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A Reappraisal of the Reliability of Local Buckling Rules Based on the Winter Curve

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Abstract
This paper contains a critical reappraisal of the reliability of local buckling rules based on the traditional "Winter curve". It makes use of a data pool of recently published and own "stub column"-type buckling tests, conducted on welded, square box sections, and uses the reliability design philosophy and evaluation methods of the Eurocode - EN 1990 as reference. The evaluation results show that the current European practice is potentially quite unconservative for the basic case studied in this paper; this practice makes use of the Winter curve in conjunction with the effective width method for buckling design, and at the same time neglects any partial safety factor - meaning that the corresponding safety factor γ_M, by which nominal strength values are divided, is set to 1.0. The paper gives an overview of the available pool of test data, discusses the background of this evaluation result in detail and describes possible amendments to the code.

1. Introduction

The resistance curve against local buckling used in international design codes is often based on the semi-empirical approach proposed by *George Winter in 1947*, widely known as the "Winter curve". This design curve reproduces the mean reduction values achieved in experiments conducted by Winter and other researchers. When applying the safety concept of LRFD-based design codes such as the Eurocodes, an additional safety factor is necessary to obtain a defined level of failure probability. Currently, this factor is set to 1.0 in the Eurocode for applications in building structures, and to 1.1 for bridge applications – values which are often criticized as possibly too optimistic. In the proposed paper, the worst-case application of square, welded box sections in centric or slightly eccentric loading with hinged edge conditions is considered. In this paper, which contains relevant excerpts from the doctoral thesis of the first author (*Schillo, 2017*), 131 international stub column tests on welded, squared box sections from steel grades S275 up to S960 were evaluated with respect to the required partial factor of safety in accordance with European building regulations (EN 1990). The evaluation of test results revealed a considerable scatter, and showed the currently employed partial factors in European standards to require additional prescriptions on fabrication tolerances and on material overstrength to guarantee the required failure probability given in EN 1990. In this paper, the individual parameters influencing the reliability of the "Winter curve" are separated and assessed. It is furthermore shown that the scatter

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of thickness has at least as high an influence as the scatter of the yield stress. Proposals are then made for the implementation of the findings into design codes.

2. Local buckling resistance prediction using the effective width method of Eurocode 3

In the current Eurocode - *EN1993-1-5 (2010)*, the reduction curve to depict local failure and calculate the effective width of the remaining load-bearing cross-section for local buckling is based on the so-called Winter-curve and is adopted in EC3. It was derived using a semi-empirical approach published by *George Winter in 1947*, with adjusted constants introduced in his paper published in *1968*. This design curve depicts the buckling reduction values achieved in the experiments conducted by Winter and other researchers as an approximated *average resistance curve*. The fact that the so-called Winter curve represents a mean predictor of the buckling resistance of slender plates has been the subject of various investigations and has been discussed in various references. *Figure 1* reproduces a figure from the Beuth commentary to DIN_18800 (*Lindner et al, 1994*), which shows older test results plotted over the plate slenderness $\bar{\lambda}_p$ and compares them to the Winter curve. The continuous line equals the Winter curve and the equation for the reduction factor ρ:

$$\rho = \frac{\bar{\lambda}_p - 0,22}{\bar{\lambda}_p^{\,2}} \leq 1,0 \qquad (1)$$

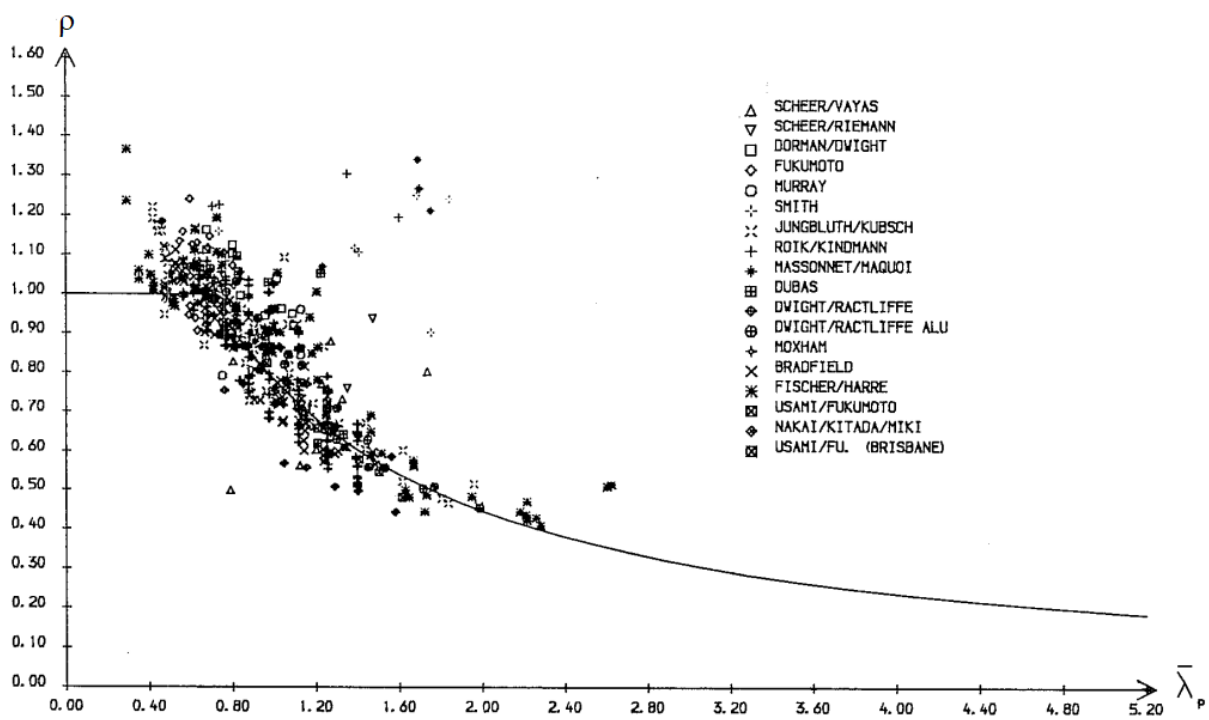

Figure 1: Results of local buckling tests, compared with the Winter resistance curve; figure taken from *Lindner et al. (1994)*

Considering these observations, it is inevitable that - when applying the reliability assessment methods prescribed by the Eurocodes (see *EN 1990, 2010*) - an additional safety factor γ_M is necessary to obtain the defined, quite low level of non-exceedance probability, which is in the proximity of 1/1000 for resistance values. However, counterintuitively, this factor is set to 1.0 for applications in building structures and can thus not cover a lower bound of the experimental results.

