Proceedings of the
Annual Stability Conference
Structural Stability Research Council
Toronto, Canada, March 25-28, 2014

Analysis and Design of Thin Metallic Shell Structural Members – Current Practice and Future Research Needs

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Abstract

The goal of this paper is to highlight promising methods in current thin metallic shell design practice and to define a future needs framework from which research can launch. Thin shell structural members are a staple in many industries – from aerospace, ship building, to offshore oil and gas to residential and commercial buildings. Shell types and geometries are numerous including ship hulls, silos, tanks, pipelines, chimneys and wind turbine towers. Despite large research investments there is still wide debate and uncertainty when designing thin shell structural members. Failure modes are complex and sensitive to initial geometric imperfections. The types of loadings vary widely– including axial, shear, flexure combined with internal or external pressure – making calculation-based methods challenging. Shell geometry – including longitudinal and transverse stiffeners and conical, tapered along a member – compound the complexity. This paper synthesizes these current approaches and organizes the most promising ideas into a framework for future research. A specific focus will be on research needed to expand current GMNIA (geometrically and materially nonlinear analysis with imperfections included) capabilities.

1. Introduction

Thin metallic shell members are widely used in many engineering structures such as silos, tanks, aerospace, wind turbines and offshore oil and gas structures. For shells of large radius to thickness, for example grain silos, buckling deformations result in large half-waves relative to the shell diameter that form suddenly, often resulting in catastrophic collapse (Jumikis 1988). Shells with smaller radius to thickness, for example cylindrical tubes used in off-shore structures, roof trusses, and spacecraft, exhibit local buckling half-waves that are smaller and more

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organized, creating a network of stiffening fold lines that resist the applied load with post-buckling reserve (Donnell 1956). The challenge is to accurately capture these geometry-defined limit states in a general design method.

Manufacturing processes vary for different shell applications – some are cold-formed, some have welded seams, and others are provided with radial stiffeners resulting in different initial stress states and geometric imperfection patterns. Because of this, a wide range of thin shell design methods are available based on the industry. Eurocode 3 provides semi-analytical approaches or the option for finite element simulation (Eurocode 3 2007). Aerospace engineering continues to use a lower bound "knockdown factor" empirical design philosophy (Weingarten 1965). The American Bureau of Shipping (ABS) provides guidance for offshore structures (American Bureau of Shipping 2005) and the American Society of Civil Engineers (ASCE)-American Wind Energy Association (AWEA) Recommended Practice gives design recommendations for wind turbine towers.

When viewing these design methods together, it becomes clear that there are missing links between thin shell analysis and design and failure criteria, and that we can do better. The discussion presented herein highlights promising methods in thin metallic shell design practice and defines a framework from which future research can launch. We begin with a review of the origins of thin shell theory.

2. Thin Shell Theory

Love was the first to formulate the basic equations that govern the behavior of thin elastic shells (Love 1888, 1944). Other formulations were later proposed by others (Reissner 1949, 1952; Sanders 1959; Flugge 1973), including Love himself to resolve some of the inconsistencies. It was however Koiter that verified Love's assumptions and derived a fully consistent shell theory (Koiter 1959). The first approximation shell theory is based on Love's original equations. Sanders-Koiter shell theory is basically an improved version of this set of equations. They are valid for linear elastic material and small deformations and are general in the sense that they reduce to equations for plates or shell membrane-only equations under appropriate simplifying assumptions (Sanders 1959, 1961; Koiter 1959). Other thin shell theories or theories that account for out-of-plane shear deformation (higher-order approximation theories) also exist but the latter are used to describe the behavior of thick shells.

Shells, depending on what the curvature of two bounding (upper and lower) surfaces, are divided into cylindrical, conical, spherical, ellipsoidal, paraboloidal, toroidal, and hyperbolic paraboloidal shells. Roughly, a shell structure is called thin if the following criterion holds

$$max\,(\mathrm{h/R}) < 1/20 \qquad (1)$$

with h the thickness and R the radius of curvature. The resemblance between shells and plates is similar to the one between arches and straight beams under that action of transverse loading. That is the efficiency lies primarily in resisting the transverse load with axial (membrane) action thus minimizing the internal shear force and bending moment. Because of the curvature of the surface, a shell structure has higher spatial stiffness and larger load-carrying capacity. The thinness on the other hand results in the potential to resist the transverse loads through membrane stresses only and better efficiency (i.e. higher stiffness to weight ratio). It is however noted that this thinness can be a point of weakness too. This has to do with increasing the potential for instability as shell gets thinner to an extent where the critical stresses may be significantly lower the shell material's proportional limit.

