



## **Shell buckling evaluation of thin-walled steel tanks filled at low liquid level according to current design codes**

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### **Abstract**

The structural stability of thin-walled steel tanks becomes a major safety issue when these operate at low levels of contained liquid. Despite numerous tank failures due to buckling of their circumferential shell, provisions in current codes do not provide cost-efficient or high-safety level solutions regarding this phenomenon. For example, the American standard API 650, which has worldwide applications, proposes only an empirical design method for stiffening the tank shell based on its thickness, height and the design wind velocity. More recent codes, such as the European standard EN1993-1-6, provide analytical relationships for evaluating the buckling resistance of shells, with stability being verified by relevant checks against appropriate design stresses. However, their provisions have not yet seen many field applications and results raise, in certain cases, doubts regarding the efficiency of the design. This paper presents a direct comparison between these two standards by attempting to evaluate the buckling resistance of two existing thin-walled steel tanks, filled at a low liquid level. Both tanks have large diameters (47m and 88m approximately), variable wall thickness, are self-supported (unanchored) and one of them supports a conical roof. The design stresses required by EN1993-1-6 were obtained from finite element simulations of the tanks (linear elastic analyses were performed) which included application of the actions specified in the provisions of the Eurocode. Results from both standards are discussed in detail, comparisons are made and discrepancies between the two standards are highlighted. Comments and conclusions to be drawn will help improve current specifications and resolve issues related to the safe design of liquid storage tanks filled at low liquid levels.

### **1. Introduction**

The safe and cost-efficient design of large diameter cylindrical steel tanks used for oil storage has always been a challenge for the civil engineering community. Their structural failure poses a severe threat to public safety (i.e. explosions), might lead to environmental degradation of the surrounding area and can induce onerous financial consequences. Such tanks are typically designed to operate at a high level of contained liquid, mainly due to overall stability reasons (i.e. resistance against sliding and overturning). As a result, current codes focus on the limitation

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of tensile stresses resulting from hydrostatic pressure and the prevention of failure modes associated with seismic excitation, such as the detachment of the bottom plate from the shell, the “elephant’s foot bulge” failure type, the detachment of piping and equipment related to the operation of the tank, the damage of the shell due to hydrodynamic pressure etc. However, specific circumstances (e.g. a global-scale financial crisis) dictate that large diameter steel tanks operate at low liquid levels. If that is the case, structural stability phenomena related to shell compression are expected to govern the design, and other actions (e.g. wind) will become critical for structural response. In order to appraise the efficiency of current design specifications in addressing the specific issue, the stability of two existing large-diameter steel tanks filled at a low liquid level will be evaluated.

## **2. Design philosophy of current codes**

For the purpose of assessing the structural stability of the two steel tanks to be discussed in this paper, two standards with worldwide application were employed. The American standard API 650 (2007) provides two empirical methods (the one-foot method and the variable design point method) for selecting the thickness of each shell course, depending on the geometry of the tank, the operational liquid level, the material used, the specific gravity of the contained fluid and the allowance for corrosion. These are based on the concept of limiting the tensile stresses in the shell due to hydrostatic pressure and do not account for buckling. This limit state is considered only indirectly, via an empirical design method that mandates stiffening of the shell (by placement of circumferential girders at specified heights) according to its thickness, height and wind design velocity. This method neglects the effect of the contained liquid and does not provide mathematical formulas for evaluating shell stability. Furthermore, the seismic design according to this standard gives emphasis to the overturning stability and sliding resistance of the tank during earthquake excitation and deals with buckling of the shell only via limitation of the maximum longitudinal compressive stress.

On the contrary, the European standard EN 1993-1-6 (2007) has a deep theoretical background and provides state-of-the-art, widely acceptable methodologies for explicitly evaluating the buckling resistance of shell structures. These involve linear bifurcation analysis methods for obtaining the critical elastic buckling load as well as analyses that include material nonlinearities and imperfections. Even though its provisions are limited to axisymmetric geometries, the standard has a wide range of applications with regard to cylindrical tanks. Another important characteristic of this specification is that it quantifies buckling resistance, by expressing it in terms of stresses calculated from analytical expressions. These take into account the relative slenderness of the shell. Stability is then verified by relevant checks against appropriate design stresses obtained from linear elastic analysis. Based on this design concept, a procedure for evaluating the buckling resistance of shells with variable wall thickness has also been developed. Most of the approaches proposed by this standard require the use of numerical methods, such as the finite element method (FEM), for analyzing the shell. The use of simplified expressions, according to the principles of mechanics, for determining the design stresses is permitted in certain cases. However, it should be noted that the standard is still very recent and its applicability on field construction has not been adequately confirmed up to today.

