Analysis and Design of Noncompact and Slender Concrete-filled Steel Tube (CFT) Beam-columns

Z. Lai¹, A. H. Varma²

Abstract
Concrete-filled steel tube beam-columns are categorized as compact, noncompact or slender depending on the slenderness ratio (width-to-thickness b/t ratio) of the steel tube wall. Current AISC Specification (AISC 360-10) uses the bilinear axial force-bending moment (P-M) interaction curve for bare steel members to design noncompact and slender CFT beam-columns. This interaction curve is shown to be over-conservative by comparisons with results from the experimental database compiled by the authors. The behavior of noncompact and slender CFT beam-columns depends on three major parameters: the tube wall slenderness ratio, axial load ratio, and member length. To explicitly investigate the effects of these three parameters, detailed 3D finite element models (FEM) are developed and benchmarked using experimental test results. Based on the parametric analysis results, revisions of the AISC 360-10 interaction curve for designing noncompact and slender CFT beam-columns in are proposed.

1. Introduction
CFT members consist of rectangular or circular steel tubes filled with concrete. These composite members optimize the best use of both construction materials, as compared to bare steel or reinforced concrete structures. The steel tube provides confinement to the concrete infill, while the concrete infill delays the local buckling of the steel tube. The behavior of CFT members under axial compression, flexure, and combined axial compression and flexure can be more efficient than that of bare steel or reinforced concrete members. Moreover, the steel tube serves as formwork for placing the concrete, which expedites and facilitates construction while reducing labor costs.

CFT members are categorized as compact, noncompact or slender depending on the slenderness ratio (width-to-thickness b/t ratio) of the steel tube wall. AISC 360-10 specifies the slenderness limits for demarcating the members, as shown in Table 1. These slenderness limits are proposed by Varma and Zhang (2009), based on the research of Schilling (1965), Winter (1968), Tsuda et al. (1996), Bradford et al. (1998, 2002), Leon (2007) and Ziemian (2010). Developments of the slenderness limits are discussed in detail by Lai et al. (2014a, 2014b).

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For a CFT column or beam, if the governing slenderness ratio is less than or equal to $\lambda_p$, the member is classified as compact; if the governing slenderness ratio is greater than $\lambda_p$ but less than or equal to $\lambda_r$, the member is classified as noncompact; if the governing slenderness ratio is greater than $\lambda_r$, the member is classified as slender. The slenderness ratio is also limited to a maximum permitted value $\lambda_{\text{limit}}$ due to: (i) the lack of experimental data for CFTs with such slender steel tubes, and (ii) potential issues with deflections and stresses in the slender tube walls due to concrete casting pressures and other fabrication processes.

For a CFT beam-column, the member is classified as compact if the governing slenderness ratio is less than or equal to $\lambda_p$ for columns or beams (whichever is smaller); the member is classified as noncompact if the governing slenderness ratio is less than or equal to $\lambda_r$ for columns or beams (whichever is smaller); if the governing slenderness ratio is greater than $\lambda_r$ for columns or beams (whichever is smaller), the member is classified as slender. The maximum permitted slenderness ratio is also limited to $\lambda_{\text{limit}}$ of columns or beams (whichever is smaller).

### Table 1: Slenderness limit for CFT members

<table>
<thead>
<tr>
<th>Loading</th>
<th>Description of Element</th>
<th>$\lambda_p$</th>
<th>$\lambda_r$</th>
<th>$\lambda_{\text{limit}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Width-to-Thickness Ratio</td>
<td>$\lambda_p$</td>
<td>$\lambda_r$</td>
<td>Maximum Permitted</td>
</tr>
<tr>
<td>Axial compression</td>
<td>Steel tube walls of Rectangular CFT</td>
<td>$b/h$</td>
<td>$2.26 \frac{E_s}{F_y}$</td>
<td>$3.00 \frac{E_s}{F_y}$</td>
</tr>
<tr>
<td></td>
<td>Members</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Steel tube wall of Circular CFT Members</td>
<td>$D/t$</td>
<td>$0.15 \frac{E_s}{F_y}$</td>
<td>$0.19 \frac{E_s}{F_y}$</td>
</tr>
<tr>
<td>Flexure</td>
<td>Flanges of Rectangular CFT Members</td>
<td>$b/h$</td>
<td>$2.26 \frac{E_s}{F_y}$</td>
<td>$3.00 \frac{E_s}{F_y}$</td>
</tr>
<tr>
<td></td>
<td>Web of Rectangular CFT Members</td>
<td>$b/t$</td>
<td>$3.00 \frac{E_s}{F_y}$</td>
<td>$5.70 \frac{E_s}{F_y}$</td>
</tr>
<tr>
<td></td>
<td>Steel tube wall of Circular CFT Members</td>
<td>$D/t$</td>
<td>$0.09 \frac{E_s}{F_y}$</td>
<td>$0.31 \frac{E_s}{F_y}$</td>
</tr>
</tbody>
</table>

As an innovative and efficient structural component, CFT members are used widely around the world in various types of structures. In China, CFT members are used in more than three hundred composite bridges. Fig.1 (a) shows a typical application of CFT members in a half-through arch bridge. The chords, webs, and bracings of the four-pipe truss are all made of CFTs. CFT members are also used as columns in composite moment or braced frames, for example in: (1) 3 Houston Center in Houston, USA, and (ii) Wuhan International Financial Center in Wuhan, China. Fig.1 (b) shows a typical application of CFT members as mega columns in composite braced frames. Both the CFT chord members in bridges and CFT columns in frames are often subjected to combined axial compression and flexure, and therefore should be designed as beam-columns.

