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Discrete Bracing Displacement Compatibility Solutions for Cold-Formed C- and Z-sections in Flexure and Torsion

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Abstract

A method is presented for calculating discrete brace forces in cold formed C- and Z- sections based on lateral and torsion displacement compatibility of the sections. By eliminating pure torsion, the calculation of brace forces is simplified with minimal loss in precision. The method also describes in detail the process to calculate the brace forces at the support locations which are commonly incorrectly applied. To facilitate the use of the method, summary tables are provided for many common brace and load combinations.

Introduction

C- and Z- sections supporting standing seam roof systems are partially braced by the standing seam system. The interaction between the purlin the roof system is complex and difficult to quantify. As a result of these complexities, one of the strategies for analysis is to largely ignore the restraining effects of the sheathing and provide discrete bracing points along the length of the member. The magnitude of these brace forces is important because they must be transferred through the structure and on large roof systems, the forces can be substantial.

The principal axes of Z-sections are rotated relative to the plane of the web and both C- and Z-sections are often loaded eccentric to the shear center. Therefore, adequate bracing is essential to ensure that C- and Z-sections can attain a reasonable capacity and minimize deflections and the corresponding second order stresses. This paper reviews the classical mechanics solutions to determining brace forces and offers some simplifications. It also compares the calculated brace forces to the discrete bracing provisions of the AISI Specification (2012) which for most bracing configurations is overly conservative. The provisions are also often misapplied when used for braces at the frame lines.

Lateral brace forces are calculated by a combination of displacement compatibility and equilibrium equations. To determine the magnitude of the braces along the span of a purlin, a displacement compatibility, aka force method, approach is used. Lateral deflection and torsion rotation of the purlin are calculated in an unbraced condition, then the brace forces are those forces, assuming the brace is rigid, that restore the member to its original position. Once the braces forces along the span are determined from displacement compatibility, the forces at the supports or frame lines are

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determined from equilibrium. Lateral and torsion deformations are treated separately and then superimposed. Flexible braces can be analyzed by incorporating braces stiffness into the compatibility analysis. However, ignoring brace flexibility will result in a conservative estimate of the brace force. Therefore, the analysis presented is based on rigid braces.

1. Load Effects requiring bracing

The loads applied to purlins that generate braces forces in C- and Z-sections are divided into 3 categories: (1) the loads applied oblique to the principal axes, (2) the torsion loads, and (3) the weak axis (downslope) loads.

1.1 Loads Oblique to Principal Axes

The deflection of a beam loaded oblique to its principal axes is calculated by reducing the load into components directed along the principal axes and superimposing the deflections in the principal axis directions. Zetlin and Winter (1955) showed that the calculation of the lateral deflection of beam loaded oblique to its principal axes could be simplified by applying a fictitious horizontal load and a modified moment of inertia. For the Z-section shown in Figure 1 with an externally applied load along the y-axis, W, the fictitious horizontal load along the orthogonal x-axis is $W(I_{xy}/I_x)$ where I_{xy} is the product of inertia and I_x is the moment of inertia of the cross section with respect to the orthogonal x and y axes. The modified moment of inertia, I_{my} is

$$I_{my} = \frac{I_x I_y - I_{xy}^2}{I_x} \tag{1}$$

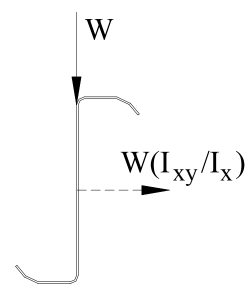


Figure 1. Fictitious Horizontal Load

Conventional deflection formulas are applied to calculate the deflection of the beam along the x-axis resulting from forces applied along the y-axis. The applied load is replaced by the fictitious load and the moment of inertia is replaced by the modified moment of inertia. For example, the mid-span x-axis deflection of a simple span Z-section shown resulting from a uniformly applied load along the y-axis is

$$\Delta = \frac{5\left(w \frac{I_{xy}}{I_x}\right) L^4}{384 E I_{my}} \tag{2}$$

At the location of the brace, displacement compatibility is enforced. That is, the brace force is the magnitude of the concentrated force in the x-direction required to cause the same x-axis displacement as the applied load in the y-direction. Again, the conventional formulas for deflection can be used with the modified moment of inertia in place of the moment of inertia in the deflection formula. For the example of a midpoint brace applied at the shear center of a simple span beam the displacement at mid-span caused by the brace is

$$\Delta = \frac{P_{L,mid} L^3}{48 E I_{my}} \tag{3}$$

Equating the two displacements and solving for the total brace force, P_L, the brace force at mid-span for a uniformly loaded simple span Z-section is

$$P_{L,mid} = \frac{5}{8}\left(w \frac{I_{xy}}{I_x}\right) L \tag{4}$$

The brace force at the mid-span in Equation 4 requires that the purlin is laterally supported at its support locations. The magnitude of the brace forces at the support locations is determined from equilibrium. Since the brace force at mid-span is generated from a load applied in the y-axis, the net sum of forces in the x-axis direction should be zero. Therefore the sum of the brace forces at the support location must be equal and opposite to the interior braces. Applying symmetry, the total brace force at each frame line is

$$P_{L,spt} = -\frac{5}{16}\left(w \frac{I_{xy}}{I_x}\right) L \tag{5}$$

1.2 Torsion

Torsion deformations are the combination of pure (St. Venant's) torsion and warping torsion. The open cross sections of C- and Z-sections have very little resistance to pure torsion and most of the resistance to deformation results from warping resistance. As a result, when using displacement compatibility to determine brace forces, warping torsion dominates the behavior and pure torsion can be reasonably eliminated from the solution. By eliminating pure torsion, the solution is greatly simplified.

