Proceedings of the
Annual Stability Conference
Structural Stability Research Council
San Antonio, Texas, March 21-24, 2017

Global lateral – torsional buckling of I-girder systems in cantilever

Andres F. Robalino[1], T. Andres Sanchez[2]

Abstract
Global lateral – torsional buckling (GLTB) is a failure mechanism that may compromise the structural stability of a system of girders connected with cross-frames or diaphragms, during construction stages. This limit state may occur at load levels lower than those required to cause lateral – torsional buckling of individual girders between bracing points; especially, in bridges with relatively long spans and short widths. Closed form solutions are available to determine the GLTB resistance of single span girder systems with simple supports and subject to various loading cases. This paper presents the results of research conducted to determine the susceptibility to GLTB of I-girder systems in cantilever, in particular, in bridges erected with the incremental launching method (ILM). The results obtained from refined finite element models show that structures constructed with the ILM may be vulnerable to GLTB as the girder group cantilevers out from the support. Recommendations are provided to conduct the GLTB strength checks of I-girder systems in cantilever using available equations.

Notation

C_b = moment gradient modifier
C_w = warping constant
E = modulus of elasticity of steel
h_o = distance between flange centroids
I_{eff} = effective moment of inertia
I = interaction factor
I_x = moment of inertia with respect to the horizontal principal axis
I_{yc} = moment of inertia of the compression flange
J = torsional constant
L_b = beam length between simple supports
L_c = cantilever length
L_g = span length
M_{crc} = lateral-torsional buckling strength of a beam in cantilever
M_{glb} = elastic global lateral-torsional buckling strength of a system with simple supports

[1] Junior Structural Engineer, ADSTREN, <arobalino@adstren.com>
[2] Structural Engineer, ADSTREN, <tasanchez@adstren.com>

M_{glc} = elastic global lateral-torsional buckling strength of a system in cantilever

M_{gls} = elastic global lateral-torsional buckling strength of an overhanging system

S = girder spacing

R = torsion parameter

t_{fc} = thinckness of the compression flange

t_{ft} = thinckness of the tension flange

t_w = web thickness

α = coefficient for loading position

γ = critical load coefficient

ϕ_b = resistance factor for bearing

ϕ_f = resistance factor for flexure

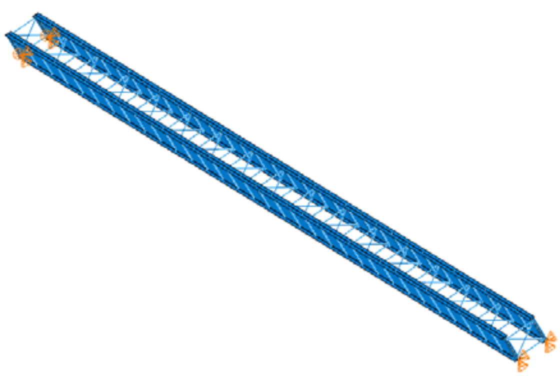
ϕ_v = resistance factor for shear

1. Background and Previous Work

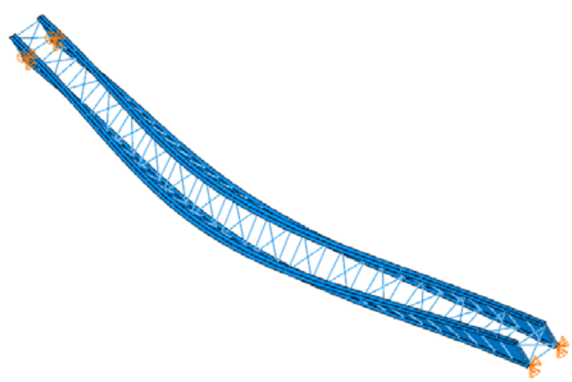
Global lateral – torsional buckling is a type of failure where an entire group of girders become unstable under vertical loads. Girder systems with a relatively long span and a narrow width are susceptible to this phenomenon. Yura et al (2008) developed the following closed form solution to determine the critical moment, M_{gl}, that may cause GLTB in a girder group

$$M_{gl} = 2C_b \frac{\pi E}{L_g} \sqrt{\frac{I_{yc} J}{1.3} + \frac{\pi^2 I_{yc}^2 h_o^2}{L_g^2} + \frac{\pi^2 I_{eff} I_x S^2}{4L_g^2}} \qquad (1)$$

This equation captures the load level and associated bending moment that would cause this type of instability in a system composed of two or more girders, simply supported and free to warp, as depicted in Figure 1.



