Stability verification of web-tapered beam-columns
– possible approaches and open questions –

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Abstract

In this paper, the case of beam-columns with linear varying web is studied. It is the purpose of this paper to: (i) review recent proposals by the authors that are in line with the Eurocodes principles for the stability verification of web-tapered columns and beams; (ii) carry out FEM numerical simulations covering several combinations of bending moment about strong axis, M_y, and axial force, N, and levels of taper; (iii) compare results to a) existing rules in EC3-1-1 (General Method); b) application of the interaction formulae for verification of beam-columns by adapting it with the previously developed rules for tapered beams and columns. Finally, (iv) to discuss of the possible approaches for the stability verification of portal frames with tapered members.

1. Introduction

Tapered steel members are commonly used over prismatic members because of their structural efficiency: by optimizing cross section utilization, significant material can be saved. However, if proper rules and guidance are not developed for these types of members, safety verification will lead to an over prediction of the material to be used. EC3 provides several methodologies for the stability verification of members and frames. Regarding non-uniform members in general, with tapered cross-section, irregular distribution of restraints, non-linear axis, castellated, etc., the following possibilities are provided in the code:

- Verification by a full nonlinear analysis which for the time being is not the preferred alternative due to its complexity;
- The General Method in EC3-1-1 (CEN, 2005) which couples the elastic stability resistance with the cross sectional resistance (subject to second order effects, if relevant). This method has been shown to not be adequate at all times, even for prismatic members (Greiner and Ofner, 2005; Simões da Silva et al., 2010);
- On the other hand, the interaction formulae in EC3-1-1 are only applicable to prismatic members.

Even when applying the possible methodologies, several difficulties are noted, such as:

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- The magnitude and shape of global and local imperfections to consider in a full nonlinear analysis or even in a second order analysis;
- The buckling curves were specifically calibrated for prismatic members, relying on the relationship between the web height and the flange width. In a tapered member this relationship varies along the member and there are no guidelines on which location to consider. This may lead to a wide range of results, either unsafe or over-safe;
- Definition of the cross-section class when it varies along the member (e.g. due to loading);

As a result, safety verification is likely to be too conservative by designer´s choice, not accounting for the advantages and structural efficiency that non-uniform members provide.

This paper aims at discussing possible approaches that could be adopted in order to achieve a competitive design for non-uniform members and giving answer for some of the above-mentioned problems, with focus on linearly web-tapered beam-columns. The paper is mainly divided in three parts:

Section 2: Isolated beams and beam-columns stability verification – recent solutions proposed by the authors;

Section 3: Isolated beam-column stability verification – the proposals presented in Section 1 are applied to the case of beam-columns by adapting the Interaction Formulae of EC3-1-1 for prismatic members. A possible adaptation of the General Method is also analyzed;

Section 4: Safety verification of steel structures with focus on frames composed of tapered members, in line with EC3-1-1. Some of the possibilities of structural analysis are presented and combined with member verification as presented in the previous sections.

2. Stability verification of web-tapered columns and beams – Ayrton-Perry procedures

The methodology for the stability verification of web-tapered beam-columns to be presented in Section 3 is based on analytically based proposals regarding flexural buckling of tapered columns and lateral-torsional buckling of tapered beams. More details may be found in Marques et al. (2012c) and Marques et al. (2012b), respectively. For this, Ayrton-Perry analytical models were built based on a linear interaction between the first order forces and second order bending moment utilizations, leading to a maximum utilization (and, consequently, to the ultimate load factor) at a certain location, denoted as the second order failure location. In these models, eigenmode conform imperfections were considered for the second order forces, leading to similar equations as those presented in EC3-1-1 for the stability verification of prismatic columns. As a result, as long as a second order failure location is known and an additional imperfection factor is considered to account for the non-uniformity either of the loading or of the cross section, the verification may be performed analogously to the rules for prismatic columns. Similarly, the model for tapered beams is also consistent with recently proposed design models for the stability verification of prismatic beams (Taras, 2010), which were also developed on the basis of Ayrton-Perry formulation.

Recent investigations have shown that this second order failure location and additional imperfection factor may be replaced in some of the terms by an "over-strength" factor φ which accounts for the relation between the ultimate resistance multiplier of the second order location, $\alpha_{ult,k}(x_c^{II})$ and the first order location, $\alpha_{ult,k}(x_c^{I})$, such that if the φ-factor is determined, the verification is always based on x_c^{I}. φ is given by

$$\varphi = \frac{\alpha_{ult,k}(x_c^{II})}{\alpha_{ult,k}(x_c^{I})} \tag{1}$$

which leads to

$$\overline{\lambda}(x_c^{II}) = \sqrt{\frac{\alpha_{ult,k}(x_c^{II})}{\alpha_{cr}}} = \sqrt{\frac{\varphi\alpha_{ult,k}(x_c^{I})}{\alpha_{cr}}} = \sqrt{\varphi}\,\overline{\lambda}(x_c^{I}) \tag{2}$$

In Eq. (2), α_{cr} is the critical load multiplier. x_c^{I} is such that the first order utilization, i.e. N_{Ed}/N_{Rk} or $M_{y,Ed}/M_{y,Rk}$ is the highest along the member. It is recommended that at least 10 locations along the member are considered to find x_c^{I}, as also proposed by Greiner et al. (2010). The verification procedures, calibrated φ-factors and second order failure locations are described in Table 1 for web-tapered columns and Table 2 to Table 4 for web-tapered beams. $\gamma_h = h_{max}/h_{min}$ and $\gamma_w = W_{,el,max}/W_{,el,min}$ are the taper ratios concerning the maximum and minimum height or elastic bending modulus and ψ is the ratio between the maximum and minimum bending moment for linear bending moment distributions and *UDL* regards uniformly distributed loading cases. A practical limit of $\gamma_h \leq 4$ and $\gamma_w \leq 6.5$ is established for application of Table 2. In addition, it should be noticed that new imperfection factors for welded cross sections were also proposed leading to more accurate levels of resistance than the ones given in the code (i.e., even for prismatic members) (Marques et al. 2012c).

Finally, these proposals yield differences of 10% relatively to the established numerical models. The procedure was then shown to be adequate for non-uniform bending moment distributions.

