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SHEAR STRENGTH OF WEB-TAPERED I-SHAPED MEMBERS

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Abstract
The shear strengths of ten slender, built-up, unstiffened, tapered I-shaped specimens were measured and compared to strength predictions computed using the AISC 2010 *Specification* method and a new method proposed by S.C. Lee, C.H. Yoo, and colleagues. The comparisons indicate that the AISC 2010 *Specification* shear strength prediction method is overly conservative and that the Lee et al. method is accurate. The method by Lee et al. takes into account certain assumptions and mechanics, including realistic boundary conditions at the web-flange connection and post-buckling strength, that were not used in the development of the AISC 2010 *Specification* shear strength equations but seem to be indicated by the experimental research.

1. Introduction

Over the last five decades, steel plate girder shear strength has been the subject of numerous research projects, most of which focused on quantifying the ultimate strength including post buckling strength (tension field action (TFA)) of web panels bounded by transverse stiffeners. Few projects have been completed on unstiffened plate girders. Similarly, few projects have been completed on the subject of tapered member shear, and none on unstiffened tapered members shear strength. However, unstiffened tapered plate girders are commonly used as bridge girders and metal building system rafters. In these applications, the web plates are often optimized, increasing the likelihood that shear strength will control the design, and thus increasing the importance of having an accurate shear strength prediction method. Therefore, the objective of this research was to determine the accuracy of shear strength prediction methods available in the literature.

2. Literature Review

The most widely cited research, published by Basler in 1961, forms the basis of the current AISC *Specification* (2010) Sections G2 and G3. Basler (1961) stated that the ultimate shear strength is the sum of the shear buckling and post-buckling strength provided by tension field action (TFA). The fundamental assumption is that, at loads below the shear buckling load, the web is subjected to a stress state with pure shear (equal compressive and tensile principal stresses), but the compressive stress does not increase after shear buckling. Therefore, to allow the tensile stress

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field to further increase, some other element must provide the equilibrating compressive force (or otherwise, vertical equilibrium is not satisfied for a free body diagram of a portion of the web). Basler (1961) reasoned that transverse shear stiffeners provide the necessary compressive force and the plate girder performs much like a Pratt truss with the diagonals in tension and vertical stiffeners in compression. Therefore, by Basler's reasoning, unstiffened plate girders have no tension field action, so the total shear strength is the shear buckling strength.

To compute the shear buckling strength, Basler started with the classical plate buckling equation which is presented in numerous textbooks including Bleich (1952), Timoshenko and Gere (1961), and Salmon, Johnson, and Malhas (2009):

$$\tau_e = \frac{k_v \pi^2 E}{12(1-\upsilon^2)(h/t_w)^2} \tag{1}$$

where

τ_e = elastic shear buckling stress

k_v = plate shear buckling coefficient for shear stress

h = web plate height

t_w = web thickness

The plate shear buckling coefficient, k_v, is a function of the web panel aspect ratio (*a*/*h*, where *a* is the clear distance between transverse stiffeners) and the type of boundary condition at the flange—simply supported (hinged), fixed, or something in between. Basler (1961) chose the most conservative boundary condition option: simply supported connection between the web and flanges. Porter et al. (1975) also used this value. Bleich (1952) provided the following equations for the simply supported web shear buckling coefficient, denoted here as k_{ss} (the first subscript to denote shear; the second subscript indicates simply supported connection at the flanges). For long panels such as the ones of interest in the present study, the shear buckling coefficient, $k_v = 5.34$ which is approximately the value adopted by the AISC *Specification* (AISC 2010) Section G2, $k_v = 5$.

$$k_{ss} = 5.34 + \frac{4}{(a/h)^2} \; for \; a/h \geq 1 \tag{2}$$

$$k_{ss} = 4 + \frac{5.34}{(a/h)^2} \; for \; a/h < 1 \tag{3}$$

Basler's equations were first adopted into the AISC *Specification* in 1963, and have been carried forward to the 2010 *Specification*, as shown below.

$$V_n = 0.6 F_y A_w C_v \tag{4}$$

where C_v is the ratio of the shear buckling strength to the plastic (full yield) shear strength.

