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Three-dimensional joist member studies using equivalent beam theory

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Abstract

When selecting steel joists for a structure, design engineers have traditionally relied on standard load tables, but this is limiting when commercial software is used. Additionally, because joists and joist girders are proprietary members, design engineers can benefit from an automated approach to obtain accurate estimates of member weights and section properties in the software analysis. Changes during the design process such as variations in loading cases can make the design of joists and joist girders tedious for the joist designers. Current structural design software packages lack the necessary capacities to estimate joist and joist girder weights and section properties accurately; however they do enable the implementation of user-defined cross-sectional property tables. For joist designers, custom tables for joists and joist girders can be used with the application of equivalent beam theory which allows joist and joist girder limit states to be analyzed by strength checks built into the design software. The custom tables for joist and joist girders are developed to represent approximate prismatic members based on the properties of typical chord sizes readily used in industry. This study presents the development and validation of custom tables for joist members. A case study illustrates the application of joist and joist girder tables in the design of a three-dimensional building with drifted snow loads

Background

Most current design software packages are limited to selecting joists and joist girders from standard load tables for simply-supported members as developed by the Steel Joist Institute (SJI). These tables do not provide the engineer of record (EOR) with member section properties or member weight. The standard load tables do not allow for the automated design of joists with non-uniform loads, such as drifted snow, pattern loading or point loads. This, combined with the reality that joists and joists girders are custom designed by different manufacturers, renders the possibility of developing all-inclusive lists of available joist properties infeasible. A more practical approach is to develop tables of approximate joist and joist girder weights and section properties based on the typical chord configurations used by joist manufacturers that can be used with current software design packages. This method was originally developed for use with JGMFs as described in Knodel et al (2011).

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Virtual Joist Property Table

Many commercial design software packages allow users to implement pre-defined user tables. For this study, the virtual joist (VJ) table was developed and formatted to be used with STAAD.Pro, although they are intended to be adaptable for multiple software packages. Appendix A has a complete list of the member properties incorporated into the VJ table. It also includes an excerpt of the VJ table which is a space-delineated text file.

The two joist properties in which the EOR and joist designer are most interested are the moment of inertia (I_{zz}) and the weight; both properties are based on chord sizes and are related to the combined chord areas (A_x). The member weight is calculated from the cross-sectional areas of both the top and bottom chords; however, this calculation does not consider the web members due to the large variability in web member selection. To account for the web member weight, the density of joist material is set to the density of steel divided by 0.85. This density increase provides a simple means to approximately account for web member weight in the VJ table without affecting calculated values of axial capacity.

Because the tables represent complex geometry of a joist with a single beam member, they do not adequately capture out-of-plane joist behavior, nor can they capture local buckling of the joist components. SJI developed this VJ table based on industry bracing standards that eliminate out-of-plane limit states as member failure modes. For these studies, the joists were considered to have continuous lateral bracing.

Equivalent Beam Theory

The application of equivalent beam has been shown to adequately capture the behavior of complex flexural systems in a single beam element. Giltner and Kassimali (2000) applied equivalent beam principles to modeling full truss systems. They evaluated two different equivalent beam methods:

1. The equivalent beam’s stiffness is calculated from the deflection of the load truss.
2. Parallel axis-theorem was used to develop the equivalent beam moment of inertia based on the truss cross-sectional area. The vertical and diagonal web members were not considered in the calculation of the equivalent moment of inertia.

In their studies, Giltner and Kassimali compared and evaluated displacement and reaction values taken from a real three-dimensional model and those from equivalent models. They found that both equivalent methods provided acceptable results and two primary advantages. Equivalent beam theory greatly reduces the computational time needed for structural design software to analyze the given member. It also reduces the time required to input the truss geometry into the software package (Knodel et al 2011).

