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New Euler-type progressive collapse curves for steel frames

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Abstract
Progressive collapse of structures has attracted the interest of the structural engineering community starting with the Ronan Point collapse of 1968. The terrorist attacks on the Alfred P. Murrah building in Oklahoma in 1995 and on the World Trade Center in New York in 2001 further increased the research interest in this field. Progressive collapse is a phenomenon usually characterized by a triggering event of local structural failure or damage which results into partial or total collapse of the structure. In certain cases, structures have shown high levels of vulnerability to the triggering event, while in other cases they have proven to be quite resilient. The research field of progressive collapse aims at quantifying and ultimately minimizing structural vulnerabilities to a wide range of triggering events. For steel structures, the governing collapse mode usually involves the instability of a member of the structure, a part of the structure or the whole structure as a system. Previous research by the authors has shown that the response of a steel frame under a key component removal scenario can be analytically calculated. Based on this analytical methodology and similarly to the Euler curve for an individual member under compression, the paper presents new Euler-type progressive collapse curves for steel frames which can identify a short-wave loss-of-stability induced collapse mode from a yielding-type induced collapse mode. One of the main advantages of the proposed new Euler-type curves is the simple elastic analysis framework of the analytical method.

1. Introduction

Progressive collapse is termed by ASCE (2005) as "the spread of an initial local failure from element to element, eventually resulting in the collapse of the entire structure, or a disproportionately large part of it" [1]. The initial damage is usually the result of an extreme event acting on the structure, such as a blast or fire. Progressive collapse of the structure is the outcome of the case where the structural system is incapable of transferring the already applying loads to the ground, resulting in damage propagation and total or partial collapse of the building. The collapse of Alfred P. Murrah building (Oklahoma, 1995) and World Trade Center (New York, 2001), both caused by terrorist attacks, are representative examples of the catastrophic aspect of the phenomenon.

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The guidelines which currently dominate the progressive collapse field are the Unified Facilities Criteria [2] and the General Services Administration guidelines [3], which propose several design methods. The indirect method implicitly considers resistance to progressive collapse "through the provision of minimum levels of strength, continuity and ductility" [2]. Direct design approaches include the *specific load resistance method*, which requires that the structure or a specific part of it has enough strength to resist specific load and the *alternate load path method* (APM). The latter includes the notion of a missing vertical load-bearing element and requires the structure to be capable of bridging over that element. In the case of steel moment frames the APM is applied by considering the column removal scenario, during which a column is removed from the model and the collapse capacity of the remaining structure is assessed. Linear static, nonlinear static and nonlinear dynamic analysis can be used under the notion of a column removal scenario. In the current framework a vertical nonlinear static "pushdown" analysis is employed, during which the vertical loads are increasing from zero until the critical load value, at which a failure mode is activated and the structure collapses.

Significant research effort has been made during the past years, attempting to achieve a reliable insight on the phenomenon [5] – [14]. However, the importance of stability in progressive collapse scenarios has not yet been appropriately highlighted and received the attention required. When the notion of a column removal scenario is employed, the vertical load previously carried by that element tries to find a new way to the ground, increasing abruptly the axial demand on the columns adjacent to the removed member. As a result, it is very possible for an adjacent column to reach its critical load and buckle. The strong connection between stability and progressive collapse becomes therefore apparent, underlining the importance of investigating the possibility of such a phenomenon to appear.

The present study expands the work done in Gerasimidis [9] and presents new Euler-type limit state curves which are applied to a series of 2D 20-storey buildings, as the result of an extensive parametric study. Pushdown analysis is employed and two collapse mechanisms are taken into consideration. These critical curves which can be derived from simple linear elastic analyses, can predict the governing collapse mechanism of a steel frame for any case of corner column removal scenario. Thus, the practicing engineers are provided with a valuable tool and the capability of assessing the vulnerability of a steel frame under a corner column removal scenario, avoiding the computational difficulties that a sophisticated, non-linear analysis would require. The results of this analytical method are verified by computational simulations conducted with the finite element software ABAQUS.

2. Analytical Progressive Collapse Method

2.1 General description of failure modes

The proposed method is applied to 2D moment resisting frames and can be utilized when the notion of a corner column removal scenario is considered, taking into account two collapse modes: the yielding-type (beam) mode and the loss-of-stability (column) mode.

