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# Design and Analysis of Liner Forms for Shaft Sinking

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**Abstract**
Access to underground mines is in many cases through vertical shafts. If unstable ground conditions are encountered, the shaft walls are lined with concrete. The cylindrical concrete walls are formed with steel plates that are reinforced with circumferential and vertical stiffeners. The structural design and analysis of these forms is discussed in this paper. It is shown that stability of the forms, when subjected to pressures from the concrete during construction, is the governing criteria for the design. It is pointed out, that in order to obtain a safe design; some input parameters for the analysis have to be based on practical experience with liner construction in vertical shafts. An illustrative example is presented.

## 1. Introduction

Shaft sinking refers to the method of excavating a vertical or near-vertical tunnel from the surface down. Shaft sinking is one of the most difficult mine development methods and only a few companies have the expertise to perform this type of work. In North America most shafts are circular and lined with concrete. A concrete liner (or wall) is required in areas where the ground is unstable in order to contain any loose material and prevent it from falling down the shaft. The walls are shaped with steel forms and the concrete is poured behind these forms. Figure 1 shows the general arrangement of the liner installation.

The structural design of these forms is discussed in this paper and a numerical example is presented to illustrate the design procedure.

The important steps in the design are as follows.

a. Determination of the geometry of the forms (based on shaft geometry)
b. Selection of the material (steel)
c. Determination of the loads on the forms
d. Estimate of the response of the forms due to the applied loads by formulating an acceptable theoretical analysis model.
e. Comparison of the resulting response with accepted yield and stability design limits
f. By choosing appropriate safety factors against failure.

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The material presented here is based on Refs. 1 to 3, and has been edited and rearranged to suit the present topic and to make this paper useful as a design aid.
This paper is intended to be of use when checking liner forms and to assess that sufficient margins of safety have been provided in the design since safety is of paramount importance in the mining industry.

## 2. Applied Loads on the Forms

The applied loads are the dead weight of the forms and the lateral pressure from the concrete on the forms when the concrete is poured into the forms.

The radial, or lateral, pressure from the concrete is estimated by using the recommendations of ACI 347 (Ref. 1 and Ref. 2). These formulas are applicable and sufficiently accurate for the present application.

$$p = 150 + 9000\ R/T \qquad \text{if } R \leq 7 \text{ feet per hour} \qquad (1)$$

$$p = C_w\, C_c\ (150 + 43500/T + 2800\ R/T) \qquad \text{if } R > 7 \text{ feet per hour} \qquad (2)$$

with a max pressure of 2000 $C_w$ $C_c$ and a min pressure of 600 $C_w$ , but never more than w.h

where
p = lateral pressure on the forms (lbs/ft2)
$C_w$ = unit weight coefficient ( $C_w = 1$ for normal weight concrete)
$C_c$ = 1.0 (for type I and III cement without retarders)
T = temperature of concrete, deg. F
R = rate of placement, feet per hour
w = unit weight of concrete, pcf
h = depth of fluid or plastic concrete from top of placement to the point of consideration, feet

A realistic estimate of the rate R of concrete pour is the critical assumption. This rate is obtained from previous experience of the shaft sinking contractor and depends on shaft diameter, thickness of liner, over break, depth of shaft and work schedule. In most cases this rate is proprietary information of the mining contractor.

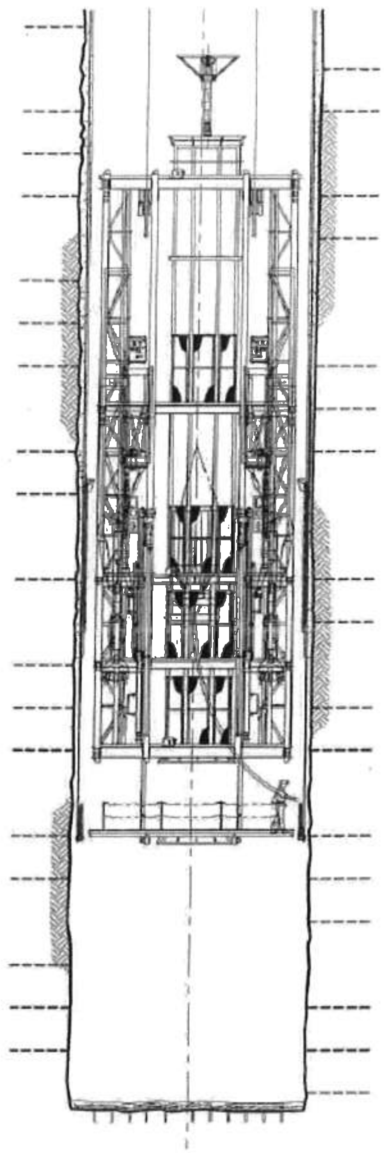


Figure 1: Shaft Sinking General Arrangement

**3. Analysis Model**
The forms are modeled as cylindrical shells with interior circumferential stiffeners. Although vertical stiffeners are also provided in practice, they can be neglected in the analysis

$$\text{if } \frac{s}{t} > 3\left(\frac{r}{t}\right)^{\frac{1}{2}} \quad \text{for light vertical stiffeners} \tag{3}$$

The analysis will therefore neglect the longitudinal stiffeners when ring response is considered. Their presence will however be considered in the shell response, since they add a degree of conservatism to the results, in that they help to increase the shell buckling limit.

