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Column design: past, present, future

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Abstract

The historical development of the different approaches for designing axially-loaded metal columns is presented and critiqued. For completeness ancient column designs of stone and wood prior to the use of iron columns are briefly mentioned. The mathematical, experimental, industrial, professional and political factors that contributed to our current status are presented. Emphasis is placed on determining the underlying factors that led to the use of various formulas. The current single column curve vs the multiple column curve controversy is evaluated. Since the first use of mild steel columns in the 1850's to the present, US design formulas for steel columns actually have changed very little. The appearances of the formulas are different but their outcomes are similar, only the implied factor of safety has been significantly affected. The fact that the early design formulas in the 1880's were based on the theory of columns with imperfections combined with test results, not buckling theory, is the major reason there has been so little change in US practice.

1. Introduction

Few topics within the field of structural engineering draw as much attention and emotion as centrally-loaded columns. Salmon (1921) published a 279 page treatise entitled *Columns* that contained a bibliography of all the important original work on this subject from the period 1747-1910, a total of 376 references. Jakkula and Stephenson (1949) collected an additional 280 references from 1910-1947. ASCE (1990) contains 2289 citations thru 1984 and SSRC (2003) developed a research inventory on centrally-loaded steel columns from 1949-2003 that contains a listing of almost two thousand papers. More recent work can be easily obtained through computer search engines. This paper contains no original research that will add to the research bank. Rather it is a historical review of past work that attempts to trace the reasons, why and how, we have arrived at our current state of steel column design. At various stages along this historical trip some personal commentary will be added. Much of the historical material was obtained from sources listed in the Appendix. Historic nomenclature has not been retained.

It can be argued that the best way to teach a topic is to present the material in the same order the material evolved with time because that is the way our brains figured it out. For example from personal observation, an engineering student trained in strength of materials that starts with a three-dimensional stress state and treats simple axial loading as a special case does not perform

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as well as one that starts with simple axial stress. Starting with the three-dimensional stress state may appear to be more efficient but lessons learned are lost on the way. This paper will begin with the design of stone columns (pillars) from about 5000 BC and end with a prediction on the future status of multiple column curves in US practice.

2. Ancient Times – 17th Century

The Greeks perfected the proportioning of the stone columns (pillars) in constructing their temples starting about 5000 BC. The design rules the Greeks used were documented by the Roman architect Vitruvius (1000 BC) in his *Ten Books on Architecture* written sometime in the 1st century BC. The following is quoted from this first recorded design manual:

> *"6. Wishing to set up columns in that temple, but not having rules for their symmetry, and being in search of some way by which they could render them fit to bear a load and also of a satisfactory beauty of appearance, they measured the imprint of a man's foot and compared this with his height. On finding that, in a man, the foot was one sixth of the height, they applied the same principle to the column, and reared the shaft, including the capital, to a height six times its thickness at the base.*
> *7. Just so afterwards, when they desired to construct a temple to Diana in a new style of beauty, they translated these footprints into term characteristic of the slenderness of women, and thus first made a column the thickness of which was only one eighth of its height, so that it might have a taller look.*
>
> *3. The columns of the upper tier should be one fourth smaller than those of the lower, because, for the purpose of bearing the load, what is below ought to be stronger than what is above, and also, because we ought to imitate nature as seen in the case of things growing; for example, in smooth-stemmed trees …."*

The Greeks built many temples containing stone columns during the period 2-6 centuries BC and portions, mainly columns, still stand with length (including the base)-to-width ratios, L/d, from 8 to 10. The ACI codes up to 1956 limited reinforced concrete columns to $L/d = 10$.

For bridge and building design up to the 17th century, designs in stone, masonry and wood were based on proper proportions, scale models, experience and tests. There was little mathematical basis used in construction. Physics, chemistry and the concept of equilibrium were reasonably understood. Our modern concepts of stress and strain were unknown.

3. Calculus, Mechanics and Euler: 1650-1800

Hooke observed the linear relationship between load and deflection in elastic beams in 1660 and a new mathematics field called calculus matured during this period. The Bernoulli family members thru three generations were the leading experts in mathematics and mechanics. Jacob Bernoulli focused on deflected shapes of elastic beams rather than bending strength and in 1705 formulated the concept that the radius of curvature at a point on the deflected was inversely proportional to the moment (M) at that point and geometrically defined curvature as $ds/d\theta$. This fundamental relationship initially based only on equilibrium and geometry is the foundation of our current understanding of flexure. The concepts of moment of inertia (I) and modulus of elasticity (E) appeared 150 years later.

The first science academies appeared in England and France around 1650 and later in Russia and Germany that promoted interaction between scientists, mathematicians and engineers within those countries. The monarchies used these academies as indicators of national pride. Euler, who was a former student of Daniel Bernoulli and graduated with a Master's degree at age sixteen, spent his professional career conducting research in mathematics and mechanics at both the St. Petersburg and Berlin Academies. While in Berlin, Euler applied his variational calculus method to investigate elastic bending curves under various loading conditions (elastica) looking for the minimum load required to maintain particular bent shapes. Euler (1744) ended up to be a book in Latin documenting his large displacement solutions: some of the bent shapes that he integrated along are shown in Fig. 1. In the Appendix of his book, he solved the problem of a cantilever bent with the load directed parallel to the straight position. He found an interesting solution for an initially straight member. There was a minimum load below which a bent shape could not be maintained. Euler did not solve the problem of a straight cantilever with length, L, that suddenly bent into the buckled shape; he determined that the column had to remain straight for loads less than the limit we now call the Euler load with an effective length of 2L for the flag-pole problem, $\pi^2 EI/4L^2$. He later described this conclusion as "very remarkable" and stated the formula could be used for wood columns, not stone, because wood could bend. Of course *EI* was an unknown quantity at that time but Euler showed that t could be determined from a simple test of a cantilever beam with the same material and cross section as the column. Thirteen years later in a more coherent paper devoted strictly to elastic columns and using small deflection theory, his classic solution for a column with pinned ends was published (Van den Broek, 1947). Euler called the unknown term equivalent to *EI* a "stiffness moment" although the units (k-in^2) are not consistent with moment units (k-in). Euler's formula for the elastic buckling load of a column was pretty much a curiosity within a small group of mathematicians. Its validity was questioned because his solution considered just bending deformations, ignoring axial and shear effects. Tests

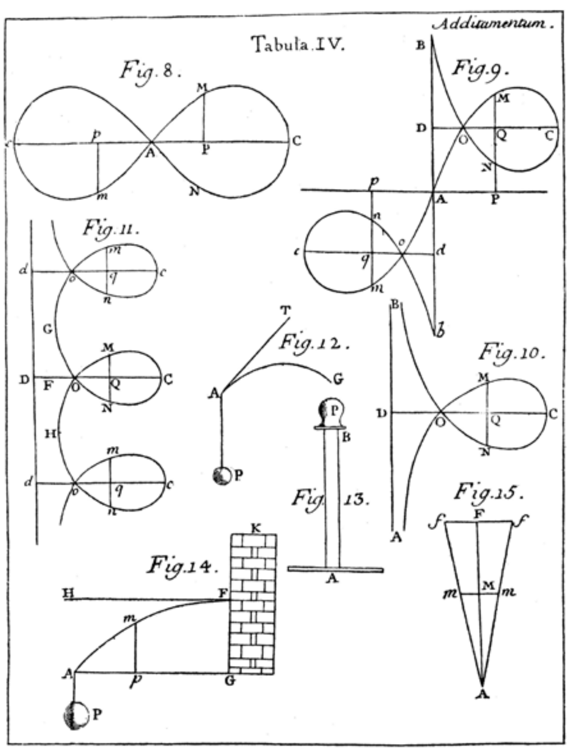


Figure 1: Leonard Euler's Elastic Curves (from Euler, 1744)

at that time did not validate Euler's theory because they violated both the end conditions and the elastic properties assumed by Euler. The formula lay dormant for more than 150 yrs. until steel construction and the airplane provided the potential for slender compression members and experiments with carefully designed end fixtures showed Euler was correct.