From Carter and Seaburg (1997), the general equation for torsion moment resisted by a cross section, T, is

$$T = GJ\theta' - EC_w\theta''' \tag{6}$$

Because the pure torsion rigidity, GJ, is small relative to the warping torsion rigidity, EC_w,, the pure torsion rigidity is considered negligible, ($GJ \approx 0$). Therefore, Equation (6) reduces to

$$T \approx EC_w\theta''' \tag{7}$$

The following equation then describes the relationship between the function describing the torque with respect to the location z along the length and the 4th derivative of the angle of rotation.

$$t(z) \approx EC_w\theta'''' \tag{8}$$

The reader will notice the parallels between the above equation and the more recognized equation that relates the 4th derivative of the deflection of a beam to the loading function.

$$w(z) = EIy'''' \tag{9}$$

Therefore, in the same way, the deflection of a beam is determined by integrating Eq. 9, Eq 8 can be integrated to approximate the torsion rotation of a beam. Note that Eq 8 should not be used to calculate the actual torsion rotations as it will overestimate the rotation for a given load. However, when the equation is used for a displacement compatibility analysis, since pure torsion is ignored in the primary load condition and the redundant brace condition, the overestimation is balanced in resulting in negligible error.

For the example of the simple span beam with a uniformly distributed torque and a torsion brace at mid-span, the mid-span rotation resulting from the uniformly distributed torque is

$$\phi \approx \frac{5tL^4}{384EC_w} \tag{10}$$

The mid-span rotation of the resulting from the mid-span brace is

$$\phi \approx \frac{T_L L^3}{48EC_w} \tag{11}$$

Equating these two rotations, the resulting brace torque is

$$T_L = \frac{5}{8} tL \tag{12}$$

The above simplified equation is compared to the full torsion equation where both warping and pure torsion is considered. The brace torque in the full equation is

$$T_L = (\alpha) tL \tag{13}$$

where

$$\alpha = \left(\frac{\frac{a^2}{GJ}\left(\frac{L^2}{8a^2} + \frac{1}{\cosh\left(\frac{L}{2a}\right)} - 1\right)}{\frac{L^2}{GJ}\left(\frac{1}{4} - \frac{a}{2L}\tanh\left(\frac{L}{2a}\right)\right)} \right) \tag{14}$$

Equation 14 is plotted in Figure 2 for a representative range of C- and Z-sections from the AISI Design Manual (2008) with depths ranging from 6 in. to 12 in. Note that if only warping torsion is considered, α = 5/8 and if only pure torsion is considered, α = 1/2. Therefore the range of solutions considering both warping and pure torsion will range between 1/2 and 5/8. For short span lengths, α matches the value of 5/8 calculated by the approximate warping-only equation. As the span length increases, there is some deviation from 5/8. The largest deviations occur for shallow purlins with thick gauges and narrow flanges. At a maximum practical span length of 50 feet, the deviations of the warping-only approximation are within 5% of the true solution. Only for ridiculously long spans on the order of 500 feet does the multiplier approach pure torsion dominated value α = ½.

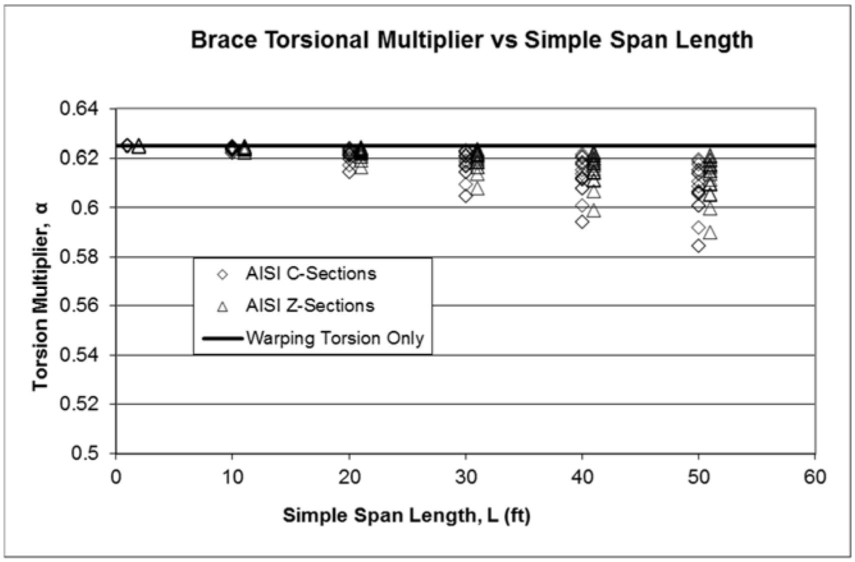


Figure 2. Torsion Multiplier Comparison

By eliminating pure torsion, the solution is greatly simplified with very little loss in accuracy. The solution will parallel the solution for the brace forces for loads oblique to the principal axes. Therefore, in developing compatibility solutions, the same factors derived for displacement compatibility can be applied to torsion compatibility.

