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Effect of Imperfections on the Ultimate Shear Strength of Tapered Girders

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Abstract
This paper presents the results of finite element analysis studies of the effect of geometrical and structural imperfections on the ultimate shear strength of web tapered plate girders. Standard imperfection models are applied to a tapered girder finite element model subjected to uniform shear stress without bending. The model is validated using available test results on web tapered girders. A parametric study is performed to investigate the variations in both elastic buckling and ultimate shear strengths with the major design parameters such as web slenderness ratio, tapering angle, tapered panel aspect ratio.

1. Introduction

Web tapered girders are usually used in bridges to achieve economy by varying the web depth according to variation of the bending moments and shear forces resulting from applied loads. This variation leads to lighter design than conventional prismatic girders. Current design codes, e.g., AASHTO (2010), are based on theoretical and experimental research on prismatic girders. Theoretical solutions of plate buckling problems are based on the simplifying assumptions of simply supported plate panels. These solutions do not consider the real boundary conditions at the web-flange and web-stiffener connections which are known from experimental investigations to be somewhere between simply supported and fixed depending on the relative slenderness of the flange and the stiffener. Finite Element Analysis has been used effectively to obtain the elastic buckling stress and the ultimate strength under a wide scope of design variables related to applied stresses and actual boundary conditions. Allowance for initial geometric imperfections and residual stresses may be easily incorporated in the finite element model. The buckling stress is obtained by solving a linear eigen-value problem with the eigen-values representing the buckling load factors and the eigen-vectors representing the buckling mode shapes. The ultimate strength is obtained by performing a nonlinear inelastic analysis up to the failure load. The finite element models used may be a single isolated panel or a complete girder model.

There are very few theoretical and experimental investigations into the structural behavior of web-tapered girders under shear and/or bending moments, e.g., Mirambell (2000), Real (2010), Studer (2013) and Bedynek (2013). Similarly, the effect of initial geometric imperfections and residual stresses on the shear strength of prismatic girders has been studied extensively by Chacon (2009), Maiorana (2009), Graciano (2011), Chacon (2009) and Chica (2013). The effect

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of such imperfections for tapered girders has received very little attention, Bedynek (2013). Consequently, there are no specific provisions in current design codes for the design of tapered girders.

This paper uses a finite element model to investigate the effect of geometric imperfections and residual stresses on the elastic shear buckling stress and ultimate stress of web tapered plate girders. The model is validated using available test results on web tapered girders. A parametric study is performed to investigate the variations in both elastic buckling and ultimate shear strengths with the major design parameters such as web slenderness ratio, tapering angle, tapered panel aspect ratio.

2. Ultimate Shear Strength

The design of plate girder sections is usually governed by flexural strength and shear strengths limit states. Local plate buckling affects the calculation of the cross section resistance related to compression flange local buckling, web bend buckling in the flexural strength limit state and web shear buckling in the shear strength limit state. Other limit states such as lateral torsional buckling, tension flange yielding, and fatigue are not covered in this paper.

Generally, a three-range design format is followed depending on the value of a slenderness parameter, λ, which equals the width-to-thickness ratio of the plate component considered. When the slenderness ratio λ is less than a value λ_p, the section can reach its plastic moment capacity and is classified as compact in the American codes AISC [2005] and AASHTO [2004], and as class 2 in the European Code EC3 [2005]. When $\lambda_p < \lambda < \lambda_r$, the section strength is limited by its yield moment and is called non-compact in the American codes AISC and AASHTO, and class 3 in the European code EC3. When $\lambda > \lambda_r$, the section strength is governed by elastic buckling and the section is slender in AISC/AASHTO and class 4 in EC3. Details of the governing equations used to calculate the cross section resistances in each case are given in the respective codes and several papers such as White [2008] and White and Barker [2008]. Summary of code provisions related to plate buckling in the three considered codes is given Abu-Hamd (2010).

The theoretical elastic buckling stress of a prismatic rectangular plate, q_{cr}, under the action of *pure shear stress* is given by the widely known formula:

$$q_{cr} = kq \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{d} \right)^2$$

where E is the modulus of elasticity, ν is Poisson's ratio, t is the thickness of the plate, d is the width of the plate, and k_q is the shear buckling factor, which depends on the plate aspect ratio α. The pure shear stress state results in *equal compressive and tensile principal stresses* in the plate. Various expressions for calculating k_q that consider the rigidity provided by the web-to-flange connection are available, e.g., Lee, et al (1996, 1998, 2012).

Slender plate girder webs, however, do not fail by elastic buckling but exhibit significant post-buckling strength when the girder behaves as a truss composed of the flanges as chords and the transversal stiffeners acting as truss verticals to maintain equilibrium in the post buckling stage with the developed inclined tensile stress state. The ultimate shear strength is reached when plastic hinges are formed in the flanges through the development of *tension field action*. Basler (1961) calculated the ultimate shear strength as the sum of the elastic buckling shear strength and the post-buckling strength provided by tension field action. This the procedure used in AISC Specification (2010) and AASHTO Code (2010) to calculate shear strength of plate girder webs.

On the other hand, real plate girders used in practice exhibit some imperfections that may prevent them from reaching the elastic buckling strength and the ultimate strength. The reduction of prismatic girders strength due to imperfections have been covered by many investigations, e.g., Chacon (2009), Maiorana (2009), Graciano (2011) and Chica (2013). For web tapered girders, however, the reduction of shear strength due to imperfections have been covered by very few investigations, e.g., Real, et al (2010) and Bedynek, et al (2013). This paper presents a numerical finite element model that can be used to investigate the effect of imperfections on both the elastic buckling strength and ultimate strength of web tapered plate girders.

3. Imperfection Models

Plate girders used in practice exhibit two significant types of imperfections:

1- *Geometrical imperfections* such initial curvature of girder axis and deviation of the cross section from theoretical shape due to dimensional tolerances, and
2- *Structural Imperfections* in the form of longitudinal residual stresses that are introduced in welded plate girders during flange cutting and web-to-flange welding.

These imperfections may reduce the strength of real plate girders from the theoretical calculated values as shown by Chacon (2009), Maiorana (2009), Graciano (2011), and Chica (2013).

3.1 Geometrical Imperfection Models:

Geometrical imperfections are usually of random nature and strongly depend on fabrication processes. Although the shape and magnitude of these imperfections may be measured in practice, their incorporation in ultimate strength calculations would be extremely hard. A much more practical way of considering geometrical imperfections is to model their shape similar to the first eigen-mode shape obtained from linear buckling analysis after normalizing its maximum amplitude according to the tolerance allowed by codes. Tsai (2006) used the following tolerance limits recommended by AASHTO/AWS D1.5:2002:

$$\text{Web:} \quad \max \delta = d_w/100 \qquad \text{for } d_w/t_w < 100$$
$$= d_w/67 \qquad \text{for } d_w/t_w > 100$$

$$\text{Flange:} \quad \max \delta_f = b_f/100$$

This model shall be used in the present study to represent initial geometric imperfections.