On the other end of the spectrum, Chern and Ostapenko (1969) used the plate buckling coefficient shown in Eq. (5) and (6) for a rotationally fixed boundary at the top and bottom of the web. For panels with large a/h, $k_v = 8.98$, a result also given by Timoshenko and Gere (1961).

$$k_{sf} = 8.98 + \frac{5.61}{(a/h)^2} - \frac{1.99}{(a/h)^3} \; for \; a/h \geq 1 \tag{5}$$

$$k_{sf} = \frac{5.34}{(a/h)^2} + \frac{2.31}{a/h} - 3.44 + 8.39\frac{a}{h} \; for \; a/h < 1 \tag{6}$$

Lee et al. (1996) proposed the following shear buckling coefficient equations for rectangular panels of I-shaped beams where Eq. (7) and Eq. (8) resemble a condition between simply supported and rotationally fixed, closer to rotationally fixed, that is a more realistic representation of the boundary condition.

$$k_v = k_{ss} + 0.8(k_{sf} - k_{ss})\left[1 - \frac{2}{3}\left(2 - \frac{t_f}{t_w}\right)\right] for \; 0.5 < \frac{t_f}{t_w} < 2 \tag{7}$$

$$k_v = k_{ss} + 0.8(k_{sf} - k_{ss}) \; for \; \frac{t_f}{t_w} > 2 \tag{8}$$

Lee et al. (1996) examined finite element analysis (FEA) results for over 300 synthetic specimens and concluded that the shear buckling coefficient is a function of the flange-to-web thickness ratio (t_f/t_w), and is between k_{ss} and k_{sf}.

Dr. S.C. Lee and Dr. C.H. Yoo published a series of papers (Lee and Yoo (1998), Lee and Yoo (1999), Yoo and Lee (2006)) in which they explain an alternative theory for post-buckling strength of stiffened rectangular plate girders. (Because the current study is concerned with unstiffened panels, some of their results are not directly applicable. However, some of their results are applicable, and some serve as the foundation of Lee et al. (2008) which is directly applicable.) In Lee et al. (1998), they performed geometric and material nonlinear FEA on hypothetical plate girders to quantify buckling, post-buckling, and overall strength. From those synthesized specimens, the researchers observed that the post-buckling strength is approximately 40% of the difference between the elastic shear buckling strength and the plastic shear strength. They proposed the following equations, with slight nomenclature changes to be more consistent with AISC variable names, which predict strengths that almost exactly match the FEA predictions.

$$V_n = V_{cr} + V_{PB} = V_{cr} + 0.4(V_p - V_{cr}) = 0.6V_{cr} + 0.4V_p \tag{9}$$

The plastic shear strength, $V_p = 0.58F_y t_w h$, is almost identical to the AISC *Specification* shear yield strength. The 0.58 factor is the theoretical value per the von Mises yield criterion, and the calculations are done in terms of the web depth, h. However the difference between V_p and the AISC *Specification* yield strength is quite small.

Introducing the variable C_v, as in the AISC *Specification*, the proposed nominal strength is $V_n = V_p(0.6C_v+0.4)$. C_v is a three-part function almost identical (slight round-off differences) to the one given in the AISC *Specification* (2010).

$$C_v = 1 \qquad h/t_w \le 1.12\sqrt{k_v E/F_y} \tag{10}$$

$$C_v = {1.10\sqrt{k_v E/F_y}}\Big/{(h/t_w)} \qquad 1.12\sqrt{k_v E/F_y} < h/t_w \le 1.4\sqrt{k_v E/F_y} \tag{11}$$

$$C_v = {(1.57 k_v E)}\Big/{(h/t_w)^2 F_y} \qquad h/t_w > 1.4\sqrt{k_v E/F_y} \tag{12}$$

Lee et al. (1998) provided several important findings. First, because the equations shown above produced shear buckling and ultimate strengths that nearly exactly matched the FEA results, the shear buckling coefficients proposed in Lee et al. (1996) are shown to be accurate.

Second, during a series of analyses intended to assess the influence of flange stiffness on post-buckling strength, Lee et al. (1998) made the very interesting discovery that web panels with no flange possess nearly the same post-buckling strength as panels with very heavy flanges. This led them to put forth a profound new theory to explain the post-buckling strength of stiffened panels.

Lee and Yoo (1999) reported experimental findings that were generated to verify the equations and theories proposed in Lee et al. (1998). During the experimental program, they tested ten plate girders to failure (a/h ranging from 1.0 to 3.0, so these were stiffened panels), with eight of them failing in shear.