Using equivalent beam theory (EBT) within the VJ table allows a multi-element flexural system, such as a joist, to be modeled as a single beam element. This simplified element has approximate beam properties equivalent to those of the more complex system. In addition, the application of EBT allows both EOR’s and joist designers to address common non-uniform loading conditions on their design members. If the section properties of the EBT accurately capture the properties of the joist, use of EBT allows the designers to move past the restrictions of simply-supported load tables when selecting an appropriate member. Panel-point loads, drifted snow loads, non-continuous uniform loads, etc. can be handled directly when selecting an appropriately sized member.

The objective of this study is to determine if the virtual joist moment of inertia and weight selected by STAAD.Pro are within an acceptable variance from the joist moment of inertia and weight designed by a proprietary joist design software. For the purpose of this study, a value within the +/-10% variance is deemed acceptable; whereas anything outside a +/-20% is considered unacceptable for design purposes. Parametric studies on simply-supported joists and fixed-end joists with uniformly distributed loads were used to determine the effectiveness of the joist tables. Tables 1 and 2 show the parameters used in the single joist studies; these parameters were established by the SJI Research Committee to represent a range of practical joist configurations and loadings.

Table 1: K-Series Joists Testing Schedule

Depth (in)	Span 1 (ft)	ASD Load Span 1 (plf)	Span 2 (ft)	ASD Load Span 2 (plf)
12	18	375	24	200
14	21	375	28	200
16	24	375	32	200
18	27	550	36	400
20	30	550	40	400
22	33	550	44	400
24	36	550	48	400
26	39	550	52	400
28	42	550	56	400
30	45	550	60	400

The following testing procedure was used in the first and second stages to verify the VJ table:

1. A member depth and span were chosen. The design was input into STAAD.Pro and the program selected a virtual joist for the given design input.
2. The virtual joist designation, moment of inertia, and member weight were recorded.
3. The same design layout was then entered into the proprietary joist design software.
4. The proprietary joist software results were recorded and the variations in program results were calculated.

Simply-Supported K-Series Joist Studies

Twenty simply-supported K-Series joists were analyzed to ensure the VJ table functioned for the simplest design case. Table 3 summarizes results. The moment of inertia and weight variances were calculated using Eq. 1(Knodel et al 2011):

$$\frac{STAAD\_value - P.Software\_value}{P.Software\_value} \times 100 \qquad (1)$$

where:
$STAAD\_value$ = moment of inertia or weight of virtual joist selected by STAAD.Pro
$P.Software\_value$ = moment of inertia or weight of joist designed by The proprietary joist software

Table 2: LH-Series Joist Testing Schedule

Depth (in)	Span 1 (ft)	ASD Load Span 1 (plf)	Span 2 (ft)	ASD Load Span 2 (plf)
18	27	600	36	400
20	30	600	40	400
24	36	600	48	400
28	42	600	56	400
32	48	600	64	400
36	54	600	72	400
40	60	600	80	400
44	66	600	88	400
48	72	600	96	400
52	78	600	104	400
56	84	600	112	400
60	90	600	120	400
64	96	600	128	400
68	102	600	136	400
72	108	600	144	400
80	120	600	160	400
88	132	600	176	400
96	144	600	192	400
104	156	600	208	400
112	168	600	224	400
120	180	600	240	400

Table 3: Simply Supported K-Series Studies

	Variance (+/-)		
	10%	15%	20%
# Considered	20	20	20
I Acceptable	100%	100%	100%
Weight Acceptable	80%	90%	100%
Both Acceptable	80%	90%	100%

All of moment of inertia approximations fell within the +/-10% variance. For the weight variances, 80% were within the +/-10% variance. All of the weight approximations fell within the +/-20% variance.

Figure 1 shows the relationship between the moment of inertia variance and depth for the K-Series tests. Figure 2 shows the weight variance vs. depth.