For the brace forces at the frame lines, it is import to consider the eccentricity of the reactions at the supports. Along the interior of the span, the torsion moments are calculated relative to the shear center. At the frame support location, it is generally assumed that the reaction is along the web of the C- or Z- section. Therefore, when the shear center does not coincide with the web of the purlin, this eccentricity causes an additional moment at the support location. The total moment along the length of the purlin is balanced by the moment at the support locations using equilibrium equations.

Using the example of the uniformly loaded simple span C- section braced at the mid-span, the total moment along the span is

$$M_z = \left(-w_x e_{sy} + w_y e_{sx}\right)L \tag{15}$$

The concentrated moment at the mid-span brace is

$$T_{L\_mid} = -\frac{5}{8}M_z = -\frac{5}{8}\left(-w_x e_{sy} + w_y e_{sx}\right)L \tag{16}$$

The concentrated moment at each support brace is

$$T_{L\_spt} = -\frac{1}{2}\left(M_z - \frac{5}{8}M_z\right) - R_y m = -\frac{3}{16}\left(-w_x e_{sy} + w_y e_{sx}\right)L - \left(-\frac{w_y m}{2}\right)L \tag{17}$$

Where m, is the eccentricity of the shear center as shown in Figure 3. The term representing the component of the applied load directed along the y-axis, w_y, is rearranged to fit the uniform solution framework presented in the following sections,

$$T_{L\_spt} = -\frac{3}{16}\left(-w_x e_{sy} + w_y(e_{sx} - m)\right)L + \frac{5}{16}\left(w_y m\right)L \tag{18}$$

Note that the 5/16 multiplier in the second term will match the multiplier in Equation 5.

1.3 Downslope forces

For C- and Z- sections used in sloped roof systems, the forces acting on the sloped purlin are resolved into components parallel and perpendicular to the web of the section as shown in Figure 3. Those forces perpendicular to the web, ie directed along they y-axis, are referred to as the downslope forces. The displacement compatibility solution for downslope forces is performed in a traditional manner. The purlin is allowed to deflect laterally according to the downslope forces and the brace force is the force that is required to restore the lateral displacement to the original position. For a beam subjected to a uniform load w, the downslope force is w_y, and the brace force at mid-span is

$$\mathrm{P_{L,mid}} = \frac{5}{8}\left(\mathrm{w}_y\right)\mathrm{L} \tag{19}$$

The forces at the frame lines again are those that satisfy equilibrium. The sum of forces at the frame line and supports equal the total downslope force. For example, the brace force at each frame line for a uniform downslope force braced at mid-span is

$$\mathrm{P_{L,spt}} = \frac{3}{16}\left(\mathrm{w}_y\right)\mathrm{L} \tag{20}$$

2. Generalized Compatibility Equation

Utilizing the general nomenclature from the AISI Specification (2012), a general equation framework is established for compatibility solutions of brace forces. The generalized equation combines unsymmetric bending effects, torsion effects, and downslope load effects. Brace forces are defined according to the span between the supports. The equations include brace forces along the interior of the span in addition to the brace forces at the support location. The total brace force at a location, P_L, is divided into a brace force at the top flange, P_{L1}, and at the bottom flange, P_{L2}. The general equations of the compatibility solution are

$$\mathrm{P_{L1}} = \mathrm{C_1 K'} - \mathrm{C_2}\frac{\mathrm{U_x}}{2} + \mathrm{C_2}\frac{\mathrm{M_z}}{\mathrm{d}} \tag{21}$$

$$\mathrm{P_{L2}} = \mathrm{C_1 K'} - \mathrm{C_2}\frac{\mathrm{U_x}}{2} - \mathrm{C_2}\frac{\mathrm{M_z}}{\mathrm{d}} \tag{22}$$

Where

$$\mathrm{K'} = \frac{\mathrm{I_{xy}}}{2\mathrm{I_x}} \tag{23}$$

$$\mathrm{M_z} = \mathrm{C_2}\left(-\mathrm{U_x e_{sy}} + \mathrm{U_y}(\mathrm{e_{sx}} - \mathrm{C_3 m})\right) + \mathrm{C_1 C_3 U_y m} \tag{24}$$

Ux and Uy are the applied loads in the x and y direction respectively and are defined according to the load cases in Tables 1-5. Coefficients C1 and C2 depend on the load case and the bracing configuration and are defined in Tables 1-5. Coefficient C3 equals zero for interior braces and equals one for braces at support locations.

Solutions to various loadings and bracing configurations are shown in Tables 1-5. Bracing configurations include midpoint brace, third point braces, single brace unsymmetrically placed and two braces symmetrically placed. Loading configurations include full uniform load distribution, partial uniform load distribution and concentrated point loads. Load cases can be superimposed but bracing configurations cannot.

