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An innovative direct analysis method based approach for the stability control of indirectly supported structures

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Abstract
This paper adopts the idea of structural control therefore; a controllable support system has been designed, fabricated, and installed in a dedicated test structure for stability and structural response control under symmetric loads. Firstly, a brief computational technique based on Direct Analysis Method is presented, highlighting stability control of additionally supported structure. Then, the paper gathers analytical data that includes numerical values, obtained from ANSYS finite element analysis. Subsequently, the paper presents the validation of the proposed technique by experimental investigation of three dimensional model of the under-slung beam, arranged of the main element reflecting behaviour of beam-column and controlling supporting system by means of stays and controllable length of the strut. Detailed design and analysis of the controllable system are carried out with the respect to control stability and obtain rational extreme bending moments distribution. Finally, paper closes with the assessment of the stability and structural control yielded by the proposed computational technique and structural solution, through their comparison with the obtained experimental results and analytical data – it is shown that both the quality and reliability of the proposal are good.

1. Introduction

It is frequently the case, especially in those days design that a structure is required to be so slender that is actually no longer suitable for the intended span. Extra supports could be added, but that is not the intention in the context of today's design of structures, namely, to span a large distance with a structure which is "too slender". Therefore, indirect intermediate supports are the only way of overcoming this problem. The concept of increasing the span with indirect intermediate supports and eliminating the real ones reflects structural form of under-slung beam.
The structural behaviour of under-slung beam may be simplified to calculation model of the beam-column with intermediate elastic restraint. A number of analytical studies of imperfect columns with intermediate elastic restraints have been performed, Al-Shawi (1998), Banfi 2002, Trahair 1999) and relationships between the forces experienced by the restraints and the load in the column have been derived. Similar analyses may be performed for beams (Trahair 1984) to obtain analogous relationships between critical bending moment and restraint stiffness. In the

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case, the problem is inherently more complex because restraints may act to limit lateral deflection, twist, or both, and beams may be subjected to variety of different forms of loading.
However, the different buckling characteristics of beams in comparison to columns leads to an appropriate minimum restraint stiffness of around 10 times the lateral bending stiffness of the beam when bracings are attached to the compression flange (Wang and Nethercot 1989). Theoretically, all structural members may be regarded as beam-column, since the common classification of elements subjected to axial load, and bending moment are merely limiting examples of beam-column (Bjorhovde 2010). Thus, the consideration of the structural behaviour of the element with the intermediate elastic restraint under simultaneous bending and compression is, therefore, the practical interest. Wang and Liew presented buckling capacities of braced columns (1991) and comprehensive set of stability criteria for Euler columns with an intermediate elastic restraint (Wang and Nazmul 2003). Gambhir considered the case of Euler strut with an added elastic central support with potential energy approach (2004). Trahair with co-workers used differential equation method (2008) for the solution of the same structural model. Saha and Banu developed the method to identify the buckling load of beam-column based on ‘Multi-segment Integration technique’ (2007).However, it has to be noted that, in the most performed studies, the authors deal with the problem either in the case when restrained stiffness approaches infinity (Goncalves, Camotim 2004) or transverse force vanishes (Davis 1990, Trahair, Rasmussen 2005, Wang, Ang 1988). Thus structural behaviour of beam-column still remains an issue for research. Currently, there is much research work on influence of element imperfections on stability performance of steel structures, from which some directions may be abstracted, such as curved member modelling, equivalent imperfection concept and reduced tangent modulus. In the study of Austin and Ross (1976) and Wen and Lange (1981, 1991), it is proposed that curved elements can be used to simulate the initial curvature with only one element for one member. Lui and Chen (1986) have proposed a more rigorous cubic lateral displacement function expressed with the usual cubic Hermite polynomial, combined with two new shape functions. These new shape functions represent respectively a symmetric first buckling mode and an asymmetric second buckling mode of a fixed ends beam. Investigating the same problem, Chan and Zhou (1998) have proposed a more relevant fifth order polynomial function for the displacement field. The curved element method is direct but complicated, and the actual member initial curvature is rather small. Therefore, it’s not suitable to use element with large bending (Zhou *et al.* 2009). Then, Chan and Zhou (1995) point out that additional moment caused by increase of lateral deflection can be considered based on self-equilibrium (Point equilibrium polynomial) straight beam element (Zhou *et al.* 2009). Chan and Gu (2000) have proposed an exact solution for an imperfect beam-column by using the stability function approach. In this study the authors have used Timoshenko’s theory (1966) and extended this theory to take into account the effect of the initial curvature along the element length.
The 2005 AISC specification provides a new method for stability design of steel structures termed the direct analysis method (DM), which provisions accounts for imperfections and nonlinear behaviour of the structure. By the time of 2005 the DM method is provided as an alternative for the classic design methods. The latest version of AISC specifications addresses the requirements for the design of structures for stability and DM is presented therein as the prior design method and effective length and first-order analysis methods become alternative. The DM is the only one of the above three procedures that is generally applicable to all types of frames. Since mainly DM is considered as major innovation in steel frames stability assessment and design (Surovek-Maleck *et al.* 2005, Deierlein 2003, and White *et al.* 2007).