### 3. Description of the Tanks

The structural adequacy of two existing large diameter steel tanks (T-776 and T-761) located in the refinery of Motor Oil Hellas S.A. (Korinth, Greece) was checked for low level of contained liquid according to current design codes. Both tanks have flat bottoms and are self-supported (not anchored to the foundation). A conical roof with a slope equal to 1/6 is supported by one of them (tank T-776), while the other tank is open-top. The geometry of the tanks and the level of contained liquid are given in Table 1. The location of the ring stiffeners (wind girders) is also included. The upper stiffener of tank T-761 is used as a walkway.

Table 1: Geometric characteristics of Tanks T-776 and T-761

Tank ID	Liquid level (m)	Tank Height <sup>1</sup> (mm)	Tank Diameter <sup>2</sup> (mm)	1 <sup>st</sup> Wind Girder Height <sup>3</sup> (mm)	2 <sup>nd</sup> Wind Girder Height <sup>3</sup> (mm)	Steel Roof
T-776	1.0	20032	46939	14860	-	Yes
T-761	0.5	19500	88430	15350	18400	No

1. The roof height is not included
2. Refers to the inside diameter
3. Height is measured from the tank bottom

Table 2: Shell course information for tanks T-776 and T-761

Tank T-776			
Course No. <sup>1</sup>	Thickness (mm)	Width (mm)	Steel Grade
1	22.25	2438	BS4360 GR50C
2	18.93	2438	BS4360 GR50C
3	16.24	2438	BS4360 GR50B
4	13.57	2438	BS4360 GR50B
5	10.9	2438	BS4360 GR50B
6	8.22	1940	BS4360 GR50B
7	8.00	1940	BS4360 GR43A
8	8.00	1940	BS4360 GR43A
9-top	8.00	1940	BS4360 GR43A
Bottom Plates	6.40	2102	BS4360 GR43A
Roof Plates	5.00	1500	BS4360 GR43A
Tank T-761			
Course No. <sup>1</sup>	Thickness (mm)	Width (mm)	Steel Grade
1	38.60	2222	E355 GRADE C
2	37.18	2222	E355 GRADE C
3	28.20	2222	E355 GRADE C
4	24.59	2222	E355 GRADE C
5	19.96	2222	E355 GRADE C
6	15.60	2222	E355 GRADE C
7	11.20	2222	E355 GRADE C
8	9.50	2222	BS4360 GR36A
9-top	9.50	1724	BS4360 GR36A
Bottom Plates	6.40	Variable	BS4360 GR36A

1. Course No.1 refers to the bottom shell course, No. 2 to the second from below, e.t.c.

Both tanks are thin-walled with variable wall thickness. The width, thickness and steel grade of each shell course (nine in total) along with relevant information regarding the bottom and roof (where applicable) are summarized in Table 2. Photographic material pertaining to the tanks is presented in Fig. 1.



Figure 1: Photographic presentation of tank T-776 (left side) and tank T-761 (right side)

#### 4. Stability evaluation according to the American Standard API 650

This section covers the application of the American standard API 650 (2007) to the studied tanks. Once the design procedure regarding shell buckling has been presented, relevant calculations for the tanks follow. Appropriate comments on the shell stability are also included.

##### 4.1 Design Procedure

The American Standard API 650 (2007) attributes buckling of the shell to wind action. For this reason, the standard proposes the attachment of stiffening rings (wind girders) around the shell of the tank. The use of a wind girder at or close to the top of the shell is mandatory for open-top tanks. The required section modulus  $Z_{top}$  (expressed in  $\text{cm}^3$ ) for the top wind girder is defined by Eq. 1:

$$Z_{top} = \frac{D^2 H_2}{17} \left( \frac{V}{190} \right)^2 \quad (1)$$

where  $D$  is the nominal tank diameter in m,  $H_2$  is the height of the tank in m and  $V$  is the design wind speed in km/h. Built-up welded, formed plate and rolled structural sections or combinations of these are permitted for use as stiffening rings. API 650 (2007) permits the “transformed” shell to be unstiffened up to a maximum height  $H_1$  as specified in Eq. 2:

$$H_1 = 9.47t \sqrt{\left( \frac{t}{D} \right)^3 \left( \frac{190}{V} \right)^2} \quad (2)$$

where  $D$  is the nominal tank diameter in m,  $t$  is the thickness of the top shell course in mm and  $V$  is the design wind speed in km/h. Afterwards, the height of the “transformed” shell is calculated by adding the “transformed” widths  $W_{tr}$  of all shell courses. The latter are obtained according to Eq. 3:

$$W_{tr} = W \sqrt{\left( \frac{t_{uniform}}{t_{actual}} \right)^5} \quad (3)$$

where  $W$  and  $t_{\text{actual}}$  are the actual width and thickness, respectively, of the considered shell course and  $t_{\text{uniform}}$  is the thickness of the top shell course. If the height of the “transformed” shell exceeds height  $H_1$ , an intermediate wind girder is required. Its location is determined so that the two unstiffened portions of the “transformed” shell satisfy the above requirement. In case height  $H_1$  is less than half the height of the “transformed” shell, a second intermediate girder is required. The required section modulus  $Z_{\text{int}}$  (expressed in  $\text{cm}^3$ ) of an intermediate wind girder is defined by Eq. 4:

$$Z_{\text{int}} = \frac{D^2 H_1}{17} \left( \frac{V}{190} \right)^2 \quad (4)$$

where  $D$  is the nominal tank diameter in m,  $H_1$  is the height defined in Eq. 2 and  $V$  is the design wind speed in km/h.