For CFT beam-columns, existing research focuses mostly on the members with compact sections. For example, Furlong (1967), Bridge (1976), Wang and Moore (1997), Cai (1991), Matsui et al. (1997), Elremaily and Azizinamini (2002), Seo et al. (2002), Varma et al. (2002, 2004), Han and
Yao (2003), Hardika and Gardner (2004). However, the research on noncompact and slender CFT beam-columns is limited.

Due to this lack of research, AISC 360-10 provides Eq. (1) and (2) for developing P-M interaction curves and designing noncompact and slender CFT beam-columns. These equations are based on those for steel columns, and do not account for the expected beneficial effects of axial compression on the flexural capacity, which is quite conservative.

\[
\begin{align*}
\text{When } \frac{P_r}{\phi_c P_n} & \geq 0.2 & \frac{P_r}{\phi_c P_n} + 8 \frac{M_r}{\phi_b M_n} & \leq 1.0 \\
\text{When } \frac{P_r}{\phi_c P_n} & < 0.2 & \frac{P_r}{2\phi_c P_n} + \frac{M_r}{\phi_b M_n} & \leq 1.0
\end{align*}
\]

where \( P_r \) is the required axial compressive strength, \( M_r \) is the required flexural strength, \( P_n \) is the nominal axial capacity, \( M_n \) is the nominal flexural capacity, and \( \phi_c \) is the resistance factor for compression (\( \phi_c = 0.75 \)). The calculations of \( P_n \) and \( M_n \) for CFT members are specified in the AISC 360-10.

In this paper, the experimental database of tests conducted on noncompact and slender CFT beam-columns is presented and discussed first. The degree of conservatism of the AISC 360-10 beam-column design equations is evaluated by using them to predict the strength of the tests in the database. Finite element models (FEMs) are then developed and benchmarked using experimental test results. The benchmarked models are used to perform comprehensive analyses that explicitly investigate the effects of slenderness ratio, axial load ratio, and member length on the behavior of CFT beam-columns. Based on the analysis results, revisions of the AISC 360-10 beam-column interaction curve for designing noncompact and slender CFT beam-columns are proposed.

2. Experimental database and comparisons with design equations

2.1 Experimental database

Several experimental databases have been developed for tests conducted on CFT members, for example, Nishiyama et al. (2002), Kim (2005), Gourley et al. (2008), and Hajjar (2013). These databases usually include tests on CFT columns, beams, beam-columns, connections, and frames. The database compiled by Gourley et al. (2008) and Hajjar (2013) are the most
comprehensive, and include the material, geometric, and loading parameters, and brief description of all tests. As part of the research by Lai et al. (2014a, 2014b), a comprehensive database of tests conducted on noncompact and slender CFT members subjected to axial compression, flexure, and combined axial compression and flexure was compiled. The database included tests from the database of Gourley et al. (2008) and Hajjar (2013), and additional tests from other databases and literature as applicable. A total of one hundred and eighty-seven tests were included in the database, including eighty-eight column tests, forty-six beam tests, and fifty-three beam-column tests. In this paper, only the part focusing on beam-column tests is presented.

Seventeen rectangular and thirty-six circular noncompact and slender CFT beam-column tests were compiled into the experimental database. Three types of loading schemes (Type-1, Type-2 and Type-3) were used in the beam-column tests, as shown in Fig.2. In Type-1 loading, concentric axial load was applied first and kept constant, and then lateral loads were applied to produce moment. In Type-2 loading, concentric axial loading was applied first and maintained constant, and then end moments were applied incrementally. In Type-3 loading, eccentric axial load was applied incrementally, so that both axial force and bending moment increased proportionally. Type-1 and Type-2 loading are fundamentally the same, therefore they are called Type-A loading hereafter. Type-3 loading is called Type-B loading hereafter. Table 2 summarizes the noncompact and slender CFT beam-column tests that were compiled into the experimental database.