In this paper DM is extended for the performance of the structural control of the indirectly supported structure (ISS). This is the first approach for the implementation of DM for structural control of the ISS which aims in assumptions approval, proposed technique experimental validation and yielding of future prospects. It should be noted that paper manly focuses on validation of analytical data, as theoretical results have been validate by analytical in the previous studies (Misiunaite, Juozapaitis 2013 and Misiunaite 2013).

2. Direct modeling of structural response control

2.1 Calculation model

The most fundamental theoretical formulation in the verification of the elastic stability of structures is the Euler formula, which defines the elastic axial buckling strength of an individual member and essentially refers to the structural analysis based on the individual member check. The theory assumes that the member is perfectly straight, behaves in-plane elastically and has a pinned ends. These assumptions all are commonly violated in real structures. In practice, elements are out-of straightness between braced points due to fabrication tolerances. Residual stresses are presented that cause inelastic behavior. Further, as a structure is loaded, deformations also occur, adding second-order forces and moments. Furthermore, it should be noted the fact that buckling always involves both an axial force and bending effects. The considering calculation model formed to implement all previously mentioned effects and moreover, the beam-column shown in Fig.2 has an intermediate restraint to cover the case when the structure is additionally supported at the mid-span as shown in Fig.1, thus obtaining the semi-continuous under-slung beam structure.

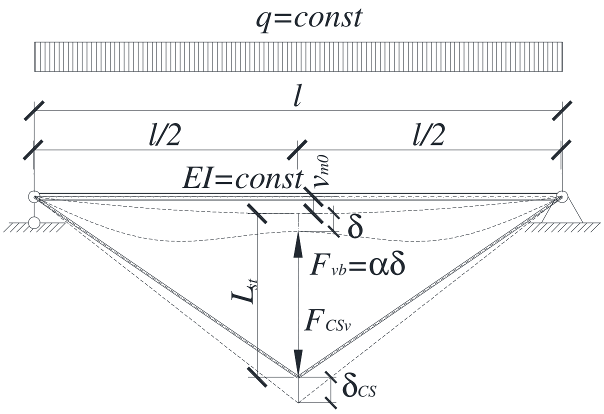

Figure 1: Scheme of considering indirectly supported under-slung beam

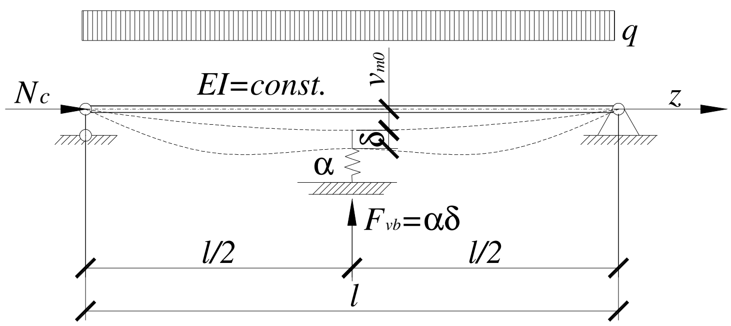

Figure 2: Generalized calculation model of imperfect beam-column with intermediate restraint

2.1 Governing equations

The governing equations for the stability verification of the presented structure in Fig.1 and its generalized calculation model in Fig. 2 derived considering the equilibrium by approach of moderately large displacement theory. Determining the equilibrium equations by moderately large displacement theory the structure is in equilibrium, in its deformed state; thus the second order effects will be accounted.

By considering simultaneous bending and compression with applying axial load N_c the deflection at z is increased by v and the differential equation of bending becomes:

$$M'' - N_c (v - v_0)'' + q = 0 \tag{1}$$

By adopting a sinusoidal function for an initial lack of straightness:

$$v_0 = v_{m0} \sin\left(\frac{\pi z}{l}\right) \tag{2}$$

the Eq. 1 can be rewritten as:

$$v(z)^{iv} + k^2 v(z)'' = \frac{q}{EI} - k^2 v_0(z)'' \tag{3}$$

General solution of the Eq. 3 is:

$$v^{l(r)} = C_{1(5)} \sin kz + C_{2(6)} \cos kz + C_{3(7)} + C_{4(8)} + v_{part} \tag{4}$$

where superscriptions l and r indicates left and right sides of the calculation model respectively. When the transverse distributed force is assumed to be constant and introducing:

$$n = \frac{(kl/2)^2}{(\pi/2)^2 - (kl/2)^2} \tag{5}$$

the particular solution can be taken as:

$$v_{part} = \frac{qz^2}{EIk^2} + n v_{m0} \sin\frac{\pi z}{l} \tag{6}$$

When combining Eq. 4 with the boundary conditions for the left side of calculation model shown in Fig. 2 $0 \le z \le l/2$ (referring to continuity and symmetry, between left and right side of the whole structure, it is allowable to consider one of the parts) .