Similarly, The European Code EC3:EN1993-1-5, Annex C (2006) recommends using the following fabrication tolerances for normalizing the mode shape:

$$\text{Web:} \quad \max \delta = d_w/200$$

$$\text{Flange:} \quad \max \theta = b_f/100$$

Where d_w= web depth, t_w= web thickness, and b_f= Flange width, δ = out-of-flatness, and θ = flange twist.

Initial geometric imperfections change the plate stability behavior from a bifurcation problem into a load-deflection problem. Chacon (2009), Maiorana (2009), Graciano (2011), and Chica (2013) investigated the reduction of shear strength due to several geometrical imperfection models and found that the difference between proposed models is small. In this paper, the influence of geometric imperfections on the ultimate shear strength of web tapered girders shall be investigated using the a.m. AASHTO/AWS recommended tolerances.

3.2 Structural Imperfection Model

Several distribution of longitudinal residual stress in welded plate girders have been proposed, e.g., ECCS Manual on Stability of Steel Structures (1976) and Barth and White (1998). Residual stresses reduce both the elastic buckling shear strength and the ultimate shear strength. Chacon (2009), Maiorana (2009), Chacon (2012), Chica (2013), and Bedynek et al (2013) investigated the reduction in the shear strength due to several residual stress models and found that the difference between proposed models is small. In this paper, the influence of residual stresses on the ultimate shear strength of web tapered girders shall be investigated using the model proposed by Barth and White (1998) based on ECCS (1976). The distribution uses a value of 0.33 F_y in the flange and 0.63*F_y in the web at the web-to-flange connection. The corresponding compressive stresses are 0.17*F_y in the flanges and 0.07*F_y in the web. The tensile stresses at flange tips due to flame cutting are taken equal to 0.18*F_y.

4. Numerical Analysis

Finite Element Analysis, Earls (2007), Ziemian (2010), may be used effectively to obtain the elastic buckling and the ultimate strength under a wide scope of design variables related to girder geometry, applied stresses and actual boundary conditions. The elastic buckling stress is obtained by solving the linear eigen-value problem:

$$\mathbf{K}_E = \lambda\, \mathbf{K}_G \tag{2}$$

Where $\mathbf{K}_E$ is the elastic stiffness matrix, $\mathbf{K}_G$ is the geometric stiffness matrix, and λ is the eigen-values which represents the buckling load factors. The corresponding eigen-vectors represent the mode shapes of the buckled plate.

The existence of geometrical imperfections and residual stresses in real girders prevent them from reaching their elastic buckling strength. The ultimate strength under these conditions may be obtained by performing a nonlinear inelastic analysis on an imperfect model with gradually increasing loads up to the load level at which the model reaches its ultimate load or becomes unstable.

4.1 Description of Girder Model

Neither an all edges simply supported plate, nor a single web panel can realistically represent the actual behavior of real plate girders used in practice. The finite element models used may be a single isolated panel or a complete girder model. Numerical solutions obtained from isolated single panel models give conservative buckling strength values as compared to results obtained from complete girder models, e.g., Maiorana (2009) and Abu-Hamd (2011). Therefore, a multi-panel girder model with realistic boundary conditions is used in this study to simulate real girders.

Fig. 1a shows the geometric configuration of the complete girder model used in the present study. It represents a bridge girder with five segments, two of which are tapered.

The finite element model used to represent one half of the girder is shown in Figs. 1b and 2. It is composed of one tapered segment with a straight segment on each side of length 1000 mm. The tapered segment has a variable length $L = \alpha * d_2$, where α= aspect ratio. The results presented in the numerical study covers the cases $\alpha = 1$ and $\alpha = 2$.

Figure 1a: Complete Girder

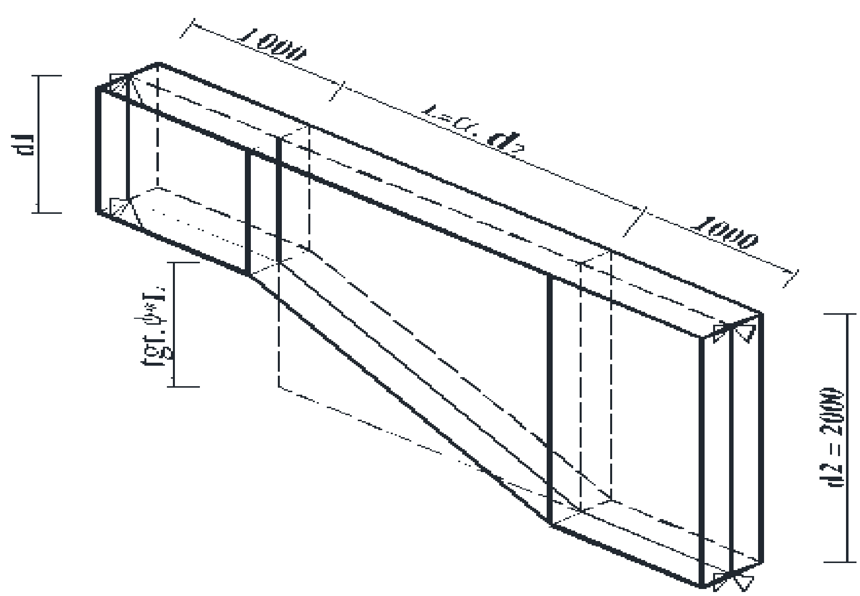


Figure 1b: Girder Model

The deeper end web depth d_2 is fixed in the numerical study at 2 meters while the smaller end depth d_1 is varied between 0.40 meter and 2 meters at 0.40 meter intervals to give different tapering angles of 0.20 and 0.4. The tapered segment length is varied between 2 and 4 meters at 2 meters intervals to give different tapered panel aspect ratios of 1, and 2, respectively.

4.2 Finite Element Model

Fig. 2 shows the finite element model made using ANSYS (2009) to represent the model girder. All plate elements were modeled with an iso-parametric finite strain shell element designated as “Shell 181” in ANSYS element library. Shell 181 is a four-noded shell element with six degrees of freedom per node and has geometric and material nonlinearities capabilities. It is well suited for linear, large rotation, and /or large strain nonlinear applications. In the construction of the finite element model, convergence was achieved by using a mesh size in the order of 50 mm for all plate elements. The displacement boundary conditions at the left end were specified to give a roller support while the right end was restrained to represent symmetry about the middle vertical plane. Lateral torsional buckling was prevented by restraining the movement in the out-of-plane direction of all nodes along the web-to-flange connection. The material properties used correspond to an elastic-plastic material with Von Mises yield criteria and isotropic hardening. The values of the material constants used are Elastic modulus E=210 GPa, yield stress Fy=350 MPa, and Poisson’s ratio ν=0.3.

