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Stiffness reduction method for the design of steel columns and beam-columns

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Abstract

A stiffness reduction approach is presented in this paper, which utilises Linear Buckling Analysis (LBA) and Geometrically Nonlinear Analysis (GNA) in conjunction with developed stiffness reduction functions for the design of columns and beam-columns in steel frames. The proposed stiffness reduction approach obviates the need to model member imperfections and to make member buckling checks. While LBA with appropriate stiffness reduction provides inelastic buckling loads of columns, GNA with stiffness reduction is performed for the prediction of beam-column failure. In addition to regular members, the accuracy and practicality of the method is illustrated for irregular members. For the latter case, results indicate that the proposed stiffness reduction method provides more accurate strength predictions in comparison to traditional design approaches. The influence of moment gradient on the development of plasticity (i.e. stiffness reduction) is accounted for by incorporating simple moment gradient factors into the stiffness reduction expressions originally derived for members under uniform bending. The accuracy of the proposed stiffness reduction approach is verified against results obtained through non-linear finite element modelling for all of the considered cases.

1. Introduction

The development of plasticity within constituent members of a steel frame, leading to the reduction in the stiffness, may result in a member force distribution that is different than that determined through elastic analysis. The conventional means of taking into account this behaviour are the use of effective length or notional load concepts (EN 1993-1-1, 2005; AISC-360-10, 2010). An alternative approach is the reduction of the stiffness of members in a frame considering corresponding member forces in the analysis. Included in the two most recent versions of the AISC-360 specification, including AISC 360-10 (2010), this approach provides a more direct way of considering the influence of the spread of plasticity in comparison to conventional methods. The approach is based on the method suggested by Surovek-Maleck and White (2005a, 2005b) attempting to capture the development of plasticity under axial load and bending. However, since the stiffness reduction scheme proposed by Surovek-Maleck and White

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(2005a) does not fully take into account factors influencing the development of plasticity, the method still requires column buckling equations for the design of members.

This paper summarises and extends the stiffness reduction approach proposed by Kucukler et al. (2014) for the design of steel members. Stiffness reduction functions are presented for pure axial load, pure bending and combined bending and axial load, which fully consider the deleterious influence of the development of plasticity, residual stresses and geometrical imperfections. For the design of columns, the implementation of linear buckling analysis (LBA) with reduced stiffness is proposed, while the use of Geometrically Nonlinear Analysis (GNA) with stiffness reduction and without modelling out-of-straightness is used for the design of beam-columns, where the failure is signified by reaching the ultimate cross-section resistance at the most heavily loaded cross-section. Unlike the method of Surovek-Maleck and White (2004a), the use of column strength equations is not required in the method proposed in this paper; instead only cross-section checks are necessary. Moreover, the proposed method in this paper accounts for the influence moment gradient on the strength of beam-columns, thereby providing more accurate strength predictions for beam-columns subjected to non-uniform bending. Since the proposed method does not require explicit modelling of member imperfections, it obviates the need to identify suitable shapes and directions. Finite element models of steel members are also developed in this paper using Geometrically and Materially Nonlinear Analyses with Imperfections (GMNIA) providing benchmark results used to assess the accuracy of the proposed stiffness reduction approach. In addition to regular members, the suggested approach is also applied to irregular members where its accuracy is compared against EN 1993-1-1 (2005) provisions.

Details of the finite element modelling adopted in this study are described in the following section. Then, the primary aspects of the proposed stiffness reduction approach and its application are addressed for pure axial load, pure bending and combined bending and axial load respectively. For all of the considered cases, the results obtained through the proposed approach are compared against those determined through GMNIA.

2. Finite element modelling

The finite element software Abaqus (2010) was used in the numerical simulations. Referred to as B31OS in the Abaqus element library, a linear Timoshenko beam element accounting for the warping degree of freedom and shear deformations was used for the development of beam element models where a generic I section is discretized through 33 integration points for each flange and web. For shell element models, a shear flexible, reduced integration, 4-noded general purpose shell element which is referred to as S4R in the Abaqus element library was used. With the exception of tapered columns, beam elements were used in all numerical simulations as the members considered in this study are not susceptible to local buckling effects. The tri-linear elastic-plastic material model shown in Fig 1 was used, where E is the Young's modulus, E_{sh} is the strain hardening modulus, f_y and ε_y are the yield stress and strain respectively and ε_{sh} is the strain at the onset of strain hardening. The parameters f_u and ε_u correspond to the ultimate stress and strain respectively. E_{sh} was assumed to be 2% of E, which was taken as 210GPa, and ε_{sh} was asumed as $10\varepsilon_y$, complying with the ECCS (1984) material model recommendations for hot rolled steel. In all numerical models S235 steel (f_y = 235MPa) was used. The ECCS (1984) residual stress patterns, shown in Fig. 2, were employed, where the residual stresses were applied through the SIGINI subroutine (Abaqus, 2010). The initial geometrical imperfections (member out-of-straightness) were assumed to be half-sine wave in shape and 1/1000 of the corresponding member length in magnitude (AISC, 2010). In all GMNIA calculations, geometrical and material nonlinearity, residual stresses and geometrical imperfections are considered.

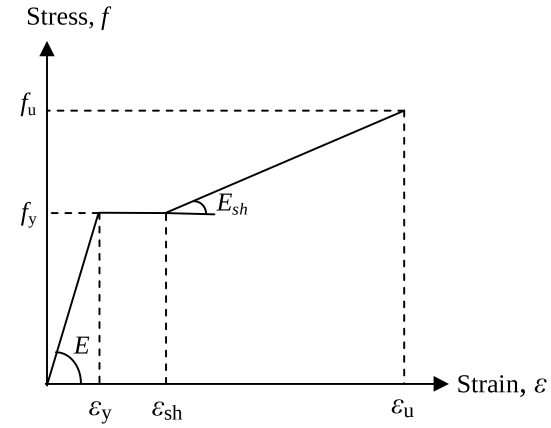


Figure 1: Material model used in finite element models

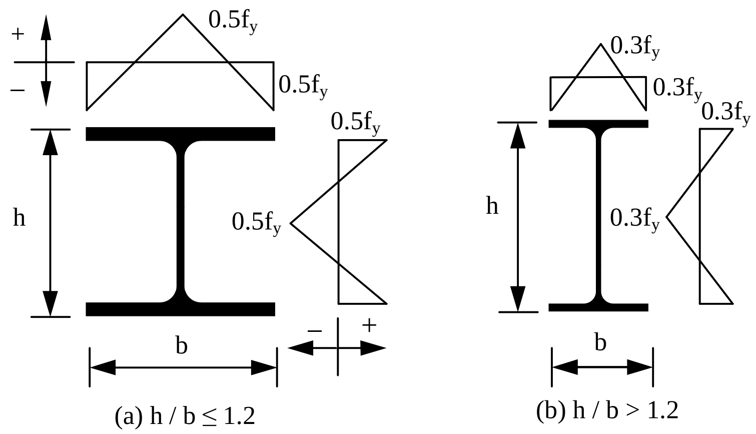


Figure 2: Residual stress patterns applied to finite element models (+ve tension,-ve compression)

3. Stiffness reduction under axial loading

The derivation of a stiffness reduction function of a steel member under axial loading and its application with LBA for the determination of inelastic buckling strength of columns are addressed in this section. The accuracy of the proposed stiffness reduction function is verified using the results obtained through GMNIA.

