem, the use of a nodal bracing stiffness equal to the ideal bracing stiffness, labeled as β_i = 20 kips/inch in Fig. 4, resulted in brace strength demands only slightly larger than 2 % in distributed plasticity simulation studies.

Although the solution by the DM is reasonably accurate, this method (often thought of as a reasonable approximation of the results from a rigorous distributed plasticity simulation analysis or a physical test, and thus providing the best design assessment for stability requirements) may not be a feasible option for assessing stability bracing demands in problems like this due to the following fact: With the DM, as well as with the simulation analysis, an appropriate magnitude and pattern of the initial geometric imperfections must be imposed on the member to estimate the maximum strength demand on a given brace in question. This means that one must consider geometric imperfections in a manner similar to the way that load combinations are considered in general strength design. For each specific brace, an imperfection needs to be identified that produces the maximum demand on that brace. Although procedures have been identified by Sharma (2010) and others to determine the "critical" imperfection for a given brace, these procedures are relatively complex and in general may require a number of trials to truly identify the critical imperfection. Of equal importance is that these procedures would generally need to be executed for each brace within the structural system. This level of effort can be tolerated for research studies; however, it is not practical for ordinary design.

In contrast, the "exact" inelastic eigenvalue buckling analysis gives similar results to the DM or DP solutions with much less computational effort. However, one should note that an eigenvalue buckling analysis only provides the designer with an estimate of the required bracing *stiffness* and the overall system strength. As has been discussed extensively (see, Yura (2001) for example), to be effective, a brace must provide sufficient stiffness *and* strength to resist the loads imparted to it by the braced member. Fortunately, Sharma (2010) and Tran (2009) have shown through numerous finite element simulations that the brace force is usually in the range of 2 to 3% of the effective flange force for nodal bracing cases approaching full bracing. In fact, 2% is often enough to allow the frame to reach 95% of its rigidly-braced capacity. Therefore, one can combine the brace stiffness requirement from an inelastic eigenvalue buckling analysis with say, a 2 to 4% brace strength rule (for frames not specifically required to sustain large inelastic cyclical loadings) for a complete determination of the bracing requirements.

Creation of a new computational bracing analysis tool is needed because no current structural analysis software provides the specific necessary elements or combination of analysis methods to produce the type of solutions illustrated in Fig. 4 for general member and/or frame geometries and general bracing arrangements. The targeted bracing analysis tool must be able to address such aspects as warping and out-of-plane bending restraint, roof and wall "shear panel" (i.e., relative) bracing combined with nodal torsional bracing, combined effects of axial and flexural loading, unequal brace spacing, web taper, and steps in the cross-section geometry. The computational tool needs to be able to solve for the in-plane elastic and/or inelastic state of a member or frame at a given design load level, or at an envelope of all the maximum internal forces based on a range of design loadings, and then determine the ideal bracing stiffness demands (i.e., the required ideal bracing stiffness) to sufficiently stabilize the structure in this elastic/inelastic state.

For speed and efficiency, the bracing tool also must be able to obtain the above solutions with minimal computational effort (i.e., a minimum number of degrees of freedom). As depicted in Fig. 5, the webs of the members in the targeted computational tool are modeled using plane stress elements for the initial planar load-deflection analysis and shell elements for the subsequent 3D inelastic eigenvalue buckling analysis. However, the flanges are modeled using beam elements.

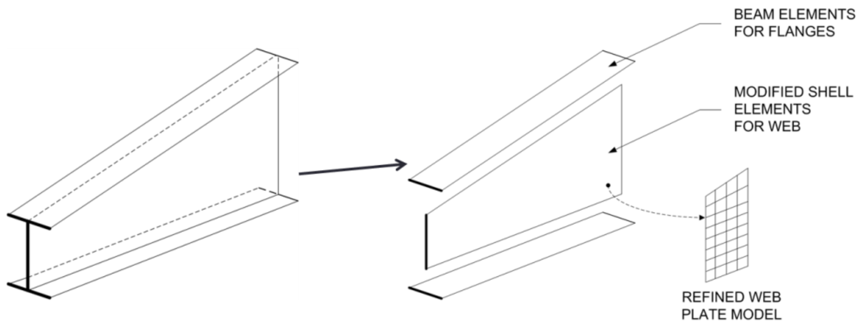

Figure 5 – Discretization scheme for the member

One additional significant problem that occurs for the above type of model is that, for I-section members having non-compact or slender webs, the eigenvalue buckling solutions are commonly dominated by web local buckling modes. However, it is well known that these types of I-section member webs generally exhibit a stable post-buckling response. That is, the web local buckling behavior tends to have a secondary effect on the overall member flexural and lateral-torsional response, and hence on the member bracing demands. Therefore, for an eigenvalue buckling analysis to provide an efficient solution, something must be done to parse the inconsequential web buckling modes from the general solution. This is accomplished by using multiple elements through the web depth to determine the web stiffness, but effectively using only *one* element through the web depth to determine the web's geometric stiffness properties. Thus, with four-node shell elements, the membrane and bending stiffness of the web is captured accurately, but the modeled web is unable to locally buckle due to the lack of internal web nodes (essentially restricting buckling to the web/flange juncture). Prior simulations in Abaqus (Simulia 2011) have shown that modifying the web in a similar manner yields elastic eigenvalue buckling results that are similar to those from load-deflection solutions.

The application of the above computational tool can provide a direction for engineers to better assess their bracing requirements and therefore, provide efficient, safe building designs.

3. Eigenvalue Buckling Solutions using SINBAD

A new software package referred to as the System for INelastic Bracing Analysis and Design (SINBAD) has been developed to address the need for a comprehensive bracing tool as described above. This section describes SINBAD's implementation and how this software may be used to provide a more efficient and accurate assessment of stiffness demands for flange bracing than possible using the AISC Specification Appendix 6 equations.

3.1 Software Overview

SINBAD is in essence a stand-alone, special purpose finite element program that performs 3D eigenvalue buckling analyses based on the elastic or inelastic state of a planar structure caused by the application of in-plane loads. The solutions from SINBAD may be used to either determine member or framing system out-of-plane buckling capacities or to determine ideal stiffnesses of the corresponding out-of-plane bracing system. The in-plane inelastic analysis capabilities of SINBAD involve a complete spread of plasticity (or plastic zone) analysis of the structure including any appropriate in-plane geometric imperfections as well as specified member nominal residual stresses. SINBAD is written in Matlab (MathWorks 2011) and includes both an analysis engine and a graphical user interface (GUI).