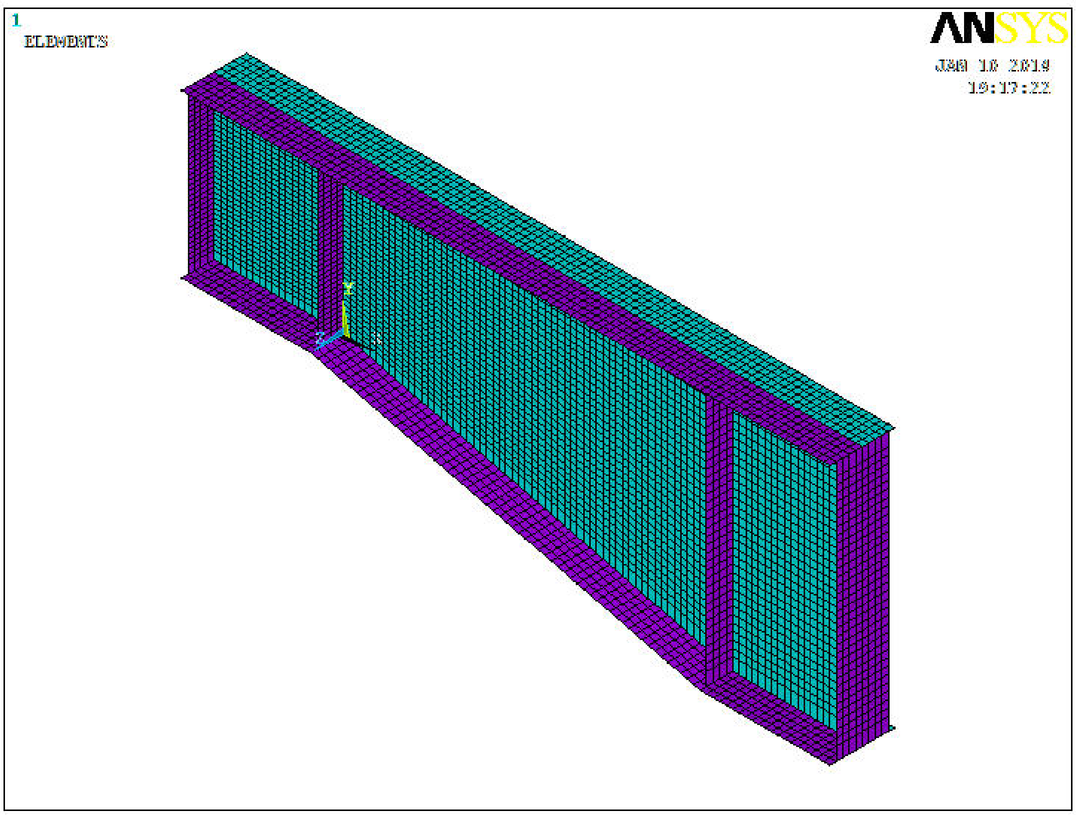


Figure 2: Finite Element Model

4.3 FEM Validation

Bedynek, et al (2013) presented an experimental and numerical study on shear strength of tapered plate girders. In the experimental part, four small-scale experimental tests of tapered steel plate girders were carried out and their test results were compared with those obtained by numerical simulation of the tests. The four tests covered four typologies used in tapered plate girders according to the direction of the resulting diagonal compression responsible for shear buckling. Typologies I and II only shall be considered here because of their practical importance. In typology I, the direction of diagonal compression is in the direction of the longer diagonal of the tapered panel, while as the direction of diagonal compression is in the direction of the shorter diagonal of the tapered panel. This effect of diagonal compression direction was also discussed by Real et al (2010). Due to the difference in length of the buckled diagonal, the shear strength of typology II is larger than that of typology I.

The proposed model in the present paper was validated by comparing its results with the experimental and numerical results presented by Bedynek, et al (2013). Three test girders of topology I were simulated using the proposed model for the same material properties of the test girders, i.e., Elastic modulus E=211.3 GPa, Poisson's ratio ν = 0.3, and yield strength Fy= 320.6 MPa. The results of the comparison are shown in Table 1.

Table 1

Comparison between Test Results and Model Results

	Girder A	Girder B	Girder C
Girder Cross Section	600/800*3.9*180*15	500/800*3.9*180*15	480/800*3.9*180*15
Length (mm)	800	1200	800
Aspect Ratio α	1	1.5	1
Tapering Angle tgt.ϕ	0.25	0.25	0.4
1) Elastic Buckling			
Test Results	225	220	265
Model Result	240	215	290
% Difference	+6.66 %	-2.32 %	+9.40 %
2) Ultimate Strength			
Test Result	392	320.5	388.2
Model Results	396	330	347
% Difference	+1 %	+3.12 %	-10.6 %

The differences between test results and model results are between -2.32 % for Girder B and +9.40 % for Girder C. Test result for the ultimate strength of girder C is equal to $0.754*F_y$ which exceeds the shear yield stress $0.577*F_y$, indicating a large amount of tension field action prior to failure. These differences are considered acceptable and may be attributed to the differences in initial imperfections and residual stress distributions.

5. Parametric Study

After validating the FE-model versus available test results, a parametric study is performed to investigate the effect of major design parameters such as web and flange slenderness, tapering angle, tapered panel aspect ratio on the ultimate shear strength in the presence of typical distributions of geometric imperfections and residual stresses. The model is subjected to two point loads at tapered panel ends. The load values are chosen to produce a shear stress of $0.577*F_y$ at each end. As these loads subjects the tapered panel to high bending stresses, balancing uniform distributed moment (m) is applied to the tapered panel. The values of the additional moments are calculated to balance the bending moment resulting from the difference in applied shear so that the tapered panel is subjected to uniform pure shear stress. Details of these loads are given in Abu-Hamd (2011).

The range of design parameters covered in this study is as follows:

1- Tapered panel aspect ratio α= 1,2
2- Tapered panel angle tgt ϕ= 0, 0.2, and 0.4.
3- Web slenderness ration at larger end $\lambda_w = d_2/t_w$ = 75 to 200 at intervals of 25.
4- Flange slenderness ratio $\lambda_f = b_f/2*t_w$ = 8,13,18 corresponding to the cases of a compact, non- compact and a slender flange, respectively.