The American Standard API 650 (2007) also provides an analytical expression for determining the maximum longitudinal compressive stress  $\sigma_c$  (in MPa) induced during earthquake excitation. This is given in Eq. 5 and Eq. 6:

$$\sigma_c = \left( w_t (1 + 0.4A_v) + 1.273 \frac{M_{rw}}{D^2} \right) \frac{1}{1000t_s} \quad \text{when} \quad J \leq 0.785 \quad (5)$$

$$\sigma_c = \left( \frac{w_t (1 + 0.4A_v) + w_a}{0.607 - 0.18667J^{2.3}} - w_a \right) \frac{1}{1000t_s} \quad \text{when} \quad J > 0.785 \quad (6)$$

where  $w_t$  is the tank and roof weight acting at the base of the shell (in N/m),  $A_v$  is the vertical acceleration coefficient obtained from the design response spectrum,  $M_{rw}$  is the ringwall moment (in Nm), namely the portion of the overturning moment that acts at the base of the shell perimeter,  $D$  is the nominal diameter of the tank (in m),  $t_s$  is the thickness of the bottom shell course (in mm),  $J$  is the anchorage ratio and  $w_a$  is the force resisting uplift in the annular region of the tank (in N/m). Structural stability is verified if this stress does not exceed a limiting stress  $F_c$  (in MPa) calculated from equations Eq. 7 and Eq. 8:

$$F_c = \frac{83t_s}{D} \quad \text{when} \quad \frac{GHD^2}{t^2} \geq 44 \quad (7)$$

$$F_c = \min \left( 0.5F_{ty}, \frac{83t_s}{2.5D} + 7.5\sqrt{GH} \right) \quad \text{when} \quad \frac{GHD^2}{t^2} < 44 \quad (8)$$

where  $D$  is the nominal diameter of the tank (in m),  $t_s$  is the thickness of the bottom shell course (in mm),  $G$  is the specific gravity of the contained liquid,  $H$  is the liquid height (in m),  $t$  is the thickness of the thinnest shell course and  $F_{ty}$  is the minimum specified yield strength of the bottom annulus.

#### 4.2 Calculations for tanks T-776 and T-761

After applying the provisions of the American Standard API 650 (2007), the results presented in Table 3 were obtained for tanks T-776 and T-761. The value for the design wind speed was  $V=190$  km/h. The assumed specific gravity of the contained liquid was  $G=0.7$ .

Table 3: Buckling assessment of tanks T-776 and T-761 according to API 650

Tank ID	Transformed shell height (mm)	Height $H_1$ (mm)	Lower unstiffened part height <sup>1</sup> (mm)	Upper unstiffened part height <sup>1</sup> (mm)	Stress $\sigma_c$ (MPa)	Stress $F_c$ (MPa)
T-776	10290	5330	5200	5090	2.2	22.2
T-761	6901	3167	2820	2981	1.2	18.9

1. Refers to the “transformed” shell

#### 4.3 Comments on the results

The results in Table 3 show that both tanks fulfill the requirements of API 650 (2007) for stiffening the circumferential shell against wind action. The length requirement of the unstiffened shell parts is satisfied marginally (the height  $H_1$  only exceeds these lengths by less than 5% in all cases), showing that the selection of the wind girder locations is optimal. Even though the standard according to which the tanks were designed is not known, the above observation suggests the use of an earlier version of API 650. Moreover, the longitudinal compressive stresses induced by earthquake loading represent only a small fraction (approximately 10% for tank T-776 and 6% for tank T-761) of those allowed.

### 5. Stability evaluation according to the European standard EN1993-1-6

This section focuses on evaluating shell stability of the tanks under consideration per EN1993-1-6 (2007). The various analysis procedures proposed by the standard are described and emphasis is given on the “stress design” concept, according to which stability is evaluated. Presentation of analysis results and appropriate checks is also included, followed by comments regarding the structural adequacy of the shell.

#### 5.1 Code provisions related to buckling

Besides the various analysis procedures specified in the standard, other crucial parameters, such as the imperfection factor, will be discussed. A procedure for stability evaluation of tanks with stepwise variable thickness will also be presented. It should also be noted that the European standard EN1993-4-2 (2007), which provides specific rules for the design of steel tanks, mandates that buckling of the shell be evaluated according to EN1993-1-6 (2007). The euro-norm EN1993-4-2 (2007) also provides a simplified design procedure for circular tanks when specific requirements are met. However, this method was not applicable to tanks T-761 and T-776 and will not be discussed in this paper.