In this paper, as stated in the introduction, 131 more recent stub column tests on welded, squared box sections fabricated from plates with steel grades S275 up to S960 (nominal yield strength values of f_y=275 to 960 N/mm²) were evaluated to assess a realistic safety factor γ_M in conformity with the above-mentioned requirements of EN 1990. The experimental database comprises the tests of the research papers found in the list of references.

3. Stub column tests

In this section, the experiments on welded sections conducted by the first author are described, see *Schillo (2017)*. An exemplary representation of the test setup is shown in *Figure 2*. Special attention was paid on the measurement and evaluation of intended and unintended eccentricities. While the data collected from previous research claim a concentric loading of the specimens, it could be observed during the tests that small eccentricities are inevitable. For the assessment of the load prediction, the eccentricity was taken into account, leading to a reduction of scatter in the results.

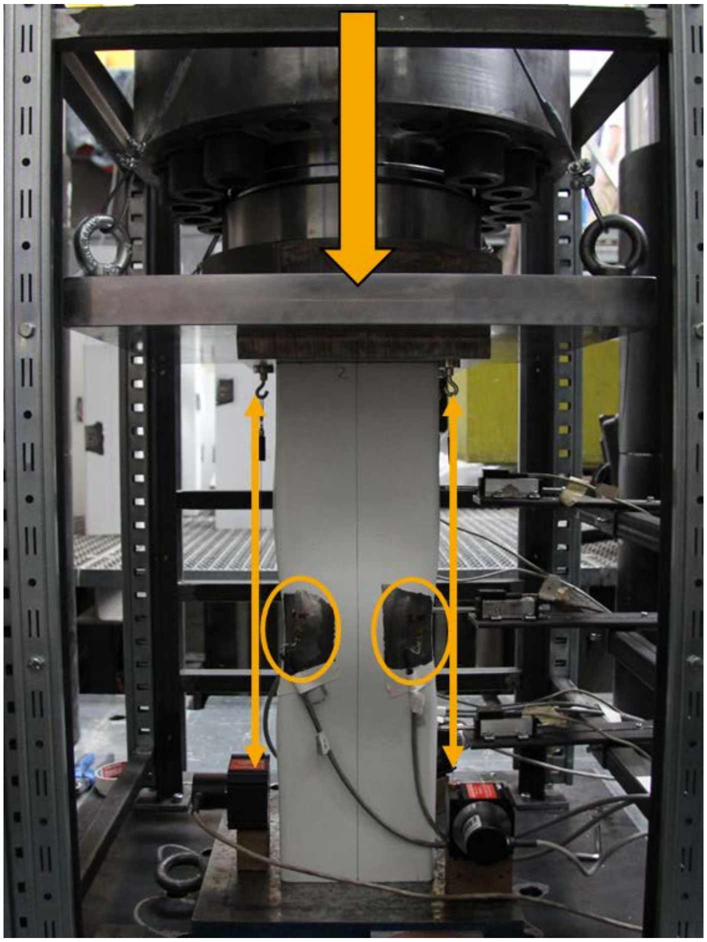

Figure 2: Tests at the laboratory of the Chair of Steel Structures at RWTH Aachen; representation of the load introduction and the measurement of rotation (through two LVDTs) and strains (through strain gauges) at each corner.

3.1 Design and fabrication

The welded sections were provided with matching weld strengths, i.e. the yield strength of the seams were similar to the yield strength of the specimens. The specimens were designed so that they covered a significant range of slenderness. The length was taken as 3*width + 50 mm, in order to avoid global buckling behaviour and allow for a representative residual stress distribution in the specimens. After sawing, the specimens were milled flat at the ends, providing for an even surface. Welded end plates were avoided in order to introduce no further residual stresses. All the plate material was fabricated according to EN 10149-2.

3.2 Test matrix and results

34 stub column tests were carried out. They covered the steel grades S500 up to S960, and values of the relative plate slenderness from 0.64 to 1.55. The denomination of specimens contains the steel grade (e.g. 960), the dimensions (e.g. 170-6 means a width of 170 mm and a thickness of 6 mm) and the sequential number (−4 means the 4th test of this specimen configuration). For each configuration (steel grade and slenderness), 4 to 5 tests were conducted, with different eccentricities of load introduction.

The experiments were carried out with intentionally and unintentionally applied eccentricities. It was aimed at at least 2 concentrically loaded tests, but the evaluation of strain gauges showed that small eccentricities are inevitable. The eccentricity was measured by 2 × 2 strain gauges at opposite faces of the specimens. The values were evaluated at a total load of 10% of the theoretical yield load, which was also used as alignment load. At this level, the specimen response is still in the elastic range and no significant 2nd order effect exists. An overview of eccentricities and the achieved ultimate load is presented in Table 1. The values of the stress ratio $\psi=\sigma_{min}/\sigma_{max}$ indicate the degree of eccentricity recorded by the strain gauges.