The results of the parametric study are presented in Figs. 3,4,5 for the elastic buckling shear strength and in Fig. 6 and 7 for the inelastic ultimate shear strength. The difference in stress reduction due to flange slenderness was found to be small so that only results corresponding to the case on non-compact flange are presented.

5.1 Elastic Shear Buckling Results

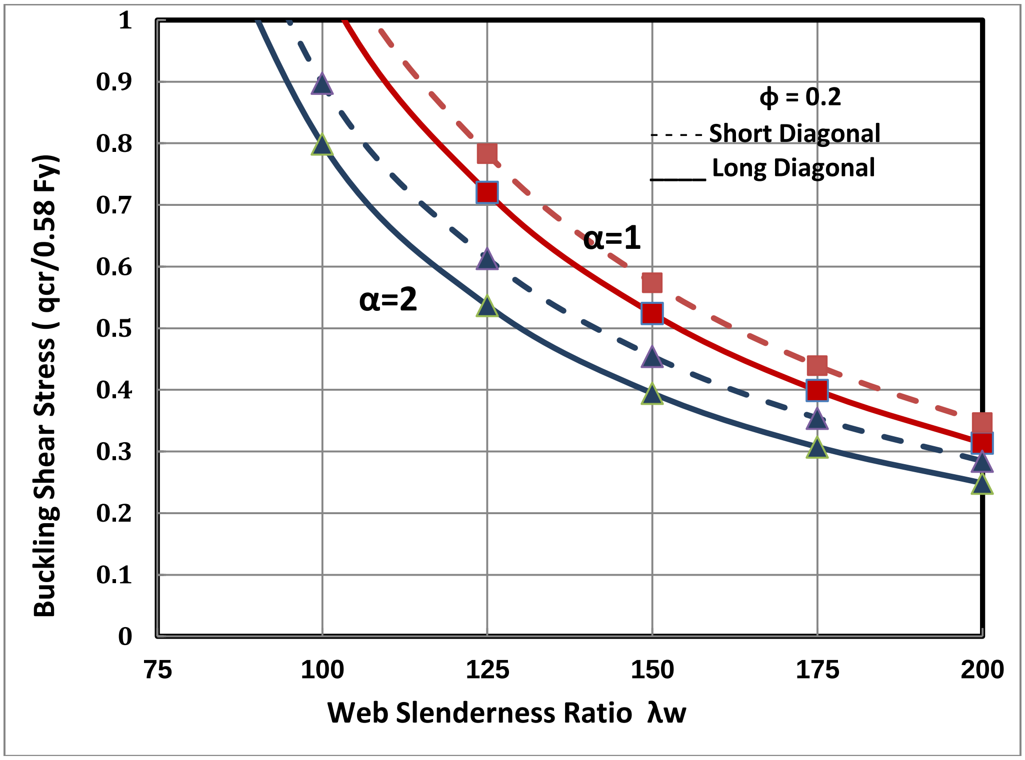
5.1.1 Effect of diagonal compression direction

Fig. 3 shows the variation of the elastic shear buckling stress ($q_{cr}/0.58*F_y$) with the web slenderness ratio λ_w for different values of the aspect ratio α and the tapering angle φ for two cases of long and short diagonal compression. Based on the results of the studied girder models, the elastic buckling stress for the case of short diagonal compression is larger than the case of long diagonal compression. The % increase ranges between 6 to 10 % for α=1 and ϕ=0.2 and between 25 to 36 % for α=2 and ϕ=0.4.

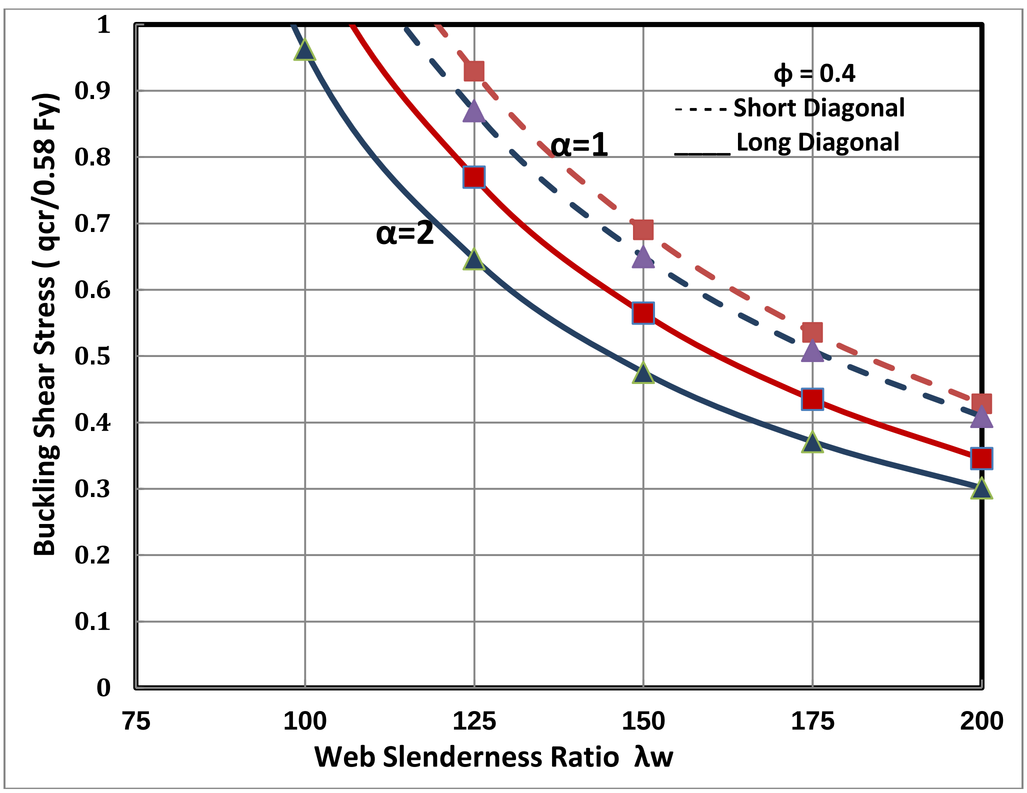
5.1.2 Reduction of Elastic shear buckling stress due to residual stresses

Figs. 4 and 5 show the variation of the elastic shear buckling stress ($q_{cr}/0.58*F_y$) against the web slenderness ratio λ_w for different values of the aspect ratio α and the tapering angle φ with and without residual stresses.

Based on the results of the studied girder models, the reduction in elastic shear buckling strength ranges between 4 and 10 % for all α and ϕ values.