Undeformed shape



Deformed Shape

Figure 1: Global lateral – torsional buckling of a twin-girder system subject to a uniformly distributed load

Equation (1) is valid for both girder groups with doubly symmetric and singly symmetric sections. This equation is derived from the classical solution for a simply supported beam subject to major-axis bending (Timoshenko and Gere 1961). For this purpose, the warping stiffness of the single beam is replaced by the warping stiffness of the entire group so that

$$EC_w = \frac{2EI_y h_o^2}{4} + \frac{2EI_x S^2}{4} \tag{2}$$

As discussed in Yura et al (2008), the first term in Eq. (2) represents a small part of the warping rigidity and does not contribute significantly to the global buckling strength, M_{gl}. Therefore, Eq. (2) may be simplified to ignore the contribution of individual girders and consider the contribution of the system twist only. As a result, the warping rigidity may be conservatively computed as

$$EC_w = \frac{EI_x S^2}{2} \tag{3}$$

The lateral-torsional buckling (LTB) resistance of girders in cantilever is a subject that has been studied in the past, and different solutions exist to determine the strength of these types of members, depending on the support and loading conditions (Timoshenko and Gere 1961, Nethercot 1973, Trahair 1983, Essa and Kennedy 1994, Dowswell 2004). For a cantilever beam subject to a uniform distributed load applied at the top flange, with a fixed support at one end and free at the other, Nethercot (1973) shows that the critical moment, M_{crc}, may be calculated as follows

$$M_{crc} = \alpha \frac{\pi}{L_b} \sqrt{EI_y GJ + \frac{\pi^2 E^2 I_y C_w}{L_b^2}} \tag{4}$$

Where

$$\alpha = \frac{A}{B_1}$$

$$A = 2.030 - \frac{9.114}{R^2} + \frac{7.245}{R} \tag{5}$$

$$B_1 = 0.934 + \frac{1.290}{R^2} + \frac{3.418}{R}$$

$$R = \sqrt{\frac{L^2 GJ}{EC_w}} \tag{6}$$

The equations for a single cantilever beam can be combined with Eqs. (1) and (3) to obtain the GLTB strength of a twin-girder system in cantilever. For this purpose, the calculation of R is modified by replacing EC_w in E. (6) by the warping stiffness term shown Eq. (3). Also, since there are two girders instead of one, the term GJ in the numerator of Eq. (6) must be replaced by $2GJ$. Therefore, the torsional parameter, R, may be expressed as

$$R = 2\sqrt{\frac{L^2 GJ}{EI_x S^2}} \tag{7}$$

Finally, the GLTB strength of a twin system in cantilever can be computed with the following equation:

$$M_{glc} = \alpha\left(2\frac{\pi E}{L_g}\sqrt{\frac{I_{yc}J}{1.3}+\frac{\pi^2 I_{yc}^2 h_o^2}{L_g^2}+\frac{\pi^2 I_{eff} I_x S^2}{4L_g^2}}\right) \tag{8}$$

The procedures described previously are valid for simply supported girder systems (Yura et al 2008) and for individual built-in cantilever beams subject to uniformly distributed loads acting on the top flange (Nethercot 1973). The combination of both procedures may be used to predict the GLTB strength of a built-in cantilever system, using Eq. (8). These criteria set the background to develop a methodology to compute the GLTB strength of a structural system with overhanging girders. In a system with overhanging girders, the cantilever is the projection over a support (sometimes referred as fulcrum) of simple supported or continuous girders. In practice, this condition exists when a steel I-girder bridge is erected with the incremental launching method (ILM).

In the ILM the bridge girders are assembled on firm soil, and then they are pushed over the obstacle until it reaches the next support; Fig. 2 shows the photograph of a steel I-girder bridge erected with ILM.

Figure 2: Bridge erected with the incremental launching method

In the next sections, studies are conducted to develop a procedure that may be used to predict the GLTB strength of steel I-girder bridges erected with the ILM. For this purpose, the developments introduced previously are modified to capture the buckling strength of a system of overhanging girders.

2. Finite Element Model Descriptions

The finite element analyses (FEA's) developed in this study are carried out in the ABAQUS v6.13 program (Simulia 2013). All the FEA's are elastic eigenvalue buckling analyses that capture the behavior of girder systems subject to uniformly distributed loads that are prone to fail due to GLTB. In the analyses, the webs and flanges are modeled with the general-purpose shell element, S4R, available in the ABAQUS element library. Twelve shell elements are used to model web, and four elements are used for the flanges. Cross-frame top and bottom chords are modeled with B31 elements, and diagonals are modeled with the truss type element T3D2, from the ABAQUS library. Similarly, girder transverse stiffeners are modeled with B31 elements. The modeling techniques implemented in this study have been applied and validated in previous research to predict the behavior of steel girder bridges (Kim 2010, Sanchez 2011, Subramanian 2015). Figure 3 shows the 3D representation of a typical FE model used in this research.