In 1795 the first engineering school based on math and science and requiring entrance exams was opened in Paris. In the first two years physics, chemistry, mathematics and mechanics were covered and the third year consisted of engineering courses. The topics were taught in large classes; professors prepared their lectures for publication into books. The first strength of materials book appeared in 1798. The educational model developed by the French was eventually adopted by all other countries and is still a model for current engineering education. The evolution of modern engineering mechanics and strength of materials during the period from 1800-1850 developed mainly by the contributions of graduates from Ecole Polytechnique. Britain was the most highly industrialized country at that time but did not establish an engineering school until 1840.

4. 19th Century Metal Column Design

The previous century's contributions to engineering mechanics focused mainly on what the eye could see, i.e. deflections and moments, and the breaking strength of stone, wood and cast iron. The concepts of stress, strain, neutral axis, E (modulus of elasticity), and I (moment of inertia) were established in early 1800 so engineers could now calculate elastic stresses in single span bending members and relate them to the properties from material tests. Modest size test machines and strain measuring devices brought the behavior of material into importance as represented in an experimental stress-strain diagram. With the ability to calculate stresses along with dependable material properties, British engineers switched from ultimate strength design with high factors of safety (up to 10 for cast iron) to a working stress concept with a factor of safety of 2 or more for the elastic limit of wrought iron.

Wrought iron could be rolled into plates that permitted the steam locomotive to be invented and the resulting free enterprise railroad boom spread quickly to Europe and America. The need for railroad bridges dominated structural engineering in the 19th century. Truss technology was used for most bridge construction but structural analysis had not kept pace with developments in strength of materials. Until Whipple in 1847 developed the method of joints for determining the force in truss members, engineers tested small models to evaluate their truss designs. Up to 1800, design engineers in England relied on experimental results for columns because there was no mathematic alternative. Experiments on small compression specimens, even slender ones, continued to cast doubt on the usefulness of Euler's theory. The theory was considered suspect, not the tests.

It is surprising then that the first published column design formula that came into public use was based on strength of materials fundamentals, not tests. In 1822 Tredgold, a carpenter by training and a self-taught civil engineer, published a book on the strength of metal members (Tredgold, Hodgkinson, 1860). Using the combined stress formula for the limit of yielding, $f_a + f_b = F_y$, axial, bending and yield stresses, respectively, he solved the problem of a rectangular column with load, P, and an eccentricity, e. The bending term was divided into two parts, one associated with the deflection due to the uniform end moments and the other due to the additional flexure,

$P\delta$, caused by the axial load acting on a bent member. He correctly calculated the initial deflection at midspan that was proportional to L^2/d where d = depth of the cross section caused by the uniform end moments, Pe, It is not clear if he even knew of Euler's work because he stated that the deflection caused by $P\delta$ followed the same shape as that caused by the initial end moments. In other words Tredgold assumed that the second order moment diagram was uniform along the length which is very conservative. That was his only error. Solving the combined stress equation for P gave:

<u>Tredgold Formula</u>

$$\frac{P}{P_y} = \frac{1}{1 + \frac{6e}{d} + \frac{3}{2}\frac{F_y}{E}\left(\frac{L}{d}\right)^2} \tag{1}$$

where P_y is the yield load. The eccentricity had to be chosen by the engineer. A plot of Eq. (1), with d = 3.46r where r is the radius of gyration for a rectangular section and e = L/1000, compared to the Euler formula is shown in Fig. (2). The correct solution for the load at first yield of an eccentrically-loaded column was derived twenty-six years later by Scheffler and is known as the secant formula:

<u>Secant Formula</u>

$$\frac{P}{P_y} = \frac{1}{1 + \frac{ed}{2r}\sec\left[\left(\frac{L}{2r}\right)\sqrt{\left(\frac{P}{P_y}\frac{F_y}{E}\right)}\right]} \tag{2}$$

As expected, the secant formula plotted in Fig. (2) shows the Tredgold formula is very conservative in predicting first yield for identical values of load eccentricity. Both formulas require an assumed e to determine P. Tredgold's formula was used for about twenty-five years until a new formula based on his solution but adjusted by test results appeared in 1858.

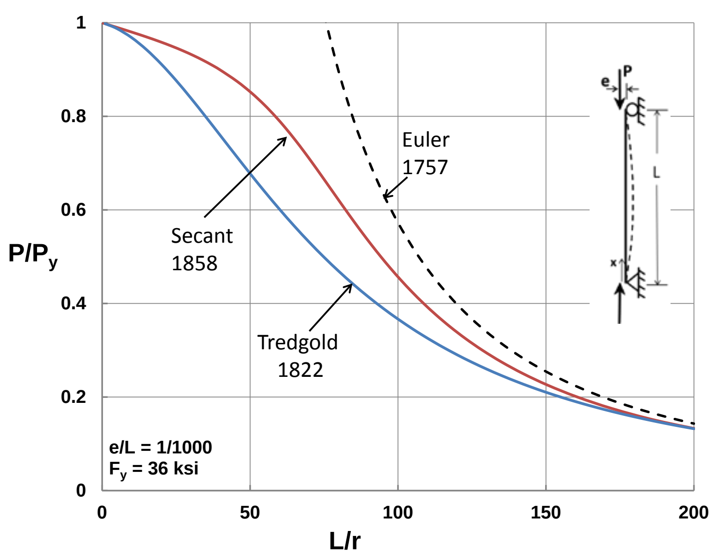


Figure 2: The First Column Design Formula

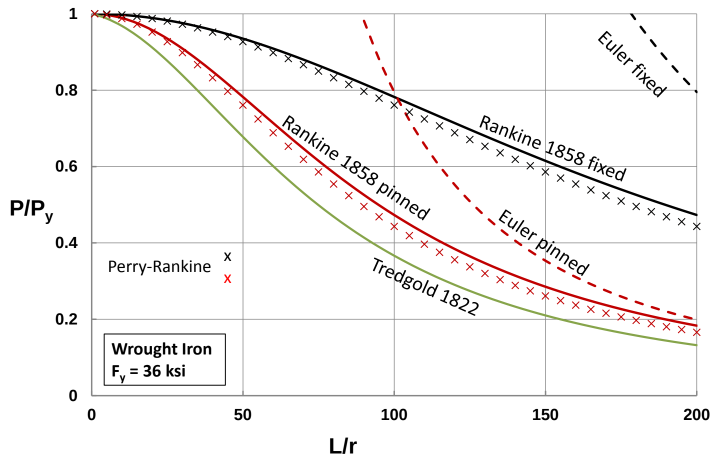


Figure 3: Original Gordon-Rankin and Perry-Rankine Formulas

In 1840, Hodgkinson published the results of an extensive test series (over 250 specimens) on cylindrical cast iron, wrought iron and wood columns: see (Tredgold, Hodgkinson 1860). The effect of rounded ends and flat ends was included. He had hoped to show that Euler's equation was valid but his statistical evaluation of the test results indicated that the failure loads were inversely proportional to $L^{1.7}$, not L^2 as proposed by Euler. Louis Gordon, who held the first engineering chair in Great Britain at Glasgow University in 1840, took Tredgold's formula based on combined stresses, Eq.(1), simplified it by eliminating the term containing *e* and generalized it for most cross sections by replacing *d* with *r*:

Gordon-Rankine Formula
$$\frac{P}{P_y} = \frac{1}{1 + a\left(\frac{L}{r}\right)^2} \tag{3}$$