SINBAD has two distinct modules: members and frames. The members module is used exclusively to assess the bracing requirements for individual members. For example, one can analyze a given physical beam or column segment for analysis as an isolated member. That is, the member module is only useful to model *one* member. In the frame module, SINBAD provides the ability to assess the bracing requirements for an entire framing system. The frame can have any generalized geometry but must have no more than two exterior columns and two rafters (one on each side of a ridge location). The input to SINBAD may be accomplished either by a set of Microsoft Excel worksheet forms or via a general application program interface.

3.2 FEA Modeling

The following sub-sections describe some of the individual components that comprise SINBAD. The inelastic buckling analysis solutions in SINBAD are conducted generally in two steps:

1. The 2D (planar) elastic/inelastic state of the structure is calculated given a prescribed loading condition, and
2. A 3D eigenvalue buckling analysis is performed based on the stress state determined in Step 1.

By limiting the stress determination to a planar solution, significant analysis time savings are realized relative to the requirements for a general 3D simulation such as that conducted in Abaqus. After the state of stress is determined, the program must "upgrade" the model to its 3D counterpart in order to assess the out-of-plane stability of the system in question. Details about how this solution is achieved efficiently are provided in the following sections.

3.2.1 Flange Elements

All flanges and stiffeners are modeled in SINBAD using 2-node cubic Hermitian beam elements with one additional internal axial degree of freedom, providing for a linear variation in the interpolated strains along the member length for both axial and bending deformations (White 1985). For the planar solution, there are a total of 7 degrees of freedom: two translations and one out-of-plane rotation at each end plus an additional axial degree of freedom at the middle of the element. The interior axial degree of freedom is removed via static condensation (see McGuire, et al. 2000, for example) to leave a total of 6 global degrees of freedom. For the 3D model, a total of 13 degrees of freedom are used for the beam element: three translations and three rotations at the member ends plus one additional axial degree of freedom at the middle of the element. Again, static condensation is performed to render 12 global degrees of freedom.

To track the spread of plasticity through the beam element, White (1985) proposed a fiber model that subdivides each element into a predefined grid. 12 fibers are used through half the width of the flange ($b_f/2$) and 2 fibers through the thickness of the flange (t_f). It is only necessary to model the grid over one-half of the flange width since the planar solution is symmetric about the plane of the structure. Tracking the spread of yielding throughout the element is performed on an "as needed" basis. That is, the fiber grid is only created when the global element has detected yielding. This reduces the memory requirements and substantially increases the computation speed. For brevity, the reader is referred to White (1985) for a further discussion and step-by-step implementation of this second-order inelastic element.

3.2.2 Web Elements

The web shell elements are created through the combination of a QM6 element for plane stress and a PBMITC element for plate bending.

The QM6 is created by starting with the general four-node isoparametric quadrilateral element (Q4) and then making the following two modifications:

1. To prevent shear locking, bending deformation modes are included by adding incompatible modes to the element's displacement field. This reduces the element's tendency to be overly stiff when displaying bending-type deformations and creates the commonly known Q6 element (Cook 2002).
2. The determinant of the Jacobian is evaluated only at the middle of the element. This modification allows the Q6 element to represent constant stress (or shear) states for shapes other than rectangles and thus, the element is able to pass all patch tests and is renamed the QM6 element.

The PBMITC element is a four-node mixed interpolation of tensorial components element originally derived by Bathe and Dvorkin (1985). The inclusion of the mixed interpolation removes shear locking of the element while avoiding spurious zero-energy modes.

After combination of the QM6 and the PBMITC elements, an additional "null" drilling degree of freedom is included at nodes that are attached to the beam elements to accommodate the three rotational nodal dofs of the 3D frame element; only two rotational dofs are required at the other shell nodes.

An important aspect of SINBAD focuses on a formulation of the geometric stiffness for the web shell finite elements such that web local buckling modes are not considered in the 3D eigenvalue buckling solutions. As discussed in Section 2, due to the nature of the web being stable in its post-buckled state plus the fact that the brace demands are usually driven by lateral bending of the flanges, removing the local web modes provides a substantial improvement in the efficiency in the solution algorithm while focusing on solving for the member or frame out-of-plane buckling. As shown in Fig. 6, the following solution scheme is proposed to eliminate the handling of local web buckling modes:

1. Determine the stress at the Gauss points within each web "sub-element" from the in-plane analysis (in Fig. 6, 16 Gauss points, each with three stress measures).
2. Use Gauss quadrature within each sub-element to integrate over the volume of each "super-element," composed of all the sub-elements through the depth of the web, to obtain a single geometric stiffness for each web super-element.

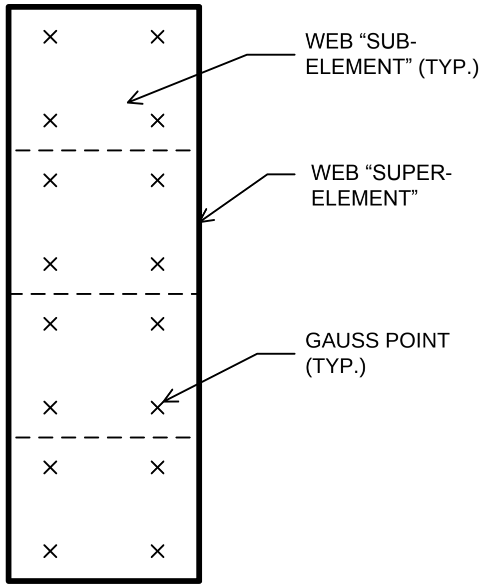

Figure 6: Web sub-elements for calculation of the geometric stiffness matrix

This process ultimately arrives at a solution that approximates the effect of "condensing" out the geometric stiffness terms associated with the internal web sub-element nodal dofs and leaving just the corner nodes of the "super-element" for the global 3D buckling solution. The global buckling solution is effectively limited to the movements associated only with the "super-element" corner nodes.

3.2.3 Bracing Elements

There are three distinct types of bracing (as termed by AISC 2010b), and any general combination of these types is addressed by SINBAD:

1. Nodal (discrete grounded) bracing
2. Relative (shear panel) bracing
3. Nodal torsional bracing

When implementing nodal bracing, SINBAD simply includes a grounded spring stiffness directly in the global stiffness matrix at the out-of-plane translational degree of freedom associated with the brace location. For relative bracing, SINBAD models the bracing as a "shear" spring by incorporating a basic 2x2 element stiffness matrix that resists the relative out-of-plane movement of two connected points. Lastly, nodal torsional bracing is modeled in a manner similar to the nodal relative bracing with the added step that the program must first divide the user input torsional bracing stiffness (in units of force x length/radian) by the web-depth squared to determine an equivalent shear spring stiffness. It is also important to note here that these spring elements are only required for the out-of-plane buckling solution. Since the 2D, nonlinear solution only deals with deflections in-plane, the degrees of freedom associated with the out-of-plane movement at the braces (or springs) is not activated and thus, not required in the planar global stiffness matrix.