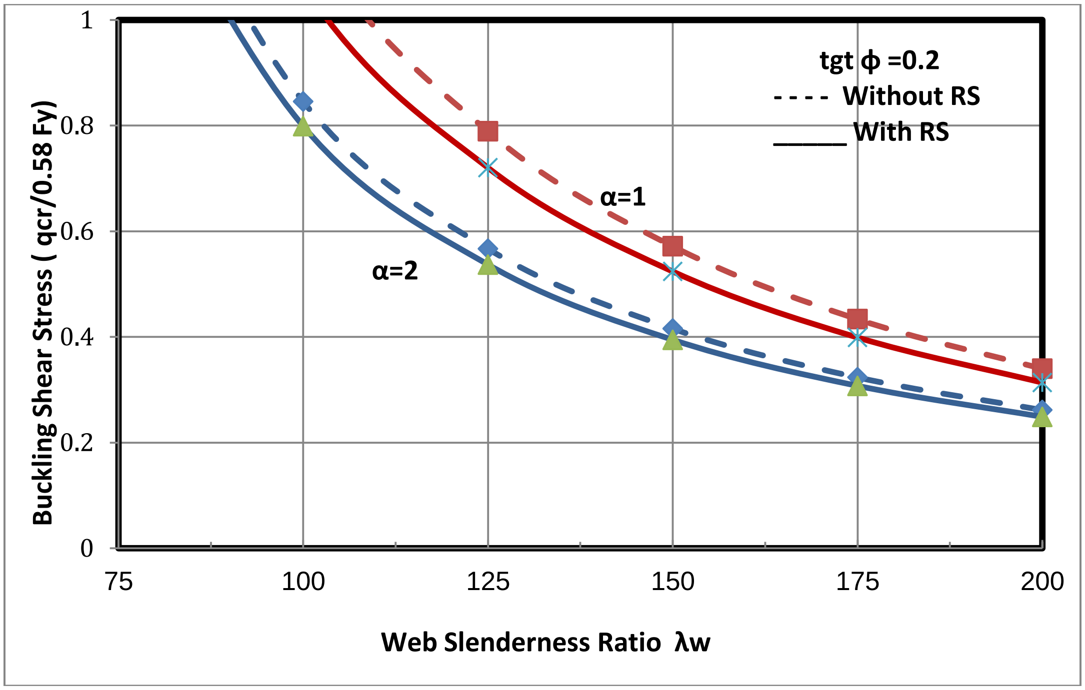


ϕ = 0.2
- - - - Short Diagonal
_____ Long Diagonal
α=1
α=2
Buckling Shear Stress (qcr/0.58 Fy)
Web Slenderness Ratio λw

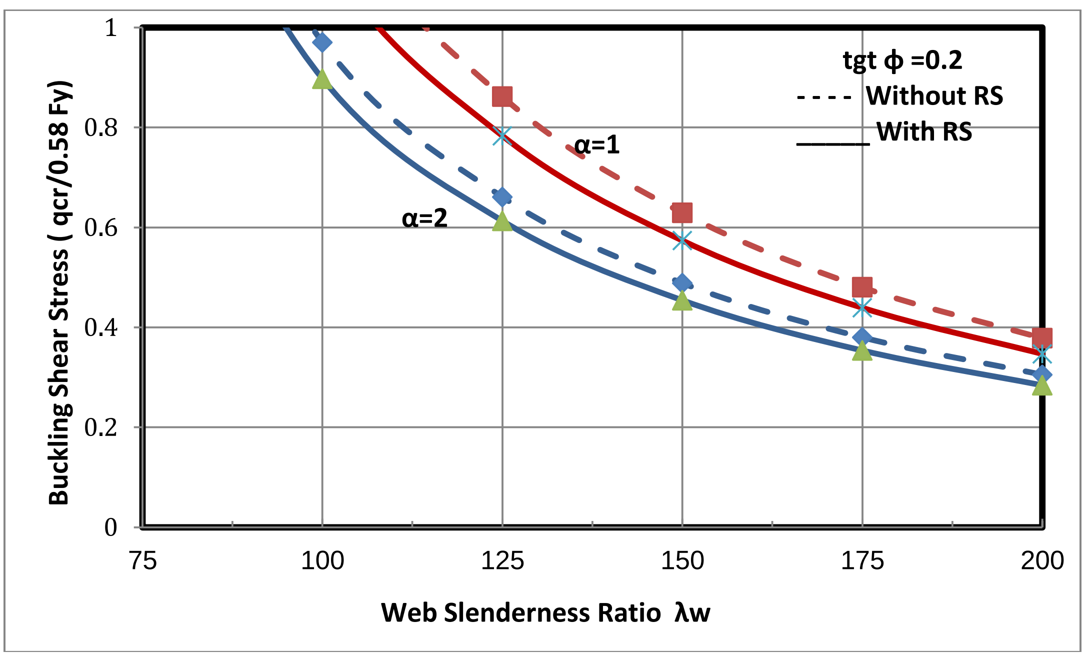


ϕ = 0.4
- - - - Short Diagonal
_____ Long Diagonal
α=1
α=2
Buckling Shear Stress (qcr/0.58 Fy)
Web Slenderness Ratio λw

Figure 3: Variation of Shear Buckling Stress according to Compression Diagonal Direction