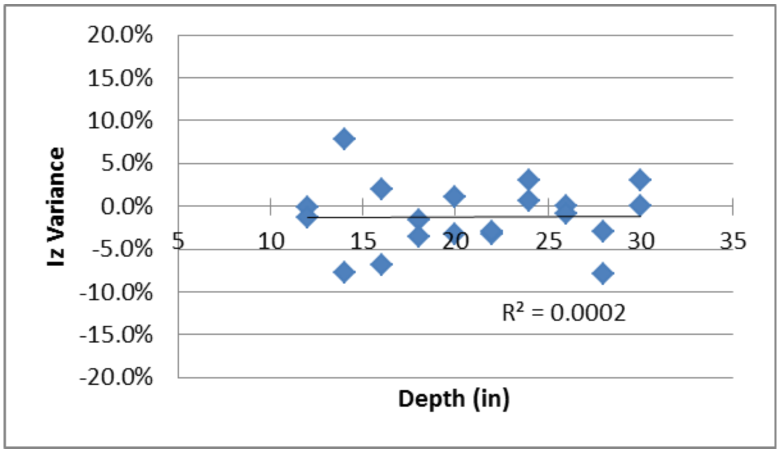

Figure 1: Moment of Inertia Variance vs. Depth for Simply-Supported K-Series Tests

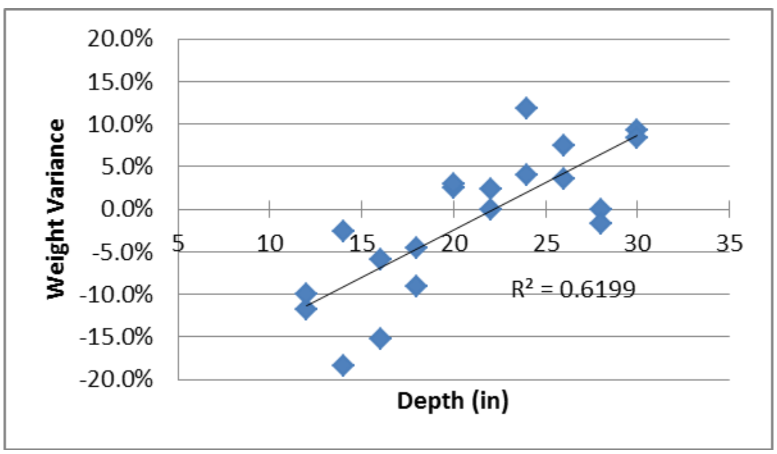

Figure 2: Weight Variance vs. Depth for Simply-Supported K-Series Tests

Figure 1 illustrates that there is no correlation between the moment of inertia variances and the joist depths. However, Figure 2 shows that all of the joists less than 20 inches in depth have negative weight variances, that is STAAD.Pro calculated a lower weight than the proprietary software. After further investigation, it was determined that for joists with depths less than 20 inches, joist manufacturers will typically use steel rod for the joist web; whereas, for all other joists depths (20 inches and greater), manufacturers will use angle. With repsect to proportion of total joist weight, the rod webbing contributes more weight to the joists than the angle webbing. To improve the weight variances presented in Figure 2, the simply-supported K-Series joists less than 20 inches in depth were re-analyzed with various incremental increases added to the joist material density.

Following the re-analysis, it was determined that the density for K-Series joists with depths less than 20 inches should be set to the density of steel divided by 0.782, which yields a density approximately 10% greater than the original joist density. Table 4 summarizes the re-analysis results. Figure 3 shows the updated weight variance vs. depth.

Table 4: Simply Supported K-Series Studies with New Density for Joists <20" in Depth

	Variance (+/-)		
	10%	15%	20%
# Considered	20	20	20
I Acceptable	100%	100%	100%
Weight Acceptable	90%	100%	100%
Both Acceptable	90%	100%	100%