The symbol *a* is a constant determined by test results. Gordon used Hodgkinson's results to find *a* = 1/3000 for wrought iron columns with fixed ends and 4/3000 for pinned ends. The effect of end conditions appears in the design formula but not *e*. The fixed case was recommended for typical design that intersects the Euler pinned case at L/r = 100 and $P/P_y = 0.8$. Gordon presented this formula in his class lecture notes probably around 1845 and Rankine, his colleague at Glasgow, used the formula in his book on applied mechanics, (Rankine 1858). A working stress, 0.25 of the breaking strength, was recommended for design. The Gordon-Rankin formula is compared to Euler and Tredgold (e = L/1000) in Fig, (3). The form of Eq. (3) is based somewhat on the combined stress at first yield concept so it could be classed as a semi-empirical equation since *a* must be determined to fit test results. The Rankine formula, as it was referred to in the 19th century, was quickly and widely adopted by practicing engineers in Britain, Europe and US. Rankine was a well-known academic and engineer. His book, *A Manual on Applied Mechanics*, was immediately called a classic and 22 editions were published, the last one in 1921. The form of the Rankine formula was simple, direct and could be adopted to fit new test results, which explains its long life well into the 20th century in the history of column design. By eliminating *e*, Gordon-Rankine could only be applicable to concentrically-loaded columns. Perry (1889) reasoned that $a = F_y/(\pi^2 EK^2)$ where *K* is the effective length of the column and his solution is shown by the markers (x) in Fig. 3 that compares favorably with the original Gordon-Rankine design recommendations for fixed and pinned columns. Inserting Perry's contribution into Eq. (3) give the following forms also identified as Rankine formulas,

<u>Rankine-Perry Format</u>

$$\frac{P}{P_y} = \frac{1}{1 + \frac{P_y}{P_{cr}}} \qquad \text{or} \qquad \frac{1}{P} = \frac{1}{P_y} + \frac{1}{P_{cr}} \tag{4}$$

where $P_{cr} = \pi^2 EI/(KL)^2$.

As indicated earlier, the mathematically correct solution for the first yield capacity of an eccentrically-loaded column, the secant formula, was published in the same year as Rankine's book but it did not have any impact on column design because of the difficulty in using Eq. (2). The load P, which appears on both sides of the formula one of which is within the secant term, had to be determined by trial and error. With only a slide rule for a computational aid, it is understandable why Eq. (2) was shunned by practicing engineers. It gained more favor in the 20th century.

Ayrton and Perry (1886) showed that some of the difficulties of the secant formula could be overcome by assuming an initial bent shape in the form of a sine curve instead of a parabola as shown in Fig. 4a. The intent was to still consider the problem as a column with end-load eccentricity. With the sine curve shape, the bending moment is:

$$P\delta = \frac{P\delta_o}{1 - \frac{P}{P_e}} \tag{4}$$

where δ_o is the initial displacement at midspan of an unloaded column, δ is the maximum calculated deflection and P_e is the Euler load = $\pi^2 EI/L^2$. Much like the secant formula, P appears on both sides of the equation but in this case when Eq. (4) is inserted in the combined stress

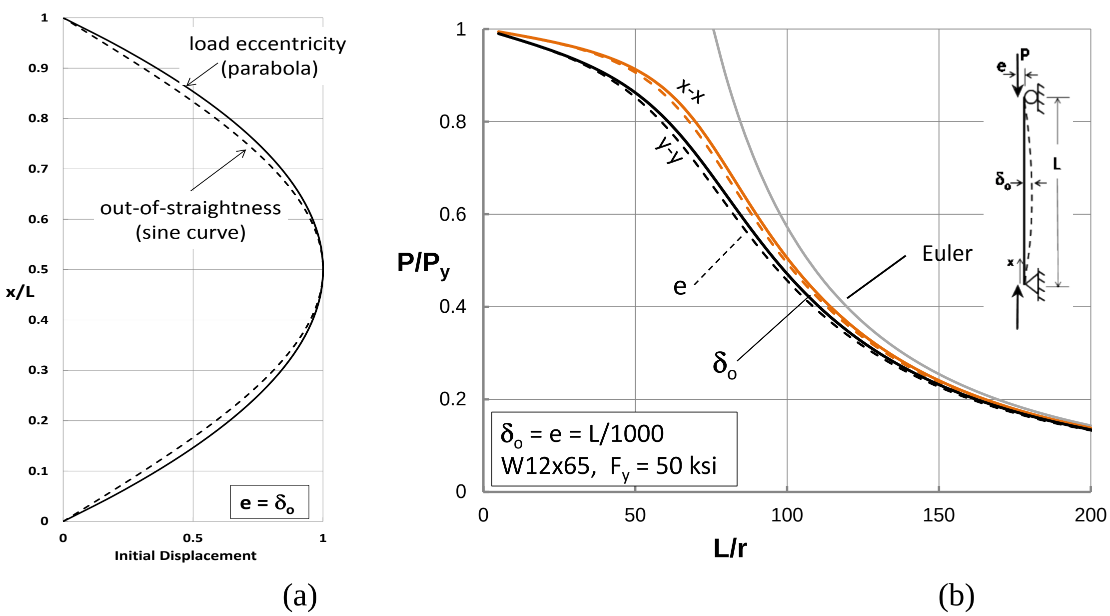


Figure 4: Effect of Load Eccentricity and Initial Out-of-Straightness

equation, the smallest root of the resulting quadratic equation gives P at first yield as:

Ayrton-Perry formula

$$\frac{P}{P_y} = \frac{1+(1+m)\frac{P_e}{P_y}}{2} - \sqrt{\left[\frac{1+(1+m)\frac{P_e}{P_y}}{2}\right]^2 - \frac{P_e}{P_y}} \tag{5}$$

and $m = \delta_o c/r^2$ where *c* is the distance to the extreme fiber. The Ayrton-Perry (solid lines) and secant (dashed lines) formulas are compared in Fig. 4 for a W12x65 section, 50 ksi material and initial imperfection of L/1000. There is only an approximate 3% difference between Eqs. (2) and (5). To eliminate this small difference they recommended use of Eq. (5) for eccentrically-loaded columns with initial curvature by adding 5/6 *e* to δ_o in defining *m*. The correctness of Eq. (4) was verified by evaluating the measured load-displacement data from the Hodgkinson tests. Figure 4 also shows that both formulas produce multiple column curves for a constant imperfection because there are two cross-section variables, *c* and *r*, in addition to *L/r*. The Ayrton-Perry formula was not used in design because of its complexity. In his *Practical Mechanics* book, (Perry 1912) still designed struts using a Rankine formula.

At this point it is useful to briefly comment on the status of engineering education and the profession of civil engineering during this period. Engineering education was still in its infancy in Great Britain and US and demand for civil engineers was high. Most colleges provided classical education and there was no BS degree. There were some polytechnic schools but the engineering courses usually covered just a one year curriculum heavily oriented to practice. Many US engineers learned their profession as apprentices to trained emigrant engineers or they went to Germany where the education emphasized theory and experimental verification. There was a major disconnect between the mathematical capabilities of the average engineer and the sophistication expressed in the derivations discussed in the previous sections. Communication was difficult; the telegraph was not invented until 1830-1840. Many times solutions were rediscovered because previous solutions were unknown to the authors. There was practically no formal engineering training in the US until Congress established land-grant universities in each state in 1862 whose purpose was “to teach such branches of learning as are related to agriculture and the mechanic arts, in order to promote the liberal and practical education of the industrial classes”. It enabled students to get a free education in a field that jobs were available. Engineering colleges grew from 4 to more than 100 by 1900 with the emphasis on practical “shop” courses. Professional societies, ICE (London) in 1812 and ASCE in 1852, evolved so engineers could discuss common problems and issues, which eventually led to the concept of organizations developing solutions that others would respect and use.