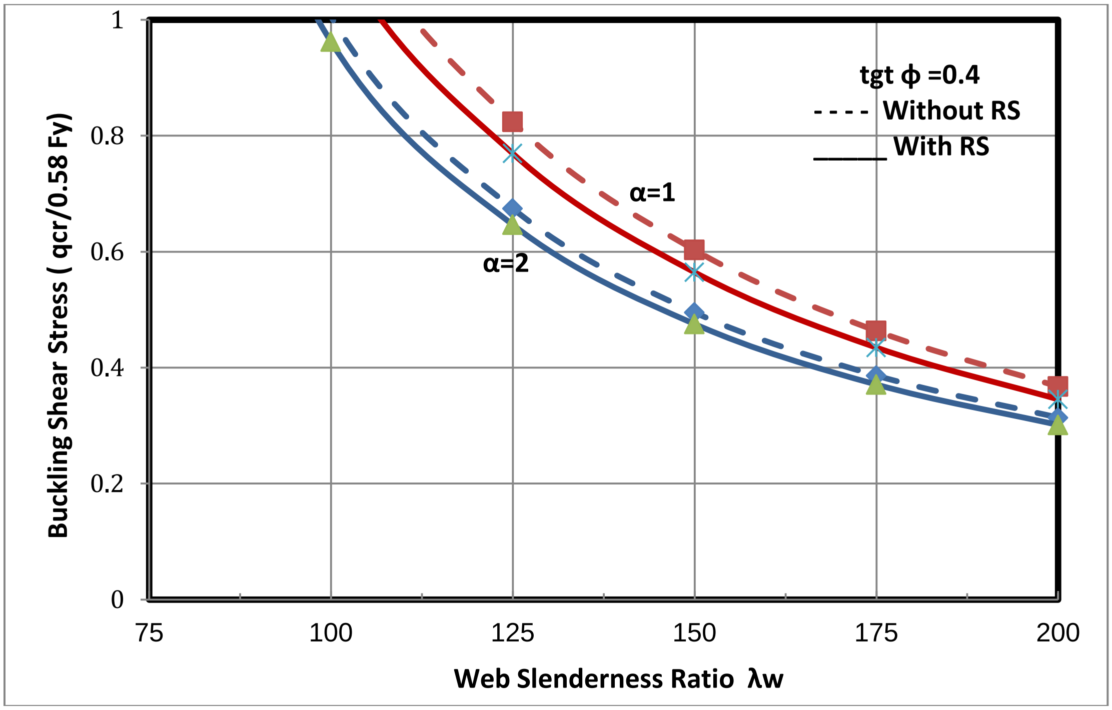


(a) Long Diagonal

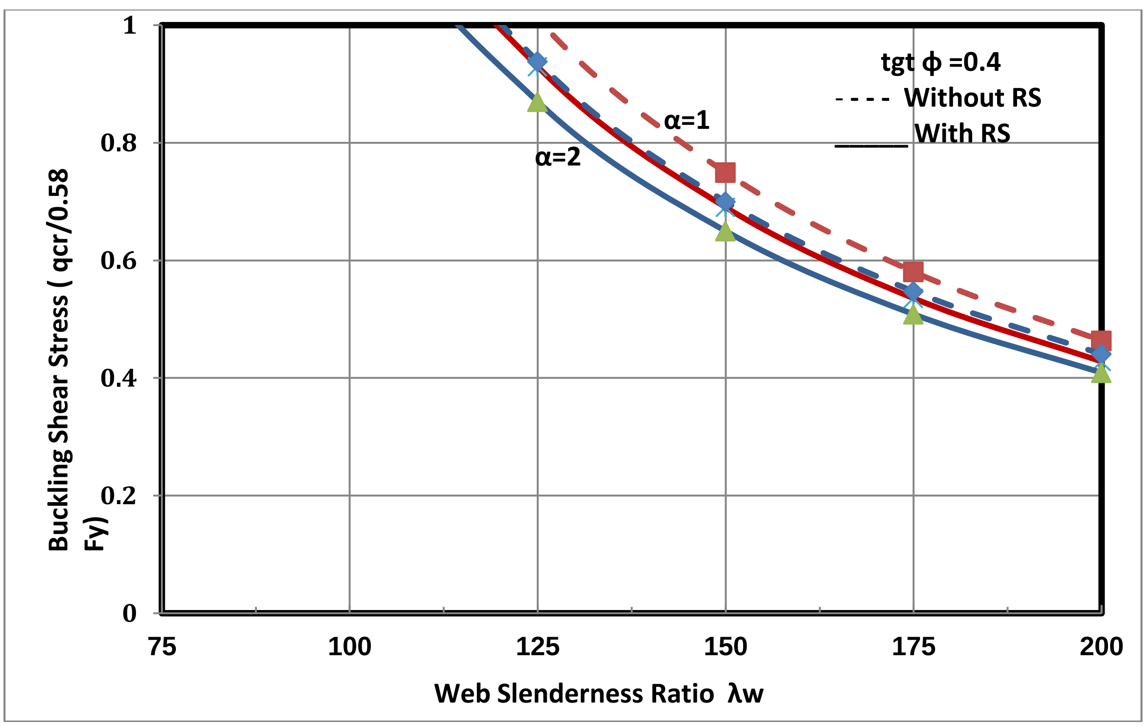


(b) Short Diagonal

Figure 4a: Reduction in Shear Buckling Stress due to Residual Stresses (tgt ϕ=0.2)

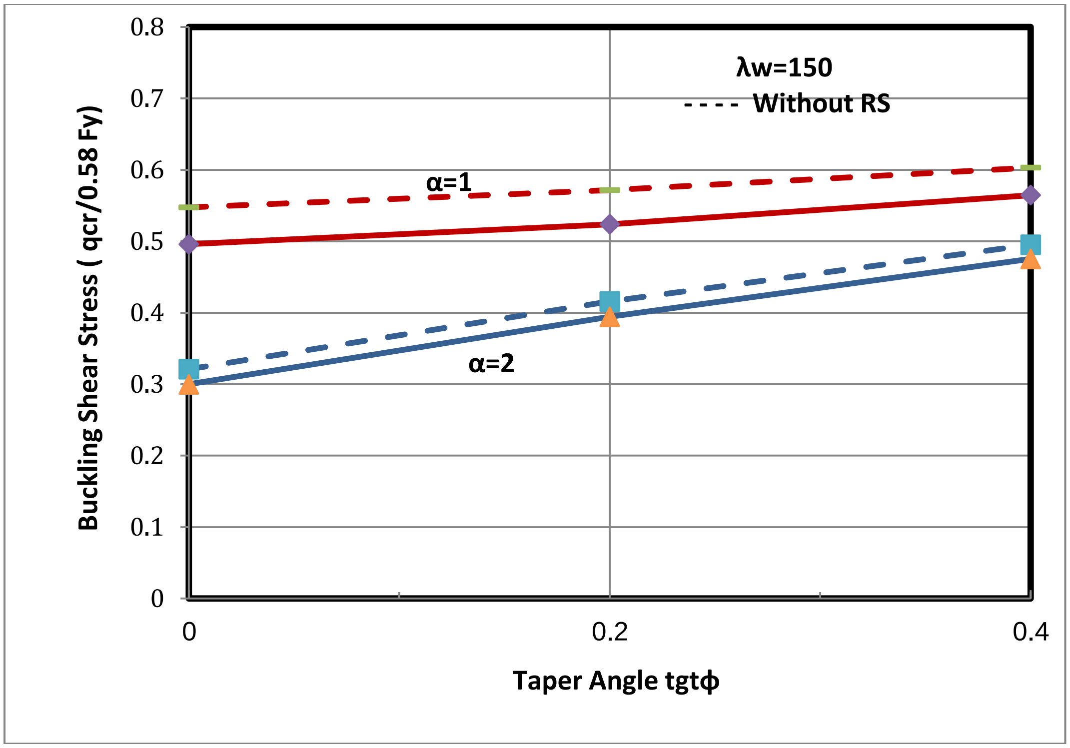


(a) Long Diagonal



(b) Short Diagonal

Figure 4b: Reduction in Shear Buckling Stress due to Residual Stresses (tgt ϕ=0.4)