3.1 Derivation of a stiffness reduction function under axial loading

For a steel member under axial loading, the stiffness reduction function is derived utilising the European column buckling curves provided in EN 1993-1-1 (2005). The stiffness reduction function under axial loading τ_N, which corresponds to the ratio between the reduced modulus E_r and the Young's modulus E, can be determined considering the ratio between the inelastic and elastic critical buckling loads of the member, $N_{cr,i}$ and N_{cr} respectively, as shown in Eq. 1 where N_{pl} is the yield load, χ is the buckling reduction factor and $\bar{\lambda}$ is the member non-dimensional slenderness $\bar{\lambda} = \sqrt{N_{pl}/N_{cr}}$. Note that the yield load is equal to the multiplication of cross-sectional area A with material yield stress f_y.

$$\tau_N = \frac{E_r}{E} = \frac{N_{cr,i}}{N_{cr}} = \frac{\chi N_{pl}}{N_{cr}} = \chi\bar{\lambda}^2 \tag{1}$$

The Perry-Robertson expression (Robertson, 1925), which is the basis of the European column buckling curves, can be used for the determination of the buckling reduction factor χ as given in Eq. 2, where α is the imperfection factor. To define five different buckling curves, five values of α are provided in Eurocode 3 (EN 1993-1-1, 2005) – see Table 1.

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} \qquad \text{where} \qquad \phi = 0.5\left[1 + \alpha(\bar{\lambda} - 0.2) + \bar{\lambda}^2\right] \tag{2}$$

Table 1: Imperfection factors α for flexural buckling from EN 1993-1-1 (2005)

Buckling curve	a_0	a	b	c	d
α	0.13	0.21	0.34	0.49	0.76

Eq. 2 can be rearranged in terms of $\bar{\lambda}$, providing Eq. 3:

$$\bar{\lambda}^2 = \frac{4\psi^2}{\alpha^2\chi^2\left[1 + \sqrt{1 - 4\psi\dfrac{\chi - 1}{\alpha^2\chi}}\right]^2} \qquad \text{where} \quad \psi = 1 + 0.2\alpha\chi - \chi \tag{3}$$

Substituting Eq. 3 into Eq. 1 and with $\chi = N_{Ed} / N_{pl}$ gives Eq. 4, where τ_N is expressed as a function of the ratio between the applied axial load N_{Ed} and the yield load $N_{pl} = Af_y$, as well as the imperfection factor α. Owing to the inclusion of the imperfection factor, the influence of geometrical imperfections and residual stresses are implicitly accounted for in the proposed expression for τ_N. The derived stiffness reduction function τ_N is illustrated in Fig. 3 for different buckling curves. For stocky members, with $\bar{\lambda} \leq 0.2$, Eurocode 3 (2005) allows for the use of the full cross-sectional resistance, in which case $N_{Ed} / N_{pl} = 1.0$ and $\tau_N = 0.04$.

$$\tau_N = \frac{4\psi^2}{\alpha^2 N_{Ed}/N_{pl}\left[1 + \sqrt{1 - 4\psi\dfrac{N_{Ed}/N_{pl} - 1}{\alpha^2 N_{Ed}/N_{pl}}}\right]^2} \qquad \text{but} \qquad \tau_N \leq 1.0$$

$$\text{where} \quad \psi = 1 + 0.2\alpha\frac{N_{Ed}}{N_{pl}} - \frac{N_{Ed}}{N_{pl}} \tag{4}$$

Since the member is assumed to be perfect, the stiffness reduction method gives the inelastic buckling load of a member in lieu of its full load-displacement path. The strength of a column can be checked using Eq. 5, where the inelastic flexural buckling load amplifier $\alpha_{cr,i}$ must be greater than or equal to 1.0. The ultimate strength of a column can be obtained by iterating N_{Ed} until satisfying $\alpha_{cr,i} = N_{cr,i} / N_{Ed} = 1.0$.

$$\alpha_{cr,i} = \frac{N_{cr,i}}{N_{Ed}} = \frac{\tau_N N_{cr}}{N_{Ed}} \geq 1.0 \quad but \quad \tau_N N_{cr} \leq N_{pl} \tag{5}$$

It is of significance to note that the use of the derived stiffness reduction function τ_N provides an exact match to the European buckling curves, thus relying on the ability of these curves to capture accurately the behaviour of real columns. A suitable buckling curve for a member is chosen through a buckling curve selection table in Eurocode 3 (2005), which has been developed on the basis of extensive experimental, numerical and probabilistic studies (ECCS, 1976). Using multiple buckling curves, τ_N takes into account different magnitudes and patterns of residual stresses and different buckling directions.

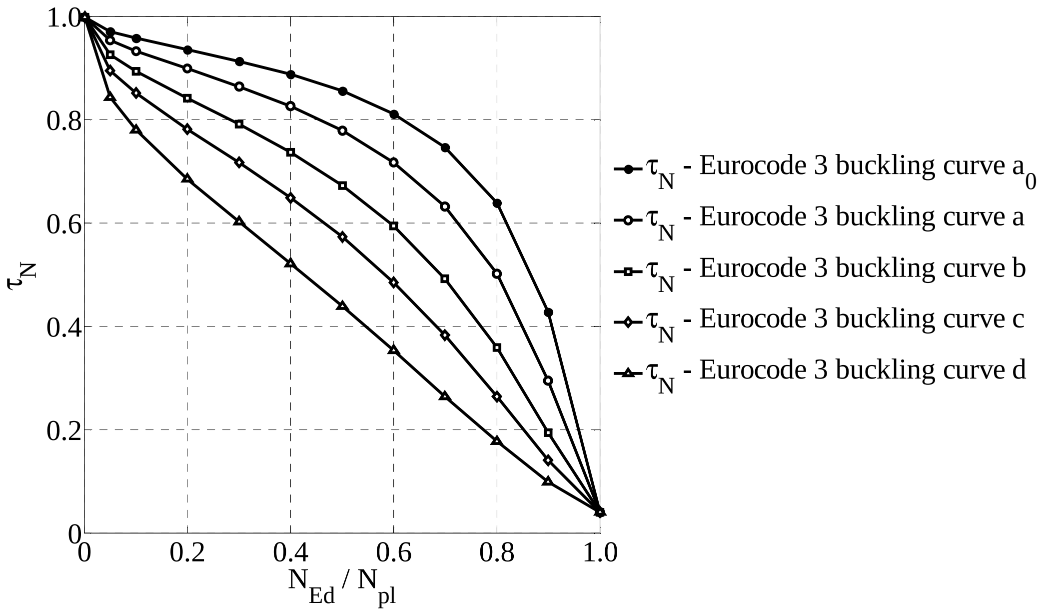


Figure 3: Proposed stiffness reduction function under axial load based on Eurocode 3 (2005) buckling curves

3.2 Application of the derived stiffness reduction function to inelastic flexural buckling

In the case of regular members, the proposed stiffness reduction function τ_N provides the same results as those obtained through the Eurocode 3 (2005) provisions. In this section, it is shown that use of the stiffness reduction function in conjunction with Linear Buckling Analysis (LBA) results in more accurate strength predictions in comparison to those obtained through Eurocode 3 (2005) for the case of columns with irregular geometry and boundary conditions. Since irregular problems can generally only be solved by making conservative assumptions or simplifications to the structural system in traditional design methods, the use of LBA in conjunction with the stiffness reduction function τ_N given in Eq. 4, which will be henceforth referred to as LBA-SR, may offer both an accurate and practical means of solving this kind of problem. The application of the proposed stiffness reduction method to two examples of irregular columns including a web-tapered column and a column with an intermediate elastic restraint is considered herein.

