



## **Effect of Girder Continuity and Imperfections on System Buckling of Narrow I-girder Systems**

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### **Abstract**

System buckling of interconnected narrow I-girder systems is an instability mode that has been recognized over the past decade. The buckling mode is relatively insensitive to the spacing between cross frames and often controls 2- and 3-girder systems that are relatively narrow compared to their lengths. Past studies have primarily focused on the critical buckling loads for simply supported systems. This paper presents parametric finite element analyses in the study of narrow doubly-symmetric I-girder systems and includes the impact of girder continuity as well as nonlinear geometrical effects. Eigenvalue buckling analyses of both simply-supported and two-span continuous I-girder systems were first conducted and the results were compared with predictions using a buckling solution developed in an earlier study. The impact of moment gradient on the buckling capacity is studied and factors reflecting the impact of girder continuity are developed. In addition, nonlinear buckling analyses were carried out to investigate the effect of the shape of imperfection on the susceptibility of I-girder systems to the 2<sup>nd</sup>-order amplification of lateral-torsional displacements.

### **1. Introduction**

Lateral torsional buckling (LTB) of a steel girder under transverse loading is a limit state that often controls in the bridge and building design specifications, involving both the twist and lateral movement of the girder sections. Intermediate bracing such as cross frames are typically used along the girder at discrete points to reduce the unbraced length of the girder thereby improving the buckling resistance by providing torsional restraint.

While the cross frames are often viewed as a brace point for lateral torsional buckling of the individual girders, system buckling of the girder system as a structural unit is also a viable stability limit state. The mode shape of the system consists of a half-sine curve buckled shape along the span length and often controls on girder systems that are relatively narrow compared to the length. Historically, this mode shape was not covered by design specifications until the failure of the Marcy Pedestrian Bridge during concrete deck placement that led to a study of behavior of narrow structural systems (Yura and Widiyanto 2005). Although the Marcy Pedestrian

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Bridge was a trapezoidal box girder system, no top lateral truss was supplied so the girder behaved in a similar manner to a twin I-girder system since the box had several internal cross frames along its length. Following the Marcy Bridge failure, a number of problematic 2- and 3-girder systems were encountered which were dangerously close to failure. A study of narrow twin girder systems with simple supports were reported in Yura et al. (2008) leading to the following expression for the elastic system buckling resistance of a twin I-girder systems interconnected with cross frames under non-composite loading conditions:

$$M_{gs} = \frac{\pi^2 s E}{L^2} \sqrt{I_{eff} I_x} \quad (1)$$

where  $E$  is the modulus of elasticity;  $I_x$  is the moment of inertia about the strong axis of a single girder;  $I_{eff}$  is the moment of inertia about the weak axis of a single girder;  $L$  is the span length; and  $s$  is the spacing between girders. For three girder systems, the girder spacing  $s$  is replaced with the distance between the two exterior girders.  $M_{gs}$  given by (1) is the total moment resistance of the girder system and would be compared with the sum of the maximum moments in the individual girders. The AASHTO specification (2015) includes equation (1); however the sum of the moments are limited to 50% of the critical buckling load to avoid moment amplification due to second order effects; however very limited study has been carried out to investigate the potential for second order amplification.

Equation (1) is applicable to simply-supported twin I-girder systems subjected to uniform moment. Many steel girder systems are continuous over multiple supports and the validity of this equation to such systems is unclear. The objective of his paper is to 1) extend the previous study of the elastic system buckling from the simply-supported systems to continuous systems using parametric FE study; 2) perform preliminary nonlinear buckling analyses (load-deflection analysis) on I-girder systems to investigate the susceptibility of I-girder systems to 2<sup>nd</sup>-order amplification of the lateral-torsional displacement and the impact of the cross-sectional shape the imperfection.