Railroads in the US were being constructed by numerous small companies and each of them had their own engineer to design the bridges. Since there were no organized civil engineering specifications or building codes, engineers would use their own judgment to establish safe working stresses and column formulas. Due to competition among the railroad companies, many column formulas appeared, mostly of the Rankine type, and some based on test results internal to individual companies. The result of this chaos was the alarming rate of bridge failures, averaging

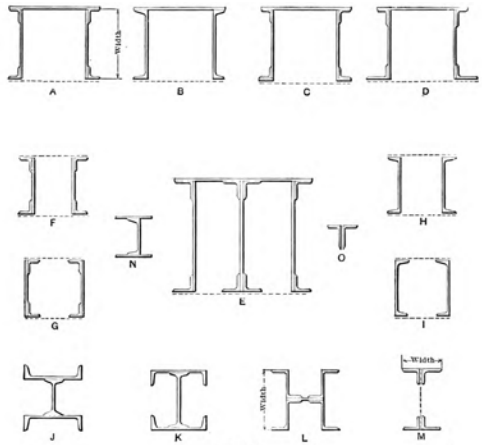


Figure 5: 19th Century Bridge Column Sections (Johnson et al 1894)

one failure every two weeks over the ten-year period 1870-1880. Part of the problem was the variety of complex cross sections in the compression members of trusses that were built up from the limited rolled shapes available, i.e. plates, angles, zees and channels. Some examples are shown in Fig, 5. The effect of shear on the strength of columns was not yet established. The development of large test machines such as the 800 kip machine at Watertown Arsenal that could handle specimens up to 30 ft long led to increased emphasis on testing. Column tests were mainly on wrought iron members with flat ends and the results affected details of lacing, width-thickness ratios of plate elements and rivet spacing. Tests were more exploratory in purpose and were not designed to check some existing theory. The flat-end testing failed to expose the shear issue with built-up members. Some railroad companies began to issue printed bridge specifications in order to force potential bidders to produce design based on similar criteria.

Structural developments during the last two decades of the 19th century had a significant impact on future column design. Bauschinger used knife-edge end fixtures that effectively provided a zero-moment end condition and proved experimentally that the Euler formula was correct. He also determined that cross sections bent into the inelastic range unload with the elastic Young's modulus. For straight members Engesser extended the Euler formula into the inelastic range by replacing E with E_T, the tangent to the slope of the stress-strain curve, for both loading and unloading elements of the cross section. He later produced a second inelastic theory, supposedly a more accurate formulation called the *double modulus theory*, that used E, not E_T for the unloading fibers during bending (buckling). However, limited tests supported the tangent modulus theory. It would take fifty years to resolve the controversy between these two theories. Jasinski in Russia introduced the concept of effective length by noting that column tables developed for pinned ends could be used if entered with the effective length. The truss era of column design began to wind down and use of metal columns in framed structures accelerated. Because of the 1871 fire, metal framed buildings began to appear in Chicago and that trend evolved into the skyscraper era that was unique to the US. The first textbook devoted to steel design, (Johnson, J. et al 1894), contained a new column curve based on test results called the Johnson Parabola plus the Euler formula that intersect with the same tangent at $P/P_y = 0.5$:

$$\frac{P}{P_y} = 1 - \frac{F_y}{4\pi^2 E}\left(\frac{KL}{r}\right)^2 \text{ for } \frac{P}{P_y} \geq 0.5; \text{ and } \frac{P}{P_y} = \frac{\pi^2 E}{F_y\left(\frac{KL}{r}\right)^2} \text{ for } \frac{P}{P_y} < 0.5 \tag{6}$$

For 36 ksi steel the *KL/r* for $P/P_y = 0.5$ is 126. The *L/r* of metal columns was generally less than 100 so only the parabolic term of Eq. (6) would be used. As shown later, Eq. (6) was adopted without change by AISC in 1960 and is still in use in Japan. Prior to the development of Eq. (6), a simple straight line formula for wrought iron columns was published by T. Johnson, based on test results that was used widely:

T. Johnson Straight Line

$$\frac{P}{A} = 42000 - 157\frac{L}{r} \tag{7}$$

where *P/A* is the ultimate column stress in psi. Eqs. (6), (7) were both curve-fit to test data but the straight line format gives results for short struts that exceed the yield strength of the material so the maximum was usually limited to the yield stress. Rankine, Parabolic and the Straight Line formulas are compared with test data in Fig. 6. Ostenfeld (see Salmon 1921, p. 231) evaluated the design formulas presented above with the best available test data using the least squares method to establish the mean error and found the Johnson Parabola-Euler formulation, Eq. (6), the most accurate.

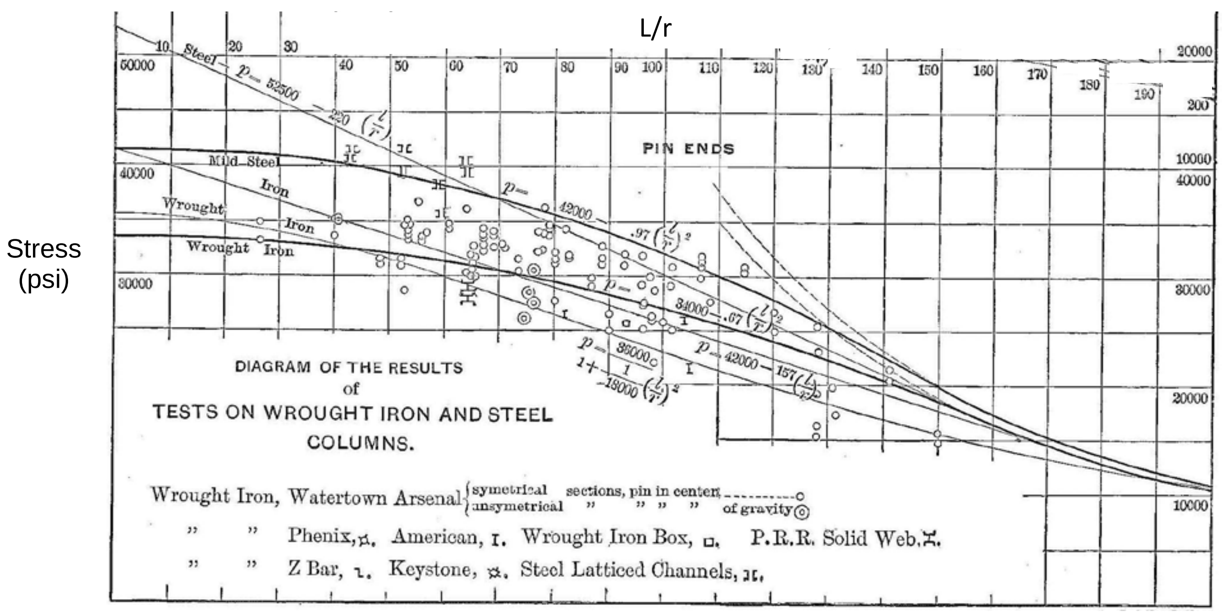


Figure 6: Test Results Compared with Rankine, Straight Line and Parabolic Column Curves (Johnson et al 1894)