After a corner column is removed from the structural system, collapse is triggered in the first mode as a result of plastic hinge formation in both ends of the beams immediately adjacent and above the removed column. Fig. 1 shows coarsely the propagation of damage in the cantilevered part above the removed member. As a result of the bending of the cantilevered part, plastic hinges first form in the left end of every beam. The load value of the last beam to form a plastic hinge is named $q = q_{lf}$. When plastic hinges form in the right end of all beams (for $q = q_{rf}$), an unstable mechanism has been formed and that part of the structure collapses in a ductile way (yielding – type mode).

The other collapse mechanism which was taken into account is the loss-of-stability mode, which involves the buckling of a column of the structure when its critical load is met ($q = q_{stability}$). The purpose of the current method is to estimate the value of the total applied load at which each mechanism is being activated, predicting therefore which one will be activated first. In other words, when $q_{stability}$ is less than q_{rf} the governing collapse mode is the loss-of-stability and vice versa.

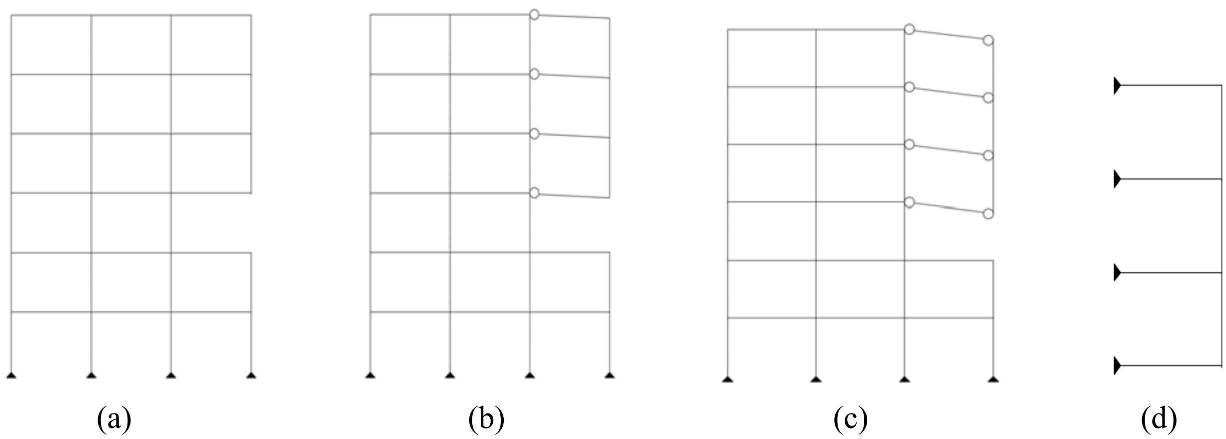

Fig. 1: (a) Initial model "A", (b) & (c) Propagation of plastic hinges, (d) Model "B"

2.2 Mathematical treatment of yielding-type mode

The flexural capacity $M_{cap,j}$ of a beam element j is given in Eq. (1):

$$M_{cap,\,j} = w_j \cdot f_y \cdot \mu \tag{1}$$

where w_j is the plastic section modulus, f_y is the yield stress and μ is the allowable ductility limit, considered as an unknown in the current framework.

The behavior of a typical beam which belongs to the cantilevered part is depicted in Fig. 2. The horizontal axis represents the uniform load acting on the beams, while the vertical axis represents the moments of the left end (below the horizontal axis – negative) and the right end (above the horizontal axis – positive). For load values $q < q_{lf,j}$ the beam behaves in a linear elastic way. At this load value a plastic hinge has been formed in the left end. When $q_{lf,j} < q < q_{rf,j}$ the moment of the left end remains practically the same, while the right end attracts more bending. For $q = q_{rf,j}$ a plastic hinge has been formed in the right end, after which the beam enters the hardening zone.

