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Buckling and design of columns with intermediate elastic torsional restraint

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Abstract

Cold-formed steel haunched portal frames are popular structures in industrial and housing applications. They are mostly used as sheds, garages, and shelters, and are common in rural areas. Cold-formed steel portal frames with spans of up to 30m (100 ft) are now being constructed in Australia. In the specific frame system analyzed herein, the column is partially restrained against twist rotation at an intermediate point where the knee brace joining the rafter and column is connected. An experimental program was carried out on a series of portal frame systems composed of back-to-back channels for the columns and rafters. It was found that changing the knee brace and knee brace-to-column connection bracket affected the buckling capacity of the column, however this was not captured in design calculations. In order to correctly predict frame behavior and ultimate loads for design purposes, the column buckling capacity must be accurately calculated. This paper presents an energy method approach to calculate the buckling load of a column with an intermediate elastic restraint. Various end conditions of the column are considered including the semi-rigidity of the column base. Displacement functions are determined based on measured experimental data. The Southwell and Meck plot methods to determine column buckling loads are discussed and results are compared to the buckling loads calculated by the energy method and to the experimental buckling loads. It is shown that the energy method outlined herein is satisfactory to determine column buckling loads.

1. Background of Experimental Program

An experimental program was carried out on a series of haunched portal frame systems composed of back-to-back channels for the columns, rafters, and knee braces. Members were connected together with double brackets bolted through the webs. The experimental setup consisted of three frames connected in parallel with purlins to simulate a free standing structure, however load was applied only to the center frame. Load was applied through a hydraulic jack, which connected to a load spreading system to distribute the load from the jack to eight points along the rafter, thus simulating a uniformly distributed vertical load. A horizontal jack was connected to the main jack, and was controlled by a transducer at the apex which measured frame sway. The horizontal jack moved equally with the frame sway, therefore maintaining the main jack in a vertical position.

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Transducers were placed at various locations on the frame to measure global movements in three directions, twist, and local deformations. This includes transducers at the column base, midheight of column, column at the knee connection, knee and knee connection brackets, eave connection, and apex connection. Additionally, strain gauges were attached to the column base to measure base reactions until a load cell was created as an improved alternative and placed under the columns in later tests to measure axial loads and bending moments. Strain gauges were placed at several locations on the column and sleeve stiffeners, when required. A photo of the experimental setup is shown in Fig. 1.



Figure 1: Experimental frame setup

A total of nine frames were tested: eight with unbraced columns and one with braced columns. Half of the unbraced column tests were with vertical loads only and half were tested with horizontal and vertical loads. The focus of this paper is frames with unbraced columns with applied vertical loads, thus only those four tests will be discussed herein. The testing program included several variations to the original frame configuration such as changes in the knee connection and the addition of sleeve stiffeners. These effects on the frame strength will be discussed. The results of the experiments are shown in Table 1, where ultimate loads, P_u , are given for the entire frame and per column. Vertical base reactions in the columns were determined by strain gauges near the column base for experiments 1 and 2, and by load cells on the column in each experiment. Experiments 1 and 2 were nominally the same frame configuration. Experiments 5 and 7 were intended to be companion tests, where the only change between them

was the addition of sleeve stiffeners to the column in experiment 7. However, the intended 3.0 mm (0.118 in) plates were unavailable at the time of testing for experiment 5, thus 3.3 mm (0.130 in) plates were substituted.

Design methods and calculations do not consider the effect of the knee brace to column connection bracket (KBC) on restraining the twist of the column. It was observed during the experiments that increasing the thickness of the bracket had a large impact on the frame ultimate load. However, as design calculations do not consider this effect, changing the thickness of the KBC bracket would be negated. Likewise, if the KBC bracket was too thin, the capacity of the column would be greatly underused and could result in an unsafe structure. Therefore the goal of this work is to determine a design method that correctly accounts for the effect of the knee to column connection.