The most widely used theory of thin shells is the linear theory which is based on the following assumptions (Kirchhoff's assumptions): (a) normals to the undeformed middle surface undergo no extension and remain straight and normal to the deformed middle surface, (b) the transverse normal stress is negligible and may be neglected. The error in adopting such assumptions scales with h/R and decreases as this ratio decreases (Novozhilov 1964). In the linear theory it is furthermore assumed that the displacements are small. This means equilibrium equations can be written in the original (reference) configuration. The governing equations obtained by adopting these assumptions are complex and are not repeated here. The interested reader is referred to classical texts on the subject such as Timoshenko et al. (1959), Urgural (1999), Ventsel and T. Krauthammer (2001) and Reddy (2007). However to provide the reader with a taste of shell mechanics we here include some qualitative but theoretical remarks on perhaps the most controlling (and difficult to characterize) shell failure mechanism that is buckling. A more practical view, one that is the intent in this paper, is provided in the next section.

The difficulty in buckling analysis of thin shells is associated with the fact that a mathematical description of the deformed state of shells is involved and lends itself to a diversity of situations at which a shell can buckle. Moreover, transition from an initial configuration at the onset of buckling to another stable buckled configuration occurs by a jump (snap-through) and through an unstable path. This is depicted in figure 1 (see arrow ***AB***). The straight line represents the initial path for a perfect shell. While the curve that crosses the straight line at ***A*** pertains to an alternative equilibrium path for the same perfect shell, the dashed line depicts the path of a real shell with real geometric and/or other imperfections. Unlike columns and flat plates under in-plane compressive forces that are relatively insensitive to slight geometric imperfections what this figure demonstrates is that (thin) shells are very sensitive to such imperfections.

For a shell structure, it follows from the above that the buckling behavior can be described via three distinct values (Ventsel and Krauthammer 2001):

- P_{c1}, the upper critical load –the load up to which the initially perfect shell remains stable with respect to small disturbances;
- P_{c2} the lower critical load –the largest load up to which the initially perfect shell remains stable with respect to both small and finite disturbances;
- P_c, the critical load of a real shell, also referred to as the buckling load

From the above the last pertains to a critical value of the load for which the original state of equilibrium for the real shell jumps to another state through an unstable path.

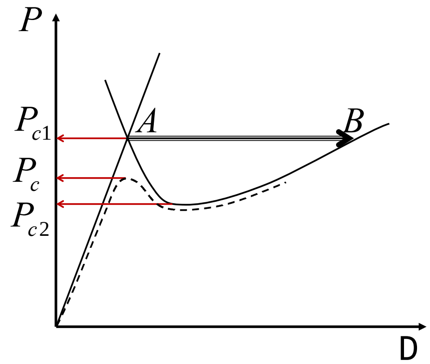

Figure 1: Load-deformation for perfect and imperfect shell structure

3. Current Thin Shell Code-Based Design Approaches

Slender shells, which characteristically have dozens of elastic buckling modes all with nearly identical buckling loads, are acutely sensitive to imperfections and the strength of such sections is often controlled by local buckling. In this section, methods for estimating the local buckling strength of slender shells under compressive meridional stresses are summarized for two popular international design standards: "Eurocode3: Design of Steel Structures - Part 1-6: Strength and Stability of Shell Structures" and "DNV-RP-C202: Buckling Strength of Shells." Both standards are based on semi-empirical methods because there is generally a lack of agreement between theoretical and experimental buckling loads for slender shells.

3.1 Eurocode

Eurocode is a LRFD based code that permits three types of buckling analysis, each increasingly versatile and increasingly complex. The first and least complex is linear elastic analysis (LA) or stress design, the second is material nonlinear analysis (MNA), and the third and most complex is geometric and material nonlinear analysis with imperfections included (GMNIA). A recent paper by Gettel (2007) describes a situation where MNA-type analysis yields unrealistic results and so this paper will focus on LA and GMNIA-type analysis. Linear elastic analysis is the least complex and most widely used of the Eurocode approaches, but it may be overly conservative and is limited to the specific situations covered by Eurocode. GMNIA-type analysis is the most complex and most versatile of the Eurocode approaches, but, in practice, GMNIA analysis results are sensitive to analysis subtleties which are not thoroughly defined in the code.

Linear elastic, or stress design, buckling analysis depends on the magnitude of imperfections. The imperfection classification is accomplished through the assignment of a fabrication quality class, and the code provides tabulated tolerances of certain geometric imperfections which are then linked to the fabrication quality classes used to assess the critical buckling stress. The code states that each of the three specified imperfection types (shown in Figure 2 below), accidental eccentricities, out-of-roundness, and dimple imperfections, may be treated separately without consideration of interactions between imperfection types.