On the other hand, application of the General Method in EC3-1-1 with one of the most common buckling curves of EC3-1-1 may lead to differences of 40% on the safe side. Unsafe results up to 25% were also achieved, clearly showing a great range of uncertainty relatively to the proposed methodologies.

Table 1: Proposed verification procedures for web-tapered I-section columns – φ approach

	Out-of-plane flexural buckling	In-plane flexural buckling
$\alpha_{ult,k}(x_c^{I})$	$N_{Rk}(x_{c,N}^{I})/N_{Ed}(x_{c,N}^{I})$ – for N_{Ed}=const. is the smallest cross section	
α_{cr}	Numerically or from literature $\approx N_{cr,z,hmin}/N_{Ed}$ (approximately the Euler load of an equivalent column with the smallest cross section)	Numerically or from literature
$\overline{\lambda}(x_c^{I})$	$\sqrt{\alpha_{ult,k}(x_c^{I})/\alpha_{cr}}$	
φ	$1 + \dfrac{ht_w}{A_{min}}\left[\dfrac{(1+4\gamma_h)(\gamma_h-1)}{10\gamma_h}\right]$	$1 + \dfrac{h_{min}t_w}{A_{min}}\dfrac{\gamma_h-1}{\gamma_h+1}$
α	Hot-rolled: 0.49 Welded: 0.64	Hot-rolled: 0.34 Welded: 0.45
η	$\alpha_z\left(\sqrt{\varphi_z}\,\overline{\lambda}(x_c^{I}) - 0.2\right)$ If welded, $\eta_z \leq 0.34$	$\alpha_y\left(\sqrt{\varphi_y}\,\overline{\lambda}(x_c^{I}) - 0.2\right)$ If welded, $\eta_y \leq 0.27$
ϕ	$0.5 \times \left(1 + \varphi \times \eta + \varphi \times \overline{\lambda}^2(x_c^{I})\right)$	
$\chi(x_c^{I})$	$\varphi/\phi + \sqrt{\phi^2 - \varphi \times \overline{\lambda}^2(x_c^{I})} \leq 1$	
Verification	$\chi(x_c^{I}) \times \alpha_{ult,k}(x_c^{I}) \geq 1$	

Table 2: Proposed verification procedure for web-tapered I-section beams – $x_{c,lim}{}^{II}$ and φ combined approach

	Lateral-torsional buckling
$\alpha_{ult,k}(x_c{}^I)$	$M_{y,Rk}(x_{c,M}{}^I)/M_{y,Ed}(x_{c,M}{}^I)$ – the minimum along the beam, e.g. 10 sections
α_{cr}	Numerically e.g. or by expressions for M_{cr} from literature, see Section 5.2.4. The multiplier α_{cr} shall afterwards be obtained with respect to the applied load.
$\overline{\lambda_{LT}}(x_c^I)$	$\sqrt{\alpha_{ult,k}(x_c^I)\big/\alpha_{cr}}$
$x_{c,lim}{}^{II}$	See Table 3
φ_{LT}	For ψ: $\quad A\cdot\psi^2 + B\cdot\psi + C \quad \geq 1$ For UDL: $\quad -0.0025 a_\gamma^2 + 0.015 a_\gamma + 1.05$ See Table 4 for A,B,C and a_γ
α_{LT}	Hot-rolled: $\quad 0.16\sqrt{\dfrac{W_{y,el}(x_{c,\lim}^{II})}{W_{z,el}(x_{c,\lim}^{II})}} \leq 0.49$ Welded: $\quad 0.21\sqrt{\dfrac{W_{y,el}(x_{c,\lim}^{II})}{W_{z,el}(x_{c,\lim}^{II})}} \leq 0.64$
η_{LT}	$\alpha_{LT}\times\big(\overline{\lambda}_z(x_{c,\lim}^{II})-0.2\big)$ If welded, $\eta_{LT} \leq \sqrt{\dfrac{W_{y,el}(x_{c,\lim}^{II})}{W_{z,el}(x_{c,\lim}^{II})}}\big(0.12\psi^2 - 0.23\psi + 0.35\big)$
$\overline{\lambda}_z(x_{c,lim}^{II})$	$\sqrt{N_{Rk}(x_{c,\lim}^{II})\big/N_{cr,z,h\,\min}}$
ϕ_{LT}	$0.5\times\left(1 + \varphi\times\eta\times\dfrac{\overline{\lambda}^2{}_{LT}(x_c^I)}{\overline{\lambda}^2{}_z(x_{c,\lim}^{II})} + \varphi\times\overline{\lambda}^2{}_{LT}(x_c^I)\right)$
$\chi_{LT}\,x_c^I)$	$\varphi\big/\phi + \sqrt{\phi^2 - \varphi\times\overline{\lambda}^2{}_{LT}(x_c^I)} \leq 1$
Verification	$\chi_{LT}(x_c^I)\times\alpha_{ult,k}(x_c^I)\geq 1$

Table 3: Calculation of $x_{c,lim,M}{}^{II}/L$ for lateral-torsional buckling of tapered I-beams

For ψ	$\big(0.75 - 0.18\psi - 0.07\psi^2\big) + \big(0.025\psi^2 - 0.006\psi - 0.06\big)(\gamma_h - 1)\geq 0$ $\text{If}\quad \psi < 0 \quad and \quad |\psi|\big/\gamma_w \geq 1 + 1.214(\gamma_h - 1),\quad x_{c,\lim}^{II}/L = 0.12 - 0.03(\gamma_h - 1)$
For UDL	$0.5 + 0.0035(\gamma_h - 1)^2 - 0.03(\gamma_h - 1)^2 \leq 0.5$