Table 1: Column ultimate loads from experiments					
Experiment	t_{KBC} (mm) (in) —	$P_u \exp(kN)$ (kip)			
		Frame Total	Column N	Column S	
1	2×2.4 (0.094)	21.8 (4.90)	10.3 (2.32)	11.5 (2.59)	
2	2×2.4 (0.094)	22.8 (5.13)	11.1 (2.50)	11.7 (2.63)	
5	2×3.3 (0.130)	29.8 (6.70)	15.9 (3.57)	13.9 (3.12)	
7^{1}	2×3.0 (0.118)	29.8 (6.70)	15.8 (3.55)	14.0 (3.15)	

1. Experiment 7 includes column sleeve stiffeners

2. Southwell and Meck Plots

The frame failure mode was lateral-torsional buckling. Failure was initiated by movement of the knee brace to column connection plate (KBC) out-of-plane (*v*-direction), followed by significant lateral and torsional displacements of the column. After the ultimate load was reached, the column continued to rotate until a distortional buckle of the compression flange and local buckle of the lip occurred approximately 680 mm (26.8 in) below the knee connection, at which time the load dropped significantly. When the load was removed and frame disassembled, the column reverted to the pre-bucked position, except for the small distortional and local buckle. Therefore, it can be concluded that the column failed in global elastic buckling, and the ultimate load, P_u , is approximately equal to the elastic buckling load.

A method for determining elastic buckling of a column from experiments was developed by Southwell in 1932 (Singer 1989). This method was originally applied to flexural buckling, where deflection of the column, δ , is measured as the column is loaded axially, *P*, at a location where the deflection is predicted to be the largest. Then the results are plotted as δ/P against δ , which yields a linear relationship, and the slope, $1/P_{cr}$, is the inverse of the critical buckling load, P_{cr} . The intercept on the δ axis reflects the initial column imperfection. The method allows buckling loads to be determined experimentally without having to fail the specimen (Mandal 2002).

It has been shown that the Southwell plot can be extended to lateral-torsional bucking by plotting a characteristic deflection of either lateral deflection v, or twist ϕ . Likewise, the elastic buckling load is the inverse slope of the plots of v/P against v or ϕ/P against ϕ . This method works as after the initial stages of loading, v and ϕ are proportional to each other so there is a direct coupling between them (Mandal 2002). This method can also be applied to cantilever columns (Ings 1987).

Meck proposed an alternative version of the Southwell plot for lateral-torsional buckling where two plots are made, v/P against ϕ with a slope of $1/\beta$, and ϕ/P against v with a slope of $1/\alpha$. The critical buckling load, P_{cr} , is the geometric mean of the two slopes, and is calculated as shown in Eq. (1). As in the Southwell plot, the intercepts on the v and ϕ axes indicate the initial imperfections. The Meck plot represents the physical interdependence of lateral flexure and torsion in lateral-torsional bucking. The Meck plot method was determined to be more reliable than the Southwell plot method for lateral-torsional buckling. However the Southwell method yields satisfactory results and can be used when only v or ϕ is measured, whereas the Meck plot requires both measurements (Mandal 2002).

$$P_{cr} = \left(\alpha\beta\right)^{0.5} \tag{1}$$



Figure 2: Elastic buckling plots of experiment 1 (a) Southwell plots for v and ϕ , and (b) Meck plots for v and ϕ

Other plotting techniques to determine elastic buckling loads are the Massey and Modified Plot methods. The Massey method plots δ/P^2 against δ . It was determined that there is no benefit to using the Massey plot over the Southwell plot (Mandal 2002). The Modified Plot method plots $P\delta$ against δ (Trahair 1969). The Massey method works if there is only one kind of imperfection (e.g. v or ϕ) and does not yield perfectly straight line plots for beams with initial imperfections of crookedness and twist. The Modified Plot method is an improvement over the Massey method in

that it has an increased range of linearity and therefore leads to more accurate determination of the buckling load (Zirakian 2010). The above plotting techniques have been extended to lateraldistortional buckling (Zirakian 2008, 2010).

Southwell and Meck plots have been created for experiments 1 and 2, and are shown in Figs. 2 and 3, respectively. As the column begins to buckle in a lateral-torsional manner, the relationships between v/P against v and ϕ/P against ϕ become linear. The same holds true for the linear relationship in the Meck plots. This linear portion, shown in the dashed red line, is then used to determine the critical buckling load of the column.



Figure 3: Elastic buckling plots of experiment 2 (a) Southwell plots for v and ϕ , and (b) Meck plots for v and ϕ

The critical buckling loads calculated from the Southwell and Meck plots are presented in Table 2. Overall both methods predict fairly well the experimental buckling load. The largest difference in the Southwell predictions is for the twist rotation for experiment 2, with a 3.4% difference. The other Southwell predictions are below a 2% difference. The Meck plot predictions for buckling for both experiments are within 1% of the experimental value. This agrees with Mandal's (2002) conclusion that the Meck plot is more reliable than the Southwell plot, yet the accuracy of the Southwell plot is satisfactory.