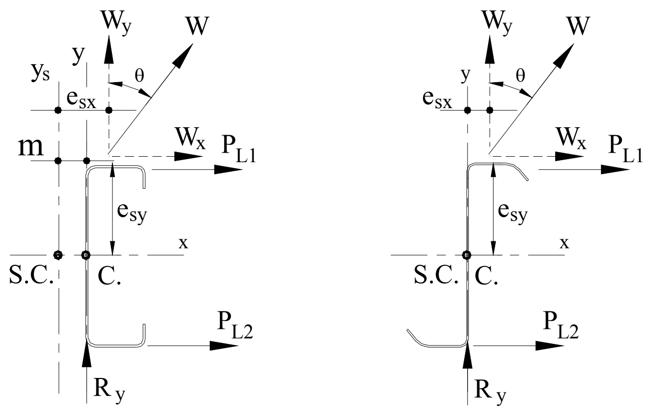


Figure 3. Nomenclature and Positive Force Directions

For distributed load cases other than those shown, the loads can be approximated by concentrated loads. The distributed loads should be divided between the braces with the concentrated load applied at the centroid of the distributed load segment between braces. For example, the triangular load distribution shown in Figure 4, can be divided into 2 concentrated loads each located at the centroid of the load bounded by the brace locations.

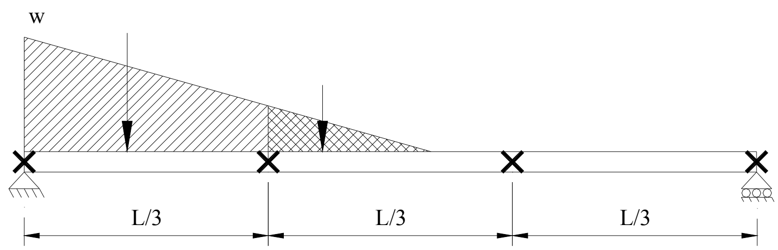


Figure 4. Approximation of Distributed Load

Compatibility solutions for multi-span systems are only provided for uniform load distributions. For multi-span systems with bracing or load configurations other than those provided, the system can be approximated as a simple span system. This approximation will result in a conservative approximation for interior braces. For braces at the frame lines, the brace forces will be conservative for low slope roofs and unconservative for roofs with steep pitches. Since interior braces are typically the greater focus of design, it is more desirable to be conservative at the interior braces.

3. Comparison to AISI Equations

The current AISI provisions for discrete bracing from Section D3.2.1 of the AISI Specification are derived from the same compatibility principles originally presented by Zetlin and Winter (1955) and revisited in this paper. However, the compatibility solution in the specification is generalized to envelop all load cases and brace configurations. While the envelope solution maintains a certain level of simplicity, the solution is overly conservative for uniform load distributions and is often misinterpreted for braces at the support locations.

For the most basic bracing configuration, a simply supported beam with a mid-span brace subjected to uniform loading, the AISI specification is 20% conservative when compared to the actual compatibility solution. As additional braces are added along the span and for multi-span systems, the method is even more conservative with percentages exceeding 50% in some cases. For large roof systems where these brace forces can accumulate in collectors, the forces can be substantial so it is desirable to improve the ability to more accurately predict the brace forces.

The other problem with the current envelope procedure in the AISI Specification is that it is misinterpreted for brace forces at the frame lines. For brace forces resulting from unsymmetric bending, the braces at the frame lines balance the interior brace forces since there is no net horizontal force from unsymmetric bending. The common interpretation of the AISI Specification is that the brace forces at the frame lines from unsymmetric bending come from the portion of the load tributary to the exterior brace. This interpretation can lead to gross errors in predicting the brace forces at the frame lines, often predicting forces in the opposite direction of the actual force. This can be problematic for bracing system details that are often tension-only or compression-only details.

4. Conclusions

The method provided in this paper to predict the brace forces in cold-formed C- and Z-sections includes effects form unsymmetric bending, torsion and downslope roof forces. The method follows a framework similar to the framework already familiar to designers accustomed to using the AISI Specification. However, rather than use simple multipliers intended to envelope all load and bracing configurations, the method presented provides coefficients specific to each loading and bracing configuration. The coefficients are organized in a table format for easy reference. The method also makes explicit recommendations for the brace forces at the support locations. The presented method more accurately predicts the brace forces for cold-formed C- and Z- sections than the current provisions in the AISI Specification which can be grossly over-conservative.

References

AISI (American Iron and Steel Institute). (2008) *Cold Formed Steel Design Manual.* Washington, DC.

AISI (American Iron and Steel Institute). (2012). *North American Specification for the Design of Cold-Formed Steel Structural Members.* Washington, DC.

Seaburg, P. A., Carter, C. J. (1997). *Steel Design Guide Series 9: Torsional Analysis of Structural Steel Members.* American Institute of Steel Construction. Chicago, IL.

Zetlin, L and G. Winter (1955), "Unsymmetrical Bending of Beams with and without Lateral Bracing." *Journal of the Structural Division*, ASCE, Vol. 81, 1955.