One objective was to investigate the restraint at the web-to-flange connection and verify the shear buckling coefficient proposed in Lee et al. (1996). Because of large initial imperfections, obvious bifurcation buckling was not observed, so it was not possible to identify the elastic shear buckling strength and thus not possible to infer the boundary conditions from the buckling load. However, the researchers inspected the final buckled shape for two specimens, finding that they resembled the buckling mode shape of a fixed-fixed column, thus implying that "the boundary condition at the flange-web juncture is very close to fixed."

Lee and Yoo (1999) also showed that the shear strength equation presented in Lee et al. (1998) was indeed very accurate, with an average measured-to-predicted ultimate shear strength ratio 1.01 (COV=4%) for the specimens that failed by shear buckling. They also concluded that through-thickness (out-of-plane) bending of the web has a significant effect near failure. Finally, probably the most important result toward the current project's objectives is the conclusion that "an anchoring system, such as the flanges, is not needed for the development of postbuckling strength." This conclusion sheds light on the source of postbuckling strength for unstiffened panels.

Yoo and Lee (2006) studied and explained the source of postbuckling strength for stiffened panels. They stated that the fundamental assumption in the classical failure theories is that the "compressive stresses that develop in the direction perpendicular to the tension diagonal do not increase any further once elastic buckling has taken place." Lee and Yoo (2006) did not state the following, but this fundamental assumption runs contrary to the fact that, upon plate buckling, the portions of the plate far from supports become more flexible so do not accept further load, but the portions of the plate near the support continue to accept additional stress. This is the basis for the effective width concepts used to develop the effective widths used in the AISC *Specification* Section E7 and explained in Salmon, Johnson, and Malhas (2008). Lee and Yoo (2006) performed material and geometric nonlinear FEA of hypothetical specimens and investigated changes in the tension and compression stress fields. They discovered that the compression stress field does, in fact, increase near the supports, which for their stiffened panels, are the flanges and stiffeners.

Lee et al. (2008) extended their previous work to long web panels such as those of interest for the current study in their paper "Ultimate Shear Strength of Long Web Panels." They performed nonlinear FEA on hypothetical plate girders with a/h ratios ranging from three to six. Their Table 2 indicates that the shear buckling strength equation (using the shear buckling coefficient from Lee et al. (1996)) accurately and slightly conservatively predicted the shear buckling prediction from the FEA.

The researchers also compared predictions from the ultimate shear strength equations from Lee and Yoo (1998) to the FEA predictions, indicating that the equations are accurate for low h/t_w ratios, but are unconservative by 12-40% for h/t_w ratios between 210 and 300 for $a/h = 6$. Their FEA results indicated that significant postbuckling strength existed in the hypothetical specimens, although the researchers did not explain the source. It seems reasonable to assume that the postbuckling strength is due to a similar compression field stress redistribution as that described in Lee and Yoo (2006), although less efficient as indicated by the fact that the Lee and Yoo (1998) equations slightly over-predict the ultimate strength compared to the strength predicted by FEA.

Because the Lee and Yoo (1998) equation over-predicted the ultimate strength for long panels, Lee et al. (2008) developed an adjustment factor, λ, to bring the equations into agreement with the FEA. When the equations from Lee and Yoo (1998) are multiplied by λ, the equations provide slightly conservative results compared to the FEA predictions, with the ratio of FEA-to-equation result ranging from 1.00 to 1.04 for a/h=6. They also observed that real plate girders have larger initial imperfections (h/120) than those used in the models, so they re-analyzed the hypothetical specimens with larger initial imperfections, indicating that a further adjustment factor is necessary. They were able to locate one directly applicable experimental test specimen, and their equation almost exactly predicted the failure load, giving an indication of its accuracy.

The following is their strength prediction equation, which account for realistic initial imperfections.

$$V_n = R\lambda V_p(0.6C_v + 0.4) \tag{13}$$

The high slenderness factor, λ is given by the following:

$$\lambda = 1.0 \qquad C_v \geq 0.3 \tag{14}$$

$$\lambda = 1.35C_v + 0.6 \qquad 0.1 < C_v < 0.3 \tag{15}$$

$$\lambda = 5.62C_v + 0.145 \qquad C_v = 0.1 \tag{16}$$

The geometric imperfection factor, R is given by the following:

$$R = 1.0 - 0.2\frac{h/t_w\sqrt{F_y/k_vE}}{1.10} \qquad h/t_w < 1.10\sqrt{\frac{Ek_v}{F_y}} \tag{17}$$