The deflection is obtained as:

$$v(z) = \frac{1}{N_c}\left[\frac{1}{2}F_{vb} l\left(\frac{z}{l} - \frac{\sin kz}{kl\cos {kl}/{2}}\right) + \frac{ql^2}{(kl)^2}\left(tg\,{kl}/{2} \sin kz + \cos kz - 1\right) - \frac{qz}{2}(l - z)\right] + n v_{m0} \sin {\pi z}/{2} \tag{7}$$

Accordingly, by $M = -EIv''$ the moment is given by:

$$M(z) = \frac{ql^2}{(kl)^2}\left(tg\,{kl}/{2} \sin kz + \cos kz - 1\right) - \frac{F_{vb} l}{2kl}\frac{\sin kz}{\cos {kl}/{2}} + EI\left(\frac{\pi}{l}\right)^2 n v_{m0} \sin {\pi z}/{l} \tag{8}$$

Restoring force F_{vb} introduced by intermediate restraint in Eqs. 7 and 8 is defined as:

$$F_{vb} = 2\left[\frac{ql}{kl}\frac{\left(1 + \left({kl}/{2}\right)^2/2 - 1/\cos {kl}/{2}\right)}{\left({kl}/{2} - tg\,{kl}/{2}\right)} + \frac{(\delta - n v_{m0})}{l} \times N_c \frac{kl}{\left({kl}/{2} - tg\,{kl}/{2}\right)}\right] \tag{9}$$

Elastic support at the middle of the considering calculation model shown in Fig.2 restrains its displacement to δ. Thus restoring force F_{vb} can be expressed as a product of δ and restraint stiffness α_1, substituting it into Eq. 9 restrained displacement may be obtained as:

$$\delta = \frac{ql^4}{EI(kl)^4} \frac{\left(1+\left(kl/2\right)^2 - 1/\cos kl/2\right)}{\alpha_1/\alpha_c - 1} - \frac{(nv_{m0})}{\alpha_1/\alpha_c - 1} \tag{10}$$

where α_c is restraint stiffness for the column with intermediate elastic restraint (Trahair 2008).
The variation with the dimensionless restraint stiffness α_c/α_L of the dimensionless buckling load on the basis of simply supported additionally elastically restrained compressed element was described by Misiunaite (2013) and was assumed to be limiting as considering element buckles in the asymmetric second mode.
It follows from Eq. 10 that intermediate restraint stiffness supposed to be non-equal and greater than α_c, otherwise restrained deflection approach infinity and turns to unrestrained one. Furthermore, the shift from the unrestrained deflection shape to restraint one asserts between the values of 2 to 4 of the relative restraint stiffness α_c/α_L depending on the slenderness parameter *kl* (Misiunaite 2013).

2.2 Direct structural response control

Very few studies have been conducted on an attempt to control structural response of the indirectly supported structures. A new approach on the flexural response control of the simple-span bridges was proposed by Juozapaitis et al. (2007). The study was performed on the case of under-deck cable-stayed bridge reconstruction with no suspension during exploitation.
This study attempts to provide brief guidelines for the direct structural control technique. Proposed technique relies on the approach of the flexural behaviour of under-slung beam, thus besides sagging moments has to consider hogging moments at the point of the indirect support. The structural control may be performed by setting the equalized extreme bending moment's criterion:

$$\left|M\left(z=0\right)\right| = \left|M\left(z=l/2\right)\right| \tag{11}$$

The indicated criterion in the considering structure may be stifled by obtaining rational displacement at the intermediate support and reflect the rational length of the strut.
As the rational geometric parameters are set the structural response of the considering structure reflects structural behaviour of the imperfect beam-column with intermediate restraint. For the case of structural response control should be found the rational value of the restoring force at the restraints which is constant for any number of the restraints.
By introducing:

$$\varphi_1(kl) = \frac{1-2\cos\frac{kl}{2}+\sin\frac{kl}{2}\sin kz+\cos\frac{kl}{2}\cos kz}{kl\left(\sin\frac{kl}{2}+\sin kz\right)} \tag{12}$$

the rational restoring force can be obtained as:

$$F_{vb,rac} = ql\varphi_1(kl) \tag{13}$$

As previously shown restoring force relays on restrained displacement at the section of indirect support and vice versa, thus the rational displacement can be obtained by introducing:

$$\varphi_2(kl) = \frac{\left[\dfrac{1}{\cos\dfrac{kl}{2}} - \dfrac{(kl)^2}{8} - 1 - F_{vb,rac}\left(tg\dfrac{kl}{2} - \dfrac{kl}{2} \right)\dfrac{kl}{ql} \right]}{(kl)^4} \tag{14}$$

as:

$$\delta_{rac} = \frac{ql^4}{16EI}\varphi_2(kl) \tag{15}$$

2.3 Analytical investigation

For the propose of realizing the direct structural and stability control of indirectly supported structures, this study besides theoretical method aims to propose a simple analytical modeling for direct structural control of ISS. In Fig. 1 presented structural scheme of under-slung beam has been modeled as three dimensional ISS using FE software ANSYS as shown in Fig.3. For the theoretical assumptions and experimental validation the model shall comply with the following principles.

1. The main beam of the structure has to be modeled to account for initial imperfections. For this particular case the spline elements have been used which reflects sinusoidal function as assumed in theoretical model. The magnitude of the initial imperfections has been modeled according the one measured of the experimental test model.
2. In order to control structural behaviour of the ISS and satisfy equalized extreme bending moments criterion set in Eq. 11, there was simulated the gap at the beam-strut connection. The value of the gap reflects the difference between displacements of supporting system and the mid-span of the beam in order to obtain the value of the rational displacement. It should be noted that the previous researches have revealed that considering the deformational behaviour of non-controlled ISS previously mentioned displacements are equal (Misiunaite 2013).
3. The prediction of the gap at the different load steps uses the relationship between transverse force and the gap presented in Fig. 4.