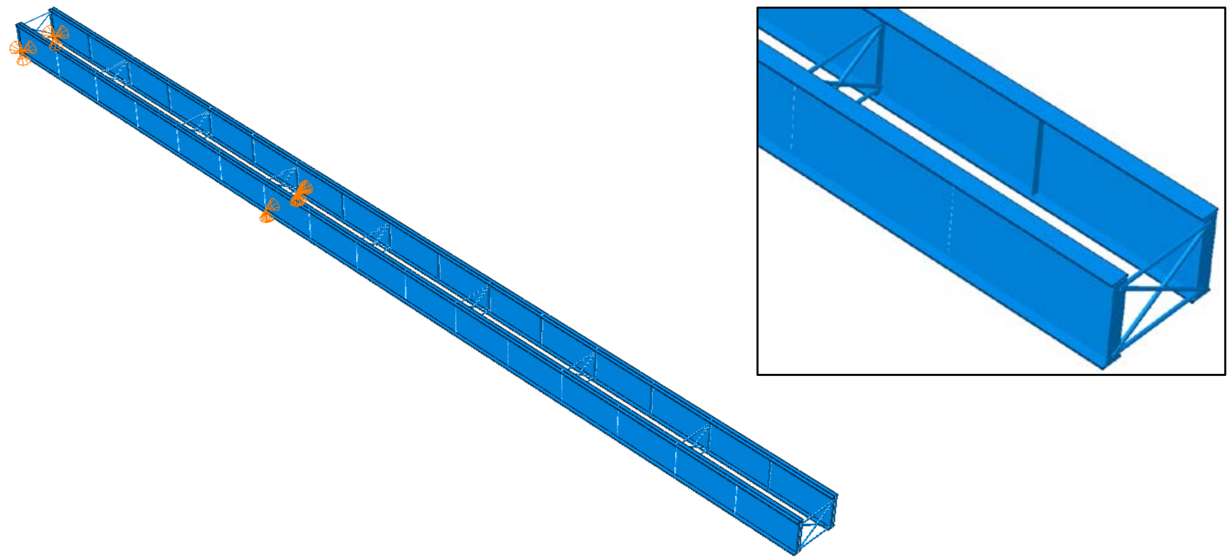


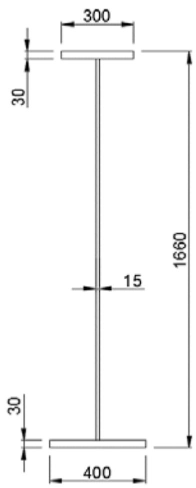
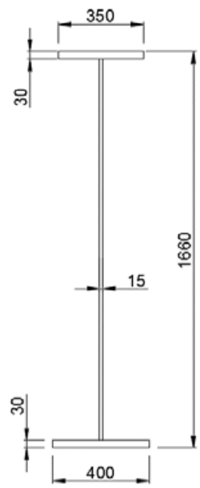
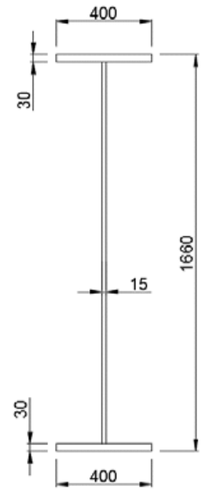
Figure 3: Three-dimensional representation of a typical girder system modeled in the ABAQUS program

3. Behavior of I-Girder Systems with Overhanging Cantilevers

As mentioned in the previous sections, systems with overhanging girders may be prone to fail due to GLTB. In the ILM, the cantilever length increases as the bridge is moved forward, increasing the potential of this failure mechanism to occur. In this section, FE analyses are conducted in a series of systems with different girder cross-sections and span lengths to investigate their behavior and determine their GLTB strength.

Table 1 summarizes the nine structural systems considered in the analyses. One doubly symmetric and two singly symmetric sections are used to model twin girder systems with total lengths of 50m, 75m, and 100m. The table shows the cross-section dimensions and the nomenclature used to identify each structure.

Table 1: Cross-section properties and model nomenclature for analyses

Cross-Section Dimensions			
Girder Length (m)	Nomenclature		
50	L50SS300	L50SS350	L50DS400
75	L75SS300	L75SS350	L75DS400
10	L100SS300	L100SS350	L100DS400

The girder cross-sections have been selected so they satisfy the dimensional requirements specified in Section 6.10.2 of AASHTO (2014). The systems studied in this research are composed of two girders connected with cross-frames at every 6.25m. The distance between girder centerlines is fixed to 2000m. The girder lengths, cross-frame spacing, and girder distance are selected so the primary failure mode is GLTB. The general configuration of the analyses is shown in Figure 4. In the models, the support at the right is moved in each step to increase the cantilever length, L_c, from $L_c/L_b = 0$ up to $L_c/L_b = 2$.