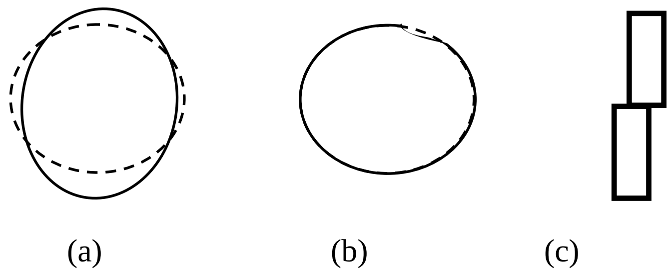

Figure 2: Imperfection types considered in Eurocode buckling quality class calculations: (a) Out-of-Roundness (b) Dimple (c) Accidental Eccentricity

The assignment of a fabrication quality class is recommended to be based on measurements of each imperfection type by “representative sample checks on the completed structure” when the structure is unloaded, though preferably with realistic boundary conditions. The buckling strength is then controlled by the lowest quality class from amongst each imperfection type. The quality class is then associated with a knockdown factor that is multiplied by the critical buckling stress determined by membrane theory. It is important to note that this tier of analysis, while relative straightforward and conservative, ignores bending effects, does not provide guidance on how to estimate the strength of stiffened shells, and does not consider specifics of imperfection shape. Specifics of imperfection shape, and also

GMNIA analysis overcomes these limitations and is based on a nonlinear finite element analysis. This type of analysis explicitly considers details such as the interaction between imperfection types, the shape of imperfections (both globally and locally), the specifics of the stress distribution (i.e. bending versus pure compression), and custom geometries which may include stiffeners. The versatility of this method is certainly appealing, however, in practice, there are many inherent challenges. For example, the code requires the modeling of “equivalent” imperfections which must result in a buckling strength that includes the combined influences of geometric imperfections, residual stresses, and non-idealized boundary conditions. While geometric imperfections can be measured explicitly, it is much more ambiguous to appropriately determine “equivalent” imperfections that meaningfully include the influences of residual stresses and non-ideal boundary conditions. These equivalent geometric imperfections are required to be chosen to have the “most unfavorable effect” on the elastic-plastic buckling ratio of the structure (a ratio of critical buckling stresses achieved through bifurcation analysis and nonlinear analysis). If possible, the code also suggests that the imperfection pattern follow expected construction details, and eigen-mode affine patterns should be used if no expected pattern is known. The subtleties and complexities of GMNIA analysis are described in much greater detail in recent research (Gettel and Schneider 2007; Schneider 2006; Hübner el al. 2006). Nevertheless, GMNIA-type analysis is certainly a promising approach to assess the buckling strength of slender shells in a versatile way.

3.2 DNV

DNV is a LRFD-based recommended practice (RP) for designing slender shells. The RP provides details on a single analysis method that is analogous to linear elastic analysis in Eurocode. In this RP, unlike Eurocode, geometric imperfections and residuals stresses do not appear as explicit parameters in the formulations for the buckling resistance. Rather, the RP provides an estimate of the buckling strength assuming that the minimum tolerance requirements are satisfied. The minimum tolerance requirements are provided in a separate document, DNV-OS-C401. Besides this, the other major differences between the DNV RP and linear elastic analysis in Eurocode is that the DNV RP provides explicit formulations to estimate buckling strength for shells loaded in bending and also for shells that are ring-, longitudinal- and orthogonal-stiffened. Although not explained nearly as thoroughly as Eurocode, the DNV RP includes a statement that allows for an analysis similar to GMNIA. The RP states that the provided methods may be substituted for more refined analyses considering real boundary conditions, pre-buckling edge disturbances, actual geometric imperfections, non-linear material behavior and residual welding classes.

4. Thin Shell Computational Analysis Tools and Methods

4.1 GMNIA and Initial Geometric Imperfections

As discussed in the previous section, GMNIA is a sophisticated method for performing geometrically and materially nonlinear analysis of an imperfect structure. Choosing an appropriate imperfection form in GMNIA analysis is not simple, and in fact, the choice of proper equivalent imperfections has the strongest influence on the accuracy of GMNIA calculation (Rotter 2002).

The accurate treatment of initial geometric imperfections in shells in computation simulations starts with measured data. The Imperfection Data Bank at TU Delft is the primary source of thin shell data for the aerospace engineering field (deVries 2009). This same idea of an imperfections database is being applied to thin-walled cold-formed steel metallic members, with a new database recently created through the Cold-Formed Steel Research Consortium (McAnallen et al. 2014).

The characterization of imperfections typically falls on statistical approaches, and historically by collecting single point measurements based on buckling mode classifications and treating them as a probability of occurrence (Schafer and Peköz 1997). Efforts associated with the TU Delft Imperfection Data Bank have proposed first order, second moment interpretations of imperfection fields (deVries 2009). In cold-formed steel, research was recently completed to apply power spectral density approach where contributions of cross-sectional mode shapes are deduced from the data and then phase angles are varied to develop random imperfection patterns with pre-established buckling mode shapes (Zeinoddini and Schafer 2013).