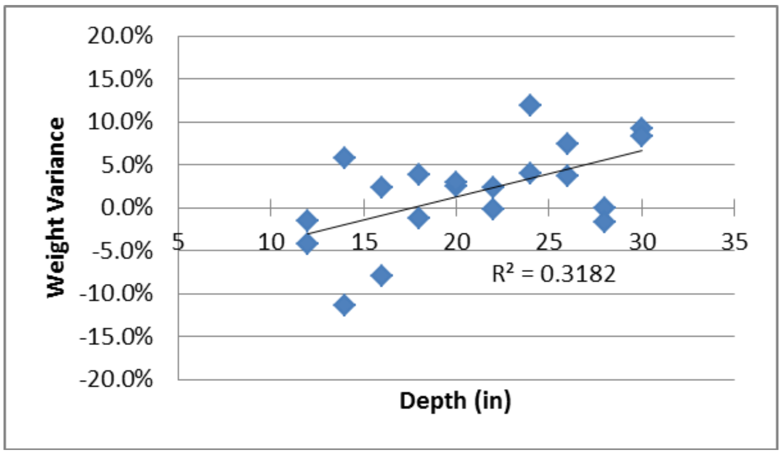

Figure 3. Weight Variance vs. Depth for Simply-Supported K-Series Studies with New Density for Joists <20" in Depth

After applying the new density to joists less than 20 inches in depth, 100% of the moment of inertia approximations and 90% of the weight variances fell within +/-10% variance. All of the weight variances fell within the +/-15% range. Figure 3 shows a positive shift in the weight variances for joists with depths less than 20 inches. The lowest weight variance shown in Figure 2, before the density adjustment, was approximately -18%; now the same variance is approximately -12% as shown in Figure 3.

Accounting for Higher Weights in the VJ Tables

The increased density for joist with depths less than 20 inches better accounted for the increased rod web weight experienced in these members; however, it is not practical from a design standpoint to have two different densities that are based on member depth. EOR's and joist designers do not necessarily know the member depth requirements prior to initial member selection. Application of a methodology that does not require any parameters of the final design selections to implement is essential in making the approach both user-friendly and universally applicable.

Thus, instead of using the increased density, the cross-sectional areas of joists with depths less than 20 inches were increased by 10% to account for the rod web weight. This area adjustment allows EOR's and joist designers to only apply one specialized density for the joist members in their design project. This is only accomplished by accepting that joists with small depths are not typically used in situations where axial capacity of the joist is a limiting factor in design. The increase in area should not greatly influence the design stresses that are calculated for these members by the structural design software.

Fixed-End K-Series Joist Studies

The same K-Series joists tested in the simply-supported tests were also analyzed with fixed-end conditions. The results of this study are shown in Table 5. Figures 4 and 5 show the moment of inertia variance and the weight variance versus depth, respectively. The increased cross-sectional areas for joists with depths less than 20" were included in these tests.

Table 5: Fixed End Supported K-Series Studies

	Variance (+/-)		
	10%	15%	20%
# Considered	20	20	20
I Acceptable	100%	100%	100%
Weight Acceptable	95%	100%	100%
Both Acceptable	95%	100%	100%