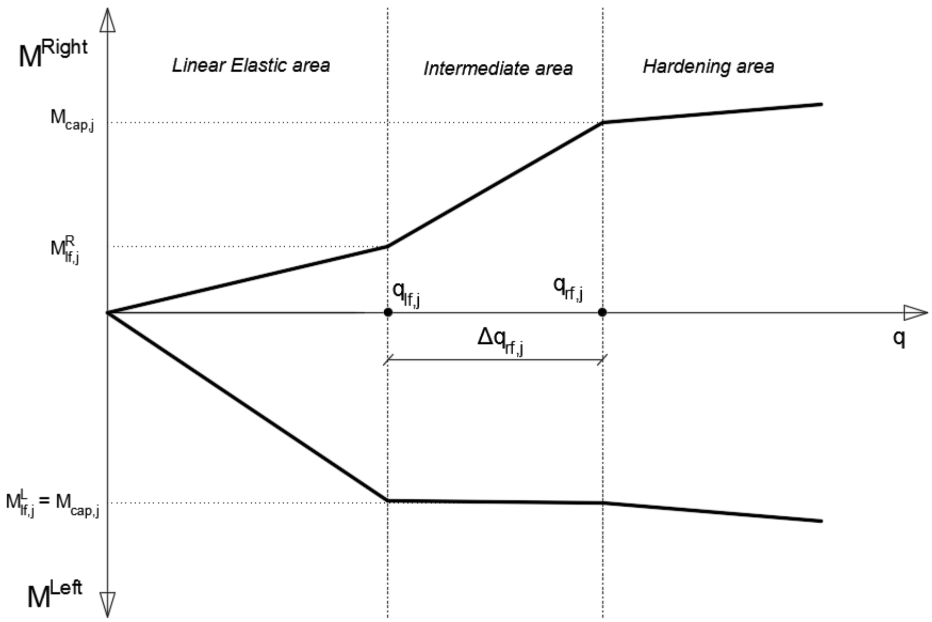

Fig.2: Moment at left and right end of a typical beam vs uniform vertical load acting on beam

The behavior described above is representative of every beam above the removed member. It has to be clarified here that when $q = q_{lf} = \max(q_{lf,j})$ the cantilevered part has formed plastic hinges at every beam left end and when $q = q_{rf} = \max(q_{rf,j})$ the cantilevered part has formed plastic hinges at every beam right end. Therefore, we can obtain the response of the cantilevered part as follows (model "A" is the initial structural system and model "B" is the cantilevered part with plastic hinges – Fig 1(a) and (d)):

1) We conduct a linear elastic analysis in model A, applying an arbitrary load $\overset{(A)}{q}_{el}$ and obtaining $\overset{(A)}{M}_{L,j}$ and $\overset{(A)}{M}_{R,j}$. The latter are the moments acting on the left and right end of every beam element j for this applied load. Since we are in the linear elastic area, through simple analogy we can calculate $q_{lf,j}$, the load at which the left plastic hinge will form, as well as $M^R_{lf,j}$, which is the acting moment on the right end at this load value.

2) After all left plastic hinges have been formed ($q = q_{lf}$) we conduct a linear elastic analysis in model B, applying an arbitrary load $\overset{(B)}{q}_{el}$ and obtaining $\overset{(B)}{M}_{R,j}$. The latter is the moment acting on the right end of every beam element j for this applied load. Assuming linear behavior of the beam right end we can calculate $\Delta q_{rf,j}$, which is the additional load that needs to be applied in beam j in order the right end plastic hinge to form.

3) The total applied load in beam j when the right end plastic hinge is formed is:

$$q_{rf,j} = q_{lf,j} + \Delta q_{rf,j} \tag{2}$$

Since the yielding-type failure mode is dictated by the last beam member to form a plastic hinge, it can be defined by a function $C_b(\alpha)$ with the following form:

$$C_b(a) = q_{rf}(a) = \max_{j \in N}\left(q_{lf,j}(a) + \Delta q_{rf,j}(a)\right) \tag{3}$$

By substituting $q_{lf,j}$ and $\Delta q_{rf,j}(a)$ in Eq. 3, we end up with the following equation:

$$C_b(a) = f_y \cdot \mu \cdot \max_{j \in N}\left(\frac{w_j}{M^{(A)}_{L,j}(a)} \cdot q^{(A)}_{el} + \frac{w_j \cdot \left(1 - \dfrac{M^{(A)}_{R,j}(a)}{M^{(B)}_{L,j}(a)}\right)}{M^{(B)}_{R,j}(a)} \cdot q^{(B)}_{el} \right) \tag{4}$$

where *(α)* represents a random corner column removal scenario. Eq. (4) provides the critical load at which the yielding-type mode will be activated.