3.2.1 Tapered column

Application of the stiffness reduction method to the design of web-tapered columns with doubly-symmetric cross-sections is investigated in this sub-section. Beam element models are used to perform LBA-SR where the tapered column was discretized through 10 uniform elements along the span. The depth of the cross-section of an element is assumed to be equal to the average of the smaller and larger depths of the corresponding discretized portion. Considering the corresponding average cross-sectional area, stiffness reduction factors τ_N obtained from Eq. 4 were applied to each corresponding discretized portion. Using the first eigenmode as an imperfect shape, GMNIA of the tapered columns modelled using shell elements were performed to assess the accuracy of the proposed approach. The residual stress pattern suggested for hot-rolled sections is used in the finite element models, assuming hot-rolled members are cut to fabricate the tapered members. Comparisons of the results obtained through LBA-SR with those obtained through GMNIA, based on an IPE 240 cross-section with different tapering ratios is displayed in Fig. 4. Note that the tapering ratio γ is equal to the ratio of the depth of the increased cross-section h_2 to that of the original IPE 240 section h_1, such that $\gamma = h_2 / h_1$. The non-dimensional slenderness values shown in Fig. 4 are calculated with respect to the original cross-sectional area A_1 and elastic buckling load of the uniform column neglecting tapering. As seen in Fig. 4, LBA-SR provides results that are in very good agreement with those obtained through GMNIA for all member slenderness and tapering ratios. Moreover, the use of LBA-SR brings about improved accuracy in comparison to the Eurocode 3 (2005) provisions with N_{cr} determined from LBA.

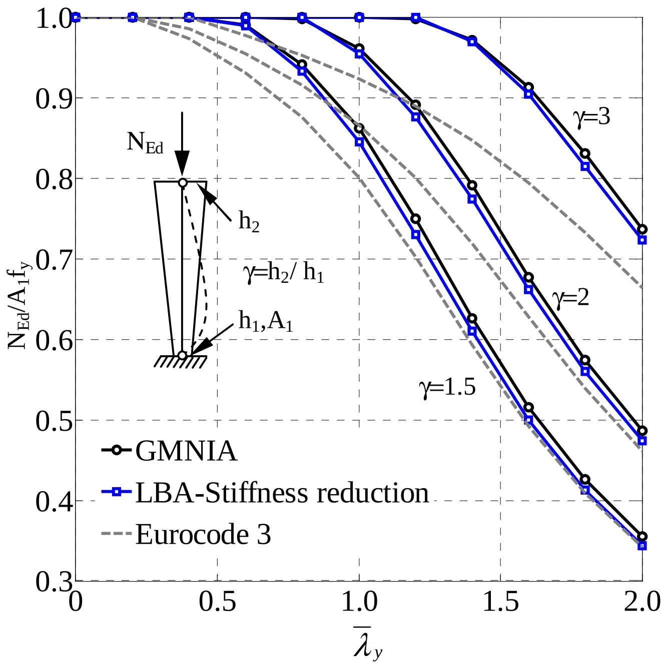
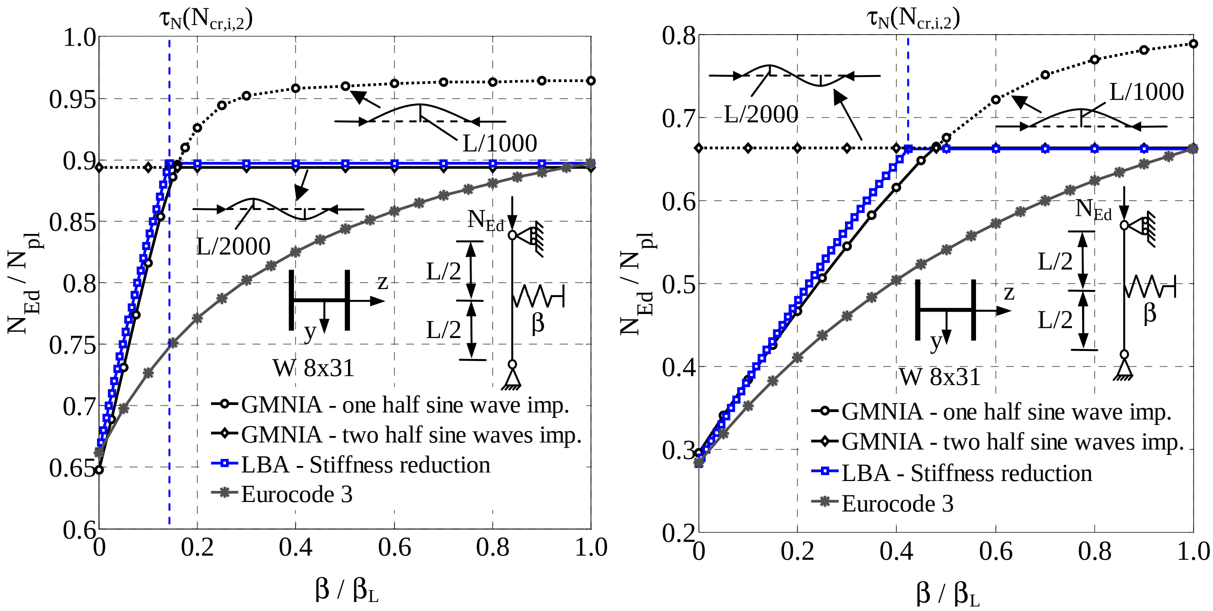


Figure 4: Comparison of the inelastic strengths of tapered columns obtained through the stiffness reduction method (LBA-SR) with those obtained through GMNIA and Eurocode 3 (2005)

3.2.2 Column with an intermediate elastic lateral restraint

For members with intermediate lateral restraints, the stiffness of the support should be taken into account when the restraint is not fully-rigid. In this sub-section, the behaviour of two columns with W8x31 (W200x46.1) cross-sections and non-dimensional member slenderness with respect to minor axis buckling $\bar{\lambda}_Z = 0.8$ and $\bar{\lambda}_Z = 1.6$, restrained by an elastic lateral restraint at the mid-height, are investigated. The non-dimensional slenderness of columns was determined neglecting the elastic restraint. The columns, fully restrained in the out-of-plane direction, were subjected to pure axial load. The stiffness of the elastic restraint β was varied so that the influence of the support stiffness on the inelastic buckling strengths could be investigated. The results obtained from linear buckling analysis with reduced member stiffness through τ_N (LBA-SR) are compared against those obtained through GMNIA and Eurocode 3 (2005) in Fig. 5. Note that $\beta_L = 16\pi^2 EI_z / L^3$ is the threshold restraint stiffness leading to elastic buckling of the column in the second mode (two half-sine wave mode). Two types of geometrical imperfections were assumed in the GMNIA calculations: a single half-sine wave imperfection scaled with L/1000 and a two half-sine wave imperfection scaled with L/2000, where L is the height of a column. For Eurocode 3 (2005) calculations, the non-dimensional slenderness of the restrained members was determined using elastic buckling loads obtained through LBA where the elastic restraint was considered.



(a) $\bar{\lambda}_Z = 0.8$ (b) $\bar{\lambda}_Z = 1.6$

Figure 5: Comparison between inelastic strengths of columns with elastic restraints obtained through the stiffness reduction method (LBA-SR) and those obtained through GMNIA and Eurocode 3 (2005)

It may be seen from Fig. 5 that the single half-sine wave imperfection is critical up to a specific threshold restraint stiffness $\beta_{L,inelastic}$ for both columns. For larger restraint stiffnesses, the two half-sine wave imperfection results in lower column strengths. As the development of plasticity in the columns increases the effectiveness of the intermediate elastic lateral restraint, the threshold stiffness forcing inelastic buckling in the second mode $\beta_{L,inelastic}$ is lower than that required for elastic buckling β_L, i.e. $\beta_{L,inelastic} \leq \beta_L$. As can be seen in Fig. 5, LBA-SR captures the increased effectiveness of the elastic restraint with the development of plasticity in the columns, providing very accurate results that are in close agreement with those obtained through GMNIA for both slenderness values. It is noteworthy that while two types of imperfection have to be considered in the case of GMNIA, LBA-SR directly captures the transition between the first and second inelastic buckling modes after exceeding the inelastic threshold stiffness $\beta_{L,inelastic}$. Since the development of plasticity in the column is not considered, the Eurocode 3 (2005) design equations, with elastic buckling loads determined through LBA, results in less accurate and overly conservative strength predictions in comparison to LBA-SR.