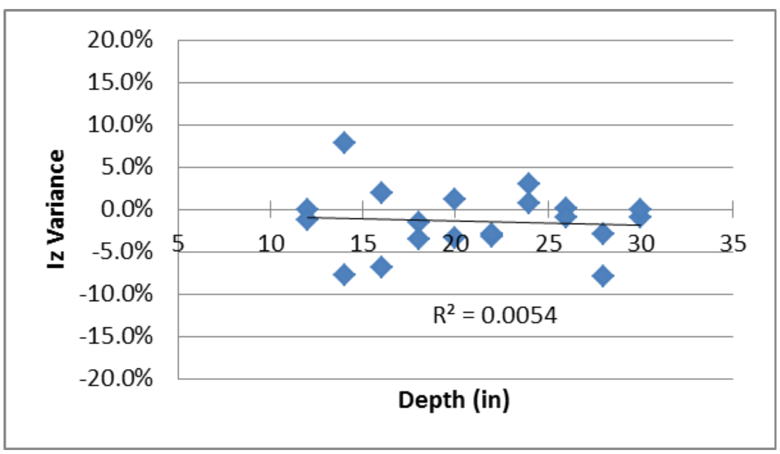

Figure 4. Moment of Inertia Variance vs. Depth for Fixed-End K-Series Tests

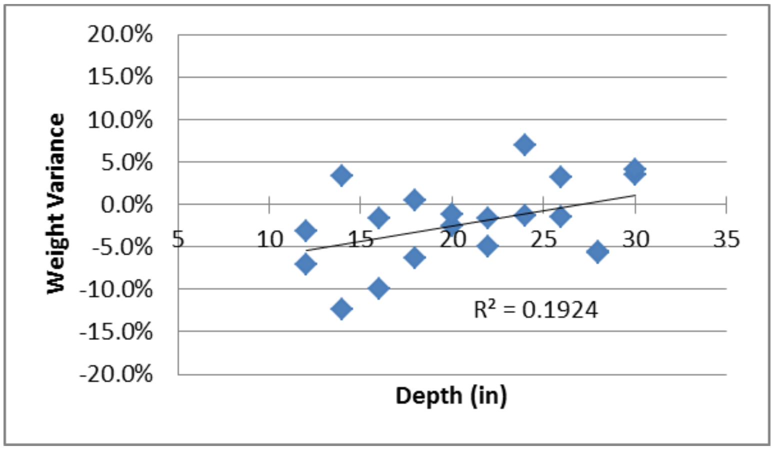

Figure 5. Weight Variance vs. Depth for Fixed-End K-Series Tests

Similar to the simply supported study, all of the moment of inertia approximations fell within a +/- 10% variance. For the weight approximations, only one virtual joist fell outside of a +/- 10% variance. Figure 4 shows almost zero correlation between the moment of inertia variance and joist depth. Figure 5 also indicates a lack of significant correlation between weight and joist depth, similar to that shown in Figure 3.The correlations in Figures 4 and 5 are not sufficient to justify more adjustments to the VJ table.

Simply-Supported LH-Series Joists Studies

The trials on the simply supported LH-Series joists used the same VJ table as the simply-supported K-Series joists. Table 6 summarizes these results.

Table 6: Simply-Supported LH-Series Studies

	Variance (+/-)		
	10%	15%	20%
# Considered	42	42	42
I Acceptable	86%	98%	100%
Weight Acceptable	90%	100%	100%
Both Acceptable	79%	98%	100%

The moment of inertia variance fell within the +/-10% variance 86% of the time. All of the moment of inertia variances fell within +/-20% range. All of the weight approximations fell within the +/-15% variance.

Figure 6 shows the relationship between the moment of inertia variance and depth for the LH-Series tests. Figure 7 shows the weight variance vs. depth.

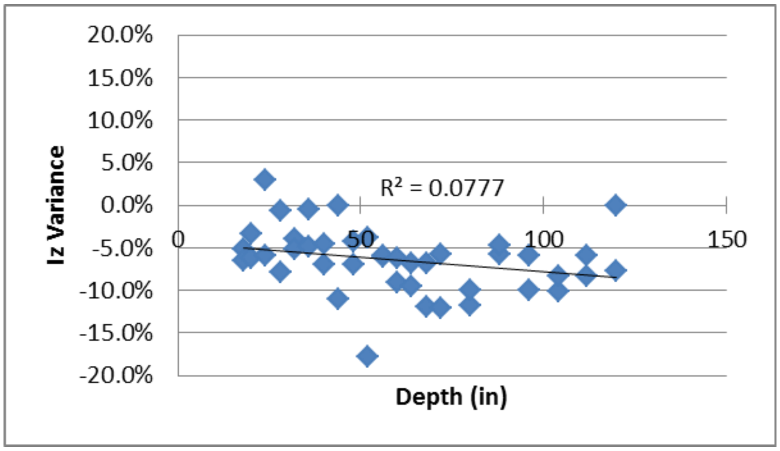

Figure 6. Moment of Inertia Variance vs. Depth for Simply-Supported LH-Series Tests