Experiment	P_u (kN) (kip)		P_{cr} (kN) (kip)	
number	Experiment	Southwell v	Southwell ϕ	Meck
1	11.5 (2.59)	11.7 (2.63)	11.4 (2.56)	11.6 (2.61)
2	11.7 (2.63)	11.6 (2.61)	12.1 (2.72)	11.8 (2.65)

Table 2: Comparison of column buckling loads from Southwell and Meck plotting methods

3. Energy Equations

3.1 Internal actions of column

Consider a portal frame, as shown in Fig. 4, with pinned bases, length of column *L*, height from column bases to apex L_a , and horizontal distance from the eaves to column L_h . The knee brace is connected to the column at $\frac{3}{4L}$ from the bottom at an angle θ with the vertical. There is a uniformly distributed load q on the rafters. For the frame analyzed experimentally herein, L=5285 mm (17.3 ft), $L_h=6800 \text{ mm}$ (22.3 ft), $L_a=6600 \text{ mm}$ (21.7 ft), and $\theta=50^\circ$. The crosssection of the column under consideration is a 2×C200-19 back-to-back lipped channel section with overall nominal depth of 203 mm (8.0 in) and nominal thickness of 1.9 mm (0.07 in).



Figure 4: Frame geometry and layout

Assuming pinned connections between all members, the frame is determinate and the horizontal reaction force at the base, H, can be determined by taking the sum of moments about the apex, as shown in Fig. 5(a). Therefore the horizontal force is calculated as shown in Eq. (2). The actions on the column in between the knee and the eaves are shown in Fig. 5(b), where F is the force in the knee brace. Taking the sum of moments about the apex yields the force in the knee brace, Eq. (3). The axial forces acting in the column are shown in Fig. 5(c). The vertical reaction force, P, is equal to qL_h , and is the column buckling load when $P=P_{cr}$. The axial force in the column in between the knee and the eaves, x, is calculated as shown in Eq. 4. This is shown as a compressive force in Fig 5(c), however in actuality is a tensile force. The axial force and moment distributions in the column can now be determined, and are shown in Fig. 6(a).

For fixed base and semi-rigid column bases, the structure is indeterminate, therefore an FE program, MASTAN2 (McGuire 2000) was used to determine the base reactions and moment distributions. A pinned base model was first created and the results matched Eqs. 2-4, validating the modeling technique. Next, models were created with fixed bases and semi-rigid bases. The

semi-rigid column base was modeled using a spring with the average spring stiffness as measured during experiments. The base reactions and moment at the knee were compared to the known actions in the pinned base case: H and M_{kp} , where $M_{kp}=\frac{3}{4}LH$ is the moment at the knee for the pinned base column. The moment distributions and horizontal reaction forces for the fixed and semi-rigid column bases are shown in Figs. 6(b) and 6(c) respectively, where the fractions are close approximations to the actions obtained from MASTAN2. The knee brace force was calculated for the fixed and semi-rigid bases using a free-body diagram similar to Fig. 5(b), resulting in the knee brace forces of $F = 15H/4\sin\theta$ for the fixed column base and F = $13H/4\sin\theta$ for semi-rigid column base.

$$H = \frac{1}{2} P \frac{L_h}{L_a} \tag{2}$$

$$F = \frac{4H}{\sin\theta} \tag{3}$$

$$x = P - F \cos\theta \tag{4}$$



(a)

Figure 5: FBD of (a) partial frame, (b) column from knee to eave, (c) axial forces in column

3.2 Work done by forces acting at the knee connection

As the knee brace moves out of the plane of the frame at an angle α , it causes the column to move laterally and twist. The knee brace force can be decomposed into forces acting out of plane, F_{v} , and forces in the plane of the frame, F_{uy} , which can then be decomposed into forces F_{u} and F_{y} , as shown in Fig. 7(a) and Eqs. 5-7. Figure 7(b) shows the knee movement from the top view of the frame. The knee brace connects to the column at the flange-web junction of the backto-back channels, as shown in Fig. 8(a). Therefore the knee brace forces act on the column at the flange-web junction, and to calculate the work done by the force in the knee brace, W_k , as calculated in Eq. 8, the displacements at the flange-web junction, v_{kc} and u_{kc} , must be determined.