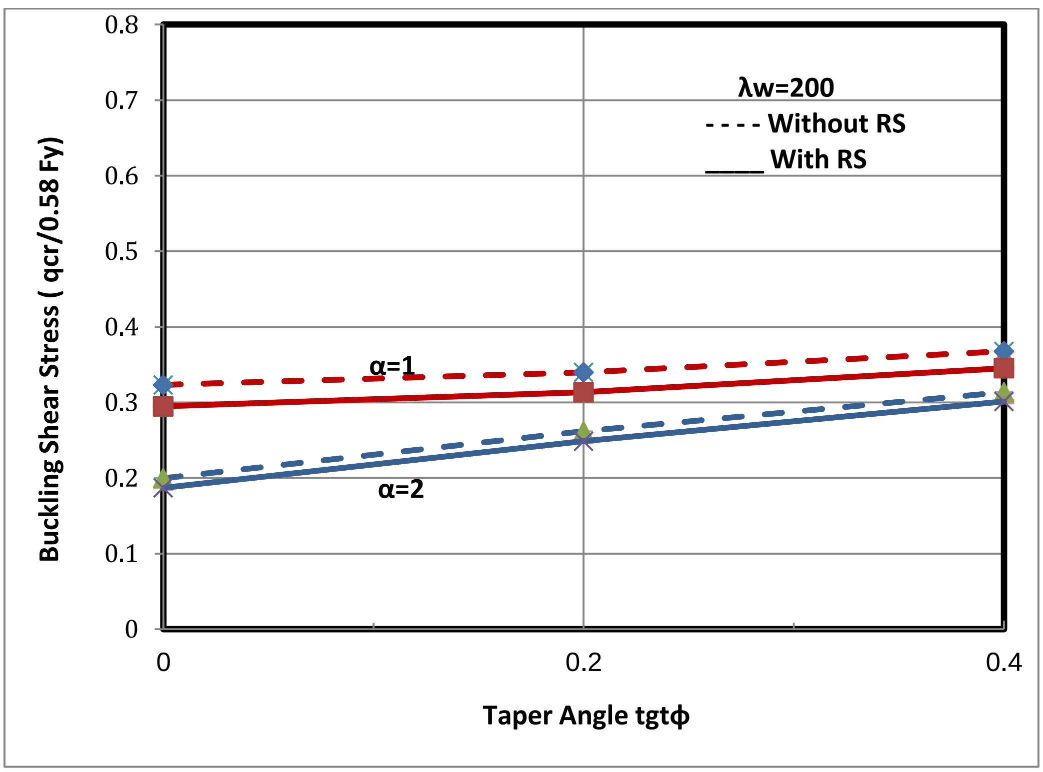
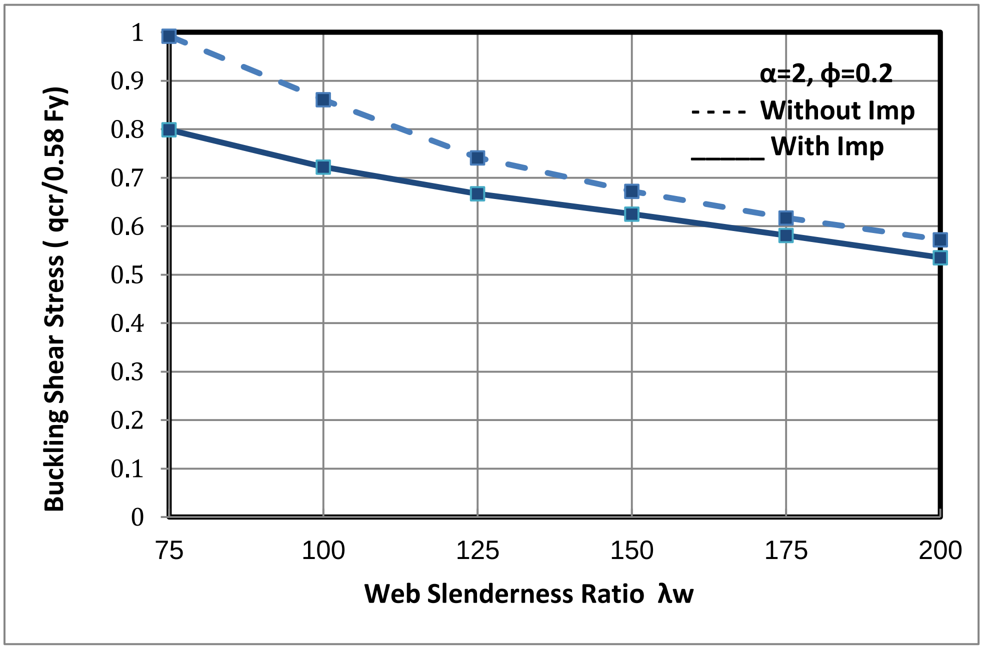


Figure 5: Reduction in Shear Buckling Stress due to Residual Stresses for different α and ϕ values

5.2 Ultimate Shear Strength

Fig. 6 and 7 show the variation of the ultimate shear stress ($q_{cr}/0.58*F_y$) against the web slenderness ratio λ_w for different values of the α=2 and the tapering angle φ =0.2 and 0.4, with and without geometric imperfection and residual stresses.

Based on the results of the studied girder models, the reduction in ultimate shear strength for girder with compact and non-compact web ranges between 15 and 24 %. For girders with slender webs, the reduction ranges between 3 and 15 %..