The basic geometric shell parameters are shown in Figure 2.

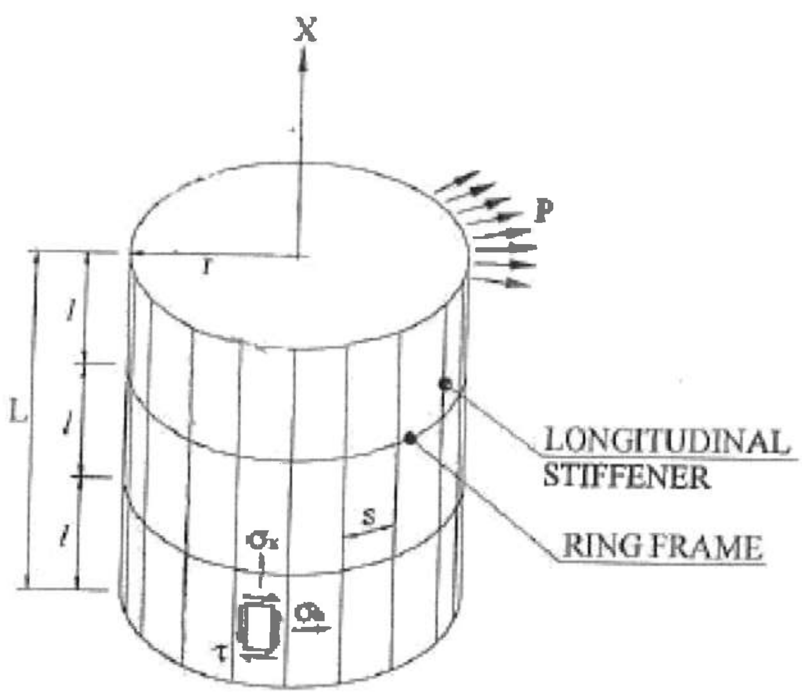



Figure 2: Cylinder Geometry (Ref. 3)

**4. Stresses in Ring Stiffened Cylindrical Shells**

*4.1 Ring Stiffened Cylinder*
In a ring stiffened cylinder (without longitudinal stiffeners) the circumferential membrane stress midway between two ring frames is

$$\sigma_h = \frac{pr}{t} - \frac{\alpha\zeta}{\alpha+1}\left(\frac{pr}{t} - \nu\sigma_x\right) \tag{4}$$

where

$$\zeta = 2\,\frac{\sinh\beta\cos\beta + \cosh\beta\sin\beta}{\sinh 2\beta + \sin 2\beta}, \quad \zeta \geq 0 \tag{5}$$

$$\beta = \frac{l}{1.56\sqrt{rt}} \tag{6}$$

$$\alpha = \frac{A_R}{l_{eo}t} \tag{7}$$

$$l_{eo} = \frac{l}{\beta}\,\frac{\cosh 2\beta - \cos 2\beta}{\sinh 2\beta + \sin 2\beta} \tag{8}$$

$\zeta$ and $l_{eo}$ may be obtained from Fig.3, or simply

$l_{eo} = l$ or $l_{eo} = 1.56\ (r\ t)^{1/2}$ , whichever is smaller

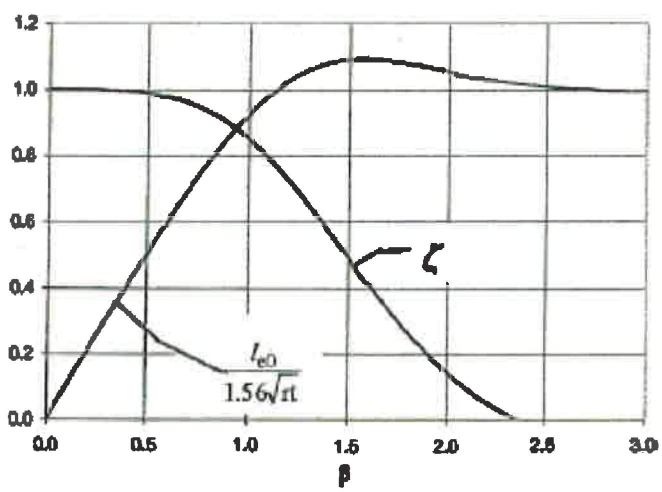



Figure 3: The Parameters $l_{eo}$ and $\zeta$ (Ref. 3)

*4.2 Ring Frame*

The circumferential membrane stress at a ring frame for a ring stiffened cylinder (without longitudinal stiffeners) is

$$\sigma_{hR} = \left(\frac{pr}{t} - \nu\sigma_x\right)\frac{1}{1+\alpha} + \nu\sigma_x \tag{9}$$

*4.3 Design Longitudinal Membrane*
The design longitudinal membrane stress is

$$б_x = б_a \tag{10}$$

where $б_a$ is due to the uniform axial force. In the present application this is the weight of the forms.