2.3 Mathematical treatment of loss-of-stability mode

The critical load $P_{R,i}$ of an axially loaded column is given in Table 1:

Table 1: Column critical load

Type of column	**Condition**	**Critical load**
Slender	$P_{Euler} < A_i \cdot f_y$	$P_{Euler} = \dfrac{\pi^2 \cdot \mathrm{E} \cdot \mathrm{I}}{(k \cdot H)^2}$
Intermediate	$P_{Euler} > A_i \cdot f_y$	$A_i \cdot f_y$
Stocky	$P_{Euler} > A_i \cdot f_y$	$A_i \cdot f_u$

where A_i is the column's cross section, I is the moment of inertia, E is the modulus of elasticity, f_y is the yield stress and f_u is the ultimate stress of the material. In case of slender columns buckling is going to be elastic, while for intermediate columns buckling is going to be inelastic. Stocky columns will suffer material failure and are considered a very rare case; however they are mentioned here for the sake of completeness.

From the linear elastic analysis of model A we obtain the values $\overset{(A)}{P_i}$, which are the axial load of every column at this applied load. For every column we can then calculate the values of $q_{stability,\,i}$ as follows:

$$q_{stability,i} = \frac{P_{R,i}}{\overset{(A)}{P_i}(a)} \cdot \overset{(A)}{q}_{el}$$

(5)

where $q_{stability,i}$ is the load at which buckling of column i will occur and $P_{R,i}$ is the critical load. Since the loss-of-stability mechanism is governed by the first column element to buckle, it can be defined by a function of the following form:

$$Cc(a) = q_{stability} = \min_{i \in \theta}\left(\frac{P_{R,i}}{\overset{(A)}{P_i}(a)} \cdot \overset{(A)}{q}_{el} \right) \tag{6}$$

where *(α)* represents a random corner column removal scenario.

2.4 Progressive collapse limit state function

The governing mechanism can be provided by comparing the two functions and defining a new progressive collapse function *R(α)*:

If $R(a) = \dfrac{Cc(a)}{Cb(a)} < 1$ => *Loss-of-stability mode*

If $R(a) = \dfrac{Cc(a)}{Cb(a)} > 1$ => *Yielding-type mode*

If we substitute Eqs. (4) and (6) into *R(α)* and solve for ductility μ, we will obtain the following equation:

$$\mu = \frac{1}{f_y} \cdot \frac{\min_{i \in \theta}\left(\dfrac{P_{R,i}}{\overset{(A)}{P_i}(a)} \cdot \overset{(A)}{q}_{el} \right)}{\max_{j \in N}\left(\dfrac{w_j}{\overset{(A)}{M}_{L,j}(a)} \cdot \overset{(A)}{q}_{el} + \dfrac{w_j \cdot \left(1 - \dfrac{\overset{(A)}{M}_{R,j}(a)}{\overset{(B)}{M}_{L,j}(a)} \right)}{\overset{(B)}{M}_{R,j}(a)} \cdot \overset{(B)}{q}_{el} \right)}, \quad \alpha \in A \tag{7}$$

where *(α)* represents a random corner column removal scenario and A is the set of the total possible corner column removal locations.

The ductility μ indicates which collapse mechanism will be activated first. If $\mu < 1$, then the governing failure mode is the loss-of-stability one, while when $\mu > 1$ the governing mechanism is the yielding-type mode. The critical ductility $\mu_{critical} = 1$ is the threshold between the two failure modes, where the collapse mechanism switches from one mode to the other. What must be explicitly mentioned here, is the fact that μ is dependent on a set of parameters which are either already known (w_j, f_y, $P_{R,i}$) or products of our decision ($\overset{(A)}{q}_{el}, \overset{(B)}{q}_{el}, \overset{(A)}{M}_{L,j}, \overset{(A)}{M}_{R,j}, \overset{(B)}{M}_{R,j}$). Therefore, we can avoid the computational cost of an analysis which would include material and geometric nonlinearities; only two simple linear elastic analyses are adequate to provide enough information about the load at which each collapse mechanism will occur.

3. Application – Numerical Examples

3.1 Mechanical description of steel frames

The 2D steel frames analyzed in the current study are designed for the Seismic Analysis Code (SAC) Steel Project and correspond to the 20-storey buildings in the cities of Los Angeles, Seattle and Boston, located in N-S direction. Both Pre- and Post-Northridge practices were taken into consideration. A detailed description of the steel frame sections can be found in [4]. All frames share the same elevation, as depicted in Fig. 3. The material used for all beams and columns is A572 Gr.50 steel with isotropic strain hardening, modulus of elasticity 210 GPa, yield strength 50 ksi (345 MPa) and ultimate strength 65 ksi (450 MPa) at uniaxial strain 18%. The graphic depiction of the material law is shown in Fig. 4.