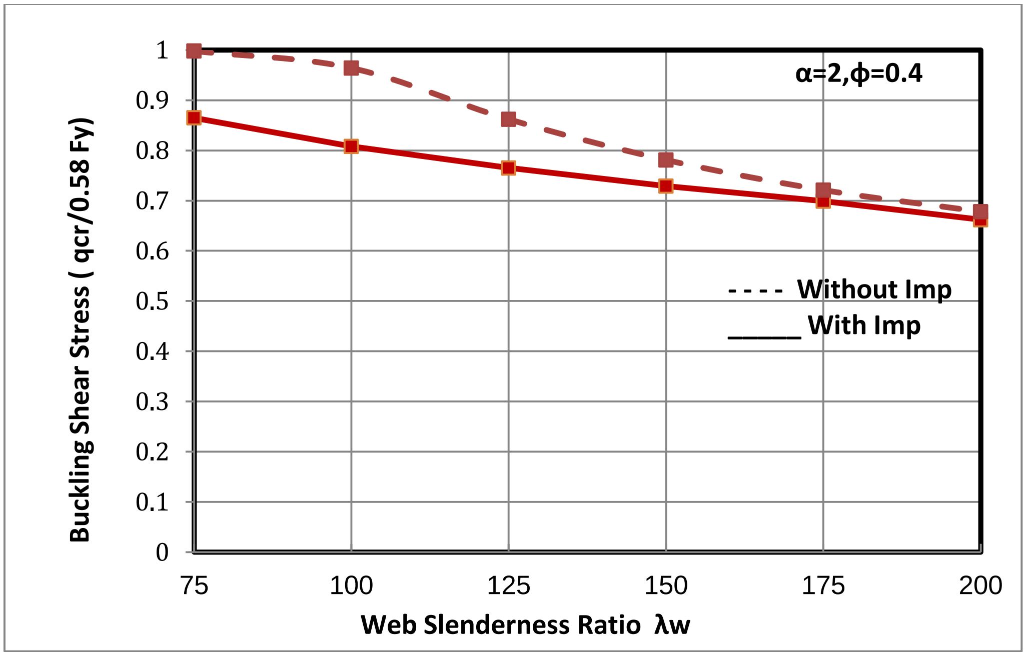
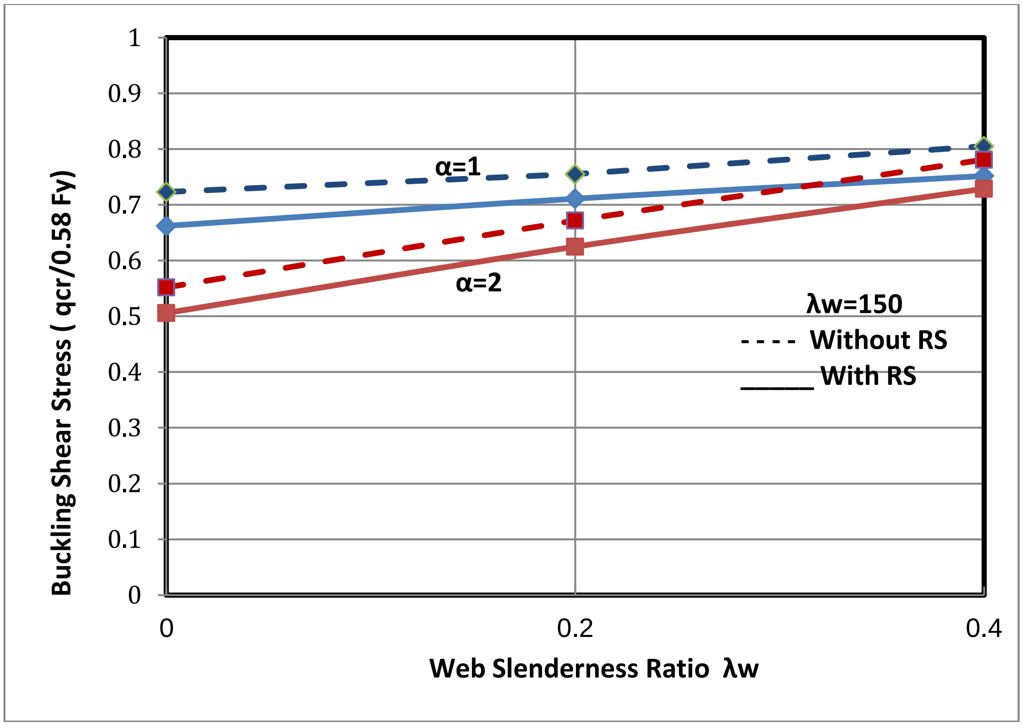


Figure 6: Reduction in Ultimate Shear Stress due to Imperfections (α=2, tgt ϕ=0.2,0.4)



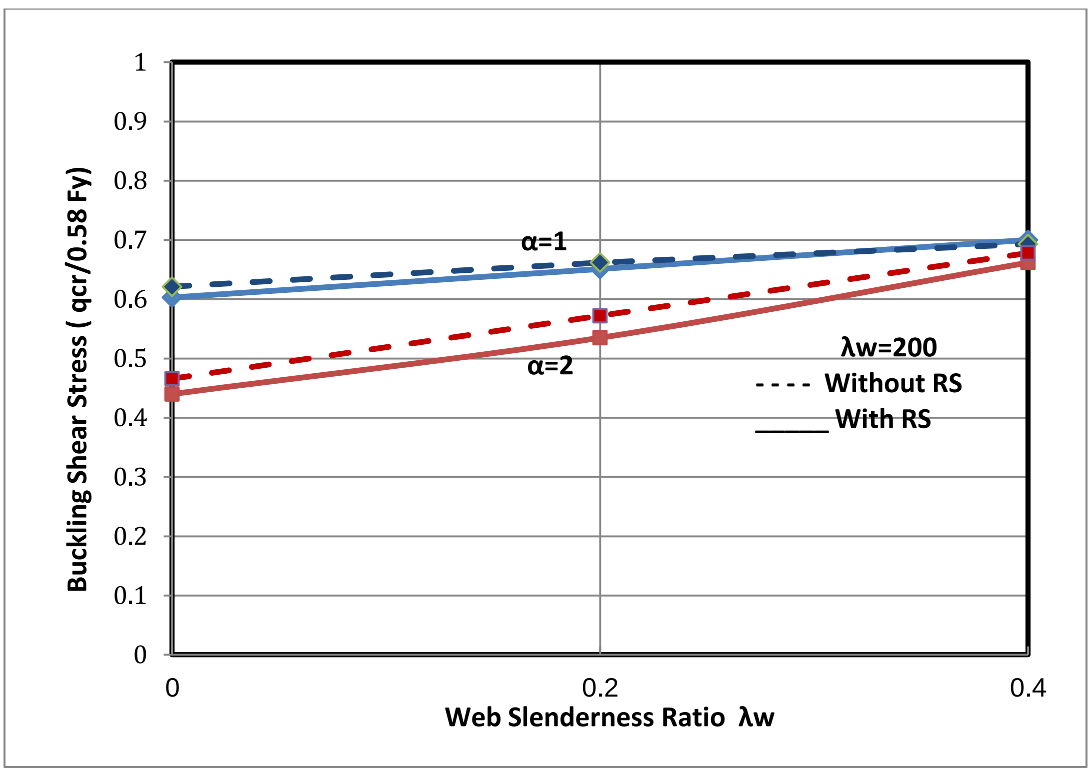


Figure 7: Reduction in Ultimate Shear Stress due to Imperfections for different α and ϕ values

6. Conclusions

This paper presents the results of finite element analysis studies of the effect of geometrical and structural imperfections on the elastic buckling strength and the ultimate shear strength of web tapered plate girders. Standard geometric imperfection and residual stress models are applied to a tapered girder finite element model subjected to uniform shear stress without bending. The model is validated using available test results on web tapered girders. A parametric study is performed to investigate the reduction due to imperfections in both elastic buckling strength and the ultimate shear strengths under different values of the major design parameters such as web slenderness ratio, tapering angle, tapered panel aspect ratio.

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