For a cylindrical shell without longitudinal stiffeners

$$б_a = \frac{N_s}{2\pi r t} \tag{11}$$

*4.4 Bending Stresses*
Bending stresses and associated shear stresses will occur in the vicinity of discontinuities such as frames. The longitudinal bending stress in the shell at a ring frame is

$$б_{xm} = \left(\frac{pr}{t} - б_{hR}\right)\sqrt{\frac{3}{1-\nu}} \tag{12}$$

where $б_{hR}$ is given in Eq. (9)

*4.5 Circumferential Bending Stress*
The circumferential bending stress in the shell at a ring frame is

$$б_{hm} = \nu\, б_{xm} \tag{13}$$

**5. Buckling Strength of Shells**
The methods of buckling analysis are semi-empirical because agreement between theoretical and experimental buckling loads for some cases is difficult to establish. This discrepancy is due to the effect of geometric imperfections and residual stresses in fabricated shell structures. Geometric imperfections and residual stresses do not appear as explicit parameters in the theoretical buckling formulas. The methods of buckling analysis are therefore based on an assumed level of imperfections.

Stiffened cylindrical shells have to be designed to avoid several buckling failure modes. The relevant modes are as follows.

a. Shell buckling: buckling of shell plating between rings and longitudinal stiffeners

b. Panel stiffener buckling: Buckling of shell plating including longitudinal stiffeners. Rings are nodal lines.

c. Panel ring buckling: Buckling of shell plating including rings. Longitudinal stiffeners act as nodal lines.

d. General buckling: Buckling of shell plating including longitudinal stiffeners and rings.

e. Local buckling of longitudinal stiffeners and rings.

In the present application we shall consider only a., c. and e. as possible failure modes.

*5.1 Elastic Buckling Strength of Unstiffened Curved Panels*

The elastic buckling strength of the curved panel between stiffeners and with aspect ratio of $l/s > 1$ is

$$f_E = C\left\{\frac{\pi^2 E}{12(1-\nu^2)}\right\}\left(\frac{t}{s}\right)^2 \tag{14}$$

The buckling coefficient is

$$C = \psi\left\{1 + \left(\frac{\rho\xi}{\psi}\right)^2\right\}^{\frac{1}{2}} \tag{15}$$

where

$$\psi = \left\{1 + \left(\frac{s}{l}\right)^2\right\}^2 \qquad \xi = 1.04\left(\frac{s}{l}\right)(Z_s)^{\frac{1}{2}} \qquad \rho = 0.6 \tag{16}$$

The curvature parameter $Z_s$ is defined as

$$Z_s = \left(\frac{s^2}{rt}\right)(1-\nu^2)^{\frac{1}{2}} \tag{17}$$

A curved panel with aspect ratio $l/s < 1$ can be taken as an unstiffened circular cylindrical shell with length $l$. The buckling radial pressure is approximately (Ref. 4)

$$p_{cr} = 0.807\left(\frac{Et^2}{lr}\right)\left\{\left[\frac{1}{1-\nu^2}\right]^3\left(\frac{t}{r}\right)^2\right\}^{\frac{1}{4}} \tag{18}$$

and the buckling stress is

$$f_E = p_{cr}\left(\frac{r}{t}\right) \tag{19}$$

*5.2 Panel Ring Buckling*

*5.2.1 Initial Assumptions*

In order to aid the designer with initial assumptions on the geometric parameters of the shell and ring stiffeners the following steps are offered as guides.

The cross sectional area of the ring frame to avoid panel ring buckling (exclusive of shell plate portion) should not be less than

$$A_{req} \geq \left(\frac{2}{(Z_l)^2} + 0.06\right) l\, t \tag{20}$$

$$Z_l = \frac{l^2}{rt}(1-\nu^2)^{\frac{1}{2}} \tag{21}$$

The effective moment of inertia of the ring frame (inclusive of the effective shell portion) should not be less than

$$I_h = \left(\frac{pr(r_0)^2 l}{3E}\right)\left[1.5 + \frac{3E z_t \partial_0}{(r_0)^2(0.5f_r - 6_{hR})}\right] \tag{22}$$

where the effective width of the shell plating to be included in the moment of inertia should be taken as the smaller of

$$l_{ef} = \frac{1.56(rt)^{\frac{1}{2}}}{1+\frac{12l}{r}} \tag{23}$$

$$\text{or} \quad l_{ef} = l$$

and the characteristic material resistance coefficient $f_r$ is $\left(\frac{f_r}{2} > 6_{hR}\right)$

and $f_r = f_T$ for fabricated ring frames

$f_r = 0.9\, f_T$ for cold formed ring frames

The torsional buckling strength fT may be taken equal to the yield strength if the following requirements are satisfied

$$h \leq 0.4\, t_w \left(\frac{E}{f_y}\right)^{\frac{1}{2}} \tag{24}$$

for flat bar ring frames and

$$h \leq 1.35\, t_w \left(\frac{E}{f_y}\right)^{\frac{1}{2}}$$

$$b \geq \frac{7h}{\left\{10 + \left(\frac{Eh}{f_y} r\right)\right\}^{\frac{1}{2}}} \tag{25, 26}$$