Table 1: Test results

Specimen (Welded, steel grade, width, thickness)		No. 1	No. 2	No. 3	No. 4	No. 5
W-S500-195-6	ψ [-]	0.92	0.96	0.68	0.95	-
	$F_{u,exp}$ [kN]	2261	2275	2216	2341	-
W-S500-250-4	ψ [-]	0.95	1	0.71	0.32	-
	$F_{u,exp}$ [kN]	1086	1083	1056	902	-
W-S700-180-4	ψ [-]	0.85	0.99	0.25	1	0.96
	$F_{u,exp}$ [kN]	2716	2686	2017	-	2785
W-S700-260-6	ψ [-]	0.96	0.97	0.69	0.33	0.95
	$F_{u,exp}$ [kN]	2666	2670	2579	2199	2661
W-S960-120-6	ψ [-]	1	0.98	-	0.74	-
	$F_{u,exp}$ [kN]	2931	2970	1970	2622	-
W-S960-170-6	ψ [-]	0.98	0.99	0.92	0.18	-
	$F_{u,exp}$ [kN]	3382	3362	3447	2241	-
W-S960-220-6	ψ [-]	0.99	0.96	0.2	0.96	-
	$F_{u,exp}$ [kN]	3178	3184	2359	3196	-
W-S960-250-6	ψ [-]	0.96	0.95	0.54	0.27	-
	$F_{u,exp}$ [kN]	-	3289	2867	2526	-

4. Evaluation of the experiments

The following section describes the procedure employed in the evaluation of the experiments conducted by the first author, as well as the experiments available from other researchers, and shows the results of this evaluation.

4.1 Evaluation procedure for experiments with eccentricity - determination of "ρ_{exp}"

The aim of the procedure described hereafter was obtaining values of ρ_{exp}, i.e. the buckling reduction factor for plate buckling, from the tests on entire sections. This is particularly relevant for eccentrically loaded plates (when a square section is loaded centrically, the plate buckling reduction factor can be determined directly and in a straightforward manner). A complete compilation of the considered tests can be found in *(Schillo, 2017).*

In order to assess the local buckling resistance, EC3 - EN 1993-1-5 generally assumes that each plate features hinged boundary conditions and thus an elastic buckling factor of k = 4 for rectangular plates in uniform compression. However, the consideration of bending on the stub column leads to an increased k-value for the overall section, since the plates adjacent to the most loaded plate act as a partial, additional clamping. This effect must be taken into account for the evaluation of ρ_{exp}. For the assessment of the elastic critical load, this value can e.g. be derived by using the open source software CUFSM, which uses the Finite-Strip-Method to calculate the critical load of a given structure, see e.g. *Schafer et al. (2006)*. With the input of Young's Modulus (210,000 MPa), the cross-section geometry and the load pattern ψ, a load amplification factor is given as output, defining the critical load but also the buckling factor k, as we know that

$$\sigma_{cr} = k \cdot \sigma_E \tag{2}$$

where σ_{cr} equals the critical stress and σ_E is the reference, Euler buckling stress. By applying EN 1993-1-5 in its conservative form, the less loaded plate on the opposite side of an eccentrically loaded box section would also be evaluated with k = 4, resulting in a smaller ρ-value than for the most loaded plate, if the modified k-value is applied only to the latter. Yet since the latter plate will actually buckle first, the load on the opposite plate will be generally lower, which can be taken into account by using a modified slenderness:

$$\bar{\lambda}_{p,red} = \bar{\lambda}_p \sqrt{\frac{\sigma_{com,Ed}}{f_y}} \tag{3}$$

The slenderness itself is determined by

$$\bar{\lambda}_p = \frac{\bar{b}/t}{28,4 \cdot \sqrt{235/f_y[N/mm^2]} \cdot \sqrt{k}} \tag{4}$$

Thus, the reduction factor $\rho_{EC}$of the most loaded plate is assessed by means of the k-factor derived from the Finite Strip Analysis, while the k-values (and subsequently the ρ-values) of the opposite and adjacent faces are set in proportional dependency to the first one. The reduction factor ρ (later to be set equal to ρ_{exp}) can be calculated using the modified Winter curve (to account for ψ-values in individual plates) as found in EN 1993-1-5. We can then calculate:

For the plate in an eccentrically loaded box section opposite of the most loaded one:

$$\rho_{less-loaded} = \rho \cdot \frac{\left(\bar{\lambda}_{p,less-loaded} - 0,22\right) / \bar{\lambda}_{p,less-loaded}^{\;2}}{\left(\bar{\lambda}_{p,most-loaded} - 0,22\right) / \bar{\lambda}_{p,most-loaded}^{\;2}} \tag{5}$$

and for the side plates (where a stress gradient is present):

$$\rho_{psi} = \rho \cdot \frac{\left(\bar{\lambda}_{p,psi} - 0,055 \cdot (3+\psi)\right) / \bar{\lambda}_{p,psi}^{\;2}}{\left(\bar{\lambda}_{p,most-loaded} - 0,22\right) / \bar{\lambda}_{p,most-loaded}^{\;2}} \tag{6}$$

As the load-carrying capacity of each plate depends on the force distribution, which correlates with the reduction factor, an iterative process is needed in order to calculate the reduction factor ρ which corresponds to the value ρ_{exp} that can be determined from an evaluation of the experimental tests with known eccentricity. The following equations has thus been fulfilled:

$$F_{u,\exp} = \frac{f_y \cdot A_{eff}}{1 + z_{eff} \cdot A_{eff} / W_{y,eff}} \cdot \left(\rho = \rho_{\exp}\right) \tag{7}$$

where the effective properties depend on the ρ-values and z_{eff} additionally depends on the load eccentricity.

4.2 Illustration of the results

When the experimental results are compared with the resistance prediction of EN 1993-1-5, the representation in *Figure 3* is obtained.

The actual slenderness $\bar{\lambda}_{p,act}$ is thereby calculated by referring to the the actual dimensions and measured yield strength of the experiments.

The figure illustrates that the deviation of the experimental results from the theoretical (Winter curve) resistance increases with slenderness, with the theoretical curve tending towards increasingly optimistic results. In *Wang et al. (2017)*, this tendency was confirmed also for cold-formed and hot-rolled hollow sections.

Another observation is the generally somewhat higher resistance recorded in the tests documented in *Schillo (2017)*. This may be attributed to the more precise evaluation possibilities, since for these experiments more information in terms of intended and unintended eccentricity was available and was considered in the evaluation.

