Proceedings of the
Annual Stability Conference
Structural Stability Research Council
Toronto, Canada, March 25-28, 2014

Effective length *K*-factors for flexural buckling strengths of web members in open web steel joists

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Abstract
The Steel Joist Institute publishes the governing specifications in the U.S. for the design of open web steel joists. For compression chord and web members, this specification employs an effective length *K*-factor approach. In many cases, these *K*-factors have been conservatively assumed equal to 1.0 for compression web members, regardless of the fact that intuition and limited experimental work suggest that smaller values could be justified. Given that such reductions could result in more economical designs without a loss in safety, this paper provides an overview of recently completed research that investigate three different methods for computing the in-plane and out-of-plane buckling behavior of compression web members. These methods include (1) a hand calculation procedure based on the use of the alignment charts, (2) computational critical load (eigenvalue) analyses based on uniformly distributed gravity loading, and (3) computational analyses using a self-equilibrating induced compression approach. The latter method is novel and allows for studying the buckling behavior of a specific member within a structural system without regard to the applied loading condition. Four different joist configurations are investigated, including an 18K3, 28K10, and two variations of a 32LH06. Based on these methods and the very limited number of joists studied, it appears promising that in-plane and out-of-plane *K*-factors of 0.75 and 0.85, respectively, could be used in computing the flexural buckling strength of web members in routine steel joist design. Several recommendations for future work, which include systematically investigating a wider range of joist configurations and connection restraint, are also provided.

1. Introduction

The Steel Joist Institute (SJI) provides separate standard specifications for the design of K-Series and LH/DLH-Series joists. These truss-like beams differ in depth and corresponding length and are intended to support vertical loads, such as gravity and/or uplift due to wind effects. The depth of K-Series joists range from 10 to 30 inches (25.4 to 76.2 cm), with spans up through 60 feet (18.3 m). LH-Series joists employ depths of 18 to 48 inches (45.7 to 121.9 cm) to provide spans up through 96 feet (29.3 m), whereas DLH-Series joists provide for spans up through 240 feet (73.1 m) by using much larger depths of 52 to 120 inches (132.1 to 304.8 cm).

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The top and bottom chords of these open web steel joists are typically constructed from back-to-back double angles that are separated by spacers and/or the ends of the web members, which are usually round bars, crimped single angles, or double angles. In general, applied vertical loading is resisted at the end supports of the joists via system bending action, which results in a non-uniform distribution of axial force in the continuous top and bottom chords. Equilibrium of the corresponding shear force distribution is resolved by axial force in the web members.

The purpose of this paper is to report on a research project (Lee, 2013) that is intended to contribute to a better understanding of the stability behavior of compression web members, with a specific focus on their in-plane and out-of-plane flexural buckling capacity. Given that the SJI employs an effective length method to define the axial compressive strength of these members, a key part of this study was to develop and employ several methods for computing effective length *K*-factors for a sample of typical joist configurations. Two of these methods are based on finite element analyses and a third method is intended for hand calculation. Given that the results of this study based on only a limited number of joists, initial and conservative recommendations for such *K*-factors are also suggested.

2. Reasonable Bounds on *K*-factors

Table 1 provides SJI current requirements on *K*-factors for use in designing web members in K- and LH/DLH-Series joists.

Table 1: SJI *K*-factors for web members.

Joist	In-plane	Out-of-plane
K-Series	1.00	1.00
LH/DLH-Series	0.75	1.00

For flexural buckling within the plane of the joist (Fig. 1), the *K*-factor could theoretically range between 0.5 and 1.0, depending on the level of end-restraint provided to the web member. This end restraint is not only a function of the rotational resistance provided by the flexural stiffness of the top and bottom chords, but is also directly related to the connection stiffness at the web-to-chord location. Given that the top and bottom chords typically have a much greater flexural stiffness than that of the web member, values much less than K=1.0 would be expected. Such values, however, would probably not always approach K=0.5 because of the significant stiffness and strength demands on the connections. In this regard, SJI's in-plane value of K=0.75 for LH/DLH-Series joists seems feasible, and the corresponding requirement of K=1.0 for K-Series would seem conservative.

In regard to flexural buckling out of the plane of the joist (Fig. 2), the appropriate *K*-factor is a function of the out-of-plane bracing and the torsional stiffness of the top and bottom chords, and to a much smaller degree the out-of-plane flexural stiffness of the tension web members. And similar to the in-plane assessment, connection stiffness also plays a significant role. As for suggesting a reasonable bound on the out-of-plane *K*-factor, it is important to note that when the joist is subject to gravity loading, the bottom chord and the web members closest to the support are in tension and the joist behaves as an underslung truss. If the flexural stiffness of these members is neglected (that is they are treated as cables), buckling of the compression web member results in a restraining force that always aligns with a chord defined by its ends; regardless of the amount of out-of-plane movement at the bottom chord and the degree of rotational restraint provided by the top chord (Fig. 2). This implies a maximum out-of-plane value of K=1.0, with the potential for smaller values depending on the out-of-plane flexural stiffness of the bottom (tension) chord and connection. By this reasoning, SJI may be conservative in their use of K=1.0, and the degree of this conservatism will be assessed by studies presented in this paper.