(25)

(26)

for flanged ring frames.

The assumed mode of deformation of the ring frame corresponds to ovalization and the initial out-of-roundness is defined by

$$w_0 = \partial_0 \cos 2\theta\,, \tag{27}$$

$$\partial_0 = 0.005\, r \tag{28}$$

*5.2.2. Buckling Stress in Ring Stiffened Cylindrical Shells*

The elastic buckling strength is expressed as

$$f_E = C_1 \left[\frac{\pi^2 E}{12(1-\nu^2)}\right]\left(\frac{t}{L}\right)^2 \tag{29}$$

Where

$$C_1 = \frac{2(1+\alpha_B)}{1+\alpha_B}\left\{1 + \frac{0.27 Z_L}{(1+\alpha_B)^{\frac{1}{2}}}\right\}^{\frac{1}{2}} - \frac{\alpha_B}{1+\alpha_B} \tag{30}$$

$$Z_L = \left(\frac{L^2}{rt}\right)(1-\nu^2)^{\frac{1}{2}} \tag{31}$$

$$\alpha_B = \frac{12(1-\nu^2)I_h}{l\, t^3} \tag{32}$$

*5.2.3 Local Buckling of Ring Stiffeners*

To prevent local buckling, ring stiffeners should be proportioned as follows

Flat bar ring frames $$h \leq 0.4\, t_w \left(\frac{E}{f_y}\right)^{\frac{1}{2}} \tag{33}$$

Flanged ring frames $$h \leq 1.35\, t_w \left(\frac{E}{f_y}\right)^{\frac{1}{2}} \tag{34}$$

## 6. Design Criteria

The final design of the forms should satisfy the following design criteria:

a. Stress limits: The estimated shell and stiffener stresses should not exceed one half of the yield stress of the material.
b. Stability limits: The estimated shell and stiffener stresses should not exceed one half of the lowest buckling stress.

## 7. Geometry of Ring Stiffener

A general ring cross section is shown in Fig. 4

The centroid of the ring frame including the effective shell width $l_{eo}$ is at point A.
The centroid of the ring frame excluding the shell is at point B.
The centroid of the stiffener flange is at point C.

Symbols not defined in the text:

$A_R$ Cross sectional area of the ring frame (exclusive of shell flange)
E Young's modulus
$I_h$ Minimum required moment of inertia of ring frame inclusive effective shell flange
L Distance between effective supports of the ring stiffened cylinder
$N_s$ Axial design load
$f_r$ Material strength
$f_T$ Torsional buckling strength
$f_y$ Material yield strength
l Distance between rings
p Lateral design pressure on the forms
$p_{cr}$ Buckling pressure
r Cylinder radius
s Distance between longitudinal stiffeners
t Shell thickness
$\partial_0$ Initial out of roundness parameter
$\nu$ Poisson's ratio = 0.3
$\Theta$ Circumferential coordinate