The ratio between the inelastic and elastic threshold stiffness values is dependent on the extent of plasticity undergone by the columns. The stockier the column, the smaller the ratio. This can be seen from Fig. 5, where the ratio $\beta_{L,inelastic} / \beta_L$ is smaller for the column with non-dimensional slenderness $\bar{\lambda}_Z = 0.8$ in comparison to that with $\bar{\lambda}_Z = 1.6$. The ratio between threshold inelastic and elastic stiffness $\beta_{L,inelastic} / \beta_L$ is equal to the stiffness reduction factor determined for the inelastic buckling load in the second mode $N_{cr,i,2}$ through Eq. 4, $\tau_N(N_{cr,i,2})$, as given in Eq. 6.

$$\beta_{L,inelastic} / \beta_L = \tau_N(N_{cr,i,2}) \tag{6}$$

4. Stiffness reduction under bending

The development of a stiffness reduction function for steel members under bending is addressed in this section. For a member subjected to constant bending and sufficiently restrained against out-of-plane instability effects, the stiffness reduction function can be developed considering its moment-curvature (M_{Ed} - φ) relationship. The cross-section geometry, material model and distribution and magnitude of residual stresses are of importance for the M_{Ed} - φ relationship, but not geometrical imperfections. Incrementally applying bending moment, the change in flexural stiffness can be determined, with the tangent stiffness EI_r defined as $EI_r = dM_{Ed} / d\varphi$. The ratio of the flexural stiffness at a particular bending moment value to the initial flexural stiffness then provides a stiffness reduction factor due to bending $\tau_M = EI_r / EI$.

As the same general stiffness reduction patterns, involving elastic, primary plastic and secondary plastic stages (Chen and Atsuta, 1976) are exhibited for all I-shaped cross-sections, expressions for stiffness reduction due to bending can be calibrated on the basis of moment-curvature relationships generated through finite element analysis. This approach was adopted by Zubydan (2010, 2011) to develop an expression for the reduction of stiffness in I-sections under pure bending, assuming the ECCS (1984) residual stress distributions illustrated in Fig. 2. This function is used in the present study and is given by Eq. 7a, 7b, 7c representing the reduction in stiffness in the elastic, primary plastic and secondary plastic stages respectively. The parameters τ_{M1}, ξ, ϕ, β, δ are given in Table 2 for major and minor axis bending of I-sections. Two sets of model parameters are provided, for h / b ≤ 1.2 and h / b > 1.2, which correspond to the different residual stress patterns shown in Fig. 2.

$$\tau_M = 1.0 \quad if \quad M_{Ed} / M_{pl} \le \phi \tag{7a}$$

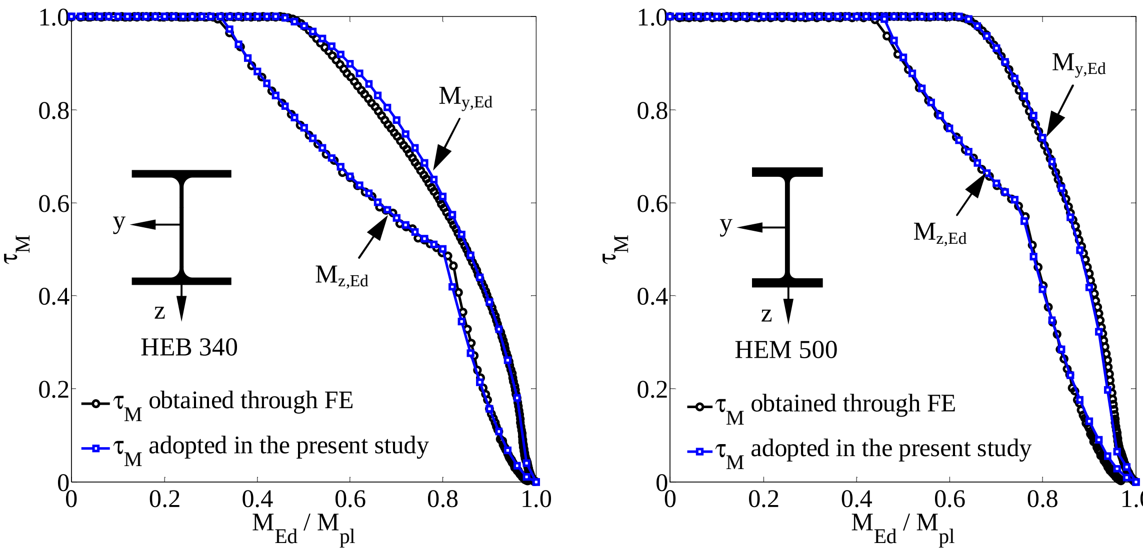
$$\tau_M = (1-\tau_{M1})\left[1-\left(\frac{M_{Ed} / M_{pl} - \phi}{\xi - \phi}\right)^{\beta}\right]^{1/\beta} + \tau_{M1} \quad if \quad \phi \le M_{Ed} / M_{pl} \le \xi \tag{7b}$$

$$\tau_M = \tau_{M1}\left[1-\left(\frac{M_{Ed}/M_{pl}-\xi}{1-\xi}\right)\right]^{1/\delta} \quad if \;\; \xi \le M_{Ed}/M_{pl} \tag{7c}$$

Table 2: Parameters for the stiffness reduction function for I-sections in bending (Zubydan, 2010, 2011)

	Major axis bending					Minor axis bending				
	τ_{M1}	ϕ	ξ	β	δ	τ_{M1}	ϕ	ξ	β	δ
h / b ≤ 1.2	0.04	$0.5W_{el,y}/W_{pl,y}$	0.98	1.5	1.0	0.5	$0.5W_{el,z}/W_{pl,z}$	0.8	0.85	0.6
h / b > 1.2	0.08	$0.7W_{el,y}/W_{pl,y}$	0.95	1.5	1.0	0.6	$0.7W_{el,z}/W_{pl,z}$	0.75	0.85	0.6

The results of finite element analysis were used to validate the ability of the stiffness reduction function given in Eq. 7 to capture accurately the influence of the spread of plasticity through the cross-section depth. Employing 313 section integration points for each cross-section (105 integration points in both flanges and web), a wide range of European IPE and HE sections were analysed. For the considered wide-range of cross-section shapes, it was observed that the variation in the values of the parameters of the stiffness reduction function τ_M provided in Table 2 is small and as shown in Fig. 6, the adopted stiffness reduction function leads to results in close agreement to the finite element predictions for different magnitudes of residual stresses and axes of bending.



(a) HEB 340 (h / b ≤ 1.2)

(b) HEM 500 (h / b > 1.2)

Figure 6: Comparison of the adopted stiffness reduction functions due to bending with those obtained through finite element analysis

5. Stiffness reduction under combined bending and axial load

The in-plane design of beam-columns through the stiffness reduction method is addressed in this section. Assumed ultimate cross-section strength interaction equations are first introduced. Then, a stiffness reduction function for combined bending and axial load is derived. The proposed design approach is based on the use of the developed stiffness reduction function in conjunction with elastic, but Geometrically Nonlinear Analysis with stiffness reduction (GNA-SR), and is validated for both major and minor axis bending and for different magnitudes of residual stresses. To consider the influence of moment gradient, a modification to the stiffness reduction function is suggested. The proposed modification is verified for a large number of beam-columns subjected to various bending moment gradients.