##### 5.1.1 Analysis procedures specified in the standard

The European standard EN1993-1-6 (2007) proposes several methods for evaluating the buckling resistance of shells, which involve global numerical analysis of the complete structure (usually via the FEM). Provisions for modeling the shell (i.e. geometry, boundary conditions, application of loads etc.) are also included.

The “stress design” concept is typically followed in practice and evaluates the stability of the shell by comparing resistance stresses, which are defined by analytical expressions, with the

appropriate design stresses related to stability (axial and circumferential compressive stresses, in-plane shear membrane stresses). The latter are obtained from linear elastic analysis (LA) of the “perfect” shell.

Another approach involves a linear elastic bifurcation analysis (LBA), which is conducted to determine the elastic critical buckling resistance ratio  $r_{Rcr}$  (selected equal to the lowest eigenvalue of the bifurcation analysis) and a materially non-linear analysis (MNA) for calculating the plastic reference resistance ratio  $r_{Rpl}$  (plastic limit load under the applied loading combinations). These are combined to calculate an overall slenderness for the shell and evaluate its design buckling strength.

Alternatively, the resistance can be evaluated by a geometrically and materially nonlinear analysis which includes imperfections. These are introduced in the simulation model by appropriate modification of the perfect shell geometry. EN1993-1-6 (2007) proposes several values for their magnitude and mandates that various imperfection patterns be investigated if the one with the most onerous effect on the resistance is not easily identifiable. It should be noted that the calculated elastic-plastic buckling resistance is evaluated by comparison with numerical results pertaining to other shells (whose resistance is known) or experimental results and is calibrated accordingly via an appropriate factor.

This standard also permits the evaluation of the buckling strength of shell structures via the “direct design” concept, in which, standard expressions derived from membrane theory are used to determine the required stresses. In this concept, a global analysis of the structure is not required. Despite being practical and easily applicable, it fails to take into account several factors that affect the buckling resistance of the shell (geometry, openings, roof etc.) and leads to approximate results.

### 5.1.2 Buckling strength evaluation according to the “stress design” concept

According to the specifications of EN1993-1-6 (2007), the buckling strength of shell structures is represented in terms of three stresses, namely the meridional design buckling stress ( $\sigma_{x,Rd}$ ), the circumferential design buckling stress ( $\sigma_{\theta,Rd}$ ) and the in-plane shear design buckling stress ( $\tau_{x\theta,Rd}$ ). These are defined (Eq. 9, Eq. 10 and Eq. 11) by reducing the characteristic yield stress  $f_{yk}$  of the material via appropriate factors related to buckling ( $\chi_x$ ,  $\chi_\theta$ , and  $\chi_\tau$  respectively):

$$\sigma_{x,Rd} = \frac{\chi_x f_{yk}}{\gamma_{MI}} \quad (9)$$

$$\sigma_{\theta,Rd} = \frac{\chi_\theta f_{yk}}{\gamma_{MI}} \quad (10)$$

$$\tau_{x\theta,Rd} = \frac{\chi_\tau f_{yk}}{\gamma_{MI} \sqrt{3}} \quad (11)$$

The factor of safety  $\gamma_{M1}$  should not have a value lower than 1.1 according to EN1993-1-6 (2007). The buckling reduction factors are expressed as a function of the relative shell slenderness  $\bar{\lambda}$  according to Eq. 12, Eq. 13 and Eq. 14:

$$\chi = 1 \quad , \text{ if } \quad \bar{\lambda} \leq \bar{\lambda}_0 \quad (12)$$

$$\chi = 1 - \beta \left( \frac{\bar{\lambda} - \bar{\lambda}_0}{\bar{\lambda}_p - \bar{\lambda}_0} \right)^\eta \quad , \text{ if } \quad \bar{\lambda}_0 < \bar{\lambda} < \bar{\lambda}_p \quad (13)$$

$$\chi = \frac{\alpha}{\bar{\lambda}^2} \quad , \text{ if } \quad \bar{\lambda}_p \leq \bar{\lambda} \quad (14)$$

where  $\alpha$  is the elastic imperfection reduction factor,  $\beta$  the plastic range factor ( $\beta=0.6$  per EN1993-1-6 2007),  $\eta$  the interaction exponent ( $\eta=1$  per EN1993-1-6 2007),  $\bar{\lambda}_0$  the squash limit relative slenderness and  $\bar{\lambda}_p$  the plastic limit relative slenderness, defined as follows (Eq. 15):

$$\bar{\lambda}_p = \sqrt{\frac{\alpha}{1-\beta}} \quad (15)$$

The relative slenderness of the shell  $\bar{\lambda}$  is expressed in three terms (Eq. 16, Eq. 17 and Eq. 18) according to the stress component of the design resistance being evaluated:

$$\bar{\lambda}_x = \sqrt{\frac{f_{yk}}{\sigma_{x,Rcr}}} \quad (16)$$

$$\bar{\lambda}_\theta = \sqrt{\frac{f_{yk}}{\sigma_{\theta,Rcr}}} \quad (17)$$

$$\bar{\lambda}_\tau = \sqrt{\frac{f_{yk}/\sqrt{3}}{\tau_{x\theta,Rcr}}} \quad (18)$$