3.3 Material Description

The material model employed in SINBAD considers that the steel remains elastic at modulus E up to the yield stress F_y (and yield strain ε_y), exhibits a yield plateau with a minimal hardening at a small modulus E_t (used for numerical purposes) from ε_y up to $\varepsilon_{st} = 10\varepsilon_y$, and then experiences strain hardening at E_{st} above ε_{st}. The tangent modulus is taken $E/100$ within the yield plateau of the material stress strain curve. It should be noted that this stiffness value is approximately equal to the bounding stiffness exhibited by typical structural steels upon cyclic loading of the material (see White 1988, for example). SINBAD uses E = 29,000 ksi for the steel elastic modulus and E_{st} = 900 ksi for the steel strain-hardening modulus.

3.4 Residual Stresses

Fig. 7 shows the residual stress pattern employed in SINBAD. This is selected as a nominal residual stress distribution that provides a close representation of the column inelastic flexural and beam inelastic lateral-torsional buckling strength curves from the AISC (2010b) Specification. The flange residual stress distribution is the same as that recommended by ECCS (1983) for rolled I-section beam members with $h/b_f > 1.2$, and the web residual compression is representative of that observed in welded I-section members with noncompact and slender webs (Avent and Wells, 1979; Nethercot, 1974). White (2008) discusses a large set of experimental data upon which the AISC lateral-torsional and flange local buckling strength curves are based, and shows that the influence of the type of I-section (rolled or welded) on the strengths is of minor significance given this data. This justifies the use of the single beam lateral-torsional buckling or column flexural buckling strength curves in AISC (no distinction in the resistances for rolled or welded cross-sections), and thus also may be used as a justification for the use of a single nominal residual stress pattern for the flanges as shown in Fig. 7. For I-section members with noncompact or slender webs, an interesting result is that the web typically cannot sustain substantial stresses in uniform axial compression over most of its depth without experiencing local buckling. Therefore, the maximum uniform web compressive residual stress is taken as the smaller value of $0.1F_y$ or the elastic local buckling stress of the web under uniform axial compression, F_{crw}, assuming simply-supported edge conditions. The web residual tension is modeled over a depth of $h/8$ at the top and bottom of the web such that the residual stress pattern in the web is self-equilibrating. Of course, the flange residual stresses shown in Fig. 7 are also self-equilibrating. For web-tapered members, the above web residual stresses are calculated at the mid-length of each tapered member. This is a simplification of the potential web residual stresses in a physical tapered member with a noncompact or slender web, which may in fact vary along the member length as a function of the web buckling resistance to the longitudinal residual compression.

4. Member Module

This section highlights the use of SINBAD to assess member bracing stiffness requirements. The results from SINBAD are compared to AISC (2010b) and to rigorous load-deflection simulations using Abaqus (Simulia 2011). The plots presented show representative member strength versus bracing stiffness results, which are commonly referred to as a “knuckle curves”. For these types of graphs, one generally plots the bracing stiffness in kips/inch for relative bracing and kips x inch/radian for torsional bracing on the abscissa and some measurement of the member strength on the ordinate ($M_{max}/(\phi M_y)$ for these plots). Knuckle curves generally exhibit a region that may be referred to as the “plateau.” The plateau is the portion of the knuckle curve where the strength of the fully-braced member has been reached and further increases in the bracing stiffness do not provide any significant additional member capacity.

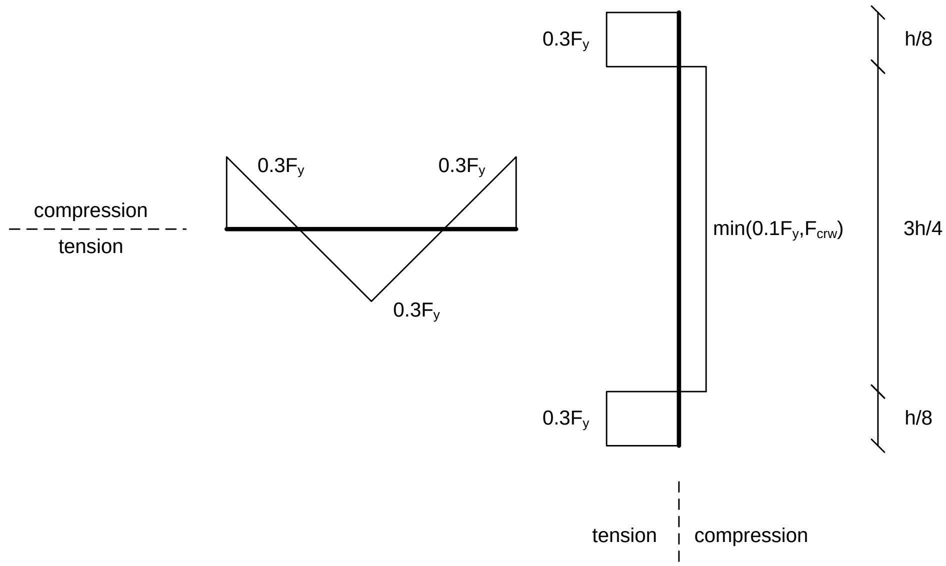

Figure 7: Residual stress pattern for flanges (left) and web (right)

4.1 Qualification of Simulations Using Abaqus

As mentioned above, Abaqus (Simulia 2011) is selected as the benchmark to assess SINBAD's ability to predict accurate bracing stiffness demands. Due to the complexity of the stability bracing systems in metal building frames combined with the wide range of factors that can influence the bracing response (as discussed in the introduction), a large number of member and frame tests is needed to develop meaningful (and generalizable) bracing data. Simulation provides the only economical approach to generate this data. Furthermore, stability bracing force demands generally are sensitive to the pattern of the geometric imperfections. Therefore, evaluating maximum required bracing forces for design generally necessitates the calculation of the critical geometric imperfection. Forcing initial geometric imperfections on physical members that cause the largest demands on the bracing system in an experimental test may prove difficult. However, critical geometric imperfections can be generated with relative ease for simulation studies. Validation of simulation methods against experimental test data for members on which detailed physical geometric imperfection and residual stress measurements have been documented provides an important validation of the simulation models. These simulation models can then be applied to provide detailed assessment of the true maximum design bracing strength and stiffness demands.