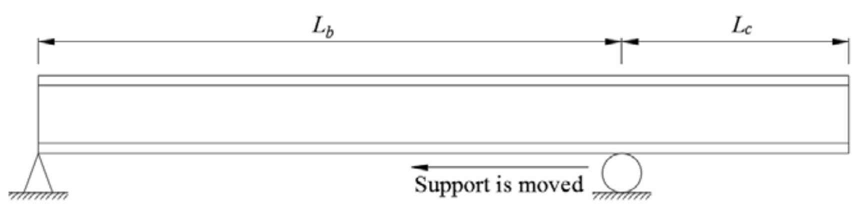
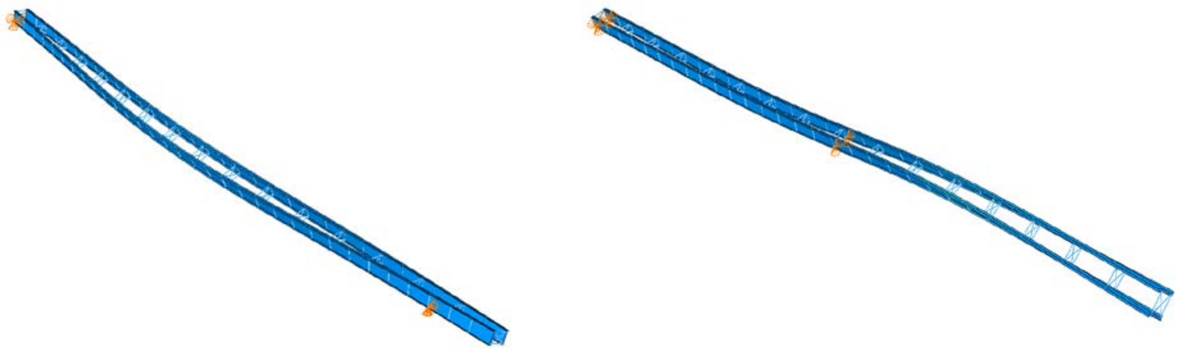


Figure 4: Configuration of models for FEA's

In each analysis, the girder system is subject to a uniformly distributed load (UDL) applied at the top flange, and the buckling load is obtained from the FEA solution. The UDL is obtained by dividing the self-weight of the structure by the total length of the girder system. Figure 5 shows the buckled shape of the L100DS400 model for $L_c/L_b = 0.188$ and $L_c/L_b = 1$. As depicted in the figure, the system's first buckling mode corresponds to GLTB. In the first case, with $L_c/L_b = 0.188$, buckling occurs in the back span, while for $L_c/L_b = 1$, the GLTB occurs in the cantilever part of the system.



L_c/L_b = 0.188 L_c/L_b = 1.0

Figure 5: Buckled shapes in the L100DS400 models

Figures 6, 7, and 8 show the results obtained from the FEA's and the predictions obtained with Eqs. (1) and (8). In these plots, the vertical axis represents the critical load factor, γ. The buckling load is equal to γ(UDL) that is, the factor that multiplied by the self-weight of the structure causes GLTB of the system. The γ factor is plotted versus L_c/L_b to observe the system behavior in terms of global buckling, as the cantilever length increases. The predictions obtained with Eq (1) corresponds to the GLTB strength of the back-span, which has a length equal to L_b. Similarly, the buckling loads obtained with Eq. (8) are the predictions for the cantilever portion of the structure if the cantilever is fully fixed at the support and with a length equal to L_c.