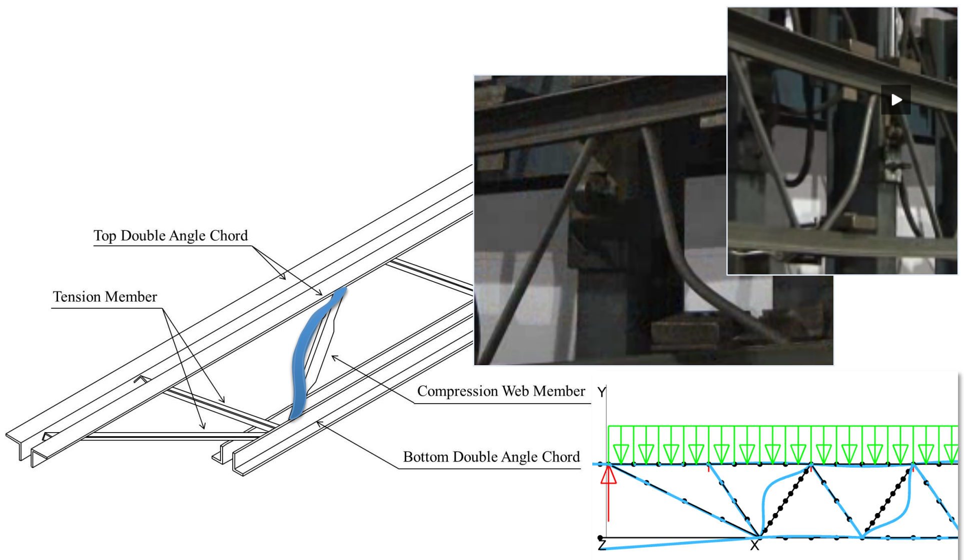


Figure 1: In-plane buckling of compression web member.

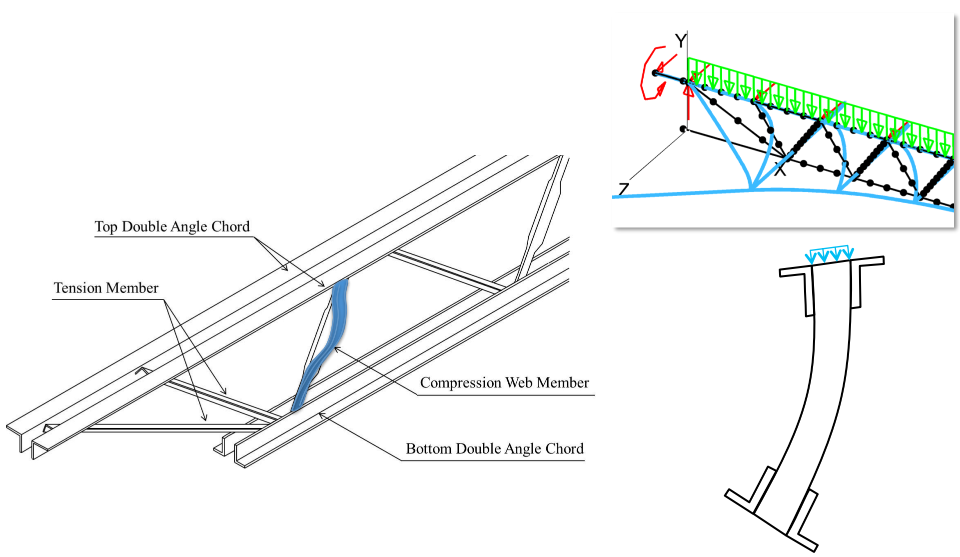


Figure 2: Out-of-plane buckling of compression web member.

3. Methods for Determining *K*-factors

As indicated earlier, three methods were developed for determining in-plane and out-of-plane K-factors for the compression web members. Two of these methods employ elastic bifurcation (critical load) analyses available in most finite element analysis software (e.g. MASTAN2, 2013). Although both of these computational methods rely on eigenvalue analysis, they are very different in their approach as will be explained below. The third approach is a hand calculation method that could be used as an alternative to or as a means for checking the results of the computational methods.

The same basic assumptions are made for all of the methods, and include

1. centroid to centroid dimensions are employed
2. torsional stiffness due warping resistance is neglected
3. material is assumed elastic
4. all web members are attached with fully restrained (rigid) connections
5. top and bottom chords are continuous
6. top chords are braced at panel points, and bottom chords are braced per SJI requirements
7. initial in-plane and out-of-plane imperfections are not modeled

3.1 Hand Calculation Method

This method assumes the compression web member is similar to an isolated column in a building in which sidesway is inhibited. With buckling controlled by the ratio of the web member's (column's) flexural stiffness to the resisting bending stiffness provided by the chords and neighboring tension members (girders), the alignment chart method presented in the Commentary to AISC *Specification for Structural Steel Buildings* (2010) can be used as a means for approximating a *K*-factor that may be appropriate for use in steel joist design. The computation of the relative stiffness *G* ratios for each end of the column, which are at the heart of the alignment chart method, may be approximated in analyzing the web member of joist as follows

$$G_{\text{end A}} = \frac{\sum_{i=1}^{m}\left(\frac{E_i I_i}{L_i}\right)_{\text{columns at end A}}}{\sum_{j=1}^{n}\left(\frac{E_j I_j}{L_j}\right)_{\text{girders at end A}}} \approx \frac{\left(\frac{EI}{L}\right)_{\text{compression web member}}}{\frac{1}{2}k_{\text{end A}}} \tag{1}$$

in which E, I, and L are the elastic modulus, moment of inertia, and length of the compression web member, and k is the resisting rotational stiffness provided by the chords and tension members located at the associated end (e.g. end A) of the compression web member. Selection of the moment of inertia I for the compression web member would be function of whether the in-plane or out-of-plane buckling is being assessed.

For in-plane buckling of the web member, the resisting rotational stiffness k at each end of the member would be taken as

$$k_{\text{end A}} = \sum_{j=1}^{n}\left(\frac{4E_j I_j}{L_j}\right)_{\text{end A}} \tag{2}$$

in which E, I, and L are the elastic modulus, in-plane moment of inertia, and length of the members neighboring the end of the compression member, and k would include chord segments to the left and right of the connection and any connected tension web members.