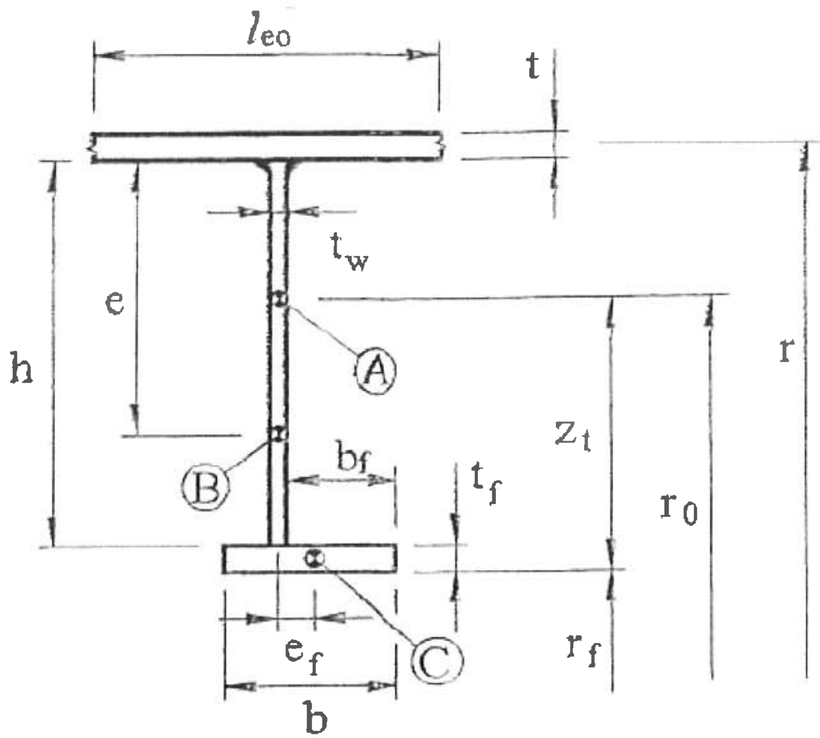



Figure 4: Stiffener Geometry (Ref. 3)

**References**

1. American Concrete Institute, Committee 347, (ACI 347-01), "Guide to Formwork for Concrete".

2. Concrete Construction, Nov.1994, Problem Clinic, "Lateral Pressure on one-sided Wall Forms".

3. Det Norske Veritas, Recommended practice DNV-RP-C202, Oct. 2010, "Buckling Strength of Shells".

4. R. Roark and R. W. Young, Formulas for Stress and Strain, McGraw Hill.

**APPENDIX**

Numerical Example

Photos courtesy of PROMAIN, Proyecto Manufacture Industrial S.A., Hermosilla,Sonora,Mexico.

DESIGN PARAMETERS:

Shaft diameter = 22 ft
Form height = 20 ft
Material ASTM A36
Representative panel geometry Fig. A-2

Concrete rate of pour = 3.0 ft per hr.
Concrete temp. = 100°F

LOADS ON FORMS:

Pressure from concrete

$$p = 150 + \frac{43500}{100} + 2800\frac{3}{100} = 670\ \text{psf} \cong 4.6\ \text{psi}$$

Vertical load = dead weight of forms estimated @ 1000 lbs / ft. circumference (or $\sigma_x = 333$ psi)

ANALYSIS MODEL: Fig. A-1

$\ell = 24$ in, $s = 23$ in, $r = 132$ in, $t = 0.25$"

$$\frac{23}{0.25} > 3\sqrt{\frac{132}{0.25}}$$

$$92 > 69$$

∴ can neglect vert. stiffeners in ring response

STRESSES IN RING STIFFENED SHELL:

a) Circular membrane stress midway between two rings

$\ell = 24$ in, $r = 132$ in, $p = 4.6$ psi, $t = 0.25$ in.

$A_R = 1.97\ in^2$

$\ell_\infty = 1.56\sqrt{132 \times 0.25} = 9$ in

$\beta = \frac{24}{9} = 2.7$

$\alpha = \frac{1.97}{9 \times 0.25} = 0.88$

From Fig. 3:

$\ell_\infty = 9$ in OK

$\zeta = 0$

$$\sigma_h = \frac{4.6 \times 132}{0.25} = 2430\ \text{psi}$$

CIRCULAR MEMBRANE STRESS IN RING:

$$\sigma_{h1} = \left[\frac{4.6 \times 132}{0.25} - 0.3 \times 333\right]\frac{1}{1+0.88} + 0.3 \times 333 =$$

$\sigma_h = 1340$ psi

LONGITUDINAL MEMBRANE STRESS:

$\sigma_x = 333$ psi

LONGITUDINAL BENDING STRESS AT RING

$$\sigma_{xm} = \left(\frac{4.6 \times 132}{0.25} - 1840\right)\sqrt{\frac{3}{1-0.3^2}} =$$

$\sigma_{xm} = 1977$ psi

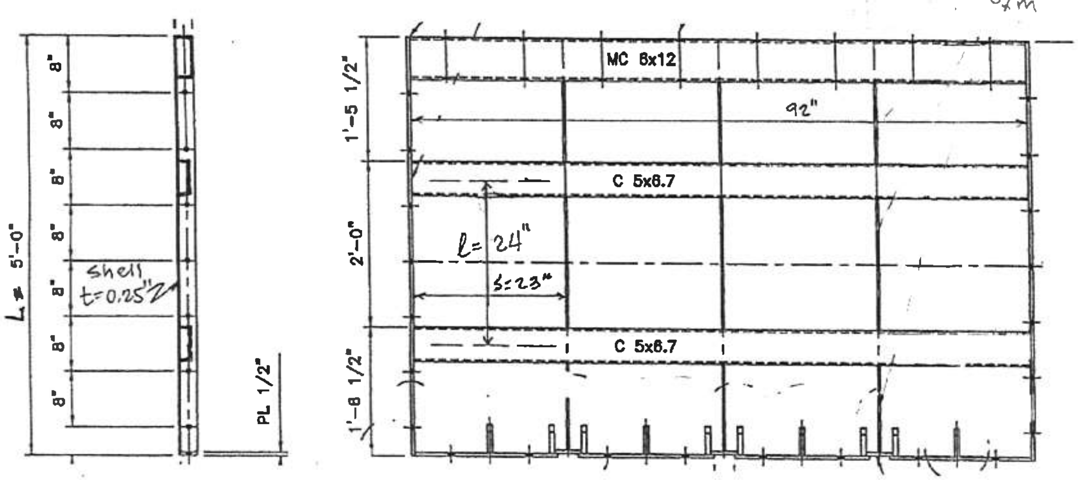



Fig. A-1

CIRCUMFERENTIAL BENDING STRESS AT RING:

$$\sigma_{hm} = 0.3 \times 1977 = 593 \text{ psi}$$

BUCKLING OF CURVED PANEL:

$$l/s = \frac{24}{23} \cong 1.0$$

$$E = 29 \times 10^6 \text{ psi}$$

$$p_{cr} = 0.807 \frac{E \times 0.25^2}{24 \times 132} \left[ \left( \frac{1}{1-\nu^2} \right)^3 \cdot \left( \frac{0.25}{132} \right)^2 \right]^{1/4} =$$

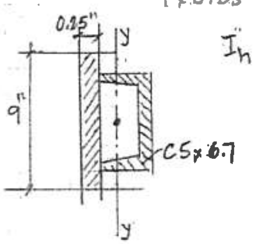
$$p_{cr} = 21.6 \text{ psi}$$

Buckling stress:

$$\sigma_{cr} = \frac{21.6 \times 132}{0.25} = 11{,}405 \text{ psi}$$

PANEL RING BUCKLING:

$$\alpha = \frac{1.97}{9 \times 0.25} = 0.88$$




$$I_h = 2.5 \text{ in}^4$$

= moment of inertia of combined section about axis y-y

L = Height of form = 15 ft

$$\alpha_B = \frac{12(1-0.3^2)\,2.5}{24 \times 0.25^3} = 72.8$$

$$Z_L = \frac{180^2}{132 \times 0.25} \sqrt{1-0.3^2} = 6.5$$

Buckling coefficient

$$C_1 = \frac{2(1+72.8)}{1+0.88} \left[ \left[ 1 + \frac{0.27 \times 16.5}{\sqrt{1+72.8}} \right]^{1/2} - \frac{72.8}{1+72.8} \right] =$$