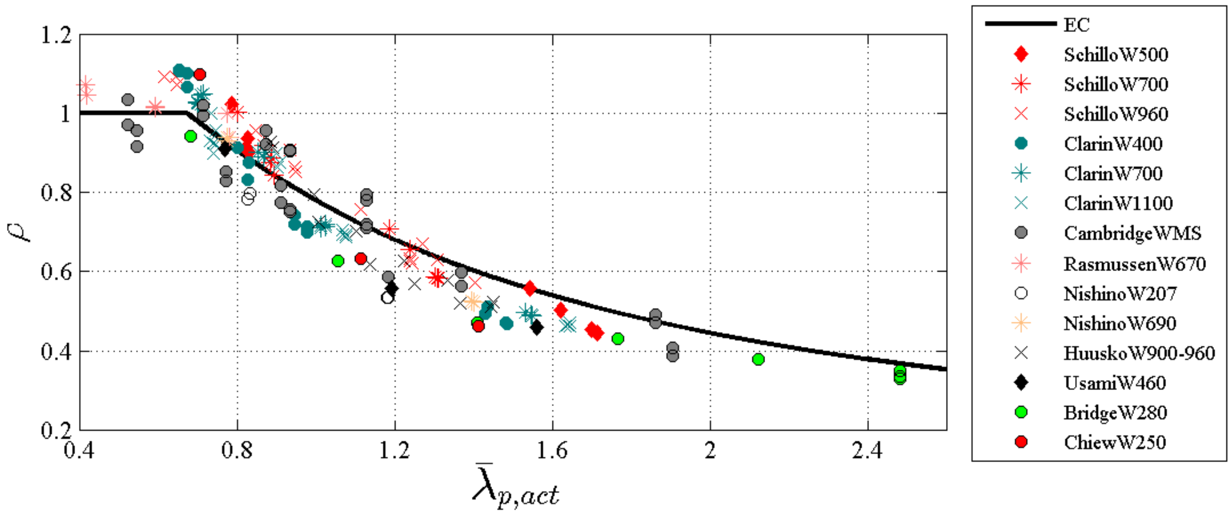

Figure 3: Results of recent stub column tests on welded box section members, compared with the Winter resistance curve

5. Reliability assessment according to EN 1990

5.1 General description

The definition and the reliability of load and resistance units is regulated in *EN 1990 (2010)* for all materials (concrete, timber, steel etc.). The aim of this standard is to provide a uniform and consistent procedure to allow for the quantification of the reliability of structural design rules across structural typologies. As some of the design procedures of Eurocode 3 (EC3) were developed well before the introduction of EN 1990, the actual applied safety concept may differ and the design might not fit to the concept prescript by this standard. For example, for global buckling, the application of the reliability assessment procedure of EN 1990 leads to inconsistent, non-homogenous levels of reliability across slenderness values and failure modes, see e.g. *Müller (2003)* or *Taras et al. (2014)*. The need of a γ_{M1}-value higher than 1 for various global instability failure modes is thus currently under discussion in the responsible committee CEN/TC 250 SC3.

This paper focuses exclusively on the assessment with regards to the corresponding partial safety factor γ_{M0} used for the effective width method for local buckling according to EN1993-1-1. The employed methodology is described in the following.

In order to define or quantify the reliability of a structure, the basic variables of action (loading) and reaction (resistance) in the structure have to be identified. The focus of this study is on the determination of a safety factor γ_M, which defines the safety margin on the resistance (or material side, “M”). Using a common procedure in the semi-probabilistic LRFD design codes found internationally, the total reliability demand is split - through “fixed” factors - among the resistance and loading side. This, in practical terms, the reliability demand of the Eurocodes leads to the request for a non-exceedance probability of the resistances corresponding to the value at around 3,09 standard deviations from the mean in a standard normal distribution, which in turn is approximately 1/1000.

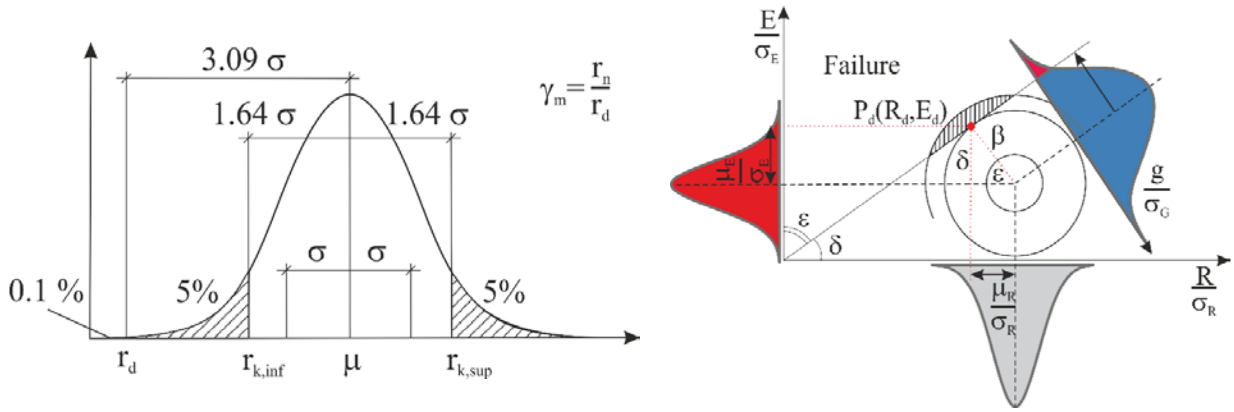

Figure 4: Reliability concept in EN 1990 and representation of the characteristic (r_k) and design (r_d) values of resistances.

The graphical interpretation of the reliability concept of EN 1990 is illustrated in *Figure 4*. Assuming a normal distribution, the characteristic (5% fractile) values R_k can be described by:

$$R_k = \mu - 1.64\sigma \tag{8}$$

In order to reach the design value, equal to a 0.1%-fractile, 3.09 standard deviations are necessary.

$$R_d = \mu - 3.09\sigma \tag{9}$$

Knowing the mean value μ and standard deviation σ of a unit or model, the Coefficient of Variation (CoV) V can be derived. The CoV is a standardized measure for the scatter of a probability distribution.

$$V = \frac{\sigma}{\mu} \tag{10}$$

For the total scatter of the resistance, it is possible to distinguish between the model uncertainty V_δ (a quantity that describes the quality of the description of the actual resistance, as measured e.g. in tests, by the chosen design resistance function or formulae) and uncertainties in the basic variables of the design formulation itself, V_{rt} (error propagation in the resistance function of the scatter of material properties and geometric data). The coefficients of variation of both lead to a common value V_r, which may finally be used to determine the design value of resistance as described above.