![Fig.2: Typical loading schemes for CFT beam-column tests](image)

Table 2(a) includes the length \(L\), width \(B\), depth \(H\), flange thickness \(t_f\), web thickness \(t_w\), governing tube slenderness ratio \((b/t_f \text{ and } H/t_w)\), and the slenderness coefficient \(\lambda_{coeff}\) obtained by dividing the governing slenderness ratio by \(\sqrt{E_s/F_y}\) for rectangular CFT beam-columns. Table 2(b) includes the length \(L\), diameter \(D\), tube thickness \(t\), tube slenderness ratio \((D/t)\), and the slenderness coefficient \(\lambda_{coeff}\) obtained by dividing the governing slenderness ratio by \(E_s/F_y\) for circular CFT beam-columns. These tables also include the measured steel yield stress \(F_y\) and concrete strength \(f'_c\) where reported by the researchers. The experimental axial load capacity \(P_{exp}\) and moment capacity \(M_{exp}\) are included along with the nominal strength \(P_n\) and \(M_n\).
### Table 2(a): Noncompact and slender rectangular CFT beam-column tests

| Reference                     | Load Type | Specimen ID | L (mm) | B (mm) | t (mm) | \( \lambda_{coeff} \) | L/D | \( F_y \) (MPa) | \( f'c \) (MPa) | \( E_c \) (GPa) | \( P_a \) (kN) | \( P_{exp} \) (kN) | \( P_{exp}/P_a \) | \( M_u \) (kN-m) | \( M_{exp} \) (kN-m) | \( M_{exp}/M_u \) |
|-------------------------------|-----------|-------------|--------|--------|-------|----------------------|-----|-----------------|-----------------|----------------|----------------|-------------------|-------------------|----------------|-------------------|
| **Momo et al. (1996)**        | Type-B    | ER4-D-4-06  | 969    | 323    | 4.38  | 71.7 17.7 323 4.38 71.7 2.56 | 3.0 262.0 41.1 30.34 4520.1 3306.0 0.73 209.7 210.0 0.96 |
| **Nakamura and Salerno (1998, 2000a)** | Type-A    | BR4-3-10-02 | 600    | 200    | 3.17  | 61.1 200 3.17 61.1 2.41 | 3.0 310.0 119 51.62 4303.0 10490.0 0.24 72.0 136.0 1.89 |
| **Uy, B. (2001)**             | Type-B    | HS16        | 630    | 210    | 5.00  | 40.0 210 5.00 40.0 2.45 | 3.0 378.0 32.0 2077.3 3106.0 0.76 249.1 77.7 0.31 |
| **Marsi and Uy (2004)**       | Type-B    | SH-C210     | 2416   | 220    | 5.00  | 42.0 220 5.00 42.0 2.59 | 3.0 119 71.6 20.0 21.16 4122.9 3062.0 0.74 363.8 76.6 0.21 |

### Table 2(b): Noncompact and slender circular CFT beam-column tests

| Reference                     | Load Type | Specimen ID | L (mm) | D (mm) | \( \lambda_{coeff} \) | L/D | \( F_y \) (MPa) | \( f'c \) (MPa) | \( E_c \) (GPa) | \( P_a \) (kN) | \( P_{exp} \) (kN) | \( P_{exp}/P_a \) | \( M_u \) (kN-m) | \( M_{exp} \) (kN-m) | \( M_{exp}/M_u \) |
|-------------------------------|-----------|-------------|--------|-------|----------------------|-----|-----------------|-----------------|----------------|----------------|-------------------|-------------------|----------------|-------------------|
| **Ichinohe et al. (1991)**    | Type-A    | CO4F5M      | 2000   | 300    | 42.5  | 70.6 0.15 67 438.0 66.2 38.51 5262.6 3069.5 0.58 197.8 329.0 1.66 |
| **Prion and Boehme (1993)**   | Type-A    | BP11        | 2120   | 152    | 1.70  | 89.4 0.15 139 328.0 92.0 45.39 1341.1 470.0 0.35 16.6 29.7 1.79 |
| **O’shea and Bridge (1997c)** | Type-B    | S12E210     | 662    | 190    | 1.13  | 168.1 0.18 3.5 165.7 41.0 17.81 1073.4 1229.0 1.14 10.2 10.5 1.02 |

### Table 2(c): Noncompact and slender circular CFT beam-column tests

| Reference                     | Load Type | Specimen ID | L (mm) | D (mm) | \( \lambda_{coeff} \) | L/D | \( F_y \) (MPa) | \( f'c \) (MPa) | \( E_c \) (GPa) | \( P_a \) (kN) | \( P_{exp} \) (kN) | \( P_{exp}/P_a \) | \( M_u \) (kN-m) | \( M_{exp} \) (kN-m) | \( M_{exp}/M_u \) |
|-------------------------------|-----------|-------------|--------|-------|----------------------|-----|-----------------|-----------------|----------------|----------------|-------------------|-------------------|----------------|-------------------|
| **O’shea and Bridge (2000)**  | Type-B    | S12E50A     | 662    | 190    | 1.13  | 168.1 0.18 3.5 165.7 41.0 17.81 1073.4 1229.0 1.14 10.2 10.5 1.02 |

2.2 Comparisons and discussions

The AISC beam-column design equations (Eq. 1 and Eq. 2) were used to predict the strength of the fifty-three test specimens in the database. Comparisons of the results are shown in Fig. 3. In Fig. 3(a) and Fig. 3(b), the comparisons are shown separately for rectangular CFT beam-columns with normal strength and high strength steel \((F_{y}>=525\ \text{Mpa})\), as specified in the AISC 360-10).
As shown, the design equations are very conservative for all specimens with normal strength steel, and slender specimens (with $\lambda_{\text{coef}} = 3.92$) with high strength steel. For example, the ratio of experimental moment capacity ($M_{\text{exp}}$) to nominal moment capacity ($M_n$) is 2.24 for the specimen with $\lambda_{\text{coef}} = 3.42$ and axial load ratio of 0.48. For other specimens with high strength steel, the design equations match the test results closely. Fig. 3 (c) shows that the design equations are conservative for all circular beam-column specimens. The maximum $M_{\text{exp}}/M_n$ ratio is 3.05 for some specimens with nominal axial load ratio of 0.7.