$$R = 0.8 + 0.2\frac{h/t_w\sqrt{F_y/k_vE} - 1.10}{1.10} \qquad 1.10\sqrt{\frac{Ek_v}{F_y}} \leq h/t_w \leq 2.20\sqrt{\frac{Ek_v}{F_y}} \tag{18}$$

$$R = 1.0 \qquad h/t_w > 2.20\sqrt{\frac{Ek_v}{F_y}} \tag{19}$$

So-called modified shear force methods such as those suggested by William & Harris (1957) and Blodgett (1966) were also investigated in this research. Concisely, the principal flange stresses due to bending are parallel to the angle of inclination, so the transverse component of each flange force reduces the required web shear; the flanges in a tapered member therefore provide shear resistance. The modified shear approach is summarized as follows: (1) the required web shear is not the entire shear at a section, but is the total shear at a section minus the transverse component of each flange force, and (2) the required web shear is compared to the web shear yielding or buckling strength computed using the AISC 2010 *Specification* or some other source.

3. Experimental Program

3.1. Test Setup

A simply supported beam specimen with a midspan point load exactly simulates the shear and moment diagrams of a metal building moment frame column and approximates the shear and moment diagrams in the portion of a metal building rafter or bridge girder between the knee and the rafter splice. Therefore, the configuration shown in Fig. 1 was chosen for all specimens. A moment end plate splice was included at midspan to allow the specimens to be more easily handled and transported into the laboratory, to prevent web local crippling, and to provide a bearing surface for the hydraulic ram. The moment end plate was flush at the bottom, extended at the top, and had two interior rows of bolts at the top to more uniformly distribute flexural stresses into the flange and web.

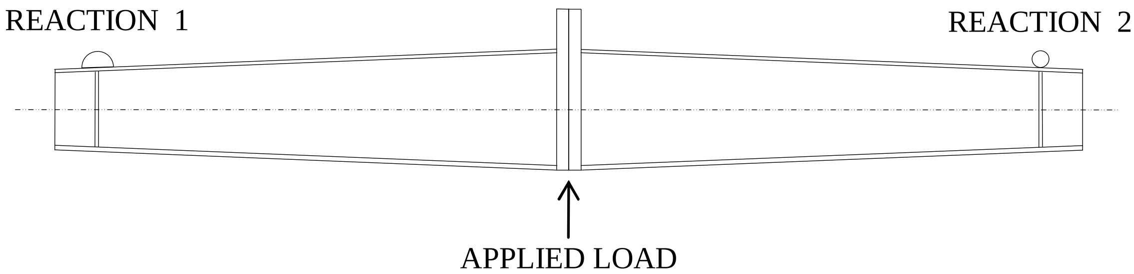

Figure 1: Specimen Elevation

3.2. Specimens

A summary of specimen dimensions is shown in Table 1. Each web was flexurally slender at midspan and had h/t_w large enough that elastic shear buckling was the anticipated behavior. Three specimens ("Tapered 4," "Tapered 5," and "Tapered 6") had different flange sizes, two of which had a larger compression flange. Taper angles varied between 5 deg. to 10 deg.

Table 1: Summary of Specimens

						Bottom Flange		Top Flange		h/t_w		a/h
Designation	Length (m)	d_{End} (mm)	$d_{Midspan}$ (mm)	Taper Angle	t_w (mm)	b_f (mm)	t_f (mm)	b_f (mm)	t_f (mm)	End	Midspan	Average
Tapered 1a	4.6	305	508	5.1	3	152	8	152	8	91	155	5.77
Tapered 1b	4.6	305	508	5.1	3	152	8	152	8	91	155	5.77
Tapered 1c	4.6	305	508	5.1	3	152	8	152	8	91	155	5.77
Tapered 2a	4.6	254	635	9.5	4	203	13	203	13	58	154	5.36
Tapered 2b	4.6	254	635	9.5	4	203	13	203	13	58	154	5.36
Tapered 2c	4.6	254	635	9.5	4	203	13	203	13	58	154	5.36
Tapered 3	3.6	333	508	5.6	3	152	16	152	16	95	150	4.38
Tapered 4	3.6	313	559	7.9	3	203	16	203	10	91	168	4.15
Tapered 5	4.0	410	581	5	4	203	13	203	19	100	144	4.18
Tapered 6	3.6	546	362	6.5	3	203	13	203	10	154	100	3.94

Combinations of taper angle, depths, web thickness, and flange sizes were selected to increase the likelihood of shear failure without having unrealistically large flanges or a/h less than three, which is the AISC *Specification* demarcation between stiffened and unstiffened panels. Each web-to-flange fillet weld was on one side only except for short segments of weld near the ends of the members. Bearing stiffeners were included at the ends of the members to prevent web local yielding and web local crippling. The left half of each specimen had the web thickness listed in Table 1; the right half had a web thickness that was a size or two larger to ensure that it did not fail, thus saving fabrication, transportation, and instrumentation expense.