The resisting rotational stiffness k for out-plane buckling may be derived as (Lee, 2013)

$$k_{\text{end A}} = \alpha - \frac{\beta}{\gamma} \quad (3)$$

where

$$\alpha = \sum_{j=1}^{n} \left[\frac{4E_j I_j \cos^2 \phi_j}{L_j} + \frac{G_j J_j \sin^2 \phi_j}{L_j} \right] \quad (3a)$$

$$\beta = \sum_{j=1}^{n} \left[\frac{4E_j I_{yj} \cos \phi_j \sin \phi_j}{L_j} - \frac{G_j J_j \cos \phi_j \sin \phi_j}{L_j} \right]^2 \quad (3b)$$

$$\gamma = \sum_{j=1}^{n} \left[\frac{4E_j I_{yi} \sin^2 \phi_j}{L_j} + \frac{G_j J_j \cos^2 \phi_j}{L_j} \right] \quad (3c)$$

in which E, G, I_y, J, and L are the elastic modulus, shear modulus, out-of-plane moment of inertia, St. Venant torsional constant, and length of the neighboring members at the end of the compression web member, and α, β, and γ would include chord segments to the left and right of the connection and any connected tension web members. The member inclination angles ϕ_j are measured counter-clockwise from the chord of the compression web member of interest (Fig. 3).

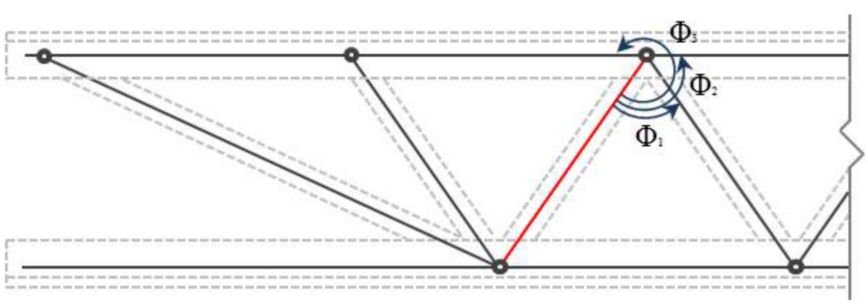


Figure 3: Member inclination angles ϕ_j

3.2 FEA with Uniformly Distributed Gravity Load

A computational model of the joist is first prepared, which includes modeling all chord and web segments between panel points with at least four finite elements and subjecting the joist to a uniformly distributed gravity load. An elastic eigenvalue analysis is then employed to determine the critical (bifurcation) axial force P_{cr} in the compression web member of interest. This axial load is then used to back-calculate an effective length K-factor from (Ziemian, 2010)

$$K = \frac{\pi}{L}\sqrt{\frac{EI}{P_{cr}}} \tag{4}$$

where E, I, and L are the elastic modulus, the moment of inertia resisting buckling, and the length of the compression web member.

One of the difficulties with this method is that the controlling bifurcation mode may not correspond to buckling of the web member of interest; in many cases, out-of-plane buckling of the top chord between panel points controlled, thereby invalidating the use of Eq. 4 for computing a K-factor for a specific web member. To resolve this problem, an iterative process was employed in which the specific web member's moment of inertia was artificially reduced such that buckling of this member defined the primary bifurcation mode of the joist. In most cases, these reductions were approximately 70% of the cross section's original (as-built) value. Given that out-of-plane buckling of the web member often controlled, the in-plane web buckling modes were determined by simply performing a two-dimensional analysis (i.e. constraining all out-of-plane degrees of freedom).

To determine if the non-buckling compression web members were providing restraining stiffness that may not be fully there (given that they may be close to buckling themselves), simultaneous buckling of all compression web members was also investigated. This variation is obviously quite time consuming because it requires finding the combination of maximum moments of inertia for the web members that would permit the joist's primary bifurcation mode to be controlled by concurrent buckling of all compression web members. The results from this exercise should provide an upper bound on the K-factors back-calculated from Eq. 4.

3.3 FEA with Self-Equilibrating Induced Compression (SEIC)

To alleviate the need for artificially reducing moments of inertia and only being able to consider uniformly distributed gravity loading, the authors developed an alternative method that also uses the results of an eigenvalue analysis to compute K-factors. Using the original finite element model employed in the previous method (section properties unaltered), gravity loading is replaced by a loading scheme in which the axial force in the web member of interest is increased until that specific member buckles. To prevent the member's neighboring chords and web members from having their stiffness modified when resisting this induced compressive force (or perhaps loaded to the point of buckling themselves), an artificial rigid truss element is added to the finite element model. By having the ends of this truss element share the nodes located at the ends of the compression web member of interest, this pinned-ended element equilibrates the induced compressive force without providing any resistance to buckling; in other words, only the neighboring chords and web members provide rotational restraint at the ends of the web member. By further defining the induced compressive load as a unit force, an eigenvalue analysis can then determine the scale factor or axial load P_{cr} that corresponds to buckling of the web member. With this load, Eq. 4 is then used to determine the corresponding effective length K-factor.

In general, most finite element analysis software does not include an option for defining induced member forces. As an alternative, the more readily available option of defining temperature loads (thermal straining) can be employed. To make this work, the temperature in only the web member of interest is increased, which results in a compressive force in the web member that is equilibrated by a tension force in the artificial truss element. This procedure is illustrated in Fig. 4, which correctly shows the non-sway buckling of a column restrained by upper and lower beams. The application of the SEIC method to study joist behavior is provided in Fig. 5.

Given that out-of-plane buckling of the web member will most likely control, a two-dimensional analysis is again used to determine the in-plane buckling loads and corresponding *K*-factors. And similar to the above study employing gravity loading, individual and simultaneous buckling of the compression web members is also investigated.