Table 4: Calculation of φ for lateral-torsional buckling of tapered I-beams

a_γ	$-0.0005\cdot(\gamma_w - 1)^4 + 0.009\cdot(\gamma_w - 1)^3 - 0.077\cdot(\gamma_w - 1)^2 + 0.78\cdot(\gamma_w - 1)$		
ψ_{lim}	$1 + 120\cdot a_\gamma + 600\cdot a_\gamma^2 - 210\cdot a_\gamma^3\big/1 + 123\cdot a_\gamma + 1140\cdot a_\gamma^2 + 330\cdot a_\gamma^3$		
φ_{LT}	$\psi < -\psi_{\lim}$	$-\psi_{\lim} \leq \psi \leq \psi_{\lim}$	$\psi > \psi_{\lim}$
A	$-0.0665\cdot a_\gamma^6 + 0.718\cdot a_\gamma^5 - 2.973\cdot a_\gamma^4$ $+5.36\cdot a_\gamma^3 - 2.9\cdot a_\gamma^2 - 2.1\cdot a_\gamma - 1.09$	$\dfrac{-11.37 + 12090\cdot a_\gamma - 8050\cdot a_\gamma^2 + 1400\cdot a_\gamma^3}{1 - 1058\cdot a_\gamma + 705\cdot a_\gamma^2 - 120\cdot a_\gamma^3} + 11.22$	$0.008\cdot a_\gamma^2 - 0.08\cdot a_\gamma - 0.157$
B	$-0.1244\cdot a_\gamma^6 + 1.3185\cdot a_\gamma^5 - 5.287\cdot a_\gamma^4$ $+9.27\cdot a_\gamma^3 - 5.24\cdot a_\gamma^2 - 2.18\cdot a_\gamma - 2$	$+0.02\cdot a_\gamma^6 - 0.133\cdot a_\gamma^5 + 0.425\cdot a_\gamma^4$ $-0.932\cdot a_\gamma^3 + 1.05\cdot a_\gamma^2 - 0.5\cdot a_\gamma - 0.1$	$-0.033\cdot a_\gamma^3 + 0.04\cdot a_\gamma^2$ $+0.48\cdot a_\gamma + 0.37$
C	$-0.0579\cdot a_\gamma^6 + 0.6003\cdot a_\gamma^5 - 2.314\cdot a_\gamma^4$ $+3.911\cdot a_\gamma^3 - 2.355\cdot a_\gamma^2 + 0.02\cdot a_\gamma + 0.3$	$0.02\cdot a_\gamma^2 - 0.14\cdot a_\gamma + 1.25$	$0.032\cdot a_\gamma^3 - 0.092\cdot a_\gamma^2$ $+0.06\cdot a_\gamma + 0.8$

3. Stability verification of web-tapered beam-columns - existing and possible approaches

3.1 Interaction formulae in EC3-1-1

The interaction formulae of clause 6.3.3 for the stability verification of <u>prismatic</u> beam-columns are presented in Eq. (3) and (4) for uniaxial bending and class 1, 2 or 3 cross sections.

$$\frac{N_{Ed}}{\chi_y\, N_{Rk}/\gamma_{M1}} + k_{yy}\,\frac{M_{y,Ed}}{\chi_{LT}\, M_{y,Rk}/\gamma_{M1}} \leq 1.0 \tag{3}$$

$$\frac{N_{Ed}}{\chi_z\, N_{Rk}/\gamma_{M1}} + k_{zy}\,\frac{M_{y,Ed}}{\chi_{LT}\, M_{y,Rk}/\gamma_{M1}} \leq 1.0 \tag{4}$$

in which N_{Ed} and $M_{y,Ed}$ are the design values of the compression force and the maximum moment about the *yy* axis along the member. The interaction factors k_{yy} and k_{zy} may be determined either by Annex A (Method 1) or Annex B (Method 2) of the same code. Finally, besides the verification of Eq. (3) and (4), an additional cross section check is required at the extremes of the member.

The adaptation of the interaction formulae to the verification of tapered member naturally leads to some questions as there is not an analytical background specifically developed for the tapered beam-column case as it was performed for prismatic members. Nevertheless, an adjustment can be fairly easily analyzed especially when considering Method 2.

In a tapered beam-column the following verifications shall be performed:

- Out-of-plane stability verification;
- In-plane stability verification;
- Cross section verification at the most heavily loaded cross section, i.e., with the highest first order utilization.

In the following, a possible approach for each of these verifications is discussed and results are analyzed further in Section 3.4 for tapered beam-columns failing in out-of-plane buckling (Eq. (4)) or cross section failure.

Firstly, regarding cross-section verification, the first order failure location of tapered beam-columns varies with varying levels of axial force relatively to the applied bending moment leading to different utilizations – see Figure 1. As a result, and also as referred for the case of tapered beams, cross section verification should be performed in a sufficient number of locations in order to find the cross section with the highest first order utilization.

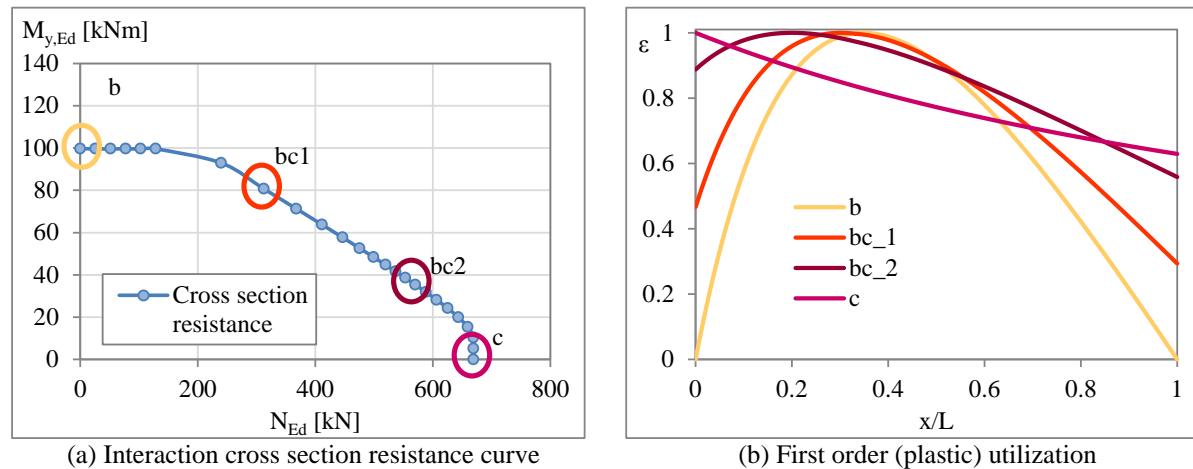(a) Interaction cross section resistance curve (b) First order (plastic) utilization
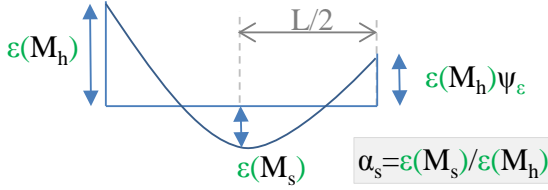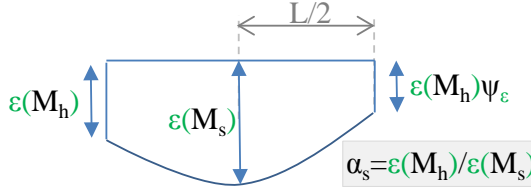Figure 1: First order failure location with varying axial force relatively to the bending moment