4. Comparisons of Measurements and Predictions

Table 2 shows the measured failure load (midspan point load) and failure description for each tapered specimen. Fig. 2 and Fig. 3 shows typical failure modes for the specimens that failed by shear buckling.

Table 2: Summary of Experimental Results

Specimen	Failure Load (kN)	Failure Description
1a	256.2	Flange Local Buckling (Flexure)
1b	269.1	Flange Local Buckling (Flexure)
1c	260.7	Flange Local Buckling (Flexure)
2a	615.6	Web Shear Buckling
2b	600.5	Web Shear Buckling
2c	576.5	Web Shear Buckling
3	376.3	Web Shear Buckling
4	378.5	Web Shear Buckling
5	507.5	Web Shear Buckling
6	303.4	Web Shear Buckling

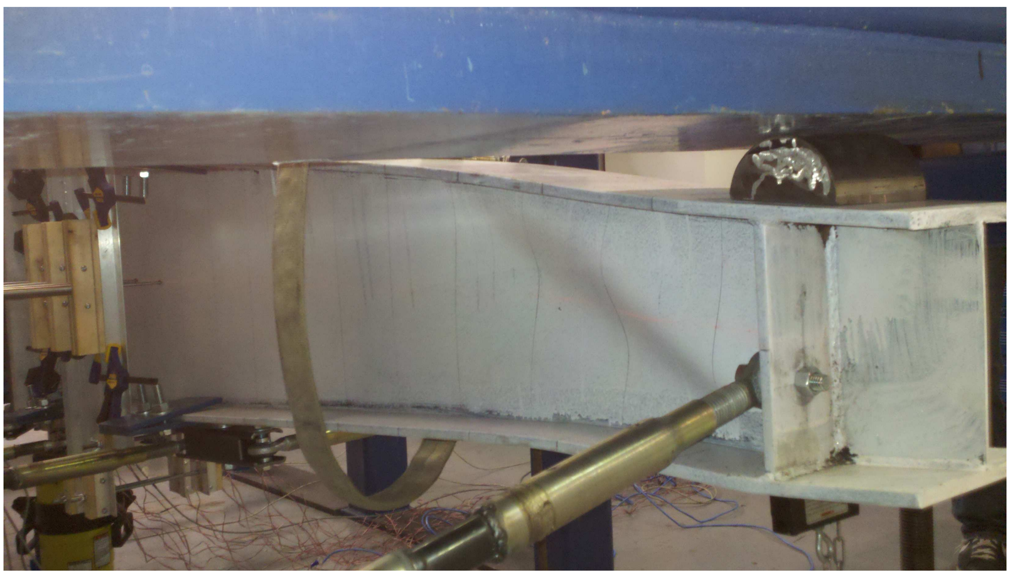

Figure 2: Tapered 2a Shear Buckling Failure

Figure 3: Tapered 4 Shear Buckling Failure

The shear failure load (midspan point load) was predicted using the web strength prediction methods in AISC (2010) and Lee et al. (2008) and the applied web shear computed using the following three methods: (i) web resists entire shear, (ii) web resists the modified shear per Williams and Harris (1957), and (iii) the web resists the modified shear per Blodgett (1966). Thus, the following six methods were investigated by comparing their predictions to measured failure loads. Table 3 shows the predicted strengths for each specimen using these methods.

- AISC *Specification* web shear strength; web resists entire shear.
- AISC *Specification* web shear strength; Williams and Harris modified shear.
- AISC *Specification* web shear strength; Blodgett modified shear.
- Lee et al. web shear strength; web resists entire shear.
- Lee et al. web shear strength; Williams and Harris modified shear.
- Lee et al. web shear strength; Blodgett modified shear.

4.1. Example Specimen

Fig. 4 and Fig. 5 show the measured load (kN) vs. midspan displacement (mm) for Tapered 3. The shear buckling load is not identifiable because of sizeable initial out-of-flatness imperfections.