Table 1. Equation Coefficients: Simple Span, Uniform Load Distribution, Single Brace

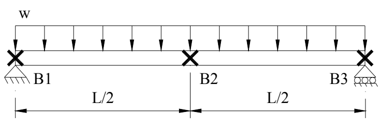
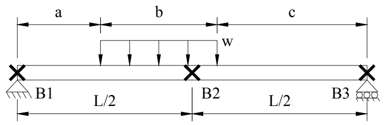
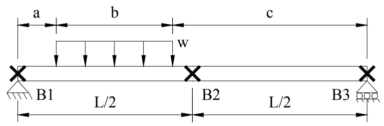
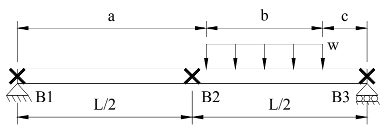
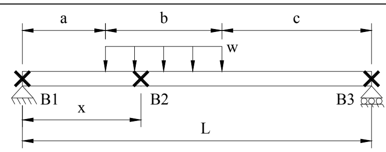
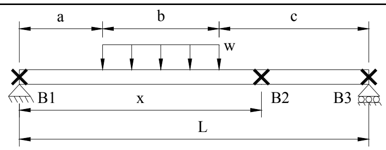
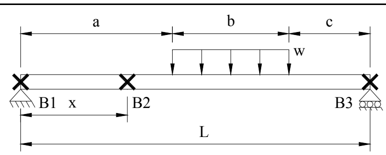
Load Case	Figure	Load, U		Brace Location B1	B2	B3
1SB-U$_1$		wL	C_1	-5/16	5/8	-5/16
			C_2	3/16	5/8	3/16
1SB-U$_2$ $a < L/2$ $L/2 < (a+b)$		wb	C_1	$-\frac{1}{2}Y_{Ui}$	Y_{Ui}	$-\frac{1}{2}Y_{Ui}$
			C_2	$\frac{1}{2}\left(2-2\left(\frac{a}{L}\right)-\frac{b}{L}-Y_{Ui}\right)$	Y_{Ui}	$\frac{1}{2}\left(2\left(\frac{a}{L}\right)+\frac{b}{L}-Y_{Ui}\right)$
1SB-U$_3$ $(a+b) < L/2$			$Y_{U2}=\frac{L}{b}\left[\frac{b}{L}\left(\frac{18a+9b-2L}{2L}\right)+\frac{L-8a}{8L}+3\left(\frac{a}{L}\right)^2+\left(\frac{a}{L}\right)^4-\left(\frac{a+b}{L}\right)^3\left(\frac{4L-a-b}{L}\right)\right]$ $Y_{U3}=\frac{L}{b}\left[\frac{3b}{2L}\left(\frac{2a+b}{L}\right)+\left(\frac{a}{L}\right)^4-\left(\frac{a+b}{L}\right)^4\right]$			
1SB-U$_4$ $L/2 < a$			$Y_{U4}=\frac{L}{b}\left[\frac{b}{L}\left(\frac{18a+9b-2L}{2L}\right)+\left(4-\frac{a}{L}\right)\left(\frac{a}{L}\right)^3-\left(\frac{a+b}{L}\right)^3\left(\frac{4L-a-b}{L}\right)\right]$			
1UB-U$_5$ $a < x$ $x < (a+b)$		wb	C_1	$-\left(\frac{L-x}{L}\right)Y_{Ui}$	Y_{Ui}	$-\left(\frac{x}{L}\right)Y_{Ui}$
			C_2	$\frac{2a+b+2xY_{Ui}}{2L}$	Y_{Ui}	$\frac{2L-2a-b}{2L}+\left(\frac{x-L}{L}\right)Y_{Ui}$
1UB-U$_6$ $x > (a+b)$			$Y_{U5}=\frac{L}{b}\left[\frac{1}{8}\left(\frac{x}{L-x}\right)^2-\frac{xb(2L-2a-b)}{4L(L-x)^2}+\frac{a(3a-2x)}{4(L-x)^2}+\frac{a^4}{8Lx^2(L-x)}+\frac{(a+b)^3(a+b-4L)}{8Lx(L-x)^2}+\frac{bL(2a+b)}{2x(L-x)^2}\right]$ $Y_{U6}=\frac{L}{b}\left[\frac{b}{L}\frac{(2a+b)(2L-x)}{4x(L-x)}+\frac{a^4-(a+b)^4}{8Lx^2(L-x)}\right]$			
1UB-U$_7$ $x < a$			$Y_{U7}=\left[\frac{4bL^2(2a+b)+2bx^2(2a+b-2L)+a^3(4L-a)+(a+b)^3(a+b-4L)}{8bx(L-x)^2}\right]$			