Secondly, regarding member verification, Eq. (3) and (4) must be adapted:

- Cross section properties to consider: according to the methodologies presented in Section, the quantities $N_{Ed}/(\chi_y N_{Rk})$, $N_{Ed}/(\chi_z N_{Rk})$ or $M_{y,Ed}/(\chi_{LT} M_{y,Rk})$ are constant along the locations of the column or beam, respectively. As a result, it is irrelevant which location is chosen and is here recommended (for simplicity reasons) the consideration of the first order failure location of the axial force acting alone $(x_{c,N}^I)$ for the utilization term regarding axial force; and the first order failure location of the bending moment acting alone $(x_{c,M}^I)$ for the utilization term regarding the bending moment;

- Adaptation of the interaction factors, k_{yy} and k_{zy}. Only Method 2 is considered for a straightforward application/adaptation of the interaction formulae to the case of tapered beam-columns and further validation. Because I-sections are susceptible to torsional deformations, according to Method 2, the interaction factors to be considered are summarized in Table 5. Finally, regarding the equivalent uniform moment factors $C_{m,y}$ and $C_{m,LT}$, Table B.3 of EC3-1-1 can be adopted provided that the diagram to be considered is the <u>bending moment first order utilization diagram</u> instead of the bending moment diagram itself, see Table 6.

Table 5: Possible interaction factors for web-tapered beam-columns according to Method 2

k_{yy}	$C_{my} \times \left(1 + \underbrace{(\bar{\lambda}_y(x_{c,N}^{II}) - 0.2)}_{\leq 0.8 \geq 0} \dfrac{N_{Ed}(x_{c,N}^I)}{\chi_y(x_{c,N}^I) N_{Rk}(x_{c,N}^I)/\gamma_{M1}} \right)$
k_{zy}	$1 - \dfrac{\overbrace{0.1\bar{\lambda}_z(x_{c,N}^{II})}^{\leq 0.1}}{C_{m,LT} - 0.25} \dfrac{N_{Ed}(x_{c,N}^1)}{\chi_z(x_{c,N}^1) N_{Rk}(x_{c,N}^1)/\gamma_{M1}}$

$$for\ \bar{\lambda}_z(x_{c,N}^{II}) < 0.4$$

$$0.6 + \bar{\lambda}_z(x_{c,N}^{II}) \leq 1 - \frac{0.1\bar{\lambda}_z(x_{c,N}^{II})}{C_{m,LT} - 0.25}\frac{N_{Ed}(x_{c,N}^1)}{\chi_z(x_{c,N}^1) N_{Rk}(x_{c,N}^1)/\gamma_{M1}}$$

Table 6: Adaptation of the equivalent uniform moment factors C_m for prismatic members

Moment **utilization** diagram	Range		C_{my} and C_{mLT}
$\varepsilon(M_h)$ ... $\varepsilon(M_h)\psi_\varepsilon$ ... $\varepsilon(M_s)$ ... $\alpha_s = \varepsilon(M_s)/\varepsilon(M_h)$	$0 \leq \alpha_s \leq 1$	$-1 \leq \psi_\varepsilon \leq 1$	$0.2 + 0.8\,\alpha_s \geq 0.4$
	$-1 \leq \alpha_s < 0$	$0 \leq \psi_\varepsilon \leq 1$	$0.1 - 0.8\,\alpha_s \geq 0.4$
		$-1 \leq \psi_\varepsilon < 0$	$0.1(1-\psi_\varepsilon) - 0.8\,\alpha_s \geq 0.4$
$\varepsilon(M_h)$ ... $\varepsilon(M_s)$ ... $\varepsilon(M_h)\psi_\varepsilon$ ... $\alpha_s = \varepsilon(M_h)/\varepsilon(M_s)$	$0 \leq \alpha_s \leq 1$	$-1 \leq \psi_\varepsilon \leq 1$	$0.95 + 0.05\,\alpha_h$
	$-1 \leq \alpha_s < 0$	$0 \leq \psi_\varepsilon \leq 1$	$0.95 + 0.05\,\alpha_h$
		$-1 \leq \psi_\varepsilon < 0$	$0.95 + 0.05\,\alpha_h(1 + 2\psi_\varepsilon)$

Finally, it should be mentioned that such adaptation of the interaction formulae leads to a similar level of reliability as for prismatic members for the analyzed cases – unrestrained beam-columns subject to axial force and major axis bending and failing in out-of-plane buckling. This occurs not only because the utilization ratios for axial force and for bending moment determined from the rules provided in Section 3 present a closer level of safety when compared to the prismatic cases; but also because for determination of C_m factors, a direct equivalency of the tapered to a prismatic member is done by considering the utilization of the applied bending moment with the resistant bending moment.

3.2 Generalized slenderness procedure for out-of-plane stability verification
For application of the general method of clause 6.3.4 of EC3-1-1, the following steps are taken (Figure 2):
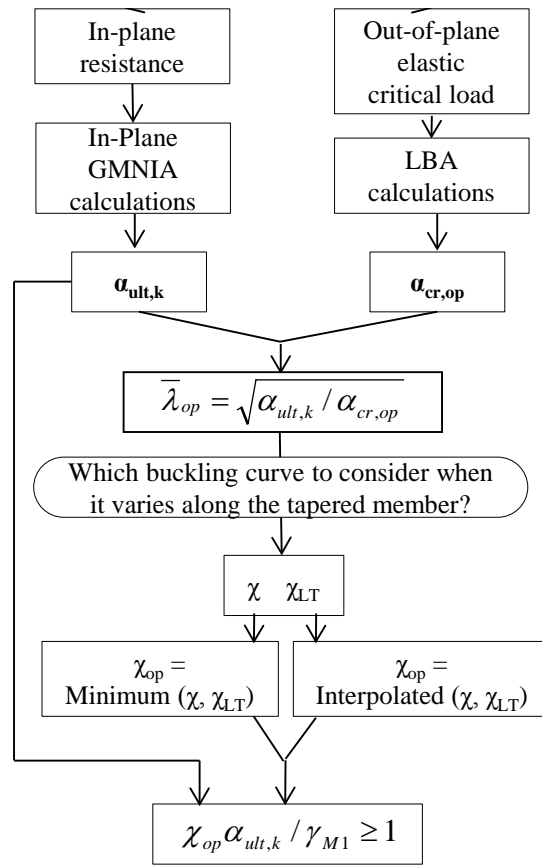