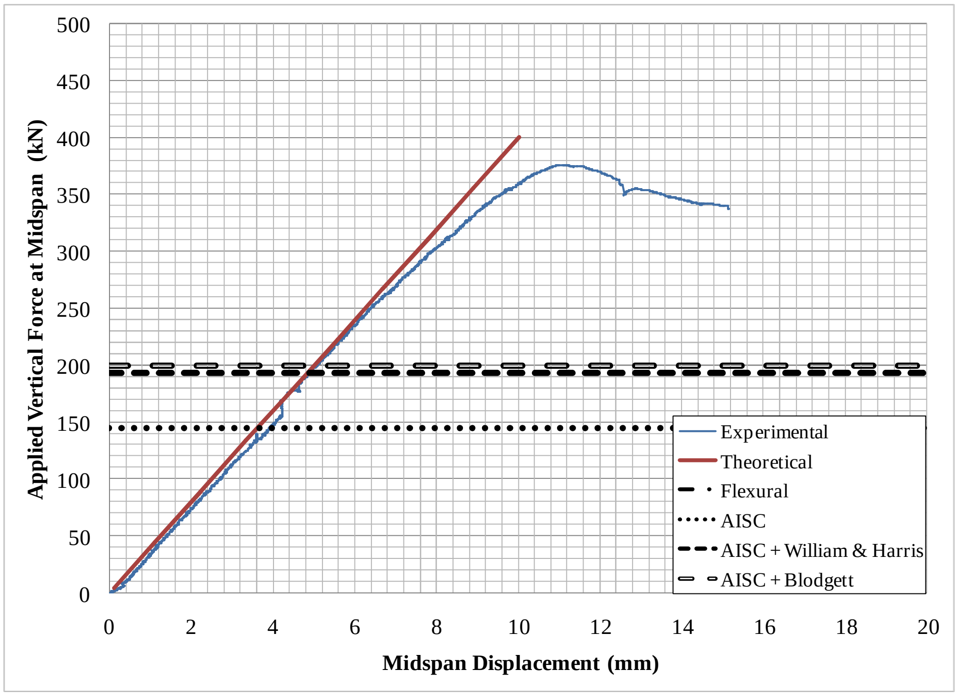

Figure 4: AISC Predicted Failure Loads

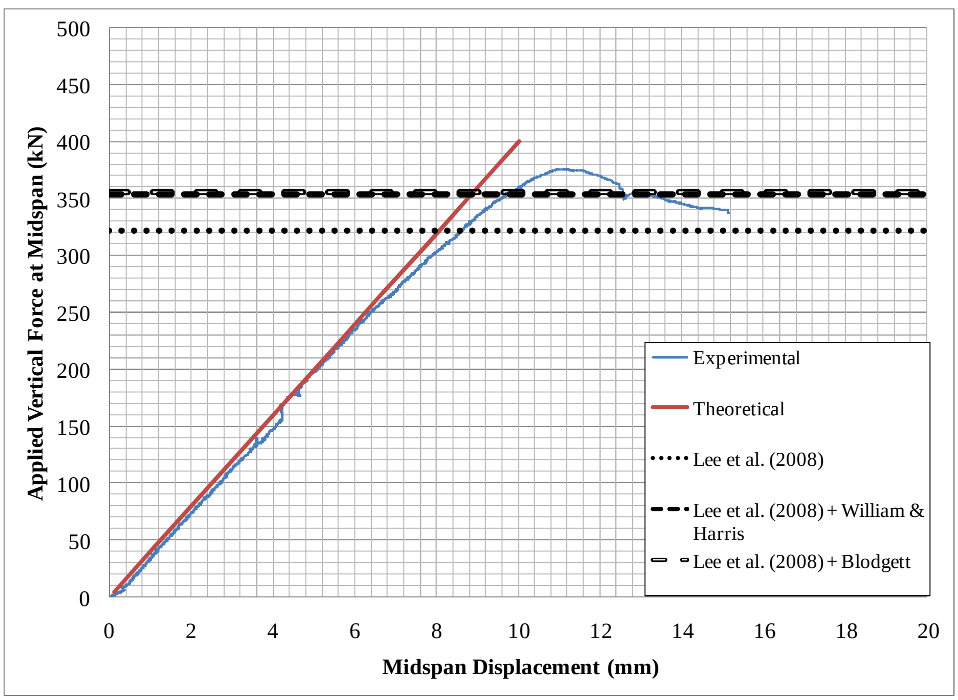

Figure 5: Lee et al. (2008) Predicted Failure Loads

Table 3: Summary of Predicted Strengths

Specimen	Predicted Failure Load (kN)					
	AISC *Specification*			Lee et al. (2008)		
	Web Resists All Shear	W&H	Blodgett	Web Resists All Shear	W&H	Blodgett
1a	134.8	189.9	206.0	387.0	418.6	423.0
1b	134.8	189.9	206.0	406.1	438.6	443.5
1c	134.8	189.9	206.0	385.2	416.4	420.8
2a	237.5	448.8	503.5	415.5	484.9	490.2
2b	237.5	448.8	503.5	423.0	493.3	498.6
2c	237.5	448.8	503.5	432.8	504.9	510.2
3	144.6	193.5	199.3	308.7	339.4	341.6
4	133.0	196.2	205.5	284.2	323.8	326.9
5	211.3	267.8	274.9	433.3	469.7	472.8
6	156.1	140.6	139.2	361.2	270.9	265.1

The figures also show the AISC 2010 *Specification* and the Lee et al. (2008) predicted shear failure loads. For this specimen, the AISC *Specification* prediction was very conservative with the following measured-to-predicted strength ratios if the web resists all shear, using the William and Harris (1957) modified shear, and Blodgett (1966) modified shear, respectively: 2.61, 1.95, and 1.89. The Lee et al. (2008) prediction was very accurate, with the following measured-to-predicted strength ratios if the web resists all shear, using the William and Harris (1957) modified shear, and Blodgett (1966) modified shear, respectively: 1.22, 1.11, and 1.10.

4.2. Summary of Comparisons

For the tapered specimens which failed by web shear buckling, the average ratio of measured ultimate strength to that predicted by the AISC *Specification* 2010 without use of a modified shear method was 2.48 with a 11.2% coefficient of variation (COV), indicating that the method is extremely conservative. When the AISC Specification web shear strength is used with the William and Harris (1957) and Blodgett (1966) modified shear methods, the average measured-to-predicted ratio was 1.71 (21.1% COV) and 1.62 (25.9% COV), respectively, indicating that both of those methods are very conservative also. Tapered 1a, b, and c failed by flexural flange local buckling, so it is not possible to compute the measured-to-predicted ratio. However, it is noteworthy that these three specimens achieved loads that averaged 1.94, 1.38, and 1.28 the predicted failure load using the AISC Specification with the web resisting all shear, the Williams and Harris modified shear, and Blodgett modified shear, respectively.