The Abaqus simulations discussed in this paper use the same residual stress pattern as described for SINBAD. Also, since load-deflection analyses are driven by initial imperfections, patterns of initial imperfections were selected within allowable erection tolerances specified in AISC's Code of Standard Practice (2010a) and the Metal Building Systems Manual (MBMA 2006). The nature of selecting and implementing the necessary initial imperfections is beyond the scope of this paper. The reader is referred to Tran (2009), Sharma (2010), Kim (2010), and Bishop (2013) for a more complete discussion of the process necessary to select the critical initial imperfections.

4.2 Example 1: W16x26 with Relative Bracing under Uniform Moment

Fig. 8 shows the results for a W16x26 loaded under uniform moment with three equally-spaced unbraced lengths, each with L_b = 4 ft. There are four relative braces, all of equal stiffness, along the compression flange of the member. The above unbraced length places this section in the inelastic buckling region for this cross-section profile. All the end conditions for these example cases are ideally simply-supported, flexurally and torsionally. That is, hinges and rollers are assumed for the in-plane end conditions and "fork" end conditions (out-of-plane flexure and warping of the flanges unrestrained) are assumed for the out-of-plane buckling analysis.

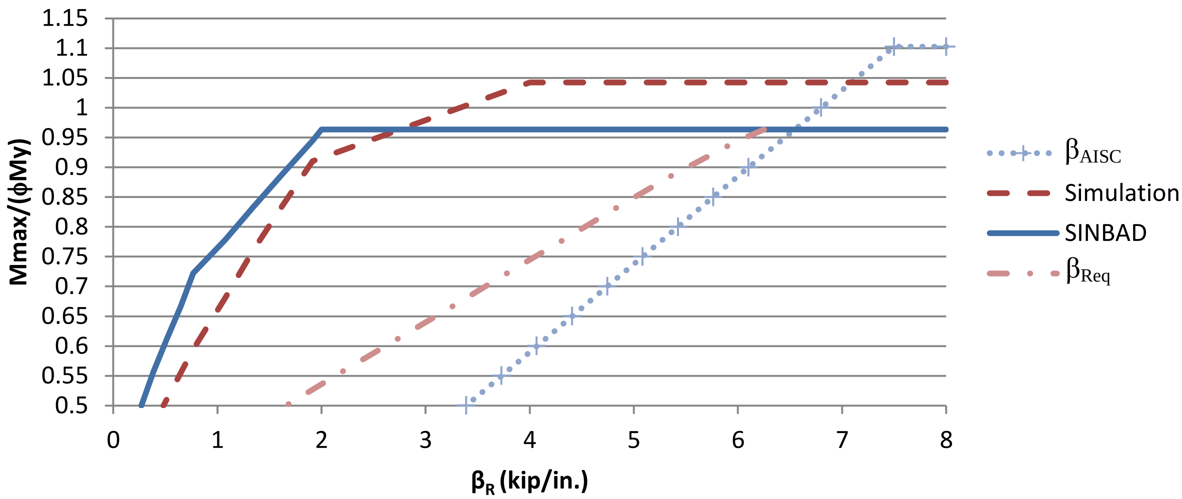

Figure 8: Example 1; W16x26, uniform moment, 4 relative braces, L_b = 4 ft

In Fig. 8 legend (and all subsequent legends in this section), the following nomenclature is used:

- "β_{AISC}" is the stiffness requirement obtained from AISC Appendix 6 for either relative bracing via Equation 1 or torsional bracing via Equation 2. Note that these equations have been rewritten to incorporate a refined estimate of the bracing stiffness based on the Commentary to Appendix 6 (AISC 2010b).

$$\beta_{br} = \psi\left(\frac{2(M_r/h_o)}{L_b}\right)C_{tR}C_d \qquad \text{(1, AISC A-6-6)}$$

$$\beta_T = 10\psi h_o^2 \left[\frac{M_r / C_b h_o}{P_{ef.eff}}\right]\left(\frac{M_r / C_b h_o}{L_b}\right)\frac{(n_T+1)}{n_T}C_{tT} \qquad \text{(2, AISC A-6-11)}$$

where $\psi = 1/\phi = 1/0.75 = 1.33$ for LRFD, L_b = spacing between the brace points, assumed constant in the development of Eq. 2, M_r = required flexural strength in the beam from the LRFD load combinations, h_o = distance between flange centroids, C_b = equivalent uniform moment factor for a given unbraced length, based on flange stresses for non-prismatic members (ad hoc extension), C_d = double curvature factor, C_{tR} = flange load height factor for relative bracing, C_{tT} = flange load height factor for torsional bracing, n_T = number of intermediate nodal torsional brace points within the member length between the rigid "end" brace locations, where both twisting and lateral movement of the beam are prevented, $P_{ef.eff}$ = effective flange buckling load = $\pi^2 EI_{f.eff} / L_b^2$, E = steel modulus of elasticity = 29,000 ksi, $I_{f.eff} = I_{yc}$ for doubly-symmetric sections or $(I_{yc} + \frac{t}{c} I_{yt})/2$ for singly-symmetric sections, c = distance between cross section centroid and compression flange centroid, t = distance between cross-section centroid and tension flange centroid, I_{yc} = lateral moment of inertia of the compression flange, and I_{yt} = lateral moment of inertia of the tension flange.

- "Simulation" refers to the member maximum strength obtained from the test simulations conducted using Abaqus.

- "SINBAD" refers to the member inelastic, out-of-plane buckling strength from SINBAD based on a given ideal bracing stiffness. In this paper, ideal bracing stiffness is defined as the stiffness required to brace a perfectly straight member such that a particular out-of-plane buckling resistance is achieved. Due to the fact that physical columns and beams have inherent geometric imperfections, immediately upon application of load, the initial deflections present in the member begin to be amplified (Timoshenko and Gere 1961). This amplification of the initial imperfections impacts the brace force demands, since brace point displacement times the bracing stiffness (assumed to remain elastic) generates the brace forces. Therefore, one finds that while theoretically it is possible to provide the ideal stiffness for a given member assuming the member is perfectly straight, practically, one must often provide some multiple of that value in an effort to keep the brace forces (and brace strength design requirements) manageable for real members where initial imperfections are unavoidable. It is common in the literature to use a multiple of $2/\phi$ where the resistance factor ϕ= 0.75 on the ideal bracing stiffness in order to limit the amplification of brace point movement and the resulting brace forces (Yura 2001).