5.1 Interaction equations

In view of their accuracy and simple form, the continuous interaction equations proposed by Duan and Chen (1990) were chosen in this study amongst numerous interaction equations defining the ultimate capacity of I-sections under combined loading. These expressions are provided in Eq. 8 and Eq. 9 for axial load plus major axis bending and for axial load plus minor axis bending respectively. Note that A_w and A_f are the areas of the web and flange respectively.

$$1/\alpha_{ult,c} = \left(\frac{N_{Ed}}{N_{pl}}\right)^{1.3} + \frac{M_{y,Ed}}{M_{y,pl}} = 1 \tag{8}$$

$$1/\alpha_{ult,c} = \left(\frac{N_{Ed}}{N_{pl}}\right)^{\gamma} + \frac{M_{y,Ed}}{M_{y,pl}} = 1 \quad \text{where} \quad \gamma = 2 + 1.2\frac{A_w}{A_f} \tag{9}$$

5.2 Derivation of stiffness reduction function for combined bending and axial load

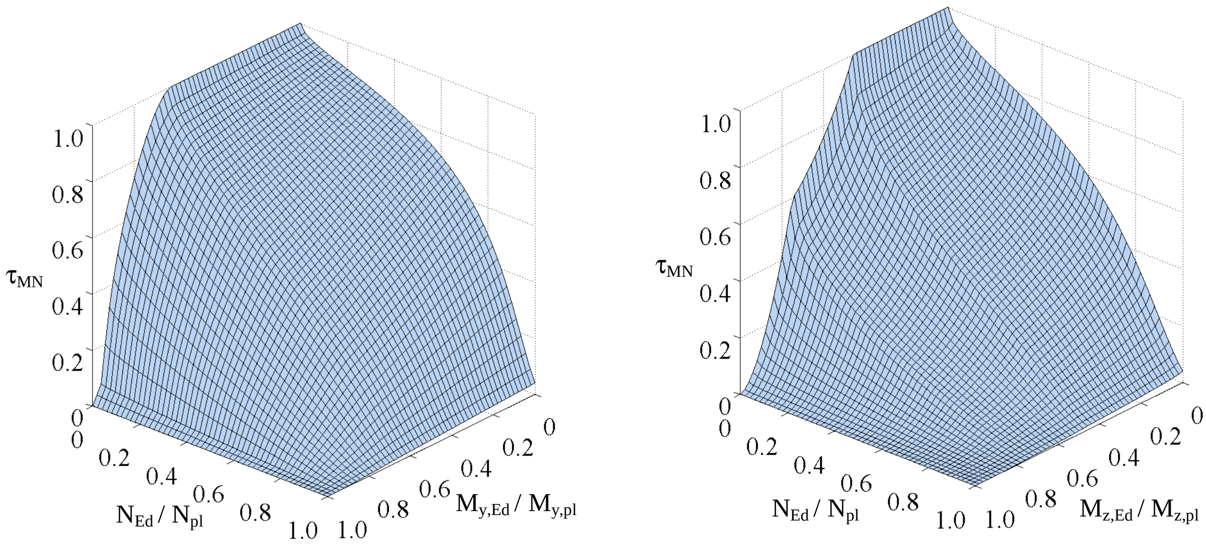
The stiffness reduction functions for pure bending τ_M and pure axial load τ_N described in the previous sections were utilised to derive a stiffness reduction considering the influence of combined bending and axial load τ_{MN}. The proposed stiffness reduction function for combined axial load and bending is given by Eq. 10.

$$\tau_{MN} = \tau_M \tau_N \left[1 - \left(\frac{N_{Ed}}{N_{pl}}\right)^{\eta}\left(\frac{M_{Ed}}{M_{pl}}\right)^{\rho}\right] \tag{10}$$

This form of equation leads to τ_{MN} having two anchor points: when the bending moment is zero $\tau_{MN} = \tau_N$ and when the axial force is zero $\tau_{MN} = \tau_M$. The factors η and ρ are provided in Table 3 for major and minor axis bending. Using the interaction equations given in Eq. 8 and Eq. 9 and the results obtained through GMNIA of simply-supported beam-columns with different European I-sections subjected to axial load plus constant bending, these factors were derived by calibration in Kucukler et al. (2014).

Table 3: Proposed values for the η and ρ parameters for the stiffness reduction function due to combined axial force and bending

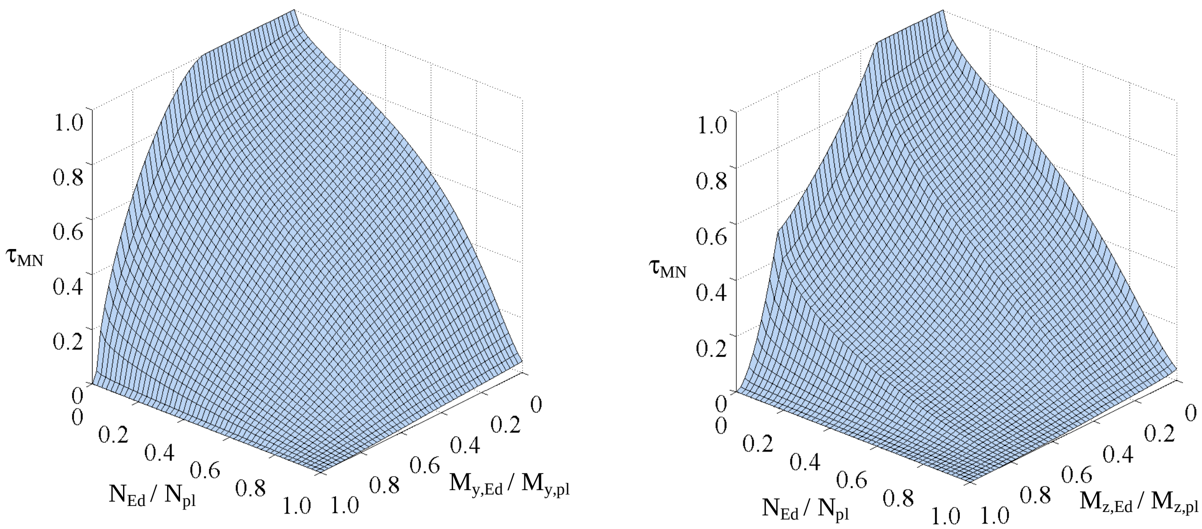
	Major axis bending		Minor axis bending	
	η	ρ	η	ρ
h / b ≤ 1.2	0.5	0.9	0.5	0.5
h / b > 1.2	0.8	1.0	0.5	0.55



(a) Major axis bending plus axial load

(b) Minor axis bending plus axial load

Figure 7: Proposed stiffness reduction function under combined bending and axial load for narrow flange cross-sections (h / b > 1.2)