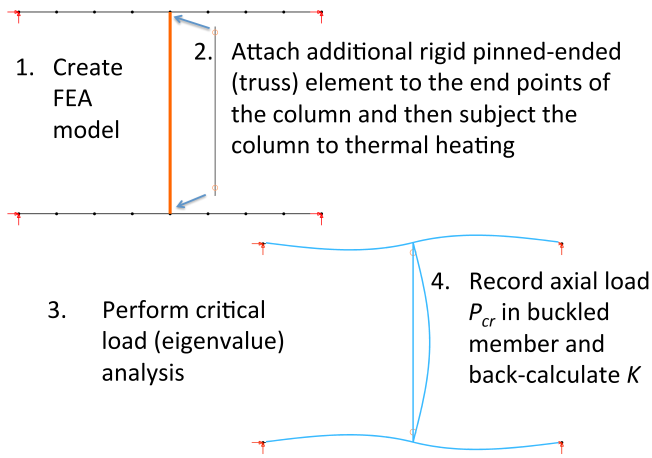


Figure 4: Beam and column assemblage used to describe SEIC method

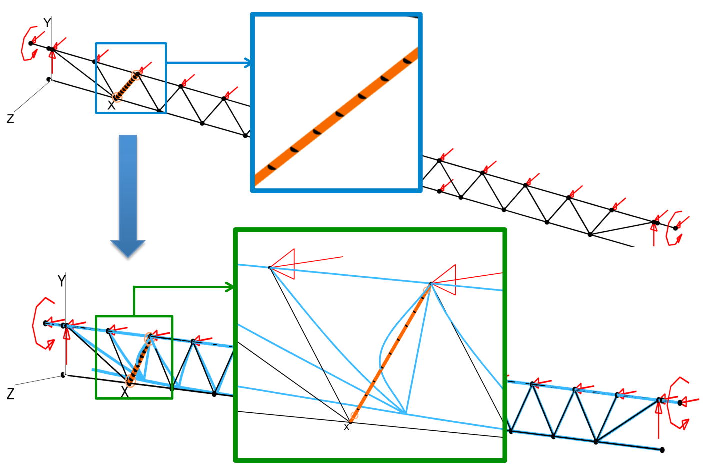


Figure 5: SEIC method applied to joist configuration (out-of-plane buckling).

4. Joist Configurations Studied

To date, only a limited number of joists have been investigated. Two K-Series joists (18K3 and 28K10) and two variations of an LH-Series joist (32LH06) were selected for study given that they encompass the range of geometry and member sections found in these joist types. As an example, the specific properties and representative finite element model for the 18K3 are provided in Fig. 6. Details for the remaining joists are provided in Appendix 1. Given that the SJI only publishes load bearing and deflection requirements for joist types, specific geometries and member sizes were taken from a comprehensive set of joists that were manufactured by a former steel joist company and experimentally tested for other purposes by Emerson (2001) and Schwarz (2002). As indicated earlier, the top chords of the joists are braced out-of-plane at all panel points and any longitudinal rotational stiffness that may be offered by the deck is neglected. Requirements for bottom chord bracing, which is specific to the joist type, are provided by SJI and brace locations are shown in the accompanying figures. In all cases, the braces are assumed rigid.

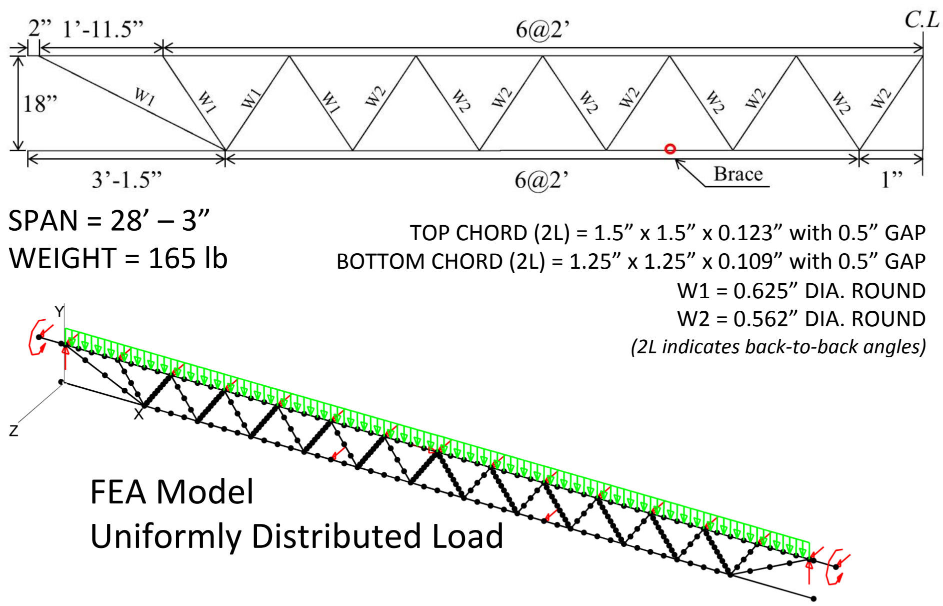
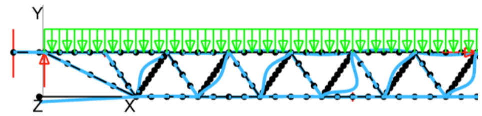


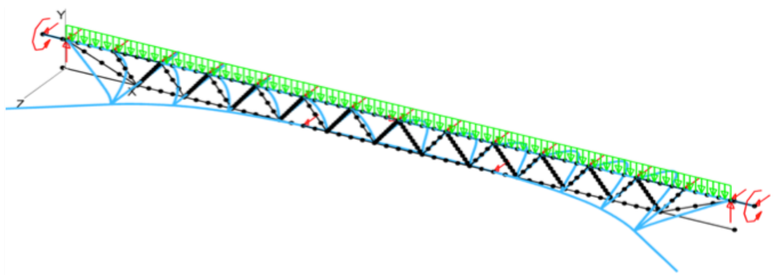
Figure 6: Geometric properties and representative finite element model (18K3)

5. Results

By employing the three methods discussed above, in-plane and out-of-plane effective length *K*-factors were determined for all compression web members within the four joist configurations studied. Results were obtained for cases of each compression web member buckling individually (Fig. 5) and for all web members buckling simultaneously (Fig. 7). An example of *K*-factors determined for the 18K3 joist is provided in Tables 2a and 2b. Tabulated results for the remaining joists investigated are provided in Appendix 1. The webs are numbered according to their location from the joist's end support (e.g. 1st compression web member is located nearest the support, and the 6th web member is at mid-span). Maximum and minimum *K*-factors are highlighted in red and green accordingly. The tables also include the as-built (original) and the artificially reduced in-plane and out-of-plane moments of inertia for each of the compression web members.



a. In-plane



b. Out-of-plane

Figure 7: Simultaneous buckling of all web members (18K3)