where  $\sigma_{x,Rcr}$ ,  $\sigma_{\theta,Rcr}$ ,  $\tau_{x\theta,Rcr}$  are the elastic critical meridional, circumferential and shear buckling stresses respectively. Analytical expressions for these are given in Annex D of the European standard EN1993-1-6 (2007), which defines three categories for cylindrical shells (short, medium-length, and long) according to the dimensionless length parameter  $\omega$  (Eq. 19):

$$\omega = \frac{l}{\sqrt{r t}} \quad (19)$$

where  $l$  is the length of the cylinder,  $t$  is the thickness of the shell and  $r$  is the radius at the middle surface of the cylinder. The critical stresses are expressed as a function of the boundary conditions, the parameter  $\omega$ , the geometric characteristics of the cylinder (thickness, radius) and Young's elastic modulus.

According to EN1993-1-6 (2007), the buckling-related stresses obtained from a linear analysis of the shell should not exceed the appropriate resistances (buckling strength verification). When these stresses coexist, an interaction check has been proposed (EN1993-1-6 2007). This check, which is omitted for shell areas close to boundaries, neglects the effect of tensile stress components.

### 5.1.3 Classification of tanks according to fabrication tolerance quality class

In order to estimate the magnitude of geometrical imperfections, which are crucial for evaluating the buckling strength of a steel shell, EN1993-1-6 (2007) defines three fabrication tolerance quality classes (Class A: excellent, Class B: high and Class C: normal fabrication). Class selection is based on representative sample measurements conducted on the unloaded, completed structure.

A clear distinction, based on the imperfection type being considered, is made among fabrication quality tolerance measurements. More specifically, these are categorized as: a) out-of-roundness measurements, which are associated with the internal diameter of the shell, b) non-intended eccentricity measurements at the joints of the connected plates, c) dimple measurements, on the meridional direction and along the circumference of the shell, including measurements across the welds and d) flatness measurements at the interface of the shell and its bottom. The fabrication quality class is assessed separately for each measurement type, according to the tolerances specified in EN1993-1-6 (2007). The lowest quality class is then assigned to the shell structure.

In case measurements exceed the tolerances specified for normal (Class C) fabrication, procedures for improving the geometry of the shell (e.g. straightening) are required (EN1993-1-6 2007). This is also acceptable for improving fabrication quality (for example Class B could be upgraded to Class A etc).

The effect of the fabrication quality class on the design buckling strength is quantified via the elastic imperfection reduction factor  $\alpha$ . More specifically, factors  $\alpha_\theta$  and  $\alpha_\tau$  (corresponding to the circumferential and shear buckling stresses) are selected directly after classification of the shell, while the buckling parameter  $\alpha_x$  (referring to the meridional buckling stress) is calculated, after selecting the appropriate value of the fabrication parameter  $Q$ , according to Eq. 20:

$$\alpha_x = \frac{0.62}{1 + 1.91 \left( \frac{l}{Q} \sqrt{\frac{r}{t}} \right)^{1.44}} \quad (20)$$

where  $r$  is the radius at the middle surface of the cylinder and  $t$  is the thickness of the shell. The proposed values for  $Q$ ,  $\alpha_\theta$  and  $\alpha_\tau$  per the fabrication quality class are summarized in Table 4:

Table 4: Effect of fabrication quality on buckling parameters

Fabrication tolerance quality class	Description	Q	$\alpha_0$	$\alpha_\tau$
Class A	Excellent	40	0.75	0.75
Class B	High	25	0.65	0.65
Class C	Normal	16	0.50	0.50

#### 5.1.4 Design Procedure for shells with variable wall thickness

The provisions of EN1993-1-6 (2007) related to the “stress design” concept discussed in section 5.1.2 refer to cylinders with constant wall thickness. In practice, however, numerous shell structures (such as the studied large-diameter oil storage tanks) are designed with their thickness decreasing in a progressive way towards the top (stepwise variable wall thickness cylinders). For this reason, a procedure for evaluating their buckling resistance has been included in Annex D of EN1993-1-6 (2007), according to which, the meridional buckling stress for a constant-thickness segment is calculated assuming the total length of the shell (the thickness of the portion under consideration remains unchanged). The elastic critical circumferential buckling stress for each segment is determined as follows: a) the variable thickness shell is initially treated as a cylinder consisting of three parts with uniform thickness. The length of each part depends on the unmodified geometry of the shell. The weighted average of the thicknesses over the length of each fictitious segment is used to calculate its thickness. b) A cylinder of uniform thickness (equal to that of the thinnest fictitious segment) is used to replace the three-part cylinder of the previous step. Its length is calculated by dividing that of the thinnest fictitious segment with the dimensionless factor  $\kappa$ . The latter depends on the geometry of the shell, as modified in the previous step and is obtained graphically. c) The elastic critical circumferential buckling stress  $\sigma_{\theta,Rcr,j}$  for each segment  $j$  with thickness  $t_j$  is then obtained from the following expression (Eq. 21):