Figure 2: Application of the General Method to non-uniform members

Relatively to this, the following comments can be made:
- It was shown in Simões da Silva et al. (2010) that the consideration in-plane local imperfections in the multiplier $\alpha_{ult,k}$ for the out-of-plane flexural buckling verification of columns gives inconsistent results with clause 6.3.1. This definition was adopted in clause 6.3.4 because the consideration of the cross section resistance load multiplier (with no local imperfections) may sometimes lead to unsafe results for the stability verification of beam-columns, even if the minimum between χ_z and χ_{LT} is considered (Ofner and Greiner, 2005),

see Figure 3. A more consistent approach is therefore to consider only resistance effects in the definition of $\alpha_{ult,k}$;

- Although it is not recommended by ECCS TC8 (2006), an interpolation between the reduction factors for flexural and lateral-torsional buckling, respectively χ_z and χ_{LT} could solve this problem. The question arises on what type of interpolation to consider in order to follow the buckling behavior of the beam-column. According to the general method in EC31-1, the reduction factor χ_{op} shall be calculated with the generalized slenderness $\overline{\lambda}_{op}$ considering the imperfection factors for lateral-torsional buckling or flexural buckling, respectively $\chi_{op,LT}$ and $\chi_{op,zz}$. However, if this approach is analyzed, it can be right away observed that $\overline{\lambda}_{op,zz} = \overline{\lambda}_{op,LT} = \overline{\lambda}_{op}$ correspond to different member lengths and therefore, to different member behaviours and particularly for the case of tapered members, to different tapering angles. EC3-1-1 suggests in addition a linear cross section interpolation. Results provided by this alternative are given and discussed in Section 3.4. Even if the interpolation is carried out between the real member $\overline{\lambda}_{zz}$ and $\overline{\lambda}_{LT}$, it will be shown that a deeper study is needed in order to establish a procedure which correctly takes into account the weight between lateral and lateral-torsional buckling phenomena.
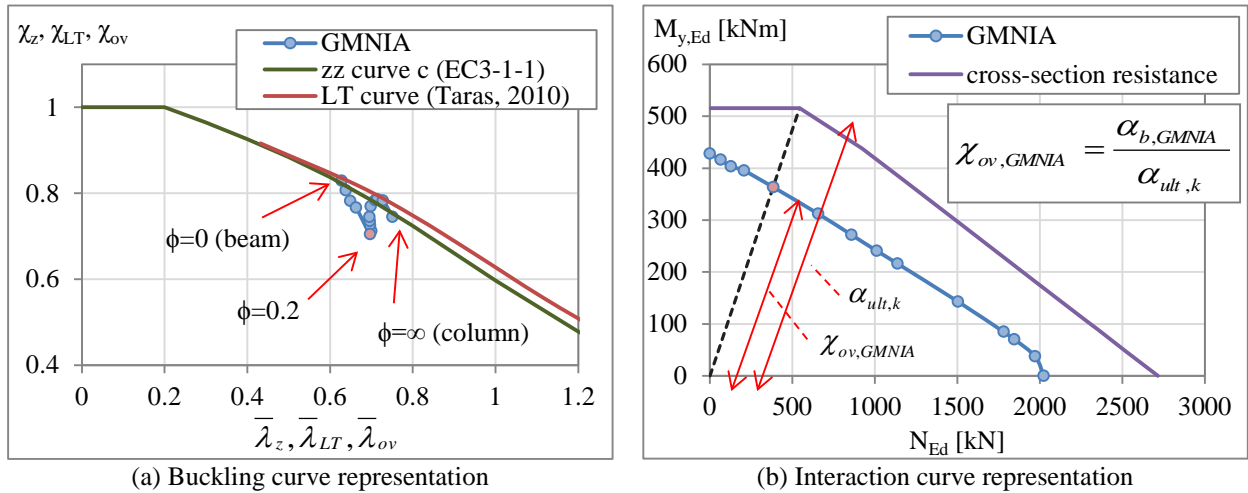

(a) Buckling curve representation   (b) Interaction curve representation

Figure 4: Example of a member which χ_{ov} is lower than both χ_z and χ_{LT}

3.3 Parametric study

The parametric study compromises 275 simply supported symmetrically web tapered beam-columns which will fail in out-of-plane buckling (with or without lateral-torsional buckling). More details may be found in Marques et al. (2012a). For the finite element models to be compared with the presented procedures, thecommercial finite element package Abaqus, version 6.10 (Abaqus, 2010) was considered.

3.3 Results and discussion

Regarding the interaction formulae, in general, the consideration of the analyzed adapted approach for tapered beam-columns leads to a resistance level between 80% and 103% of the GMNIA resistance, with an average of 93% and a coefficient of variation of CoV=5.66%. In Figure 5, to have a common basis, the generalized reduction factors are compared: $\chi_{ov}^{GMNIA} = \alpha_b^{GMNIA}/\alpha_{ult,k}$ and $\chi_{ov}^{interaction} = \alpha_b^{interaction}/\alpha_{ult,k}$, in which α_b is the resistance multiplier

obtained numerically or by the interaction approach and $\alpha_{ult,k}$ is the cross section resistance multiplier.
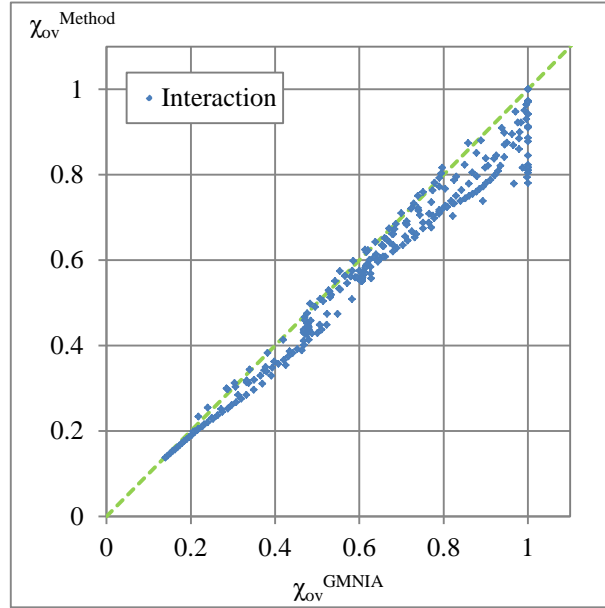