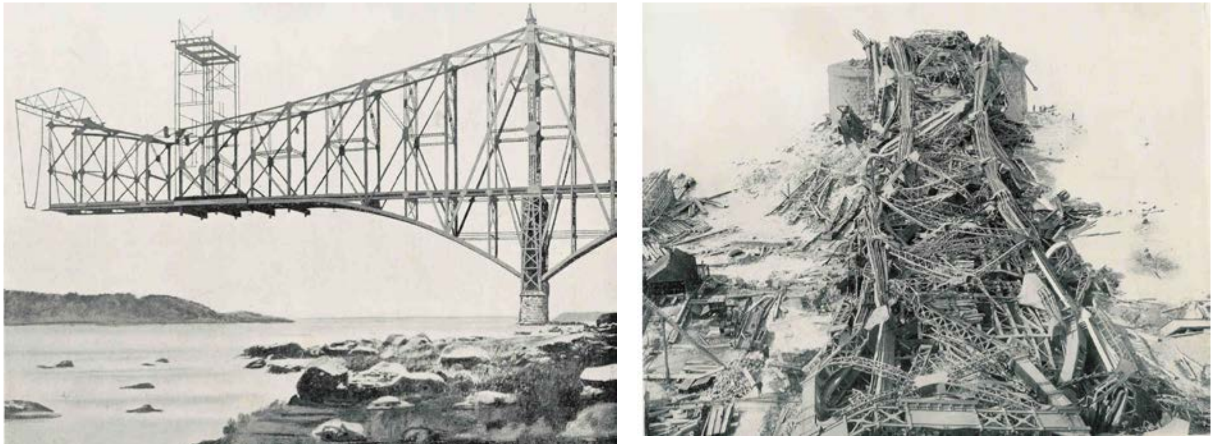
By the end of the 19th century all the fundamental mechanics principles related to design of an individual column, both elastic and inelastic, were established, but the concept of stability was not fully developed. Column design was considered a bending problem, not a buckling problem. The word *buckling* did not exist in English language technical publications. The iron column curve was viewed as three zones of behavior: crushing (yielding) for very short columns (usually referred to as crippling strength), elastic bending for very long slender columns and combined bending and crushing in the *L/r* range of 20 – 120 where most columns reside. The effective length was known for three end-support cases: pin-pin (1.0), fixed-fixed (0.5) and pin-fixed (0.7). The Euler flagpole with K = 2 was either forgotten or not applicable to members in trusses. The engineering education system transitioned from the British practical approach to the calculus based German approach of science tempered with experiment. Steel, with its more reliable properties compared to wrought iron, became generally available for structural members after 1880. Initially steel was rolled into rails. Engineers could calculate the forces in assumed pin-connected truss members and the stresses and defections in simple beams.

5. 1900-WWII

In 1900 the American Railway Engineering and Maintenance-of-Way Association was formed, now known as AREA, for "the advancement of knowledge pertaining to the scientific and economical location, construction, operation and maintenance of railroads". This was the first technical organization designed to benefit an industry rather than a particular entity and was a model for other industry-type groups established later such as ACI and AISC. Voting membership in AREA was interdisciplinary but limited to any civil, mechanical and electrical engineer with at least five years' railroad-related experience. Individuals exclusively associated with the sale or promotion of railroad patents and appliances were not eligible for membership. The 1900 Proceedings lists 114 different railroads and 359 members, including some from Canada and Mexico. A subcommittee on iron and steel bridges issued the first consensus draft AREA specification in 1905 and recommended a straight-line type formula, $F_a = 16000\text{-}70\ L/r$, that was more conservative than those used by individual railroads and limited to $L/r < 100$ for main members and < 120 for lateral members. Steel with a yield point of 30 ksi was required, not wrought iron. The specification was approved in 1906 after responding to criticism of the column formula by explaining that the test data available showed wide variations and compression members in trusses had greater eccentricities than columns compressed in a test machine. In the discussions associated with this first consensus steel bridge specification (AREA 1906),T. Cooper wrote:

> "Common Specifications: I do not believe it desirable or possible to get up one common specification. Different experiences and special studies of existing work frequently bring out new requirements that may appear useless to others. For unity of ideas and harmonious agreement, there must be sacrifices and compromises. I never could accept the idea, now so prevalent, that a majority vote can settle intelligently any scientific question of difference of view."

One year later the Quebec railroad bridge designed by Cooper across the St. Lawrence Seaway collapsed during erection as shown in Fig. 7. It was to be the longest cantilever bridge in the world. The bottom built-up compression chords near the pier were showing sign of distress by accelerating mid-length displacements that were being monitored over a three-week period as construction continued. The bridge was exhibiting behavior as predicted by Eq. (4), increased defections as the critical load of the columns is approached. The investigation faulted the failure on the effect of shear on the strength of built-up compression members and the high working stress specified by Cooper (20% higher than the more liberal individual specifications published) coupled with a 7-10% underestimate of the dead weight. The failure had a significant effect on column design. The demise of individual specifications started and the introduction of industry standards accelerated. It was fairly standard practice during that time to check the end details of columns by testing. A comment by Cooper that there was no test machine with sufficient compressive capacity to determine the strength of the large sections used in the Quebec Bridge prompted the Department of Commerce to order a 10 million pound test in 1910. Prior to that time the largest machine in the US was at Watertown Arsenal in Massachusetts with a capacity of 800 kips. The Bureau of Standards (NBS) had a 2.3 million pound machine installed in 1912.



(a) Just prior to collapse (b) Nineteen thousand tons of steel

Figure 7: Quebec Bridge Failure, Aug. 29, 1907 (Tarkov 1986)

5.1 ASCE Column Committees (1909-1933)

Two years after the collapse, ASCE organized a Committee on Steel Columns and Struts along with AREA and NBS that planned and executed a large test program with an objective to determine the strength of steel columns and their safe working values. A total of 213 tests were conducted over the next nine years and neither objective was fulfilled. According to the 1919 committee report:

> "numerous theoretical formulas have been promulgated from time to time, all of them based on entirely elastic material of uniform grade. At the present time it seems useless to the committee to attempt to write a rational formula, from a theoretical standpoint, with the possibility of variations of 28 per cent in the strength of the material."

Typical plots of column test results are represented by Fig, 6. In hindsight, the obvious solution for this situation is to nondimensionalize the results with the measured yield stress of the material assuming material tests were performed. Basquin (1924) at NBS evaluated all the tests in the ASCE program to determine if the results conformed to the tangent modulus theory. He states that material tests were not conducted on the individual component sections of the built-up shapes. He used the test data from the smallest $L/r \leq 50$ specimen (stub columns) of each group of identical cross sections to get a compressive stress-strain response that would provide tangent modulus data to predict the capacity of the other longer lengths of the group. The approach had merit but the fact that all the tests specimens had flat ends and unknown eccentricities required too many assumptions to develop a proper tangent modulus evaluation. The one positive result of this experimental effort was the observation that some of the columns failed with twist, not just bending deformation.

A second ASCE committee on Steel Column Research spent 1923-1933 trying to develop a rational solution to the column problem by testing an additional 107 different columns with pinned ends and deliberate eccentricities. They concluded that the secant formula was the best theoretical solution that was adaptable to various orientations of end eccentricities compared to the Ayrton-Perry initial out-of-straightness approach. Realizing that the secant formula was challenging for use in design as discussed above, a parabolic formula for bridges with a working tension stress of 18,000 psi was presented as a reasonable fit to the secant solution for $L/r < 140$: $P/A = 15{,}000 - 0.25\,(L/r)^2$.

5.2 Steel Specifications

Steel had replaced wrought iron in structures as more types of sections were rolled making fabrication easier. Bethlehem Steel held the patent on the H column shape and in 1908 began shipping what we now call W- shape members that weighted up to 290 lb/ft. With I beams and slender beams also available, the all steel frame became common place because of the reduced fabrication and erection costs. The truss was still the main structural system, just oriented vertically for buildings. The railroad boom was over and the automobile provided an alternative means of travel. Highway bridges were now needed but the reduced live loads compared to locomotives were generating more slender columns.

AREA and AASHTO

In the draft 1919 AREA Specification, the Johnson parabola, Eq. (6), was recommended with a with a safety factor equal to about 2.75. Turneaure (1920) found that the parabola provided a good fit to a screened test-data base limited to solid cross sections with $A \geq 4$ in^2, measured mill F_y and pinned bases. Small specimens and built-up sections were not considered. This was the first time a measured F_y was considered in an evaluation of an empirical column formula. The column subcommittee also noted that working stress formulas for higher strength steels could not just be increased in proportion to F_y because the Euler stress portion of the column curve is the same for all steels. The AREA Association rejected the draft proposal in 1919 as too complicated so the subcommittee responded by fitting two straight lines to approximate the parabola, one horizontal from 0-50 L/r and the other sloped to intersect the parabola at L/r = 150 that was approved. The AREA column subcommittee had also worked with a group from ASCE dealing with highway bridges that would morph into AASHTO. Both groups agreed on the linear approximations but for all practical purposes they had adopted Eq. (6) for column design.