## 2. Linear Eigenvalue Analyses on I-girder Systems

Three-dimensional parametric FE eigenvalue buckling analyses were first conducted to investigate the elastic system buckling resistance of both the simply-supported and two-span continuous I-girder systems. This section presents the details of the FEA model and the results from the parametric eigenvalue buckling analyses.

### 2.1 Finite Element Model

The general-purpose FE program ANSYS (2015) was used for this study. This software program features the ANSYS Parametric Design Language (APDL) allowing the user to vary the geometric parameters for the FE model with ease to facilitate the parametric study. Fig. 1 depicts the three-dimensional isometric view of the FE model for a two-span continuous twin I-girder system considered in this study.

The steel was assumed to be linear elastic material in the model, with the Young's Modulus  $E = 29,000$  ksi and Poisson's ratio  $\nu = 0.3$ .

The girder cross section was modelled using the SHELL281 element, which is a 3D 8-node shell element with three translational and three rotational degrees of freedom at each node. The element has quadratic displacement shape functions, which are well suited to model either straight or curved girder geometries. At the cross section of all girders, the web was meshed with four elements along the depth whereas two elements were used for each flange. Transverse stiffeners attached to the steel girders at each bracing location were also modelled with the SHELL281 element. All cross frames on the girders were modelled using the 3D space truss element LINK180. This element is a two-node uniaxial tension-compression element with three translational degrees of freedom at each node. The shell elements for the steel girders and the truss elements for the cross frame diagonals shared the coincident nodes at the interfaces. This simplified model proved effective and accurate in previous steel girder buckling research studies (Helwig 1994; Quadrato 2010; Battistini et al. 2016).

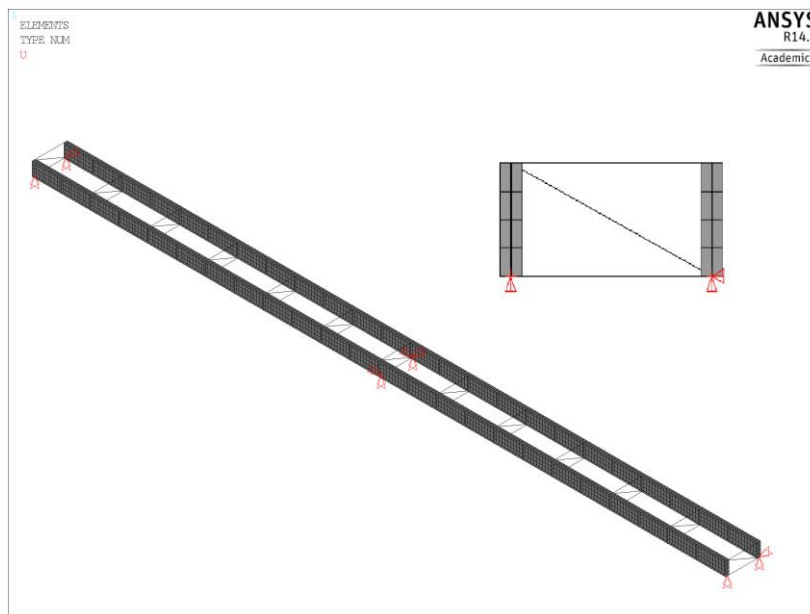


Figure 1: FE model for a two-span continuous twin I-girder system

## 2.2 Geometric Parameters

For both the simply-supported and two-span continuous girders evaluated in the parametric FE study, a cross section with a flange-to-depth ratio of 1/4 was selected for all analyses, which is representative to common bridge design practices (Stith 2010). As shown in Fig. 2, the cross section consisted of two 14'' $\times$ 1.5'' flanges and a 56'' $\times$ 0.625'' web plate. Although most continuous girders would have different sized flanges in the negative moment region, the decision was made to maintain a prismatic section in this trial study; however, the behavior of non-prismatic sections is also being considered in a subsequent study.