Figure 5: Results given by the interaction approach

Figure 6 illustrates 2 examples of application of the General Method. Results are given regarding curves *a*, *b* and *c*. *GM num* and *GM CS* illustrate the cases in which $\alpha_{ult,k}$ is obtained from the GMNIA in plane analysis and from the cross section resistance, respectively. Differences between the two approaches for the given example can go up to 8%. Focusing now on the level of safety given by the general method, from Figure 6 it can also be seen that the safety provided by the method may be either too conservative (up to 25%) or too unconservative (up to 20%). This is because there is not a clear decision on which curve to adopt.

(a) HEB300 | γ_h=2 | Ψ =0.25 | $\bar{\lambda}_z (x_{h\min}) = 1$
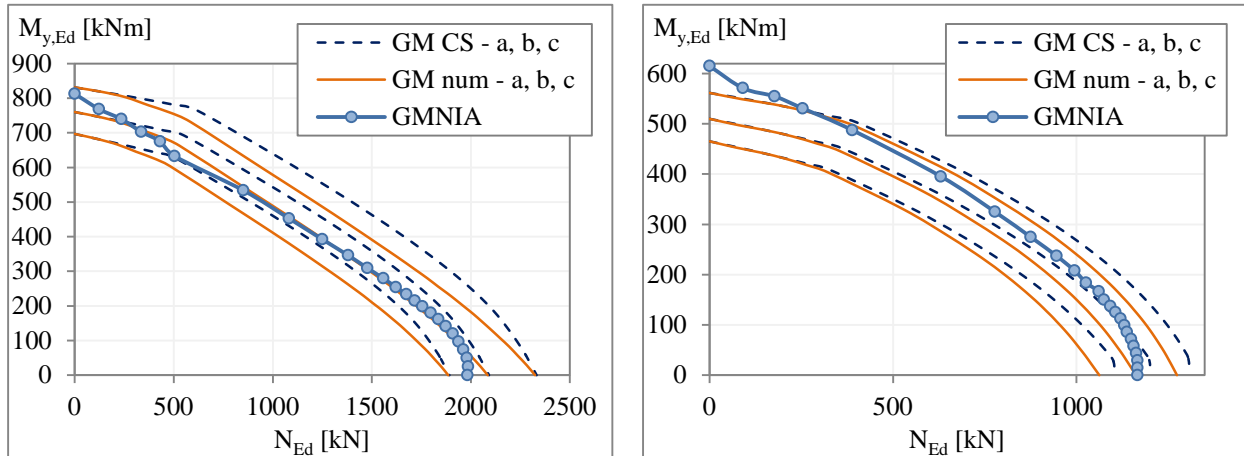
(b) HEB300 | γ_h=1.5 | Ψ =0 | $\bar{\lambda}_z (x_{h\min}) = 1.5$

Figure 6: Results of the general method considering different assumptions for $\alpha_{ult,k}$

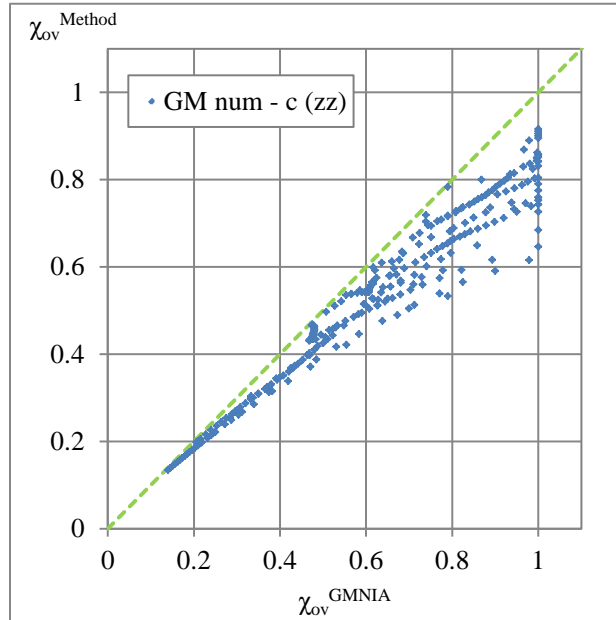Results of the general method are plotted against numerical results in Figure 7 (flexural buckling curve – *c*).

Figure 7: Results of the general method for out-of-plane flexural buckling – curve *c*

Finally, Figure 8 illustrates numerical results of a beam-column plotted over the buckling curve and the *M-N* interaction curve for analysis of the proposed cross section linear interpolation in the code. For comparison, results of the interaction approach are also plotted. When Figure 8(a) is analyzed it is noticed that the *GM_cs* interpolation leads to unsafe levels of resistance. This can be confirmed in Figure 8(b). This occurs because a linear interpolation of the cross section resistance curve between the reduction factors for flexural and lateral-torsional buckling is a simple shift of that curve, not properly accounting for the buckling phenomena. Therefore, more thought needs to be given to a proper interpolation procedure.
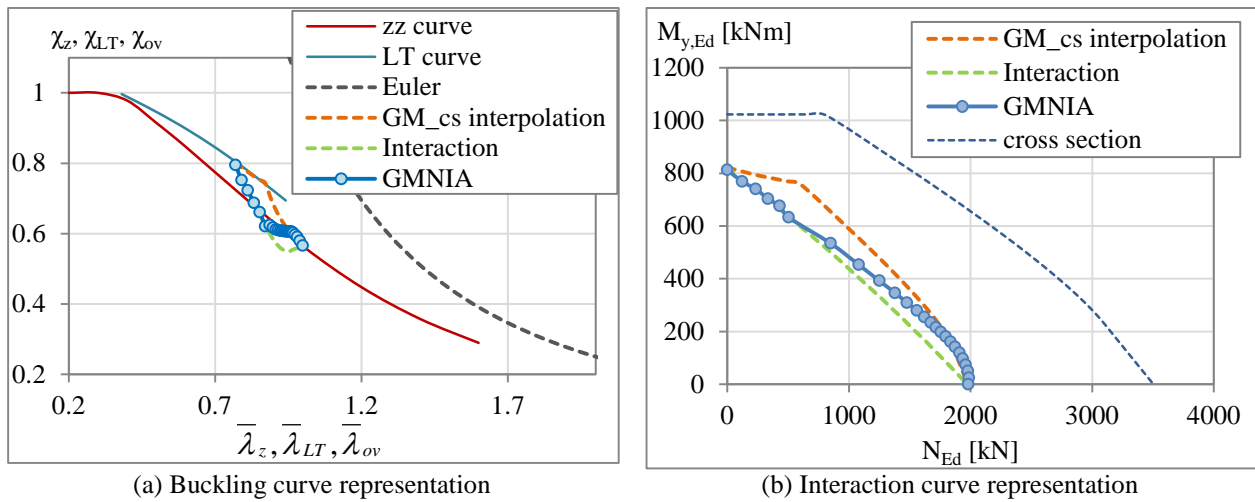