For the 6 different frames, both the analytical method and FEM simulation were conducted for all the possible corner column loss scenarios, removing every time the column from each story. The total number of progressive collapse scenarios analyzed was 120. To facilitate the following discussion, the notation of C-P-N is used for every progressive collapse analysis conducted. In this notation ‘C’ represents the city to which the model belongs (Boston, Los Angeles, Seattle), ‘P’ stands for Pre- or Post- Northridge guidelines, and ‘N’ depicts the floor from which the corner column was subtracted.

3.2 Computational Modeling of Steel Frames and description of the analysis

All simulations were carried out using the finite element software ABAQUS. The element type assigned in both columns and beams was the Timoshenko (shear flexible) beam element B32OS. The only exception was the exterior columns in the Los Angeles models which have a box section, and thus were assigned beam element B32. Beams and columns were meshed in a way that each one of them comprises 10 elements, resulting in a total amount of 2,410 elements and 4,717 nodes per model. Base nodes are considered to be pinned, and lateral support is provided in the 2 basement floors.

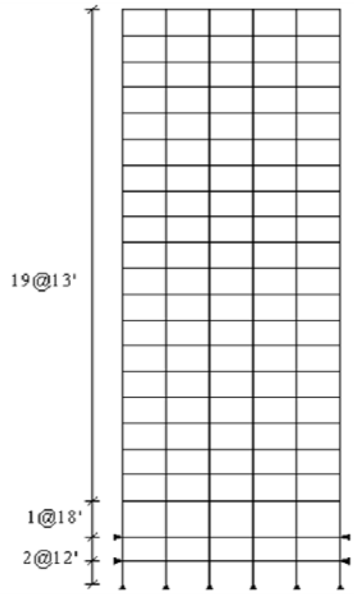

Fig.3: Model Building Elevation

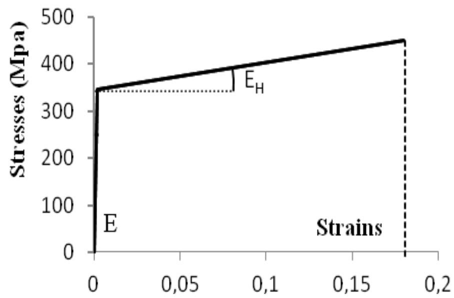

Fig.4: Material law

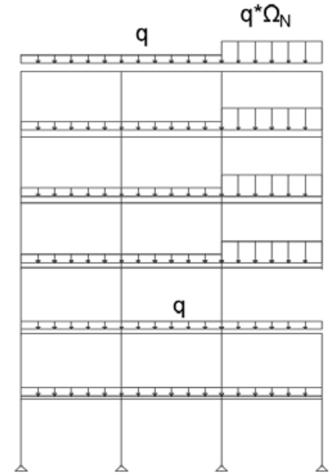

Fig.5: Dynamic Increase Factor

Pushdown analysis was employed, accounting for both material and geometric nonlinearities. Vertical uniform load was applied in every bay incremented from zero until a critical value, at which a collapse mechanism was obtained. For the particular frames the vertical loads of the roof were multiplied by the factor 0.88, according to the loading information provided in [4]. The loading pattern follows the guidelines described in [2], where the Dynamic Increase Factor Ω_N is employed to account for the dynamic nature of the phenomenon, multiplying the nominal gravity loads. The increased gravity loads were applied to those bays immediately adjacent to the removed element and at all floors above the removed element. For every primary structural element (beam) in this area, the ratio θ_{pra}/θ_y was obtained. θ_{pra} is the plastic rotation angle given in the acceptance criteria tables in ASCE 41 for the appropriate structural response level (Collapse Prevention or Life Safety) and θ_y is the yield rotation of the beam, given in Equation 5-1 in ASCE 41. These parameters were obtained through the following equations:

$$\theta_{pra} = 0.0337 - 0.00086 \cdot (h_{beam} / 2) \tag{8}$$

$$\theta_y = \frac{Z_{pl} \cdot f_y \cdot l}{6EI} \tag{9}$$

where h_{beam} is the depth of the beam cross section, Z_{pl} is the plastic modulus, f_y is the yield stress, l is the length of the beam, E is the modulus of elasticity and I is the moment of inertia. Ω_N was then determined for every column removal scenario using the smallest ratio of θ_{pra}/θ_y calculated, as shown in Eq. (10):

$$\Omega_N = 1.08 + 0.76 / (\theta_{pra} / \theta_y + 0.83) \tag{10}$$

For every model the dynamic increase factor was calculated for each column removal scenario; since the differences among the Ω_N values were not significant, the maximum value of Ω_N was applied to all column removal scenarios within the same model.