Table 2. Equation Coefficients: Simple Span, Concentrated Load, Single Brace

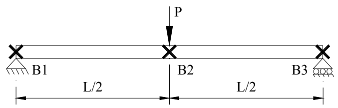
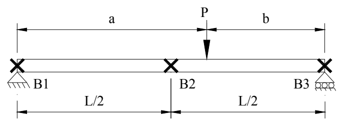
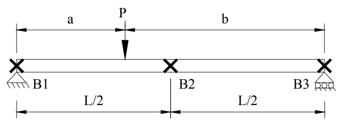
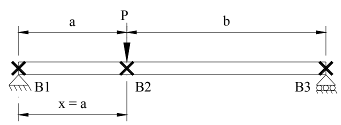
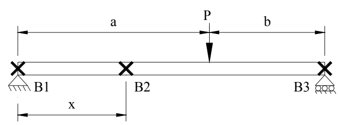
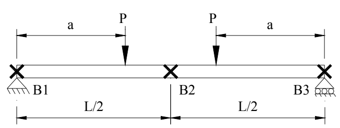
Load Case	Figure	Load, U		Brace Location		
				B1	B2	B3
1SB-P_1			C_1	-1/2	1	-1/2
			C_2	0	1	0
1SB-P_2 a > L/2			C_1	$-\left(\frac{3}{2}\left(\frac{b}{L}\right)-2\left(\frac{b}{L}\right)^3\right)$	$3\left(\frac{b}{L}\right)-4\left(\frac{b}{L}\right)^3$	$-\left(\frac{3}{2}\left(\frac{b}{L}\right)-2\left(\frac{b}{L}\right)^3\right)$
			C_2	$-\frac{1}{2}\left(\frac{b}{L}\right)+2\left(\frac{b}{L}\right)^3$	$3\left(\frac{b}{L}\right)-4\left(\frac{b}{L}\right)^3$	$1-\frac{5}{2}\left(\frac{b}{L}\right)+2\left(\frac{b}{L}\right)^3$
1SB-P_3 a < L/2			C_1	$-\left(\frac{3}{2}\left(\frac{a}{L}\right)-2\left(\frac{a}{L}\right)^3\right)$	$3\left(\frac{a}{L}\right)-4\left(\frac{a}{L}\right)^3$	$-\left(\frac{3}{2}\left(\frac{a}{L}\right)-2\left(\frac{a}{L}\right)^3\right)$
			C_2	$1-\frac{5}{2}\left(\frac{a}{L}\right)+2\left(\frac{a}{L}\right)^3$	$3\left(\frac{a}{L}\right)-4\left(\frac{a}{L}\right)^3$	$-\frac{1}{2}\left(\frac{a}{L}\right)+2\left(\frac{a}{L}\right)^3$
1UB-P_4 x = a		P	C_1	-b/L	1	-a/L
			C_2	0	1	0
1UB-P_5 x < a x < (a+b)			C_1	$-\frac{b(L-x)}{2Lx}\left(\frac{L^2-x^2-b^2}{(L-x)^2}\right)$	$\frac{b}{2x}\left(\frac{L^2-x^2-b^2}{(L-x)^2}\right)$	$-\frac{b}{2L}\left(\frac{L^2-x^2-b^2}{(L-x)^2}\right)$
			C_2	$\frac{b}{2L}\left(2-\frac{L-x}{x}\left(\frac{L^2-x^2-b^2}{(L-x)^2}\right)\right)$	$\frac{b}{2x}\left(\frac{L^2-x^2-b^2}{(L-x)^2}\right)$	$\frac{1}{2L}\left(2a-b\left(\frac{L^2-x^2-b^2}{(L-x)^2}\right)\right)$
1UB-P_6			C_1	$-3\left(\frac{a}{L}\right)+4\left(\frac{a}{L}\right)^3$	$6\left(\frac{a}{L}\right)-8\left(\frac{a}{L}\right)^3$	$-3\left(\frac{a}{L}\right)+4\left(\frac{a}{L}\right)^3$
			C_2	$1-3\left(\frac{a}{L}\right)+4\left(\frac{a}{L}\right)^3$	$6\left(\frac{a}{L}\right)-8\left(\frac{a}{L}\right)^3$	$1-3\left(\frac{a}{L}\right)+4\left(\frac{a}{L}\right)^3$

Table 3. Equation Coefficients: Simple Span, Third Point Braces

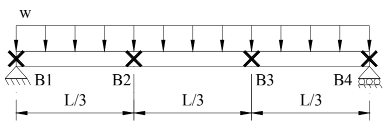
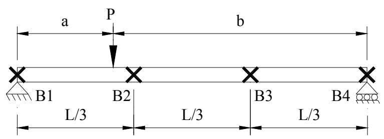
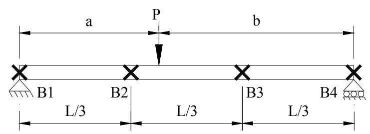
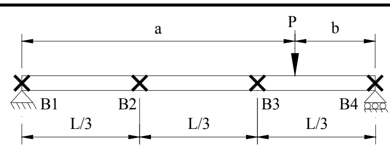
Load Case	Figure	Load, U		Brace Location B1	B2	B3	B4
3DB-U_1		wL	C_1	-11/30	11/30	11/30	-11/30
			C_2	4/30	11/30	11/30	4/30
3DB-P_1 a < L/3 L/2 < (a+b)		P	C_1	$-\frac{2}{3}Y_1-\frac{1}{3}Y_2$	Y_1	Y_2	$-\frac{1}{3}Y_1-\frac{2}{3}Y_2$
			C_2	$\frac{b}{L}-\frac{2}{3}Y_1-\frac{1}{3}Y_2$	Y_1	Y_2	$\frac{a}{L}-\frac{1}{3}Y_1-\frac{2}{3}Y_2$
			$Y_1=\frac{3}{5}\left(\frac{a}{L}\right)\left[8-27\left(\frac{a}{L}\right)^2\right]$		$Y_2=\frac{3}{5}\left(\frac{a}{L}\right)\left[18\left(\frac{a}{L}\right)^2-2\right]$		
3DB-P_2 a > L/3 a<2L/3			C_1	$-\frac{2}{3}Y_1-\frac{1}{3}Y_2$	Y_1	Y_2	$-\frac{1}{3}Y_1-\frac{2}{3}Y_2$
			C_2	$\frac{b}{L}-\frac{2}{3}Y_1-\frac{1}{3}Y_2$	Y_1	Y_2	$\frac{a}{L}-\frac{1}{3}Y_1-\frac{2}{3}Y_2$
			$Y_1=\frac{3}{5}\left[\frac{64}{3}-40\frac{a}{L}+21\left(\frac{a}{L}\right)^3-24\left(\frac{b}{L}\right)^3\right]$		$Y_2=\frac{3}{5}\left[40\left(\frac{a}{L}\right)-\frac{56}{3}+21\left(\frac{b}{L}\right)^3-24\left(\frac{a}{L}\right)^3\right]$		
3DB-P_3 a>2L/3			C_1	$-\frac{2}{3}Y_1-\frac{1}{3}Y_2$	Y_1	Y_2	$-\frac{1}{3}Y_1-\frac{2}{3}Y_2$
			C_2	$\frac{b}{L}-\frac{2}{3}Y_1-\frac{1}{3}Y_2$	Y_1	Y_2	$\frac{a}{L}-\frac{1}{3}Y_1-\frac{2}{3}Y_2$
			$Y_1=\frac{3}{5}\left(\frac{b}{L}\right)\left[18\left(\frac{b}{L}\right)^2-2\right]$		$Y_2=\frac{3}{5}\left(\frac{b}{L}\right)\left[8-27\left(\frac{b}{L}\right)^2\right]$		