- "β_{Req}" is a "SINBAD" generated curve in which the abscissa is scaled by the larger of $3/\phi$ times the result determined from an *elastic* eigenvalue buckling analysis or $2/\phi$ times the result determined from an *inelastic* eigenvalue buckling analysis at a given strength level. The coefficients $2/\phi$ and $3/\phi$ are needed as discussed above and to parallel the approach taken in AISC Appendix 6 of scaling the ideal bracing stiffness to determine a required design bracing stiffness, where ϕ is the resistance factor taken equal to 0.75 (AISC 2010b). The justification of this curve is detailed below.

The first item of note from Fig. 8 is the difference in the strength of the member reached when full bracing is provided; i.e., the plateau strength comparisons from the SINBAD, simulation, and AISC results. In this case, SINBAD reaches a strength limit at $M_{max}/(\phi M_y)$ = 0.96, Abaqus suggests $M_{max}/(\phi M_y)$ = 1.05 and the AISC equations suggest $M_{max}/(\phi M_y)$ = 1.1. The following is a discussion on why this variance in the capacity for rigid bracing occurs.

The main difference between the results from a general eigenvalue buckling analysis (SINBAD in this case) and the AISC Specification is due to the lateral movement of the compression flange. The inelastic eigenvalue buckling analysis is based on the state of the structure associated with in-plane deformations. That is, the member is assumed to displace solely in-plane until the critical buckling load is reached. At this point, the member then displaces laterally out-of-plane an undetermined amount. Conversely, the AISC design curves are a fit to experimental data. Therefore, due to the imperfect nature of the initial geometry in real test specimens, lateral movement of the compression flange initiates immediately with the onset of load. This lateral movement increases as the load increases, exacerbating the compression on one side of the compression flange and relieving compression on the other. Therefore, a potentially higher load may be reached in certain cases in experimental tests when compared to an eigenvalue buckling analysis. The ASCE-WRC Plastic Design Guide (ASCE 1971) discusses this phenomenon in detail and documents these observations for compact-web sections with relatively close brace spacing subjected to uniform moment.

One key source of differences between the results from the simulation results from Abaqus and the AISC Specification is due to the initial conditions; i.e., the magnitude and location of initial imperfections and the magnitude and distribution of residual stresses. The residual stresses, as discussed in Part 3, are representative of those determined from Avent and Wells (1979) and Nethercot (1974). However, many experiments on which the design curves are based did not have their residual stresses recorded. Also, as was alluded to earlier yet the details were omitted, the initial imperfections determination may not be as unfavorable as those employed in the simulation. Due to the current erection tolerances specified by AISC (2010a) and MBMA (2006), it is common to take some combination of these tolerances to create a "worst case scenario" for the pattern of initial imperfections. Where one might suggest that a procedure that invokes the worst possible pattern is overly conservative, it may be difficult to justify a more unconservative pattern. Statistically, it is improbable for an experimental specimen to have the worst case imperfection pattern as well as the worst-case residual stress pattern. Therefore, the imperfections and residual stresses used in the simulation will often be higher than those realized in experiments, leading to a solution predicting less capacity. In addition, for the uniform inelastic bending case, the representation of the member response as a continuum is a clear idealization of the physical response generally observed from experiments (ASCE 1971).

Next, one may compare the stiffness suggested from β_{Req} versus that suggested by β_{AISC}. One of the reasons the AISC prediction is high relative to the results from SINBAD (larger bracing stiffness required to achieve a given beam moment resistance) is because the AISC equation for the relative bracing stiffness requirement (Eq. 1) ignores the contribution of the member to the resistance of brace point movement (Yura 2001). That is, all out-of-plane displacements must be resisted by the provided bracing, with no consideration of the member's own inherent ability to resist brace point movement. The two plots from SINBAD (including both the ideal stiffness and the scaled ideal stiffness) consider the member's ability to resist out-of-plane brace movement.

As introduced earlier, β_{Req} is produced by combining the larger of 3/0.75 times the ideal *elastic* bracing requirement from SINBAD or 2/0.75 times the ideal *inelastic* bracing requirement from SINBAD. (For clarity, the bracing stiffness required from an elastic eigenvalue buckling analysis using SINBAD is not shown on the plot.) β_{Req} is a reasonable metric since it incorporates values of bracing stiffness that would buckle the member elastically (using 3/0.75 times the ideal stiffness), yet reduces this scaling of the stiffness requirements to 2/0.75 as the member begins to yield. Also, it should be noted that β_{Req} has been calibrated to a large set of results. Therefore, the conservatism realized in this specific case is not necessarily representative of the level of conservatism in other cases.

4.3 Example 2: W16x26 with Torsional Bracing under Uniform Moment

Example 2 is also a W16x26 loaded with uniform moment. However, the member now has 3 interior torsional braces, all with equal stiffness, in lieu of relative bracing. Again, L_b = 4 ft and the member is classified in the inelastic lateral-torsional buckling region. The results for Example 2 are shown in Fig. 9.