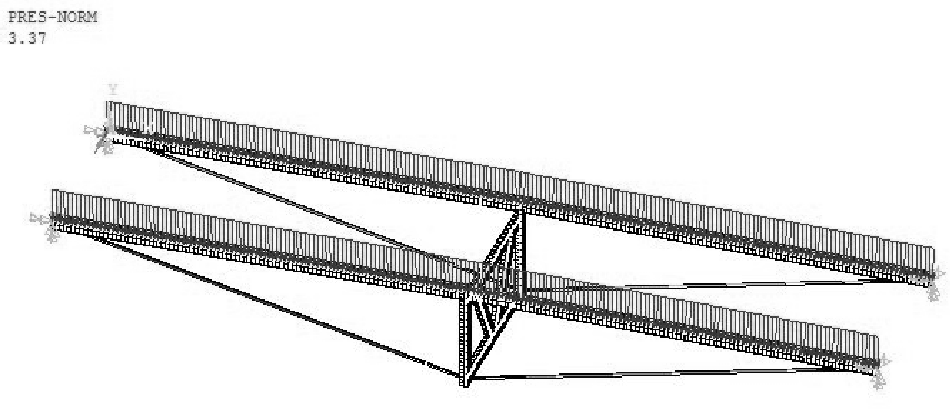

Figure 3: Analytical under-slung beam calculation model

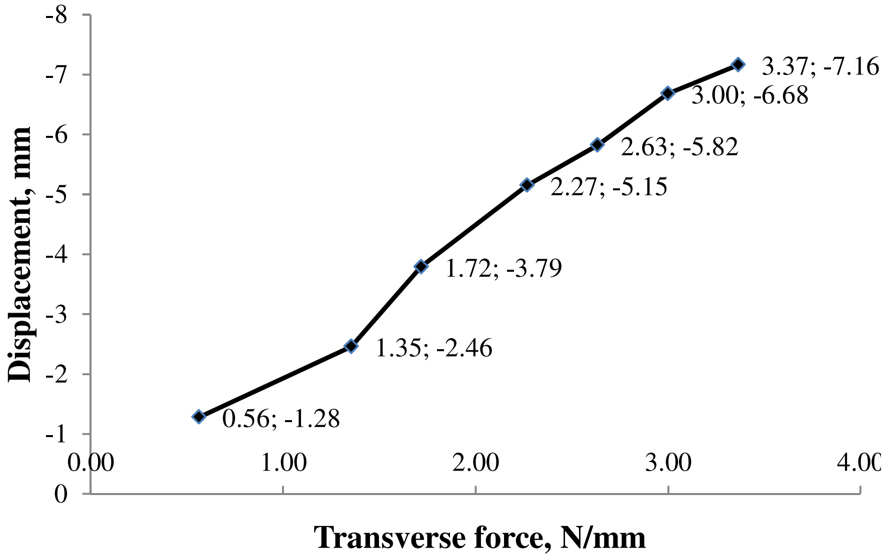

Figure 4: Relationship between transverse force and the gap

Table 1 presents the direct structural control results based on extreme bending moments equalizing criterion. The columns titled M2 and M13 indicate, respectively extreme sagging and hogging bending moments (see Fig. 8). The obtained differences between extreme bending moments are given in the last column of the table. It can be seen the minor constant error of 2%, which is obtained due to measuring point of M13. The measuring point is placed slightly (25 mm) away from the beam-strut connection accordingly to stress-strain gauges in experimental test model as shown in Fig. 8. Thus for the future comparison analysis the analytical data has been obtained at the same point instead of at the middle point of the structure. Accordingly to previously mentioned assumptions it could be maintained that the criterion of the equalized extreme bending moments during analytical investigation has been satisfied. Moreover, the last two rows indicate the differences between flexural behaviour of the considered ISS with and without structural control. It could be seen that in the structure without structural control the hogging moment is 22% greater than sagging and increase the stress in the structure as well as destabilize its behaviour.

Table 1: Analytical data (bending moments)

	ANSYS Data (Nmm)				**Criterion**
Load steps	M2	M4	M6	M13	M13/M2
0	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0%
1*	-1.91E+05	-1.46E+05	3.80E+04	1.88E+05	2%
4*	-4.07E+05	-3.11E+05	1.05E+05	4.01E+05	2%
7*	-6.09E+05	-4.64E+05	1.59E+05	6.00E+05	2%
9*	-8.20E+05	-6.01E+05	2.17E+05	8.06E+05	2%
10*	-9.67E+05	-6.78E+05	2.57E+05	9.45E+05	2%
11*	-1.12E+06	-8.11E+05	2.99E+05	1.09E+06	2%
12*	-1.27E+06	-9.22E+05	3.44E+05	1.24E+06	2%
No Control	**-1.18E+06**	**-7.76E+05**	**5.27E+05**	**1.44E+06**	-22%

Table 2 gives the results of the deformational behaviour of ISS. The last column of the table indicates the difference between displacements of cable staying system and at the point of indirect support. The differences reflect the values of the simulated gap performing FE analysis. Moreover the obtained values of the gap will be used for the calculations of rational displacement during experimental investigation.