After ASCE issued their report in 1933 recommending a parabolic formula, both AREA and AASHTO adopted it for $L/r \leq 140$. For larger L/r the secant formula was specified with two different K factors: 0.75 for riveted ends and 0.875 for pinned ends.

AISC

AISC was established in 1920 to further the interests of structural steel in its use, fabrication and erection thru a standard specification and code of practice. The first AISC Standard Specification for the Design, Fabrication and Erection of Structural Steel for Buildings (AISC Spec) was issued in 1923. The first Code of Practice was issued in 1924 that contained a provision that compression members could not deviate laterally more than L/1000 where L is the distance between lateral supports. The AISC Spec was developed by a committee composed of two academics, two building designers and one bridge engineer. (The current AISC Spec Committee of approximately forty individuals consists of equal numbers of fabricators, academics, structural engineers and steel industry representatives.) The first AISC Manual was issued in 1925 that contained among other design aids, the 11 page 1923 Specification and a 25 page Commentary. The explanation of general column behavior in the Commentary was 1880 vintage reflecting shear failure at the ends of compression members, not the eccentrically-loaded, first-yield bending approach advocated in the 1920's.

The column formula in the first AISC Spec was a Rankine-type formula and the L/r of main members was limited to 120 as shown in Fig. 8. The working stress formula has been converted

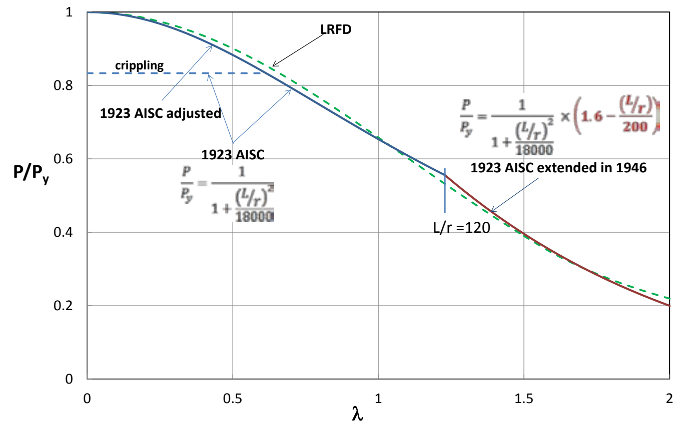


Figure 8: The 1st AISC Column Curve Compared to the 2018 Curve

to an ultimate load formula so comparison can be made with the current AISC-LRFD formula shown by the green-dash line. The Rankine formula was limited to the crippling level shown by the dashed horizontal line. If the crippling cutoff is ignored as shown by the "1923 AISC adjusted" legend and the 1946 formula extending $120 < L/r < 200$ for main members is added to the 1923 formula, then the complete Rankine formulation for $L/r < 200$ compares very favorably with the 2018 LRFD column formula.

After ASCE recommended a parabolic curve in 1933, the 1936 AISC Specification also switched to a parabolic curve for compression members as shown in Fig. 9 and parabolic-type column curves persisted in the AISC Specification for the next fifty years until 1985 when LRFD was introduced. The 1936 version used a factor of safety (FS) = 1.94 for the entire range of L/r. If the FS in the 1963 Specification that varies from 1.67 at $L/r = 0$ to 1.94 in the elastic range is applied to the 1936 formula, the results of the adjusted 1936 formulation shown dashed-red and the 1963 column curve in solid-blue are almost identical. The 1963 parabola is exactly the same as the 1894 Johnson parabola, Eq. (6). The comparisons in Figs. 8 and 9 show little change has occurred in the AISC column curves in over 95 years. Column design in general has remained unchanged from 1858 to the present, 160 years.

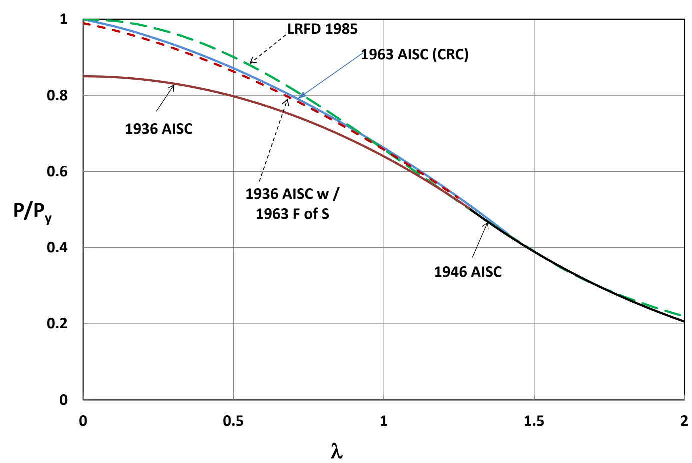


Figure 9: Parabolic-Type Column Formulas in AISC (1936-1985)

One factor did change in 1936 that has a major effect on column design, namely, how combined stresses are treated. Prior to 1936 axial and bending stresses were combined in the classic strength of materials fashion, $f_a + f_b$ = maximum stress. But what is the limit of the maximum stress? In all US steel specifications prior to 1936 the limit was set at the working stress permitted for the column since that allowable stress is less than the bending allowable. That limit would appear to be very conservative if axial stress was very low and bending stress dominates. The limit would be yielding if instability were not a consideration. The 1936 AISC Specification adopted a recommendation by (Jones 1938), a member of the specification committee and a founding member of The Column Research Council (SSRC), that combined stresses be handled by an interaction equation as follows:

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = 1 \tag{8}$$

where F_a and F_b are the allowable axial and bending stresses, respectively. While Eq. (8) checks at the two extremes, axial alone and bending alone, the equation itself is not derivable from mechanics principles because both terms can have potential unstable equilibrium issues (buckling).

5.3 Structural Mechanics Developments

At the beginning of the 20^{th} century, the elastic Euler theory had been confirmed experimentally and extended into the inelastic range. Tests on single members eccentrically loaded about the strong axis sometimes showed a twisting mode of failure also noted in tests on angle struts. The words *buckling, unstable equilibrium, bifurcation and critical load* now appeared in English technical literature. Mechanics and structural analysis methods improved greatly with slope deflection, moment distribution, and area-moment procedures. Timoshenko arrived in the US in 1922 and through his teaching and clear writing of classic textbooks on strength of materials, elasticity and elastic stability brought an immediate upgrade to the quality of US engineering schools. Stability functions extended elastic analysis methods to include the effects of axial load in computing moments and deflections. Laterally-loaded members with axial loads were solved to produce 2^{nd} order moment diagrams. Small frames could be analyzed but not very efficiently; matrix methods coupled with the mainframe computer would not be developed until 1955.

Tangent modulus buckling theory was gaining acceptance and was verified by Shanley after WWII. Bleich was a strong proponent of a buckling approach to design of compression members as illustrated by the following statement in Bleich (1924):

> "In this sense we must consider every buckling problem, both in the elastic range as well as in the inelastic range of buckling, as a problem of unstable equilibrium between internal and external forces. Naturally, this does not preclude calculation, when necessary, of material stresses due to deformation, in addition to investigations of buckling strength. In originally bent columns, or when the load is not applied axially such stresses may even precede buckling. That kind of calculation has nothing to do with the actual buckling problem. They are parallel problems and any attempt to include the buckling question and determination of stress in a single formula must be regarded from the start as a failure."