$$F_{\rm v} = F \sin \alpha \tag{5}$$

$$F_{uy} = F \cos \alpha \tag{6}$$

$$F_{u} = F_{uy} \sin \theta = F \cos \alpha \sin \theta \tag{7}$$

$$W_k = F_v v_{kc} + F_u u_{kc} \tag{8}$$



Figure 6: Column force and moment distributions for (a) axial force and moment for pinned base column, (b) moment for fixed base column, and (c) moment for semi-rigid column base

The displacements of the column as the knee brace forces it to twist and displace laterally are shown in Fig. 7(c). The displacements of the column at the flange-web junction are calculated in Eqs. 9-10, where v_k and ϕ_k are the lateral displacement of the column and the column twist, respectively, at the location of the knee connection (${}^{3}\!/_{4}L$ from the floor), d_s is the distance from the load application point to the shear center of the column, and using small angle approximations for ϕ_k . Substituting Eqs. 9-10 into Eq. 8 yields Eq. 11 to calculate the work done on the column by the knee connection.

$$v_{kc} = v_k + d_s \sin \phi_k \cong v_k + \phi_k d_s \tag{9}$$

$$u_{kc} = d_s \left(1 - \cos \theta_k\right) \cong 1/2 \, \phi_k^2 \, d_s \tag{10}$$

$$W_{k} = F \left(v_{k} + \phi_{k} d_{s} \right)^{2} / L_{k} + 1/2 F \sin \theta \, \phi_{k}^{2} \, d_{s}$$
(11)



Figure 7: Work done by knee connection (a) column forces at knee connection, (b) knee movement out of plane, (c) column movement at knee connection



Figure 8: Knee to column connection (a) experimental setup with sleeve stiffener in column, and (b) drawing showing KBC plate area (mm)

3.3 Strain energy in KBC connection plate

The knee brace to column connection plate, KBC, acts as a torsional spring to the column. The strain energy stored in the KBC plate, U_s , helps to restrain the column against twist at that location, and is calculated in Eq. 12, where k_{ϕ} is the torsional stiffness of the plate and $\Delta \theta_{kc}$ is the total rotation of the plate. The plate stiffness is calculated using plate bending theory (Timoshenko 1989) where the plate is bent in only one direction. The KBC bracket is composed of 2 plates bolted together, thus the plate stiffness is double that of a single plate. The plate stiffness is calculated in Eq. 13, where *h* is the plate height, *b* is the plate width (Fig. 8(b)), *E* is Young's modulus, t_k is the thickness of a single KBC connection plate, *v* is Poisson's ratio, and *D* is the plate flexural rigidity. The total rotation of the plate, Eq. 14, is the sum of the column rotation at the knee, ϕ_k , and the rotation due to the knee brace movement, α , as shown in Fig. 7(b). Using small angle approximations, $\tan \alpha \approx \alpha$.

$$U_s = 1/2 k_{\phi} (\Delta \theta_{kc})^2 \tag{12}$$

$$k_{\phi} = 2\left[\frac{M_x}{\Delta\theta_x}\right] = 2\left[\frac{h\left[-D\frac{\partial^2 w}{\partial x^2}\right]}{\Delta\theta_x}\right] = 2\left[\frac{hD^{\Delta\theta_x}}{\Delta\theta_x}\right] = 2\left[\frac{Et_x^3}{12(1-v^2)}\right] \left(\frac{h}{b}\right)$$
(13)

$$\Delta \theta_{kc} = \phi_k + \frac{v_{kc}}{L_k} \sin \theta \tag{14}$$

3.4 Sleeve stiffeners

Experiment 7 had sleeve stiffeners placed inside the column and rafters at the knee connection, as pictured in Fig. 8(a). The sleeve stiffeners were 1800 mm long, connected at the knee so that $\frac{1}{3}$ of the stiffener was above the knee while $\frac{2}{3}$ was below. The stiffener was connected to the column by bolts through the web at the knee connection only. Strain gauges on the inside of the stiffener and on the outside of the column channels were placed to determine the moment carried by the column and stiffeners. Strain gauges were located at the knee connection, 300 mm above and 500 mm below the knee connection.