The confinement factor (as will be discussed later in Eq.3) of CFT beam-columns with high strength steel is greater than that of CFT beam-columns with normal strength steel. This is the reason that design equations are less conservative for CFT beam-columns with high strength steel. This paper is limited to CFT beam-columns with normal strength steel, since AISC does not permit the use of CFT members with high strength steel ($F_y \geq 525$ Mpa) due to the lack of comprehensive research.

For CFT members with normal strength steel, there are two primary reasons for the over-conservatism of the AISC beam-column design equations. The first reason is that the AISC 360-10 is conservative in evaluating the axial and flexural capacity of noncompact and slender CFT columns and beams. As mentioned in Section 2.1, eighty-eight column tests and forty-six beam tests on noncompact and slender CFT members were included in the database. The results of these tests are compared with the $P_n$ and $M_n$ calculated according to AISC 360-10, as shown in Fig.4. The ordinate represents the ratio of experiment to calculated value ($P_{\text{exp}}/P_n$ or $M_{\text{exp}}/M_n$).
while the abscissa represents the normalized nondimensional slenderness coefficient ($\lambda_{\text{coeff}}$). As shown, the $P_{\text{exp}}/P_n$ and $M_{\text{exp}}/M_n$ ratios are greater than 1.0 for all the specimens. The maximum $P_{\text{exp}}/P_n$ ratio is 1.74, and the maximum $M_{\text{exp}}/M_n$ ratio is 1.72. The comparisons indicate that the AISC design equations for calculating the strength of noncompact and slender columns and beams are very conservative.

The second reason is that AISC 360-10 uses the bilinear interaction curve for bare steel members to design CFT beam-columns. Similar to steel beam-columns, the slenderness ratio governs the local buckling behavior of the steel tubes. Similar to steel or reinforced beam columns, the behavior of CFT beam-columns is dependent on the axial load ratio and member length. The axial load ratio $\alpha$ ($\alpha=P/P_n$, where $P$ and $P_n$ is the applied axial load and nominal axial capacity of the CFT columns, respectively) governs the axial load-bending moment (P-M) interaction behavior of the CFT beam-column. When the axial load ratio ($\alpha$) is low, i.e., below the balance point on the P-M interaction curve, flexural behavior dominates the response. When $\alpha$ is high, i.e., above the balance point, axial compression behavior dominates the response. The member length determines the effects of secondary moments and imperfections.

However, the effects of slenderness ratio, axial load ratio and member length on the behavior of CFT beam-columns are different than that of bare steel beam-columns or reinforced concrete columns. The shape of the P-M interaction curve for CFT beam-columns is influenced by the confinement factor $\xi$ defined in Eq. 3, where $A_s$ and $A_c$ are the total areas of the steel tube and concrete infill, respectively. CFT beam-columns with larger $\xi$ values have P-M interaction curves that are more similar to those for steel columns, while beam-columns with smaller $\xi$ have P-M interaction curves that are more similar to those for reinforced concrete columns, as shown in Fig. 5.

$$\xi = \frac{A_s f_y}{A_c f_c}$$

(3)

Therefore, two tasks should be completed in order to improve the AISC 360-10 design equations for noncompact and slender CFT beam-columns. The first task is to improve the design equations for calculating $P_n$ and $M_n$, and the second task is to improve the bilinear interaction diagram. The first task is discussed in detail by the authors in separate articles. This paper focuses on the second task.

Attempts are made by several researchers to improve the interaction curve for CFT beam-columns, namely, Hajjar and Gourley (1996), Choi (2004) and Han (2007). However, the findings from those researchers are based on the research on CFT members with compact sections. Therefore they may not be applicable to noncompact and slender CFT members. In order to complete the second task, the effects of slenderness ratio, axial load ratio and member length on the behavior of noncompact and slender CFT beam-columns, as well as the effect of the confinement factor on the shape of the interaction curve should be investigated explicitly. This is accomplished in this paper by performing comprehensive finite element analysis using ABAQUS (6.12).
Fig. 4: Comparisons of the calculated (based on AISC 360-10) and experimental results for noncompact and slender: (a) rectangular CFT columns; (b) circular CFT columns; (c) rectangular CFT beams; and (d) circular CFT beams

Fig. 5: Effect of the confinement factor $\xi$ on the shape of CFT beam-column interaction curve

3. Development of FEM
In this part, detailed 3D finite element models (FEM) are developed and benchmarked using experimental test results from the database presented in Section 2. The FEM accounts for steel plasticity and local buckling, concrete cracking and compression plasticity, geometric imperfections, contact between steel tube and concrete infill, as well as interaction between local buckling and global column buckling.