Table 4. Equation Coefficients: Simple Span, Two Symmetric Braces

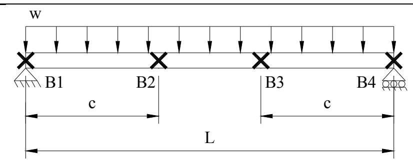
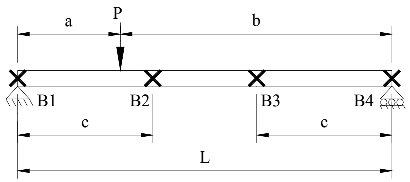
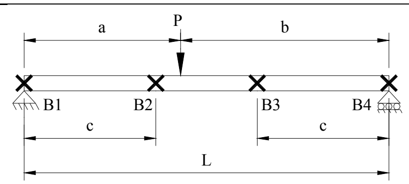
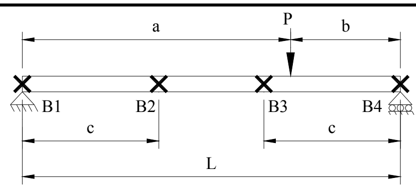
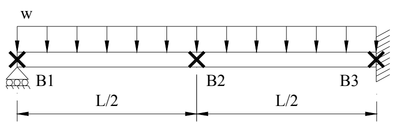
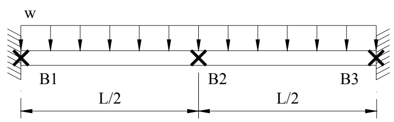
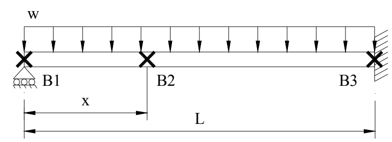
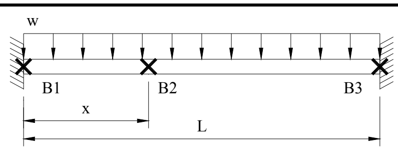
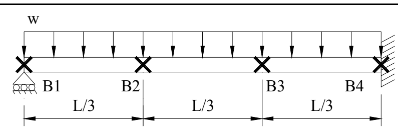
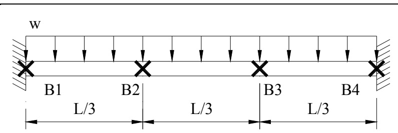
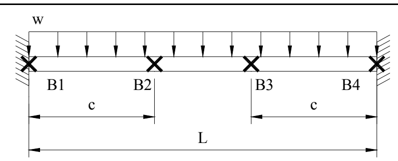
<table>
<tr><th rowspan="2">Load Case</th><th rowspan="2">Figure</th><th rowspan="2">Load, U</th><th></th><th colspan="4">Brace Location</th></tr>
<tr><th></th><th>B1</th><th>B2</th><th>B3</th><th>B4</th></tr>
<tr><td rowspan="2">2SB-U_1</td><td rowspan="2">
</td><td rowspan="2">wL</td><td>C_1</td><td>-Y</td><td>$Y = \frac{11}{162}\frac{(L/c)^2}{(3-4(c/L))}$</td><td>Y</td><td>-Y</td></tr>
<tr><td>C_2</td><td>$\frac{1}{2} - Y$</td><td>Y</td><td>Y</td><td>$\frac{1}{2} - Y$</td></tr>
<tr><td rowspan="3">2SB-P_1
a < c</td><td rowspan="3">
</td><td rowspan="9">P</td><td>C_1</td><td>$Y_1\left(\frac{c}{L}-1\right) - Y_2\left(\frac{c}{L}\right)$</td><td>$Y_1$</td><td>$Y_2$</td><td>$Y_2\left(\frac{c}{L}-1\right) - Y_1\left(\frac{c}{L}\right)$</td></tr>
<tr><td>C_2</td><td>$\left(1-\frac{a}{L}\right) + Y_1\left(\frac{c}{L}-1\right) - Y_2$</td><td>$Y_1$</td><td>$Y_2$</td><td>$\frac{a}{L} - Y_1\left(\frac{c}{L}\right) + Y_2\left(\frac{c}{L}-1\right)$</td></tr>
<tr><td colspan="5">$Y_1 = \frac{a}{c^2}\left[c - \frac{(a^2-c^2)(2L-3c)}{(3L-4c)(L-2c)}\right]$ $Y_2 = \frac{a}{c^2}\left[\frac{(a^2-c^2)(L-c)}{(3L-4c)(L-2c)}\right]$</td></tr>
<tr><td rowspan="3">2SB-P_2
c < a
a<(L-c)</td><td rowspan="3">
</td><td>C_1</td><td>$Y_1\left(\frac{c}{L}-1\right) - Y_2\left(\frac{c}{L}\right)$</td><td>$Y_1$</td><td>$Y_2$</td><td>$Y_2\left(\frac{c}{L}-1\right) - Y_1\left(\frac{c}{L}\right)$</td></tr>
<tr><td>C_2</td><td>$\left(1-\frac{a}{L}\right) + Y_1\left(\frac{c}{L}-1\right) - Y_2$</td><td>$Y_1$</td><td>$Y_2$</td><td>$\frac{a}{L} - Y_1\left(\frac{c}{L}\right) + Y_2\left(\frac{c}{L}-1\right)$</td></tr>
<tr><td colspan="5">$Y_1 = \frac{2(b)(L^2-b^2-c^2)(L-c)^2 - a(L^2-a^2-c^2)(L^2-2c^2)}{Lc(3L-4c)(L-2c)^2}$ $Y_2 = \frac{2(a)(L^2-b^2-c^2)(L-c)^2 - b(L^2-b^2-c^2)(L^2-2c^2)}{Lc(3L-4c)(L-2c)^2}$</td></tr>
<tr><td rowspan="3">2SB-P_3
a>(L-c)</td><td rowspan="3">
</td><td>C_1</td><td>$Y_1\left(\frac{c}{L}-1\right) - Y_2\left(\frac{c}{L}\right)$</td><td>$Y_1$</td><td>$Y_2$</td><td>$Y_2\left(\frac{c}{L}-1\right) - Y_1\left(\frac{c}{L}\right)$</td></tr>
<tr><td>C_2</td><td>$\left(1-\frac{a}{L}\right) + Y_1\left(\frac{c}{L}-1\right) - Y_2$</td><td>$Y_1$</td><td>$Y_2$</td><td>$\frac{a}{L} - Y_1\left(\frac{c}{L}\right) + Y_2\left(\frac{c}{L}-1\right)$</td></tr>
<tr><td colspan="5">$Y_1 = \frac{b}{c^2}\left[\frac{(b^2-c^2)(L-c)}{(3L-4c)(L-2c)}\right]$ $Y_2 = \frac{b}{c^2}\left[c - \frac{(b^2-c^2)(2L-3c)}{(3L-4c)(L-2c)}\right]$</td></tr>
</table>