(a) Major axis bending plus axial load

(b) Minor axis bending plus axial load

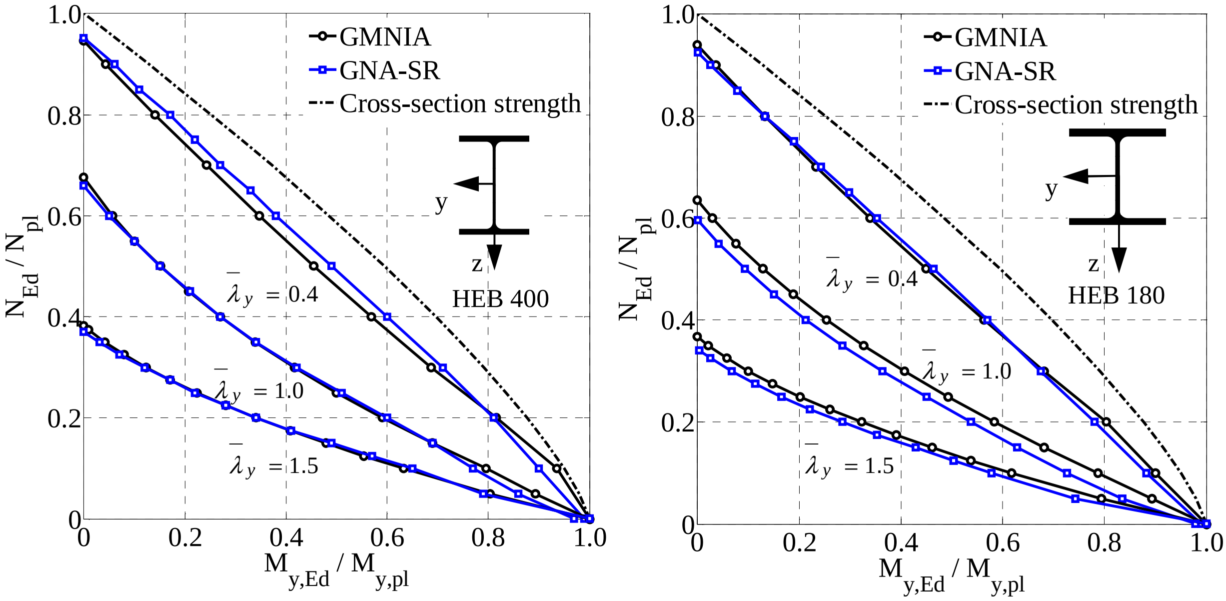
Figure 8: Proposed stiffness reduction function under combined bending and axial load for wide flange cross-sections (h / b ≤ 1.2)

Stiffness reduction functions for combined bending and axial load τ_{MN} for beam-columns with narrow and wide flange sections are illustrated in Fig 7 and Fig 8 respectively. As seen from the figures, the incorporation of the stiffness reduction functions developed for pure axial load τ_N and pure bending τ_M into τ_{MN} enables the consideration of different patterns of stiffness reduction for major and minor axes bending and different magnitudes of residual stresses.

5.3 Application of the stiffness reduction function for combined bending and axial load

This section investigates the accuracy of the proposed stiffness reduction function τ_{MN} for the in-plane design of beam-columns restrained in the out-of-plane direction. Geometrically Nonlinear Analysis with stiffness reduction (GNA-SR) is performed on a series of simply-supported beam-columns subjected to axial load and constant bending moment. The analysis is elastic but Geometrically Nonlinear, where geometrical imperfections are not explicitly modelled, but the Young's modulus of the material is multiplied by the stiffness reduction factor provided in Eq. 10 to account for the spread of plasticity, residual stresses and geometrical imperfections. The attainment of the maximum load carrying capacity of the member is signified when the section forces at the most heavily loaded cross-section become equal to corresponding cross-section resistance given in Eq. 8 and Eq. 9.

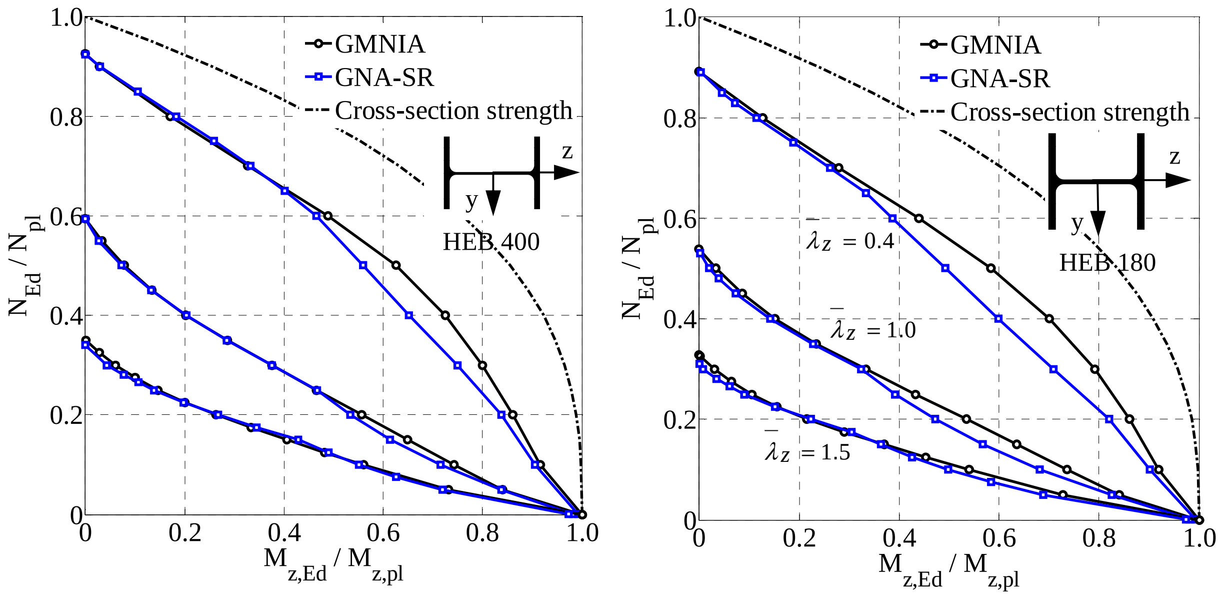
The results obtained through GNA-SR are compared against those obtained through GMNIA for simply-supported beam-columns with HEB 400 (h / b > 1.2) and HEB 180 (h / b ≤ 1.2) cross-sections, subjected to axial load plus constant major axis bending in Fig 9a and Fig 9b respectively. Three non-dimensional slenderness values, $\bar{\lambda}$ =0.4, 1.0 and 1.5, were considered, to cover the response of beam-columns with low, intermediate and high slenderness. It can be seen from Fig. 6 that GNA-SR provides accurate results for the different slenderness values and different ratios of bending to axial force. There only exists slight overestimation of the strength in the case of the beam-column subjected to major axis bending with HEB 400 cross-section and member slenderness $\bar{\lambda}_y$ =0.4. For the case of columns with the HEB 180 cross-section, the slight underestimation of the strength of the beam-columns with member slenderness $\bar{\lambda}_z$ =1.0 and 1.5 results principally from the underestimation of the strong axis flexural buckling strength of columns with wide-flange cross-section (e.g. HEB 180) given by the Eurocode 3 (2005) flexural buckling curves.



(a) HEB 400 (h / b > 1.2) (b) HEB 180 (h / b ≤ 1.2)

Figure 9: Comparison of the results obtained through Geometrically Nonlinear Analysis with stiffness reduction (GNA-SR) with those obtained through GMNIA for simply supported beam-columns subjected to constant major axis bending and compression

Fig. 10a and Fig. 10b illustrate the results obtained through GNA-SR and GMNIA for simply-supported beam-columns with the HEB 400 (h / b > 1.2) and HEB 180 (h / b ≤ 1.2) cross-sections subjected to axial load plus constant minor axis bending respectively. Cross-section strengths determined through Eq. 9 are also shown in the figures. As can be seen from the figures, the results obtained through GNA-SR, which were calculated using the cross-section strength given in Eq. 9, are in good agreement with those obtained through GMNIA. Spread of plasticity transforms the convex interaction surface observed for a member with low slenderness ($\bar{\lambda}_Z$ =0.4) to a concave shape for members with intermediate-to-high slenderness ($\bar{\lambda}_Z$ =1.0 and 1.5). This transformation is accurately captured by GNA-SR as can be identified in the figures



(a) HEB 400 (h / b > 1.2)

(b) HEB 180 (h / b ≤ 1.2)

Figure 10: Comparison of the results obtained through Geometrically Nonlinear Analysis with stiffness reduction (GNA-SR) with those obtained through GMNIA for simply supported beam-columns subjected to constant minor axis bending and compression

In addition to the results considered herein, comparison of the results obtained through GNA-SR and GMNIA for a large number of beam-columns with a wide-range of European cross-section shapes subjected to both major and minor axes bending was made in Kucukler et al. (2014), where the accuracy of the proposed stiffness reduction method was verified.