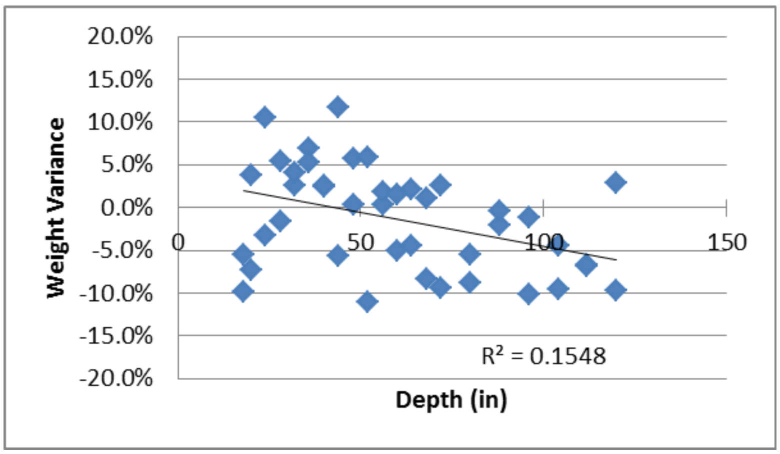

Figure 7. Weight Variance vs. Depth for Simply-Supported LH-Series Tests

Both Figures 6 and 7 show downward trends, but insignificant correlation, in the variances as the joist depth increases. The correlations for the moment of inertia and weight variances to joist depth are minimal. There is little evidence here to support adjustment of the VJ table. Figure 6 does show that for a majority of the LH-Series joists tested, STAAD selected virtual joist members with smaller moments of inertia than what the proprietary joist software calculated. Further investigation into this difference yielded that the LH-Series joists have high interaction check values for the top chord in the end panels. Therefore, even in the simply supported case, the top chord experiences combined axial and bending forces that need to be accounted for in the design.

Three-Dimensional Building Case Study

In order to demonstrate the effectiveness of the VJ tables in a more realistic design situation, a three-dimensional building model was developed to implement the VJ table. One goal for this case study was to determine the VJ table functionality with non-uniform member loads; in this instance, triangular drifted snow loads were applied to selected joists. The building geometry is shown in Figure 6 (SJI 2007). Table 7 shows the design loads and the sub-set of ASCE7-10 (2010) load combinations considered. To account for drifted snow, a six foot high mechanical screen was added to the north face of the building. In addition, the wind loads were only considered from the east and west directions, which are resisted by the joist girder moment frames shown in Section A-A of Figure 8. A joist spacing of five feet was also used in the building model.