In the basic procedure of EN 1990 (as detailed in Annex D of that standard), it is assumed that the basic variables for the investigated problem (here: local buckling) follow a normal or log-normal distribution, as indicated in *Figure 4*, and that they are independent of each other. This is a rather simplified assumption, as e.g. the yield strength of steel is known to be thickness-dependent and thus its scatter cannot be entirely independent of the scatter of the plate thickness. Furthermore, the scatter of certain quantities is often neglected in evaluations of the partial safety factors, e.g. the Young's Modulus *E*, which is usually not determined in the material certificates and thus not available for evaluation purposes. While for E the assumption of 210,000 N/mm^2 is commonly accepted as natural constant, the precise magnitudes of the local and global imperfections are often also neglected as basic variable as well, since they do not enter common design rules directly - this is true e.g. for the Winter curve. This is due to the fact that reliable data is hard to assess with conventional measurement techniques and thus seldomly available. As this is, however, a major variable in stability issues, the capturing of scatter in imperfection amplitudes is shifted to the model uncertainty V_δ.

5.2 Model uncertainty: term V_δ

As mentioned above, the model uncertainty is accounted for by the CoV of the resistance model error, V_δ. The model means here the resistance function or equation (1), respectively. For its derivation, the experimental results r_e are compared with the theoretical results of the resistance model r_t. The deviation between both is characterised by the mean deviation b:

$$b = \frac{\sum r_{e,i}}{\sum r_t^2} \tag{11}$$

For each experiment i, the corresponding dispersion can subsequently be estimated:

$$\delta_i = \frac{r_{e,i}}{b \cdot r_{t,i}} \tag{12}$$

Using some interim steps for a set of n experiments:

$$\Delta_i = \ln\left(\delta_i\right) \tag{13}$$

$$\overline{\Delta} = \frac{1}{n}\sum_{i=1}^{n} \Delta_i \tag{14}$$

the Variance s can be calculated by:

$$s_D{}^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(\Delta_i - \overline{\Delta}\right)^2 \tag{15}$$

And knowing the Variance, finally the value of V_δ may be assessed by:

$$V_\delta = \sqrt{\exp\left(s_\Delta{}^2\right)} \tag{16}$$

5.3 Error propagation in the theoretical resistance function: term V_{rt}

The basic variables describe the main parameters influencing the resistance. In the case of local buckling, these are the geometric properties like the plate width b and the thickness t, as well as the yield strength f_y. The imperfections and the Young's Modulus are neglected as scattering variables, as discussed in the previous section

V_{rt} is the resulting CoV that represents the "error propagation", i.e. the propagation of the scatter of individual basic variables into the scatter of the theoretical resistance function. In order to derive V_{rt}, EN 1990 Annex D gives several possibilities. For simple additive or multiplicative resistance functions, the following formulation may be used, i.e. a simple square of the sum of roots:

$$V_{rt}{}^2 = \sum_{i=1}^{j} V_{x,i}{}^2 \tag{17}$$

If the resistance function r_t is of complex character (e.g. a highly non-linear function of the various basic variables, as is usually the case for buckling formulae), the sensitivity of the function for the various basic variables must be assessed at the given "design point", using partial derivatives:

$$V_{rt}{}^2 = \frac{1}{r_t^2\left(\underline{X}_m\right)} \cdot \sum_{i=1}^{j} \left(\frac{\partial r_t}{\partial x_i} \cdot \sigma_i \right)^2 \tag{18}$$

5.4 Resulting CoV: V_r

The resulting CoV may be assessed through the sum of the squares of V_δ and V_{rt} are each small:

$$V_r{}^2 = V_{rt}{}^2 + V_\delta{}^2 \tag{19}$$

For larger values of scatter, the following equation should be used instead:

$$V_r{}^2 = \left(1 + V_\delta{}^2\right) \cdot \left(1 + V_{rt}{}^2\right) - 1 \tag{20}$$

For the studies conducted by *Schillo (2017)* and the summary shown in this paper, the local buckling reduction curves is a complex function. This is due to the fact that the dependencies of each basic variables in the resistance function (1) are changing and the impact on the result differs in dependence of the slenderness. This was explained in detail in section 4.1.

*5.5 Partial safety factors: γ_M and γ_M**

One common definition of the partial safety factor γ_M (valid, as given here, for a large number of test results) makes use of both the characteristic (5% fractile) and design values:

$$\gamma_M = \frac{r_k}{r_d} = \frac{b \cdot r_t(\underline{X}_m) \cdot \exp\left(-1.645Q - 0.5Q^2\right)}{b \cdot r_t(\underline{X}_m) \cdot \exp\left(-3.09Q - 0.5Q^2\right)} \tag{21}$$

where r_k and r_d are the characteristic (5%) and design value (1/1000 non-exceedance probability), respectively.

The Q -value depends on the scatter of the model and basic variables, (see Equ. (11) and Equ. (12), respectively):

$$Q = \sqrt{\ln\left(V_r^2 + 1\right)} \tag{22}$$

The definition of γ_M given above is the theoretical definition found in EN 1990 Annex D, and may be applied in this form for materials and failure scenarios for which no standardized, nominal value of resistance is used.
However, in normal practice for steel structures design, the designer does not know - nor make use of - the statistical distribution of the basic variables affecting the design problem. Nominal values, often taken from production standards as the minimum guaranteed values of strength values, or as mean, “idealized” values in the case of geometric quantities, are used instead. Thus, the calculations are carried out with these nominal values. This leads to a corrected safety factor γ_M*:

$$\gamma_M * = \frac{r_{nom}}{r_d} \tag{23}$$

where r_{nom} is the result of the evaluation of the resistance function with nominal values of the basic variables.

All γ_{Mi} values found in EC3 are derived on the basis of calculated γ_M^* values, i.e. by referring to nominal strength and geometric values.

6. Application and discussion of the reliability assessment procedure

6.1 On the coefficients of variation of the basic variables $V_{rt,i}$

The coefficients of variation (CoVs) are without dimension and describe the scatter of a probability distribution. For each parameter of the local buckling equation (e.g. b, t, f_y), a coefficient can be derived.