5.4 Experiments

With the addition of large test machines at government facilities and universities, a large number of column tests were performed during 1900 to 1945. Jakkula, Stephenson (1947) summarized all these studies that involved 30 different types of cross sections, concentric and eccentric loading, type of end fixture (flat, roller-pin-knife edge, spherical). A total of 567 steel column and 636 aluminum and stainless steel tests are categorized and discussed. The aluminum and stainless steel tests focused on aircraft applications and the non-linear stress-strain curves for these materials were prime candidates for the use of the tangent modulus buckling theory for compression members. The tests on mild steel columns, with a linear stress strain up to yield based on test coupons, did not support inelastic buckling theory for explaining the scatter shown in the test results. Obviously end eccentricities appeared to be a better explanation thus supporting the eccentric column bending approach. Tests on mild steel columns executed by the 2nd ASCE column committee all had rounded ends more closely approximating the perfect pinned end conditions assumed by Euler. The results of those tests supported the secant formula as the best fit to the data, not buckling theory. The Committee suggested a practical parabolic working stress formula, $F_a = 15000 - 0.25\ (L/r)^2$ for material with a tensile working stress of 18000 psi, that closely approximated the secant formula.

6. Buckling and Plastic Design: 1946 – 1975

Prior to 1946 steel column design was based on the yield limit of compression members with assumed imperfections. The destruction of structures in WWII enabled engineers to understand the behavior of structures, not just members, beyond the first yield limit and the concepts of plastic design intersected with inelastic stability. Shanley showed that there was a mistake in the derivation of the reduced or double modulus buckling load; P had been held constant during buckling in the derivation of the reduced modulus load, when in fact P varies for buckling in the inelastic range. With the tangent modulus theory confirmed as the proper inelastic buckling approach, there was renewed interest in the possibility that inelasticity could provide a simple practical basis for column design.

6.1 SSRC

SSRC (originally known as CRC, Column Research Council) was founded as an extension of the ASCE Committees on Columns mentioned in Section 5.1 but with a much broader scope related to compression elements. The goals were to financially support research and develop practical design procedures that were as simple as possible and consistent with accurate predictions of structural strength. SSRC financially supported the development of Bleich (1952) and requested that he develop a recommendation for the design of metal columns. The approach taken by Bleich is documented in Chapter 1 of his book and his recommendation was predictable based on his comment 28 years earlier cited in *Section 5.3*. He criticized the secant formula and its first-yield limit especially for bending in the weak direction. He concluded that column strength should be viewed as a stability problem, not as an eccentrically-loaded bending stress problem especially for columns as part of a frame. The uncertainties of accidental eccentricities or initial crookedness are best taken care of by the factor of safety. In Germany, mild steel was assumed to have a proportional limit of 0.8 Fy based on von Karman's 1910 column tests in which is used the compressive stress-strain curve from a stub column test of the small column cross section. For a column as part of a structure, Bleich recommended that the tangent modulus stress be replaced for practical use by a parabola in the inelastic range of behavior:

$$F_{cr} = F_y - \frac{F_p}{\pi^2 E}\left(F_y - F_p\right)\left(\frac{KL}{r}\right)^2 \tag{9}$$

where F_p is the proportional limit. SSRC adopted Bleich approach but replaced F_p with $(F_y - F_{rc})$

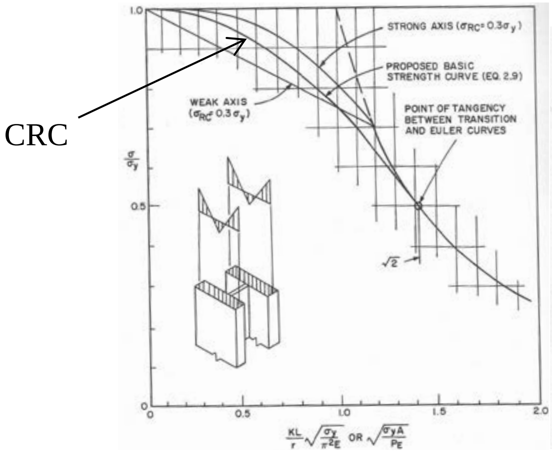


Figure 10: CRC Column Curve, Eq. (6)

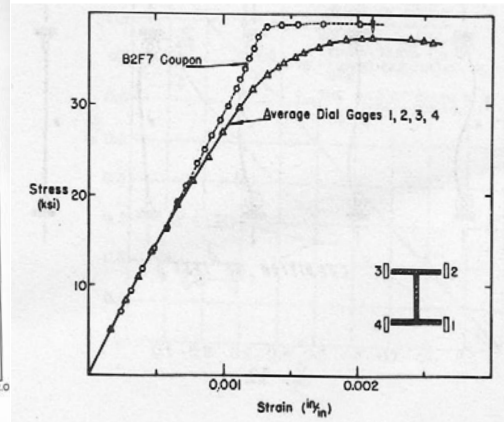


Figure 11: Stub Column and Coupon Tests (Johnson 1952)

where F_{rc} is taken as $0.5F_y$ that gives the Johnson parabola, Eq. (6), but with a KL factor. The Johnson parabola was adopted by AISC in 1961 but without the K factor. Two years later AISC did adopt the K factor concept. The CRC column curve appears to represent the average of the column buckling solutions for a section with residual stresses as described in the next section. This was the first time a buckling model was used for inelastic column design in place of the imperfection model used in the previous 100 years.

6.2 Residual Stresses

In 1951, Huber conducted the first stub column tests on full size W shapes to determine the compressive stress-strain curve shown in Fig. 11 along with the results of a tensile coupon test cut from the same specimen. The coupon test shows linear behavior up to F_y whereas the stub column has a significantly lower proportional limit. This observation shows the effect of cooling residual stresses formed during the steel section rolling process. Independent determination of residual stress by cutting the cross section into strips and measuring the change in length show a distribution of compressive stress at the flange tips that average about 0.3 F_y for steel with F_y = 36 ksi but cooling residual stresses do not actually vary with F_y. A residual stress distribution is shown in Fig. 12 that is typically used in theoretical analyses. The residual stress distribution indicates that its effect on column strength would be more detrimental for weak-axis buckling than for strong axis. Wang et al (1952) showed that the use of tangent modulus theory for the tension coupon data of an ideal elastic (E=29000 ksi) plastic (E=0 ksi) stress-strain curve, coupled with the initial residual stress distribution would cause first yielding at the tips and the remaining elastic core would provide the stiffness for stability. With a measured or assumed residual stress pattern, a column curve can be generated by the following procedure: apply a uniform strain to the column, determine how much of the cross section has locally yielded, calculate the I_x and I_y of the elastic core, determine P from a summation of cross-section stresses and determine L from the elastic Euler equation $L^2 = \pi^2 EI/P$. Figure 10 shows the x-x and y-y column curves determined by buckling theory. The theory using a perfectly straight member with residual stresses provides an additional explanation for the test scatter in the inelastic region.

Measured residual stress patterns and magnitudes vary widely. Allen, (1980) tabulates eleven patterns for the W8x31 that range between 11-18 ksi for the compressive residual stress at the flange tips. A single specimen that was rotary straightened had a 1.5 ksi value. They also show (Fig. 15) the significant impact of the straight line or parabolic shape with the same residual stress value at the flange tips. Figure 13 shows the pattern of a rotary straightened member. In the 1950-1970 era when most residual stress data was obtained, few sections were straightened continuously. Present practice rotarizes most rolled sections up to 420 lb/ft. It is expected that strength reductions of hot-rolled columns caused by the presence of residual stresses would be significantly reduced by present practice.