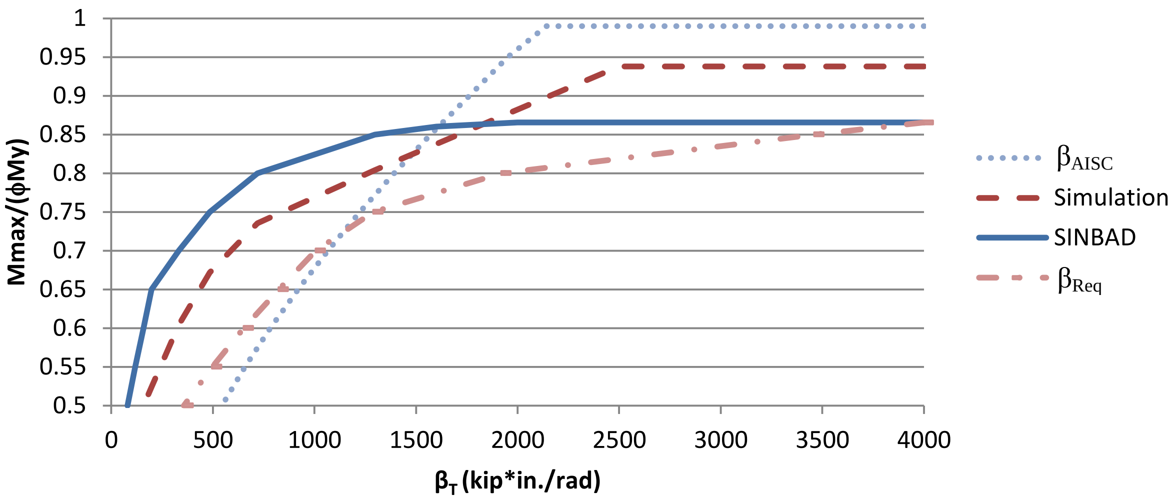

Figure 9: Example 2; W16x26, uniform moment, 3 torsional braces, L_b = 4 ft

When looking at the bracing requirement for a particular load level, say at $M_{max}/(\phi M_y)$ = 0.65, the Abaqus simulation suggests that a bracing requirement of 450 kip x in./rad is required at each brace. The prediction for the ideal bracing requirement from SINBAD is closer to 200 kip x in./rad. However, to keep brace point displacements small to limit the brace force, some multiple of the ideal stiffness is required (see the discussion at the beginning of this section). β_{Req} suggests that a stiffness closer to 820 kip x in./rad. The requirements from AISC would suggest that around 900 kip x in./rad is required. The slight variation between the two requirements at this load level is due to the different approximations associated with each of these calculations.

5. Frame Module

In this section a 90 ft clear-span frame is analyzed via SINBAD and compared to the simulation results from Abaqus. Kim (2010) and Sharma (2010) studied this frame in-depth and the reader is referred to these theses for a more detailed structure description and presentation of their findings. Fig. 10 and Table 1 provide a synopsis of the section properties pertinent to this study for various locations along the frame. Specifically, location r1 is of importance in the following discussion. The frame is subjected to an applied, uniformly distributed vertical load on the horizontal projection of the roof.

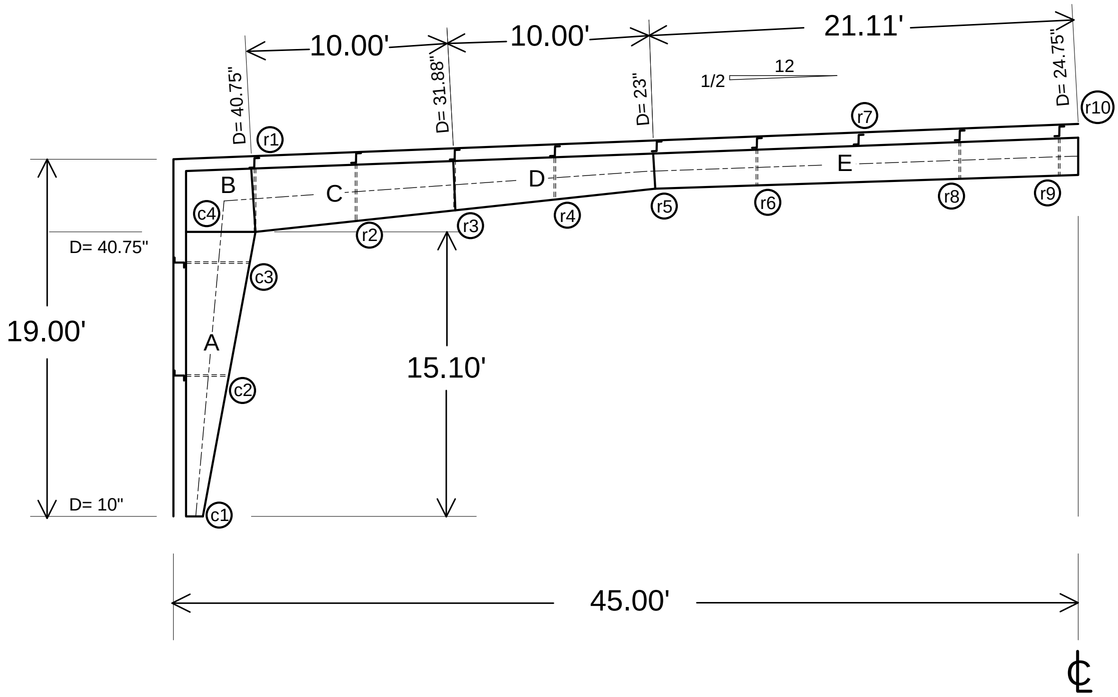

Figure 10: Elevation view of ninety foot clear-span frame (from Kim 2010)

Table 1: 90' Clear-span frame geometry (from Sharma 2010)

Length	Location	Web				Inside Flange			Outside Flange		
		d (in)	t_w (in)	h/t_w*	h_c/t_w	b_f (in)	t_f (in)	$b_f/2t_f$	b_f (in)	t_f (in)	$b_f/2t_f$
A	c1	10.00	7/32	42	36	6.0	1/2	6.0	6.0	3/8	8.0
	c2	25.27		112	103						
	c3	37.49		167	157						
	c4	40.75		182	172						
B			7/32								
C	r1	40.75	1/4	160		6.0	3/8	8.0	6.0	3/8	8.0
	r2	36.31		142							
	r3	31.88		125							
D	r3	31.88	3/16	166		6.0	3/8	8.0	6.0	3/8	8.0
	r4	27.44		142							
	r5	23.00		119							
E	r5	23.00	5/32	142		6.0	3/8	8.0	6.0	3/8	8.0
	r6	23.42		145							
	r7	23.80		148							
	r8	24.25		150							
	r9	24.67		153							
	r10	24.75		154							

*For F_y = 55 ksi, the webs are compact for $h/t_w \leq 86$ and they are slender for $h/t_w \geq 130$.

For this example, the wall panel bracing and all the cable bracing are set to the values provided by the design engineer; the torsional bracing stiffness is then varied in the analysis. However, to use SINBAD correctly given a *provided* relative bracing stiffness, one would first need to divide the provided stiffness by 2/0.75 or 3/0.75 to perform the analysis. After the analysis is complete, the designer would then scale the relative bracing by 2/0.75 or 3/0.75 to bring the relative bracing stiffness back up to the level provided. This procedure is required since SINBAD provides an assessment of the ideal bracing stiffness.

In this analysis, for simplicity, it is assumed that each torsional brace has the same stiffness. As such, the knuckle curve corresponding to the torsional bracing stiffness at any point along the frame is as shown in Fig. 11. In general, a given ratio of the different bracing stiffnesses can be specified, and then all the stiffnesses can be scaled while maintaining this ratio.