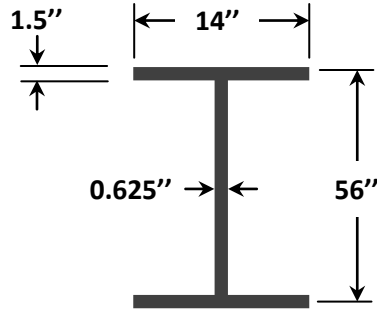


Figure 2: Cross Section Used in Parametric FE Study

Although most cross frame types have two diagonals (e.g. K-type cross frame and X-type cross frame), the single diagonal “tension-only” Z-type cross frame was used for the FE model as shown illustrated at upper right corner of Fig. 1. It conservatively disregards the compression diagonal due to the potentially low buckling resistance of the slender steel angle. Because the diagonals were truss elements with nodes at the ends, these members are not buckling elements and therefore the stiffness of the diagonal will be the same whether in tension or compression. Steel angles sized 4’×4’×1/2” with a sectional area of 3.75 in<sup>2</sup> were applied for all cross frame diagonals in compliance with common TxDOT practices. Recent work (Battistini et al. 2016) has shown that cross frames comprised of single angles can have significant reductions in stiffness due to eccentric connections. Although the cross frames in this study were single angles, because a “tension-only” cross frame was represented with the single diagonal, no reduction in stiffness was necessary.

To the ensure that the I-girder systems is adequately braced, trial eigenvalue analyses were conducted with increasing sectional area of cross frame diagonals from the designated value. The results were compared in terms of buckling strength between individual analyses. The cross frames were regarded to have provided adequate stiffness as no notable differences were observed between them.

### 2.3 Bridge Layout

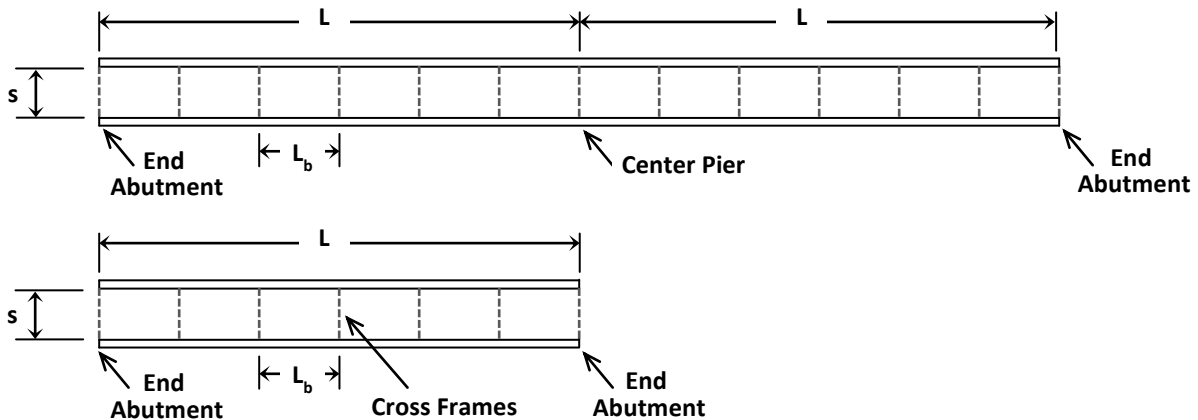


Figure 3: Bridge Layout for Both the Simply-Supported and Continuous I-Girder Systems

Fig. 3 illustrates the schematic diagram of the structural layout for both the simply-supported and the continuous twin I-girder systems. The girder was modelled as perfectly straight without end skew or imperfection. The cross frames were placed between the two girders with varying number for different structural layouts but at the same interval of  $L_b = 20$  ft.