After post-processing the experimental data, it was determined that at the knee connection, the sleeve stiffeners carried approximately $\frac{1}{3}$ of the bending moment in the column, while at 300 mm above and 500 mm below the knee connection the stiffeners carried approximately 1/13 and $\frac{1}{4}$, respectively, that of the column. To account for the effects of the stiffeners, the moment distribution of the column (Fig. 6(c)) can be altered around the knee connection, as the stiffeners reduced the moment carried by the column. An approximate distribution was determined where between 100 mm above to 300 mm below the knee connection, the moment in the column was equal to $\frac{2}{3}$ that of the unstiffened column to account for the stiffeners taking around $\frac{1}{3}$ of the moment. Between 300 mm to 600 mm below the knee connection, the moment in the column was equal to $\frac{3}{4}$ that of the unstiffened column to account for the stiffeners carrying roughly $\frac{1}{4}$ of the column moment.

Due to the connection design of the stiffeners to the column, the sleeve stiffeners were only partially effective. Consequently it would be difficult to calculate effective section properties of the combined column and stiffener. During the experiments, the stiffeners popped out of the column channel at buckling and had subsequently no effect on the buckled shape. Therefore altering the moment distribution based on the experimental relationship between column and stiffener moment is the preferred method to account for sleeve stiffener effects.

3.5 Total energy

The total energy is the sum of the flexural, torsional, and warping strain energy stored in the column, the potential energy associated with the internal axial force N and bending moment M, the work done by the forces acting at the knee brace connection, and the spring energy stored in the KBC connection plate (Eq. 15). The distribution of internal actions N and M are as shown in Fig. 6, where N = P between 0 and $\frac{3}{4}L$, and N = x between $\frac{3}{4}L$ and L. For the pinned base column, the moment distribution along the column height z, is M(z) = Hz for $0 \le z \le \frac{3}{4}L$, and M(z) = -3Hz + 3HL for $\frac{3}{4}L \le z \le L$. Similar relationships can be calculated for the fixed base and semi-rigid base column conditions.

$$V_{T} = \frac{1}{2} \int_{0}^{L} \left(EI_{y} v''^{2} + EI_{w} \phi''^{2} + GJ \phi'^{2} \right) dz + \frac{1}{2} \int_{0}^{L} M(2\phi v'') dz - \frac{1}{2} \int_{0}^{L} N(v'^{2} + r_{2}^{2} \phi'^{2}) dz - F(v_{k} + \phi_{k} d_{s})^{2} / L_{k} + \frac{1}{2} F \sin \theta \phi_{k}^{2} d_{s} + \frac{1}{2} k_{\phi} (\Delta \theta_{kc})^{2}$$
(15)

3.6 Displacement Functions

Solutions to the energy equations are highly dependent on the assumed displacement fields. The more accurately the displacement functions reflect the true displacements, the more precise the solutions will be (Timoshenko 1989). Standard cases are columns with pinned bases and fixed bases. Displacement functions for a pin-ended column, v_p and ϕ_p , can be represented by a sine function, as shown in Eqs. 16-17. Displacement functions for fixed-pinned column, v_f and ϕ_f , can be represented by a limited power series as shown in Eqs. 18-19 (Trahair 1993).

$$v_p = A_v \sin\left(\frac{\pi z}{L}\right) \tag{16}$$

$$\phi_p = A_\phi \sin\!\left(\frac{\pi z}{L}\right) \tag{17}$$

$$v_f = A_v \left(\frac{z^2}{L^2} - \frac{z^3}{L^3} \right)$$
(18)

$$\phi_f = A_{\phi} \left(\frac{z^2}{L^2} - \frac{z^3}{L^3} \right)$$
(19)

Displacements at various locations along the column were measured during the experiments with LVDTs. A typical column displaced shape with the LVDTs is shown in Fig. 9. Displacements in the *v*-direction were measured on the column at 2000 mm (6.56 ft) from the base, at the knee connection, and at the top of the column at the eave connection. Twist displacements in the ϕ -

direction were measured 150 mm (5.91 in) from the column base, on the column at 2000 mm (6.56 ft) from the base, and at the knee connection. From the buckled column shape shown in Fig 9, the displaced shape can be characterized as a sine curve where the max displacement occurs closer to the knee connection than half-way along the column length, and therefore can be represented as a combination of sine curves. Displacement functions, v_e and ϕ_e , were fitted to the experimental measurements and are given in Eqs. 20-21.