Table 5. Equation Coefficients: Multi-Span, Uniform Load

Load Case	Figure	Load, U		Brace Location			
				B1	B2	B3	
1SB-EX-U		wL	C_1	-5/28	4/7	-11/28	
			C_2	11/56	4/7	13/56	
1SB-IN-U			C_1	-1/4	1/2	-1/4	
			C_2	1/4	1/2	1/4	
1US-EX-U			C_1	$-Y\left(\frac{(L-x)^2(2L+x)}{2L^3}\right)$	$Y=\frac{L^2}{4}\left[\frac{(L+2x)}{(L-x)(3L+x)x}\right]$	$-Y\left(\frac{(3L^2-x^2)^2x}{2L^3}\right)$	
			C_2	$\frac{3}{8}-Y\left(\frac{(L-x)^2(L+2x)}{2L^3}\right)$	Y	$\frac{5}{8}-Y\left(\frac{(3L^2-x^2)^2x}{2L^3}\right)$	
1US-IN-U			C_1	$-Y\left(\frac{(L-x)^2(L+2x)}{L^3}\right)$	$Y=\frac{L^2}{8x(L-x)}$	$-Y\left(\frac{x^2(3L-2x)}{L^3}\right)$	
			C_2	$\frac{1}{2}-Y\left(\frac{(L-x)^2(L+2x)}{L^3}\right)$	Y	$\frac{1}{2}-Y\left(\frac{x^2(3L-2x)}{L^3}\right)$	
				B1	B2	B3	B4
2SB-EX-U			C_1	$-\frac{342}{1404}$	$\frac{531}{1404}$	$\frac{450}{1404}$	$-\frac{639}{1404}$
			C_2	$\frac{184.5}{1404}$	$\frac{531}{1404}$	$\frac{450}{1404}$	$\frac{238.5}{1404}$
2SB-IN-U_1			C_1	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$
			C_2	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$
2SB-IN-U_2			C_1	-Y	$Y=\frac{(L-c)^2}{4c(2L-3c)}$	Y	-Y
			C_2	$\frac{1}{2}-Y$	Y	Y	$\frac{1}{2}-Y$