Variables not included here are the residual stresses due to fabrication processes and the inevitable imperfections of plated material. The first issue here is the assessment of appropriate data for the basic variables: In many tests from literature, these values are either not documented or different measuring techniques impede comparability. Furthermore, the variation would not be independent of other variations: e.g. residual stress would depend on the yield strength, while the imperfections are depending on width and thickness. In this case of depending variations, eq. (19) or (20) would not be applicable. The influence is nonetheless indirectly captured in the test results, displayed in a considerable scatter along the resistance model and thus influencing the CoV of the Model V_δ.

*Table 2 s*hows the mean values and the values of the CoVs for the individual *geometric* basic variables adopted in this study, while *Table 3* shows the same for the yield strength. Thereby, the plate width, thickness and length were considered as scattering geometry parameters. The assumed CoV values are comparable with widely assumed values from the literature, see e.g. *Taras et al. (2014).* For the yield strength, the so-called “over-strength” factor, i.e. the ratio between the mean value and the nominal value of f_y, is of particular relevance: it clearly decreases with increasing yield strength, leading to a loss of “safety” inherently given by a high “over-strength”.

Table 2. Assumed scatter - geometry

Basic Variable	V_i	mean value	CoV [-]
Plate thickness	V_t	t_{nom}	0.05
Plate width	V_b	h_{nom}	0.005
Plate length	V_l	l_{nom}	0.005

Table 3. Assumed scatter - yield strength

nominal f_y *[N/mm²]*	$f_{y,mean}$	V_{fy}
$f_y \leq 300$	1,25 $f_{y,nom}$	0.070
$300 < f_y \leq 450$	1,15 $f_{y,nom}$	0.055
$450 < f_y \leq 600$	1,10 $f_{y,nom}$	0.045
$f_y > 600$	1,05 $f_{y,nom}$	0.033

Studying the sensitivity of the scatter of the basic variables (b, t, f_y) on the resulting coefficient of variation for the model (V_{rt}), equation (18) is used. In *Figure 5* the $V_{rt,i}$ values for each test i, in dependence of the nominal slenderness $\bar{\lambda}_{p,nom}$, are shown.
It can be observed that:

1. the influence of the scatter of the plate width is negligible.
2. the scatter of the yield strength is of high influence for stocky sections, but its influence decreases with increasing slenderness of the plate.
3. the scatter of the plate thickness is absolutely decisive in the relevant slenderness range.
4. the often-assumed sum function (equation (19)) for V_{rt} , which would result in $V_{rt} = 0.086$ with the assumed scatter bands, is unconservative in the relevant slenderness range.

The reduced influence of plate width and yield strength, as well as the enhanced influence of the plate thickness can be derived from the resistance equation r_t:

$$r_t = b \cdot t \cdot \rho \cdot f_y \tag{23}$$

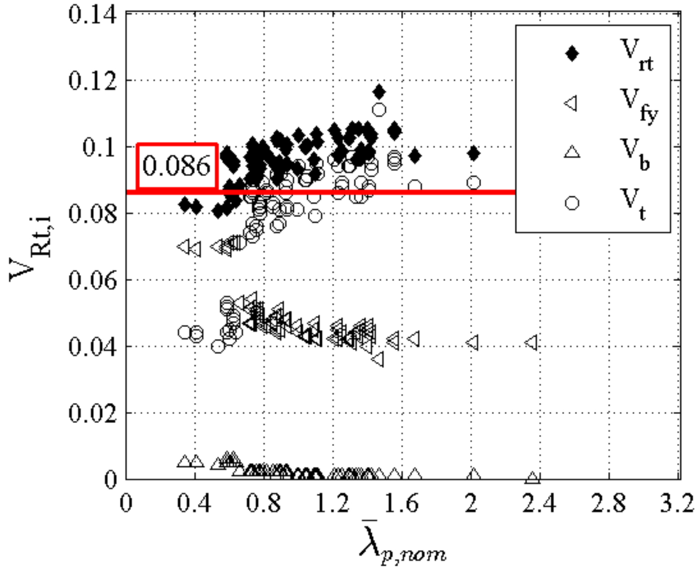

Figure 5: Sensitivity analysis: $V_{rt,i}$ for the basic variables V_i

Considering that

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} \tag{24}$$

it is evident that the slenderness $\bar{\lambda}_p$ is approximately proportional to the b/t -ratio times the square root of the yield strength and (see equation (1)):

$$\bar{\lambda}_p \xleftrightarrow[proportional]{} \frac{b}{t}\sqrt{f_y} \tag{25}$$

The second term in Equ. (1), $0{,}22/\bar{\lambda}_p^2$, is small. By substituting in equation (23), we obtain:

$$r_t = b \cdot t \cdot \frac{t}{b} \cdot \frac{1}{\sqrt{f_y}} \cdot f_y \xleftrightarrow[proportional]{} t^2 \cdot \sqrt{f_y} \tag{26}$$

This highlights that the resistance of slender plates is approximately proportional to the square of the plate thickness, while it is only approximately proportional to the square root of the yield strength. This implies that the scatter of the plate thickness is far more relevant than other parameters, including the yield strength, in determining the scatter of the buckling resistance.

In conclusion, the assumption of a simple additive function for V_{rt} proved to be unconservative and thus in the following studies equation (18) was used in the following, leading to accurate CoVs for each experiment.

The use of constant Coefficients of Variation, using the values of *Table 2* and *Table 3*, in some cases leads to unrealistic assumptions: an example is illustrated on the left hand side of *Figure 6*. If we assume the CoV V_t for the plate thickness to be a constant value at 5%, and a mean plate thickness of 6 mm, the standard deviation would result in 0.3 mm. For the yield strength, the 5%-fractile is computed, meaning that the lower limit for the actual thickness t_{act} values would be t_{mean}-1.64σ = 5.5 mm. Taking into account that the tolerance limit given in EN 10029 *(2010)* can be taken (for execution Class B) as 0.3 mm, everything below 5.7mm would be below the limit. Since 0.3 mm equals the assumed standard deviation, 16% of the samples of the normal distributed unit would be below tolerance. This is obviously not in compliance with the quality management of the steel producers.