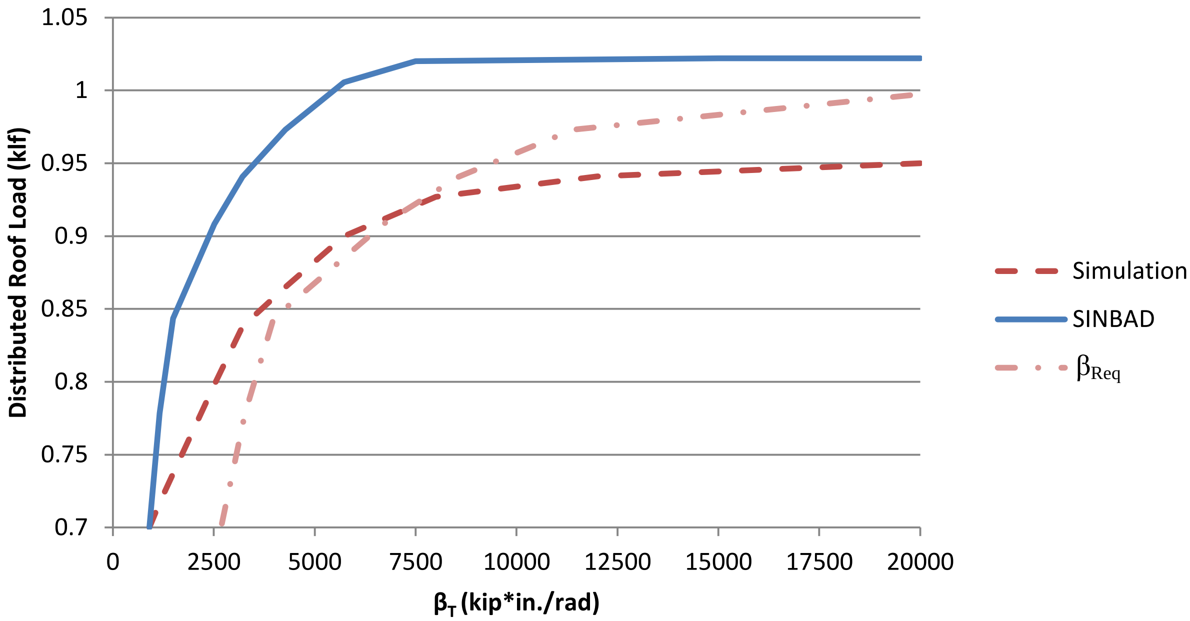

Figure 11: 90’ clear-span frame knuckle curve

The torsional bracing stiffness requirement at location r1 in the knee region, determined from an adaptation of the AISC Appendix 6 provisions, is 32700 kip x in./rad (Sharma 2010). From Fig. 11, to reach a distributed load of say, 0.9 klf, the Abaqus model requires approximately 5700 kip x in./rad where the required beta from SINBAD suggests 6200 kip x in./rad. The significant difference here between the simulation, SINBAD, and the adapted Appendix 6 stiffness requirements can be largely attributed to the combination of bracing types present in the framing system. As mentioned previously, the AISC equations do not consider the reduction in bracing stiffness requirements that may be possible if multiple bracing types are employed simultaneously. SINBAD has the capability to include the roof/wall panels as well as any X-bracing in the framing system, bracing inherently engaged in the physical structure upon out-of-plane movement, along with the torsional bracing to develop a complete bracing system requirement. β_{Req} may slightly under-predict the stiffness requirement from the simulation, yet still suggests bracing stiffness levels that are able to allow the frame to reach greater than 95% of its intended capacity.

6. Recommendations for Design

This section provides a summary of the steps necessary to use SINBAD to determine required bracing stiffnesses for complex bracing systems.

6.1 Design for all Bracing Types

- Set the elastic modulus for design, $E^* = 0.9E$; required for designs according to AISC's Appendix 1 (2010b). AISC Appendix 1 is invoked for the in-plane analysis to allow the use of SINBAD to determine the bracing requirements from an inelastic analysis independent of individual Specification section checks.
- Determine the knuckle curve for the *inelastic* structure.
    - Set the yield strength for design, $F_y^* = 0.9F_y$; required for designs according to Appendix 1.
    - Set all bracing as rigid and *iterate the load level* until an eigenvalue equal to one is reached; this gives the rigidly braced strength. Note: if an eigenvalue ≥ 1 cannot be reached, then consider either adding bracing, changing the size and spacing of bracing, or increase the section properties of the member that is being braced and rerun the analysis.
    - Pick different values of load level (ensuring they are less than the rigid load level) and *iterate the bracing stiffness* until an eigenvalue equal to one is reached.
    - Continue this process until sufficient data points are captured to create the inelastic buckling knuckle curve.
- Determine the knuckle curve for the *elastic* structure.
    - Start with the minimum load level obtained from the inelastic analysis and *iterate the bracing stiffness* at each load level until an eigenvalue of one is reached.
    - Continue this process until sufficient data points are captured to create the elastic buckling knuckle curve.
- Determine β_{Req}.
    - Each point on the β_{Req} curve is determined by taking the larger of 3/0.75 times the elastic stiffness curve or 2/0.75 times the inelastic stiffness curve at each load level.
    - Continue this process until β_{Req} has been created and added to the knuckle curve plots.
- To determine the required stiffness for a given load level: Enter the knuckle curve at the appropriate load level and move horizontally until intersection with β_{Req}. The value on the abscissa will be the required stiffness for the relative or torsional brace.

6.2 Consideration of Bracing Strength Requirements

Sharma (2010) conducted extensive studies of the bracing strength and stiffness requirements using full nonlinear finite element test simulations of complete metal building frames. He determined that the strength requirements were rarely more than 4% of the corresponding member internal moment for torsional bracing (as long as the bracing stiffness was reasonably close to the "knuckle" of the knuckle curve or larger). In fact he found that usually around 2% of the internal moment was enough to get the member to within 95% of the member's capacity. Lastly, he noted that the comparable relative bracing limit was around 1%.

In this research, the key concept is to determine the ideal stiffness directly and use a multiple of that stiffness as the torsional or relative stiffness requirement. However, if the bracing stiffness is too low, then substantial amplification of system geometric imperfections can occur. This can lead to excessive brace point movement and correspondingly excessive brace forces. Since brace point movement is directly proportional to the brace force (i.e., the brace strength requirement), it is imperative to limit the out-of-plane movement of the brace points. Yura (2001) largely championed the procedure necessary to scale the ideal bracing stiffness accordingly in order to limit these brace forces. The AISC Specification (2010b) embodies this by using a factor of $2/\phi$ in the bracing stiffness equations. In this paper, all the suggested recommendations require a minimum multiple of the ideal stiffness of $2/\phi$ based on the inelastic structure and $3/\phi$ based on the elastic structure. The computational tool, SINBAD, provides an efficient and rigorous calculation of the ideal bracing stiffness values.