Table 2a: In-plane effective length K-factors (18K3)

Case	Method	Compression Web Number (joist support to mid-span) 1st	2nd	3rd	4th	5th	6th
Individual buckling of web members	Original I_x (10^{-3} in^4)	7.49	7.49	4.90	4.90	4.90	4.90
	Hand Calculation	0.52	0.51	0.51	0.51	0.51	0.51
	FEA – SEIC	0.52	0.51	0.51	0.51	0.51	0.51
	Reduced I_x (10^{-3} in^4)	6.75	5.10	3.97	2.74	1.71	0.56
	FEA – Dist. Load	0.51	0.51	0.51	0.51	0.51	0.50
Simultaneous buckling of web members	Reduced I_x (10^{-3} in^4)	6.23	5.10	3.97	2.78	1.70	0.56
	Hand Calculation	0.51	0.51	0.51	0.51	0.51	0.51
	FEA – Dist. Load	0.52	0.51	0.52	0.51	0.52	0.52
	FEA – SEIC	0.51	0.51	0.51	0.50	0.50	0.50

Table 2b: Out-of-plane effective length K-factors (18K3)

Case	Method	Compression Web Number (joist support to mid-span) 1st	2nd	3rd	4th	5th	6th
Individual buckling of web members	Original I_y (10^{-3} in^4)	7.49	7.49	4.90	4.90	4.90	4.90
	Hand Calculation	0.66	0.69	0.66	0.66	0.66	0.66
	FEA – SEIC	0.69	0.69	0.66	0.65	0.65	0.65
	Reduced I_y (10^{-3} in^4)	6.23	4.65	3.53	2.35	1.32	0.39
	FEA – Dist. Load	0.63	0.61	0.60	0.58	0.56	0.53
Simultaneous buckling of web members	Reduced I_y (10^{-3} in^4)	6.67	4.97	3.60	2.35	1.25	0.47
	Hand Calculation	0.69	0.64	0.63	0.59	0.55	0.52
	FEA – Dist. Load	0.67	0.65	0.62	0.60	0.56	0.60
	FEA – SEIC	0.67	0.64	0.62	0.59	0.55	0.53

All methods indicate that the in-plane K-factors are all very close to the fixed-fixed boundary case of K=0.5. As illustrated by the relative stiffness G ratios obtained for the Hand Calculation method (Table 3), this can be attributed to the web members having substantially smaller flexural stiffness than that of the chords.

Table 3: Details for in-plane Hand Calculation method (18K3)

Compression Web Number (from joist support to mid-span)	Member End	k (kip-in/rad)	G	K
1st	Top	1523.3	0.011	0.52
	Bottom	440.65	0.039	
2nd	Top	1509.0	0.009	0.51
	Bottom	795.87	0.018	
3rd	Top	1509.0	0.007	0.51
	Bottom	781.54	0.014	
4th	Top	1509.0	0.005	0.51
	Bottom	781.54	0.010	
5th	Top	1509.0	0.005	0.51
	Bottom	781.54	0.010	
6th	Top	1509.0	0.003	0.51
	Bottom	781.54	0.006	

A summary of the results for all methods and joist configurations investigated is provided in Tables 4a (individual buckling) and 4b (simultaneously buckling). Discussion of these results and those presented below will be provided in the next section of this paper.

As indicated earlier, bottom chord bracing was provided at two or three locations per SJI requirements. Given that such bracing may have an impact on the out-of-plane *K*-factors computed by the two methods that are based on FEA eigenvalue analyses, the SEIC method study was fully repeated for the case of individual buckling of the compression web members. A summary of these results is provided in Table 5.

Table 4a: Summary of *K*-factors for individual buckling of all web members.

Joist	Method	in-plane		out-of-plane	
		min.	max.	min.	max.
18K3	Hand Calculation	0.51	0.52	0.66	0.69
	FEA – Dist. Load	0.50	0.51	0.53	0.63
	FEA – SEIC	0.51	0.52	0.65	0.69
28K10	Hand Calculation	0.51	0.55	0.66	0.80
	FEA – Dist. Load	0.51	0.53	0.55	0.72
	FEA – SEIC	0.51	0.55	0.72	0.83
32LH06_L1	Hand Calculation	0.51	0.54	0.66	0.73
	FEA – Dist. Load	0.50	0.52	0.61	0.70
	FEA – SEIC	0.51	0.54	0.69	0.81
32LH06_L2	Hand Calculation	0.51	0.54	0.66	0.78
	FEA – Dist. Load	0.50	0.51	0.61	0.71
	FEA – SEIC	0.51	0.55	0.69	0.87
All Joists	All Methods	0.50	0.55	0.53	0.87

Table 4b: Summary of K-factors for simultaneous buckling of all web members.

Joist	Method	in-plane min.	in-plane max.	out-of-plane min.	out-of-plane max.
18K3	Hand Calculation	0.51	0.51	0.52	0.69
	FEA – Dist. Load	0.51	0.52	0.56	0.67
	FEA – SEIC	0.50	0.51	0.53	0.67
28K10	Hand Calculation	0.50	0.53	0.53	0.73
	FEA – Dist. Load	0.51	0.53	0.55	0.77
	FEA – SEIC	0.50	0.53	0.54	0.79
32LH06_L1	Hand Calculation	0.50	0.53	0.52	0.72
	FEA – Dist. Load	0.50	0.52	0.63	0.74
	FEA – SEIC	0.50	0.53	0.55	0.80
32LH06_L2	Hand Calculation	0.50	0.51	0.53	0.71
	FEA – Dist. Load	0.50	0.51	0.61	0.72
	FEA – SEIC	0.50	0.51	0.55	0.78
All Joists	All Methods	0.50	0.53	0.52	0.80

Table 5: Comparison of K-factors with different bottom chord bracing conditions (SEIC, individual).