It should be emphasised that the stiffness of the beam-column is reduced from the onset of the analysis on the basis of the applied bending and compression prior to any deformation in the application of the proposed stiffness reduction method. Provided the ultimate load-carrying capacity of the most heavily loaded cross-section is not exceeded ($\alpha_{ult,c} \leq 1.0$), the beam-column can be classified as adequate to withstand the applied forces. To determine the capacity of the beam-column, the applied bending and/or axial load may be increased until reaching the ultimate capacity of the most heavily loaded section ($\alpha_{ult,c} = 1.0$).

5.1 Moment gradient effect

For a beam-column with varying bending moment along the length, the required stiffness reduction may be different due to a different pattern of the development of plasticity in comparison to that required for beam-columns subjected to constant bending. The influence of moment gradient thus should be taken into account to achieve accurate results for beam-columns subjected to non-uniform bending moment. The moment gradient effect on the development of plasticity was considered through the incorporation of moment gradient factors into the calculation of the stiffness reduction factors in this study. There exist different equations for the determination of moment gradient factors in the literature (Austin, 1961, Kirby and Nethercot 1979). Unlike the equation developed by Austin (1961), the equation proposed by Kirby and Nethercot (1979) can be applied to members with non-linear moment gradients. The formula put forward by Kirby and Nethercot (1979) for the determination of the moment gradient factor is provided in Eq. 11 where M_{max} is the absolute value of maximum moment along the unsupported member length, M_A is the absolute value of moment at the quarter point of the unsupported member length, M_B is the absolute value of moment at centerline of the unsupported member length and M_C is the absolute value of moment at the three-quarter point of the unsupported member length.
2

$$C_m = \frac{2.5M_{max} + 3M_A + 4M_B + 3M_C}{12.5M_{max}} \tag{11}$$

It should be noted that while the equation of Austin (1961) was developed for the consideration of moment gradient effects for the in-plane design of beam-columns which is associated with the problem investigated herein, the formula proposed by Kirby and Nethercot (1979) was derived for the determination of the lateral-torsional buckling moments of beams with varying bending moments along the length. Nevertheless, the latter provides almost identical results to the former for linearly varying moments and it is shown in this section that the incorporation of the moment gradient factors proposed by Kirby and Nethercot (1979) into the stiffness reduction functions provides accurate results for various types of moment gradients. Another point that needs to be emphasised is that in the implementation of GNA-SR, the actual loads and moments are considered and the moment gradient factors are only used in the determination of the stiffness reduction factors so as to take into account the influence of moment gradient on the development of plasticity.

It is proposed to adopt Eq. 11 in the stiffness reduction method developed herein to consider bending moment gradient effects. Specifically, in the calculation of τ_M and τ_{MN}, the maximum bending moment values along the member length M_{Ed} need to be factored by C_m, as shown in Eq. 12 a,b,c and Eq. 13.

$$\tau_M = 1.0 \quad if \;\; C_m M_{Ed} / M_{pl} \le \phi \tag{12a}$$

$$\tau_M = (1-\tau_{M1})\left[1-\left(\frac{C_m M_{Ed} / M_{pl} - \phi}{\xi - \phi}\right)^{\beta}\right]^{1/\beta} + \tau_{M1} \quad if \;\; \phi \le C_m M_{Ed} / M_{pl} \le \xi \tag{12b}$$

$$\tau_M = \tau_{M1}\left[1-\left(\frac{C_m M_{Ed}/M_{pl}-\xi}{1-\xi}\right)\right]^{1/\delta} \quad if\ \ \xi \le C_m M_{Ed}/M_{pl} \tag{12c}$$

$$\tau_{MN} = \tau_M \tau_N \left[1-\left(\frac{N_{Ed}}{N_{pl}}\right)^{\eta}\left(\frac{C_m M_{Ed}}{M_{pl}}\right)^{\rho}\right] \tag{13}$$

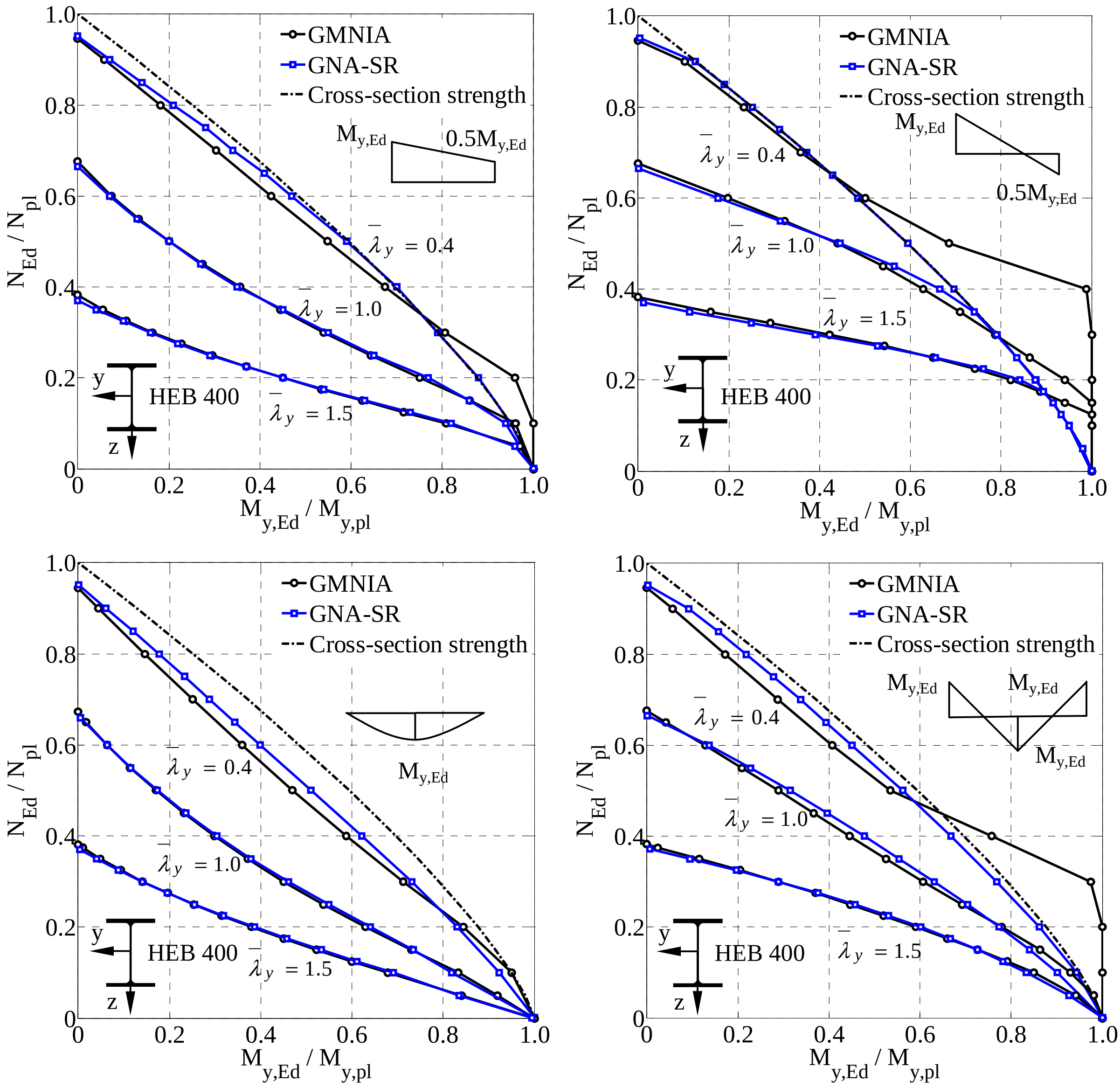


Figure 11: Comparison of the results obtained through Geometrically Nonlinear Analysis with stiffness reduction (GNA-SR) with those obtained through GMNIA for simply supported beam-columns subjected to varying major axis bending and compression