3.3 New Euler-type progressive collapse curves for the six frames

The results of the analytical method are presented in Fig. 6-7. These graphs illustrate the ductility limit state curve for every model. The vertical axis of the graph depicts the ductility factor μ as presented in Eq. (7), while the horizontal axis represents the floors above the column removal. Each point in the graph simulates a distinct column removal scenario. The dashed line ($\mu = 1$) represents the threshold of change between the two collapse mechanisms. When $\mu > 1$ failure is governed by the yielding-type mode, while when $\mu < 1$ the collapse mode is the loss-of-stability one.

A first observation of the results is that the ductility factor μ is indeed descending as the column removal moves from the top floors to the bottom ones. Therefore, when a column is removed in the upper floors the governing mechanism is yielding type failure, while when it is removed in the lower ones loss-of-stability dominates the phenomenon. Buckling requires a minimum number of floors in order to appear, although this number is not consistent between the frames examined.

Another useful remark is that the majority of the frames are predicted to fail due to loss-of-stability. Out of the 120 models analyzed, 75 were governed by column mode while 45 were governed by beam mode. The aforementioned note illustrates the strong correlation between stability and progressive collapse, highlighting the importance of investigating possible instability phenomena when a progressive collapse analysis is conducted.

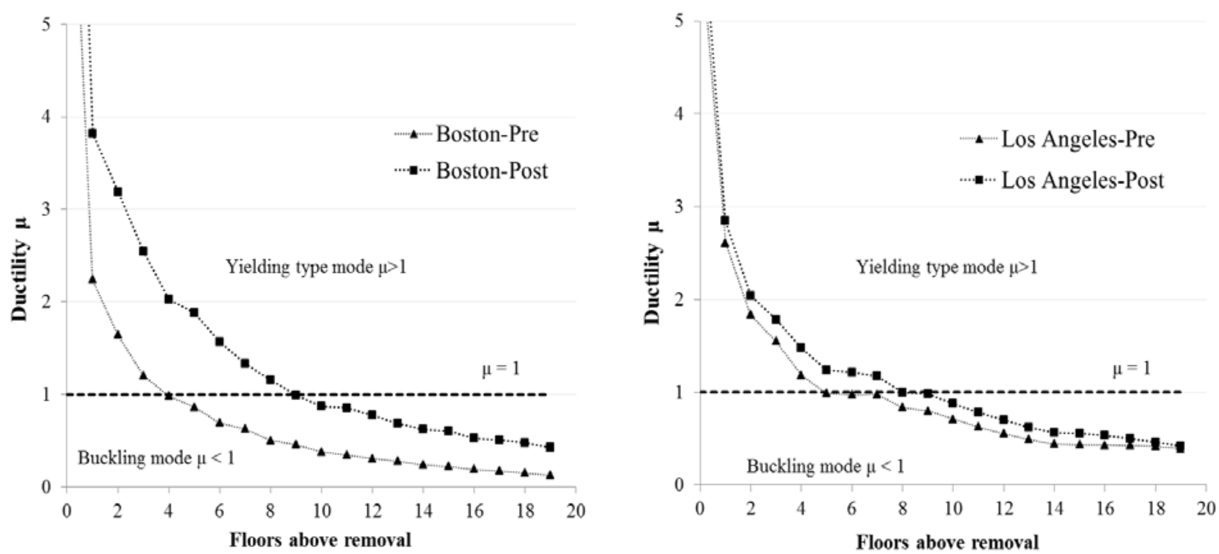

Fig.6: Euler-type curve for Boston-Pre, Boston-Post, Los Angeles-Pre and Los Angeles-Post models