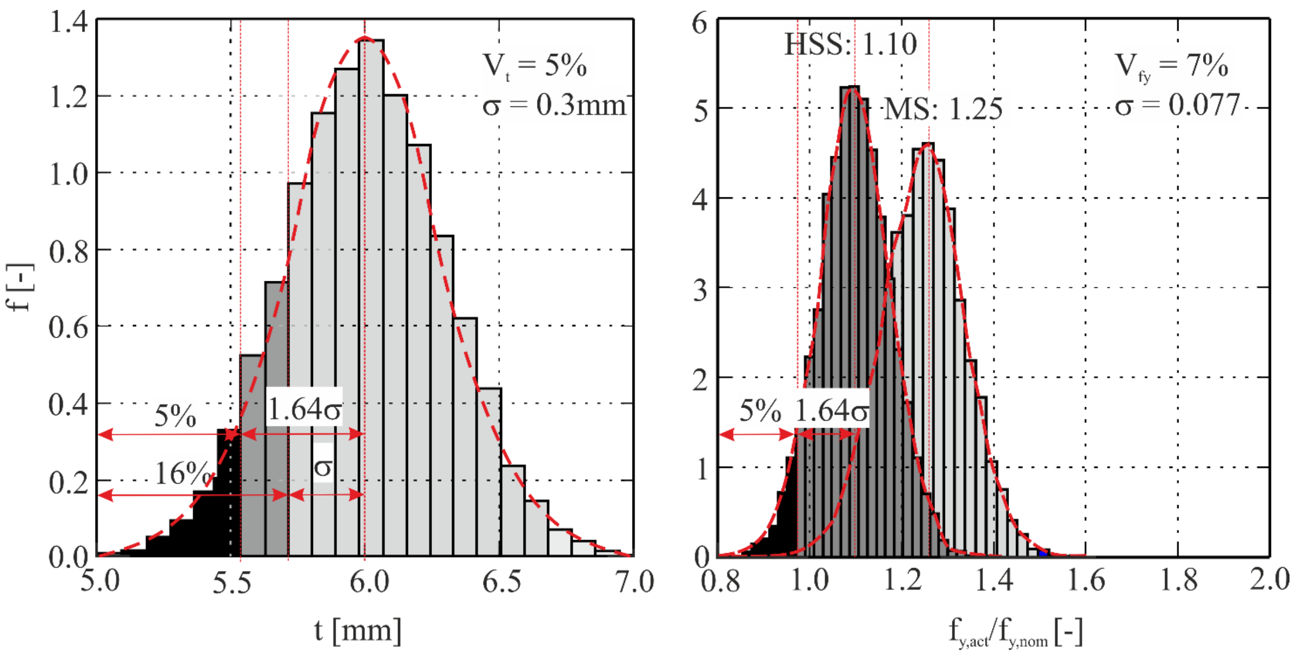

Figure 6: Example of unreasonable assumption for V_t (thickness, left) and V_{fy} (yield strength, right)

For V_t, it was therefore assumed that the maximum value would be 5%, but that a lower value would be possible assuming the distance between the tolerance limit for Class B and the mean value equals 3σ, as given in Equ. (27).

$$V_t = \min\left(0,05; 0,3 \cdot \frac{1}{t_{mean}} \cdot \frac{1}{3}\right) \tag{27}$$

The same exercise can be carried out for V_{fy}, as shown in *Figure 6* on the right hand side. It is commonly known that the over-strength $f_{y,mean}/f_{y,nom}$ reduces with increasing nominal values of yield strength. For S235, a value of 1.25 can be assumed, see *Table 3*, with a corresponding CoV of 5%; during the evaluation of EC3 rules, however, often a value of 7% was assumed. Values for high strength steel were only recently assessed, e.g. in *Schillo (2017)*. Assuming an over-strength ratio for HSS of 1.1, with the "old" assumption of a CoV of 7%, the 5%-fractile limit for $f_{y,act}/f_{y,nom}$ would be at 0.97: this would be lower than the nominal value. It was for this reason found more sensible to assume lower V_{fy}-values, leading to limits that are still in compliance with the quality requirements in steel production.

7. Evaluation results - required values of the partial safety factor γ_M*

In this section, the EN1990-compatible reliability evaluation of the experimental results shown in *Figure 3* is summarized. As was stated previously, the graphical evaluation of the plot itself already indicated lower deviations from the resistance curve for the tests conducted by the first author. In an attempt to reduce the scatter and thus the determined values of the required partial safety factor γ_M*, the experimental data set was thus split in two: one set with all collected (recent) test data on high-strength or mild steel welded box sections, and a reduced set containing only the results of the tests conducted at RWTH by the first author. On the left hand side of Figure 7, the achieved values of the nominal buckling reduction factor ρ_{nom} for the two subsets are shown in comparison to the Winter curve (continuous line) and the Winter curve reduced by the resulting, required value of γ_M*=1.30. *This is therefore the value of the partial safety factor that would be required to fully comply with EN 1990 on the basis of the presented test results.* The right hand side subplot shows the γ_M* values for each individual test, indicating that the RWTH/Schillo-results generally lead to lower values. This is discussed further below.