Table 2: Analytical data (displacement)

ANSYS DATA (mm)								
Load steps	I10	I9	I8	I5	I4	I3	I2	GAP
1*	-2.05	-3.07	-3.05	-2.72	-3.05	-3.07	-2.05	**-1.21**
4*	-4.35	-6.51	-6.44	-5.75	-6.44	-6.51	-4.35	**-2.52**
7*	-6.49	-9.70	-9.59	-8.54	-9.59	-9.70	-6.49	**-3.70**
9*	-8.69	-12.98	-12.81	-11.40	-12.81	-12.98	-8.69	**-4.88**
10*	-10.19	-15.20	-14.98	-13.32	-14.98	-15.20	-10.19	**-5.67**
11*	-11.75	-17.52	-17.25	-15.32	-17.25	-17.52	-11.75	**-6.46**
12*	-13.34	-19.88	-19.55	-17.35	-19.55	-19.88	-13.34	**-7.25**
No Control	**-10.54**	**-14.79**	**-12.88**	**-10.12**	**-12.88**	**-14.79**	**-10.54**	

3. Testing Program

An experimental research program to validate proposed direct structural control of indirectly supported structures (ISS) methodology was undertaken by the authors at VGTU. It investigated ISS performance with direct design method provision, variations of strut length in order to control structural behaviour of the considering system.

3.1 Test model

The structure under observation is a three dimensional mono-strut under-slung beam (Fig. 5) consisting of two parallel situated substructures (Fig. 1).

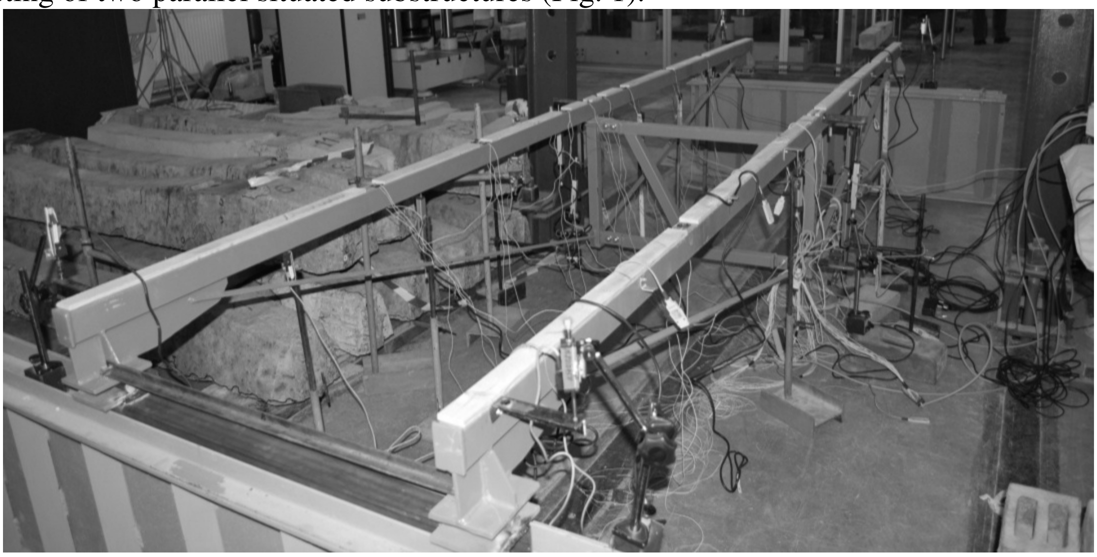

Figure 5: View of Test model

The main element of interest – beam works as a simply supported beam-column element with an intermediate elastic restraint by means of strut which stiffness relays on the axial force and displacement of the strut and varies according loads. The struts with the height of 1/8 of the span are situated at the middle of each substructure of the three dimensional system and connected to the beam with the possibility to control their length. Supporting system of each substructure is composed of two stay cables directly anchored to the beam at the support sections and deviated by the strut at the midspan. The main parameters of the sub-structure are presented in Table 3.

Table 3: Main parameters of the three dimensional test model

Parameter		**Test model**
Overall dimensions	Total length, mm	4000
	Left side, mm	2000
	Right side, mm	2000
	Length of the strut, mm	470
	Width, mm	800
Rigid beams	Cross-section area 60x40x2, mm^2	374
	Moment of inertia, mm^4	184100
	Modulus of elasticity, N/mm^2	210 000
Struts	Cross-section area □ 40x40x2, mm^2	294
	Modulus of elasticity, N/mm^2	210 000
Stay cables	Cross-section area — 25x4, mm^2	100
	Modulus of elasticity, N/mm^2	210 000

Avoiding lateral torsional buckling the under - slung beam has been made of rectangular cold formed hollow section less susceptible to torsional deformations. Moreover, in order to obtain the effect of geometrical nonlinearity of the beam the cross section has been chosen such that the slenderness parameter at the maximum loading would be 2,0-2,5 (Misiunaite 2013). The block of lateral bracings has been provided to simplify the spatial behaviour of the test model to in-plain behaviour of beam-columns.

The boundary conditions of the test model represent the simply-supported element with intermediate elastic restraint. Each substructure of under-slung beam model is free for in plane rotations due to provided steel rollers. One side of each substructure is free for longitudinal movement and the other one is restricted to 55 mm by providing controllers at the supports.