The continuous twin I-girder systems consisted of two equal spans with four selected span length values of  $L = 120$  ft, 140 ft, 160 ft, and 180 ft, resulting in the span-to-depth ratios of 25.7, 30, 34.3, and 38.6 and the cross frame numbers of 13, 15, 17, and 19 from end to end. Meanwhile, four girder spacing values were selected with  $s = 5'$ ,  $7'$ ,  $9'$  and  $11'$ . Consequently, a total number of 16 analyses were conducted for the study of continuous I-girder systems. These selected values are representative of those commonly encountered in design practice. The girder systems were pinned at the center pier with restraints of vertical, lateral, and longitudinal translations whereas roller type supports were provided at the two end supports preventing the vertical and lateral translations also as illustrated in Fig. 1 and Fig. 3.

For the parallel of parametric study, four span length values of  $L = 100$  ft, 120 ft, 140 ft, and 160 ft were selected for the simply-supported twin I-girder system, corresponding to the span-to-depth ratios of 21.4, 25.7, 30, and 34.3 and the cross frame numbers of 6, 7, 8, and 9. Meanwhile, only one value  $s = 7'$  was selected for the girder spacing, giving a total number of 4 analyses in this case. Pin type support was provided for the girder system at one end and roller type support on the other end.

While cross frames were placed at the ends of the girders, the warping deformation of the cross section of each individual girder was not restrained. Distributed loads were applied along the girder length at the mid-height in the FE Model despite the fact that concrete dead loads and construction loads are naturally applied on the top flange. The adverse impact of this liberal assumption is mitigated by a number of factors that are conservatively neglected such as warping restraint at supports and “tipping restraint” (Helwig et al. 1997) and (Yura et al. 2008). Although results are not shown in this paper due to length constraints, the impact of top flange loading was considered and was found to be relatively small.

### 2.3 Results

Fig. 4 and Fig. 5 depict the buckled shapes for both the simply-supported and two-span continuous twin I-girder systems obtained from the eigenvalue buckling analyses. The buckling of the simply-supported I-girder system occurs in a manner similar to the LTB of an individual girder. The whole bridge unit underwent both the twist and lateral movement leading to a buckled shape akin to a sinusoidal wave of a half-wave length whereas the buckled shape of the two-span continuous I-girder system can be likewise roughly characterized by a sinusoidal wave of a full-wave length with inflection points at center pier and end supports formed by twist and lateral movement also.