$$\sigma_{\theta,Rcr,j} = \left( \frac{t_a}{t_j} \right) \sigma_{\theta,Rcr,eff} \quad (21)$$

where  $t_a$  is the thickness and  $\sigma_{\theta,Rcr,eff}$  is the elastic critical circumferential buckling stress of the equivalent single cylinder of step b). Moreover, for long cylinder segments ( $\omega_j > 1.63r/t_j$ ) a limiting value for  $\sigma_{\theta,Rcr,j}$  is specified (EN1993-1-6 2007). d) Once  $\sigma_{\theta,Rcr,j}$  has been determined, the design resistance  $\sigma_{\theta,Rd,j}$  for each segment is calculated according to the procedure described in section 5.1.2. Besides the buckling verification checks for uniform cylinders (section 5.1.2), the provisions for shells with variable wall thickness in Annex D of EN1993-1-6 (2007) additionally mandate that the elastic critical circumferential buckling stress be greater than or equal to the circumferential stress obtained from shell analysis. The same procedure is proposed for calculating the elastic critical shear buckling stress in each segment of the shell. For this purpose, the relevant expressions for in-plane shear should be used instead of those pertaining to circumferential compression.

#### 5.2 Buckling strength evaluation of tanks T-776 and T-761 according to EN1993-1-6

The buckling strength of the tanks was evaluated according to the “stress design” concept, as permitted by EN1993-1-6 (2007). This was selected over the other concepts for simplicity reasons and for reducing computational effort. Global analysis of the structures via numerical methods was conducted as required.

### 5.2.1 Description of the FE Models used for analyzing the Tanks

The linear elastic analyses mandated by EN1993-1-6 (2007) for obtaining the stresses associated with buckling of the shell were performed by applying the FEM. A separate 3D finite element model was created for each tank. The commercial software STAAD.Pro V8i (2007) was used to perform the required analyses. To simulate the shell and the bottom of each tank, quadrilateral (4-node) and triangular (3-node) plate elements, which incorporate membrane action as well as bending and have six degrees of freedom per node, were used. This satisfies the modeling requirements of EN1993-1-6 (2007). The same element type was selected to model the steel roof plates (tank T-776 only). The remaining structural members (wind girders, top curb angles, roof trusses etc.) were simulated by beam elements. The as-built dimensions and thicknesses, as well as the appropriate material properties, were given to all elements of the models. Geometrical eccentricities were also incorporated. The FE mesh was selected to account for the change in shell thickness and the location of stiffeners. Fig. 2 presents the FE models created to simulate the studied tanks. Linear elastic supports (translational springs) were used to model the foundation of the tanks. The constants of the support springs were determined from the soil factor.

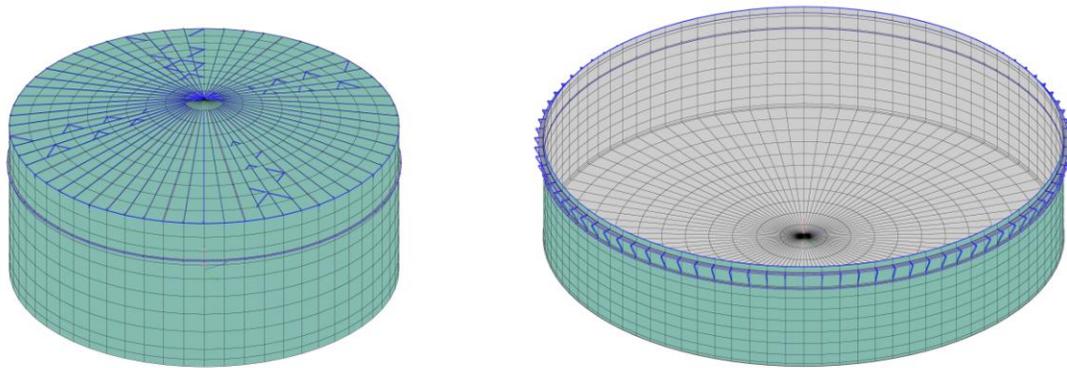


Figure 2: Simulation of tank T-776 (left side) and T-761 (right side) via the FEM

The following actions were imposed to the tanks: a) Self-weight of the tank per EN1991-1-1 (2002) b) Weight of the contained liquid per EN1991-1-1 (2002) c) Live load per EN1991-1-1 (2002) (where applicable) d) Wind loading per EN1991-1-4 (2005) e) Temperature loads per EN1991-1-5 (2004) f) Earthquake loading per EN1998-1 (2005) and EN1998-4 (2007). The imposed actions were combined according to the provisions of EN1990 (2002). It should be mentioned that the specific gravity of the contained liquid was conservatively assumed to be 0.7.