3.2 Test setup and loading protocols

In the testing process, only the vertical loads have been initiated. The weights of approximately 24.5 kg have been used to invoke the transverse distributed load. Loading has been performed by steps. For this stage of the study only symmetric loading has been considered. The first load step reflects the self-weight of the wooden deck used for transverse load distribution; the total load of 2692 kg has been applied in 12 steps with the approximate increment of 250 kg. Each step of loading has been performed in a short period of about 2 min.

The structural control of the system by means of variation of the strut length has been performed 7 times, at the load steps: 1*, 4*, 7*, 9*, 10*, 11* and 12*. With the increase of load the length of the strut has been decreased with the incremental magnitude according load-gap relationship used for the analytical investigation and shown in Fig. 4. For the first approach to structural control of the ISS the mechanical control method has been used with the attempt to initiate active structural control of the system. Therefore there was used the indicator at the place of the strut to measure simulated rational displacement by means of decreasing strut length (Fig. 6).

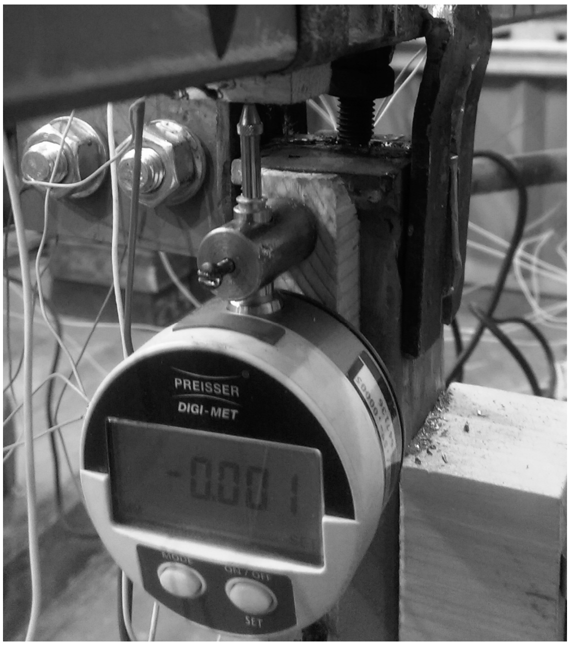

Figure 6: Simulation of ration displacement

Previously described loading scheme is shown in Fig. 7.

Load steps	q, N/mm
0	0
P	0.10
1/1*	0.56
2	0.81
3	1.05
4/4*	1.17
5	1.35
6	1.54
7/7*	1.72
8	1.90
9/9*	2.27
10/10*	2.63
11/11*	3.00
12/12*	3.37

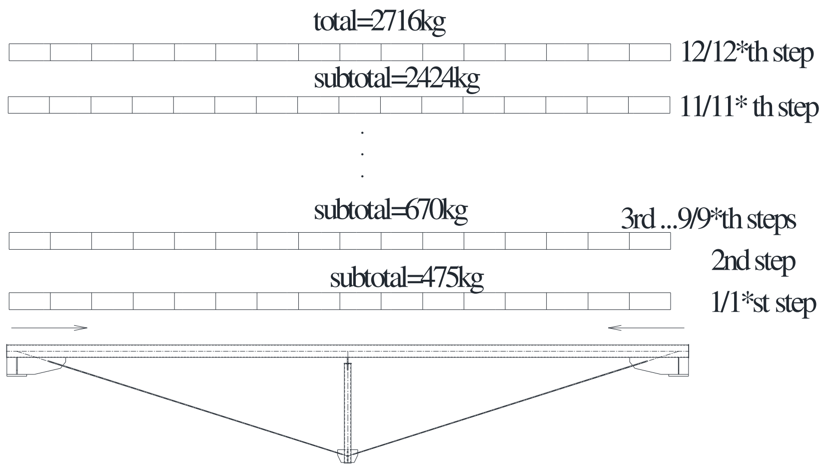

Figure 7: Load steps

In case to perform comparison between structural behaviour of controlled and non-controlled structural behaviour of ISS there was made testing of the same structural model without decreases of the strut for the maximum transverse loading of 3.37 N/mm.

4. Experimental validation

The obtained controlled bending moments of the tested model are summarized in Table 4. Noticing the symmetric conditions of the structure Table 4 presents just half of the results. The placement of the stress-strain gauges can be seen in Fig. 8. Columns titled of M2 and M13 indicates considering extreme bending moments, the last column gives the result of the controlling procedure. It can be observed that at all load steps when the structural control has been simulated were achieved sufficient results. The minor errors indicated in the last column of the Table 4 can be explained by the placement of the stress-strain gauges. It can be seen in the Fig. 8 that stress-strain gauges due to construction of the beam-strut connection at the intermediate support have been placed 25 mm away from the center, thus caused minor disagreement between extreme bending moments. Meanwhile, it can be seen (Fig. 8) that structural control caused the decrease of the extreme bending moment at the intermediate support in comparison with the flexural behaviour of the same structure without structural control at the maximum loading of 3.37 N/mm, thus can be related with the decrease in stress of the structure. Consequently, decrease in stress of the structure refers to its more stable performance. In addition the experimental results presented in Table 4 validates theoretical proposal of the structural control of ISS and criterion set for the extreme bending moments equalization.