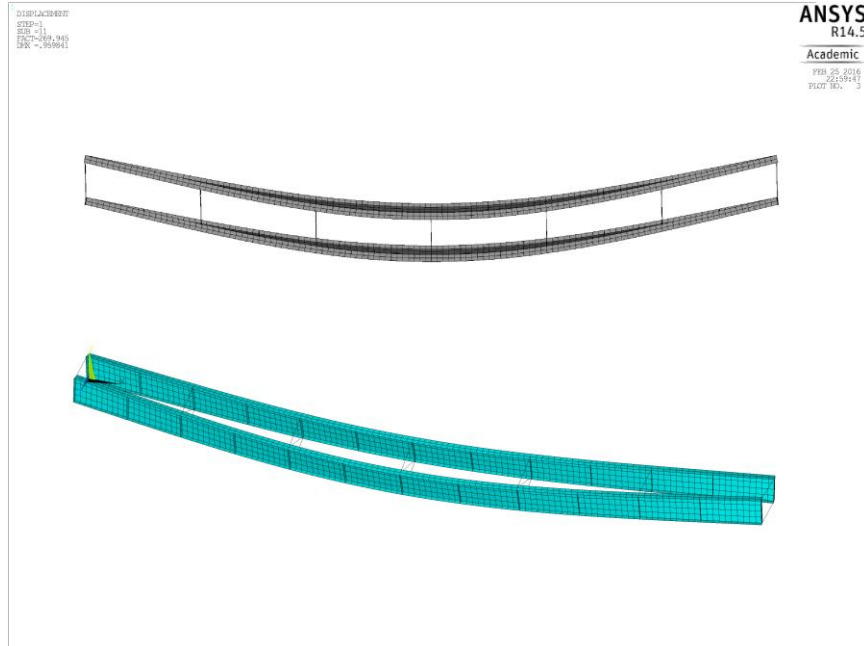


Figure 4: Buckling Shape for Simply-Supported I-girder systems

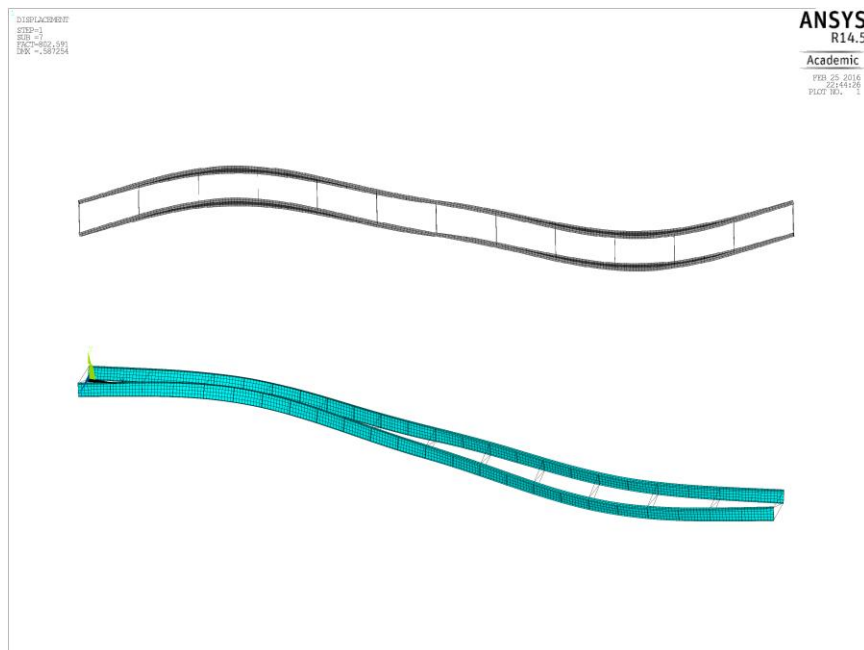


Figure 5: Buckling Shape for Two-Span Continuous I-girder systems

Table 1 and Table 2 tabulate the results from the parametric FE study for both the simply-supported and two-span continuous I-girder systems in comparison with the predictions by Eq. 1 given the corresponding parameters. Analogous to the moment gradient for the classic solution for LTB of an individual girder, A  $C_{bs}$  factor was defined by a fraction of  $M_{cr}$ , the maximum girder moment at buckling calculated by FEA, to  $M_{gs}$ , the capacity predicted by Eq. 1, as given by the following equation:

$$C_{bs} = \frac{M_{cr}}{M_{gs}} \quad (2)$$

Table 1: Comparison of FEA Results and Predictions for Continuous I-girder Systems

	$L$ (ft)	120	140	160	180
	$L/D$	25.7	30	34.3	38.6
$s=5'$	$M_{cr}$ (k-ft)	8,618	6,471	5,050	4,063
	$M_{gs}$ (k-ft)	3,708	2,724	2,086	1,648
	$C_{bs}$	<b>2.32</b>	<b>2.38</b>	<b>2.42</b>	<b>2.47</b>
$s=7'$	$M_{cr}$ (k-ft)	11,690	8,697	6,812	5,453
	$M_{gs}$ (k-ft)	5,191	3,814	2,920	2,307
	$C_{bs}$	<b>2.25</b>	<b>2.30</b>	<b>2.33</b>	<b>2.36</b>
$s=9'$	$M_{cr}$ (k-ft)	14,669	11,054	8,594	6,867
	$M_{gs}$ (k-ft)	6,674	4,903	3,754	2,966
	$C_{bs}$	<b>2.20</b>	<b>2.25</b>	<b>2.29</b>	<b>2.32</b>
$s=11'$	$M_{cr}$ (k-ft)	17,487	13,283	10,356	8,281
	$M_{gs}$ (k-ft)	8,157	5,993	4,588	3,625
	$C_{bs}$	<b>2.14</b>	<b>2.22</b>	<b>2.26</b>	<b>2.28</b>