The linear elastic response spectrum (behaviour factor  $q=1$ ) according to EN1998-1 (2005) for ground type C was used for the seismic analysis of the tanks. The peak ground acceleration for Korinthos, Greece (0.24g) was incorporated in the spectrum. The vertical component of the earthquake motion was also included. The importance factor was selected equal to 1.40 (importance class IV). 5% damping was assumed for both tanks (EN1998-4 2007). It should be noted that, because the tanks are filled at a very low level, the effect of the contained liquid on the dynamic response of the system is negligible, as the major portion of its mass will be oscillating in the convective mode (“out-of-phase” motion). For this reason, simulation of the

contained liquid with 8-noded solid elements (an approach commonly followed by researchers (Cho, Song and Lee 2001; Greiner and Kettler 2005; Liu 1981; Maraveas 2011) in seismic analysis of liquid-storage tanks) was not deemed necessary. The lateral force method, which is permitted for seismic analysis of such structures according to EN1998-4 (2007), was applied instead. Calculation of the hydrodynamic pressure was conducted per EN1998-4 (2007). However, its magnitude is small and does not affect the seismic response of the tanks in a major way.

### 5.2.2 Analysis Results and buckling strength verification

The FEM analysis results, namely the meridional compression stress ( $\sigma_{x,Ed}$ ), the circumferential compression stress ( $\sigma_{\theta,Ed}$ ) and the in-plane shear stress ( $\tau_{x\theta,Ed}$ ), for both tanks are summarized in Table 5. The presented stresses refer to the middle of each shell course and correspond to the loading combination that produces the most onerous effect on the shell. In case a tensile stress was developed, the value “0.00” was inserted. Additionally, the maximum meridional compressive stresses from seismic loading were 3.4MPa and 2.1MPa for tanks and T-776 and T-761 respectively.

Table 5: Analysis stresses for tanks T-776 and T-761

Table 5: Analysis stresses for tanks T-776 and T-761			
Tank T-776		Analysis Stresses (MPa)	
Course No. <sup>1</sup>	$\sigma_{x,Ed}$	$\sigma_{\theta,Ed}$	$\tau_{x\theta,Ed}$
1	1.95	4.36	1.04
2	3.64	3.05	0.91
3	5.48	4.36	0.65
4	7.12	5.13	0.21
5	8.40	6.07	0.56
6	9.16	7.51	1.80
7	6.71	7.55	0.17
8	5.10	6.60	1.09
9-top	1.30	4.30	0.13
Tank T-761		Analysis Stresses (MPa)	
Course No. <sup>1</sup>	$\sigma_{x,Ed}$	$\sigma_{\theta,Ed}$	$\tau_{x\theta,Ed}$
1	2.86	2.27	1.82
2	0.00	5.48	0.70
3	0.00	5.05	0.71
4	0.63	6.18	0.57
5	0.90	7.51	0.36
6	1.79	10.10	0.05
7	3.84	0.00	1.82
8	1.66	9.75	0.53
9-top	0.30	6.78	0.26

1. Course No.1 refers to the bottom shell course, No. 2 to the second from below, etc.

Based on the analysis stresses and the buckling strength evaluation procedure described in sections 5.1.2 and 5.1.4, the safety factor related to buckling is calculated for each shell course. Because imperfection measurements have not been carried out for the tanks as of today, relevant results for all fabrication quality classes are presented (Table 6).

Table 6: Safety factors against buckling for tanks T-776 and T-761

Tank T-776		Fabrication Quality Class		
Course No. <sup>1</sup>	Class A	Class B	Class C	
1	0.36	0.30	0.21	
2	0.67	0.54	0.37	
3	0.50	0.39	0.26	
4	0.46	0.34	0.22	
5	0.41	0.28	0.17	
6	0.32	0.20	0.11	
7	0.55	0.47	0.27	
8	0.39	0.27	0.16	
9-top	0.98	0.85	0.66	

Tank T-761		Fabrication Quality Class		
Course No. <sup>1</sup>	Class A	Class B	Class C	
1	1.10	0.88	0.57	
2	0.49	0.41	0.30	
3	0.78	0.65	0.46	
4	0.70	0.58	0.41	
5	0.68	0.57	0.40	
6	0.61	0.47	0.31	
7	1.01	0.57	0.32	
8	0.60	0.41	0.56	
9-top	1.23	0.98	0.67	

1. Course No.1 refers to the bottom shell course, No. 2 to the second from below, etc.

### 5.3 Comments on the analysis results and evaluation of stability

The results presented in Table 6 show that the tanks violate the requirements set by EN1993-1-6 (2007) regarding the limit state of buckling, irrespective of fabrication quality class. Only three shell courses in tank T-761 (Class A fabrication quality) have a factor of safety that exceeds unity, while values as low as 0.32 have been calculated even for excellent fabrication. Factors of safety drop in a consistent way (approximately 20% to 30%) as fabrication quality deteriorates. According to the provisions of this standard, the operation of the tanks at low level of contained liquid is not safe and stiffening of the shell is necessary. It should also be stated that all design stresses used in the buckling verification checks are induced from loading combinations in which wind is the leading variable action.