Figure 9: Buckled columns during experiments (a) experiment 7 and (b) experiment 5

$$v_e = A_v \left(\sin\left(\frac{\pi z}{L}\right) - 0.2 * \sin\left(\frac{2\pi z}{L}\right) \right)$$
(20)

$$\phi_e = A_{\phi} \left(\sin\left(\frac{\pi z}{L}\right) - 0.2 * \sin\left(\frac{2\pi z}{L}\right) \right)$$
(21)

4. Solution and Comparison to Buckling Loads

The column buckling load can be solved by minimizing the total energy of the column, $\delta V_T = 0$, where V_T is given in Eq. 15. For non-trivial solutions, Eq. 22 must be solved.

$$\frac{\partial V_T}{\partial A_V} = \frac{\partial V_T}{\partial A_{\varphi}} = 0 \tag{22}$$

The buckling load was determined for various boundary conditions with the corresponding displacement functions. The moment distributions and base reaction forces considered are those shown in Fig. 6 while the displacement functions are those given in Eqs. 16-21. Several thicknesses of KBC plates were considered for the semi-rigid column base to reflect the various plates tested in the experiments. The results of the energy analyses are presented in Table 3.

Table 3: Results of energy analysis					
Disp. function	t_{KBC} (mm) (in)	P_{cr} (kN) (kip)			
Pin-pin	2×2.4 (0.094)	9.06 (2.04)			
Fixed-pin	2×2.4 (0.094)	22.2 (4.99)			
Experimental	2×2.4 (0.094)	12.7 (2.86)			
Experimental	2×3.0 (0.118)	15.4 (3.46)			
Experimental	2×3.3 (0.130)	17.1 (3.84)			
	Table 3: ResultsDisp. functionPin-pinFixed-pinExperimentalExperimentalExperimentalExperimental	Table 3: Results of energy analysisDisp. function t_{KBC} (mm) (in)Pin-pin 2×2.4 (0.094)Fixed-pin 2×2.4 (0.094)Experimental 2×2.4 (0.094)Experimental 2×3.0 (0.118)Experimental 2×3.3 (0.130)			

Increasing the thickness of the KBC plate from 2.4 mm (0.094 in) to the next available size of 3.0 mm (0.118 in) increases the buckling load by 21.3%. Therefore the rotational restraint provided by the KBC bracket has a significant effect in restraining the column twist. This was displayed in experiments as movement of the KBC bracket was immediately followed by column buckling.

The critical buckling load obtained from the energy analysis (12.7 kN or 2.86 kip) is 9.48% and 7.76% greater than the estimated buckling loads from the Meck plotting method for experiments 1 and 2 respectively, as given in Table 2. As was previously discussed, the Meck plotting method predicts the column ultimate load within 1%. As the column failed by elastic lateral-torsional buckling, the column ultimate load is approximately equal to the column buckling load.

A comparison of the column ultimate loads in experiments with those determined by the energy analysis using Eqs. 20-21 for the displacement field are shown in Table 4. The difference between the experimental ultimate loads and energy method buckling loads is less than 10% for all experiments. The percentage difference between the experiments and the energy method could be further reduced by refining the displacement functions. Displacements for lateral deflection and twist were measured at four locations along the column. Measuring the deflections at additional locations would improve the calculated displacement function. However, as the difference is less than 10%, the aforementioned energy method is considered suitable to predict

the critical buckling load for similar structures to that of the experimental program discussed herein, where the failure mode is elastic lateral-torsional buckling.

Table 4. Comparison of buckning loads from energy method and experiments						
Experiment	$t_{\rm KBC} ({\rm mm})$	Sleeve	P_u (kN) (kip)	P_{cr} (kN) (kip)	% difference	
number	(in)	stiffeners	Experiments	Energy	70 uniterence	
1	2×2.4 (0.094)	No	11.5 (2.59)	12.7 (2.86)	9.45	
2	2×2.4 (0.094)	No	11.7 (2.63)	12.7 (2.86)	7.87	
5	2×3.3 (0.130)	No	15.9 (3.57)	17.1 (3.84)	7.02	
7	2×3.0 (0.118)	Yes	15.8 (3.55)	16.7 (3.75)	5.39	

Table 4: Comparison of buckling loads from energy method and experiments

7. Conclusions

An energy method approach to calculate the buckling load of a column with an intermediate elastic restraint is presented. The effects of base end conditions, column moment distribution, and the rotational restraint of the knee brace to column connection is discussed. A method to account for internal sleeve stiffeners in the column is presented. Appropriate displacement functions are determined based on experimental data. Southwell and Meck plots of the experimental data are presented and were deemed sufficient for determining column buckling loads for the type of structure tested herein. The energy method approach estimates the experimental column buckling load within 10% and is therefore a suitable method for calculating column buckling loads in lieu of conventional design methods which do not account for the torsional spring effect of connection elements.

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