Joist	Bottom Chord Bracing	Compression Web Number (from joist support to mid-span) 1st	2nd	3rd	4th	5th
18K3	2 locs. @ ⅓ L pts.	0.69	0.69	0.66	0.65	0.65
	All panel pts.	0.66	0.67	0.64	0.64	0.64
	% difference	4.35	2.90	3.03	1.54	1.54
28K10	3 locs. @ ¼ L pts.	0.83	0.78	0.75	0.72	0.72
	All panel pts.	0.80	0.76	0.75	0.72	0.72
	% difference	3.61	2.56	0.00	0.00	0.00
32LH06_L1	3 locs. @ ¼ L pts.	0.81	0.77	0.74	0.74	0.69
	All panel pts.	0.75	0.76	0.74	0.74	0.69
	% difference	7.41	1.30	0.00	0.00	0.00
32LH06_L2	3 locs. @ ¼ L pts.	0.87	0.76	0.73	0.73	0.69
	All panel pts.	0.82	0.75	0.74	0.73	0.68
	% difference	5.75	1.32	1.37	0.00	1.45

6. Summary of Results

Three independent methods of analysis were employed to provide insight for defining in-plane and out-of-plane effective length K-factors for use in computing the flexural buckling strength of compression web members in open web steel joists. These approaches include a hand calculation method that is based on the alignment charts, and two methods that employ bifurcation (eigenvalue) analysis that are based on the finite element method. The latter methods differ in the loading applied, with one using uniformly distributed gravity loading and the other using a self-equilibrating axial loading scheme. All three methods provided similar results for the limited number of joist configurations investigated. In all cases, the computed in-plane and out-of-plane effective length K-factors are less than 1.0, although it should be emphasized that only joists with some degree of out-of-plane bottom chord bracing were studied.

For buckling of web members in the plane of the joist, the computed effective length factors ranged from K=0.50 to K=0.55. This indicates a significant amount of rotational restraint exists at the ends of the compression web members, which can be shown to be the byproduct of assuming fully restrained (rigid) connections and having top and bottom chords with in-plane flexural stiffnesses that are much larger than that of the web members. The overall performance of K-Series joists was similar to that of LH-Series joists, which indicates that the SJI may be justified in allowing *K*-factors for both types of joists to be less than unity (see Table 1). Unless the connections are specifically designed for the demands associated with using *K*-factors close to the rigid-rigid case of K=0.5, the SJI's value of K=0.75 for LH-Series joists seems reasonable.

A much wider range of out-of-plane effective length factors (K=0.52 to K=0.87) is observed, with most values between K=0.5 and K=0.75. Given that web buckling out of the plane of the joist is primarily resisted by the relatively smaller torsional stiffness of the top and bottom chords, *K*-factors larger than the in-plane values would be expected. Because the ends of the web members are almost always located (pinched) between the angles comprising the top and bottom chords, the SJI's assumption of K=1.0 (see Table 1) for out-of-plane buckling appears to be conservative, and similar studies of additional joists may indicate that a value of K=0.85 or K=0.90 could be used in routine design.

Several additional observations can be made from the results of this study. In general, the in-plane and out-of-plane *K*-factors obtained do not appear to be impacted by whether the compression web members are buckling individually or all simultaneously. Providing additional bottom chord bracing has only a small influence of the computed out-of-pane *K*-factors. And, in-plane and out-of-plane *K*-factors less than unity always prevailed with the largest values typically computed for the compression web member closest to the joist end supports (often referred to as the first compression web member).

7. Future Work

A key part of this study was the development of three independent methods of computing effective length *K*-factors for computing the flexural buckling strength of web members. Unfortunately, only a limited number of joist configurations could be studied. Although the authors are confident that the results of this study provide significant insight for suggesting *K*-factors for routine design, a wider range of joist configurations should be investigated before any all-purpose conclusions are made. The following list provides suggestions for what a more comprehensive study should include:

a. Only two conditions of bottom chord bracing are considered in the above study, including SJI's bracing requirements and all panel points braced. In many cases, the SJI permits no bottom chord bracing, and by no means has the authors proven that out-of-plane *K*-factors less than unity would prevail. Hence, such a condition should be studied.
b. Web to chord connections most likely permit some degree of relative rotation between these members, and this resulting loss in stiffness will increase the *K*-factors obtained in this study. With this in mind, it is recommended that the web-to-chord connection stiffness be systematically varied from pinned to fully restrained, and thereby provide *K*-factors for several degrees of partially restrained connection stiffness.
c. Before a steel joist achieves a strength limit state, yielding in the chords or compression web members may alter the relative stiffness between these members, and thereby may increase or decrease the computed *K*-factors. Given that the study reported in this paper only considered elastic behavior, performing inelastic eigenvalue analyses or employing

a stiffness reduction τ-factor with the use of the alignment charts to compute *K*-factors could provide valuable additional insight.

d. This study focused on determining *K*-factors for computing flexural buckling strengths. In many cases, the web members are single angles and flexural-torsional buckling may need to be considered.