## 6. Comparison between the two standards

The method proposed by API 650 (2007) for stiffening the shell of tanks is practical, easily applicable and well-suited for design purposes. However, information regarding its background is not provided and, therefore, a direct evaluation of this method via the principles of mechanics is not possible. Furthermore, a procedure for determining the critical stress-state pertaining to buckling of the shell is not specified. Buckling resistance is not quantified by means of analytical expressions and it is not possible to evaluate the safety level provided by the specific approach. It should also be noted that the effect of the contained liquid and is not accounted for. This observation suggests that buckling of the shell is considered for the empty tank according to this design method. Combination of wind with other actions that might affect stability of the shell is neglected.

As discussed in section 5.1.1, EN1993-1-6 (2007) proposes several methodologies consistent with contemporary design concepts for evaluating buckling of shell structures. However, application of this standard requires significant computational effort, especially for large-scale tanks. The theoretical orientation of the specific code in conjunction with the sophisticated mathematical formulas used in the “stress design” concept render its implementation impractical. EN1993-1-6 (2007) sets conservative requirements by limiting resistance stresses to a minor fraction of the yield stress (in certain cases less than 1%). This is reflected on the results for the studied tanks. The proposed approach for variable thickness tanks produces questionable results. Commenting on Eq. 21, which refers to the circumferential and shear buckling resistances, the authors of EN1993-1-6 (2007) state that it “may seem strange in that the resistance appears to be higher in thinner plates”. The explanation given in EN1993-1-6 (2007), according to which the “whole cylinder bifurcates at a single critical external pressure” and higher resistances at thinner courses result in a constant utilization ratio along the height of the shell (stresses are lower at thicker sections for uniform loading) is not convincing because, eventually, buckling evaluation is performed for each shell segment separately. The validity of this method should be further investigated. Even though the remaining analysis methods (LBA, materially non-linear analysis, etc) are not presented here, results related to shell buckling are not expected to be more favorable. Future work should include such analyses for the same or similar tanks in order for the above statement to be verified. Moreover, the efficiency of the proposed methods in the design of tanks cannot be properly evaluated, because the standard has only recently been published and limited insight can be obtained from its application on field construction.

Based on the above, it can be stated that the two standards approach the problem of shell buckling with different philosophies. Comparison of results shows major incompatibilities between them. Both tanks satisfy the requirements set by API 650 (2007) related to shell stability. In contrast with this, the provisions of EN1993-1-6 (2007) pertaining to the limit state of buckling are not met for either tank. A reasonable explanation for this is associated with consideration of geometrical imperfections. Their effect is meticulously accounted for in EN1993-1-6 (2007), while the authors of API 650 (2007) have selected to omit it. Another reason for the discrepancies in the results is the difference in application of the wind load. While the provisions of EN1991-1-4 (2005) mandate the use of coefficients for determining wind pressure distribution on circular tanks, API 650 (2007) assumes uniform wind loading. Furthermore, the design wind velocity (190km/h  $\approx$  53m/sec) used in the API 650 (2007) method is higher than that specified in EN1991-1-4 (2005) (33m/sec). Despite the above observations, the large contradictions in shell stability evaluation are not justifiable (i.e. the extremely low safety factors obtained from EN1993-1-6 (2007) are not comparable with results from the API 650 (2007) method).

The calculated maximum meridional compressive stress  $\sigma_c$  for seismic loading per API 650 (2007) matches well that obtained from the linear elastic analysis of the shell. Stability verification under seismic action per API 650 (2007) involves this stress only, contrary to EN1993-1-6 (2007) that accounts for all stress resultants. The maximum allowable stress  $F_c$  (approximately equal to 15% of the yield stress) is much higher of that specified in EN1993-1-6 (2007). However, these differences are of minor importance for tanks filled at low level, because seismic loading is rarely critical for shell stability.

## 7. Conclusions

Application of current specifications to tanks T-776 and T-761 shows that buckling of shell structures cannot be determined with satisfactory accuracy. The contradictory results render the safety of steel tanks filled at low liquid level doubtful. As of today, selection of the design standard is crucial and is expected to have major consequences on the structural adequacy appraisal of thin-walled tanks. The efficiency of current codes should be evaluated by comparison with data from small-scale and, if possible, full scale experiments. Based on the conclusions drawn, improvements in future editions should be made. Moreover, design concepts used to evaluate stability that include state-of-the-art methods for analyzing the shell should also lead to cost-effective and practical design solutions. The theoretical background of empirical design methods for stiffening the shell should be investigated and decisions regarding the adequacy of the provided safety level should be made. As a general recommendation, such methods should set a minimum level for the contained liquid and determine the maximum duration of tank operation filled at that level, based on the occurrence probability of the critical load.

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