(a) Buckling curve representation

(b) Interaction curve representation

Figure 8: Results of the analyzed methodologies: HEB300 | $\gamma_h=2$ | $\Psi=0.25$ | $\bar{\lambda}_z(x_{h\min})=1$

4. Structural analysis and safety verification of steel structures according to EC3

4.1 Introduction

There are many alternatives to study stability aspects. The designer will choose which method to adopt according to the complexity of the problem; the precision of results; the level of safety to be achieved or even the simplicity of application of the method to the problem itself. Figure 9 describes the available possibilities for the analysis of a structure according to EC3-1-1.
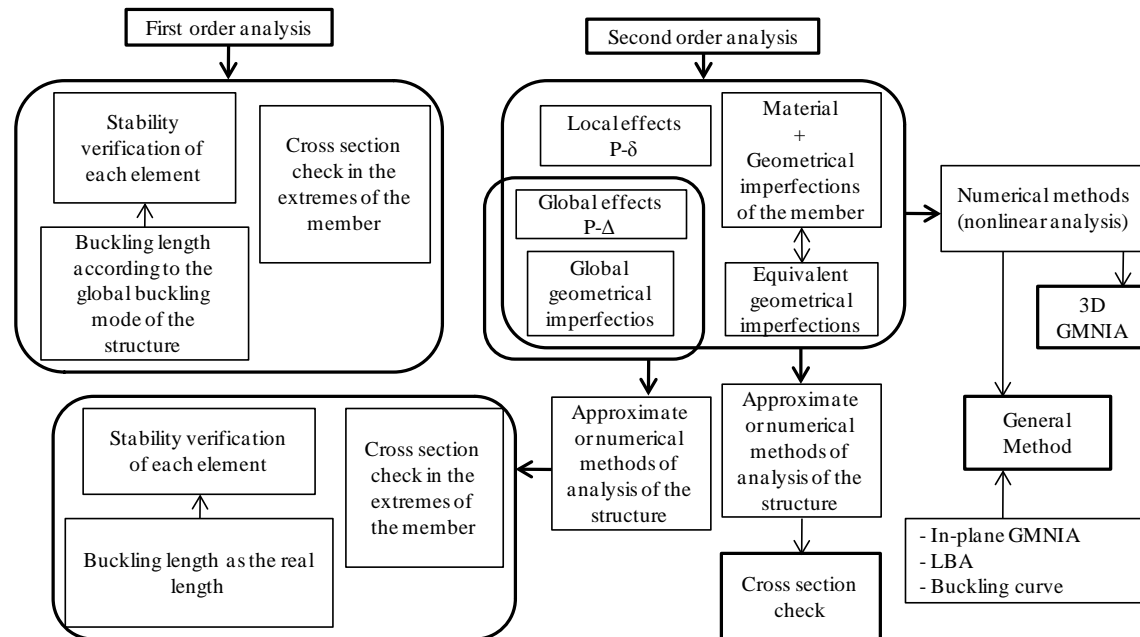

Figure 9: Methods of analysis

Provided that a second order analysis is required, following EC3-1-1, there are mainly three levels of analysis.

- Level 1: Second order analysis accounting for all the effects and imperfections – global and local (clause 5.2.2 a) of EC3-1-1). It becomes only necessary to check the cross-section resistance of the member;
- Level 2: Second order analysis considering only global effects and global geometrical imperfections (clause 5.2.2 b) of EC3-1-1). This method is the most commonly used. The stability verification of the members according to clauses 6.3.1 to 6.3.3 is carried out considering the buckling length of the member as the non-sway buckling length;
- Level 3: First order analysis of the structure (clause 5.2.2 c) of EC3-1-1). Neither imperfections nor second order effects are included in the analysis of the structure and, as a result clauses 6.3.1 to 6.3.3 of EC3-1-1 must be verified considering the buckling length of the member defined according to the global buckling mode of the structure.

In Section 4.2, an example of a frame composed of tapered members is given in order to illustrate the implementation of the several approaches (or even combination of those, as it may appear more practical).

4.2 Stability verification of frames composed of tapered members

In Section 2, design procedures for respectively flexural buckling of columns and lateral-torsional buckling of beams were developed for the case of isolated members with fork

conditions. However, members in real structures often do not exhibit these idealized boundary conditions. Due to this, several procedures exist on how to tackle the problem, either by considering all the relevant imperfections in the structural analysis or by extracting the member from the real structure by adequate buckling lengths in order to perform its stability verification separately from the global structure. These methods are brought into this section for the design of structural systems (focus on frames) with tapered members. As a starting point, only straight tapered members buckling out-of-plane between points that are braced in both flanges (i.e. in which both lateral and torsional deformation is prevented) are considered. Partial bracing is not contemplated in this analysis. As a result, for the considered cases, the buckling lengths may be assumed to be approximately equal to the member length, i.e., an approximation to fork conditions. In summary, in the scope of the present study, some of the possibilities of structural analysis and member verification are now presented for the case of frames with tapered members, assuming that, as a starting point, isolated member verification procedures are provided in the scope of Section 3.