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Appendix 1

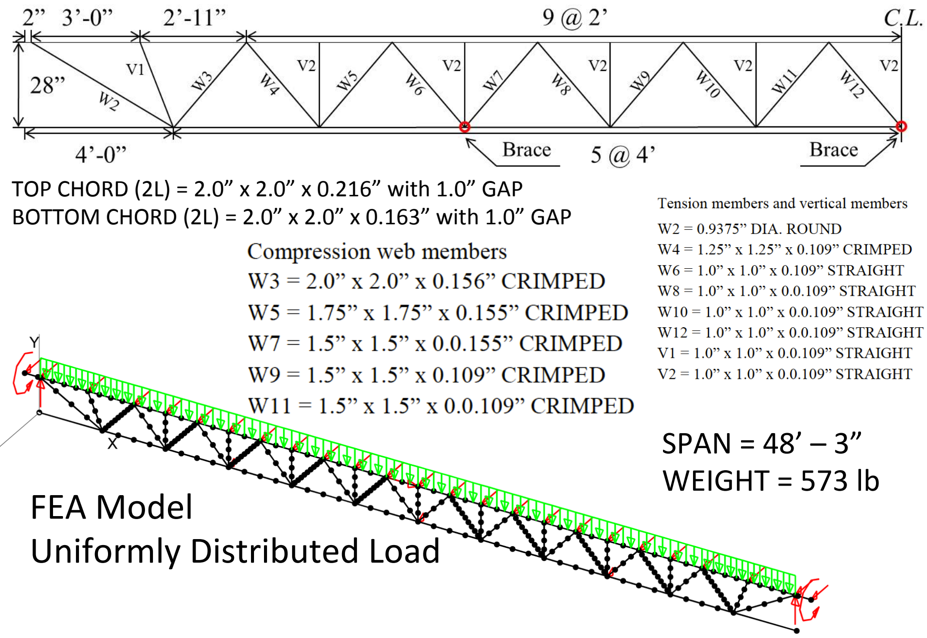


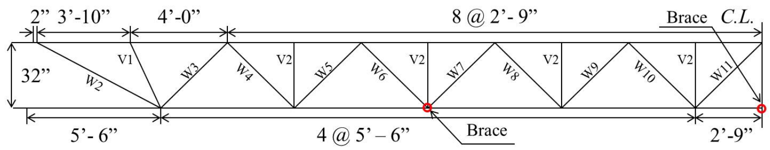
Figure A1: Geometric properties and representative finite element model (28K10)

Table A1a: In-plane effective length *K*-factors (28K10)

Case	Method	Compression Web Number (from joist support to mid-span) 1st	2nd	3rd	4th	5th
Individual buckling of web members	Original I_x (10^{-2} in^4)	12.94	8.17	4.77	3.56	3.56
	Hand Calculation	0.55	0.52	0.51	0.51	0.51
	FEA – SEIC	0.55	0.52	0.51	0.51	0.51
	Reduced I_x (10^{-2} in^4)	8.41	6.37	4.53	2.85	1.18
	FEA – Dist. Load	0.53	0.52	0.51	0.51	0.51
Simultaneous buckling of web members	Reduced I_x (10^{-2} in^4)	7.95	6.05	4.53	2.88	1.18
	Hand Calculation	0.53	0.52	0.51	0.51	0.50
	FEA – Dist. Load	0.53	0.52	0.52	0.52	0.51
	FEA – SEIC	0.53	0.52	0.51	0.51	0.50

Table A1b: Out-of-plane effective length *K*-factors (28K10)

Case	Method	Compression Web Number (from joist support to mid-span) 1st	2nd	3rd	4th	5th
Individual buckling of web members	Original I_y (10^{-2} in^4)	10.03	8.16	6.34	4.97	4.97
	Hand Calculation	0.80	0.74	0.68	0.66	0.66
	FEA – SEIC	0.83	0.78	0.75	0.72	0.72
	Reduced I_y (10^{-2} in^4)	7.03	4.60	3.15	1.60	0.63
	FEA – Dist. Load	0.72	0.66	0.64	0.59	0.55
Simultaneous buckling of web members	Reduced I_y (10^{-2} in^4)	7.20	4.20	3.00	1.60	0.58
	Hand Calculation	0.73	0.66	0.64	0.58	0.53
	FEA – Dist. Load	0.77	0.66	0.66	0.60	0.55
	FEA – SEIC	0.79	0.70	0.67	0.60	0.54



TOP CHORD (2L) = 2.5” x 2.5” x 0.212” with 1.0” GAP
BOTTOM CHORD (2L) = 2.0” x 2.0” x 0.216” with 1.0” GAP

Compression Web Members

W3 = 2.0” x 2.0” x 0.187” CRIMPED
W5 = 1.75” x 1.75” x 0.155” CRIMPED
W7 = 1.5” x 1.5” x 0.0.155” CRIMPED
W9 = 1.5” x 1.5” x 0.155” CRIMPED
W11 = 1.5” x 1.5” x 0.0.155” CRIMPED

Tension and vertical members

W2 = 1.5” x 1.5” x 0.109” DOUBLE ANGLE
W4 = 1.25” x 1.25” x 0.109” CRIMPED
W6 = 1.0” x 1.0” x 0.109” STRAIGHT
W8 = 1.0” x 1.0” x 0.0.109” STRAIGHT
W10 = 1.0” x 1.0” x 0.0.109” STRAIGHT
V1 =1.0” x 1.0” x 0.0.109” STRAIGHT
V2 = 1.0” x 1.0” x 0.0.109” STRAIGHT

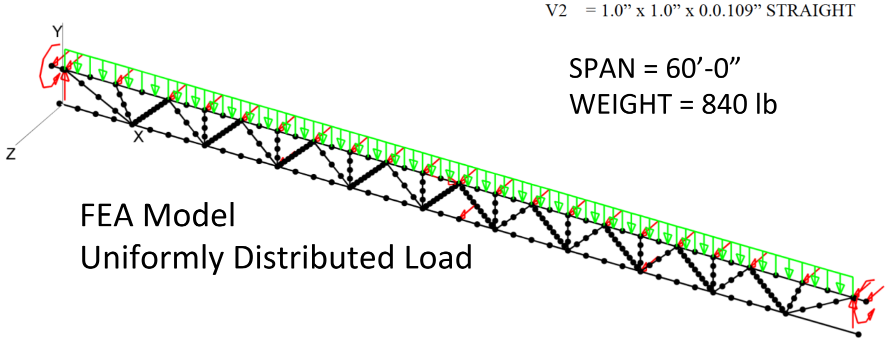


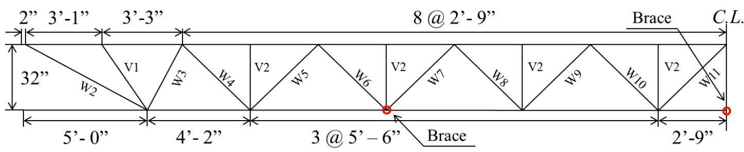
Figure A2: Geometric properties and representative finite element model (32LH06_L1)

Table A2a: In-plane effective length *K*-factors (32LH06_L1)