Table 2: Comparison of FEA Results and Predictions for Simply-Supported I-girder Systems

	$L$ (ft)	100	120	140	160
	$L/D$	21.4	25.7	30	34.3
$s=7'$	$M_{cr}$ (k-ft)	8,157	5,830	4,360	3,383
	$M_{gs}$ (k-ft)	7,475	5,191	3,814	2,920
	$C_{bs}$	<b>1.09</b>	<b>1.12</b>	<b>1.14</b>	<b>1.16</b>

As viewed in Table 2, the  $C_{bs}$  values for the simply-supported I-girder system range from 1.09 to 1.16, indicating the value  $C_b = 1.12$  for uniformly distributed load for the LTB of an individual girder is also applicable to the simply-supported twin girder system. The results also illustrate that the impacts of the span-to-depth ratio and girder spacing on the  $C_{bs}$  value are limited for I-girder systems for the I-girder system, the maximum and minimum  $C_{bs}$  values for continuous I-girder systems being 2.47 and 2.14. Therefore, constant  $C_{bs}$  values of 1.1 and 2.0 might be utilized for estimating the critical system buckling loads for simply-supported and continuous I-girder systems, respectively.

### 3. Large Displacement Analysis and Nonlinear Behavior

The elastic critical buckling load, which is mathematically described by the bifurcation of a continuous system (e.g., differential equations), only serves as a theoretical upper limit to the system buckling resistance of the girder system. The bridge structural unit might undergo excessive 2<sup>nd</sup>-order amplification of the torsional-lateral displacement before the elastic critical buckling load is actually approached. It is therefore necessary to perform a large-displacement load-deflection FEA study to capture the nonlinear behavior of the bridge unit with the non-composite dead load gradually applied to a target value under the elastic critical buckling load, which is nevertheless an important indicator of the structural susceptibility to the 2<sup>nd</sup>-order

amplification effect (Sanchez and White 2012). The nonlinear behavior of the I-girder system is associated with many factors such as the shape of imperfection, skew, horizontal curvature, load pattern. This section presents a preliminary study of the effect of the shape of the imperfection on 2<sup>nd</sup>-order amplification. As noted earlier in this paper, the AASHTO (2015) specification limits the critical load to 50% of the value predicted by Eq. (1) due to concerns of second order amplification; however very little work has been carried out to study this impact.

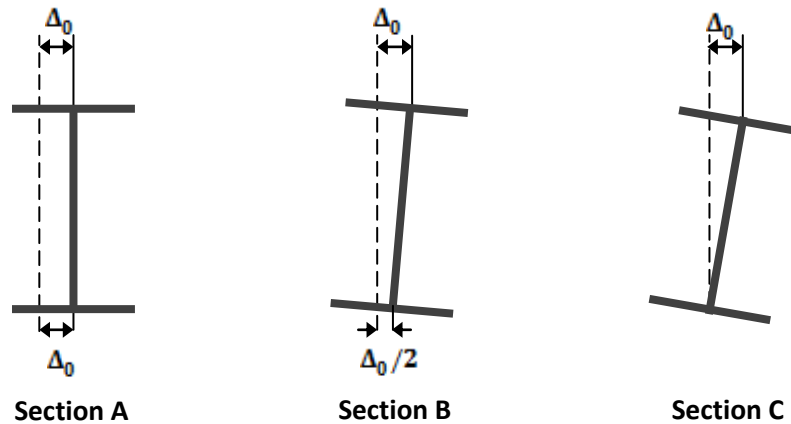


Figure 6: Schematic Diagram of the Cross-sectional Shapes of Imperfection at mid-span