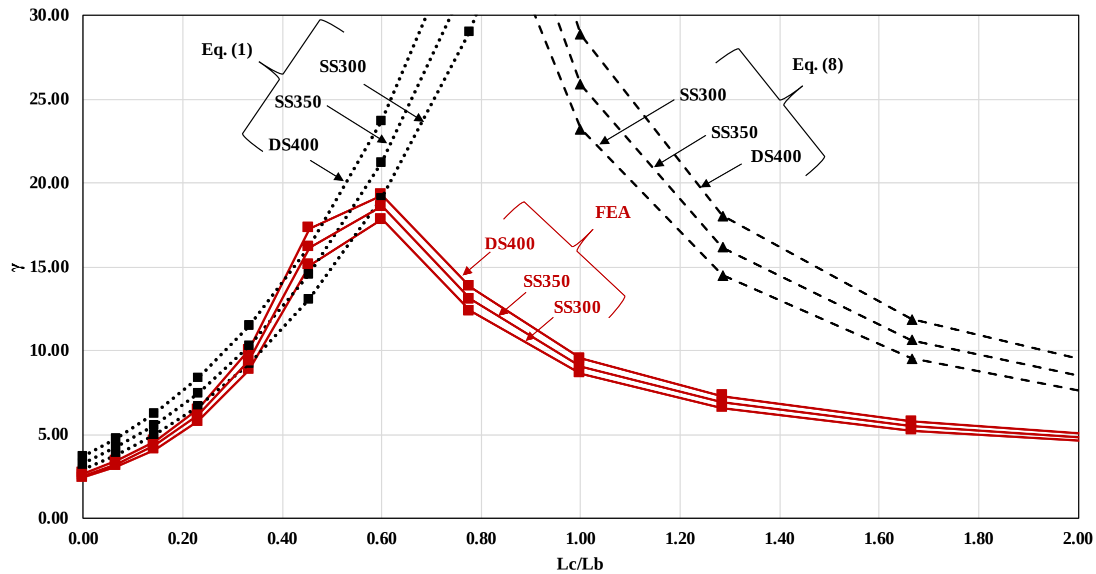


Figure 6: Buckling load predictions for the systems with L = 50m

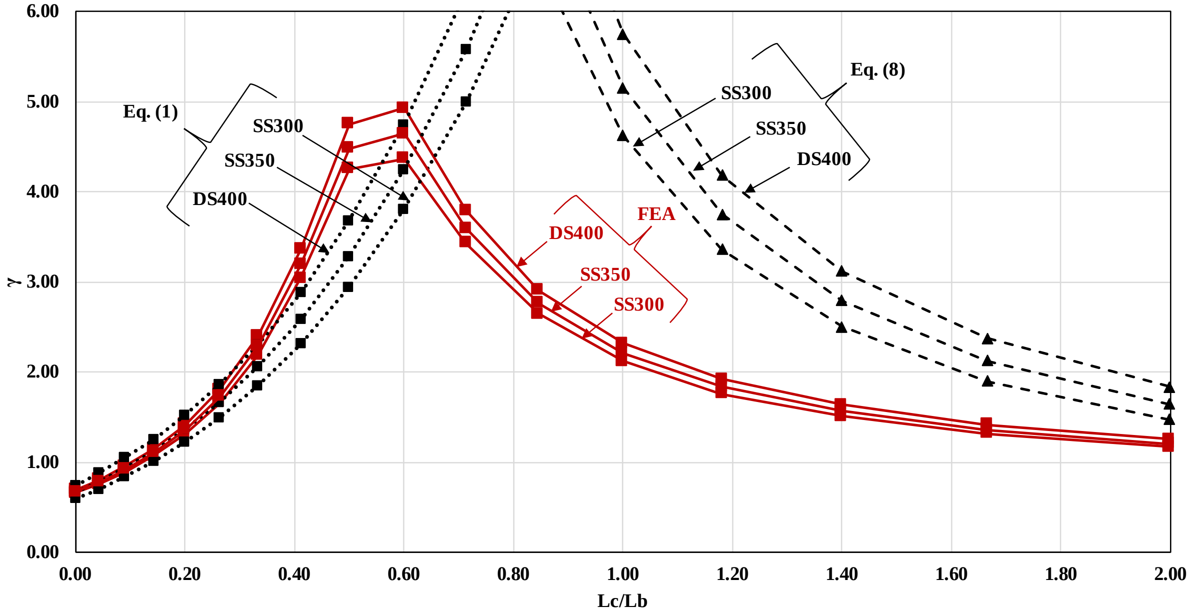


Figure 7: Buckling load predictions for the systems with L = 75m

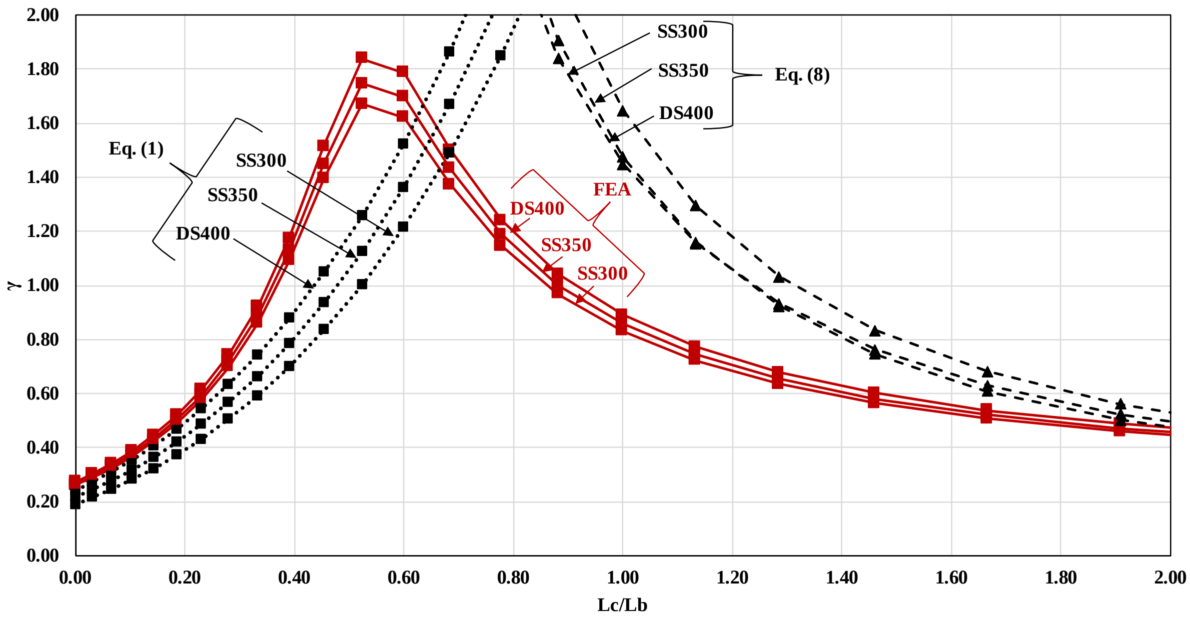


Figure 8: Buckling load predictions for the systems with L = 100m

The buckling load predictions obtained from the FEA show that the GLTB strength of the systems increase, as the L_c/L_b ratio is incremented, up to approximately 0.6. When this ratio is larger than this value, the buckling strength of the system decreases rapidly. From these observations, it is inferred that for L_c/L_b values between 0 and 0.6, the global buckling of the structures occurs in the back-span. As the cantilever length increases beyond this value, buckling occurs in the cantilever portion of the two-girder system. Another aspect shown in the plots is that Eq. (1) provides accurate predictions for the cases where buckling of the back-span controls. The predictions obtained with Eq. (8) in the region where cantilever buckling controls are significantly larger than the FEA benchmark. The reason is that Eq. (8) was developed for single beams in cantilever. In girder systems, such as the discussed in this paper, the torsion stiffness of the group is larger than that of single girders. However, the procedure presented in the next section is used to calibrate this equation, so the GLTB resistance of twin-girder systems in cantilever may be predicted accurately.