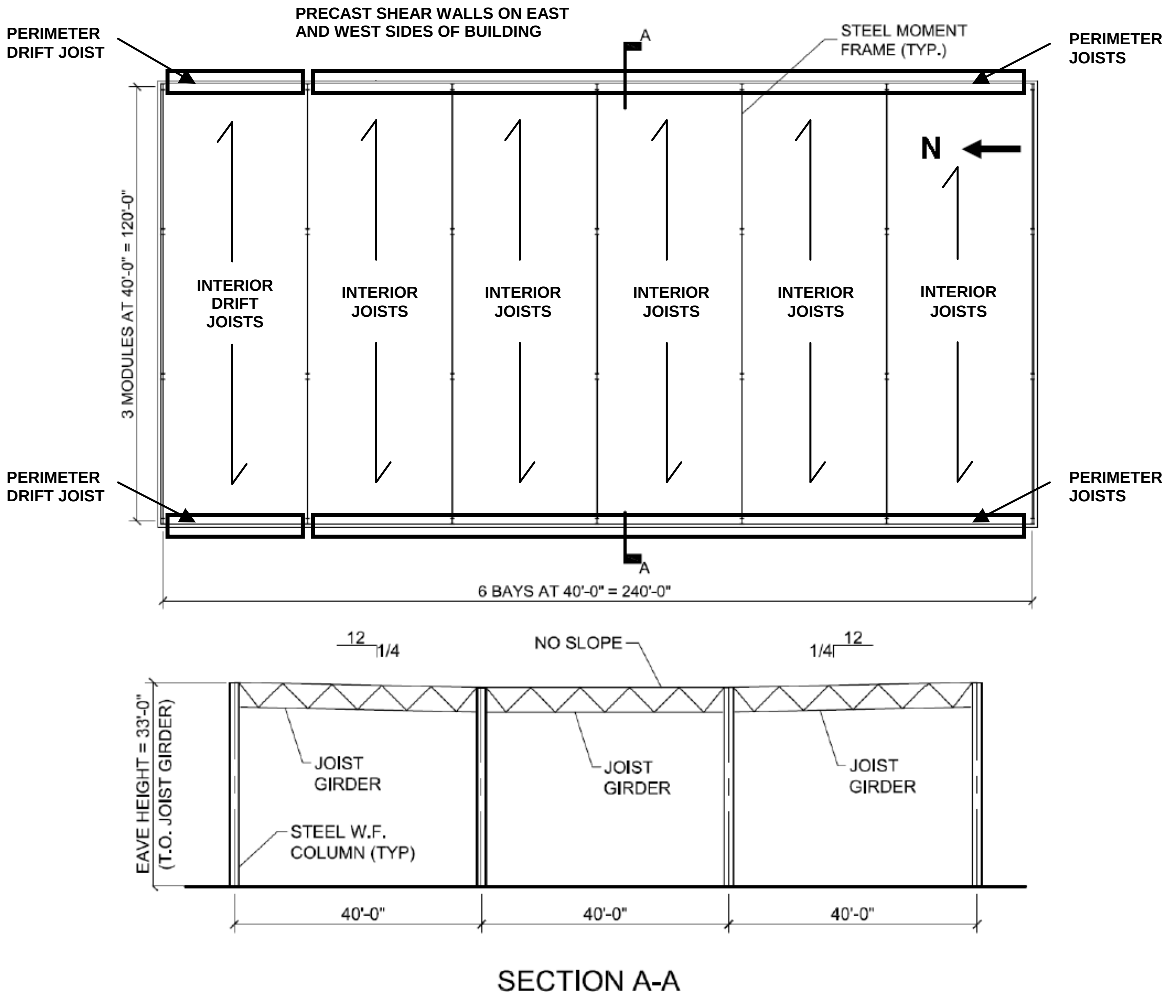

Figure 8. Building Model Dimensions and Layout

Table 7. Design Loads and Load Combinations

Load Type	Load (psf)	Load Combinations
Dead Load	20	D + S
Balanced Snow Load	32	D + W
Drifted Snow Load[1]	80	D + 0.75S + 0.75W
Uniform Wind Load	25	0.6D + W

1. Acts over 17' drift length in the north bays.

Case Study Results – Joist Selection

As shown in Figure 8, there are four different types of joists that need to be designed in this case study: Perimeter Drift Joists, Interior Drift Joists, Perimeter Joists, and Interior Joists. All of the joists were considered pinned-pinned assuming no joist extensions and standard joist support conditions. The results of the case study are shown in Table 8.

Table 8. Joist Type Selection Results

	STAAD.Pro			The proprietary joist software			Variances	
Joist Type	Member Designation	Moment of Inertia (in^4)	Weight (lbs)	Member Designation	Moment of Inertia (in^4)	Weight (lbs)	Moment of Inertia Variance (%)	Weight Variance (%)
Perimeter Drift	VJ20-6	115.0	230	20K130/80	114.8	258	0.2	-10.8
Interior Drift	VJ20-28	209.3	430	20K260/160	221.7	441	-5.6	-2.5
Perimeter	VJ20-2	98.5	195	20K130/80	104.3	238	-5.6	-18.1
Interior	VJ20-22	186.8	385	20K260/160	183.5	374	1.8	2.9

Table 8 shows that for two of the designed joist types, both the moment of inertia and weight approximations fell well within +/-10% variance. For the perimeter joist type, the moment of inertia approximation also fell within the +/-10% variance and the weight variance fell within the +/-20% variance. The perimeter drift joist had a weight variance fall just outside of the +/-10% variance. Similar to the simply-supported joist cases, this case study provided reasonably accurate agreement between the commercial selections using the commercial software and the proprietary joist manufacturer designs. These results demonstrate the VJ table functionality with both drifted snow and uniform loading conditions. Even though the weight variance for the perimeter type joist is outside the +/-10% variance, the aggregate joist weight estimated by STAAD of 56.4 kips was within 0.7% of the proprietary software estimate of 56.0 kips. It should be noted that the developed joists were designed for strength requirements only.