A simply-supported twin I-girder system was considered. It had a span length of 140 ft and a girder spacing of 7 ft, with the exact geometric parameters and boundary conditions used in section 2. Identical imperfections were imposed to both girders with a single wave distribution along the length and maximum value  $\Delta_0 = L/1000 = 1.68''$  at mid-span. As illustrated in Fig.6, three different types of cross-sectional shapes of imperfections were examined. Section A has a “pure sweep” type of shape of imperfection which consists of lateral sweep at both the top and bottom flanges with same magnitudes of  $\Delta_0$ . Section B consists of a top flange sweep of  $\Delta_0$  and a bottom flange sweep of  $\Delta_0/2$ . In Section C, the bottom flange remains straight while the top flange displaces. Uniformly distributed load was applied to the bridge unit gradually up to 75% of the elastic critical buckling load of 1.78 kip/ft for analyses. The imperfect shape shown in Section C is consistent with past studies that have shown this shape is critical for the design of cross frames and diaphragms (Wang and Helwig, 2005).



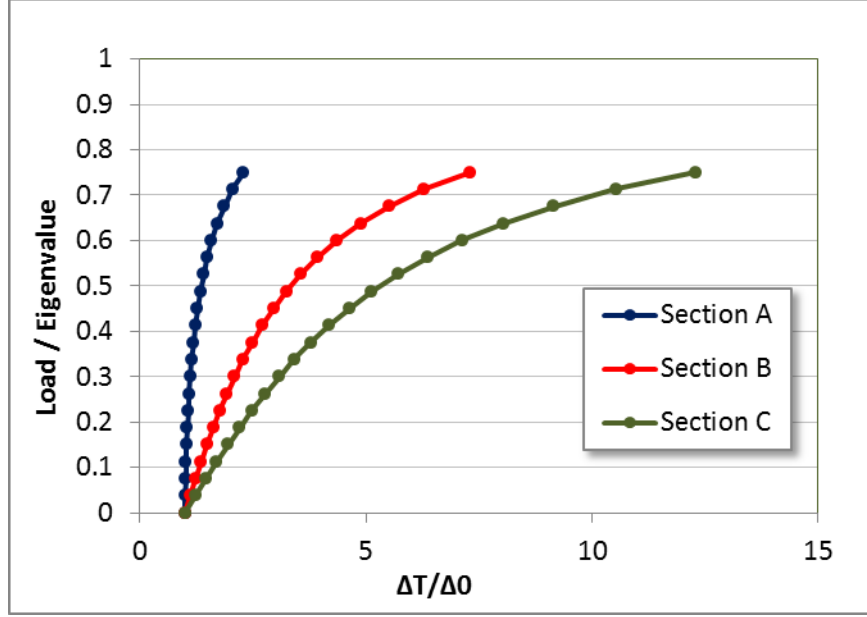


Figure 7: Normalized Load vs Lateral Displacement Curves at Mid-Span

As depicted in Fig. 7, the three analyses with different shapes of imperfection were compared in terms of maximum lateral displacement at mid-span under the normalized load. The “twist-dominant” Section C was found to be the most critical shape of the imperfection. However, during the erection of the girder system, the shape of imperfection of the girders would be altered by the installation process of the cross frames. The ability of a rectangular cross frame to resist distortion is significantly greater than the torsional rigidity of a girder. Therefore, the girder will be forced to rotate once a cross frame is fitted up. The final cross sectional shape of imperfection of the girder will become “sweep-dominant” afterwards, significantly mitigating the 2<sup>nd</sup>-order amplification of the torsional-lateral displacement. Significant study of the impact of the likely shape of the imperfect girder with cross frame installed was carried out as part of this study and the computer simulations confirmed that the likely shape is more consistent with a pure sweep. Due to length restrictions, this data are not shown in this paper but will be presented in a subsequent publication. Therefore, the results showed that the load deformation curve on girder with cross frames installed will be similar to the curve shown for “Section A” in Fig. 7. In-as-much, the AASHTO (2015) restriction to 50% of the critical load is likely overly-conservative and the permissible load can be increased.