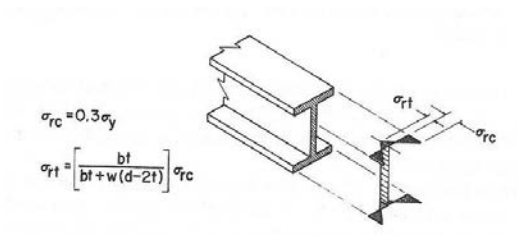


Figure 12: Ketter Residual Stress Pattern

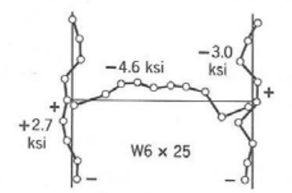


Figure 13: Residual Stress –Rotary Straightened

6.3 Effective Length Factors

When effective length factors were required by AISC, it was quite a shock for engineers. No computer programs were available to conduct a buckling analysis. The alignment chart shown in Fig. 14 was a design aid provided in SSRC (1960) and is based on an elastic stability analysis of an assemblage of members rigidly framed at both ends of the column. The sidesway model assumed there could be relative lateral displacement between the ends of the column. The AISC Specification Commentary lists nine assumptions associated with the use of the chart and over time techniques were developed that address these assumptions to achieve a more accurate evaluation of K. Recent AISC Commentaries have included some of these techniques. One important one is related to the notion that a single column in an unbraced frame cannot sidesway. All the columns in a story contribute to the stiffness, which is known as the story stiffness technique or **Σ**P concept. With the availability of computer programs that can perform 2nd order inelastic analyses, the use of K factors in the design office will be diminishing.

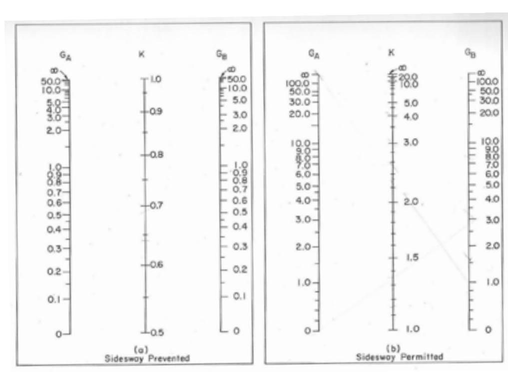


Figure 14: Alignment Chart (SSRC 1960)

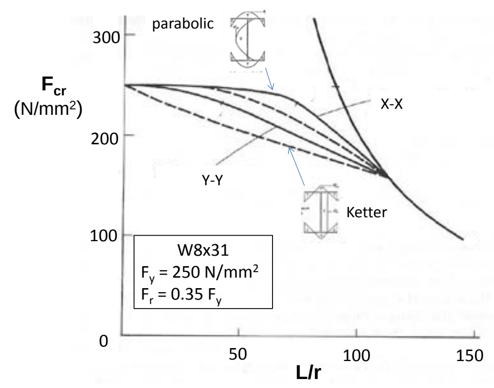


Figure 15: Effect of Residual Pattern (Allen 1980)

6.4 Plastic Design

Research on plastic design that started in Great Britain initially focused on continuous beams and introduced M_p = the plastic moment Z = plastic section modulus, , f = shape factor =M_p/M_y and the concept of moment redistribution as cross sections attain Mp in indeterminate structures. After WWII plastic analysis and design was extended to small frames and later to braced frames. Plastic analysis methods were simple to apply since the structure becomes statically determinate at the collapse load. Plastic design was oversold as a replacement for elastic design methods and its demise by 1970 was the result of two major factors. First, elastic structural analysis became easy because of the computer where thousands of simultaneous equations could be solved in seconds for unbraced frames. Second, axial load stability effects that were initially ignored proved troublesome and made the plastic design method complicated. The plastic moment capacity, including reduction for the presence of axial stress has endured, and the design principal of ductility to permit the structure to adjust for changing conditions remain important concepts especially in forensic investigations and seismic design. In the US the plastic design research focused on stability issues associated with beam-columns. The 15 year effort is presented in Chapter 6 of the SSRC (1966) and ASCE (1971). The lasting effect of that research is the analytical technique developed for the 2^{nd} order inelastic analysis of unbraced frames.

7. Multiple Column Curves

Figures 4 and 10 show that both eccentrically-loaded columns and straight columns with residual stresses produce different column curves for the strong and weak axes. The availability of computer programs for analysis of columns with residual stresses and initial out-of- straightness became a reality in 1960. In Europe ECCS initiated an experimental and analytical approach to develop multiple column curves that were statically reliable for predicting the maximum strength of an imperfect column. Over 1000 column tests on three different sections at three different L/r at five different laboratories provided the lower bound statistics (mean–2 std dev) to fit Ayrton-Perry design curves. The tests were on unstraightened members to retain the cooling residual stresses and the out-of-straightness. A summary of the effort is given in the ECCS Introductory Report of the Second International Colloquium on Stability in 1976. The original work developed three design curves; two additional curves were added later.

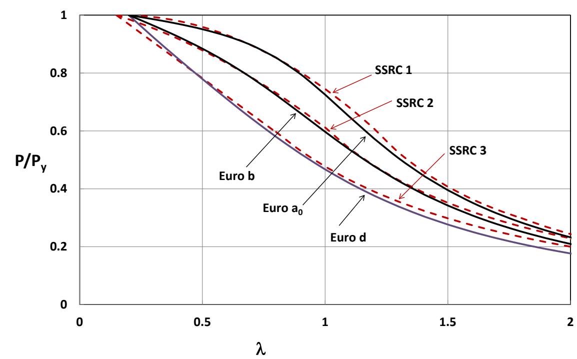


Figure 16: ECCS and SSRC Multiple Column Curves

In the US a purely analytical approach started in 1968 by Bjorhovde produced three column curves from a base of 110 column curves. The study included more welded sections than rolled sections. SSRC (1976) published slightly adjusted Bjorhovde curves shown as SSRC 1, 2, 3 in Fig.16 along with three of the five ECCS curves. The ECCS and SSRC curves are almost identical. In 1984 SSRC withdrew support for the three curves for LRFD because the out-of-straightness of 1/1000 used in their development was an extreme value, Mean values are used in LRFD developments.

8. Maximum Strength and LRFD

Discussions for a column curve in the first LRFD Specification spanned a period of five years, 1976-1984. The multiple column curve approach was not considered because all the research work for the new interaction equation based on frame behavior was completed and it involved a single analytical column curve. The buckling approach in ASD was eliminated and reliability analyses on the various proposals was conducted by Galambos using a maximum strength column curve he produced based on a W8x31 with Fy = 36 ksi, out-of-straightness = L/1000, Ketter residual stress pattern and end restraint at both ends corresponding to G =10. The SSRC 2 curve was considered but eventually rejected because Tide among others noted that designs in the higher slenderness range would be less competitive. In 1982 Fleischer fit a curve to the Galambos data but it lay dormant for two years until Tide tweaked Fleischer's formula, which was approved in 1984. Figure 17 shows the Fleischer-Tide fit to the Galambos data shown by red markers. The secant formula with e = L/1000 and its first-yield limit compares very closely with the LRFD formula.

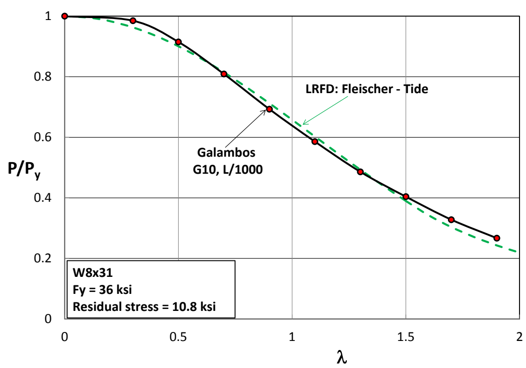


Figure 17: LRFD Column Data and Curve Fit

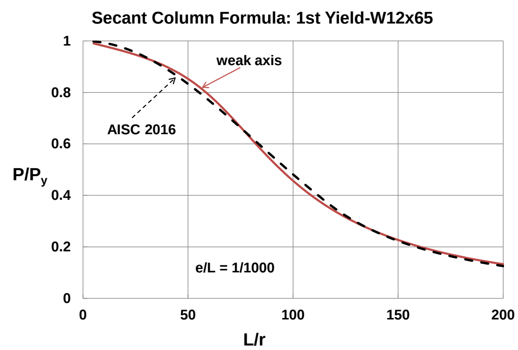


Figure 18: LRFD Compared to Secant Formula

Summary

A lot work went into proving that an 1858 formula was very good.

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