3.1 FEM details
3.1.1 Material model
The steel material behavior was defined using Von Mises yield surface, associated flow rule, and kinematic hardening. An idealized bilinear curve as shown in Fig. 6 (a) was used to specify the
steel stress-strain behavior. Steel yield stress as listed in Table 2 were used for each specimen. The elastic modulus was assumed to be 200 Gpa, and the tangent modulus $E_t$ was assumed to be $E_s/100$.

The concrete material was modeled using the damaged plasticity (CDP) material model developed by Lee and Fenves (1998). This model accounts for multiaxial behavior using a special compression yield surface developed earlier by Lubliner et al. (1998) and modified by Lee and Fenves (1998) to account for different evolution of strength in tension and compression.

It is well known that concrete has non-associated flow behavior in compression (Chen and Han 1995). The CDP model accounts for this non-associated flow using the Drucker-Prager hyperbolic function as the flow potential $G$, a dilation angle $\psi$, and an eccentricity ratio $e$. Increasing the dilation angle will result in larger volumetric dilation of the concrete, and potentially better confinement, and resulting increase in strength and ductility. The behavior of confined concrete has a significant effect on the behavior of CFT columns and beam-columns. The value of the dilation angle is important from the perspective of modeling. For rectangular CFT beam-columns, the dilation angle was defined as 15°. For circular CFT beam-columns, extensive concrete dilation occurs in the compressive region. Changing the dilation values alone in the FEM analysis was incapable of providing sufficient dilation. Instead, different compressive stress-strain curve was used, as will be discussed later. The dilation angle was also assumed to be 15°. The selected value of the dilation angle and concrete stress-strain curves are reasonable for all specimens, as shown later in the comparisons of the FEM and experimental results.

Other parameters required to define the plasticity are: the eccentricity ratio $e$, the ratio of biaxial compressive strength to uniaxial strength $f'_{bc}/f'_c$, and the ratio of compressive to tensile meridians of the yield surface in $\Pi$ (deviatoric stress) space $K_c$. The default value of 0.1 was specified for the eccentricity ratio. The default value of $e$ indicates that the dilation angle converges to $\psi$ reasonably quickly with increasing value of compression dominant pressure ($p$). The ratio of the biaxial-to-uniaxial compressive strength is assumed to be equal to 1.16 based on Kupfer and Gerstle (1971). The ratio of the tensile-to-compressive meridian is assumed to be equal to 0.67 based on Chen and Han (1995).

Two types of stress-strain curve were used to specify the compressive behavior of the concrete infill, depending on the concrete strength. According to AISC 360-10, concrete with the compressive strength $f'_c$ greater than 68.5 MPa is defined as high strength concrete. Under axial compression, high strength concrete develops the compressive stress linearly up to $f'_c$. There is no significant dilation before the compressive strength is reached. Therefore, the empirical stress-strain curve proposed by Popovics (1973) was used to define the compressive behavior of the concrete infill for CFT beam-columns with high strength concrete, as shown in Fig. 6 (b). For CFT beam-columns filled with normal strength concrete, the concrete infill has significant volumetric significant amount of dilation after the compressive stress exceeds 0.70$f'_c$. Thus, elastic perfectly plastic curve was used.

The tension behavior was specified using a stress-crack opening displacement curve that is based on fracture energy and empirical models, as proposed by Wittmann (2002), as shown in Fig.6(c).
3.1.2 Contact
General contact was used to simulate the normal and tangential interaction between concrete infill and steel tube. For interaction in the normal direction, the hard contact pressure-overclosure relationship with penalty constraint method was used. This enforced no penetration between steel tube wall and concrete infill. For interaction in the tangential direction, penalty friction formulation was used, with a Coulomb friction coefficient of 0.55, and maximum interfacial shear stress of 0.41 Mpa (60 psi, as suggested by AISC 360-10). There was no additional bond (adhesive or chemical) between the steel tube and the concrete infill in the model. The steel tube and concrete infill are free to slip relative to each other if the applied interfacial shear stress ($\tau_{app}$) is greater than 0.55 times the contact pressure ($p$).

3.1.3 Geometric imperfection, boundary conditions, element type, and analysis method
The shape of the geometric imperfection was developed by conducting eigenvalue buckling analysis. The first mode shape as shown in Fig. 7 (a) with the magnitude of 0.1 $t$ (thickness of the steel tube) was incorporated into the model as the imperfection. The boundary conditions and constraints used for the FEM were designed to simulate those achieved in the experiments, i.e., by using coupling constrains as shown in Fig. 7 (b).