Case	Method	Compression Web Number (from joist support to mid-span) 1st	2nd	3rd	4th	5th
Individual buckling of web members	Original I_x (10^{-2} in^4)	15.12	7.84	4.54	4.54	4.54
	Hand Calculation	0.54	0.51	0.51	0.51	0.51
	FEA – SEIC	0.54	0.52	0.51	0.51	0.51
	Reduced I_x (10^{-2} in^4)	8.77	6.59	5.32	2.41	0.50
	FEA – Dist. Load	0.52	0.51	0.51	0.50	0.50
Simultaneous buckling of web members	Reduced I_x (10^{-2} in^4)	8.65	6.39	4.36	2.40	0.50
	Hand Calculation	0.53	0.51	0.51	0.50	0.50
	FEA – Dist. Load	0.52	0.51	0.51	0.50	0.50
	FEA – SEIC	0.53	0.51	0.51	0.51	0.50

Table A2b: Out-of-plane effective length *K*-factors (32LH06_L1)

Case	Method	Compression Web Number (from joist support to mid-span) 1st	2nd	3rd	4th	5th
Individual buckling of web members	Original I_y (10^{-2} in^4)	11.07	8.92	6.72	6.72	6.72
	Hand Calculation	0.71	0.73	0.71	0.71	0.66
	FEA – SEIC	0.81	0.77	0.74	0.74	0.69
	Reduced I_y (10^{-2} in^4)	9.96	7.13	4.37	2.08	0.57
	FEA – Dist. Load	0.70	0.70	0.65	0.61	0.70
Simultaneous buckling of web members	Reduced I_y (10^{-2} in^4)	10.52	7.58	4.70	2.25	0.59
	Hand Calculation	0.71	0.72	0.67	0.60	0.52
	FEA – Dist. Load	0.74	0.72	0.68	0.63	0.70
	FEA – SEIC	0.80	0.75	0.70	0.62	0.55



TOP CHORD (2L) = 2.5” x 2.5” x 0.212” with 1.0” GAP
BOTTOM CHORD (2L) = 2.0” x 2.0” x 0.216” with 1.0” GAP

Compression Web Members

W3 = 2.0” x 2.0” x 0.187” CRIMPED
W5 = 1.75” x 1.75” x 0.155” CRIMPED
W7 = 1.5” x 1.5” x 0.0.155” CRIMPED
W9 = 1.5” x 1.5” x 0.155” CRIMPED
W11 = 1.5” x 1.5” x 0.0.155” CRIMPED

Tension and vertical members

W2 = 1.5” x 1.5” x 0.109” DOUBLE ANGLE
W4 = 1.25” x 1.25” x 0.109” CRIMPED
W6 = 1.0” x 1.0” x 0.109” STRAIGHT
W8 = 1.0” x 1.0” x 0.0.109” STRAIGHT
W10 = 1.0” x 1.0” x 0.0.109” STRAIGHT
V1 =1.0” x 1.0” x 0.0.109” STRAIGHT
V2 = 1.0” x 1.0” x 0.0.109” STRAIGHT

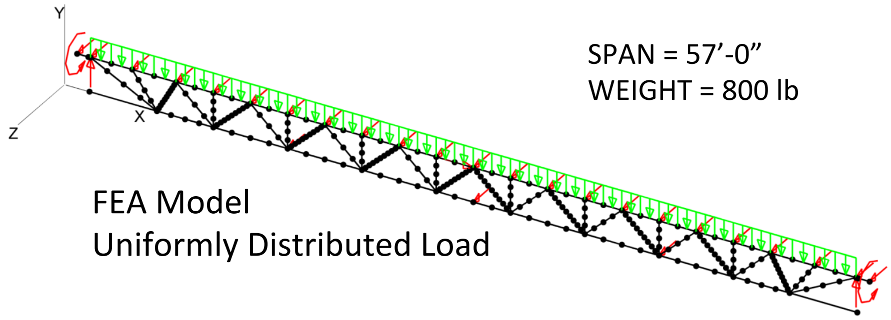


Figure A3: Geometric properties and representative finite element model (32LH06_L2)

Table A3a: In-plane effective length *K*-factors (32LH06_L2)

Case	Method	Compression Web Number (from joist support to mid-span) 1st	2nd	3rd	4th	5th
Individual buckling of web members	Original I_x (10^{-2} in^4)	15.12	7.84	4.54	4.54	4.54
	Hand Calculation	0.54	0.51	0.51	0.51	0.51
	FEA – SEIC	0.55	0.52	0.51	0.51	0.51
	Reduced I_x (10^{-2} in^4)	4.08	6.28	4.36	2.27	0.50
	FEA – Dist. Load	0.51	0.51	0.51	0.50	0.50
Simultaneous buckling of web members	Reduced I_x (10^{-2} in^4)	3.99	6.39	4.36	2.40	0.50
	Hand Calculation	0.51	0.51	0.51	0.50	0.50
	FEA – Dist. Load	0.51	0.51	0.51	0.50	0.50
	FEA – SEIC	0.51	0.51	0.51	0.51	0.50

Table A3b: Out-of-plane effective length *K*-factors (32LH06_L2)

Case	Method	Compression Web Number (from joist support to mid-span) 1st	2nd	3rd	4th	5th
Individual buckling of web members	Original I_y (10^{-2} in^4)	11.07	8.92	6.72	6.72	6.72
	Hand Calculation	0.78	0.73	0.71	0.71	0.66
	FEA – SEIC	0.87	0.76	0.73	0.73	0.69
	Reduced I_y (10^{-2} in^4)	4.87	7.13	5.04	2.22	0.61
	FEA – Dist. Load	0.71	0.69	0.69	0.61	0.70
Simultaneous buckling of web members	Reduced I_y (10^{-2} in^4)	4.87	7.50	5.04	2.15	0.60
	Hand Calculation	0.69	0.71	0.67	0.60	0.53
	FEA – Dist. Load	0.72	0.71	0.70	0.61	0.71
	FEA – SEIC	0.78	0.75	0.70	0.61	0.55