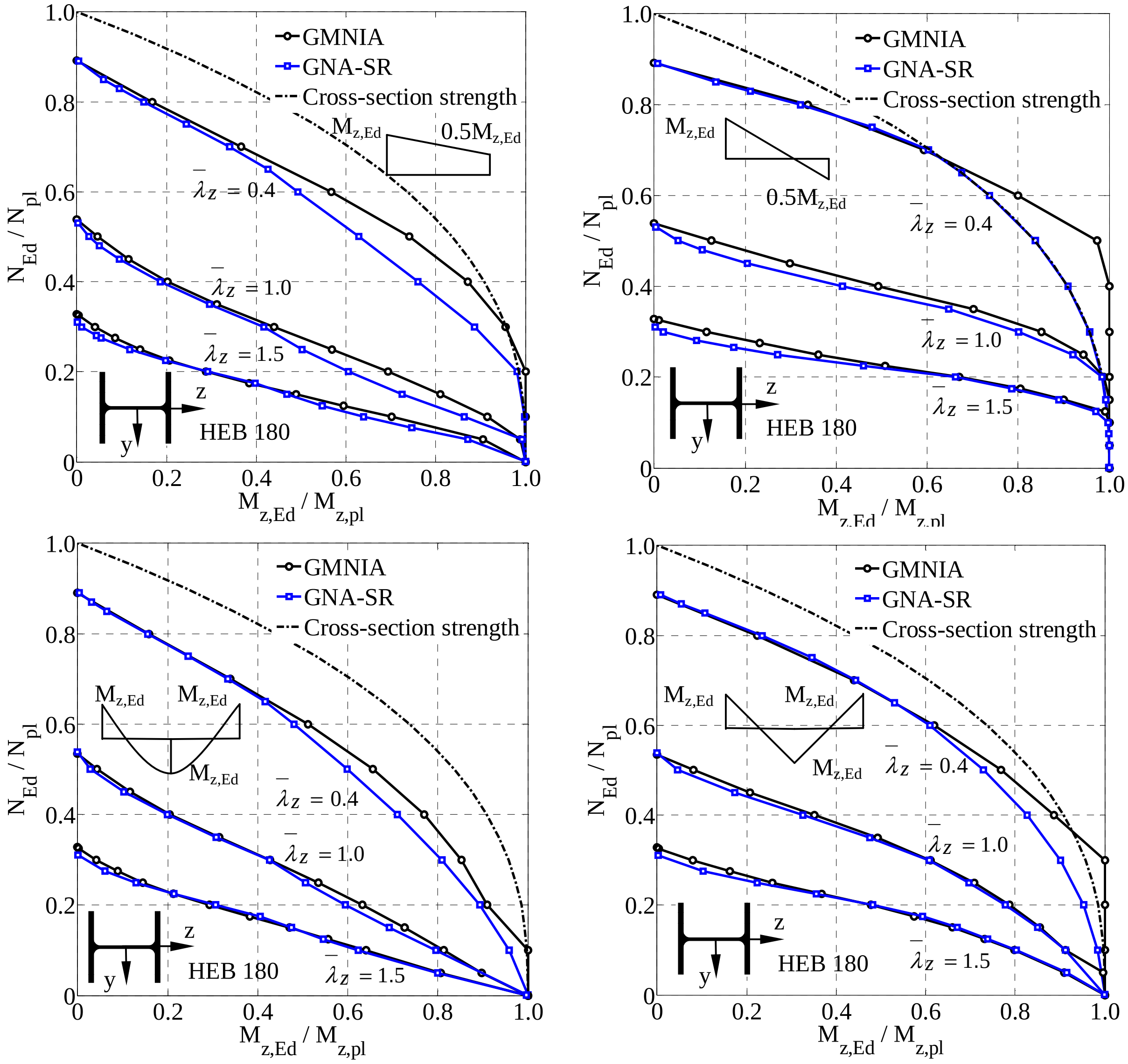


Figure 12: Comparison of the results obtained through Geometrically Nonlinear Analysis with stiffness reduction (GNA-SR) with those obtained through GMNIA for simply supported beam-columns subjected to varying minor axis bending and compression

Following the procedure described, the strength of a series of beam-columns subjected to compression and varying moment along the member length were calculated through GNA with the stiffness reduction function τ_{MN} accounting for the moment gradient (GNA-SR) – Eq. 11, Eq. 12 a,b,c and Eq. 13. Comparison of the results obtained through GNA-SR and GMNIA for simply-supported beam-columns with an HEB 400 cross-section subjected varying major axis moment along the length is illustrated in Fig. 11. It is seen from the figure that for various bending moment gradients, the incorporation of C_m into τ_{MN} leads to GNA-SR results that are in a very good correlation with those of GMNIA. Note that the results from GMNIA for stocky members with low axial loads exceed the cross-section strength due to the occurrence of strain hardening and that the maximum moments have been limited to M_{pl} in Fig.11. For the case of simply-supported beam-columns with an HEB 180 cross-section subjected to varying minor axis bending and compression, GNA-SR also provides accurate results for various moment gradients and different member slendernesses, as can be seen in Fig. 12. As observed in Fig. 11, capacities in excess of the cross-section resistance are also evident in Fig. 12 due to the effects of strain-hardening.

6. Conclusions

This study focused on a stiffness reduction method for the determination of the capacities of steel columns and beam-columns. Stiffness reduction functions were derived for pure compression, pure bending and combined bending and compression. In the case of the stiffness reduction function for pure compression, the Eurocode 3 (2005) column buckling curves were utilised. Taking into account residual stresses and material nonlinearities, stiffness reduction functions for pure bending were determined considering the moment-curvature response of European cross-sections. Stiffness reduction expressions for combined bending and axial load were derived through calibration to GMNIA results. The implementation of Linear Buckling Analysis with the derived stiffness reduction function due to axial loading (LBA-SR) is proposed for the design of columns, and this provides an exact match to the European column buckling curves for regular members. LBA-SR was also applied for the determination of the inelastic buckling strengths of irregular members, leading to strength predictions that are more accurate than those obtained through the Eurocode 3 (2005) provisions. Without modelling geometrical imperfections, the use Geometrically Nonlinear Analysis with the stiffness reduction function due to combined bending and compression (GNA-SR) is recommended for the design of beam-columns. According to GNA-SR, failure of a beam-column is signified by reaching the ultimate cross-section resistance at the most heavily loaded cross-section. The accuracy of GNA-SR was verified for beam-columns subjected to major or minor axis bending with wide and narrow flange cross-sections and different slenderness values. To account for the influence of bending moment gradient on the development of plasticity along the length of a beam-column, the incorporation of moment gradient factors into the stiffness reduction expressions was proposed. The accuracy of this proposal was validated for various shapes of bending moment diagrams, for both major and minor axis bending and for both narrow and wide flange sections.

One of the significant aspects of the proposed stiffness reduction approach is that it can be applied through conventional structural analysis software to perform LBA-SR and GNA-SR. Thus, it offers a very practical way of determining the strength of steel members. In contrast to previous approaches, the stiffness reduction scheme proposed in this study enables the design of members without the need of using column strength equations; instead only cross-section checks are required. This may bring about significant improvements in terms of both accuracy and practicality, particularly when designing steel frames. Future work will be concerned with the extension of the proposed stiffness reduction method to the design of steel frames.

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