The steel tube was modeled using a fine mesh of 4-node S4R shell elements with six degrees of freedom per node, and reduced integration in the plane of the elements. These elements have a

Fig.6: (a) idealized bilinear curve for the steel tube; (b) compressive stress-strain curve for concrete (40 Mpa); and (c) tensile stress-crack opening displacement curve for concrete (40 Mpa)
general thick-shell formulation that reduces mathematically to thin-shell (discrete Kirchhoff) behavior as the shell thickness becomes small. Transverse shear deformation is allowed in these elements. Three section points through the thickness were used to compute the stresses and strains through the thickness. Gauss quadrature was used to calculate the cross-sectional behavior of the shell element. The concrete infill was modeled using eight-node brick (solid) elements with reduced integration (C3D8R). This is a computational effective element for modeling the concrete infill.

The fracture behavior of concrete in tension makes it impossible to obtain converged results in standard (predictor-corrector) analysis methods like full Newton or modified Newton-Raphson. Even the arc-length based techniques like modified-Riks methods developed to predict the behavior of structures undergoing large deformations and instability cannot provide converged results. Implicit dynamic analysis based analysis methods also become unstable and cannot provide results after significant cracking. Therefore, the explicit dynamic analysis method was used. The primary advantage and reason for using this method is that it can find results close or even slightly beyond failure, particularly when brittle materials (like concrete in tension) and failures are involved. The explicit dynamic analysis method was used to perform quasi-static analyses simulating the experiments.

3.2 Benchmarking the FEM
The developed FEM was used to model the experimental tests in the database. The authors considered modeling all the tested specimens. However, this was not possible because: 1) some specimens were tested under cyclic loading (which is beyond the scope of this paper); or 2) some key information was not reported in the papers (i.e., specimen length, loading protocol, boundary conditions, etc.) for some specimens. As a result, thirty-nine beam-column specimens were modeled.

Comparisons of the strength from the FEM analysis and experimental tests are shown in Fig.8, with ordinate being the ratio of experiment to FEM value, and abscissa being the slenderness coefficient. For CFT beam-columns with Type-A loading where the axial load was kept constant, only comparisons of the flexural strength were required, as shown in Fig.8 (a) and Fig. 8 (c). The FEM flexural capacity was defined as the maximum moment value in the analysis. For CFT beam-columns with Type-B loading, comparisons of both flexural and axial capacity were shown. In this case, flexural capacity was defined as the moment value corresponding to
maximum applied load, including second order effect; and the axial capacity was defined as the maximum applied axial load in the FEM analysis. Fig. 9 shows the comparisons of moment-curvature or moment-midspan deflection curve for selected beam-column tests. As shown in Fig. 8 and Fig. 9, the finite element model is able to predict behavior of rectangular and circular CFT beam-columns quite reasonably.

(a) Comparisons of the flexural strength of rectangular beam-columns, Type A and Type B loading

(b) Comparisons of the axial compressive strength of rectangular beam-columns, Type B loading

(c) Comparisons of the flexural strength of circular beam-columns, Type A and Type B loading

(d) Comparisons of the axial compressive strength of circular beam-columns, Type B loading

Fig. 8: Comparisons of the strength for CFT beam-columns from the FEM analysis and experimental tests

(a) BRA4-2-5-02

(b) BRA4-2-5-04

Fig. 9: Comparisons of moment-curvature or moment rotation curves for selected beam-column tests from FEM and experimental tests
4. Parametric studies

The steel tube slenderness ratio and axial load ratio determine the cross section strength of CFT beam-columns. These two parameters combined with the member length control the strength of a CFT beam-column. In this part, the effects of these parameters are investigated by conducting comprehensive analyses using the benchmarked FEM. Prototype specimens from the database were selected first; then parametric studies were performed by changing the thickness, applied axial load, and member length of the prototype specimens. For rectangular CFT beam-columns, the test specimen BRA4-2-5-02 by Nakahara and Sakino (1998, 2000a) was selected as the prototype specimen. The steel yield strength \( F_y \) and the concrete compressive strength \( f'c \) of this specimen is 253 Mpa and 47.6 Mpa, respectively. Type-2 loading was used to load the specimen. For circular CFT beam-columns, the test specimen C06F3M by Ichinohe et al. (1991) was selected as the prototype specimen. For this specimen, the steel yield strength \( F_y \) is 420 Mpa and the concrete compressive strength \( f'c \) is 64.3 Mpa. Type-1 loading was used to load the specimen. Details of these two specimens are shown in Table 2.

4.1 Effect of slenderness ratio and axial load ratio

Seven different slenderness ratios were selected for both prototype specimens. For specimen BRA4-2-05-02, the slenderness coefficients are 2.49, 2.68, 2.90, 3.17, 3.87, 4.36 and 4.98. For specimen C06F3M, the slenderness coefficients are 0.11, 0.16, 0.20, 0.22, 0.24, 0.28 and 0.31. These slenderness coefficients cover the whole slenderness range for noncompact and slender sections. The designated slenderness ratio was implemented by changing the tube wall thickness.
For every beam-column with the same slenderness ratio, analyses with different axial loads ratios were conducted. Beam-columns with axial load ratio of 0 were subjected to bending moments only to get the flexural capacity $M_f$. Beam-columns with axial load ratio of 1.0 were subjected to axial compressive force only to get the axial capacity $P_f$. The applied axial force was calculated as the product of the axial load ratio and the nominal axial strength $P_n$ (calculated by the AISC design equations). Therefore, the designated axial load ratios were nominal values, since $P_n$ may not be the same as actual axial strength $P_f$. For specimen BRA4-2-05-02, five nominal axial load ratios were used (0, 0.2, 0.4, 0.6 and 1.0). For specimen C06F3M, six nominal axial load ratios were used (0, 0.2, 0.4, 0.6, 0.8 and 1.0). As a result, thirty-five rectangular and forty-two circular beam-column analyses were performed.