A review of the bracing strength requirements associated with the suggested β_{Req} values scaled from the SINBAD ideal bracing stiffness values, indicates that for cases including both full bracing as well as partial bracing (up to certain limits), the percentage of equivalent flange force is often under 2% for torsional bracing and is closer to 1% for relative bracing. This evidence corroborates the findings by Sharma (2010).

7. Conclusions

SINBAD incorporates a number of attributes of metal building frames that lie outside the scope of AISC's Appendix 6, namely:

- Consideration of web-tapered members,
- Unequal brace stiffness,
- Unequal brace stiffness,
- Warping and lateral bending restraint from joints and continuity across brace points,
- Combination of bracing types, and
- Reduced demands in non-critical regions.

This paper builds on prior work by Yura (1993, 1995, 1996, 2001), Yura and Helwig (2009), Tran (2009), Kim (2010), and Sharma (2010) and aims to present a method that is based on sound theory yet is practical for everyday design use. The use of an inelastic eigenvalue buckling tool (SINBAD) calibrated to extensive simulation studies provides the engineer with a more accurate means with which to incorporate the above attributes into a safe structural design for the complex bracing configurations that can occur in general structural steel members as well as entire structural systems.

Acknowledgments

The authors would like to thank the Metal Building Manufacturers Association (MBMA) for sponsoring this research. Special thanks are extended to the MBMA Research Steering Committee, which has been instrumental in helping the authors to make this research project a success. The views expressed in the paper are solely those of the authors and do not necessarily represent the positions of the aforementioned organizations or individuals.

References

AISC (2010a). "Code of Standard Practice for Steel Buildings and Bridges", American Institute of Steel Construction, Inc. Chicago, IL. AISC 303-10.

AISC (2010b). "Specification for Structural Steel Buildings", American Institute of Steel Construction, Inc. Chicago, IL. ANSI/AISC 360-10.

ASCE-WRC (1971). "Plastic Design in Steel – A Guide and Commentary", Manual 41, Welding Research Council and the American Society of Civil Engineers.

Avent, R. R. and Wells, S. (1979), "Factors Affecting the Strength of Thin-wall Welded Columns", Report Submitted to Metal Building Manufacturers Association, Cleveland, OH, December.

Bathe, K. and Dvorkin, E. (1985), "A Four-node Plate Bending Element Based on Mindlin/Reissner Plate Theory and a Mixed Interpolation," International Journal for Numerical Methods in Engineering, 21, Pp. 367-383.

Bishop, C.D. (2013). "Flange Bracing Requirements for Metal Building Systems", Doctoral Dissertation, Georgia Institute of Technology, Atlanta, GA, *awaiting publication*.

Cook, R., Malkus, D., Plesha, M., and Witt, R. (2002), Concepts and Applications of Finite Element Analysis, John Wiley and Sons, New York.

ECCS (1983). "Ultimate Limit State Calculations of Sway Frames with Rigid Joints," Technical Working Group 8.2, Systems, Publication No. 33, European Commission for Constructional Steelwork, November, P. 20.

Kaehler, R.C., White, D.W., and Kim, Y.D. (2011). "Frame Design Using Web-Tapered Members", AISC Steel Design Guide 25, American Institute of Steel Construction, Chicago, IL.

Kim, Y. D. (2010). "Behavior and Design of Metal Building Frames Using General Prismatic and Web-Tapered Steel I-Section Members", Doctoral Dissertation, Georgia Institute of Technology, Atlanta, GA.

MBMA (2006). Metal Building Systems Manual, Metal Building Manufacturers Association, Cleveland, OH.

MathWorks (2011). MATLAB, Software and Help Documentation, Version R2011b(7.13.0.564).

McGuire, W., Gallagher, R., and Ziemian, R. (2000), Matrix Structural Analysis, Second Edition, New York: John Wiley & Sons, Inc.

Nethercot, D. A. (1974), "Buckling of Welded Beams and Girders", Proc. International Association for Bridge and Structural Engineering, Vol. 57, 291-306.

Sharma, A. (2010). "Flange Stability Bracing Behavior in Metal Building Frame Systems", In Partial Fulfillment of the Requirements for the Degree, Master of Science in the School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA.

Simulia (2011). Abaqus, Software and Users' Manuals, Version 6.11.

Timoshenko, S. and Gere, J. (1961), Theory of Elastic Stability, New York: McGraw-Hill.

Tran, D. Q. (2009). "Towards Improved Flange Bracing Requirements for Metal Building Frame Systems", In Partial Fulfillment of the Requirements for the Degree, Master of Science in the School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA.

White, D.W. (1985). "Material and Geometric Nonlinear Analysis of Local Planar Behavior in Steel Frames Using Interactive Computer Graphics", In Partial Fulfillment of the Requirements for the Degree, Master of Science, Cornell University, Ithaca, NY, August.

White, D.W. (1988). "Analysis of Monotonic and Cyclic Stability of Steel Frame Subassemblages," In Partial Fulfillment of the Requirements for the Degree, Doctor of Philosophy, Cornell University, Ithaca, NY, January.

White, D.W., and Kim, Y.D. (2006). "A Prototype Application of the AISC (2005) Stability Analysis and Design Provisions to Metal Building Structural Systems", Report Prepared for Metal Building Manufacturers Association, January.

Yura, J.A. (2001), "Fundamentals of Beam Bracing," Engineering Journal, AISC, 38(1), 11-26.

Yura, J.A. (1996), ''Winter's bracing approach revisited,'' Engineering Structures 8(10), 821-825.

Yura, J.A. (1995), "Bracing for Stability – State-of-the-Art," Is Your Structure Suitably Braced?, 1995 Conference, Structural Stability Research Council.

Yura, J.A. (1993), "Fundamentals of Beam Bracing," Is Your Structure Suitably Braced?, 1993 Conference, Milwaukee, Wisconsin, Structural Stability Research Council, Bethlehem, PA, 1-40.

Yura, J. A. and Helwig, T. A. (2009). "Bracing for Stability," Short course sponsored by the Structural Stability Research Council, North American Steel Construction Conference, Phoenix, AZ, April.