#### 4. Conclusions

This paper presents results of a parametric FE study including both elastic eigenvalue buckling and nonlinear buckling analyses on the system buckling of I-girder system to address the absence of such provision in current design specifications. It has been shown that the system buckling critical load given by Eq. 1 is capable of providing reasonable estimate of the I-girder system with moment gradient values for the system mode denoted  $C_{bs}$ . Constant value of 1.10 is recommended for simply supported systems and 2.0 for continuous girder systems. These values are applicable for uniformly distributed loads, which are the most common load for which the system mode would be critical. Although results were not shown in this paper, the impact of load position on the cross section was insignificant on the system buckling mode. The preliminary

load-deflection analysis on the system buckling of the I-girder system reveals that the shape of the imperfection has profound effect on the 2<sup>nd</sup>-order amplification of the torsional-lateral displacement of the structure. Imperfections with a “pure sweep” have much smaller amplification compared to shapes consisting of a straight bottom flange and swept top flange. However, because the cross frames are generally fabricated with geometries close to square (the exception being geometrical “drops” based upon skewed/curved/camber requirements), the resulting imperfection on girders systems is very close to a pure sweep. As a result, the current limitation of 50% of the critical load that is in AASHTO (2015) is over-conservative and can be increased. The results presented in this paper provide insight into future studies of other important factors that affecting the susceptibility of I-girder system to the 2<sup>nd</sup>-order amplification effect such as imperfection distribution, curvature, etc. all of which will be documented in a future publication.

## References

- AASHTO (2015), “LRFD Bridge Design Specifications”, 7<sup>th</sup> Ed. Washington DC.
- ANSYS (2015), Version 14.5 Academic Research, General Purpose Finite Element Software, Canonsburg, PA.
- Battistini, A., Wang, W., Helwig, T., Engelhardt, M., and Frank, K.; (2016) “Stiffness Behavior of Cross Frames in Steel Bridge Systems,” *ASCE Journal of Bridge Engineering*, Published online: [http://ascelibrary.org/doi/abs/10.1061/\(ASCE\)BE.1943-5592.0000883](http://ascelibrary.org/doi/abs/10.1061/(ASCE)BE.1943-5592.0000883).
- Helwig, T.A. (1994). “Lateral Bracing of Bridge Girders by Metal Deck Forms.” *Ph.D. Dissertation Submitted to University of Texas*. Austin, TX.
- Helwig, T.A., Frank, K.H., and Yura, J.A. (1997). “Lateral-Torsional Buckling of Singly Symmetric I-Beams.” *Journal of Structural Engineering*, 123 (9) 1172-1179.
- Quadrato, C.E. (2010). “Stability of Skewed I-shaped Girder Bridges Using Bent Plate Connections.” *Ph.D. Dissertation Submitted to University of Texas*. Austin, TX.
- Sanchez, T.A., White, D.W. (2012). “Stability of Curved Steel I-Girder Bridges During Construction” *Transportation Research Record: Journal of the Transportation Research Board*, (2268) 122-129.
- Stith, J.C. (2010). “Predicting the Behavior of Horizontally Curved I-Girders During Construction.” *Ph.D. Dissertation Submitted to University of Texas*. Austin, TX.
- Wang, L. and Helwig, T.A., “Critical Imperfections for Beam Bracing Systems,” *ASCE Journal of Structural Engineering*, Vol. 131, No. 6, pp. 933-940, June 2005.
- Yura, J.A., Widiyanto, J.A. (2005). “Lateral buckling and bracing of beams—A re-evaluation after the Marcy bridge collapse.” *Proceedings of Structural Stability Research Council*, Montreal. 277-294.
- Yura, J.A., Helwig, T.A., Herman, R., Zhou, C. (2008). “Global Lateral Buckling of I-Shaped Girder Systems.” *Journal of Structural Engineering*, 134 (9) 1487-1494.