Fig.10 (a) and Fig.10 (b) shows the effect of steel tube slenderness ratio (with axial load ratio $\alpha = 0.2$) on the behavior of rectangular and circular CFT beam-columns, respectively. As the steel tube thickness decreases, the stiffness and strength of the CFT beam-columns decrease also. This conclusion also applies to CFT beam-columns with other axial load ratios. The corresponding figures are not shown in this paper for brevity.

Fig.11 (a) and Fig.11 (b) shows the effect of axial load ratio on the behavior of rectangular and circular CFT beam-columns, respectively. The thickness for the rectangular and circular beam-columns was 2.8mm and 5.7mm, respectively. When the axial load ratio is less than 0.4, the stiffness and strength increase as axial load ratio increases. When the axial load ratio is greater than 0.4, the stiffness and strength decreases as axial load ratio increases. The stiffness and strength of the members with axial load ratio of 0.4 are greater than the members with other axial load ratios. Also, the ductility decreases with increasing axial load ratio for rectangular beam-columns (i.e., the curve drops more rapidly after failure occurs). These conclusions are also applicable to CFT beam-columns with other thickness. Again, the corresponding figures are not shown in this paper for brevity.

Fig.12 shows the comparisons of the analysis results with interaction curves calculated using Eq.1 and Eq.2. These comparisons indicate that the axial and flexural strength of noncompact and slender CFT members are overestimated by the AISC 360-10, and that the AISC interaction curve is too conservative. The degree of conservatism increases as slenderness ratio increases.

Thickness decreases from 2.8 mm ($\lambda_{coeff} = 2.49$) to 1.4 mm ($\lambda_{coeff} = 4.98$)

Thickness decreases from 5.7 mm ($\lambda_{coeff} = 0.11$) to 2.0 mm ($\lambda_{coeff} = 0.31$)
Fig. 11: Effect of axial load ratio on the behavior of CFT beam-columns: (a) rectangular; and (b) circular

Fig. 12: Comparisons of the analysis results with AISC 360-10 interaction curve for CFT beam-columns: (a) rectangular; and (b) circular

Fig. 13: Effect of length on the behavior of CFT beam-columns: (a) rectangular; and (b) circular

4.2 Effect of member length

The member length determines the effect of second-order moments on the behavior of beam-columns. This effect is more significant for CFT members with greater slenderness ratio. Therefore, it is investigated for the rectangular and circular prototype beam-columns with the minimum permitted thickness of 1.4 mm ($\lambda_{\text{coeff}} = 4.98$) and 2.0 mm ($\lambda_{\text{coeff}} = 0.31$), respectively. Three different length-to-depth ratios ($L/D$) were used for both rectangular ($L/B=3.0$, 10.0, and 20.0) and circular members ($L/D=6.7$, 13.3, and 20.0).

Fig. 13 shows the moment-curvature response for members with different length. As shown, the stiffness of both rectangular and circular CFT beam-columns is not influenced by the length. For
rectangular members, the strength is not influenced by the length; however, failure occurs earlier as length increases due to the earlier occurrence of local buckling. For circular members, the strength increases slightly as length increases due to the steel strain hardening (local buckling did not occur in those circular beam-columns). The effects of length for CFT members with other axial load ratios (0.2, 0.4, and 0.6) are similar to the behavior shown in Fig.13.

The flexural capacity of CFT members without any axial load applied ($\alpha = 0$) is not influenced by the length. The axial capacity of CFT members without any moment applied ($\alpha = 1.0$) decreases as length increases. For example, the axial capacity for the rectangular members with $L/B=3.0$, 10.0 and 20.0 is 2153.1 kN, 2020.2 kN and 1986.5 kN, respectively; and the axial capacity for the circular members with $L/D=6.7$, 13.3 and 20.0 is 4225.2 kN, 4107.2 kN and 3825.6 kN, respectively. Fig.14 shows the comparisons of the analysis results along with AISC 360-10 interaction curve. As shown, the AISC interaction curve is too conservative.

![Fig.14: Comparisons of the analysis results with AISC 360-10 interaction curve for CFT beam-columns with different length: (a) rectangular; and (b) circular](image)

5. Proposed interaction equations
As discussed in Section 2.2, the over-conservatism of the AISC beam-column equations for designing noncompact and slender rectangular and circular CFT beam columns is due to: (i) the conservative estimation of the axial and flexural strength ($P_n$ and $M_n$) by the AISC 360-10 design equations; and (ii) the bilinear interaction curve which is the same as that used for steel beam-columns. The primary focus of this paper is to improve the bilinear interaction curve. Therefore, the conservatism of the interaction curve due to (i) were eliminated by using the actual axial and flexural strength ($P_f$ and $M_f$) from the finite element analysis to normalize the results from the parametric studies shown in Section 4, as shown in Fig.15 and Fig.16.