Conclusions and Recommendations

Similar to previous verification of the use of EBT and Virtual Joist Girder Tables for use in JGMF design (Knodel et al 2011), the studies show that a similar approach, which minor modifications to account for higher webbing weight in lower depth joists, provide a reasonable approximation for the EOR when estimating stiffness and weight of steel joists. The case study provides a preliminary proof-of-concept and illustrates the use of these tables in a more practical problem.

Further study is necessary to validate the use of EBT on joists, however. As stated earlier, equivalent beam theory allows complex models to be simplified and modeled as a single element. However, by simplifying a complex model, there is a possibility that certain failure mechanisms such as combined flexure and axial forces are not addressed in the simplified element. This can be seen in the LH-Series studies. By using the VJ Table and EBT, STAAD picked LH-Series joists with smaller moments of inertia than the same joists designed by the joist design software. The STAAD.Pro analysis failed to address the combined flexure and axial forces that these large joists experience when uniformly loaded. Thus, the members selected by STAAD had smaller top chords; this results in lower moments of inertia and member weights.

This design phenomena did not occur in the previous study (Knodel et al 2011) because the joist girders were only loaded at panel points; there was no inter-panel point loading of the top chord. Further structural modeling will need to be done to address this combined flexure and axial forces in the top chord. The next portion of this research will address methods to account for joist component interaction that cannot be captured by the EBT.

Acknowledgements

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Appendix A

Equivalent Beam Properties in Virtual Joist Table

A_x	Total Area of Top and bottom Chords
D	Girder Depth
TD	Web Thickness
B	Flange Width
TB	Flange Thickness
I_{zz}	Joist Girder Strong-Axis Moment of Inertia
I_{yy}	Joist Girder Weak-Axis Moment of Inertia
I_{xx}	Torsional Constant
S_z	Elastic Section Modulus About Strong-Axis
S_y	Elastic Section Modulus About Weak-Axis
A_y	Shear Area in Y Direction
A_z	Shear Area in Z Direction
P_z	Plastic Section Modulus About Strong-Axis
P_y	Plastic Section Modulus About Weak-Axis
HSS	Warping Constant
DEE	Depth of Web
BSD	Minimum Bearing Seat Depth

Virtual Joist Table Excerpt

Virtual_Joist_Table_STAAD.Pro.txt - Notepad

File Edit Format View Help

```
UNITS INCHES
GENERAL
VJ10-1
1.115  10.000 0.333 3.000 0.150 20.969 0.616 0.016 4.842 0.410 0.279 0.279 4.842 1.000 13.278 1.250
VJ10-2
1.151  10.000 0.333 3.500 0.127 21.331 0.833 0.014 4.812 0.476 0.288 0.288 4.812 1.000 17.758 1.500
VJ10-3
1.188  10.000 0.333 3.000 0.150 22.398 0.616 0.019 5.512 0.410 0.297 0.297 5.512 1.000 13.261 1.250
VJ10-4
1.224  10.000 0.333 3.500 0.127 22.825 0.833 0.017 5.478 0.476 0.306 0.306 5.478 1.000 17.736 1.500
VJ10-5
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VJ10-6
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VJ10-7
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VJ10-8
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VJ10-9
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VJ10-10
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VJ10-11
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VJ10-12
1.688  10.000 0.333 4.000 0.143 30.366 1.411 0.027 7.621 0.706 0.422 0.422 7.621 1.000 28.782 1.750
VJ10-13
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VJ10-14
1.813  10.000 0.333 4.500 0.141 32.100 1.989 0.029 7.568 0.884 0.453 0.453 7.568 1.000 40.000 2.000
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VJ10-16
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VJ10-17
1.938  10.000 0.333 4.500 0.141 34.078 1.989 0.031 8.629 0.884 0.484 0.484 8.629 1.000 39.444 2.000
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