4. Proposed Procedure

Essa and Kennedy (1994) provide a method to calculate the elastic critical moment of an overhanging beam with a back-span. The procedure is based on the following equation:

$$M_{cr} = M_c + I\left(M_b - M_c\right) \tag{9}$$

Equation (9) is used to compute the critical moment, M_{cr}, that would cause LTB of the beam. In this equation, M_c represents the critical moment of the cantilever segment, and M_b is the critical moment of the back-span. The interaction factor, I, is a function of the ratio of the back-span to the cantilever span. It is obtained by curve fitting, and it captures the change in failure mode, as depicted in Figures 6, 7, and 8 for a twin-girder system.

The elastic critical moment of a two-girder structure, M_{gls}, may be obtained by replacing Eqs. (1) and (8) in Eq. (9) so that

$$M_{gls} = M_{glc} + I_{gl}\left(M_{glb} - M_{glc}\right) \tag{10a}$$

By recognizing that $M_{glc} = \alpha M_{glb}$, Eq. (9) may be expressed as

$$M_{gls} = M_{glb}\left[I_{gl} + \alpha\left(1 - I_{gl}\right)\right] \tag{10b}$$

The process to derive an expression for I_{gl} is based on the results shown in Figs. (6) to (8). For example, in Fig. (6), for DS400, with L_c/L_b = 0.23, the critical load factor γ is 8.34, as predicted by Eq. (1). For the same case, the γ factor obtained from the FEA is 6.48. Therefore, the ratio between Eq. (1) and the benchmark is 8.34/6.48=1.29. This value is equal to the expression shown in brackets in Eq. (10b), that is $I_{gl}+\alpha(1-I_{gl})$=1.29. The α coefficient depends on the cross-section properties and can be computed per Eq. (4). Thus, the value of I_{gl} for this case is equal to 0.71. Following the same procedure, different values I_{gl} are obtained for other cases. As a result, a set of two equations is derived to calculate Mgls, as function of the L_c/L_b ratio:

For $L_c/L_b \leq 0.55$,

$$M_{gls} = M_{glb}\left[2.414\left(\frac{L_c}{L_b}\right)^2 - 0.373\left(\frac{L_c}{L_b}\right) + 0.736\right] \tag{11a}$$

For $L_c/L_b > 0.55$,

$$M_{gls} = M_{glb}\left[-0.045\left(\frac{L_c}{L_b}\right)^2 - 0.350\left(\frac{L_c}{L_b}\right) + 0.026\right] \tag{11b}$$

The predictions obtained with these equations are plotted together with the results obtained from the FEA's in Figs. 9, 10, and 11.