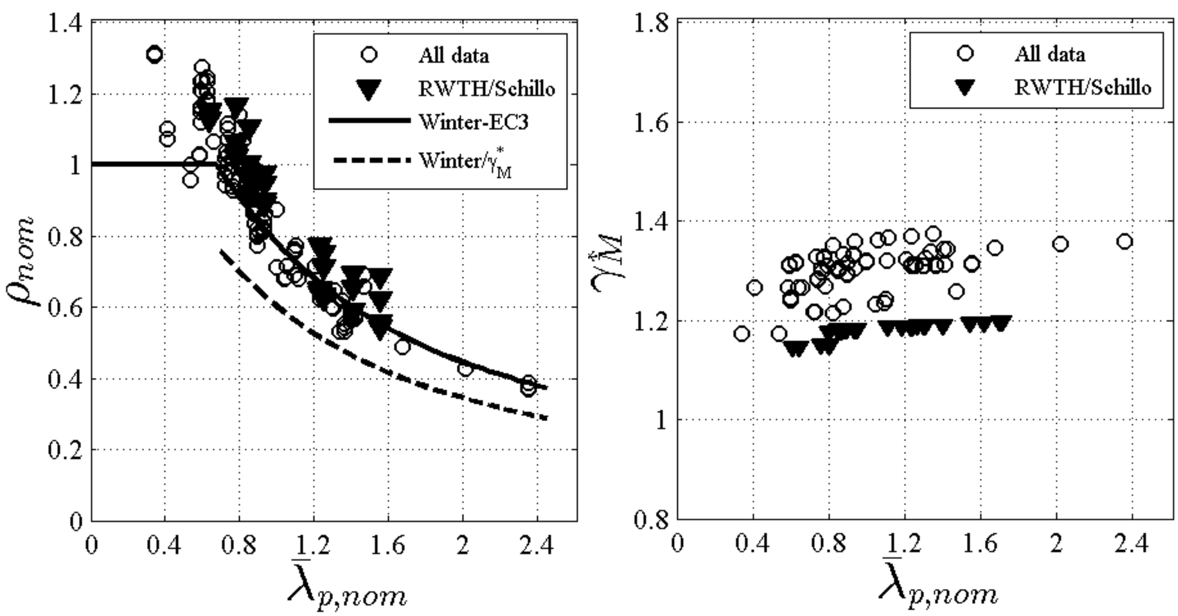

Fig. 7: Comparison of predicted ultimate load with experimental results and resulting γ_M* values: evaluation of all data (γ_M*=1.30) vs. only the RWTH results from *Schillo (2017)*

For the evaluation of γ_M*, the variable CoVs for the yield strength V_{fy} were taken from *Table 3* while the geometric parameters were taken in accordance with *Table 2.* As only two different plate thickness were used in the experiments, this results in V_t = 0.017 for the 6 mm and V_t = 0.025 for the 4 mm plates. The model uncertainty was calculated to be V_δ=0.074. With the V_{rt} values calculated for each experiment, the final, average γ_M* value for the subset of RWTH tests was shown to be considerably lower with 1.18.

A possible explanation for this lower partial safety factor is that more information was available for the RWTH tests, in particular concerning eccentricities, and thus a more precise computation could be conducted. In any case, the tendency of increasing optimism and non-conservatism in the resistance prediction with increasing slenderness was confirmed in both subsets.

8. Alternative resistance function and corresponding partial safety factor γ_{M*}

Following the chain of thoughts of the previous sections, a best-fit function was derived exemplarily, in order to display the mean resistance function of the considered stub columns tests. The proposed curve is characterized by:

$$\rho = 2.235 \cdot \exp(-1.582 \cdot \bar{\lambda}_p) + 0.288 \tag{28}$$

Since in this function the mean value of reported test results is approximated very closely, the procedure introduced in section 5 combined with the assumptions shown in section 6 coherently lead to a very low required safety factor of 1.06 when considering the tests conducted at RWTH. A comparison between the Winter curve and the proposed best-fit function is shown in Figure 8.

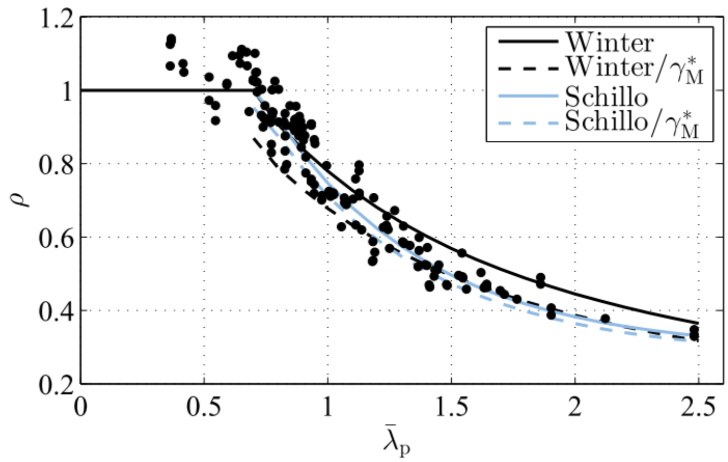

Figure 8: Comparison of Winter curve and best-fit curve with their corresponding γ_M* values

The new curve shows only insignificant deviations from current practice using the Winter curve (without γ_M*) in the stocky area, while both curves tend towards the same level of reliability with increasing slenderness.

9. Summary and conclusions

In *Schillo (2017)*, a database of stub column tests on slender, welded box sections and square hollow sections was gathered, which highlighted a slenderness-dependent unconservativeness in the Winter curve-based resistance prediction of Eurocode 3 - EN 1993-1-5. The data revealed a high scatter across the investigated steel grades (S275 to S960). A reliability assessment was conducted using EN 1990, which defines required limits for the probability of failure of structures that correspond to a non-exceedance probability for the resistances of structural members of approximately 1/1000.

The resulting values of the partial safety factor γ_M*, determined under consideration of all available test data, reaches a value of 1.30. This is by far higher than the value of 1.00 currently used in most European countries for the buckling check using the effective width method. The tests conducted by the first author, on the other hand, showed less scatter and a higher relative resistance than the remaining tests in the collected database, as here more information in terms of eccentricities in loading was available. Using only these tests for the reliability analysis, but under the same assumptions regarding CoVs as before, the resulting γ_M* value could be reduced to 1.18.

Based on the evaluation results presented in this paper, a codified γ_M value for local buckling of either 1.15 or 1.20 may be appropriate for the loading case studied in this paper (constant compression in equally supported plates in square, welded box sections). While this is clearly the most severe case encountered in practice, it is also the most basic case of plate buckling, and the reliability discrepancy shown in this paper cannot be discounted. Among the alternative ways to address this discrepancy to the EN1990 requirements, the amendment or replacement of the Winter curve itself, by a more appropriate curve displaying the mean curve of the considered stub column tests, is currently also under discussion in the responsible committee CEN/TC250/SC3 Working Group 5, Plated Structures. By using the mean curve, the corresponding, required partial safety factor can be significantly reduced to 1.06.

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