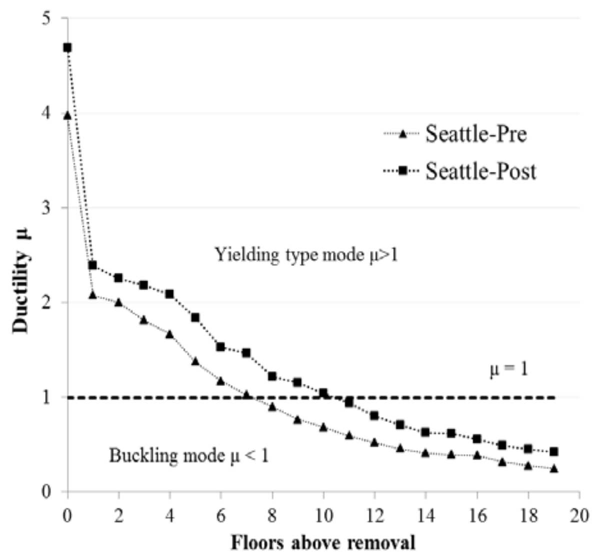

Fig.7: Euler-type curve for Seattle-Pre and Seattle-Post models

3.4 Computational Results

The main interest of the computational analysis was to compare the results of the complex nonlinear analysis conducted by FEM software with the outcome of the proposed method, which employs only key property features of the system and two elastic analyses of arbitrary load.

The behavior of the structures in Abaqus was monitored after specific criteria were set. Loss-of-stability is considered to be met when the axial load in critical column is almost equal to inelastic buckling load and at the same time the horizontal displacement in the column middle point increases abruptly. The yielding type of collapse is the governing failure mode when the moment on both ends of the cantilevered part beams have reached their flexural capacity and at the same time buckling has not yet appeared.

3.4.1 Loss-of-stability failure mode

Two typical examples of loss-of-stability modes are presented here, to illustrate the aforementioned buckling criteria. The first is the Los Angeles-Pre-9 scenario and the second is the Boston-Pre-16 scenario, where in both models the critical column is the adjacent to the removed one. The deformed shapes of the two models are depicted in Fig. 8 and 9, while the axial load of the buckled columns are derived and compared to the theoretical compression capacity of the respective sections (inelastic buckling load) in Table 2.

It is apparent in both cases that inelastic buckling is the governing failure mode of the structure. The axial load of critical columns reaches the inelastic buckling load value, while the horizontal displacement of the column middle point displays an excessive increase. A useful observation is that although in the Boston-Pre-16 model the column is removed from the 16th floor, which is considered to be in the upper floors of the structure and yielding type mode would be most probably expected, finite element analysis proves that loss-of-stability will be activated first. This is another important finding that stability may occur in unexpected areas of the structure and therefore should not be overlooked.

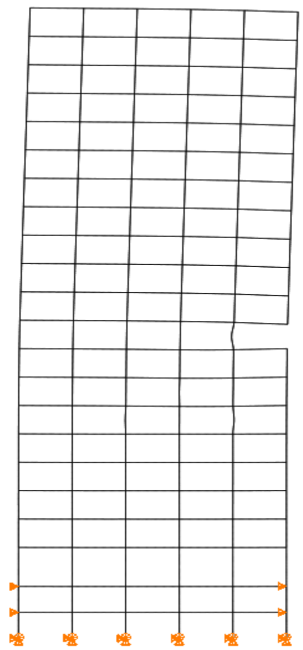

Fig.8: Deformed shape – Los Angeles-Pre-9 model

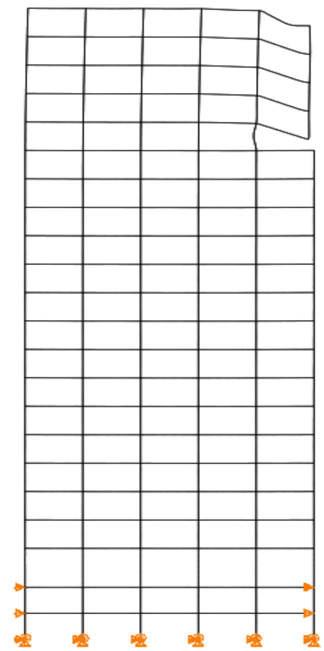

Fig.9: Deformed shape - Boston-Pre-16 model

Table 2: Comparison of Abaqus and theoretical inelastic buckling loads

Case	Buckled Column			
	Floor	Section	N_{INEL} (kN)	N_{ABAQUS} (kN)
Los Angeles – Pre - 9	9	W24x229	14957	14771
Boston – Pre -16	16	W30x99	6477	6353

3.4.2 Yielding-type failure mode

The yielding-type progressive collapse is experienced by the structure through the formation of plastic hinges at both ends of the beams above the column removal. Seattle-Pre-16 model is employed as a typical example of such a collapse mode (Fig. 10). Table 3 shows the section for every beam element, its flexural capacity and the acting moment at $q = q_{rf}$.