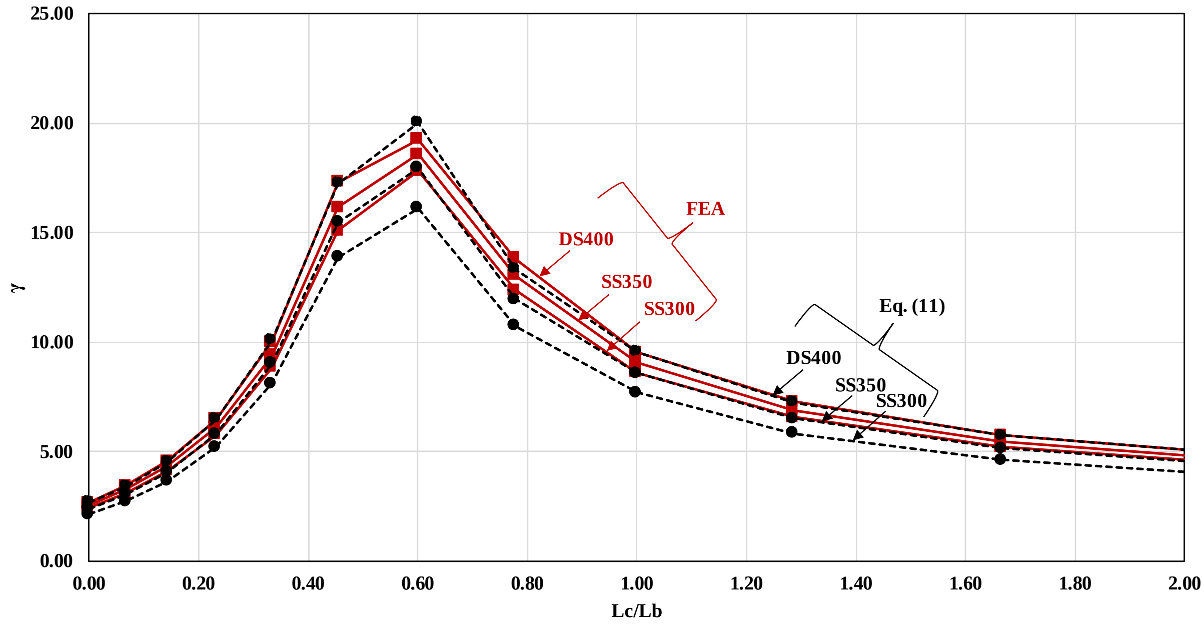


Figure 9: Buckling load predictions for the systems with L = 50m obtained with the proposed method

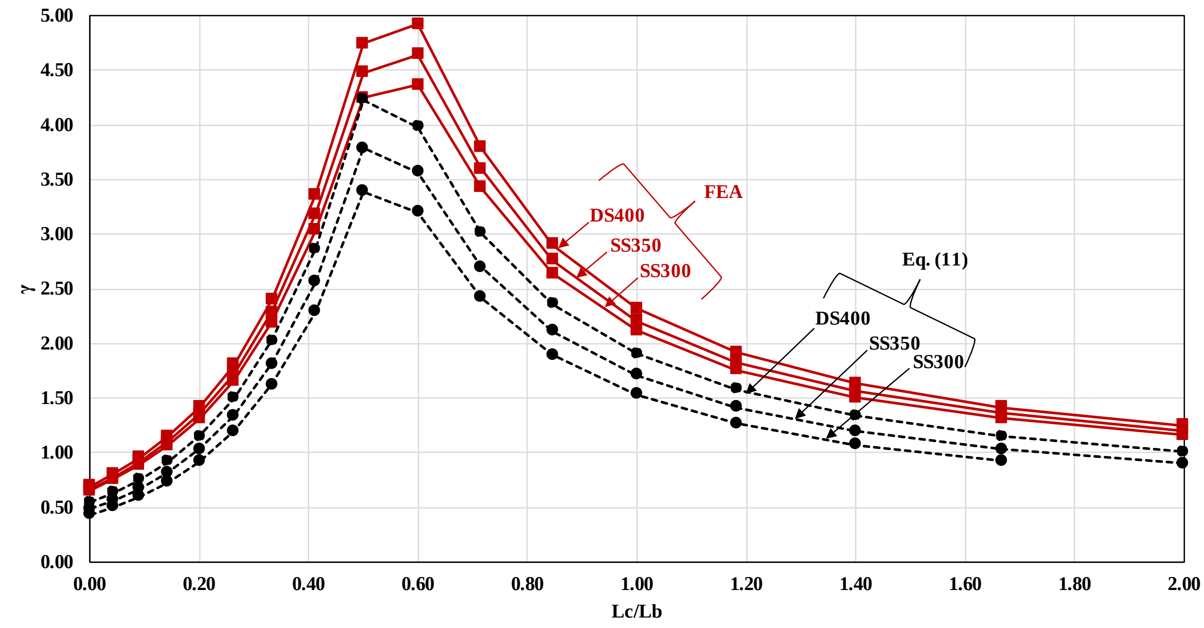


Figure 10: Buckling load predictions for the systems with L = 75m obtained with the proposed method