Table 4: Experimental data (bending moments)

	Experimental Data (Nmm)				**Criterion**
Load steps	M2	M4	M6	M13	M13/M2
0	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0%
1*	-1.91E+05	-1.46E+05	3.84E+04	1.95E+05	-2%
4*	-4.18E+05	-3.18E+05	1.07E+05	4.17E+05	0%
7*	-5.94E+05	-4.79E+05	1.62E+05	6.05E+05	-2%
9*	-7.77E+05	-6.07E+05	2.23E+05	8.26E+05	-6%
10*	-9.93E+05	-6.98E+05	2.64E+05	9.59E+05	4%
11*	-1.12E+06	-8.24E+05	3.13E+05	1.13E+06	0%
12*	-1.24E+06	-9.20E+05	3.48E+05	1.24E+06	-1%
No Control	**-1.18E+06**	**-7.98E+05**	**5.10E+05**	**1.44E+06**	-22%

Table 5 indicated good agreements between experimental data and previously obtained analytical data using FE software it concludes in simple modeling and design for ISS direct structural control.

Table 5: Experimental validation (bending moments)

	Errors (ANSYS Data/Experimental Data)							
Load steps	M2	M4	M6	M13	M14	M22	M24	M26
1*	0%	0%	1%	4%	3%	1%	0%	0%
4*	3%	2%	2%	4%	4%	2%	3%	3%

7*	-3%	3%	2%	1%	1%	2%	3%	-4%
9*	-6%	1%	3%	2%	2%	2%	0%	-4%
10*	3%	3%	3%	1%	1%	3%	3%	4%
11*	0%	2%	4%	3%	3%	4%	2%	1%
12*	-3%	0%	1%	0%	1%	1%	0%	-2%
No Control	0%	3%	-3%	1%	1%	-3%	3%	1%

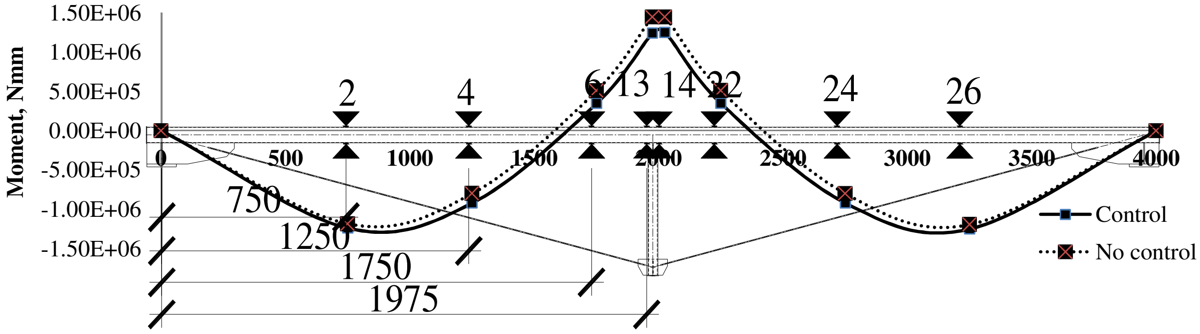

Figure 8: Comparison of experimental results of bending moment control

Table 5 presents experimentally obtained displacements at the considering points indicated in Fig. 9. Displacement sensor I5 indicates displacement of the supporting system which corresponds to the assumption of its equalization with the displacement at the beam-strut connection for the ISS structures without structural control. At this study the difference between the results at those points gives the variable magnitude of the rational displacement, which was simulated as the gap during analytical investigation. The last column of the Table 5 indicates this difference and it can be seen that it slightly differs from the analytically assumed one and used during experimental investigation. The minor errors between magnitudes of the rational displacements validate the assumptions made for the analytical investigation. From the two last rows of the Table 5 and Fig. 9 can be seen that structurally controlled structure deforms more than the same structure without structural control. Deformational increase refers to decrease in maximum stresses of the structure and more stable structural behaviour.

Table 5: Experimental data (displacement)

EXPERIMENTAL DATA (mm)								
Load steps	I10	I9	I8	I5	I4	I3	I2	GAP
1*	-2.04	-3.10	-3.16	-2.73	-3.16	-3.10	-2.04	**-1.28**
4*	-4.44	-6.50	-6.11	-5.6	-6.11	-6.50	-4.44	**-2.46**
7*	-6.59	-9.70	-9.35	-8.67	-9.35	-9.70	-6.59	**-3.79**
9*	-8.32	-12.67	-12.64	-11.58	-12.64	-12.67	-8.32	**-5.15**
10*	-9.68	-14.53	-14.90	-13.12	-14.90	-14.53	-9.68	**-5.82**
11*	-11.99	-16.63	-16.91	-15.21	-16.91	-16.63	-11.99	**-6.68**
12*	-13.40	-19.73	-19.73	-17.18	-19.73	-19.73	-13.40	**-7.16**
No Control	**-10.79**	**-14.44**	**-12.73**	**-10.07**	**-12.73**	**-14.44**	**-10.79**	

Table 6 gives the validation of the analytical results obtained using FE software in Section 2.3.