In Fig.15, the results are identified by the confinement factor $\zeta$ instead of the slenderness ratio. This is because the confinement factor includes the effects of both slenderness ratio and material strength ratio ($F_y/f'_c$), and it determines the shape of the interaction curve for CFT beam-columns. Fig.16 shows that the shape of interaction curves is not influenced significantly by the member length up to length-to-depth ratio of 20.0 (which is good for practical design). Therefore, the interaction curve could be improved by focusing on the effect of $\zeta$ only.

Several design methods could be used to improve the bilinear interaction curve used by AISC 360-10, as shown in Fig.17. Method A uses polynomial equation to represent the shape of the P-M curve defined by analysis and tests. Method B simplifies Method A by using a trilinear curve
(Line ACDB), which is similar to the approach discussed in the commentary of AISC 360-10, Section I5. Method C further simplifies Method B by using a bilinear curve (Line ADB). Method C is recommended because: i) it captures the basic P-M behavior of noncompact and slender CFT beam-columns; and ii) the bilinear form of Method C is similar to the current AISC 360-10 interaction curve, and is convenient for design. For Method C, Point A is the axial strength, Point B is the flexural strength, and Point D corresponds to the largest increase in moment capacity, i.e., $M_{FEM}/M_f$ ratio.

To use Method C, factors $\beta_1$ and $\beta_2$ for the ordinate and abscissa of Point D need to be determined. Fig.15 shows that the maximum $M_{FEM}/M_f$ ratio increases as $\xi$ increases. For rectangular CFT beam-columns, the axial load ratio corresponding to the maximum $M_{FEM}/M_f$ ratio decreases from 0.35 to 0.30 as $\xi$ increases. For circular CFT beam-columns, the axial load ratio corresponding to the maximum $M_{FEM}/M_f$ ratio increases from 0.40 to 0.50 as $\xi$ increases. Therefore, $\beta_1$ for rectangular and circular beam-columns is selected conservatively as 0.30 and 0.40. Statistical analysis on the results from parametric studies shows that the $M_{FEM}/M_f$ ratio is related to $\xi$ as:

For rectangular beam-columns:
$$\beta_2 = M_{FEM}/M_f = -2.6\xi + 2.1$$ (4)

For circular beam-columns:
$$\beta_2 = M_{FEM}/M_f = -1.0\xi + 1.7$$ (5)

with the minimum value of $\beta_2$ limited to 1.0.
The AISC 360-10 interaction curve (Eq.1 and Eq.2) for noncompact and slender CFT beam-columns can be now updated as:

When $\frac{P_r}{\phi_r P_n} \geq \beta_1$

$$\frac{P_r}{\phi_r P_n} + \frac{1-\beta_1}{\beta_2} \frac{M_r}{\phi_b M_n} \leq 1.0$$  \hspace{1cm} (6)

When $\frac{P_r}{\phi_r P_n} < \beta_1$

$$\frac{1-\beta_2}{\beta_1} \frac{P_r}{\phi_r P_n} + \frac{M_r}{\phi_b M_n} \leq 1.0$$  \hspace{1cm} (7)

where $\beta_1$ is 0.30 and 0.40 for rectangular and circular CFT beam-columns, respectively; and $\beta_2$ is calculated using Eq.4 and Eq.5. Comparisons of the interaction curve (dashed line) using Eq.6 and Eq.7 with analysis results are shown in Fig.15. As shown, the updated interaction curve compares favorably with the analysis results.

Fig.17: Different design methods for defining beam-column interaction curve

6. Conclusions

AISC 360-10 specifies over-conservative equations for the design of noncompact and slender rectangular and circular CFT beam-columns, as shown by the comparisons with tests results from the experimental database compiled by the authors. The over-conservatism of the design equations is due to the conservative evaluations of axial and flexural strength, and the use of bilinear P-M interaction curve for steel beam-columns. This paper focuses on improving the bilinear interaction curve by conducting analytical parametric studies using finite element models benchmarked against experimental data.

The bilinear interaction curve was improved by including factors $\beta_1$ and $\beta_2$ in the updated versions (Eq.6 and Eq.7). In these equations, $\beta_1$ is defined conservatively as 0.30 and 0.40 for rectangular and circular CFT beam-columns, respectively. $\beta_2$ is calculated using Eq.4 and Eq.5. The values of $\beta_1$ and the equations for $\beta_2$ were determined using the results from parametric studies focusing on the effects of axial load ratio, confinement factor, and member length. Of these, the member length was found to have negligible effect on the shape of the interaction curve, up to L/B or L/D ratio of 20. The updated interaction curve preserves the bilinear form of the current AISC 360-10 interaction curve, while capturing the basic behavior of noncompact and slender CFT beam-columns. Comparisons with the analysis results showed that the updated interaction curve was able to predict the strength of the CFT beam-columns quite well.
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