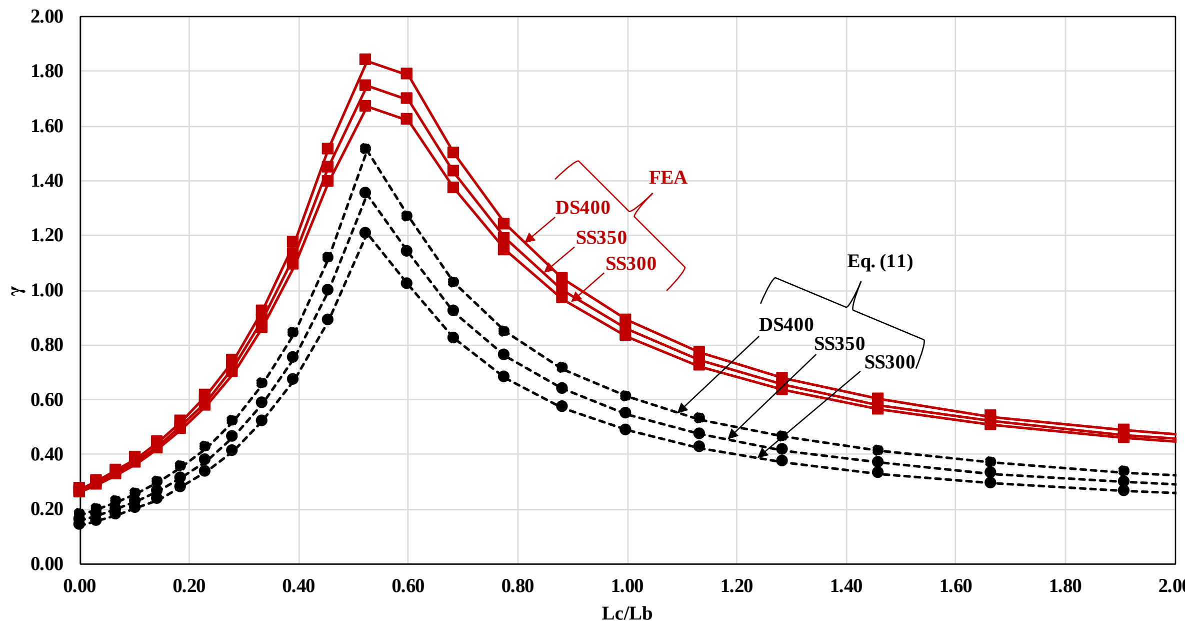


Figure 11: Buckling load predictions for the systems with L = 100m obtained with the proposed method

The results presented in the figures show that the predictions obtained with Eq. (11) are acceptable representations of the FEA results. For the case of L = 50m, the predictions calculated with the proposed method match the 3D FEA benchmarks. For L = 75m and L = 100m, the Eq. (11) results are conservative and follow the same trend as the expected responses. These results demonstrate that the GLTB strength of a system with overhang beams, as in the case of steel I-girder bridges erected with the ILM, may be computed using the proposed equations. The predictions obtained with Eq. (11) are accurate to conservative, so they may be used in cases where the structural stability of a girder system in cantilever is a concern. If the GLTB strength calculated with these equations is larger than the required bending strength, the system is stable. If the global buckling strength of the structure is less than the required strength, it is suggested to conduct an elastic 3D FE buckling analysis to determine the expected strength of the system and verify its structural integrity.

5. Conclusions

This paper presents studies conducted in I-girder systems in cantilever. This scenario is commonly found during the construction of steel girder bridges with the incremental launching method. As the bridge moves forward, the possibility of having a failure due to global lateral-torsional buckling increases. The methodology proposed in this paper consists in calculating the global buckling strength of two-girder systems by using two simple equations that are function of the cantilever span-to-back span ratio. The results show that the predictions obtained with the proposed equations are accurate to conservative, so they may be used in practice to conduct a quick check of the structural stability of these types of bridges. Further research is currently being conducted to refine the procedure and obtain a better approximation of the global lateral-torsional buckling strength of I-girder systems in cantilever.

Acknowledgments
The research presented in this paper is part of the construction engineering studies conducted for RIPCONCIV Cia. Ltda. and KUBIEC-CONDUIT S.A. The authors would like to acknowledge the financial support of these organizations. The opinions, findings, and conclusions expressed in this paper are the authors' and do not necessarily reflect the views of the above organizations.

References

AASHTO (2014). AASHTO LRFD Bridge Design Specifications. 7th Edition, American Association of State Highway and Transportation Officials, Washington, DC.

Dowswell, B. (2004). "Lateral-Torsional Buckling of Wide Flange Cantilever Beams." Engineering Journal, American Institute of Steel Construction, Chicago, IL, second quarter, Vol. 41, No. 2, 7pp.

Essa, H. S., Kennedy, D, J. L., (1994). "Design of Cantilever Steel Beams: Refined Approach." Journal of Structural Engineering, American Society of Civil Engineers, Reston, VA, Vol. 120, No. 9, 2623-2636.

Kim, Y. D. (2010). "Behavior and Design of Metal Building Frames Using General Prismatic and Web-Tapered Steel I-Section Members." Doctoral Dissertation, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA, 562pp.

Nethercot, D. A. (1973). "The Effective Lengths of Cantilevers as Governed by Lateral Buckling." The Structural Engineer, Vol. 51, No. 5, 8pp.

Sanchez, T. A. (2011). "Influence of Bracing Systems on the Behavior of Curved and Skewed Steel I-Girder Bridges during Construction." Doctoral Dissertation, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA, 360pp.

Simulia (2013). ABAQUS/Standard Version 6.13, Dassault Systèmes, Inc. Providence, RI, http://www.3ds.com/products-services/simulia/products/abaqus/

Subramanian, L., White, D.W. (2015). "Evaluation of Lateral Torsional Buckling Resistance Equations in AISC and AASHTO." Proceedings of the Annual Stability Conference, Structural Stability Research Council, Nashville, TN, 20pp.

Timoshenko, S.P., Gere, J.M., "Theory of Elastic Stability," 2nd edition, New York, McGraw-Hill. 1961.

Trahair, N. S. (1983). "Lateral Buckling of Overhanding beams." Instability and Plastic Collapse of Steel Structures, Granada Publishing, London, England, 13pp.

Yura, J., Helwig, T., Herman, R., and Zhou, C. (2008). "Global Lateral Buckling of I-Shaped Girder Systems." Journal of Structural Engineering, American Society of Civil Engineers, Reston, VA, Vol. 134, No. 9, 1487-1494.