4.2 Finite Strip Method

For open cross section thin-walled members the finite strip method has proven to be an efficient numerical method for use in stability analysis and design, and its use has been integrated into design specifications (Schafer 2008). For closed members the basic lower-order plate bending finite strip coupled to a plane stress finite strip for membrane deformations as utilized in most available finite strip codes (e.g. CUFSM, Schafer and Adany 2006) is adequate for stability analysis. However, for curved closed sections (classical shells) the method is approximate as faceted sides are employed and thus the shear transfer transverse to the strips is not exact. With adequate discretization the stability results of a model with faceted sides are acceptable for practice. Formulations for curved finite strips are available (Cheung and Tham 1997) but not implemented in widely available codes.

4.3 Modal Identification

Extension of the constrained finite strip method (Li et al. 2013), and the ability to decompose and identify buckling and deformation modes that cFSM enables, has only recently been extended to closed sections. Based on the shear formulation of (Ádány 2014) cFSM's algorithms have been extended as explained in (Ádány and Schafer 2014) and integration into the open source finite strip code CUFSM is underway. Similar capabilities exist within the context of Generalized Beam Theory (Gonçalves et al 2009).

Generalized Beam Theory (GBT) – an extension of Vlasov's classical beam theory, which takes in account local and distortional deformations – has seen a number of applications since Schardt (1989) has published a book on GBT. In short, GBT is based on the linear superposition of pre-determined deformation modes.

Recently, the research group led by Camotin, Silvestre, Gonçalvez e Dinis has published several papers on the subject. Among them is Silvestre (2007), where the author presents a formulation of GBT to analyze the elastic buckling behavior of circular hollow sections; the formulation represents an advance in the understanding and analysis of local and global buckling behavior of circular hollow sections. Silvestre (2007) does mention that this paper is just the first step to in the future develop a non-linear GBT formulation and a formulation for ply orthotropic material to analyze circular hollow sections.

4.4 Extreme Loads Modeling

A great number of researchers have carried out numerical finite element analysis to understand concrete-filled circular hollow steel columns. The interest in concrete-filled circular hollow steel columns is justified because the construction system avoids formwork, the thickness of the section is generally reduced and concrete-filled columns have good fire performance. Other references shall be consulted to learn more about how the material concrete shall be modeled, however, note that the concrete confinement has to be carefully taken into account.

In the realm of non-filled circular hollow steel under fire conditions analysis, Tondini et al. (2012) outlined a numerical investigation of high-strength steel circular columns subjected to fire, the results were compared to experimental data. Scullion et al. (2012) validated three-dimensional finite element models developed for the analysis of elliptical hollow section steel columns in fire, the information about the FE model does contribute to understanding how to develop your own model. Ellobody (2013), however, presents one of the most complete and updated references on how to perform a consistent FE analysis of thin-walled columns circular at ambient and fire conditions.

5. Future Research Directions

Assembling this thin-shell literature review was thought provoking and highlighted two major tracks for future research investment – collapse simulation and calculation-based design. Work is needed to improve thin-shell collapse simulation with accurate consideration of a member's initial state, i.e., proper treatment of manufacturing plastic strains, residual stresses, and initial imperfections. This could be achieved with manufacturing-to-failure computational modeling, for example, simulating roll forming and welding before compressing a member to collapse (Mackerle 2004).

Merging the spectral density approach described at the end of Section 4.1 with TU Delft's shell imperfections database could provide improved treatment of initial geometric imperfections. The thin shell research community should make a concerted effort to collect and organize imperfection data required to generate the spectral signatures. Coupling GMNIA collapse simulations with buckling modal decomposition tools, e.g., a non-linear generalized beam theory formulation and the constrained finite strip method, would improve our understanding of how buckling modes influence load-deformation response and drive limit states. As a general statement, it is recommended that we put more research energy into moving accurate collapse simulation from research into practice, which would free engineers from calculation-based design limitations and open doors to optimization and innovation.

Accessible collapse simulation will continue to evolve, however in the shorter term, a general calculation-based strength prediction method is needed that considers local and global buckling limit states and the benefits of stiffeners. It is recommended that the American Iron and Steel Institute's Direct Strength Method (AISI 2012) be evaluated as a capacity prediction platform for thin shells, especially because it would bring cross-sectional buckling analysis and local-global buckling interaction to thin shell design. Research is underway in North America, e.g., Chatterjee et al. (2014), and through the European-sponsored Simple Tools Sell Steel (STSS) program (Taras and Boissonnade 2014).

Acknowledgments

This paper was authored by a SPECification team assembled by the Structural Stability Research Council, Task Group 5 – Thin-Walled Structures. M. Tootkaboni further acknowledges support from National Science Foundation under grant # CMMI-1235238.

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