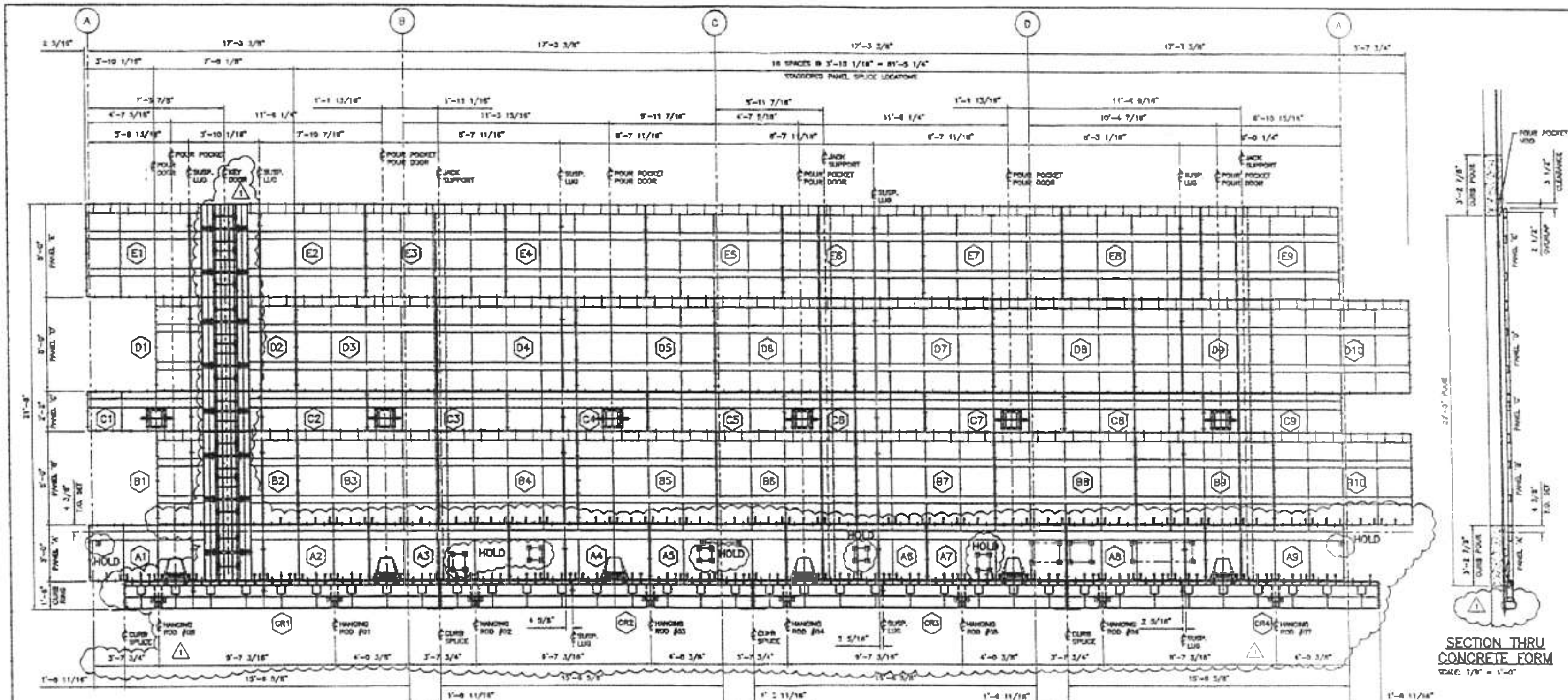
$$C_1 = 6.6$$

Buckling stress

$$f_B = C_1 \frac{\pi^2 E}{12(1-\nu^2)} \left( \frac{0.25}{180} \right)^2 = 3910 \text{ psi}$$

SUMMARY

| | |
|---|---|
| Circ. membrane in shell | 2490 psi |
| Circ. membrane in ring | 1,340 psi |
| Long. membrane in shell | 333 psi |
| Circ. bending at ring | 593 psi |
| Long. bending at ring | 1977 psi |
| Buckling stress of shell | 11,405 psi |
| Buckling stress of panel ring | 3,910 psi |



## ELEVATION – CURB & PANEL ASSEMBLY

SCALE: 3/8" = 1'-0"

## SECTION THRU CONCRETE FORM

SCALE: 1/8" = 1'-0"

NOTE: ALL DIMENSIONS IN ARC LENGTHS TAKEN ON 11'-0" RADIUS.

### GENERAL NOTES:

- FABRICATOR SHALL PRE-ASSEMBLE CURB RING, PANELS 'A' & PANELS 'B' TO CHECK TOLERANCES, INSERT LOCATIONS AND ASSEMBLY FIT UP PRIOR TO SHIPPING TO SITE.
- PANELS TO BE IDENTIFIED SEQUENTIALLY FROM THE ASSEMBLY "START LOCATION" FROM 'A' PANEL TO TOP OF CURB THROUGH TO 'E' PANEL.
- OUTSIDE SURFACES OF COMPLETE AND ASSEMBLED CURB RING AND PANELS ARE TO BE SMOOTH AND FREE OF DEFECTS AND/OR MISALIGNMENT AT PANEL AND/OR CURB SPLICES. FABRICATION TOLERANCES OF PLUS OR MINUS 1/16" WOULD APPLY.
- ALL CIRCUMFERENTIAL MEASUREMENTS OF ASSEMBLED FORM PANELS FROM 'A' PANEL TO 'E' PANEL SHALL BE EQUAL AND REMAIN WITHIN THE FABRICATION TOLERANCE.
- POUR POCKET FORMS SHALL BE CLEARLY IDENTIFIED AND MATCH MARKED SPECIFIC TO THEIR SUITED LOCATIONS WITH CORRESPONDING MARKINGS ON CURB RING.
- CURB SEGMENTS TO BE CLEARLY IDENTIFIED AND MATCH MARKED AT SPLICE LOCATIONS FOR RE-ASSEMBLY IN THE FIELD.
- BREAK ALL SHARP CORNERS AND GRIND EDGES FREE OF SLAG.
- ALL HOLES TO BE Ø13/16" U.N.O.
- MAXIMUM ROCK OVERBREAK 12". IF OVERBREAK EXCEED 12", ENGINEER TO REVIEW AND PROVIDE PROCEDURE.

### FABRICATION NOTES:

1. FABRICATION TOLERANCE: PLUS OR MINUS – 1/16" U.N.O.
2. PANEL RADIUS: PLUS 3/16" MINUS 0"
3. CURB RADIUS: PLUS 3/16" MINUS 0"
4. CURB-TOP RING FLATNESS PLUS OR MINUS 1/16" AFTER FULL ASSEMBLY AND TORQUE.
5. ROUNDNESS OF ASSEMBLED FORM PLUS OR MINUS 1/8" ON THE DIAMETER.
6. INSERT LUG PLATES TO BE TRIMMED AS REQUIRED TO CLEAR FORM STIFFENER PLATES. SOME STIFFENER PLATES TO BE RELOCATED TO SUIT INSERTS LUG AS NOTED.
7. ALL INTERIOR SURFACES SHALL BE BRUSH BLASTED AND COATED WITH ONE COAT SHOP PRIMER. ALL EXTERIOR SURFACES EXPOSED TO CONCRETE SHALL BE COVERED WITH FORM OIL.