Consider the simply supported frame of Figure 10, prevented from out-of-plane and torsional deformations at the end of each rafter and column.
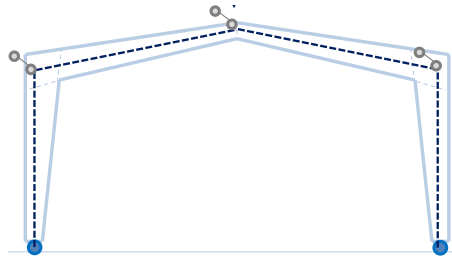

Figure 10: Frame with tapered columns and tapered rafters

Firstly, the definition of the local and global imperfections is analyzed. Assuming elastic global analysis and that α_{cr} of the frame is $\alpha_{cr}<10$, second order analysis needs to be performed. The sway imperfections ϕ are obtained from clause 5.3.2 of EC3-1-1. Local imperfections e_0 shall be obtained from Table 5.1 of EC3-1-1. Because non-uniform members either tapered or with non-uniform loading do not exactly exhibit a sinusoidal shape for the buckling mode, it is questionable whether the local member imperfections should be modeled with a bow shape or not and if the given amplitudes in the code can safely be used for members with other shapes than prismatic.

Secondly, 2 methods for the structural analysis of the frame are presented.

a) In-plane and out-of-plane member verification procedure exists (Level 2):

If proper in-plane verification procedure is available, only P-Δ effects and global (ϕ) imperfections are required in the frame analysis. The verification is then performed as follows:

1. Out-of-plane verification check (for each member):
   - Determine χ_z and χ_{LT} considering the buckling length equal to the member length;
   - Obtain the second order forces from P-Δ effects and imperfections (ϕ);
   - Perform the out-of-plane check considering the second order forces and either an interaction or generalized slenderness approach.
2. In-plane verification:
   - Determine χ_y considering the buckling length equal to the member length;

- Consider χ_{LT} from step no. 1;
- Obtain the second order forces from P-Δ effects and imperfections (ϕ);
- Perform the in-plane check considering the second order forces and either an interaction or generalized slenderness approach;
3. Perform a cross section check (considering the calculated second order forces) at a sufficient number of sections, e.g. 10 sections per member.

b) Only out-of-plane member verification procedure is provided (Level 1 and Level 3)
Considering that only out-of-plane member stability may be checked individually, the global and local in-plane second order effects and imperfections of the frame need to be accounted for in the structural analysis. For this, the second order analysis of the frame must contemplate the imperfections of Figure 11. As referred, for this alternative, local member imperfections from Table 5.1 of EC3-1-1 shall be proven to be adequate.
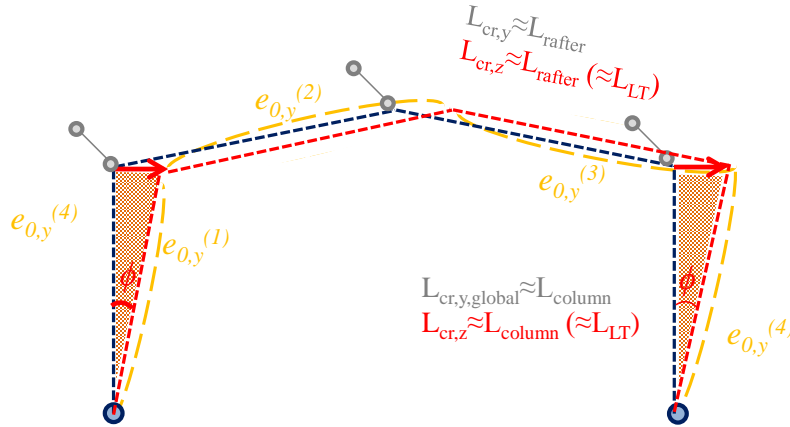This example illustrates the combination of level 1 and 3 of analysis.

Figure 11: In-plane global and local imperfections

After definition of the local and global in-plane imperfections, the verification of the frame is performed as follows:
1. Out-of-plane verification check (for each member):
   - Determine χ_z and χ_{LT} considering the buckling length equal to the member length;
   - Obtain the second order forces from P-Δ effects and imperfections (ϕ) <u>only</u>;
   - Perform the out-of-plane check considering the second order forces and either an interaction or generalized slenderness approach.
2. In-plane verification:
   - Obtain the second order forces from P-Δ and P-δ effects and imperfections (ϕ and $e_{0,y}$);
   - To include the torsional effects, reduce $M_{y,Rk}$ by $\chi_{LT}\,M_{y,Rk}$ (to be in line with the interaction formula 6.61 of EC3-1-1), considering χ_{LT} of each member;
   - Perform a cross section check (considering the calculated second order forces and the reduced moment capacity) at a sufficient number of sections, e.g. 10 sections per member.

5. Conclusions
In this paper, the stability verification of web-tapered beam-columns was discussed.
Firstly, regarding out-of-plane buckling of beam-columns it was seen that the General Method, which is the current alternative for the stability verification of such members, not only does not

provide clear guidelines of which curve to be considered, but also may lead to a high (and random) spread regarding the level of safety. Because of this, based on recent proposals for the stability verification of web-tapered columns and beams, simple adaptations of both the interaction formulae of clause 6.3.3 and the general method of clause 6.3.4 were analyzed:

- The interaction formula is applied considering the utilization of the forces N_{Ed}/N_{Rk} and $M_{y,Ed}/M_{y,Rk}$ at an arbitrary position and the respective reduction factors at the same position. For simplicity reasons it is recommended to consider $x_{c,N}^{I}$ and $x_{c,M}^{I}$, respectively. The interaction approach leads to results that are mostly on the safe side. Maximum differences of 20% relatively to the numerical results are achieved;

- A "modified" General Method is considered such that the generalized slenderness is calculated with the cross section resistance load multiplier for $\alpha_{ult,k}$. The interpolation between the flexural buckling and lateral-torsional buckling reduction factors calculated with the generalized slenderness were analyzed. It was seen that a deeper analysis needs to be carried out to provide a proper interpolation procedure considering the stability behavior of the member and also to provide limits between the stability and cross section resistance for the low slenderness range.

In the future, adequate verification procedures for the isolated member shall be developed, both for in-plane and out-of-plane buckling mode, and further adopted to the stability verification of the global structure.

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