Table 6: Experimental validation (displacements)

Errors (ANSYS Data/Experimental Data)					
Load steps	I10	I9	I8	I5	I6
1*	-1%	1%	4%	0%	-4%
4*	2%	0%	-5%	-3%	-3%
7*	1%	0%	-3%	1%	1%
9*	-5%	-2%	-1%	2%	-1%
10*	-5%	-5%	-1%	-2%	-5%
11*	2%	-5%	-2%	-1%	-4%
12*	0%	-1%	1%	-1%	-1%
No Control	**2%**	**-2%**	**-1%**	**-1%**	**-2%**

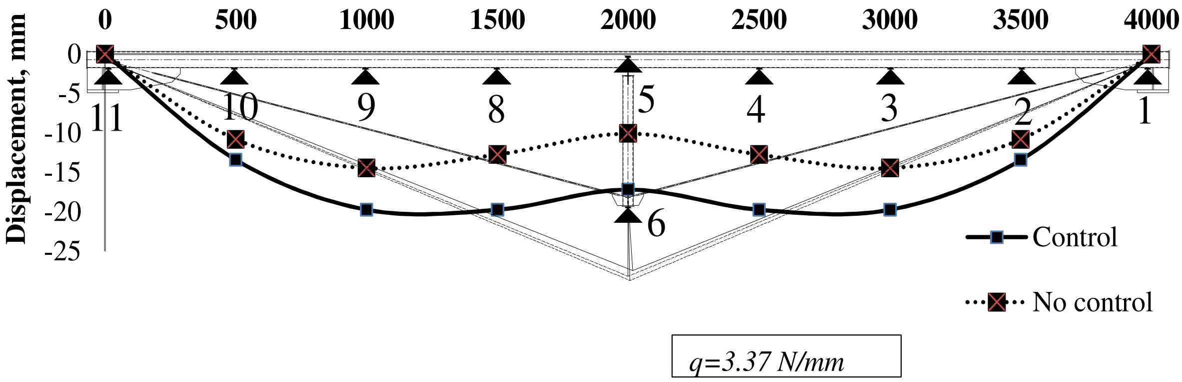

Figure 9: Comparison between controlled and non-controlled ISS displacements

Fig. 10 indicates the relationship obtained between displacement at the intermediate support and applied transverse load. The nature of the relationship curve reflects the one assumed for the rational displacement magnitude simulation used in analytical investigation and shown in Fig. 4. It can be seen that the curve represents linear relationship between transversal load and displacement. Drift at the point of 1.35 N/mm loading can be explained by differences in load increments at the beginning of the experimental investigation. Obtained linear relationship between displacement and the load as well as between load and rational displacement magnitude (Fig. 4) gives better prediction of structural behaviour of ISS during the process of control.

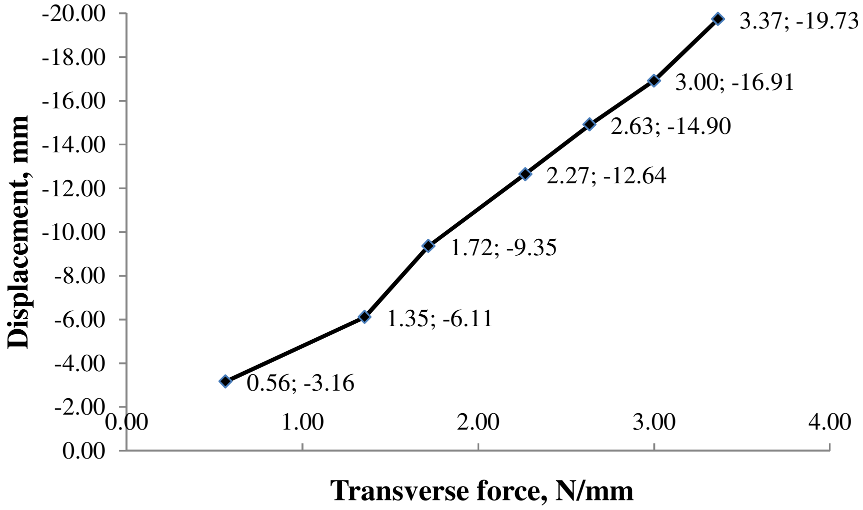

Figure 10: Relationship between transverse force and the controllable displacement

5. Conclusions

In this study, a three dimensional model of indirectly supported structure was tested under symmetric loading in order to validate proposal of structural response technique. The test results showed the possibility to stabilize structural behaviour of the considering structures by decreasing the stress when the extreme bending moments equalization criterion is used. Through the comparison analysis of the obtained analytical and experimental results, a good agreement has been achieved which approves the possibility directly control the structural behaviour of indirectly supported structures (ISS). Additionally, the structural control proposal presented in this paper includes theoretical methodology for generalized calculation model reflecting structural behaviour of the ISS, governing equations for obtaining flexural and deformational behaviour and detailed explanation of structural control procedure. It should be noted that this study was the first attempt to control structural behaviour of ISS by means of equalizing extreme moments and it contains many simplifications. Future prospects of this investigation focus on passive and active structural control of such structures. Passive structural control can be provided just for maximum design load using beam-strut connection based on friction or viscoelastic characteristics. For the more significant structures or in the case of seismic design the proposal can be used for active structural control using hydraulic systems for beam-strut connection and described computational method and obtained load – rational displacement magnitude relationship for development of control software.

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