For those specimens which failed by shear buckling the average ratio of measured ultimate strength to that predicted by Lee et al. (2008) without use of a modified shear method was 1.32 with a 9.57% COV, indicating the method to be conservative. When the Lee et al. (2008) web shear strength is used with the Williams and Harris (1957) and Blodgett (1966) modified shear methods, the average measured-to-predicted ratio was 1.16 (6.77% COV) and 1.15 (6.59% COV), indicating that both of those methods are quite accurate and slightly conservative.

4.3. Summary and Conclusions

An experimental program was conducted to investigate the accuracy of shear strength prediction methods for unstiffened tapered I-shaped members. Ten plate girder specimens were loaded to failure in the laboratory to obtain measured failure loads. Failure load predictions were generated using the AISC *Specification* and Lee et al. (2008) web shear strength equations coupled with three methods for distributing the applied shear force to the web for a total of six shear strength prediction methods. Measured and predicted failure loads were compared to assess the accuracy of the prediction methods.

The AISC *Specification* method under-predicted the failure loads by a wide margin, with average measured-to-predicted ratios of 2.48, 1.71, and 1.62 when the predictions considered the web to resist all of the shear, the Williams and Harris (1957) modified shear, and the Blodgett (1966) modified shear, respectively.

The Lee et al. (2008) method very accurately and slightly conservatively predicted the failure loads, with average measured-to-predicted ratios of 1.32, 1.16, and 1.15 when the predictions considered the web to resist all of the shear, the Williams and Harris (1957) modified shear, and the Blodgett (1966) modified shear, respectively.

The Lee et al. (2008) shear strength equations much more accurately predicted the shear strength than did the AISC 2010 *Specification* equations for the ten tapered plate girder specimens tested in this research.

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