3.5 Validation of the new Euler-type curve

Table 4 shows the comparison between computational analysis and the Euler-type curve method, for the six frames analyzed. The numbers indicate the range of floors where the specific collapse mechanism appears.

When the collapse mode is switching from one to another Abaqus will provide a vague picture of which mechanisms is activated first. Therefore, a thorough investigation of $q_{stability,i}$ and $q_{rf,i}$ is required in order to get a clear insight.

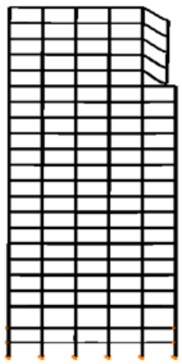

Fig.10: Deformed shape Seattle-Pre-16 Model

Table 3: Beam's cross section, flexural capacity and acting moment at t= $t_{critical}$

Floor	Section	Mrd (kN-m)	$M_{t=tcritical}$ (kN-m)	
			left	right
20	W21x57	729	-720	721
19	W21x57	729	-718	717
18	W21x57	729	-718	717
17	W24x62	870	-857	857
16	W24x62	870	-858	858

An absolute consistency between the analytical and computational results is observed. The Euler-type curves are validated for all the cases, being capable of predicting not only the exact mode of collapse, but also (in the vast majority of loss-of-stability failure) the location of the buckled column as well. Even in cases where the Euler-type predicts a value of ductility μ slightly greater or smaller than 1, the predicted mode of collapse is evaluated by finite element results.

Table 4: Comparison between Abaqus and Euler-type curve method results

			Buckling mode	*Yielding type mode*
Boston	Pre-Northridge	FEM	Floors 1-16	Floors 17-20
		Euler-type curve	Floors 1-16	Floors 17-20
	Post-Northridge	FEM	Floors 1-11	Floors 12-20
		Euler-type curve	Floors 1-11	Floors 12-20
Los Angeles	Pre-Northridge	FEM	Floors 1-15	Floors 16-20
		Euler-type curve	Floors 1-15	Floors 16-20
	Post-Northridge	FEM	Floors 1-12	Floors 13-20
		Euler-type curve	Floors 1-12	Floors 13-20
Seattle	Pre-Northridge	FEM	Floors 1-12	Floors 13-20
		Euler-type curve	Floors 1-12	Floors 13-20
	Post-Northridge	FEM	Floors 1-9	Floors 10-20
		Euler-type curve	Floors 1-9	Floors 10-20

5. Limitations and Approximations of the Analytical Method

One major approximation of the proposed analytical method is the consideration only of 2D steel moment resisting frames. 3D effects can drastically change the results of a progressive collapse analysis, since it is possible in many cases for loss-of-stability to occur in columns which do not belong to the same planar frame as the removed member does. Therefore, a future step would consider including 3D effects in the analytical proposed method, accounting also for the composite action between slab and frame beams.

Another approximation is the consideration of bilinear behavior in the right end of the beam, before plastic hinge is formed at this point. Although this behavior is more accurate on the left equivalent, acting moments on the right end follow a slight nonlinear path for a limited load range around $q = q_{lf}$, before returning to the linear path of the intermediate area (Fig. 2). However, this approximation is not considered to be crucial and this is proved by Table 4, where the Euler-type curve results are validated in every case by finite element nonlinear analyses.

Another limitation of the method is that it takes into consideration only two out of all possible failure modes that may be activated. For instance, another collapse mechanism that could be activated following the event of a column removal is the global loss-of-stability failure mode [13]. Enriching the proposed method by taking into account more failure modes would be another important step towards a more global treatment of the phenomenon and is considered a necessary future step.

6. Conclusions

A simplified, analytical method was proposed in the current study, which is capable of capturing the collapse mechanism of a 2D steel moment resisting frame structure when the notion of corner column removal scenario is employed. The yielding type mode and the loss-of-stability mode were taken into account and new Euler-type ductility curves are proposed. The latter can be obtained based only on geometry and material information, as well as 2 simple linear elastic analyses with a single loading step. Ductility values $\mu > 1$ indicate yielding type failure mode, while $\mu < 1$ declares that the governing mechanism is stability governed. The proposed method was applied to a series of 20-storey steel moment resisting frames and the results were verified by nonlinear static analyses conducted with the finite element software Abaqus. Euler-type curves were able to predict accurately and at no computational cost the exact mode of collapse, the load value at which each mechanism will appear and the exact location of the buckled column, if loss-of-stability is the governing mechanism.

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