### STRUCTURAL STEEL NOTES:

1. STEEL DETAILS, FABRICATION AND ERECTION SHALL CONFORM TO THE REQUIREMENTS OF AISC CODE OF STANDARD PRACTICE FOR STEEL BUILDINGS & BRIDGES.
2. STRUCTURAL STEEL SHALL BE NEW MATERIAL CONFORMING TO THE REQUIREMENTS OF ASTM 48/A6M LATEST EDITION:
   - W SHAPES – A572 GR. 50
   - T SHAPES – GR. A36
   - CHANNELS – GR. A36
   - STRUCTURAL HSS – A500 GR. B
   - ROUND HSS – A500 GR. B
   - PLATES – GR. A36
   - ANGLES – GR. A36
   - PIPES – A53 GR. B
3. ALL WELDING SHALL CONFORM TO THE REQUIREMENTS OF A.W.S. STANDARD D1.1/D1.1M LATEST EDITION STRUCTURAL WELDING CODE-STEEL & STANDARD B2.1 LATEST EDITION WELDING PROCEDURE & PERFORMANCE QUALIFICATION.
4. BOLTED CONNECTIONS SHALL USE HIGH STRENGTH BOLTS CONFORMING TO ASTM SPEC. A325 AND TORQUED TO MANUFACTURER'S RECOMMENDATIONS. SIZE OF BOLT TO BE 3/4" DIA. MIN. UNLESS OTHERWISE NOTED ON THE PROJECT DRAWINGS.
5. CONNECTIONS SHALL BE BEARING TYPE AND SHALL BE DESIGNED TO RESIST THE FACTORED LOADS AND FORCES SHOWN ON THE DRAWING.
6. WHERE NO LOADS AND FORCES ARE INDICATED, BEAM END CONNECTIONS SHALL BE DESIGNED TO RESIST A MINIMUM SHEAR REACTION OF HALF THE U.D.L. CAPACITY OF THE BEAM ASSUMING FULL LATERAL SUPPORT.
7. WHERE NO LOADS AND FORCES ARE INDICATED, BRACING END CONNECTIONS SHALL BE DESIGNED TO RESIST A MINIMUM SHEAR VALUE OF HALF THE TENSILE CAPACITY OF THE MEMBER.
8. CONNECTIONS SHALL HAVE A MINIMUM OF TWO BOLTS.

Exp. 31/[illegible]

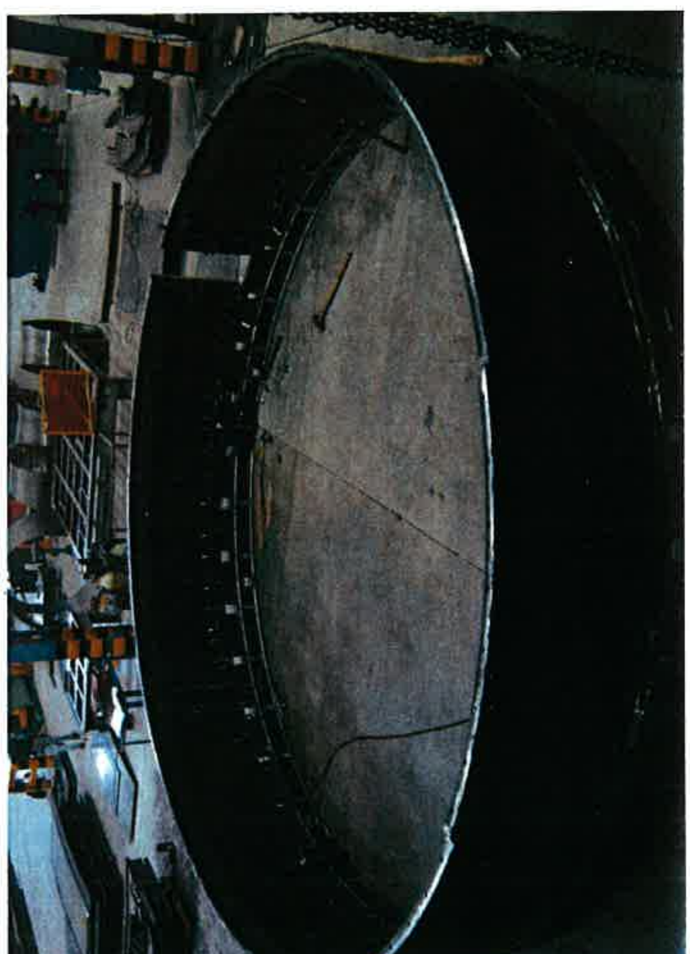
-FOR CONSTRUCTION-

**Cementation**
Cementation Group - Americas

| Rev. | Description | Date | By |
|---|---|---|---|
| 1 | REVISED HANGING RODS AND KEY DOOR [illegible] PLATES & ADDED STIFF.'S | 24/08/12 | [illegible] |
| 0 | ISSUED FOR CONSTRUCTION | 27/04/12 | [illegible] |

Figure A-2

AS NOTED | ATF | 19/09/11

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